JEE ADVANCED 2019

Comprehensive Physics

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Mc Graw Hill Education

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A Word to the Reader

Comprehensive Physics—JEE Advanced is highly useful for aspirants appearing for JEE Advanced. It is as per the syllabus of JEE Advanced.

Divided into 29 core chapters, it covers the entire gamut of the subject through a combination of conceptual theory supported by solved problems and practice exercises. Each chapter opens with a review of the basic concepts, formulae, laws and definition that is linked to the concepts and problems of that chapter. There are plenty of fully solved questions of all types as per the latest pattern and syllabus.

Questions in each chapter have been classified as follows:

Section I : Multiple Choice Questions with **Only One** correct choice.
Section II : Multiple Choice Questions with **One or More** correct choices.
Section III : Multiple Choice Questions based on passage.
Section IV : Matching of Column type questions.
Section V : Assertion-Reason type questions.

Key Features

- Review of Basic Concepts
- Solved problems in increasing level of difficulty
- Tips for quick solution of MCQs given in each chapter
- 'Integer Answer Type Questions' at the end of each chapter
- Two fully solved model test papers based on the latest pattern of the JEE Advanced examination
- Solutions to JEE Advanced Physics papers of 2012, 2013, 2014 and 2015.

New Feature

An interactive CD of 10 full length Mock Papers with Answer Key and Complete Solutions.

THE PUBLISHERS

Syllabus

General: Units and dimensions, dimensional analysis; least count, significant figures; Methods of measurement and error analysis for physical quantities pertaining to the following experiments: Experiments based on using Vernier calipers and screw gauge (micrometer), Determination of g using simple pendulum, Young's modulus by Searle's method, Specific heat of a liquid using calorimeter, focal length of a concave mirror and a convex lens using u-v method, Speed of sound using resonance column, Verification of Ohm's law using voltmeter and ammeter, and specific resistance of the material of a wire using meter bridge and post office box.

Mechanics: Kinematics in one and two dimensions (Cartesian coordinates only), projectiles; Uniform Circular motion; Relative velocity.

Newton's laws of motion; Inertial and uniformly accelerated frames of reference; Static and dynamic friction; Kinetic and potential energy; Work and power; Conservation of linear momentum and mechanical energy.

Systems of particles; Centre of mass and its motion; Impulse; Elastic and inelastic collisions.

Law of gravitation; Gravitational potential and field; Acceleration due to gravity; Motion of planets and satellites in circular orbits; Escape velocity.

Rigid body, moment of inertia, parallel and perpendicular axes theorems, moment of inertia of uniform bodies with simple geometrical shapes; Angular momentum; Torque; Conservation of angular momentum; Dynamics of rigid bodies with fixed axis of rotation; Rolling without slipping of rings, cylinders and spheres; Equilibrium of rigid bodies; Collision of point masses with rigid bodies.

Linear and angular simple harmonic motions.

Hooke's law, Young's modulus.

Pressure in a fluid; Pascal's law; Buoyancy; Surface energy and surface tension, capillary rise; Viscosity (Poiseuille's equation excluded), Stoke's law; Terminal velocity, Streamline flow, equation of continuity, Bernoulli's theorem and its applications.

Wave motion (plane waves only), longitudinal and transverse waves, superposition of waves; Progressive and stationary waves; Vibration of strings and air columns; Resonance; Beats; Speed of sound in gases; Doppler effect (in sound).

Thermal physics: Thermal expansion of solids, liquids and gases; Calorimetry, latent heat; Heat conduction in one dimension; Elementary concepts of convection and radiation; Newton's law of cooling; Ideal gas laws; Specific heats (C_v and C_p for monoatomic and diatomic gases); Isothermal and adiabatic processes, bulk modulus of gases; Equivalence of heat and work; First law of thermodynamics and its applications (only for ideal gases); Blackbody radiation: absorptive and emissive powers; Kirchhoff's law; Wien's displacement law, Stefan's law.

Electricity and magnetism: Coulomb's law; Electric field and potential; Electrical potential energy of a system of point charges and of electrical dipoles in a uniform electrostatic field; Electric field lines; Flux of electric field; Gauss's law and its application in simple cases, such as, to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Capacitance; Parallel plate capacitor with and without dielectrics; Capacitors in series and parallel; Energy stored in a capacitor.

Electric current; Ohm's law; Series and parallel arrangements of resistances and cells; Kirchhoff's laws and simple applications; Heating effect of current.

Biot–Savart's law and Ampere's law; Magnetic field near a current-carrying straight wire along the axis of a circular coil and inside a long straight solenoid; Force on a moving charge and on a current-carrying wire in a uniform magnetic field.

Magnetic moment of a current loop; Effect of a uniform magnetic field on a current loop; Moving coil galvanometer, voltmeter, ammeter and their conversions.

Electromagnetic induction: Faraday's law, Lenz's law; Self and mutual inductance; RC, LR and LC circuits with d.c. and a.c. sources.

Optics: Rectilinear propagation of light; Reflection and refraction at plane and spherical surfaces; Total internal reflection; Deviation and dispersion of light by a prism; Thin lenses; Combinations of mirrors and thin lenses; Magnification.

Wave nature of light: Huygen's principle, interference limited to Young's double-slit experiment.

Modern physics: Atomic nucleus; Alpha, beta and gamma radiations; Law of radioactive decay; Decay constant; Half-life and mean life; Binding energy and its calculation; Fission and fusion processes; Energy calculation in these processes.

Photoelectric effect; Bohr's theory of hydrogen-like atoms; Characteristic and continuous X-rays, Moseley's law; de Broglie wavelength of matter waves.

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1 Chapter Units and Dimensions

REVIEW OF BASIC CONCEPTS

1.1 THE SI SYSTEM OF UNITS

The internationally accepted standard units of the fundamental physical quantities are given in Table 1.1.

Table 1.1 Fundamental SI Units

Physical Quantity	*Name of the Unit*	*Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol
Angle in a plane	radian	rad
Solid angle	steradian	sr

Table 1.2 Dimensional Formulae of some Physical Quantities

Physical Quantity	*Dimensional Formula*	*Physical Quantity*	*Dimensional Formula*
Area	$M^0L^2T^0$	Heat energy	ML^2T^{-2}
Volume	$M^0L^3T^0$	Entropy	$ML^2T^{-2}K^{-1}$
Density	$ML^{-3}T^0$	Specific heat	$M^0L^2T^{-2}K^{-1}$
Velocity	M^0LT^{-1}	Latent heat	$M^0L^2T^{-2}$
Acceleration	M^0LT^{-2}	Molar specific heat	$ML^2T^{-2}K^{-1}$ mol^{-1}
Momentum	MLT^{-1}	Thermal conductivity	$MLT^{-3}K^{-1}$
Angular momentum	ML^2T^{-1}	Wien's constant	M^0LT^0K
Force	MLT^{-2}	Stefan's constant	$ML^0T^{-3}K^{-4}$
Energy, work	ML^2T^{-2}	Boltzmann's constant	$ML^2T^{-2}K^{-1}$
Power	ML^2T^{-3}	Molar gas constant	$ML^2T^{-2}K^{-1}$ mol^{-1}
Torque, couple	ML^2T^{-2}	Electric charge	TA
Impulse	MLT^{-1}	Electric current	A
Frequency	$M^0L^0T^{-1}$	Electric potential	ML^2T^{-3} A^{-1}
Angular frequency	$M^0L^0T^{-1}$	Electric field	MLT^{-3} A^{-1}
Angular acceleration	$M^0L^0T^{-2}$	Capacitance	$M^{-1}L^{-2}T^4A^2$
Pressure	$ML^{-1}T^{-2}$	Inductance	$ML^2T^{-2}A^{-2}$
Elastic modulii	$ML^{-1}T^{-2}$	Resistance	$ML^2T^{-3}A^{-2}$
Stress	$ML^{-1}T^{-2}$	Magnetic flux	$ML^2T^{-2}A^{-1}$

(*Contd.*)

Physical Quantity	*Dimensional Formula*	*Physical Quantity*	*Dimensional Formula*
Moment of inertia	ML^2T^0	Magnetic flux density or Magnetic induction field	$ML^0T^{-2}A^{-1}$
Surface tension	ML^0T^{-2}	Permeability	$MLT^{-2}A^{-2}$
Viscosity	$ML^{-1}T^{-1}$	Permittivity	$M^{-1}L^{-3}T^4A^2$
Gravitational constant	$M^{-1}L^2T^{-2}$	Planck's constant	ML^2T^{-1}

1.2 PRINCIPLE OF HOMOGENEITY OF DIMENSIONS

Consider a simple equation,

$$A + B = C.$$

If this is an equation of physics, i.e. if A, B and C are physical quantities, then this equation says that one physical quantity A, when added to another physical quantity B, gives a third physical quantity C. This equation will have no meaning in physics if the nature (i.e. the dimensions) of the quantities on the left-hand side of the equation is not the same as the nature of the quantity on the right-hand side. For example, if A is a length, B must also be a length and the result of addition of A and B must express a length. In other words, the dimensions of both sides of a physical equation must be identical. This is called the *principle of homogeneity of dimensions*.

1.3 USES OF DIMENSIONAL ANALYSIS

Dimensional equations provide a very simple method of deriving relations between physical quantities involved in any physical phenomenon. The analysis of any phenomenon carried out by using the method of dimensions is called *dimensional analysis*. This analysis is based on the principle of homogeneity of dimensions explained above.

There are four important uses of dimensional equations:

1. Checking the correctness of an equation.
2. Derivation of the relationship between the physical quantities involved in any phenomenon.
3. Finding the dimensions of constants or variables in an equation.
4. Conversion of units from one system to another.

EXAMPLE 1.1

The distance x travelled by a body varies with time t as

$$x = at + bt^2, \quad \text{where } a \text{ and } b \text{ are constants.}$$

Find the dimensions of a and b.

SOLUTION

The dimensions of each term on the right hand of the given equation must be the same as those of the left hand side. Hence

Dimensions of at = dimensions of x

or $$[a] = \frac{[x]}{[t]} = \frac{[L]}{[T]} = [LT^{-1}] = [M^0LT^{-1}]$$

Dimensions of bt^2 = dimensions of x

or $$b = \frac{x}{t^2} = \frac{[L]}{[T^2]} = [LT^{-2}] = [M^0\,LT^{-2}]$$

EXAMPLE 1.2

The pressure P, volume V and temperature T of a gas are related as

$$\left(P + \frac{a}{V^2}\right)(V - b) = cT$$

where a, b, and c are constants. Find the dimensions of $\frac{a}{b}$.

SOLUTION

Dimensions of $\frac{a}{V^2}$ = dimensions of P.

$\therefore$ Dimensions of a = dimensions of PV^2

Also dimensions of b = dimensions of V

$\therefore$ Dimensions of $\frac{a}{b} = \frac{[PV^2]}{[V]}$

$= [PV] = [ML^{-1}\,T^{-2}] \times [L^3]$

$= [ML^2\,T^{-2}]$

NOTE

1. Trigonometric function (sin, cos, tan, cot etc) are dimensionless. The arguments of these functions are also dimensionless
2. Exponential functions are dimensionless. Their exponents are also dimensionless

EXAMPLE 1.3

When a plane wave travels in a medium, the displacement y of a particle located at x at time t is given by

$$y = a\,\sin(bt + cx)$$

where a, b and c are constants. Find the dimensions of $\frac{b}{c}$.

SOLUTION

Terms bt and cx must be dimensionless. Hence

$$[b] = \frac{1}{[t]} = [\mathrm{T}^{-1}]$$

and $$[c] = \frac{1}{[x]} = \frac{1}{[\mathrm{L}]} = [\mathrm{L}^{-1}]$$

$$\therefore \quad \left[\frac{b}{c}\right] = [\mathrm{LT}^{-1}] = \mathrm{M}^0\ \mathrm{LT}^{-1}]$$

Note that the dimensions of a are the same as those of y.

EXAMPLE 1.4

In the expression

$$P = \frac{a^2}{b} e^{-ax}$$

P is pressure, x is a distance and a and b are constants. Find the dimensional formula for b.

SOLUTION

$$\left[\frac{a^2}{b}\right] = [P]$$

Also ax is dimensionless. Hence $[a] = [\mathrm{L}^{-1}]$.

$$\therefore \quad [b] = \frac{[a^2]}{[P]} = \frac{[\mathrm{L}^{-2}]}{[\mathrm{ML}^{-1}\ \mathrm{T}^{-2}]}$$

$$= [\mathrm{M}^{-1}\ \mathrm{L}^{-1}\ \mathrm{T}^2]$$

The principle of homogeneity of dimensions can also be used to find the dependence of a physical quantity on other physical quantities.

EXAMPLE 1.5

The time period (t) of a simple pendulum may depend upon m the mass of the bob, l the length of the string and g the acceleration due to gravity. Find the dependence of t on m, l and g.

SOLUTION

Let $$t \propto m^a l^b g^c$$

or $$t = k\ m^a l^b g^c,$$

where k is a dimensionless constant.

Writing the dimensions of each quantity, we have

$$[\mathrm{T}] = [\mathrm{M}]^a\ [\mathrm{L}]^b\ [\mathrm{LT}^{-2}]^c$$

or $$[\mathrm{M}^0\ \mathrm{L}^0\ \mathrm{T}] = [\mathrm{M}^a \mathrm{L}^{b+c} \mathrm{T}^{-2c}]$$

According to the principle of homogeneity of dimensions, the dimensions of all the terms on either side of this equation must be the same. Equating the powers of M, L and T, we have

$a = 0$, $b + c = 0$ and $-2c = 1$, which give $b = \frac{1}{2}$ and $c = -\frac{1}{2}$.

Hence $$t = k\ m^0\ l^{1/2}\ g^{(-1/2)}$$

$$\Rightarrow \quad t = k\sqrt{\frac{l}{g}}$$

Thus t is independent of the mass of the bob and is directly proportional to $\sqrt{l}$ and inversely proportional to $\sqrt{g}$.

The dimensional method can also be used to convert a physical quantity from one system to another. The method is based on the fact that the magnitude of a physical quantity X remains the same in every system of its measurement, i.e.

$$X = n_1\ u_1 = n_2\ u_2 \qquad (1)$$

where u_1 and u_2 are the two units of measurement of quantity X and n_1 and n_2 are their respective numerical values. Suppose M_1, L_1 and T_1 are the fundamental units of mass, length and time in one system of measurement and M_2, L_2 and T_2 in the second system of measurement. Let a, b and c be the dimensions of mass, length and time of quantity X, the units of measurement u_1 and u_2 will be

$$u_1 = \left[\mathrm{M}_1^a\ \mathrm{L}_1^b\ \mathrm{T}_1^c\right]$$

and $$u_2 = \left[\mathrm{M}_2^a\ \mathrm{L}_2^b\ \mathrm{T}_2^c\right]$$

Using these in Eq. (1), we have

$$n_1\left[\mathrm{M}_1^a\ \mathrm{L}_1^b\ \mathrm{T}_1^c\right] = n_2\left[\mathrm{M}_2^a\ \mathrm{L}_2^b\ \mathrm{T}_2^c\right]$$

$$\Rightarrow \quad n_2 = n_1\left[\left(\frac{\mathrm{M}_1}{\mathrm{M}_2}\right)^a \left(\frac{\mathrm{L}_1}{\mathrm{L}_2}\right)^b \left(\frac{\mathrm{T}_1}{\mathrm{T}_2}\right)^c\right] \qquad (2)$$

Knowing (M_1, L_1, T_1), (M_2, L_2, T_2), (a, b and c) and the value of n_1 in system 1, we can calculate the value of n_2 in system 2 from Eq. (2).

EXAMPLE 1.6

Dyne is the unit of force in the c.g.s. system and newton is the unit of force in the SI system. Convert 1 dyne into newton.

SOLUTION

The dimensional formula of force is

$$[F] = [\mathrm{M}^1\ \mathrm{L}^1\ \mathrm{T}^{-2}]$$

Therefore, $a = 1$, $b = 1$ and $c = -2$

System 1 (Given System)	*System 2 (Required System)*
$n_1 = 1$ dyne	$n_2 = ?$ (number of newtons)
$M_1 = 1g$	$M_2 = 1$ kg
$L_1 = 1$ cm	$L_2 = 1$ m
$T_1 = 1$ s	$T_2 = 1$ s

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

$$= 1\left(\frac{1\,g}{1\,kg}\right)^1\left(\frac{1\,cm}{1\,m}\right)^1\left(\frac{1\,s}{1\,s}\right)^{-2}$$

$$= 1\left(\frac{1\,g}{10^3\,g}\right)^1 \times \left(\frac{1\,cm}{10^2\,cm}\right)^1 \times 1$$

$$= 1 \times 10^{-5}$$

Hence there are 1×10^{-5} newtons in 1 dyne, i.e. 1 dyne = 10^{-5} N.

EXAMPLE 1.7

Convert 72 kmh^{-1} into ms^{-1} by using the method of dimensions.

SOLUTION

Given System	*Required System*
$n_1 = 72$ units	$n_2 = ?$
$L_1 = 1$ km	$L_2 = 1$ m
$T_1 = 1$ h	$T_2 = 1$ s

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^1\left(\frac{L_1}{L_2}\right)^{-1}$$

$$= 72 \times \left(\frac{1000\,m}{1\,m}\right) \times \left(\frac{60 \times 60\,s}{1\,s}\right)^{-1}$$

$$= \frac{72 \times 1000}{60 \times 60} = 20$$

Hence 72 km h^{-1} = 20 ms^{-1}

NOTE

Sometimes it is more convenient to use units rather than dimension. For example

$$72\ km\ h^{-1} = \frac{72\ km}{1\ h} = \frac{72 \times 1000\ m}{60 \times 60\ s} = 20\ ms^{-1}$$

EXAMPLE 1.8

If the units of force, energy and velocity are 10 N, 100 J and 5 ms^{-1}, find the units of length, mass and time.

SOLUTION

$$[MLT^{-2}] = 10\ N \quad (1)$$

$$[ML^2\ T^{-2}] = 100\ J \quad (2)$$

$$[LT^{-1}] = 5\ ms^{-1} \quad (3)$$

Dividing Eq. (2) by Eq. (1), we get

$$L = \frac{100\ J}{10\ N} = \frac{100\ Nm}{10\ N} = 10\ m$$

Using L = 10 m in Eq. (3), we get T = 2 s. Using T = 2 s and L = 10 m in either Eq. (1) or Eq. (2), we get M = 4 kg.

1.4 LIMITATIONS OF DIMENSIONAL ANALYSIS

Though the dimensional method is a simple and a very convenient way of finding the dependence of a physical quantity on other quantities of a given system, it has its own limitations, some of which are listed as follows:

1. In more complicated situations, it is often not easy to find out the factors on which a physical quantity will depend. In such cases, one has to make a guess which may or may not work.
2. This method gives no information about the dimensionless constant which has to be determined either by experiment or by a complete mathematical derivation.
3. This method is used only if a physical quantity varies as the product of other physical quantities. It fails if a physical quantity depends on the sum or difference of two quantities. Try, for instance, to obtain the relation $S = ut + \frac{1}{2}at^2$ using the method of dimensions.
4. This method will not work if a quantity depends on another quantity as sin or cos of an angle, i.e. if the dependence is by a trigonometric function. The method works only if the dependence is by power functions only.
5. This method does not give a complete information in cases where a physical quantity depends on more than three quantities, because by equating the powers of M, L and T, we can obtain only three equations for the exponents.

1.5 SIGNIFICANT FIGURES

The number significant figure in any measurement indicates the degree of precision of that measurement. The degree of precision is determined by the least count of the measuring instrument. Suppose a length measured by a metre scale (of least count = 0.1 cm) is 1.5 cm, then it has two significant figures, namely 1 and 5. Measured

with a vernier callipers (of least count = 0.01 cm) the same length is 1.53 cm and it then has three significant figures. Measured with a screw gauge (of least count = 0.001 cm) the same length may be 1.536 cm which has four significant figures.

It must be clearly understood that we cannot increase the accuracy of a measurement of changing the unit. For example, suppose a measurement of mass yields a value 39.4 kg. It is understood that the measuring instrument has a least count of 0.1 kg. In this measurement, three figures 3, 9 and 4 are significant. If we change 39.4 kg to 39400 g or 39400000 mg, we cannot change the accuracy of measurement. Hence 39400 g or 39400000 mg still have three significant figures; the zeros only serve to indicate only the magnitude of measurement.

Estimation of Appropriate Significant Figures in Calculations

The importance of significant figures lies in calculation to find the result of addition or multiplication of measured quantities having a different number of significant figures. The least accurate quantity determines the accuracy of the sum or product. The result must be rounded off to the appropriate digit.

Rules for Rounding off

The following rules are used for dropping figures that are not significants

1. If the digit to be dropped is less than 5, the next (preceding) digit to be retained is left unchanged. For example, if a number 5.34 is to be rounded off to two significant figures, the digit to be dropped is 4 which is less than 5.
2. If the digit to be dropped is more than 5, the preceding digit to be retained is increased by 1. For examples 7.536 is rounded off as 7.54 to three significant figures.
3. If the digit to be dropped happens to be 5, then
   (a) the preceding digit to be retained is increased by 1 if it odd, or
   (b) the preceding digit is retained unchanged if it is even.
4. Hence the next digit, namely 3, is not changed. The result of the indicated rounding-off is therefore, 5.3.

For example, 6.75 is rounded off to 6.8 to two significant figures and 4.95 is rounded off to 5.0 but 3.45 is rounded off to 3.4.

Rule for Finding Significant Figures

1. For addition and subtraction, we use the following rule.
   Find the sum or difference of the given measured quantities and then round off the final result such that it has the same number of digits after the decimal place as in the least accurate quantity (i.e., the quantity which has the least number of significant figures)

EXAMPLE 1.9

Four objects have masses 2.5 kg, 1.54 kg, 3.668 kg and 5.1278 kg. Find the total mass up to appropriate significant figures.

SOLUTION

$M = 2.5 + 1.54 + 3.668 + 5.1278 = 12.8358$ kg

In this example, the least accurate quantity is 2.5 kg. This mass is accurate only up to the first decimal place in kg. Hence the final result much be rounded off to the first decimal place in kg. The correct result up to appropriate significant figures is $M = 12.8$ kg.

2. We use the following rule to determine the number of significant figures in the result of multiplication and division of various physical quantities.
   Do not worry about the number of digits after the decimal place. Round off the result so that it has the same number of significant figures as in the least accurate quantity.

EXAMPLE 1.10

A man runs 100.2 m in 10.3 s. Find his average speed up to appropriate significant figure.

SOLUTION

$$\text{Average speed } (v) = \frac{100.2\,\text{m}}{10.3\text{s}} = 9.728155\ \text{ms}^{-1}$$

The distance 100.2 m has four significant figures but the time 10.3 s has only three. Hence the value of the result must be round off to three significant figures. The correct result is $v = 9.73\ \text{ms}^{-1}$

1.6 LEAST COUNTS OF SOME MEASURING INSTRUMENTS

1. Least count of metre scale = 1 mm = 0.1 cm
2. Vernier constant (or least count) of vernier callipers = value of 1 main scale division – value of 1 vernier scale division = 1 M.S.D. – 1 V.S.D

   Let the value of 1 M.S.D = a unit

   If n vernier scale divisions coincide with m main scale divisions, then value of

$$1.\text{V.S.D} = \frac{m}{n} \text{ of 1 M.S.D}$$

$$= \frac{ma}{n} \text{ unit}$$

$$\therefore \quad \text{Least count} = a - \frac{ma}{n} = \left(1 - \frac{m}{n}\right) a \text{ unit}$$

3. Least count of a micrometer screw is found by the formula

   Least count =

$$\frac{\text{Pitch of screw}}{\text{Total number of divisions on circular scale}}$$

   where pitch = lateral distance moved in one complete rotation of the screw.

1.7 ORDER OF ACCURACY: PROPORTIONATE ERROR

The order of accuracy of the result of measurements is determined by the least counts of the measuring instruments used to make those measurements. Suppose a length x is measured with a metre scale, then the error in x is $\pm \Delta x$, where Δx = least count of metre scale = 0.1 cm. If the same length is measured with vernier callipers of least count 0.01 cm, then $\Delta x = 0.01$ cm.

Fractional or proportionate error is defined as $\frac{\Delta x}{x}$.

Maximum percentage error = $\frac{\Delta x}{x} \times 100$.

1. **Error in sum:** Suppose a quantity is given by

$$a = x + y$$

   Then $\Delta a = \Delta x + \Delta y$ is the maximum error and

$$\frac{\Delta a}{a} = \frac{\Delta x + \Delta y}{(x + y)}$$

2. **Error in Difference:** If $a = x - y$, then the maximum error is

$$\Delta a = \Delta x + \Delta y$$

   We take the worst case in which errors add up.

$$\frac{\Delta a}{a} = \frac{\Delta x + \Delta y}{(x - y)}$$

3. **Error in product and division:** Suppose we determine the value of a physical quantity u by measuring three quantities x, y and z whose true values are related to u by the equation

$$u = x^{\alpha} y^{\beta} z^{-\gamma}$$

   Let the expected small errors in the measurement of quantities x, y and z be respectively $\pm \Delta x$, $\pm \Delta y$ and $\pm \Delta z$ so that the error in u by using these observed quantities is $\pm \Delta u$. The numerical values of Δx, Δy and Δz are given by the least count of the instruments used to measure them.

   Taking logarithm of both sides we have

$$\log u = \alpha \log x + \beta \log y - \gamma \log z$$

   Partial differentiation of the above equation gives

$$\frac{\Delta u}{u} = \alpha \frac{\Delta x}{x} + \beta \frac{\Delta y}{y} - \gamma \frac{\Delta z}{z}$$

   The proportional or relative error in u is $\Delta u/u$. The values of Δx, Δy and Δz may be positive or negative and in some uses the terms on the right hand side may counteract each other. This effect cannot be relied upon and it is necessary to consider the worst case which is the case when all errors *add* up giving an error Δu given by the equation:

$$\left(\frac{\Delta u}{u}\right)_{\max} = \alpha \frac{\Delta x}{x} + \beta \frac{\Delta y}{y} + \gamma \frac{\Delta z}{z}$$

   Thus to find the maximum proportional error in u, multiply the proportional errors in each factor (x, y and z) by the numerical value of the power to which each factor is raised and then add all the terms so obtained.

   The sum thus obtained will give the maximum proportional error in the result of u. When the proportional error of a quantity is multiplied by 100, we get the percentage error of that quantity.

EXAMPLE 1.11

In an experiment for determining density (ρ) of a rectangular metal block, a student makes the following measurements.

Mass of block (m) = 39.3 g
Length of block (x) = 5.12 cm
Breadth of block (y) = 2.56 cm
Thickness of block (z) = 0.37 cm

The uncertainty in the measurement of m is $\pm$ 0.1 g and in the measurement of x, y and z is $\pm$ 0.01 cm. Find the value of ρ (in g cm^{-3}) up to appropriate significant figures, stating the uncertainty in the value of ρ.

SOLUTION

$$\rho = \frac{m}{xyz} = \frac{39.3}{5.12 \times 2.56 \times 0.37}$$

$$= 8.1037 \text{ g cm}^{-3}$$

$$\left(\frac{\Delta\rho}{\delta}\right)_{\max} = \frac{\Delta m}{m} + \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$= \frac{0.1}{39.3} + \frac{0.01}{5.12} + \frac{0.01}{2.56} + \frac{0.01}{0.37}$$

$$= 0.0353$$

$\therefore \Delta\rho = 0.0353 \times \rho = 0.0353 \times 8.1037 = 0.286 \text{ gcm}^{-3}$

Round off error $\Delta\rho$ to the first significant figure as $\Delta\rho = 0.3 \text{ gcm}^{-3}$.

Hence $\rho = 8.1037\ \text{gm}^{-3}$ is not accurate to the fourth decimal place. In fact, it is accurate only up to the first deciamal place. Hence the value of ρ much be rounded off as 8.1 and the result of the measurement is written as

$$\rho = (8.1 \pm 0.3)\ \text{g cm}^{-3}$$

EXAMPLE 1.12

The time period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$. The measured value of L is 20.0 cm using a scale of least count 1 mm and time t for 100 oscillations is found to be 90 s using a watch of least count 1 s. Find the value of g (in m s^{-2}) up to appropriate significant figure, stating the uncertainity in the value of g.

SOLUTION

If t is the time for n oscillations, then $T = \frac{t}{n}$. Given $T = 2\pi\sqrt{\frac{L}{g}}$. Squaring, we get

$$g = \frac{4\pi^2 Ln^2}{t^2} \qquad (1)$$

Putting $L = 0.200$ m, $n = 100$ and $t = 90$ s,

$$g = \frac{4 \times (3.14)^2 \times 0.2 \times (100)^2}{(90)^2}$$

$$= 9.74\ \text{m s}^{-2}$$

From Eq. (1), the relative error in g is

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta t}{t}$$

Note that there is no error in counting the number (n) of oscillations. Thus

$$\frac{\Delta g}{g} = \frac{0.1\,\text{cm}}{20.0\,\text{cm}} + \frac{2 \times 1\text{s}}{90\text{s}}$$

$$= 0.005 + 0.022 = 0.027$$

$$\therefore \qquad \Delta g = 0.027 \times 9.74 = 0.26\ \text{m s}^{-2}$$

Rounding off Δg to the first significant figure, we get $\Delta g = 0.3\ \text{m s}^{-2}$. Hence the value of g must be rounded off as $g = 9.7\ \text{m s}^{-2}$. Hence

$$g = (9.7 \pm 0.3)\ \text{m s}^{-2}$$

EXAMPLE 1.13

In the measurement of a physical quantity $X = \frac{A^2 B}{C^{1/3} D^3}$, the percentage errors in the measurements of quantities A, B, C and D are 1%, 2%, 3% and 4% respectively. Find the percentage error in the measurement of X.

SOLUTION

$$\frac{\Delta X}{X} = 2\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{1}{3}\frac{\Delta C}{C} + 3\frac{\Delta D}{D}$$

$$= 2 \times 1\% + 2\% + \frac{1}{3} \times 3\% + 3 \times 4\%$$

$$= 17\%$$

NOTE

1. Errors always add; they never cancel other.
2. The quantity which is raised to the highest power contributes the maximum error and hence it must be measured to a high degree of accuracy.

Applications

1. For a simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$\Rightarrow \qquad \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$$

2. For a sphere of radius r,

Surface area $A = 4\pi r^2 \Rightarrow \frac{\Delta A}{A} = \frac{2\Delta r}{r}$

Volume $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{\Delta V}{V} = \frac{3\Delta r}{r}$

3. Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\Rightarrow \qquad \frac{\Delta g}{g} = \frac{2\Delta R}{R} + \frac{\Delta M}{M}$$

4. For resistances connected in series

$$R_s = R_1 + R_2 \Rightarrow \frac{\Delta R_s}{R_s} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

5. For resistances connected in parallel

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \quad -\frac{\Delta R_p}{R_p^2} = -\frac{\Delta R_1}{R_1^2} - \frac{\Delta R_2}{R_2^2}$$

$$\Rightarrow \qquad \frac{\Delta R_p}{R_P^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

6. Kinetic energy K and linear momentum p are related as

$$K = \frac{p^2}{2m} \quad \Rightarrow \quad \frac{\Delta K}{K} = \frac{2\Delta p}{p}$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. The pressure P is related to distance x, Boltzmann constant k and temperature θ as

$$P = \frac{a}{b}\, e^{-ax/k\theta}$$

The dimensional formula of b is

(a) $[M^{-1}L^{-1}T^{-1}]$ (b) $[MLT^2]$
(c) $[M^0L^2T^0]$ (d) $[M^0L^0T^0]$

2. The magnitude of induced emf e in a conductor of length L rotating in magnetic field B is given by

$$e = \frac{1}{a}\,(\pi BL^2)$$

The dimensional formula of a is

(a) $[M^0L^0T]$ (b) $[ML^2T^{-2}]$
(c) $[M^2LT^{-1}]$ (d) $M^0L^0T^0]$

3. Two resistors $R_1 = [3.0 \pm 0.1]\ \Omega$ and $R_2 = (6.0 \pm 0.3)\ \Omega$ are connected in parallel. The resistance of the combination is

(a) $(2.0 \pm 0.4)\ \Omega$ (b) $(2.00 \pm 0.08)\ \Omega$
(c) $(2.0 \pm 0.2)\ \Omega$ (d) $(2.00 \pm 0.04)\ \Omega$

4. If the resistances in Q. 3 above were connected in series, the maximum percentage error in the resistance of the combination will be

(a) 1.1% (b) 2.2%
(c) 3.3% (d) 4.4%

5. Which of the following pairs of physical quantities do not have the same dimensions?

(a) Pressure and Young's modulus
(b) Emf and electric potential
(c) Heat and work
(d) Electric dipole moment and electric flux.

6. Which of the following pairs of physical quantities have different dimensions?

(a) Impulse and linear momentum
(b) Planck's constant and angular momentum
(c) Moment of inertia and moment of force
(d) Torque and energy

7. In the expression $A = A_0\, e^{-a/kT}$, k is Boltzmann constant and T is the absolute temperature. The dimensions of a are the same as those of

(a) energy (b) time
(c) acceleration (d) velocity

8. A cube has a side of 1.2 cm. The volume of the cube up to appropriate significant figures is

(a) 1.728 cm^3 (b) 1.73 cm^3
(c) 1.7 cm^3 (d) 17.3 cm^3

9. The quantities L/R and RC (where L, C and R stand for inductance, capacitance and resistance respectively) have the same dimensions as those of

(a) velocity (b) acceleration
(c) time (d) force

10. Which one of the following has the dimensions of $ML^{-1}T^{-2}$?

(a) torque (b) surface tension
(c) viscosity (d) stress

11. The dimensions of angular momentum are

(a) MLT^{-1} (b) ML^2T^{-1}
(c) $ML^{-1}T$ (d) ML^0T^{-2}

12. According to the quantum theory, the energy E of a photon of frequency ν is given by

$$E = h\nu$$

where h is Planck's constant. What is the dimensional formula for h?

(a) $M\ L^2\ T^{-2}$ (b) $M\ L^2\ T^{-1}$
(c) $M\ L^2\ T$ (d) $M\ L^2\ T^2$

13. The dimensions of Planck's constant are the same as those of

(a) energy
(b) power
(c) angular frequency
(d) angular momentum

14. The volume V of water passing any point of a uniform tube during t seconds is related to the cross-sectional area A of the tube and velocity u of water by the relation

$$V \propto A^{\alpha} u^{\beta} t^{\gamma}$$

which one of the following will be true?

(a) $\alpha = \beta = \gamma$ (b) $\alpha \neq \beta = \gamma$
(c) $\alpha = \beta \neq \gamma$ (d) $\alpha \neq \beta \neq \gamma$

15. The frequency n of vibrations of uniform string of length l and stretched with a force F is given by

$$n = \frac{p}{2l}\sqrt{\frac{F}{m}}$$

where p is the number of segments of the vibrating string and m is a constant of the string. What are the dimensions of m?

(a) $M\ L^{-1}\ T^{-1}$ (b) $M\ L^{-3}\ T^0$
(c) $M\ L^{-2}\ T^0$ (d) $M\ L^{-1}\ T^0$

16. If velocity (V), acceleration (A) and force (F) are taken as fundamental quantities instead of mass (M), length (L) and time (T), the dimensions of Young's modulus would be
(a) FA^2V^{-2} (b) FA^2V^{-3}
(c) FA^2V^{-4} (d) FA^2V^{-5}

17. The dimensions of permittivity (ε_0) of vacuum are
(a) $M^{-1}\ L^{-3}\ T^4\ A^2$ (b) $ML^{-3}\ T^2\ A^2$
(c) $M^{-1}\ L^3\ T^4\ A^2$ (d) $ML^3\ T^2\ A^2$
IIT, 1998

18. What are the dimensions of permeability (μ_0) of vacuum?
(a) $MLT^{-2}\ A^2$ (b) $MLT^{-2}\ A^{-2}$
(c) $ML^{-1}\ T^{-2}\ A^2$ (d) $ML^{-1}\ T^{-2}\ A^{-2}$
IIT, 1998

19. The dimensions of $1/\sqrt{\mu_0 \varepsilon_0}$ are the same as those of
(a) velocity (b) acceleration
(c) force (d) energy

20. The dimensions of specific heat are
(a) $MLT^{-2}\ K^{-1}$ (b) $ML^2\ T^{-2}\ K^{-1}$
(c) $M^0L^2T^{-2}\ K^{-1}$ (d) $M^0LT^{-2}\ K^{-1}$

21. What are the dimensions of latent heat?
(a) $ML^2\ T^{-2}$ (b) $ML^{-2}\ T^{-2}$
(c) $M^0\ LT^{-2}$ (d) $M^0\ L^2T^{-2}$

22. What are the dimensions of Boltzmann's constant?
(a) $MLT^{-2}\ K^{-1}$ (b) $ML^2T^{-2}\ K^{-1}$
(c) $M^0LT^{-2}\ K^{-1}$ (d) $M^0L^2T^{-2}\ K^{-1}$

23. The dimensions of potential difference are
(a) $ML^2T^{-3}\ A^{-1}$ (b) $MLT^{-2}\ A^{-1}$
(c) $ML^2T^{-2}\ A$ (d) $MLT^{-2}\ A$

24. What are the dimensions of electrical resistance?
(a) $ML^2T^{-2}\ A^2$ (b) $ML^2\ T^{-3}\ A^{-2}$
(c) $ML^2\ T^{-3}\ A^2$ (d) $ML^2\ T^{-2}\ A^{-2}$

25. The dimensions of electric field are
(a) $MLT^{-3}\ A^{-1}$ (b) $MLT^{-2}\ A^{-1}$
(c) $MLT^{-1}\ A^{-1}$ (d) $MLT^0\ A^{-1}$

26. The dimensions of magnetic induction field are
(a) $ML^0\ T^{-1}\ A^{-1}$ (b) $M^0L\ T^{-1}\ A^{-1}$
(c) $MLT^{-2}\ A^{-1}$ (d) $ML^0T^{-2}\ A^{-1}$

27. What are the dimensions of magnetic flux?
(a) $ML^2\ T^{-2}\ A^{-1}$ (b) $ML^2\ T^{-2}\ A^{-2}$
(c) $ML^{-2}\ T^{-2}\ A^{-1}$ (d) $ML^{-2}\ T^{-2}\ A^{-2}$

28. The dimensions of self inductance are
(a) $ML^2\ T^{-2}\ A^{-1}$ (b) $ML^2\ T^{-2}\ A^{-2}$
(c) $ML^{-2}\ T^{-2}\ A^{-1}$ (d) $ML^{-2}\ T^{-2}\ A^{-2}$

29. The dimensions of capacitance are
(a) $M^{-1}\ L^{-2}\ TA^2$ (b) $M^{-1}\ L^{-2}\ T^2\ A^2$
(c) $M^{-1}\ L^{-2}\ T^3\ A^2$ (d) $M^{-1}\ L^{-2}\ T^4\ A^2$

30. If velocity (V), force (F) and energy (E) are taken as fundamental units, then dimensional formula for mass will be
(a) $V^{-2}F^0E$ (b) V^0FE^2
(c) $VF^{-2}E^0$ (d) $V^{-2}F^0E$

31. Frequency (n) of a tuning fork depends upon length (l) of its prongs, density (ρ) and Young's modulus (Y) of its material. Then frequency and Young's modulus will be related as
(a) $n \propto \sqrt{Y}$ (b) $n \propto Y$
(c) $n \propto \frac{1}{\sqrt{Y}}$ (d) $n \propto \frac{1}{Y}$

32. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 = permittivity of free space and E = electric field) are
(a) MLT^{-1} (b) ML^2T^{-2}
(c) $ML^{-1}T^{-2}$ (d) ML^2T^{-1}
IIT, 2000

33. Of the following quantities, which one has dimensions different from the remaining three
(a) Energy per unit volume
(b) Force per unit area
(c) Product of voltage and charge per unit volume
(d) Angular momentum

34. If the time period t of a drop of liquid of density d, radius r, vibrating under surface tension s is given by the formula $t = \sqrt{d^a r^b s^c}$ and if $a = 1$, $c = -1$, then b is
(a) 1 (b) 2
(c) 3 (d) 4

35. In the measurement of a physical quantity $X = \frac{A^2 B}{C^{1/3} D^3}$. The percentage errors introduced in the measurements of the quantities A, B, C and D are 2%, 2%, 4% and 5% respectively. Then the minimum amount of percentage of error in the measurement of X is contributed by:
(a) A (b) B
(c) C (d) D

36. Which of the following has the dimensions $ML^{-1}\ T^{-1}$?
(a) Surface tension (b) Coefficient of viscosity
(c) Bulk modulus (c) Angular momentum

37. Pressure gradient dp/dx is the rate of change of pressure with distance. What are the dimensions of dp/dx?

(a) $ML^{-1}T^{-1}$ (b) $ML^{-2}T^{-2}$
(c) $ML^{-1}T^{-2}$ (d) $ML^{-2}T^{-1}$

38. If E, M, J and G respectively denote energy, mass, angular momentum and gravitational constant, then $\frac{EJ^2}{M^5G^2}$ has the dimensions of
(a) length (b) angle
(c) mass (d) time

IIT, 1990

39. If e, ε_0, h and c respectively represent electronic charge, permittivity of free space, Planck's constant and speed of light, then $\frac{e^2}{\varepsilon_0 hc}$ has the dimensions of
(a) current (b) pressure
(c) angular momentum (d) angle

40. If L, R, C and V respectively represent inductance, resistance, capacitance and potential difference, then the dimensions of $\frac{L}{RCV}$ are the same as those of
(a) current (b) $\frac{1}{\text{current}}$
(c) charge (d) $\frac{1}{\text{charge}}$

41. If E and B respectively represent electric field and magnetic induction field, then the ratio $\frac{E}{B}$ has the dimensions of
(a) displacement (b) velocity
(c) acceleration (d) angle

42. If C and V respectively represent the capacitance of a capacitor and the potential difference between its plates, then the dimensions of CV^2 are
(a) ML^2T^{-2} (b) $ML^3T^{-2}A^{-1}$
(c) $ML^2T^{-1}A^{-1}$ (d) $M^0L^0T^0$

43. If h and e respectively represent Planck's constant and electronic charge, then the dimensions of $\left(\frac{h}{e}\right)$ are the same as those of
(a) magnetic field (b) electric field
(c) magnetic flux (d) electric flux

44. If energy E, velocity V and time T are chosen as the fundamental units, the dimensional formula for surface tension will be
(a) EV^2T^{-2} (b) $EV^{-1}T^{-2}$
(c) $EV^{-2}T^{-2}$ (d) $E^2V^{-1}T^{-2}$

45. A gas bubble from an explosion under water oscillates with a period proportional to $P^a d^b E^c$ where P is the static pressure, d is the density of water and E is the energy of explosion. Then a, b and c respectively are
(a) $-\frac{5}{6}, \frac{1}{2}, \frac{1}{3}$ (b) $\frac{1}{2}, -\frac{5}{6}, \frac{1}{3}$
(c) $\frac{1}{3}, \frac{1}{2}, -\frac{5}{6}$ (d) 1, 1, 1

IIT, 1981

46. In a system of units in which the unit of mass is a kg, unit of length is b metre and the unit of time is c second, the magnitude of a calorie is
(a) $\frac{4.2c}{ab^2}$ (b) $\frac{4.2c^2}{ab^2}$
(c) $\frac{abc}{4.2}$ (d) $\frac{4.2}{abc}$

47. The error in the measurement of the radius of a sphere is 1%. The error in the measurement of the volume is
(a) 1% (b) 3%
(c) 5% (d) 8%

48. If the error in the measurement of the volume of a sphere is 6%, then the error in the measurement of its surface area will be
(a) 2% (b) 3%
(c) 4% (d) 7.5%

49. The moment of inertia of a body rotating about a given axis is 6.0 kg m^2 in the SI system. What is the value of the moment of inertia in a system of units in which the unit of length is 5 cm and the unit of mass is 10 g?
(a) 2.4×10^3 (b) 2.4×10^5
(c) 6.0×10^3 (d) 6.0×10^5

50. A quantity X is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$ where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of
(a) resistance (b) charge
(c) voltage (d) current

IIT, 2001

51. The coefficient of viscosity (η) of a liquid by the method of flow through a capillary tube is given by the formula

$$\eta = \frac{\pi}{8}\frac{R^4}{l}\frac{P}{Q}$$

where R = radius of the capillary tube,
l = length of the tube,
P = pressure difference between its ends, and

Q = volume of liquid flowing per second.

Which quantity must be measured most accurately?

(a) R (b) l

(c) P (d) Q

52. The mass m of the heaviest stone that can be moved by the water flowing in a river depends on v, the speed of water, density (d) of water and the acceleration due to gravity (g). Then m is proportional to

(a) v^2 (b) v^4

(c) v^6 (d) v^8

53. The speed (v) of ripples depends upon their wavelength (λ), density (ρ) and surface tension (σ) of water. Then v is proportional to

(a) $\sqrt{\lambda}$ (b) λ

(c) $\frac{1}{\lambda}$ (d) $\frac{1}{\sqrt{\lambda}}$

54. The period of revolution (T) of a planet moving round the sun in a circular orbit depends upon the radius (r) of the orbit, mass (M) of the sun and the gravitation constant (G). Then T is proportional to

(a) $r^{1/2}$ (b) r

(c) $r^{3/2}$ (d) r^2

55. If the velocity of light (c), gravitational constant (G) and planck's constant (h) are chosen as fundamental units, the dimensions of time in the new system will be

(a) $c^{-5/2}G^2h^{-1/2}$ (b) $c^{-3/2}G^{-2}h^2$

(c) $c^2G^{-2}h^{1/2}$ (d) $c^{-5/2}G^{1/2}h^{1/2}$

56. The amplitude of a damped oscillator of mass m varies with time t as

$$A = A_0 e^{(-at/m)}$$

The dimensions of a are

(a) ML^0T^{-1} (b) M^0LT^{-1}

(c) MLT^{-1} (d) $ML^{-1}T$

57. A student measures the value of g with the help of a simple pendulum using the formula

$$g = \frac{4\pi^2 L}{T^2}$$

The errors in the measurements of L and T are ΔL and ΔT respectively. In which of the following cases is the error in the value of g the minimum?

(a) ΔL = 0.5 cm, ΔT = 0.5 s

(b) ΔL = 0.2 cm, ΔT = 0.2 s

(c) ΔL = 0.1 cm, ΔT = 1.0 s

(d) ΔL = 0.1 cm, ΔT = 0.1 s

58. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with a uncertainty of ± 0.01 mm. Take g = 9.8 m/s^2 (exact). The Young's modulus obtained from the reading is

(a) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2

(b) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2

(c) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2

(d) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

IIT, 2007

59. In a vernier callipers, one main scale division is x cm and n divisions of the vernier scale coincide with (n – 1) divisions of the main scale. The least count (in cm) of the callipers is

(a) $\left(\frac{n-1}{n}\right)x$ (b) $\frac{nx}{(n-1)}$

(c) $\frac{x}{n}$ (d) $\frac{x}{(n-1)}$

IIT, 2007

60. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm

Least count for time = 0.1 s

Student	*Length of the pendulum (cm)*	*Number of oscillations (n)*	*Total time for (n) oscillations (s)*	*Time period (s)*
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g. i.e. $\left(\frac{\Delta g}{g} \times 100\right)$ for student I, II and III, respectively,

(a) $E_1 = 0$ (b) E_I is minimum

(c) $E_I = E_{II}$ (d) E_{II} is minimum

IIT, 2008

61. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(a) 0.9% (b) 2.4%

(c) 3.1% (d) 4.2%

IIT, 2011

ANSWERS

1. (c)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (c)
7. (a)	**8.** (c)	**9.** (c)	**10.** (d)	**11.** (b)	**12.** (b)
13. (d)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (b)
19. (a)	**20.** (c)	**21.** (d)	**22.** (b)	**23.** (a)	**24.** (b)
25. (a)	**26.** (d)	**27.** (a)	**28.** (b)	**29.** (d)	**30.** (d)
31. (a)	**32.** (c)	**33.** (d)	**34.** (c)	**35.** (c)	**36.** (b)
37. (b)	**38.** (b)	**39.** (d)	**40.** (b)	**41.** (b)	**42.** (a)
43. (c)	**44.** (c)	**45.** (a)	**46.** (b)	**47.** (b)	**48.** (c)
49. (b)	**50.** (d)	**51.** (a)	**52.** (c)	**53.** (d)	**54.** (c)
55. (d)	**56.** (a)	**57.** (d)	**58.** (b)	**59.** (c)	**60.** (b)
61. (c)					

SOLUTIONS

1. The exponent is dimensionless. Hence

$$[a] = \left[\frac{k\theta}{x}\right] = \left[\frac{\text{J K}^{-1} \times \text{K}}{\text{m}}\right]$$

$$= \text{J m}^{-1}$$

$$= [\text{ML}^2\text{T}^{-2}] \times [\text{L}^{-1}]$$

$$= \text{MLT}^{-2}$$

Also $[P] = \dfrac{[a]}{[b]}$

$$\Rightarrow \quad [b] = \frac{[a]}{[P]} = \left[\frac{\text{ML}^1\text{T}^{-2}}{\text{ML}^{-1}\text{T}^{-2}}\right] = [\text{M}^0\text{L}^2\text{T}^0]$$

So the correct choice is (c).

2. $a = \dfrac{\pi BL^2}{e} = \dfrac{[\text{M}^0\text{L}^0\text{T}^0] \times [\text{ML}^0\text{T}^{-2}\text{A}^{-1}] \times [\text{L}^2]}{[\text{ML}^2\text{T}^{-3}\text{A}^{-1}]}$

$$= [\text{M}^0\text{L}^0\text{T}^1]$$

3. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$$\Rightarrow \quad R = \frac{R_1 R_2}{R_1 + R_2} = \frac{3.0 \times 6.0}{9.0} = 2.0\ \Omega$$

$$\therefore \quad + \frac{\delta R}{R^2} = + \frac{\delta R_1}{R_1^2} + \frac{\delta R_2}{R_2^2}$$

$$= \frac{0.1}{(3)^2} + \frac{0.3}{(6)^2} = 0.019$$

$\therefore\ \delta R = 0.019 \times R^2 = 0.019 \times (2)^2 = 0.076 \simeq 0.08\ \Omega$

Hence the correct choice is (b).

4. $R = R_1 + R_2 = 3.0 + 6.0 = 9.0\ \Omega$

$\delta R = \delta R_1 + \delta R_2 = 0.1 + 0.3 = 0.4$

$\therefore$ Maximum percentage error $= \dfrac{\delta R}{R} \times 100$

$$= \frac{0.4}{9.0} \times 100$$

$$= 4.4\%$$

So the correct choice is (d).

5. The correct choice is (d).

6. The correct choice is (c).

7. Since kT has dimensions of energy, the correct choice is (a).

8. $V = L^3 = 1.2\ \text{cm} \times 1.2\ \text{cm} \times 1.2\ \text{cm}$

$$= 1.728\ \text{cm}^3$$

Since there are two significant figures in $L = 1.2$ cm, the volume is accurate only up to two significant figure.

Hence the correct choice is (c).

9. L/R is the time constant of an L-R circuit and CR is the time constant of a C-R circuit. The dimension of the time constant is the same as that of time. Hence the correct choice is (c).

10. $\text{ML}^{-1}\text{T}^{-2}$ are the dimensions of force per unit area. Out of the four choices, stress is the only quantity that is force per unit area. Hence the correct choice is (d).

11. The angular momentum **L** of a particle with respect to point whose position vector is **r** is given by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where **p** is the linear momentum of the moving particle.

$\therefore$ Dimensions of **L** = dimension of **r** $\times$ dimensions of **p**

$$= \text{L} \times \text{MLT}^{-1} = \text{ML}^2\text{T}^{-1}$$

Thus the correct choice is (b).

12. Dimensions of $h = \dfrac{\text{dimension of } E}{\text{dimension of } \nu} = \dfrac{\text{ML}^2\text{T}^{-2}}{\text{T}^{-1}}$

$$= \text{ML}^2\,\text{T}^{-1}$$

Thus the correct choice is (b).

13. The correct choice is (d).

14. The dimensions of the two sides of proportionality are

$$\text{L}^3 = \text{L}^{2\alpha}(\text{LT}^{-1})^{\beta}\,\text{T}^{\gamma} = \text{L}^{2\alpha+\beta}\,\text{T}^{\gamma-\beta}$$

Equating the powers of dimensions on both sides, we have

$$2\alpha + \beta = 3$$

$$\gamma - \beta = 0$$

which give $\beta = \gamma$ and $\alpha = \frac{1}{2}(3 - \beta)$, i.e. $\alpha \neq \beta = \gamma$.

Thus the correct choice is (b).

15. Squaring both sides of the given relation, we get

$$n^2 = \frac{p^2}{4l^2} \cdot \frac{F}{m} \quad \text{or} \quad m = \frac{p^2 F}{4 l^2 n^2}$$

$\therefore$ dimensions of m

$$= \frac{\text{dimensions of } F}{\text{dimensions of } l^2 \times \text{dimensions of } n^2}$$

($\because$ p is a dimensionless number)

$$= \frac{\mathrm{MLT^{-2}}}{\mathrm{L^2} \times \left(\mathrm{T^{-1}}\right)^2} = \mathrm{ML^{-1}\,T^0}$$

Hence the correct choice is (*d*).

16. Dimensions of Young's modulus Y are $\mathrm{ML^{-1}\,T^{-2}}$. The dimension of V, A and F in terms of M, L and T are

$$(V) = \mathrm{LT^{-1}},\ (A) = \mathrm{LT^{-2}}$$

and $(F) = \mathrm{MLT^{-2}}$

Let $(Y) = (V^a A^b F^c)$

Putting dimensions of Y, V, A and F. We have

$$(\mathrm{ML^{-1}\,T^{-2}}) = (\mathrm{LT^{-1}})^a \times (\mathrm{LT^{-2}})^b \times (\mathrm{MLT^{-2}})^c$$

or $\mathrm{M^1\,L^{-1}\,T^{-2}} = \mathrm{M^c\,L^{a+b+c}\,T^{-a-2b-2c}}$

Equating powers of M, L and T we have

$$c = 1,\ a + b + c = -1$$

and $-a - 2b - 2c = -2$

which give $a = -4$, $b = 2$ and $c = 1$.

Thus $(Y) = (FA^2V^{-4})$

Thus the correct choice is (c).

17. According to Coulomb's law of electrostatics, force F between two charges q_1 and q_2 a distance r apart in vacuum, is given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

or $$\varepsilon_0 = \frac{1}{4\pi F} \cdot \frac{q_1 q_2}{r^2}$$

$$\therefore \text{ Dimensions of } \varepsilon_0 = \frac{\mathrm{Q^2}}{\mathrm{MLT^{-2}} \times \mathrm{L^2}}$$

$$= \mathrm{M^{-1}\,L^{-3}\,T^2\,Q^2}$$

$$= \mathrm{M^{-1}L^{-3}T^4A^2} \left(\because \mathrm{A} = \frac{\mathrm{Q}}{\mathrm{T}}\right)$$

The correct choice is (a).

18. The force per unit length between two long wires carrying currents I_1 and I_2 a distance r apart in vacuum, is given by

$$f = \frac{\mu_0}{4\pi} \cdot \frac{I_1 I_2}{r} \quad \text{or} \quad \mu_0 = \frac{4\pi r f}{I_1 I_2}$$

$$\therefore \text{Dimensions of } \mu_0 = \frac{\mathrm{L \times MLT^{-2} \times L^{-1}}}{\mathrm{A^2}}$$

$$= \mathrm{MLT^{-2}\,A^{-2}}$$

Therefore, the correct choice is (b).

19. Dimensions of

$$\frac{1}{\sqrt{\mu_0\,\varepsilon_0}} = \frac{1}{\left(\mathrm{MLT^{-2}\ A^2 \times M^{-1}L^{-3}T^4A^{-2}}\right)^{\frac{1}{2}}}$$

$$= \frac{1}{\left(\mathrm{L^{-2}T^2}\right)^{\frac{1}{2}}} = \mathrm{LT^{-1}}$$

which are the dimensions of velocity. Hence the correct choice is (a).

20. The heat energy content H of a body of mass m at temperature θ is given by $H = ms\theta$ where s is the specific heat. Therefore

$$s = \frac{H}{m\theta}$$

$\therefore$ Dimensions of s

$$\frac{\text{dimensions of heat energy}}{\text{dimension of mass} \times \text{dimension of temperature}}$$

$$= \frac{\mathrm{ML^2T^{-2}}}{\mathrm{M \times K}} = \mathrm{M^0L^2T^{-2}K^{-1}}$$

Thus the correct choice is (c).

21. Latent heat L is the amount of heat energy H required to change the state of a unit mass without producing any change in temperature. Thus

$$L = \frac{H}{m}$$

$$\therefore \quad \text{Dimensions of } L = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{M}}$$

$$= \mathrm{L^2T^{-2}} = \mathrm{M^0L^2T^{-2}}$$

Thus the correct choice is (d).

22. According to the law of equipartition of energy, the energy per degree of freedom of a gas atom or molecule at a temperature θ kelvin is given by

$$E = \frac{1}{2}\ k\theta \ \text{ or } k = \frac{2E}{\theta}$$

where k is the Boltzmann's constant.

$$\therefore \quad \text{Dimensions of } k = \frac{\text{dimensions of } E}{\text{dimension of } \theta}$$

$$= \frac{\mathrm{ML^2T^{-2}}}{\mathrm{K}} = \mathrm{ML^2T^{-2}K^{-1}}$$

23. The potential difference V between two points is the amount of work done in moving a unit charge from one point to the other.

Thus, $$V = \frac{\text{work done}}{\text{charge moved}} = \frac{W}{q}$$

$\therefore$ Dimensions of $V = \dfrac{ML^2T^{-2}}{Q} = ML^2\,T^{-2}\,Q^{-1}$

$= ML^2\,T^{-3}\,A^{-1}$ ($\because$ $Q = AT$)

Hence the correct choice is (a).

24. From Ohm's law, resistance R is given by

$$R = \frac{\text{potential difference}}{\text{current}}$$

$\therefore$ Dimensions of $R = \dfrac{ML^2T^{-3}A^{-1}}{A}$

$= ML^2T^{-3}A^{-2}$

Thus the correct choice is (b).

25. Force F experienced by a charge q in an electric field E is given by

$$F = qE \text{ or } E = \frac{F}{q}$$

$\therefore$ Dimensions of $E = \dfrac{\text{dimensions of } F}{\text{dimensions of } Q} = \dfrac{MLT^{-2}}{AT}$

$= MLT^{-3}A^{-1}$.

26. The force F, experienced by a charge q moving with speed v perpendicular to the direction of a uniform magnetic induction field B is given by

$$F = qvB \text{ or } B = \frac{F}{qv}$$

$\therefore$ Dimensions of $B = \dfrac{MLT^{-2}}{Q \times LT^{-1}} = ML^0T^{-1}Q^{-1}$

$= ML^0T^{-2}A^{-1}$ ($\because$ $Q = AT$)

Hence the correct choice is (d)

27. The magnetic flux ϕ linked with a circuit of area A in a magnetic induction field B is given by

$$\phi = BA\cos\theta$$

where θ is the angle between the field and area vectors.

$\therefore$ Dimensions of ϕ = dimensions of BA

($\because$ $\cos\theta$ is dimensionless)

$= ML^0\,T^{-2}\,A^{-1} \times L^2$

$= ML^2\,T^{-2}\,A^{-1}$

Thus the correct choice is (a).

28. The self inductance L of a coil in which the current varies at a rate $\dfrac{dI}{dt}$ is given by

$$e = -L\,\frac{dI}{dt}$$

where e is the e.m.f. induced in the coil. Now, the dimensions of e.m.f. are the same as those of potential difference, namely, $ML^2\,T^{-3}\,A^{-1}$.

Now, $$L = -\frac{e}{\dfrac{dI}{dt}}$$

$\therefore$ Dimensions of L

$$= \frac{\text{dimensions of } e}{\text{dimensions of } I \,/\, \text{dimensions of } t}$$

$$= \frac{ML^2T^{-3}A^{-1}}{A/T} = ML^2\,T^{-2}\,A^{-2}$$

Thus the correct choice is (b).

29. When a capacitor of capacitance C is charged to a potential difference V, the charge Q on the capacitor plates is given by

$$Q = CV \text{ or } C = \frac{Q}{V}$$

$\therefore$ Dimensions of $C = \dfrac{\text{dimensions of } Q}{\text{dimensions of } V}$

$= \dfrac{AT}{ML^2T^{-3}A^{-1}}$

$= M^{-1}\,L^{-2}\,T^4\,A^2$

Hence the correct choice is (d).

30. Let $(M) = V^a\,F^b\,E^c$

Putting the dimensions of V, F and E, we have

$$(M) = (LT^{-1})^a \times (MLT^{-2})^b \times (ML^2T^{-2})^c$$

or $$M^1 = M^{b+c}\,L^{a+b+2c}\,T^{-a-2b-2c}$$

Equating the powers of dimensions, we have

$$b + c = 1$$
$$a + b + 2c = 0$$
$$-a - 2b - 2c = 0$$

which give $a = -2$, $b = 0$ and $c = 1$. Therefore $(M) = (V^{-2}\,F^0\,E)$.

Thus the correct choice is (d).

31. Let $n \propto l^a\,\rho^b\,Y^c$

Putting dimensions of all the quantities, we have

$$(T^{-1}) \propto L^a\,(ML^{-3})^b\,(ML^{-1}\,T^{-2})^c$$

Equating powers of M, L and T on both sides, we get

$$b + c = 0,\ a - 3b - c = 0 \text{ and } -2c = -1$$

which give $a = -1$, $b = -\dfrac{1}{2}$ and $c = \dfrac{1}{2}$. Thus

$$n \propto l^{-1}\,\rho^{1/2}\,Y^{1/2}$$

Hence the correct choice is (a).

32. We know that

$$F = \frac{q_1\,q_2}{(4\pi\varepsilon_0)r^2} \text{ and } E = \frac{F}{q}$$

$\therefore$ $$\varepsilon_0 = \frac{q_1\,q_2}{4\pi F\,r^2}$$

Hence $$\frac{1}{2}\,\varepsilon_0\,E^2 = \frac{q_1\,q_2}{8\pi F\,r^2} \times \frac{F^2}{q^2}$$

$$= \frac{q_1\,q_2}{8\pi\,q^2} \times \frac{F}{r^2}$$

$\therefore$ Dimensions of $\dfrac{1}{2}\,\varepsilon_0\,E^2$ = dimensions of $\dfrac{F}{r^2}$ =

$$\frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

Hence the correct choice is (c).

33. Energy per unit volume, force per unit area and product of voltage and charge density all have dimensions of ML^2T^{-2}, but the dimensions of angular momentum are ML^2T^{-1}. Hence the correct choice is (d).

34. Given $t = d^{a/2}\, r^{b/2}\, s^{c/2}$. Substituting dimensions, we have

$$(T) = (ML^{-3})^{a/2}\ (L)^{b/2}\ (MT^{-2})^{c/2}$$
$$= M^{(a+c)/2} \cdot L^{(-3a/2 + b/2)}\ T^{-c}$$

Equating powers of L, we have, $-\frac{3}{2}a + \frac{b}{2} = 0$. Given $a = 1$.

$\therefore -\frac{3}{2} + \frac{b}{2} = 0$ or $b = 3$, which is choice(c).

35. Given $X = \frac{A^2 B}{C^{1/3} D^3}$

Taking logarithm of both sides, we have

$$\log X = 2 \log A + \log B - \frac{1}{3} \log C - 3 \log D$$

Partially differentiating, we have

$$\frac{\Delta x}{x} = 2\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{1}{3}\frac{\Delta C}{C} - 3\frac{\Delta D}{D}$$

$$\text{Percentage error in } A = 2\ \frac{\Delta A}{A} = 2 \times 2\% = 4\%$$

$$\text{Percentage error in } B = \frac{\Delta B}{B} = 2\%$$

$$\text{Percentage error in } C = \frac{1}{3}\ \frac{\Delta C}{C} = \frac{1}{3} \times 4\% = \frac{4}{3}\%$$

$$\text{Percentage error in } D = 3\ \frac{\Delta D}{D} = 3 \times 5\% = 15\%$$

We find that the minimum percentage error is contributed by C. Hence the correct choice is (c).

36. The correct choice is (b)

37. The correct choice is (b)

38. Dimensions of J and G are ML^2T^{-1} and $M^{-1}L^3\ T^{-2}$ respectively.

39. Dimensions of ε_0 and h are $M^{-1}L^{-3}\ T^4\ A^2$ and ML^2T^{-1} respectively.

40. RC has the dimensions of time (T). V has the dimensions of emf which has the dimensions of $L\ \frac{dI}{dt}$.

41. The force **F** on a particle of charge q moving with a velocity **v** in **E** and **B** fields is given by

$$\mathbf{F} = q\ (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Hence the dimensions of E are the same as those of vB.

42. Energy stored in a capacitor of capacitance C having a potential difference V between its plates is given by

$$U = \frac{1}{2} CV^2$$

Hence, the dimensions of CV^2 = dimensions of energy. Hence the correct choice is (a).

43. Dimensions of $\left(\frac{h}{e}\right) = \frac{(ML^2T^{-1})}{AT} = ML^2\ T^{-2}\ A^{-1}$

Dimensions of $B = MT^{-2}\ A^{-1}$

Magnetic flux = $B \times$ area

44. Let surface tension $\sigma = E^a\ V^b\ T^c$. Using the dimensions of σ, E, V and T and equating powers of M, L and T, find the values of a, b and c. The correct choice is (c).

45. The correct choice is (a).

46. Let n_1 be the magnitude (i.e. numerical value) of a physical quantity when the fundamental units are (M_1, L_1, T_1) and n_2 the magnitude of the same physical quantity when the fundamental units are (M_2, L_2, T_2), then, it is obvious that

$$n_1\ (M_1^x\ L_1^y\ T_1^z) = n_2\ (M_2^x\ L_2^y\ T_2^z) \qquad \text{(i)}$$

where x, y and z are the dimensions of the physical quantity in mass, length and time respectively. Now, we know that 1 calorie = 4.2 joule = 4.2 kg m^2s^{-2}. Therefore, in the first system of units $n_1 = 4.2$, $x = 1$, $y = 2$ and $z = -2$. Hence, in the second system of units in which $M_2 = a$ kg, $L_2 = b$ m and $T_2 = c$ s, we have from (i)

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^x \left(\frac{L_1}{L_2}\right)^y \left(\frac{T_1}{T_2}\right)^z$$
$$= n_1 \left(\frac{1\,\text{kg}}{a\,\text{kg}}\right)^x \left(\frac{1\,\text{m}}{b\,\text{m}}\right)^y \left(\frac{1\,\text{s}}{c\,\text{s}}\right)^z$$
$$= 4.2 \left(\frac{1}{a}\right)^1 \left(\frac{1}{b}\right)^2 \left(\frac{1}{c}\right)^{-2}$$
$$= \frac{4.2c^2}{ab^2}$$

47. $V = \frac{4}{3}\pi r^3$. Taking logarithm of both sides, we have

$$\log V = \log 4 + \log \pi + 3 \log r - \log 3$$

Differentiating, we get

$$\frac{\Delta V}{V} = 3\frac{\Delta r}{r} = 3 \times 1\% = 3\%$$

48. $\frac{\Delta V}{V} = 3\frac{\Delta r}{r}$ or $6\% = 3\frac{\Delta r}{r}$ or $\frac{\Delta r}{r} = 2\%$.

Now surface area $s = 4\pi r^2$ or $\log s = \log 4\pi + 2 \log r$

$$\therefore \quad \frac{\Delta s}{s} = 2\frac{\Delta r}{r} = 2 \times 2\% = 4\%.$$

49. The dimensions of moment of inertia are (ML^2). We have

$$n_1(u_1) = n_2(u_2)$$

or $$n_1(M_1 L_1^2) = n_2(M_2 L_2^2)$$

$$\therefore \quad n_2 = \frac{n_1(M_1 L_2^2)}{(M_2 L_2^2)} = n_1 \left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^2$$

Given $n_1 = 6.0$, $M_1 = 1$ kg, $L_1 = 1$ m, $M_2 = 10$ *g* and $L_2 = 5$ cm. Therefore,

$$n_2 = 6.0 \times \left(\frac{1\text{ kg}}{10\text{ g}}\right) \times \left(\frac{1\text{ m}}{5\text{ cm}}\right)^2$$

$$= 6.0 \times \left(\frac{1000\text{ g}}{10\text{ g}}\right) \times \left(\frac{100\text{ cm}}{5\text{ cm}}\right)^2$$

$$= 6.0 \times 100 \times (20)^2 = 2.4 \times 10^5$$

50. The capacitance of a parallel plate capacitor is given by $C = \varepsilon_0 A/d$. Hence the dimensions of $\varepsilon_0 L$ are the same as those of capacitance.

$$\therefore \text{ Dimension of } \varepsilon_0 L\frac{\Delta V}{\Delta t}$$

$$= \frac{\text{dimension of } C \times \text{dimensions of } V}{\text{time}}$$

$$= \frac{\text{dimension of } Q}{\text{time}} \qquad (\because\ Q = CV)$$

Hence the correct choice is (d).

51. The correct choice is (a).

The maximum permissible error in η is given by the relation

$$\frac{\Delta\eta}{\eta} = 4\frac{\Delta R}{R} + \frac{\Delta l}{l} + \frac{\Delta P}{P} + \frac{\Delta Q}{Q}$$

It is clear that the error in the measurement of R is magnified four times on account of the occurrence of R^4 in the formula. Hence the radius (R) of the capillary tube must be measured most accurately. Thus the quantity which is raised to the highest power needs the most accurate measurement.

52. Take $m \propto v^a d^b g^c$ and show that $a = 6$.

53. Take $v \propto \lambda^a \rho^b \sigma^c$ and show that $a = -\frac{1}{2}$.

54. Take $T \propto r^a M^b G^c$ and show that $a = -\frac{3}{2}$.

55. The correct choice is (d).

56. The exponent is a dimensionless number. Hence at/m is dimensionless. Therefore,

$$\text{Dimension of } a = \frac{\text{dimension of } m}{\text{dimension of } t} = \frac{\text{M}}{\text{T}}$$

$$= \text{M L}^0 \text{T}^{-1}$$

57. The proportionate error in the measurement of g is

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

Hence Δg will be minimum if ΔL and ΔT are minimum. Thus the correct choice is (d).

58.
$$Y = \frac{FL}{Al} = \frac{4MgL}{\pi d^2 l} \qquad (1)$$

where

$M = 1.0$ kg (exact), $g = 9.8$ ms^{-2} (exact)

$L = 2$ m (exact), $l = 0.8$ mm $= 0.8 \times 10^{-3}$ m

$\Delta l = \pm 0.05$ mm, $d = 0.4$ mm $= 0.4 \times 10^{-3}$ m

$\Delta d = \pm 0.01$ mm

Substituting the values of M, g, L, d and l in Eq. (1) we get

$$Y = 2.0 \times 10^{11}\ \text{Nm}^{-2}$$

From Eq. (1) the proportionate uncertainty in Y is given by

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

Since the values of M, g and L are exact, $\Delta M = 0$, $\Delta g = 0$ and $\Delta L = 0$. Hence

$$\frac{\Delta Y}{Y} = \frac{2\Delta d}{d} + \frac{\Delta l}{l} = \frac{2 \times 0.01\text{mm}}{0.4\text{mm}} + \frac{0.05\text{mm}}{0.8\text{mm}}$$

$$= 0.05 + 0.0625$$

$$= 0.1125$$

$$\therefore \quad \Delta Y = 0.1125 \times Y = 0.1125 \times 2.0 \times 10^{11}$$

$$= 0.225 \times 10^{11}\ \text{Nm}^{-2}$$

Since the value of Y is correct only up to the first decimal place, the value of ΔY must be rounded off to the first decimal place. Thus $\Delta Y = 0.2 \times 10^{11}$ Nm^{-2}. Therefore, the result of the experiment is

$$Y + \Delta Y = (2.0 \pm 0.2) \times 10^{11}\ \text{Nm}^{-2}$$

Hence the correct choice is (b).

59. Vernier constant = value of 1 M.S.D – value of 1 V.S.D.

Now n V.S.D = $(n-1)$ M.S.D = $(n-1)\,x$ cm

$$\therefore \quad 1\text{ V.S.D} = \left(\frac{n-1}{n}\right) x\text{ cm}$$

$$\therefore \quad \text{V.C.} = x\text{ cm} - \left(\frac{n-1}{n}\right) x\text{ cm} = \frac{x}{n}\text{ cm}$$

Hence the correct choice is (c).

60. $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = \frac{4\pi^2 l}{T^2}$ Therefore,

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$$

For student I, $E_I = \frac{\Delta g}{g}100$

$$= \left(\frac{0.1}{64.0} + \frac{2\times 0.1}{128.0}\right) \times 100$$

$$= \frac{5}{16}\%$$

For student II, $E_{II} = \left(\frac{0.1}{64} + \frac{2\times 0.1}{64.0}\right) \times 100$

$$= \frac{15}{32}\%$$

For student III, $E_{III} = \left(\frac{0.1}{20.0} + \frac{2\times 0.1}{36.0}\right) \times 100$

$$= \frac{19}{18}\%$$

Thus the percentage error is minimum for student I.

61. Least count of screw gauge

$$= \frac{\text{pitch}}{\text{number of divisions on circular scale}}$$

$$= \frac{0.5 \text{ mm}}{50} = 0.01 \text{ mm}$$

$\therefore$ diameter of ball = 2.5 mm + 20 × 0.01 mm = 2.7 mm

Density $\rho = \frac{M}{\frac{4\pi}{3}r^3}$

Maximum relative error in ρ is

$$\therefore \quad \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta r}{r}$$

$$= 2\% + 3 \times \frac{0.01}{2.7} \times 100$$

$$= 2\% + 1.1\% = 3.1\%$$

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Which of the following are *not* a unit of time?
(a) parsec (b) light year
(c) micron (d) second

2. Choose the pair of physical quantities which have identical dimensions.
(a) Impulse and linear momentum
(b) Planck's constant and angular momentum
(c) Moment of inertia and moment of force
(d) Young's modulus and pressure

3. The dimensions of energy per unit volume are the same as those of
(a) work (b) stress
(c) pressure (d) modulus of elasticity

4. When a wave traverses a medium, the displacement of a particle located at x at time t is given by

$$y = a \sin (bt - cx)$$

where a, b and c are constants of the wave. Which of the following are dimensionless quantities?
(a) y/a (b) bt
(c) cx (d) $\frac{b}{c}$

5. In Q.4, the dimensions of b are the same as those of
(a) wave velocity (b) wave frequency
(c) particle frequency (d) wavelength

6. In Q.4, the dimensions of $\frac{b}{c}$ are the same as those of
(a) wave velocity (b) angular frequency
(c) particle velocity (d) wave frequency

7. The Van der Waal equation for n moles of a real gas is

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are Van der Waal constants. The dimensions of
(a) a are the same as those of PV^2
(b) b are the same as those of V
(c) $\frac{a}{V}$ are the same as those of RT
(d) bP are the same as those of RT.

8. In Q.7, which of the following have the same dimensions as those of PV?

(a) nRT (b) $\frac{a}{V}$

(c) Pb (d) $\frac{ab}{V^2}$

9. In Q.7, the dimensions of nRT are the same as those of

(a) pressure (b) energy

(c) work (d) force

10. Which of the following are dimensionless?

(a) Boltzmann constant (b) Planck's constant

(c) Poisson's ratio (d) relative density

11. For a body in uniformly accelerated motion, the distance x of the body from a reference point at time t is given by

$$x = at + bt^2 + c$$

where a, b and c are constants of motion.

(a) The dimensions of c are the same as those of x, at and bt^2.

(b) The dimensional formula of b is $[M^0\ LT^{-2}]$.

(c) $\frac{a}{b}$ is dimensionless.

(d) The acceleration of the body is $2b$.

12. The side of a cube is $L = (1.2 \pm 0.1)$ cm. The volume of the cube is

(a) $(1.728 \pm 0.003)\ cm^3$ (b) $(1.73 \pm 0.02)\ cm^3$

(c) $(1.7 \pm 0.4)\ cm^3$ (d) $(1.7 \pm 0.3)\ cm^3$

13. Two resistances $R_1 = (3.0 \pm 0.1)\ \Omega$ and $R_2 = (6.0 \pm 0.2)\ \Omega$ are to be joined together.

(a) The maximum resistance obtainable is $(9.0 \pm 0.3)\ \Omega$

(b) The maximum resistance obtainable is $(9.0 \pm 0.2)\ \Omega$

(c) The minimum resistance obtainable is $(2.0 \pm 0.3)\ \Omega$

(d) The minimum resistance obtainable is $(2.0 \pm 0.2)\ \Omega$

14. A physical quantity P is given by

$$P = \frac{a^3b^2}{d\sqrt{c}}$$

The percentage errors in the measurements of a, b, c, and d are 1%, 3%, 4%, and 3% respectvely.

(a) The maximum percentage error in P is 14%

(b) The maximum percentage error in P is 10%

(c) The maximum error is contributed by the measurement of b.

(d) The minimum error is contributed by the measurement of c.

15. When a plane wave travels in a meduim, the diplacement y of a particle located at x at time t is given by

$$y = a \sin (bt - cx)$$

where a, b, and c are constants.

(a) The unit a is the same as that of y.

(b) The SI unit of b is Hz.

(c) The dimensional formule of c is $[M^0L^{-1}T^0]$

(d) The dimensions of $\frac{b}{c}$ are the same those of velocity.

ANSWERS AND SOLUTIONS

1. Choices (a), (b) and (c) are units of length

2. The dimensions of moment of inertia are ML^2T^0 and of moment of force are ML^2T^{-2}. All other pairs in (a), (b) and (d) have identical dimensions.

3. Dimensions of energy per unit volume are = dimensions of energy / dimensions of volume = $ML^2T^{-2}/L^3 = ML^{-1}T^{-2}$. Stress, pressure and modulus of elasticity all have the dimensions of $ML^{-1}T^{-2}$. The dimensions of work are ML^2T^{-2}. Hence the correct choices are (b), (c) and (d).

4. Since the sine function is dimensionless, sin ($bt - cx$) is dimensionless. Therefore, y and a must have the same dimensions, i.e. y/a is dimensionless. Since the argument of a sine function (or any trigonometric function) must be dimensionless, bt and cx are also dimensionless. Hence the correct choice are (a), (b) and (c).

5. Since bt is dimensionless, the dimensions of b = dimensions of $1/t = T^{-1}$, which are the dimensions of angular frequency as well as wave frequency. Hence the correct choices are (b) and (d).

6. Dimensions of bt = dimensions of cx. Therefore

Dimensions of $\frac{b}{c}$ = dimensions of $\frac{x}{t} = LT^{-1}$.

Hence the correct choices are (a) and (c).

7. Expanding Van der Waal equation we get

$$PV - Pb + \frac{a}{V} - \frac{ab}{V^2} = nRT$$

From the principle of homogeneity, it follows that all the four choices are correct.

8. The correct choices are (a), (b), (c) and (d).

9. The dimensions of nRT = dimensions of PV

$$= ML^{-1}T^{-2} \times L^3 = ML^2T^{-2}$$

which are dimensions of energy as well as work. Hence the correct choices are (b) and (c).

10. The correct choices are (c) and (d).

11. From the principle of homogeneity of dimensions, the dimensions of c must be the same as those of x *at* and bt^2. Therefore, choice (a) is correct. Also dimension of bt^2 = dimension of x. Hence $[b] = [LT^{-2}]$. Hence choice (b) is also correct. Velocity of the body is

$$v = \frac{dx}{dt} = \frac{d}{dt}[at + bt^2 + c] = a + 2bt$$

and acceleration is

$\frac{dv}{dt} = \frac{d}{dt}(a + 2\,bt) = 2b,$ which is choice (d)

choice (c) is wrong since $\frac{a}{b}$ has dimension of time [T]

12. Volume of cube ($V = L^3$) = 1.2 cm × 1.2 cm × 1.2 cm = 1.728 cm^3. Now L = (1.2 ± 0.1) cm has two significant figures. Hence the value of V must be rounded to two segnificant figures. The correct value of V = 1.7 cm^3. Now

$$V = L^3$$

$$\therefore \quad \frac{\Delta V}{V} = \frac{3\Delta L}{L} = \frac{3 \times 0.1}{1.2} = 0.25$$

$$\therefore \quad \Delta V = 0.25 \times V = 0.25 \times 1.7\ cm^3 = 0.425\ cm^3$$

The error in V is in the first decimal place. Hence the value of ΔV should be rounded off as ΔV = 0.4 cm^3. Thus the correct result is $V \pm \Delta V$ = (1.7 ± 0.4) cm^3, which is choice (c).

13. The maximum value is obtained when the resistances are joined in series. Therefore, the maximum value is

$$R_s = R_1 + R_2 = 3.0 + 6.0 = 9.0\ \Omega$$

Error in $R_s = \Delta R_s = \Delta R_1 + \Delta R_2 = 0.2 + 0.1 = 0.3\ \Omega$

$$\therefore \quad R_s \pm \Delta R_s = (9.0 \pm 0.3)\ \Omega$$

Thus choice (a) is correct and choice (b) is wrong. The minimum value is obtained when the resistances are joined in parallel.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \quad R_p = \frac{R_1R_2}{R_1 + R_2} = \frac{3.0 \times 6.0}{3.0 + 6.0} = 2.0\ \Omega$$

Now $\quad R_p = \frac{R_1R_2}{R_s} \qquad (\because R_1 + R_2 = R_s)$

$$\therefore \quad \frac{\Delta R_p}{R_p} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_s}{R_s}$$

$$= \frac{0.1}{3.0} + \frac{0.2}{6.0} + \frac{0.3}{9.0}$$

$$= 0.033 + 0.033 + 0.033$$

$$= 0.099 \simeq 0.1$$

$$\therefore \quad \Delta R_p = 0.1 \times R_p = 0.1 \times 2\ \Omega = 0.2\ \Omega$$

$\therefore$ Minimum value is $(R_p \pm \Delta R_p) = (2.0 \pm 0.2)\ \Omega$.

Hence choice (c) is wrong and choice (d) is is correct.

14. $$\log P = 3 \log a + 2 \log b - \log d - \frac{1}{2}\log c$$

$$\therefore \left(\frac{\Delta P}{P}\right)_{max} = \frac{3\Delta a}{a} + \frac{2\Delta b}{b} + \frac{\Delta d}{d} + \frac{1}{2}\frac{\Delta c}{c}$$

$$= 3 \times 1\% + 2 \times 3\% + 3\% + \frac{1}{2} \times 4\%$$

$$= 3\% + 6\% + 3\% + 2\% = 14\%$$

Hence the correct choices are (a), (c) and (d).

15. The value of any trignomatric function is a dimensionless number. Hence choice (a) is correct. The argument of a trignometric function is also dimensionless. Hence ($bt - cx$) is dimensionless. Hence b has dimension [T^{-1}] the same as that of frequency and c has dimension of [L^{-1}]. Thus choices (b), (c) and (d) are all correct.

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 and 2 are based on the following passage.

Passage-I

The dimensional method is a very convenient way of finding the dependence of a physical quantity on other physical quantities of a given system. This method has its own limitations. In a complicated situation, it is often not easy to guess the factors on which a physical quantity will depend. Secondly, this method gives no information about the dimensionless proportionality constant. Thirdly, this

method is used only if a physical quantity depends on the product of other physical quantities. Fourthly, this method will not work if a physical quantity depends only on another quantity as a trignometric or exponential function. Finally, this method does not give complete information in cases where a physical quantity depends on more than three quantities in problems in mechanics.

1. The dimensional method cannot be used to obtain denpendence of
   (a) the height to which a liquid rises in a capillary tube on the angle of contact
   (b) speed of sound in an elastic medium on the modulus of elactricity.
   (c) height to which a body, projected upwards with a certain velocity, will rise on time t.
   (d) the decrease in energy of a damped oscillator on time t.

2. In dimensional method, the dimensionless proportionality constant is to be determined
   (a) experimentally
   (b) by a detailed mathematical derivation
   (c) by using the principle of dimensional homogeneity.
   (d) by equating the powers of M, L and T.

Questions 3 to 5 are based on the following passage.

Passage-II

In the study of physics, we often have to measure the physical quantities. The numerical value of a measured quantity can only be approximate as it depends upon the least count of the measuring instrument used. The number of significant figures in any measurement indicates the degree of precision of that measurement. The importance of significant figures lies in calculation. A mathematical calculation cannot increase the precision of a physical measurement. Therefore, the number of significant figures in the sum or product of a group of numbers cannot be greater than the number that has the least number of significant figures. A chain cannot be stronger than its weakest link.

3. A bee of mass 0.000087 kg sits on a flower of mass 0.0123 kg. What is the total mass supported by the stem of the flower upto appropriate significant figures?
   (a) 0.012387 kg (b) 0.01239 kg
   (c) 0.0124 kg (d) 0.012 kg

4. The radius of a uniform wire is $r = 0.021$ cm. The value π is given to be 3.142. What is the area of cross-section of the wire upto appropriate significant figures?
   (a) 0.0014 cm^2 (b) 0.00139 cm^2
   (c) 0.001386 cm^2 (d) 0.0013856 cm^2

5. A man runs 100.5 m in 10.3 s. Find his average speed upto appropriate significant figures.
   (a) 9.71 ms^{-1} (b) 9.708 ms^{-1}
   (c) 9.7087 ms^{-1} (d) 9.70874 ms^{-1}

ANSWERS AND SOLUTIONS

1. The correct choices are (a), (c) and (d). The height of a liquid in a capillary tube depends on $\cos\theta$, where θ is the angle of contact. The height S to which a body rises is given by $S = ut + \frac{1}{2}at^2$, which is a sum of two terms ut and $\frac{1}{2}at^2$. The energy of a damped oscillator decreases exponentially with time.

2. The correct choices are (a) and (b).

3. Total mass = 0.000087 + 0.0123 = 0.012387 kg. The mass of the bee is accurate upto sixth decimal place in kg, whereas the mass of the flower is accurate only upto the fourth decimal place. Hence the sum must be rounded off to the fourth decimal place. Therefore the correct choice is (c).

4. $A = \pi r^2 = 3.142 \times (0.021)^2 = 0.00138562$ cm^2. Now, there are only two significant figures in 0.021 cm. Hence the result must be rounded off to two significant figure as $A = 0.0014$ cm^2, which is choice (a).

5. Average speed $= \frac{100.5\text{ m}}{10.3\text{ s}} = 9.708737$ ms^{-1}

The distance has four significant figures but the time has only three. Hence the result must be rounded off to three signigicant figure to 9.71 ms^{-1}. Thus the correct choice is (a).

SECTION IV

Matrix Match Type

1. Match the physical quantities in column I with their SI units in column II

Column I	***Column II***
(a) Stefan's constant	(p) $JK^{-1}\ mol^{-1}$
(b) Universal gas constant	(q) Fm^{-1}
(c) Electrical permittivity	(r) Hm^{-1}
(d) Magnetic permeability	(s) $Wm^{-2}\ K^{-4}$

ANSWER

1. (a) → (s) (b) → (p)
(c) → (q) (d) → (r)

2. Match the measurements in column I with the number of significant figures in column II.

Column I	***Column II***
(a) 62.028	(p) 3
(b) 0.034	(q) 4
(c) 0.002504	(r) 2
(d) 1.25×1.0^7	(s) 5

ANSWER

2. (a) → (s) (b) → (r)
(c) → (q) (d) → (p)

3. Match the quantities in column I with their order of magnitude given in column II

Column I	***Column II***
(a) 2.6×10^4	(p) 10^5
(b) 3.9×10^4	(q) 10^{-23}
(c) 2.8×10^{-24}	(r) 10^{-24}
(d) 4.2×10^{-24}	(s) 10^4

ANSWER

3. Use the following method to find the order of magnitude. For example, if a quantity is $x = 4.3 \times 10^3$, take its logarithm to the base 10. Log $x = 3.633$ and round if off as log $x = 4$. So the order of magnitnde of x is 10^4.

(a) → (s) (b) → (p)
(c) → (r) (d) → (q)

4. Match the physical quantities in column I with dimensions expressed in mass (M), length (L), time (T), and charge (Q) given in column II.

Column I	***Column II***
(a) Angular momentum	(p) $M\ L^{-2}\ T^{-2}$
(b) Latent heat	(q) $M\ L^2\ Q^{-2}$
(c) Torque	(r) $M\ L^2\ T^{-1}$
(d) Capacitance	(s) $M\ L^3\ T^{-1}\ Q^{-2}$

(e) Inductance
(f) Resistivity
(t) $M^{-1} L^{-2} T^2 Q^2$
(u) $M^0 L^2 T^{-1}$

IIT, 1983

ANSWER

4. (a) → (r)
(b) → (u)
(c) → (p)
(d) → (t)
(e) → (q)
(f) → (s)

SOLUTION

(a) Angular momentum $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times (m\vec{v})$

$\therefore \quad [L] = [L \times M\ L\ T^{-1}] = [M\ L^2\ T^{-1}]$

(b) Latent heat $= \dfrac{Q}{m} = \dfrac{\text{heat energy}}{\text{mass}} \dfrac{[M\ L^2\ T^{-2}]}{[M]} = [M^0\ L^2\ T^{-2}]$

(c) Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$\therefore \quad [\tau] = [L \times M\ L\ T^{-2}] = [M\ L^2\ T^{-2}]$

(d) $Q = CV \quad \Rightarrow \quad C = \dfrac{Q}{V} = \dfrac{Q}{W/Q} = \dfrac{Q^2}{W}$

$\therefore \quad [C] = \dfrac{Q^2}{[M\ L^2\ T^{-2}]} = [M^{-1}\ L^{-2}\ T^2\ Q^2]$

(e) $|V| = \dfrac{LdI}{dt} \longrightarrow \dfrac{W}{Q} = \dfrac{LQ}{T^2}$

$\therefore \quad [L] = \dfrac{WT^2}{Q^2} = \dfrac{[M\ L^2\ T^{-2}] \times [T^2]}{Q^2} = [M\ L^2\ Q^{-2}]$

(f) $\rho = \dfrac{RA}{L} = \dfrac{VA}{IL} = \dfrac{WA}{QIL} = \dfrac{[M\ L^2\ T^{-2} \times L^2]}{\left[Q \times \dfrac{Q}{T} \times L\right]} = M\ L^3\ T^{-1}\ Q^{-2}$

5. Column I gives three physical quantities. Select the appropriate units given in Column II.

Column I	***Column II***
(a) Capacitance	(p) Ohm second
(b) Inductance	(q) $(\text{coulomb})^2\ (\text{joule})^{-1}$
	(r) $\text{coulomb}\ (\text{volt})^{-1}$
(c) Magnetic induction or Magnetic field	(s) $\text{newton}\ (\text{ampere metre})^{-1}$
	(t) $\text{volt second}\ (\text{ampere})^{-1}$

IIT, 1990

SOLUTION

5. (a) From $Q = CV$, unit of $C = \dfrac{\text{unit of } Q}{\text{unit of } V} = \text{coulomb}\ (\text{volt})^{-1}$

From $U = \dfrac{Q^2}{2C}$, unit of $C = (\text{coulomb})^2\ (\text{volt})^{-1}$

(b) From $|e| = L\dfrac{dI}{dt}$, unit of L = volt second (ampere)$^{-1}$ = ohm second

(c) From $F = BIL$, unit of B = newton (ampere metre)$^{-1}$

Hence (a) $\rightarrow$ (q), (r)

(b) $\rightarrow$ (p), (t)

(c) $\rightarrow$ (s)

6. Some physical quantities are given in Column I and some possible SI units in which these quantities may be expressed are given in Column II. Match the physical quantities in Column I with the units in Column II and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	***Column II***
(a) GM_eM_s G – universal gravitational constant, M_e – mass of the earth, M_s – mass of the Sun	(p) (volt) (coulomb) (metre)
(b) $\dfrac{3RT}{M}$ R – universal gas constant, T – absolute temperature, M – molar mass	(q) (kilogram) (metre)^3 (second)$^{-2}$
(c) $\dfrac{F^2}{q^2B^2}$ F – force, q – charge, B – magnetic field	(r) (metre)^2 (second)$^{-2}$
(d) $\dfrac{GM_e}{R_e}$ G – universal gravitational constant M_e – mass of the earth, R_e – radius of the earth	(s) (farad) (volt)^2 (kg)$^{-1}$

IIT, 2007

SOLUTION

6. (a) $F = \dfrac{Gm_1m_2}{r}$. Therefore the SI unit of G is Nm^2 kg^{-2}.

$\therefore$ SI unit of GM_eM_s = (Nm^{2}kg^{-2}) $\times$ kg^2 = Nm^2 = kg ms^{-2} $\times$ m^2 = kg m^3 s^{-2}.

(b) SI unit of $\dfrac{3RT}{M} = \dfrac{\text{SI unit of } PV}{\text{SI unit of } M} = \dfrac{\text{SI unit of work}}{\text{kg}}$ = Nm kg^{-1} = kg ms^{-2} $\times$ m $\times$ kg^{-1} = m^2 s^{-2}

(c) From $\vec{F} = q\,(\vec{v} \times \vec{B})$, we find that the SI unit of $\dfrac{F}{qB}$ = SI unit of v. Hence SI unit of $\dfrac{F^2}{q^2B^2}$ = (ms^{-1})^2 = m^2 s^{-2}

(d) SI unit of $\dfrac{GM_e}{R_e} = \dfrac{\text{Nm}^2\,\text{kg}^{-2} \times \text{kg}}{\text{m}} = \dfrac{\text{kgms}^{-2} \times \text{m}^2 \times \text{kg}^{-2} \times \text{kg}}{\text{m}}$ = m^2 s^{-2}

(p) Since volt $\times$ coulomb = work, SI unit of (volt) (coulomb) (metre) = SI unit of work $\times$ metre = Nm $\times$ m = Nm^2 = kg ms^{-2} $\times$ m^2 = kg m^{3}s^{-2}

(s) Since farad = $\dfrac{\text{coulomb}}{\text{volt}}$ and coulomb $\times$ volt = work, the SI unit of (farad)(volt)^2 (kg^{-1}) = (coulomb) $\times$ (volt) $\times$ kg^{-1} = SI unit of work $\times$ kg^{-1} = kg ms^{-2} $\times$ m $\times$ kg^{-1} = m^{2}s^{-2}

Hence the correct choices are as follows
(a) → (p), (q) (b) → (r), (s) (c) → (r), (s) (d) → (r), (s)

SECTION V

Assertion-Reason Type Questions

In the following questions, **Statement-1(Assertion)** is followed by **Statement-2 (Reason)**. Each question has the following four options out of which only **one** choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1
The order of accuracy of measurement depends on the least count of the measuring instrument.

Statement-2
The smaller the least count the greater is the number of significant figures in the measured value.

2. Statement-1
The dimensional method cannot be used to obtain the dependence of the work done by a force **F** on the angle θ between force **F** and displacement $\boldsymbol{x}$.

Statement-2
All trignometric functions are dimensionless.

3. Statement-1
The mass of an object is 13.2 kg. In this measurement there are 3 significant figures.

Statement-2
The same mass when expressed in grams as 13200 g has five significant figures.

SOLUTIONS

1. The correct choice is (b).

2. Work done is $W = Fx \cos \theta$. since $\cos \theta$ is dimensionless, the dependence of W on θ cannot be determined by the dimensional method. Hence, the correct choice is (a)

3. The correct choice is (c). The degree of accuracy (and hence the number of significant figures) of a measurement cannot be increased by changing the unit.

2 Chapter Motion in One Dimension

REVIEW OF BASIC CONCEPTS

2.1 SCALAR AND VECTOR QUANTITIES

A scalar quantity has only magnitude but no direction, such as distance, speed, mass, area, volume, time, work, energy, power, temperature, specific heat, charge, potential, etc.

A vector quantity has both magnitude and direction, such as displacement, velocity, acceleration, force, momentum, torque, electric field, magnetic field, etc.

2.2 POSITION VECTOR AND DISPLACEMENT VECTOR

The position vector of a particle describes its instantaneous position with respect to the origin of the chosen frame of reference. It is a vector joining the origin to the particle and is denoted by vector $\mathbf{r}$.

For one-dimensional motion (say along x-axis), $\mathbf{r} = x\hat{\mathbf{i}}$, where x is the distance of the particle from origin O.

For two-dimensional motion (say in the x–y plane), $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$, where (x, y) are the x and y coordinates of the particle.

For three-dimensional motion, $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$.

Displacement vector

If $\mathbf{r}_1$ is the position vector of a particle at time t_1, and $\mathbf{r}_2$ at time t_2, then the displacement vector is given by

$$\mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1$$

Vector $\mathbf{s}$ is the resultant of vectors $\mathbf{r}_2$ and $-\mathbf{r}_1$ (Fig. 2.1).

The displacement vector is a vector joining the initial and the final positions of the particle after a given interval of time and its direction is from the initial to the final position.

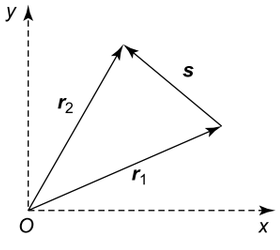

Fig. 2.1

EXAMPLE 2.1

A particle moves from A to B along a circle of radius R. Find the path length and the magnitude of the displacement in terms of R and θ. [see Fig. 2.2]

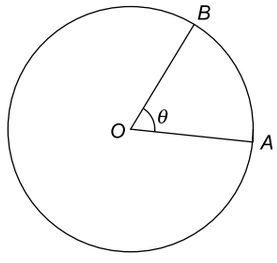

Fig. 2.2

SOLUTION

Path length = arc $AB = R\theta$

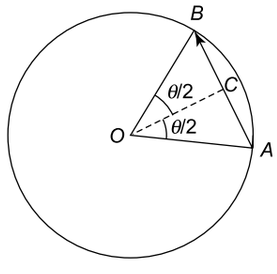

Fig. 2.3

As shown in Fig. 2.3, the magnitude of displacement is

$$s = AB = AC + CB$$
$$= 2\,AC$$
$$\Rightarrow \quad s = 2R \sin(\theta/2)$$

2.3 INSTANTANEOUS VELOCITY AND AVERAGE VELOCITY

The rate of change of displacement with time at a given instant is called instantaneous velocity and is given by

$$v = \frac{dx}{dt}$$

The average velocity in a given time interval is defined as

$$v_{av} = \frac{\text{total displacement}}{\text{time interval}}$$

EXAMPLE 2.2

The position of a particle moving along x-axis is given by $x = 2t - 3t^2 + t^3$, where x is in metre and t in second.

(a) Find the velocity of the particle at $t = 2$ s.

(b) Find the average speed of the particle in the time interval from $t = 2$ s to $t = 4$ s.

SOLUTION

(a) $x = 2t - 3t^2 + t^3$

$$v = \frac{dx}{dt} = 2 - 6t + 3t^2$$

$\therefore$ v at $(t = 2\text{ s}) = 2 - 6 \times 2 + 3 \times (2)^2 = 2\ \text{ms}^{-1}$

(b) Position at $t = 2$ s is

$$x_1 = 2 \times 2 - 3 \times (2)^2 + (2)^3 = 0$$

Position at $t = 4$ s is

$$x_2 = 2 \times 4 - 3 \times (4)^2 + (4)^3 = 24\ \text{m}$$

$\therefore$ Displacement is $x_2 - x_1 = 24 - 0 = 24$ m. Time interval $= 4 - 2 = 2$ s. Therefore,

Average velocity $= \frac{24}{2} = 12\ \text{ms}^{-1}$

2.4 INSTANTANEOUS ACCELERATION

The rate of change of velocity with time at a given instant is called instantaneous acceleration and is given by

$$a = \frac{dv}{dt}$$

2.5 EQUATIONS OF ONE DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

Let x_0 be the position of a particle at time $t = 0$ and let u be its velocity at $t = 0$. It is given a constant acceleration a for time t. As a result it moves in a straight line to a position x and acquires a velocity v.

The particle suffers a displacement $s = x - x_0$ in time t. The equations of motion of the particle are

$$v = u + at \tag{2.1}$$

$$x = x_0 + ut + \frac{1}{2}at^2$$

or

$$s = ut + \frac{1}{2}at^2 \tag{2.2}$$

and $v^2 - u^2 = 2\ a(x - x_0)$

or

$$v^2 - u^2 = 2as \tag{2.3}$$

1. While solving numerical problems of bodies moving in a straight horizontal direction, we will consider only the magnitudes of u, v, a and s and take care of their direction by assigning positive or negative sign to the quantity. For example, $+a$ will mean acceleration $-a$ will mean retardation (or deceleration).
2. In the case of a body falling vertically under gravity or projected vertically upwards, we use the following sign conventions.
   Quantities directed vertically upwards are taken to be positive and those directed vertically downwards are taken to be negative. Since the acceleration due to gravity is directed downward for a body moving vertically up or falling vertically down, we take $a = -g = -9.8\ \text{ms}^{-2}$ in Eqs. (2.1), (2.2) and (2.3).
3. Displacement in the nth second is given by
   s_n = displacement in n seconds – displacement in $(n-1)$ seconds

$$= u_n + \frac{1}{2}a(n)^2 - u(n-1) - \frac{1}{2}a(n-1)^2$$

$$\Rightarrow \quad s_n = u + \frac{a}{2}(2n-1) \tag{2.4}$$

Applications

(i) If a body moving with constant acceleration, starts from A with initial velocity u and reaches B with a velocity v, then the velocity midway between A and B is

$$v' = \sqrt{\frac{u^2 + v^2}{2}}$$

(ii) A body starting from rest has an acceleration a for a time t_1 and comes to rest under a retardation b for a time t_2. If s_1 and s_2 are the distances travelled in t_1 and t_2,

(a) $\frac{s_1}{s_2} = \frac{b}{a} = \frac{t_1}{t_2}$

(b) Total distance travelled $(s_1 + s_2) = \frac{1}{2}\left(\frac{ab}{a+b}\right)T^2$, where $T = t_1 + t_2$.

(c) Maximum velocity attained is $v_{max} = \left(\frac{ab}{a+b}\right)T$

(d) Average velocity over the whole trip is

$$v_{av} = \frac{v_{max}}{2}.$$

(iii) At time $t = 0$ a body is thrown vertically upwards with a velocity u. At time $t = T$, another body is thrown vertically upwards with the same velocity u. The two bodies will meet at time

$$t = \frac{T}{2} + \frac{u}{g}$$

(iv) A body is dropped from rest and at the same time another body is thrown downward with a velocity u from the same point, then
(a) the acceleration of each body is g,
(b) their relative velocity is always u,
(c) their separation will be x after a time $t = \frac{x}{u}$.

(v) From the top of a building, body A is thrown upwards with a certain speed, body B is thrown downwards with the same speed and body C is dropped from rest from the same point. If t_1, t_2 and t_3 are their respective times of reach the ground, then

$$t_3 = \sqrt{t_1 t_2}$$

(vi) A body of mass m is dropped from a height h on a heap of sand. If it penetrates a depth x in the sand.
(a) the average retardation in sand is given by

$$a = \frac{gh}{x}$$

because loss in PE (mgh) = work done against the resistive force of sand (max).
(b) total average force exerted by sand is

$$F = mg + ma = m(g + a)$$

(vii) A body is thrown vertically upward with a velocity u. If the resistive force due to air-friction produces a constant acceleration (or retardation) a ($< g$)
(a) the net acceleration during upward motion $= g + a$,
(b) the net acceleration during downward motion $= g - a$,
(c) the maximum height attained is

$$h = \frac{u^2}{2(g+a)}$$

(d) the time taken to reach the maximum height is

$$t_1 = \sqrt{\frac{2h}{(g+a)}} = \frac{u}{(g+a)}$$

(e) the time of descent is

$$t_2 = \sqrt{\frac{2h}{(g-a)}} = \frac{u}{\left(g^2 - a^2\right)^{1/2}}$$

(f) the speed with which the body hits the ground is

$$v = \sqrt{2(g-a)h} = u\sqrt{\frac{(g-a)}{(g+a)}}$$

NOTE

Note that t_1 is less than t_2.

Some useful tips

(i) If a body, starting from rest, moves with a constant acceleration, the distances covered by it in 1s, 2s, 3s …… are in the ratio $1^2 : 2^2 : 3^2 \ldots = 1 : 4 : 9 \ldots\ldots$
(ii) If a body starts from rest and moves with a contant acceleration, the distance covered by it in the lst, 2nd, 3rd….seconds are in the ratio 1 : 3 : 5…
(iii) A body is projected upward with a certain speed. If air resistance is nelected, the speed with which it hits the ground = the speed of projection.
(iv) If a body is projected upwards with a velocity u, the maximum height attained is proportional to u^2 and the time of ascent is proportional to u.
(v) For a freely falling body,
(a) velocity $\propto$ time
(b) distance fallen $\propto$ (time)^2
(c) velocity $\propto \sqrt{\text{distance fallen}}$

2.6 GRAPHICAL REPRESENTATION

1. *Displacement – time (x – t) graphs (Fig. 2.4)*
Fig. 2.4(a) : Body at rest
Fig. 2.4(b) : Body in uniform motion
Fig. 2.4(c) : Body subjected to acceleration ($a > 0$)
Fig. 2.4(d) : Body subjected to retardation ($a < 0$)
Fig. 2.4(e) : Body accelerating and then decelerating

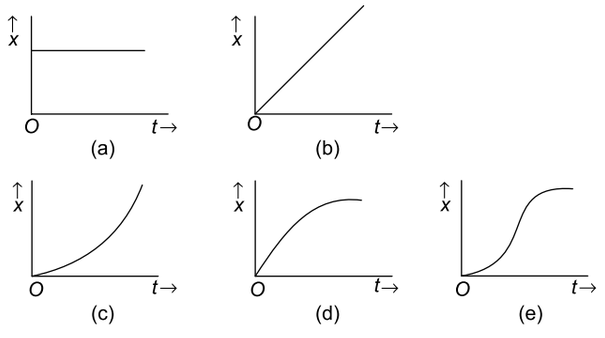

Fig. 2.4

NOTE

The slope of $x - t$ graph gives velocity for uniform motion [Fig. 2.4(b)]. For non-uniform motion [Fig. 2.4(c), (d) and (e)], the slope of the tangent to the curve at a point gives velocity at that instant.

2. Velocity-time ($v - t$) graphs for uniformly accelerated motion (Fig. 2.5)

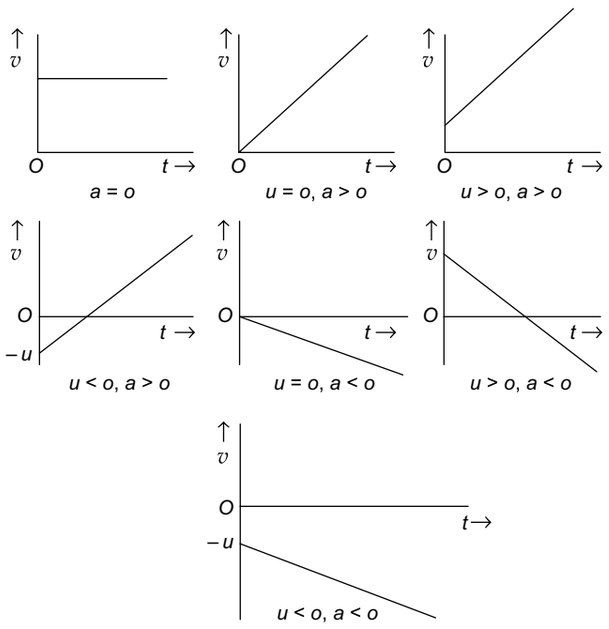

Fig. 2.5

NOTE

Acceleration = slope of ($v - t$) graph
Displacement = area under ($v - t$) graph

3. Acceleration – time ($a - t$) graph [Fig. 2.6]

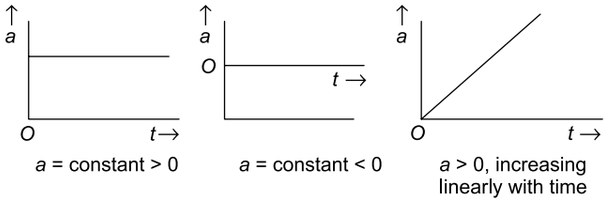

Fig. 2.6

4. ($a - t$), ($v - t$), ($x - t$) graphs for free fall [Fig 2.7]

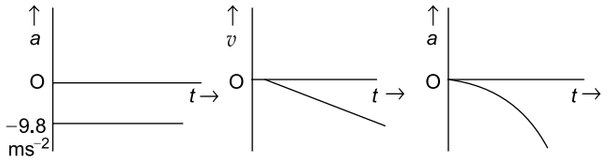

Fig. 2.7

EXAMPLE 2.3

The displacement x (in metres) of a body varies with time t (in seconds) as

$$x = -\frac{2}{3}t^2 + 16t + 2$$

How long does the body take to come to rest?

SOLUTION

$$v = \frac{dx}{dt} = -\frac{4}{3}t + 16$$

$$0 = -\frac{4}{3}t + 16 \quad \Rightarrow \quad t = 12 \text{ s}$$

EXAMPLE 2.4

From the top of a building 40 m tall, a ball a thrown vertically upwards with a velocity of 10 ms $^{-1}$. (a) After how long will the ball hit the ground? (b) After how long will the ball pass through the point from where it was projected? (c) With what velocity will it hit the ground? Take $g = 10$ ms^{-2}.

SOLUTION

(a) $s = -40$ m, $u = +10$ ms^{-1}, $a = -10$ ms^{-2}

Now $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad -40 = 10t + \frac{1}{2} \times (-10)t^2$$

$$\Rightarrow \quad t^2 - 2t - 8 = 0$$

$$\Rightarrow \quad (t + 2)(t - 4) = 0$$

$\Rightarrow$ $t = -2$ s or 4 s. The negative value of t is not possible.

Hence the ball will hit the ground after 4 s.

(b) $s = 0$, $u = +10$ ms^{-1} and $a = -10$ ms^{-2}. Therefore $0 = 10t - 5t^2 \quad \Rightarrow \quad t = 2$ s

(c) $v = u + at = 10 - 10 \times 4 = 10 - 40 = -30$ ms^{-1}.

The negative sign indicates the velocity v is directed downwards.

EXAMPLE 2.5

Figure 2.8 shows the velocity – time graph of a body moving in a straight line. Find (a) the distance travelled by the body in 20 s, (b) the displacement of the body in 20 s and (c) the average velocity in the time interval $t = 0$ to $t = 20$ s.

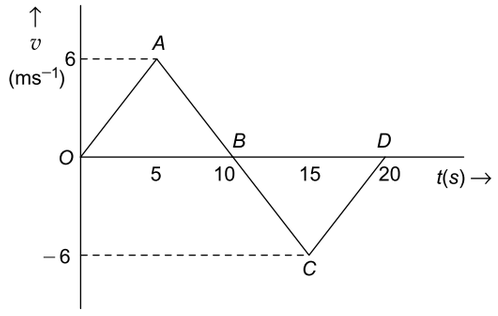

Fig. 2.8

SOLUTION

(*a*) Distance = area under speed – time graph which is shown in Fig. 2.9.

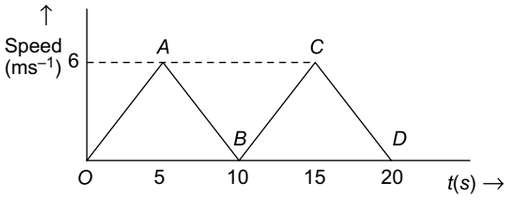

Fig. 2.9

∴ Distance travelled in 20 s = area of *OAB* + area of *BCD* in Fig. 2.9

$$= \frac{1}{2}\times 6\times 10 + \frac{1}{2}\times 6\times 10 = 60 \text{ m}$$

(*b*) Displacement in 20 s = area of *OAB* – area of *BCD* in Fig. 2.8

$$= 30 - 30 = 0$$

(c) Average velocity $= \dfrac{\text{displacement}}{\text{time}} = 0$

EXAMPLE 2.6

A body moving in a straight line covers a distance of 14 m in the 5th second and 20 m in the 8th second. How much distance will it cover in the 15th second?

SOLUTION

$$S_n = u + \frac{a}{2}(2n-1)$$

$$S_5 = u + \frac{9a}{2} \qquad (1)$$

$$S_8 = u + \frac{15a}{2} \qquad (2)$$

Solving (1) and (2), we get $a = 2$ ms^{-2} and $u = 5$ ms^{-1}.

$$\therefore \quad S_{15} = u + \frac{29a}{2} = 5 + \frac{29\times 2}{2} = 34 \text{ m}$$

2.7 RELATIVE VELOCITY IN ONE DIMENSION

If two bodies *A* and *B* are moving in a straight line with velocities v_A and v_B respectively, the relative velocity of *A* with respect to *B* is defined as

$$v_{AB} = v_A - v_B$$

The relative velocity of *B* with respect to *A* will be

$$v_{BA} = v_B - v_A$$

EXAMPLE 2.7

A police van moving on a highway with a speed of 10 ms^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 30 ms^{-1}. If the muzzle speed of the bullet is 140 ms^{-1}, with what speed will the bullet hit the thief's car?

SOLUTION

Speed of police van is $v_V = 10$ ms^{-1}, speed of thief's car is $v_C = 30$ ms^{-1}. Relative velocity of bullet with respect to van is $v_{BV} = 140$ ms^{-1}. Let v_B be the velocity of the bullet. $v_{BV} = v_B - v_V$. Hence $v_B = v_{BV} + v_V = 140 + 10 = 150$ ms^{-1}. The bullet will hit the car with a speed

$$v_{BC} = v_B - v_C = 150 - 30 = 120 \text{ ms}^{-1}.$$

EXAMPLE 2.8

From the top of a tower 60 m tall, a body is thrown vertically down with a velocity of 10 ms^{-1}. At the same time, another body is thrown vertically upward from the ground with a velocity of 20 ms^{-1}. (a) After how long will the two bodies meet? (b) At what height above the ground do they meet? Take $g = 10$ ms^{-2}. [see Fig. 2.10]

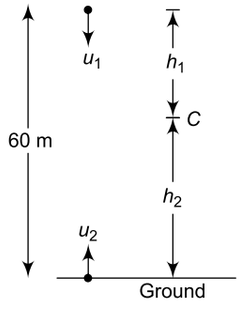

Fig. 2.10

SOLUTION

(*a*) Suppose the bodies meet at *C* and let *t* be the time at which they meet.

For body 1: $s = -h_1$, $u_1 = -10$ ms^{-1}, $a = -10$ ms^{-2}

$$\therefore \quad -h_1 = -10t + \frac{1}{2}\times(-10)t^2$$

Which gives $h_1 = 5t\,(t+2)$ (1)

For body 2: $s = +h_2, u_2 = +20\ \text{ms}^{-1}, a = -10\ \text{ms}^{-2}$

$\therefore \quad h_2 = 20t - 5t^2 = 5t(-t+4) \qquad (2)$

Adding (1) and (2), $h_1 + h_2 = 30t$ or $60 = 30t \Rightarrow t = 2$ s.

(*b*) Using $t = 2$ s in Eq. (2), $h_2 = 5 \times 2\ (4-2) = 20$ m

Alternative method

Relative velocity of body 1 with respect to body 2 is

$$u_{12} = u_1 - u_2 = -10 - (20) = -30\ \text{ms}^{-1}$$

Relative displacement of body 1 with respect of body 2 is

$$s_{12} = -h_1 - (h_2) = -(h_1 + h_2) = -60\ \text{m}$$

Relative acceleration is $a_{12} = a_1 - a_2 = -g - (-g) = 0$. Using

$$s = ut + \frac{1}{2}at^2, \text{ we have } -60 = -30\ t$$

$$\Rightarrow \quad t = 2\ \text{s}.$$

$$h_2 = 20t - 5t^2 = 20 \times 2 - 5 \times (2)^2 = 20\ \text{m}$$

EXAMPLE 2.9

The driver of train *A* moving at a speed of 30 ms^{-1} sights another train *B* moving on the same track at a speed of 10 ms^{-1} in the same direction. He immediately applies brakes and achieves a uniform retardation of 2 ms^{-2}. To avoid collision, what must be the minimum distance between trains *A* and *B* when the driver of *A* sights *B*?

SOLUTION

Relative initial velocity *A* w.r.t. *B* is,

$$u_{AB} = u_A - u_B = 30 - 10 = 20\ \text{ms}^{-1}$$

Relative retardation of *A* w.r.t *B* is

$$a_{AB} = a_A - a_B = -2 - 0 = -2\ \text{ms}^{-2}$$

To avoid collision, the relative final velocity of *A* w.r.t. *B* v_{AB} must be zero. Minimum relative displacement of *A* w.r.t. *B* is s_{AB} which is found by using the relation $v^2 - u^2 = 2as$ which gives

$$0 - (20)^2 = 2 \times (-2)\ s_{AB} \Rightarrow s_{AB} = 100\ \text{m}$$

2.8 SOLVING PROBLEMS INVOLVING NON-UNIFORM ACCELERATION

(a) *Finding velocity and displacement if the dependence of acceleration on time is given.*

Use $a = \frac{dv}{dt}$ which gives $dv = a\,dt$. Substitute the given expression for a in terms of t and integrate both sides.

$$\int_u^v dv = \int_0^t a\,dt$$

and obtain the expression for $v(t)$.

To find displacement x, use $v = \frac{dx}{dt}$ which gives $dx = v(t)\,dt$.

Substitute the expression for v and integrate both sides

$$\int_{x_0}^{x} dx = \int_0^t v\,dt$$

EXAMPLE 2.10

A particle starts from rest at $x = 0$. Its acceleration at time $t = 0$ is 5 ms^{-2} which varies with time as shown in Fig. 2.11. Find (a) the maximum speed of the particle and (b) its displacement in time interval from $t = 0$ to $t = 2$ s.

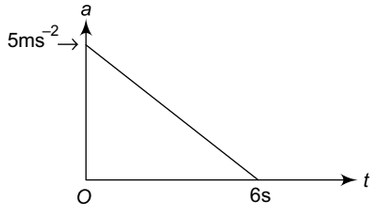

Fig. 2.11

SOLUTION

(*a*) Slope of graph is $m = -\frac{5}{6}\text{ms}^{-2}$ per second.

Intercept $c = 5\ \text{ms}^{-2}$. Using $y = mx + c$, the acceleration a varies with time as

$$a = -\frac{5}{6}t + 5$$

$$\Rightarrow \quad \frac{dv}{dt} = -\frac{5}{6}t + 5$$

Integrating $\int_0^v dv = \int_0^t \left(-\frac{5}{6}t + 5\right) dt$

$$\Rightarrow \quad v = -\frac{5}{12}t^2 + 5t \qquad (1)$$

It follows from graph that deceleration becomes zero at $t = 6$ s. Hence v will be maximum at $t = 6$ s. Using $t = 6$ s in Eq. (1) gives $v_{\max} = 15\ \text{ms}^{-1}$.

(*b*) From $v = \frac{dx}{dt}$, we have $dx = v\,dt$. Integrating

$$\int_0^x dx = \int_0^t v\,dt = \int_0^2 \left(-\frac{5}{12}t^2 + 5t\right) dt$$

$$\Rightarrow \qquad x = \left| -\frac{5}{12}\times\frac{t^3}{3} + 5\times\frac{t^2}{2}\right|_0^2$$

$$= -\frac{5}{12}\times\frac{2^3}{3} + 5\times\frac{2^2}{2} = \frac{80}{9}\text{ m}$$

(c) *Finding velocity and displacement if the dependence of acceleration on displacement is given*

Use $\quad a(x) = \dfrac{dv}{dt} = \dfrac{dx}{dt}\times\dfrac{dv}{dx} = v\dfrac{dv}{dx}$

$$\left(\because v = \frac{dx}{dt}\right)$$

or $\quad v\,dv = a(x)\,dx$

Substitute the given expression of $a(x)$ in terms of x and integrate.

$$\int_v^v v\,dv = \int_{x_0}^{x} a(x)\,dx$$

Hence we get an expression for $v(x)$ in terms of x.

To find displacement, we use $v(x) = \dfrac{dx}{dt}$

$$\Rightarrow \quad \frac{dx}{v(x)} = dt \text{ and integrate}$$

$$\int_{x_0}^{x}\frac{dx}{v(x)} = \int_0^t dt$$

where v is the expression obtained above.

EXAMPLE 2.11

A particle is moving along the x-axis with an acceleration $a = 2x$ where a is in ms^{-2} and x is in metre. If the particle starts from rest at $x = 1$ m, find its velocity when it reaches the position $x = 3$ m.

SOLUTION

$$a = v\frac{dv}{dx} \quad\Rightarrow\quad 2x = v\frac{dv}{dx} \quad\Rightarrow\quad v\,dv = 2x\,dx$$

$$\therefore \quad \int_0^v v\,dv = 2\int_1^3 x\,dx$$

$$\Rightarrow \qquad \frac{v^2}{2} = 2\left|\frac{x^2}{2}\right|_1^3 = 3^2 - 1 = 8$$

which gives $v = 4\ \text{ms}^{-1}$.

(c) *Finding velocity and displacement if the dependence of acceleration on velocity is given*

$$a = \frac{dv}{dt} \quad\Rightarrow\quad \frac{dv}{a} = dt$$

Integrating

$$\int_u^v \frac{dv}{a} = \int_0^t dt$$

Hence we obtain the expression for $v(t)$.

$$v = \frac{dx}{dt} \quad\Rightarrow\quad dx = v\,dt$$

Integrate

$$\int_{x_0}^{x} dx = \int_0^t v\,dt$$

Hence we get an expression for x.

EXAMPLE 2.12

A particle initially (i.e. at $t = 0$) moving with a velocity u is subjected to a retarding force which decelerates it at a rate $a = -k\sqrt{v}$ where v is the instantaneous velocity and k is a positive constant. (a) Find the time taken by the particle to come to rest. (b) Find the distance the particle travels during this time.

SOLUTION

(*a*) $a = \dfrac{dv}{dt} \quad\Rightarrow\quad -k\sqrt{v} = \dfrac{dv}{dt} \quad\Rightarrow\quad \dfrac{dv}{\sqrt{v}} = -k\,dt$

Integrating

$$\int_u^v \frac{dv}{\sqrt{v}} = -k\int_0^t dt$$

$$\Rightarrow\ 2\left(v^{1/2} - u^{1/2}\right) = -kt$$

Putting $v = 0$, we get $t = \dfrac{2\sqrt{u}}{k}$

(b) To find the stopping distance, we use

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\Rightarrow \quad -k\sqrt{v} = v\frac{dv}{dx} \quad\Rightarrow\quad dx = -\frac{\sqrt{v}\,dv}{k}$$

Integrating

$$\int_0^x dx = -\frac{1}{k}\int_u^0 \sqrt{v}\,dv$$

$$\Rightarrow \qquad x = \frac{2u^{3/2}}{3k}$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. A ball is dropped from the top of a tower. In the last second of its fall, the ball covers a distance 9/25 times the height of the tower. If $g = 10\ \text{ms}^{-2}$, the height of the tower is
(a) 75 m (b) 100 m
(c) 125 m (d) 150 m

2. A ball is thrown vertically upwards from the foot of a tower. It crosses the top of the tower twice after an interval of 4 s and reaches the foot of the tower 8 s after it was thrown. What is the height of the tower? Take $g = 10\ \text{ms}^{-2}$.
(a) 60 m (b) 80 m
(c) 100 m (d) 120 m

3. Two cars travelling on a straight road cross a kilometer stone A at the same time with velocities $20\ \text{ms}^{-1}$ and $10\ \text{ms}^{-1}$ with constant accelerations of $1\ \text{ms}^{-2}$ and $2\ \text{ms}^{-2}$ respectively. If they cross another kilometer stone B at the same instant, the distance between A and B is
(a) 600 m (b) 800 m
(c) 1000 m (d) 1200 m

4. The acceleration a of a body moving with initial velocity u changes with distance x as $a = k^2\sqrt{x}$, where k is a positive constant. The distance travelled by the body when its velocity becomes $2u$ is

(a) $\left(\frac{3u}{2k}\right)^{3/4}$ (b) $\left(\frac{3u}{2k}\right)^{4/3}$

(c) $\left(\frac{3u}{2k}\right)^{3/2}$ (d) $\left(\frac{3u}{2k}\right)^{2/3}$

5. A particle is moving along the x-axis. Its instantaneous velocity when it is at a distance x from the origin is $v = \sqrt{p - qx^2}$, where p and q are positive constants. The acceleration of the particle at that instant is

(a) zero (b) $-\frac{2qx}{(p-q)}$

(c) $-\frac{2px}{(p-q)}$ (d) $-qx$

6. The velocity of a particle moving along the x-axis is given by $v = k\sqrt{x}$ where k is a positive constant. The acceleration of the particle is

(a) $\frac{\sqrt{k}}{2}$ (b) $\frac{k}{2}$

(c) $\frac{k^2}{2}$ (d) $2k^2$

7. The velocity of a particle at time t (in second) is related to its displacement x (in metre) as $v = \sqrt{3x + 4}$. The initial velocity of the particle is
(a) $1\ \text{ms}^{-1}$ (b) $2\ \text{ms}^{-1}$
(c) $3\ \text{ms}^{-1}$ (d) $4\ \text{ms}^{-1}$

8. A car, starting from rest, has a constant acceleration of $3\ \text{ms}^{-2}$ for some time and then has a constant retardation of $2\ \text{ms}^{-2}$ for some time and finally comes to rest. The total time taken is 15 s. The maximum velocity of car during its motion is
(a) $12\ \text{ms}^{-1}$ (b) $15\ \text{ms}^{-1}$
(c) $18\ \text{ms}^{-1}$ (d) $21\ \text{ms}^{-1}$

9. A freely falling body, falling from a tower of height h covers a distance $h/2$ in the last second of its motion. The height of the tower is (take $g = 10\ \text{ms}^{-2}$) nearly
(a) 58 m (b) 50 m
(c) 60 m (d) 55 m

10. Ball A is rolled in a straight line with a speed of $5\ \text{ms}^{-1}$ towards a bigger ball B lying 20 m away. After collision with ball B, ball A retraces the path and reaches its starting point with a speed of $4\ \text{ms}^{-1}$. What is the average velocity of ball A during the time interval 0 to 6 s?
(a) zero (b) $2\ \text{ms}^{-1}$
(c) $4\ \text{ms}^{-1}$ (d) $5\ \text{ms}^{-1}$

11. A train is moving southwards at a speed of $30\ \text{ms}^{-1}$. A monkey is running northwards on the roof of the train with a speed of $5\ \text{ms}^{-1}$. What is the velocity of the monkey as observed by a person standing on the ground?
(a) $35\ \text{ms}^{-1}$ in the southward direction
(b) $35\ \text{ms}^{-1}$ in the northward direction
(c) $25\ \text{ms}^{-1}$ in the southward direction
(d) $25\ \text{ms}^{-1}$ in the northword direction

12. A police van moving on a highway with a speed of 36 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 108 km h^{-1}. If the muzzle speed of the bullet is

140 ms^{-1}, with what speed will the bullet hit the thief's car?

(a) 120 ms^{-1} (b) 130 ms^{-1}
(c) 140 ms^{-1} (d) 150 ms^{-1}

13. Car A is moving with a speed of 36 km h^{-1} on a two-lane road. Two cars B and C, each moving with a speed of 54 km h^{-1} in opposite directions on the other lane are approaching car A. At a certain instant when the distance AB = distance AC = 1 km, the driver of car B decides to overtake A before C does. What must be the minimum acceleration of car B so as to avoid an accident?

(a) 1 ms^{-2} (b) 2 ms^{-2}
(c) 3 ms^{-2} (d) 4 ms^{-2}

14. The driver of a train A moving at a speed of 30 ms^{-1} sights another train B moving on the same track towards his train at a speed of 10 ms^{-1}. He immediately applies brakes and achieves a uniform retardation of 4 ms^{-2}. To avoid head-on collision, what must be the minimum distance between the trains?

(a) 100 m (b) 200 m
(c) 300 m (d) 400 m

15. A bullet is fired vertically upwards. After 10 seconds it returns to the point of firing. If g = 10 ms^{-2}, the location of the bullet after 7 seconds from the time of firing will be the same as that after

(a) 2 s (b) 2.5 s
(c) 3 s (d) 3.5 s

16. A body, starting from rest, moves in a straight line with a constant acceleration a for a time interval t during which it travels a distance s_1. It continues to move with the same acceleration for the next time interval t during which it travels a distance s_2. The relation between s_1 and s_2 is

(a) $s_2 = s_1$ (b) $s_2 = 2s_1$
(c) $s_2 = 3s_1$ (d) $s_2 = 4s_1$

17. In Q.16, if v_1 is the velocity of the body at the end of first time interval and v_2 that at the end of the second time interval, the relation between v_1 and v_2 is

(a) $v_2 = v_1$ (b) $v_2 = 2v_1$
(c) $v_2 = 3v_1$ (d) $v_2 = 4v_1$

18. A body dropped from the top of a tower hits the ground after 4 s. How much time does it take to cover the first half of the distance from the top of the tower?

(a) 1 s (b) 2 s
(c) $2\sqrt{2}$ s (d) $\sqrt{3}$ s

19. Figure 2.12 shows the displacement-time *(x-t)* graph of a body moving in a straight line. Which one of the graphs shown in Fig. 2.13 represents the velocity-time (v-t) graph of the motion of the body.

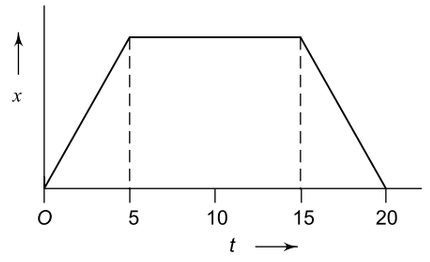

Fig. 2.12

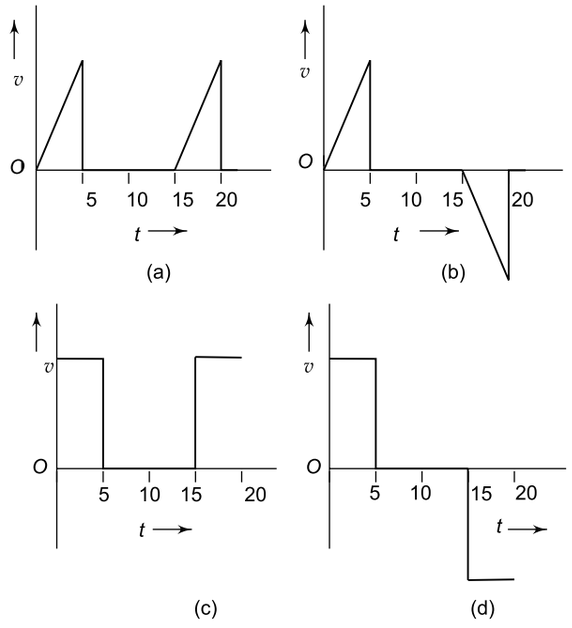

Fig. 2.13

20. A stone is dropped from a height of 125 m. If g = 10 ms^{-2}, what is the ratio of the distances travelled by it during the first and the last second of its motion?

(a) 1 : 9 (b) 2 : 9
(c) 1 : 3 (d) 4 : 9

21. A bullet is fired vertically upwards with an initial velocity of 50 ms^{-1}. If g = 10 ms^{-2}, what is the ratio of the distances travelled by the bullet during the first and the last second of its upward motion?

(a) 9 : 1 (b) 9 : 2
(c) 3 : 1 (d) 9 : 4

22. A body moving in a straight line with constant acceleration of 10 ms^{-2} covers a distance of 40 m

in the 4th second. How much distance will it cover in the 6th second?

(a) 50 m (b) 60 m
(c) 70 m (d) 80 m

23. A body, moving in a straight line with an initial velocity of 5 ms^{-1} and a constant acceleration, covers a distance of 30 m in the 3rd second. How much distance will it cover in the next 2 seconds?

(a) 70 m (b) 80 m
(c) 90 m (d) 100 m

24. A body, moving in a straight line, with an initial velocity u and a constant acceleration a, covers a distance of 40 m in the 4th second and a distance of 60 m in the 6th second. The values of u and a respectively are

(a) 10 ms^{-1}, 5 ms^{-2} (b) 10 ms^{-1}, 10 ms^{-2}
(c) 5 ms^{-1}, 5 ms^{-2} (d) 5 ms^{-1}, 10 ms^{-2}

25. A car, starting from rest, has a constant acceleration a_1 for a time interval t_1 during which it covers a distance s_1. In the next time interval t_2, the car has a constant retardation a_2 and comes to rest after covering a distance s_2 in time t_2. Which of the following relations is correct?

(a) $\frac{a_1}{a_2}=\frac{s_1}{s_2}=\frac{t_1}{t_2}$ (b) $\frac{a_1}{a_2}=\frac{s_2}{s_1}=\frac{t_1}{t_2}$

(c) $\frac{a_1}{a_2}=\frac{s_1}{s_2}=\frac{t_2}{t_1}$ (d) $\frac{a_1}{a_2}=\frac{s_2}{s_1}=\frac{t_2}{t_1}$

26. In Q.25, the average speed of the car during its entire journey is given by

(a) $\frac{1}{2}a_1t_1 = \frac{1}{2}a_2t_2$ (b) $\frac{1}{2}(a_1t_1+a_2t_2)$

(c) $\frac{1}{4}(a_1+a_2)(t_1+t_2)$ (d) zero

27. In Q.25, if the total distance covered by the car is s, the maximum speed attained by it will be

(a) $\left(2s.\frac{a_1a_2}{a_1+a_2}\right)^{\frac{1}{2}}$ (b) $\left(2s.\frac{a_1a_2}{a_1-a_2}\right)^{\frac{1}{2}}$

(c) $\left(\frac{s}{2}.\frac{a_1a_2}{a_1+a_2}\right)^{\frac{1}{2}}$ (d) $\left(\frac{s}{2}.\frac{a_1a_2}{a_1-a_2}\right)^{\frac{1}{2}}$

28. A car, starting from rest, is accelerated at a constant rate α until it attains a speed v. It is then retarded at a constant rate β until it comes to rest. The average speed of the car during its entire journey is

(a) zero (b) $\frac{\alpha v}{2\beta}$
(c) $\frac{\beta v}{2\alpha}$ (d) $\frac{v}{2}$

29. The displacement of a body from a reference point, is given by

$$\sqrt{x} = 2t+3$$

where x is in metres and t in seconds. This shows that the body is

(a) at rest (b) accelerated
(c) decelerated (d) in uniform motion

30. In Q.29, what is the initial velocity of the body?

(a) 2 ms^{-1} (b) 3 ms^{-1}
(c) 6 ms^{-1} (d) 12 ms^{-1}

31. In Q.29, what is the acceleration of the body?

(a) 2 ms^{-2} (b) 3 ms^{-2}
(c) 6 ms^{-2} (d) 8 ms^{-2}

32. A car, starting from rest, at a constant acceleration covers a distance s_1 in a time interval t. It covers a distance of s_2 in the next time interval t at the same acceleration. Which of the following relations is true?

(a) $s_2 = s_1$ (b) $s_2 = 2s_1$
(c) $s_2 = 3s_1$ (d) $s_2 = 4s_1$

33. A car moving at a speed v is stopped in a certain distance when the brakes produce a deceleration a. If the speed of the car was nv, what must be the deceleration of the car to stop it in the same distance and in the same time?

(a) $\sqrt{n}\,a$ (b) na
(c) n^2a (d) n^3a

34. Two balls are dropped from the same point after an interval of 1 second. What will be their separation 3 seconds after the release of the second ball? Take $g = 10$ ms^{-2}.

(a) 25 m (b) 30 m
(c) 35 m (d) 40 m

35. A bullet is fired vertically upwards with an initial velocity of 50 ms^{-1}. It covers a distance h_1 during the first second and a distance h_2 during the last 3 seconds of its upward motion. If $g = 10$ ms^{-2}, h_1 and h_2 will be related as

(a) $h_1 = 3h_2$ (b) $h_1 = 2h_2$
(c) $h_1 = h_2$ (d) $h_1 = \frac{h_2}{3}$

36. A ball is thrown vertically downward with a velocity u from the top of a tower. It strikes the ground with a velocity $3u$. The time taken by the ball to reach the ground is given by

(a) $\frac{u}{g}$ (b) $\frac{2u}{g}$
(c) $\frac{3u}{g}$ (d) $\frac{4u}{g}$

37. In Q.36, the height of the tower is given by

(a) $\frac{u^2}{g}$ (b) $\frac{2u^2}{g}$

(c) $\frac{3u^2}{g}$ (d) $\frac{4u^2}{g}$

38. A 150 m long train having a constant acceleration crosses a 300 m long platform. It enters the platform at a speed of 40 ms^{-1} and leaves it at a speed of 50 ms^{-1}. What is the acceleration of the train?

(a) 0.6 ms^{-2} (b) 0.8 ms^{-2}
(c) 1.0 ms^{-2} (d) 1.2 ms^{-2}

39. In Q.38, how long will the train take to cross the platform?

(a) 6 s (b) 8 s
(c) 10 s (d) 12 s

40. The motion of a body is given by the equation

$$\frac{dV(t)}{dt} = 6.0 - 3V(t)$$

where $V(t)$ is the speed (in ms^{-1}) at time t (in second), If the body was at rest at $t = 0$, the magnitude of the initial acceleration is

(a) 3 ms^{-2} (b) 6 ms^{-2}
(c) 9 ms^{-2} (d) zero

41. In Q.40, the speed of the body varies with time as

(a) $V(t) = (1 - e^{-3t})$

(b) $V(t) = 2\ (1 - e^{-3t})$

(c) $V(t) = \frac{2}{3}\left(1 - e^{\frac{-3t}{2}}\right)$

(d) $V(t) = \frac{3}{2}\left(1 - e^{\frac{-2t}{3}}\right)$

42. In Q.40, the speed of the body when the acceleration is half the initial value is

(a) 1 ms^{-1} (b) 2 ms^{-1}
(c) 3 ms^{-1} (d) 4 ms^{-1}

43. Two stones are thrown up simultaneously with initial speeds of u_1 and u_2 ($u_2 > u_1$). They hit the ground after 6 s and 10 s respectively. Which graph in Fig. 2.14 correctly represents the time variation of $\Delta x = (x_2 - x_1)$, the relative position of the second stone with respect to the first upto $t = 10$ s? Assume that the stones do not rebound after hitting the ground.

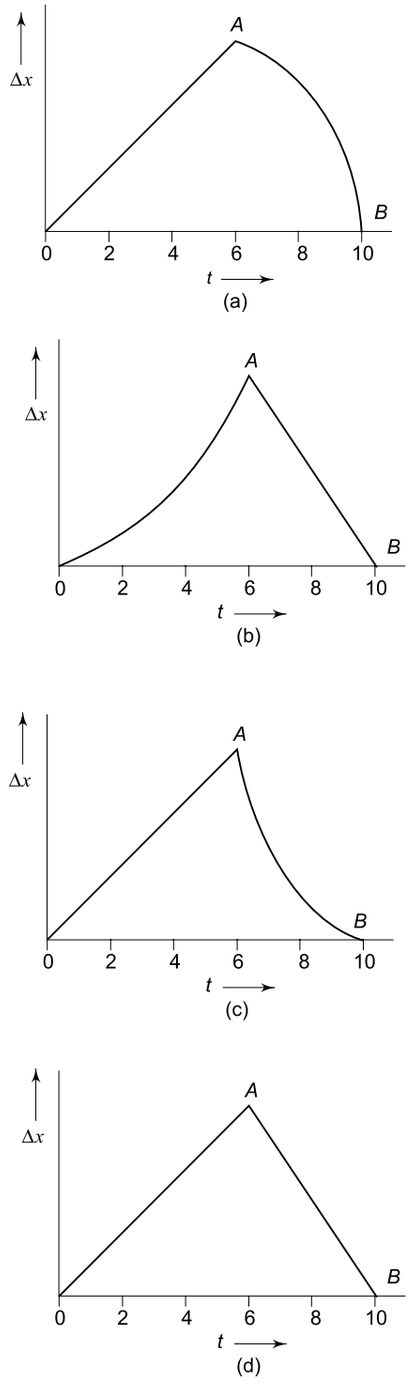

Fig. 2.14

44. A body starts from rest at time $t = 0$ and undergoes an acceleration as shown in Fig. 2.15.

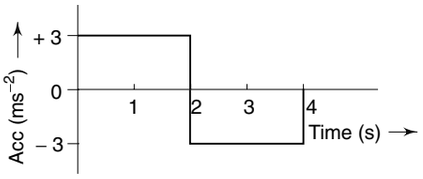

Fig. 2.15

Which of the graphs shown in Fig. 2.16 represents the velocity-time (v–t) graph of the motion of the body from $t = 0$ s to $t = 4$ s?

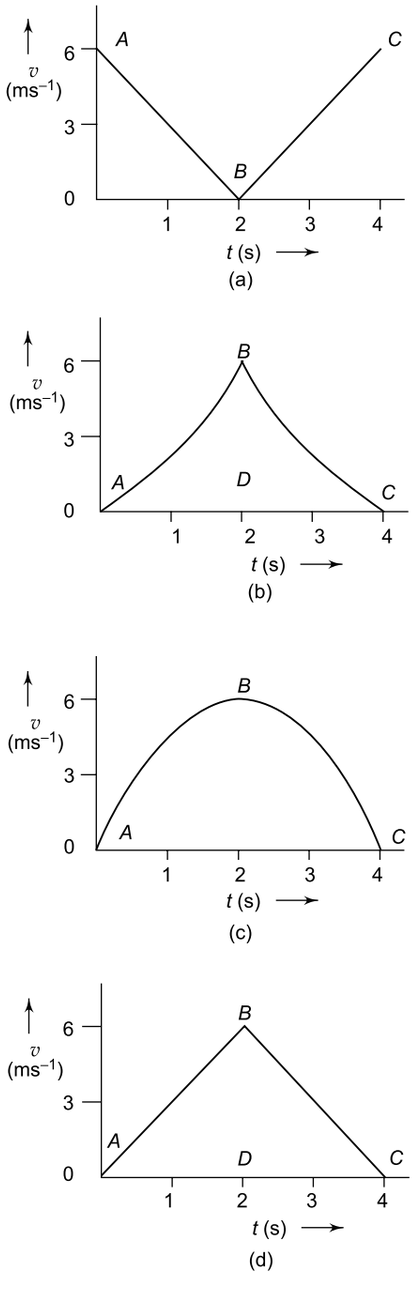

Fig. 2.16

45. In Q.44 above, what is the velocity of the body at time $t = 2.5$ s?
(a) 2.5 ms^{-1} (b) 3.5 ms^{-1}
(c) 4.5 ms^{-1} (d) 5.5 ms^{-1}

46. In Q.44, how much distance does the body cover from $t = 0$ to $t = 4$ s?
(a) 6 m (b) 9 m
(c) 12 m (d) 15 m

47. In Q.44, which of the graphs shown in Fig. 2.17 represents the displacement-time (x – t) graph of the motion of the body from $t = 0$ s to $t = 4$ s?

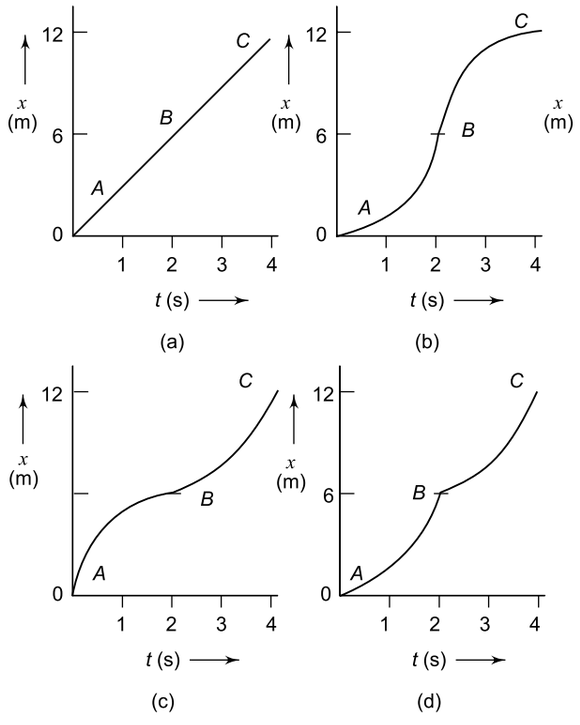

Fig. 2.17

48. A body, moving in a straight line, covers half the distance with a speed V, the remaining part of the distance was covered with a speed V' for half the time and with a speed V'' for the other half of the time. What is the average speed of the body?

(a) $\dfrac{2V(V'+V'')}{(2V+V'+V'')}$ (b) $\dfrac{V(V'+V'')}{(2V+V'+V'')}$

(c) $\dfrac{2V'V''}{(V+V'+V'')}$ (d) $\dfrac{V'V''}{(V+V'+V'')}$

49. A particle moving in a straight line covers half the distance with a speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speeds of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is

(a) 4.0 m/s (b) 5.0 m/s
(c) 5.5 m/s (d) 4.8 m/s

IIT, 1992

50. Figure 2.18 shows the velocity–time ($v - t$) graphs for one dimensional motion. But only some of these can be realized in practice. These are
(a) (i), (ii) and (iv) only
(b) (i), (ii) and (iii) only
(c) (ii) and (iv) only
(d) all

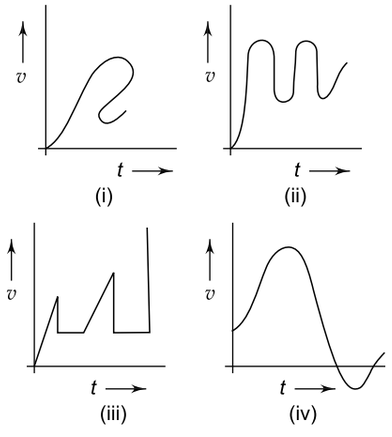

Fig. 2.18

51. A stone dropped from a building of height h reaches the ground after t seconds. From the same building if two stones are thrown (one upwards and the other downwards) with the same velocity u and they reach the ground after t_1 and t_2 seconds respectively, then the time interval t is

(a) $t = t_1 - t_2$ (b) $t = \dfrac{t_1 + t_2}{2}$

(c) $t = \sqrt{t_1 t_2}$ (d) $t = \sqrt{t_1^2 - t_2^2}$

52. Displacement (x) of a particle is related to time (t) as $x = at + bt^2 - ct^3$
where a, b and c are constants of motion. The velocity of the particle when its acceleration is zero is given by

(a) $a + \dfrac{b^2}{c}$ (b) $a + \dfrac{b^2}{2c}$

(c) $a + \dfrac{b^2}{3c}$ (d) $a + \dfrac{b^2}{4c}$

IIT, 1988

53. A ball is dropped vertically from a height h above the ground. It hits the ground and bounces up vertically to a height $h/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h as (see Fig. 2.19)

IIT, 2000

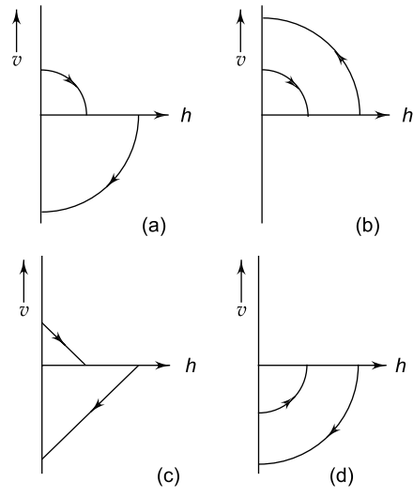

Fig. 2.19

54. A car, starting from rest, accelerates at a constant rate of 5 ms^{-2} for some time. It then retards at a constant rate of 10 ms^{-2} and finally comes to rest. If the total taken is 6 s, what is the maximum speed attained by the car?
(a) 5 ms^{-1} (b) 10 ms^{-1}
(c) 20 ms^{-1} (d) 40 ms^{-1}

55. In Q.54, what is the total distance travelled by the car is 6 s?
(a) 60 m (b) 80 m
(c) 100 m (d) 120 m

56. The distance x covered by a body moving in a straight line in time t is given by the relation

$$2x^2 + 3x = t$$

If v is the velocity of the body at a certain instant of time, its acceleration will be
(a) $-v^3$ (b) $-2v^3$
(c) $-3v^3$ (d) $-4v^3$

57. The distance x covered by a body moving in a straight line in time t is given by

$$x^2 = t^2 + 2t + 3$$

The acceleration of the body will vary as

(a) $\dfrac{1}{x}$ (b) $\dfrac{1}{x^2}$

(c) $\dfrac{1}{x^3}$ (d) $\dfrac{1}{x^4}$

58. A body is thrown vertically up with a velocity u. It passes three points A, B and C in its upward journey

with velocities $\frac{u}{2}$, $\frac{u}{3}$ and $\frac{u}{4}$ respectively. The ratio $\frac{AB}{BC}$ is

(a) $\frac{20}{7}$ (b) 2

(c) $\frac{10}{7}$ (d) 1

59. A body is thrown vertically up with a velocity u. It passes a point at a height h above the ground at time t_1 while going up and at time t_2 while falling down. Then the relation between u, t_1 and t_2 is

(a) $t_1 + t_2 = \frac{2u}{g}$ (b) $t_2 - t_1 = \frac{2u}{g}$

(c) $t_1 + t_2 = \frac{u}{g}$ (d) $t_2 - t_1 = \frac{u}{g}$

60. In Q. 59 above, the relation between t_1, t_2 and h is

(a) $t_1 t_2 = \frac{2h}{g}$ (b) $t_1 t_2 = \frac{h}{g}$

(c) $(t_1 + t_2)^2 = \frac{2h}{g}$ (d) $(t_1 + t_2)^2 = \frac{h}{g}$

61. A body dropped from a height H above the ground strikes an inclined plane at a height h above the ground. As a result of the impact, the velocity of the body becomes horizontal. The body will take the maximum time to reach the ground if

(a) $h = \frac{H}{4}$ (b) $h = \frac{H}{2\sqrt{2}}$

(c) $h = \frac{H}{2}$ (d) $h = \frac{H}{\sqrt{2}}$

62. A body of density ρ enters a tank of water of density ρ' after falling through a height h. The maximum depth to which it sinks in water is

(a) $\frac{h\rho'}{(\rho - \rho')}$ (b) $\frac{h\rho}{(\rho - \rho')}$

(c) $\frac{h\rho}{\rho'}$ (d) $\frac{h\rho'}{\rho}$

63. A body, falling freely under gravity, covers half the total distance in the last second of its fall. If it falls for n seconds, then the value of n is

(a) 2 (b) 3

(c) $2 - \sqrt{2}$ (d) $2 + \sqrt{2}$

64. In 1.0 s, a particle goes from point A to point B, moving in a semicircle of radius 1.0 m as shown in Fig. 2.20. The magnitude of the average velocity of the particle is

(a) 3.14 ms^{-1}
(b) 2.0 ms^{-1}
(c) 1.0 ms^{-1}
(d) zero

IIT, 1999

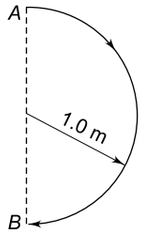

Fig. 2.20

65. A body of mass m_1, projected vertically upwards with an initial velocity u reaches a maximum height h. Another body of mass m_2 is projected along an inclined plane making an angle of 30° with the horizontal and with speed u. The maximum distance travelled along the incline is

(a) $2h$ (b) h

(c) $\frac{h}{2}$ (d) $\frac{h}{4}$

66. The displacement x of a particle moving in one dimension is related to time t by the equation

$$t = \sqrt{x} + 3$$

where x is in metres and t in seconds. The displacement of the particle when its velocity is zero is

(a) zero (b) 4 m (c) 1 m (d) 0.5 m

67. A particle initially (i.e, at $t = 0$) moving with a velocity u is subjected to a retarding force, as a result of which it decelerates at a rate

$$a = -k\sqrt{v}$$

where v is the instantaneous velocity and k is a positive constant. The time T taken by the particle to come to rest is given by

(a) $T = \frac{2\sqrt{u}}{k}$ (b) $T = \frac{2u}{k}$

(c) $T = \frac{2u^{3/2}}{k}$ (d) $T = \frac{2u^2}{k}$

68. A particle starts from rest. Its acceleration at time $t = 0$ is 5 ms^{-2} which varies with time as shown in Fig. 2.21. The maximum speed of the particle will be

(a) 7.5 ms^{-1} (b) 15 ms^{-1}
(c) 30 ms^{-1} (d) 2.5 ms^{-1}

IIT, 2004

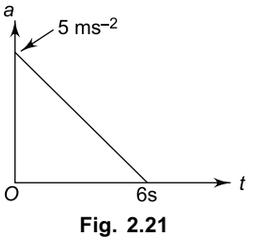

Fig. 2.21

69. Figure 2.22 shows the variation of velocity (v) of a body with position (x) from the origin O. Which of the graphs shown in Fig. 2.23 correctly represents the variation of the acceleration (a) with position (x)?

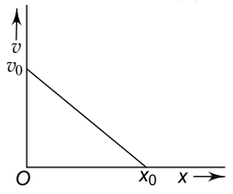

Fig. 2.22

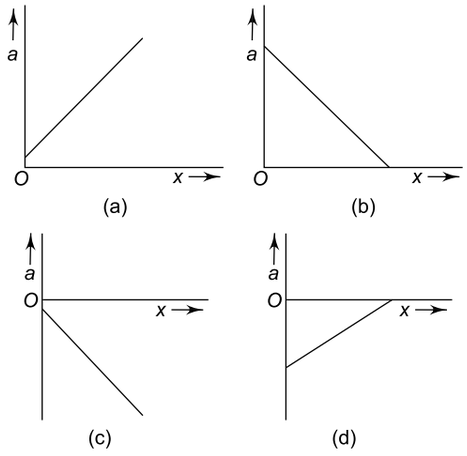

Fig. 2.23

IIT, 2005

70. The velocity (v) of a body moving along the postive x-direction varies with displacement (x) from the origin as $v = k\sqrt{x}$, where k is a constant. Which of the graphas shown in Fig. 2.24 correctly represents the displacement-time ($x - t$) graph of the motion?

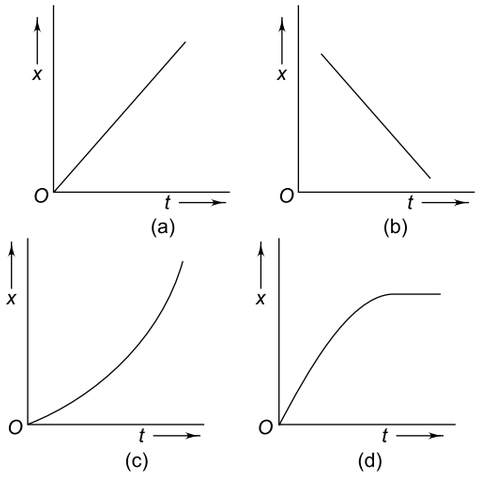

Fig. 2.24

IIT, 2006

71. Figure 2.25 shows the acceleration – time ($a - t$) graph of a body moving in a straight line. Which graph in Fig. 2.26 shows the velocity – time ($v - t$) of the motion of the body? Assume that $x = 0$ and $v = 0$ at $t = 0$.

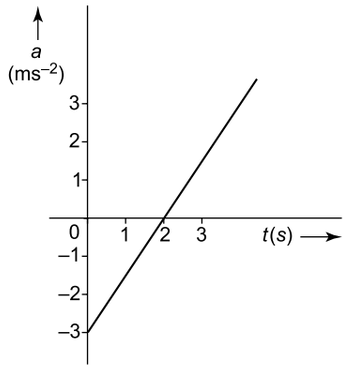

Fig. 2.25

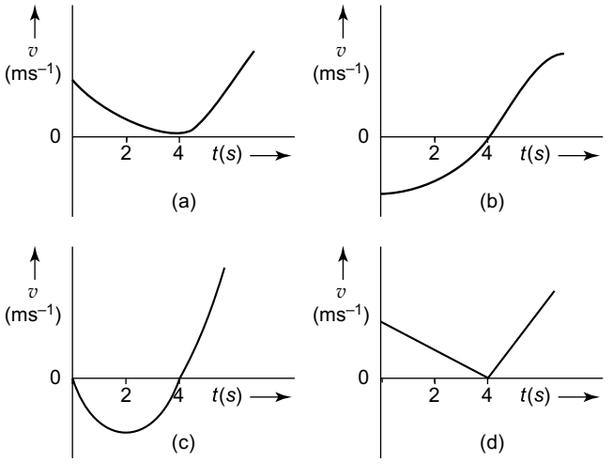

Fig. 2.26

72. In Q. 71, the displacement of the body in the interval $t = 2$s to $t = 4$s is

(a) 2 m (b) – 2 m

(c) 4 m (d) – 4 m

73. A particle of mass m moving with initial velocity u enters a medium at time $t = 0$. The medium offers a resistive force $F = kv$ wher k is a constant of the medium and v is the instantaneous velocity. The velocity of the particle varies with time t as

(a) $v = u + \frac{kt}{m}$ (b) $v = u - \frac{kt}{m}$

(c) $v = u\,e^{-kt/m}$ (d) $v = u\,e^{kt/m}$

74. In Q.73, the position x of the particle varies with time t as (assume that $x = 0$ at $t = 0$)

(a) $x = \dfrac{mut}{k}$

(b) $x = \dfrac{mu}{k}\left(t - \dfrac{1}{2}t^2\right)$

(c) $x = \dfrac{mu}{k}\left(1 - e^{-kt/m}\right)$

(d) $x = \dfrac{mu}{2k}\left(1 - e^{-kt/m}\right)$

75. In Q. 73, the velocity of the particle becomes $\dfrac{u}{2}$ at time t given by

(a) $t = \dfrac{m}{k}\ln(2)$ (b) $t = \dfrac{m}{2k}\ln(2)$

(c) $t = \dfrac{m}{k}$ (d) $\dfrac{m}{2k}$

ANSWERS

1. (c)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (c)
7. (b)	**8.** (c)	**9.** (a)	**10.** (b)	**11.** (c)	**12.** (a)
13. (a)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (c)
19. (d)	**20.** (a)	**21.** (a)	**22.** (b)	**23.** (c)	**24.** (d)
25. (d)	**26.** (a)	**27.** (a)	**28.** (d)	**29.** (b)	**30.** (d)
31. (d)	**32.** (c)	**33.** (c)	**34.** (c)	**35.** (c)	**36.** (b)
37. (d)	**38.** (c)	**39.** (c)	**40.** (b)	**41.** (b)	**42.** (a)
43. (a)	**44.** (d)	**45.** (c)	**46.** (c)	**47.** (b)	**48.** (a)
49. (d)	**50.** (c)	**51.** (c)	**52.** (c)	**53.** (a)	**54.** (c)
55. (a)	**56.** (d)	**57.** (c)	**58.** (a)	**59.** (a)	**60.** (a)
61. (c)	**62.** (a)	**63.** (d)	**64.** (b)	**65.** (a)	**66.** (a)
67. (a)	**68.** (b)	**69.** (d)	**70.** (c)	**71.** (c)	**72.** (d)
73. (c)	**74.** (c)	**75.** (a)			

SOLUTIONS

1. Let h be the height of the tower and t be the time taken by the ball to hit the ground. Then

$$h = \frac{1}{2}gt^2 \qquad (1)$$

and $\quad h' = \dfrac{1}{2}g(t-1)^2$

Now $h' = h - \dfrac{9h}{25} = \dfrac{16h}{25}$. Hence

$$\frac{16h}{25} = \frac{1}{2}g(t-1)^2 \qquad (2)$$

From Eqs. (1) and (2), we get $t = 5$ s. Therefore

$$h = \frac{1}{2} \times 10 \times (5)^2 = 125 \text{ m}$$

2. Let $h = AB$ be the height of the tower and P be the highest point reached (Fig. 2.27).The time taken by the ball to go from B to $P = 4/2 = 2$ s and the time taken to go from A to $P = 8/2 = 4$ s. Therefore, time taken by the ball to go from A to B is $t = 4 - 2 = 2$ s.

If u is the velocity of projection, then

$$0 = u - 10 \times 4 \Rightarrow u = 40 \text{ ms}^{-1}$$

$$\therefore \quad h = ut + \frac{1}{2}gt^2$$

$$= 40 \times 2 + \frac{1}{2}(-10)(2)^2$$

$$= 60 \text{ m}$$

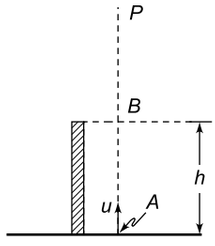

Fig. 2.27

3. Let s be the distance between A and B. Then, for the first car

$$s = 20t + \frac{1}{2} \times 1 \times t^2 = 20t + \frac{1}{2}t^2 \qquad (1)$$

For the second car,

$$s = 10t + \frac{1}{2} \times (2) \times t^2 = 10t + t^2 \qquad (2)$$

Equating (1) and (2), we get $t = 20$ s. Using this value of t in either (1) or (2) gives $s = 600$ m.

4. $a = \dfrac{dv}{dt} = \dfrac{dv}{dx} \cdot \dfrac{dx}{dt} = \dfrac{dv}{dx} \cdot v$

$\Rightarrow \quad v\,dv = a\,dx = k^2 \sqrt{x}\;dx$

$\therefore \quad \int_u^{2u} v\,dv = k^2 \int \sqrt{x}\;dx$

$\Rightarrow \quad x = \left(\dfrac{3u}{2k}\right)^{4/3}$, which is choice (b).

5. $a = \dfrac{dv}{dt} = \dfrac{dv}{dx} \cdot \dfrac{dx}{dt} = v\dfrac{dv}{dx}$

$\dfrac{dv}{dx} = \dfrac{d}{dx}\left[(p - qx^2)^{1/2}\right]$

$= \dfrac{1}{2}(p - qx^2)^{-1/2} \times (-2qx)$

$= \dfrac{-qx}{v} \qquad [\because\ v = (p - qx^2)^{1/2}]$

Hence $a = -qx$, which is choice (d).

6. The acceleration is

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx}$$

Given $v = k\sqrt{x}$. Therefore

$$a = k\sqrt{x}\,\frac{d}{dx}(k\sqrt{x})$$

$$= k\sqrt{x} \times \frac{k}{2\sqrt{x}} = \frac{k^2}{2}$$

So the correct choice is (c).

7. Given $v^2 = 3x + 4$. Comparing with $v^2 = u^2 + 2ax$, we have $u^2 = 4$ which gives $u = 2$ ms^{-1}.

8. Let t_1 be the time during which the car accelerates. The velocity at the end of t_1 is

$$v = u + at_1 = 0 + 3t_1 = 3t_1$$

The time during which the car decelerates is $t_2 = (t - t_1)$ where t is the total time taken for the car to come to rest. During time t_2, the initial velocity is $v = 3t_1$ and the final velocity is zero. Hence

$$0 = 3t_1 - 2t_2$$
$$= 3t_1 - 2(t - t_1)$$
$$= 5t_1 - 2t$$

$\Rightarrow \quad t_1 = \dfrac{2}{5}t = \dfrac{2}{5} \times 15 = 6$ s.

The car attains the maximum velocity at the end of t_1 after which it decelerates. Hence

$v_{max} = 3t_1 = 3 \times 6 = 18$ ms^{-1}, which is choice (c).

9. We have $\dfrac{h}{2} = \dfrac{1}{2}\,g\,(2n - 1)$ so that $n = \dfrac{1}{2}\left(\dfrac{h}{g} + 1\right)$.

Hence h is given by

$$h = \frac{1}{2}\,gn^2 = \frac{1}{2}\,g\left\{\frac{1}{4}\left(\frac{h}{g} + 1\right)^2\right\}$$
$$= \frac{1}{8}\,g\left(\frac{h^2}{g^2} + \frac{2h}{g} + 1\right) = \frac{1}{8}\left(\frac{h^2}{g} + 2h + g\right)$$

Simplifying and putting $g = 10$ ms^{-2}, we get

$$h^2 - 60h + 100 = 0$$

The positive root of this quadratic gives $h \simeq 58$ m. Hence the correct choice is (a).

10. The time taken by ball A to reach ball $B = \dfrac{20}{5} = 4$ s.

During the time interval 0 to 6 s, ball A covers a distance of 20 m upto ball B (which takes 4 s) and in the next 2 s, it covers a distance of 4 ms^{-1} × 2 s = 8 m in the opposite direction.

$\therefore$ Net displacement = 20 m – 8 m = 12 m

$\therefore$ Average velocity = $\dfrac{12\,\text{m}}{6\,\text{s}}$ = 2 ms^{-1} which is choice (b).

11. Suppose we choose the direction from south to north as the positive direction. Then the velocity of the train moving southwards = – 30 ms^{-1}. Velocity of the monkey running northwards = + 5 ms^{-1}. Therefore, the velocity of the monkey as observed by a person in the ground = – 30 + 5 = – 25 ms^{-1}. The negative sign indicates that the direction of this velocity is southwards. Hence the correct choice is (c).

12. Speed of the police van = 36 km h^{-1} = 10 ms^{-1}. Since the gun is in motion with the van and the bullet is fired in the direction in which the van is moving, the net speed of the bullet = speed of the gun (i.e. van) + the muzzle speed of the bullet = 10 + 140 = 150 ms^{-1}. Now, the speed of the thief's car = 108 km h^{-1} = 30 ms^{-1}. The bullet is chasing the thief's car with a speed of 150 ms^{-1} and the thief's car is speeding away at 30 ms^{-1}. Hence the bullet will hit the car with a speed which is the relative speed of the bullet with respect to the car = 150 – 30 = 120 ms^{-1}. Thus the correct choice is (a).

13. Let us suppose that cars A and B are moving in the positive x-direction.Then car C is moving in the negative x-direction. Therefore, v_A = + 36 kmh^{-1} = + 10 ms^{-1}, v_B = + 54 km h^{-1} = + 15 ms^{-1} and v_C = – 54 km h^{-1} = – 15ms^{-1}. The relative velocity B with respect to A is $v_{BA} = v_B - v_A$ =15 – 10 = 5 ms^{-1}. The relative velocity of C with respect to A is v_{CA} = $v_C - v_A$ = –15 – 10 = – 25 ms^{-1}. At time $t = 0$, the distance between A and B = distance between A and C = 1 km = 1000 m. The car C will cover a distance

AC = 1000 m and just reach car A at a time t given by

$$t = \frac{AC}{|v_{CA}|} = \frac{1000 \text{ m}}{25 \text{ ms}^{-1}} = 40 \text{ s}$$

Car B will overtake car A just before car C does and avoid an accident, if it acquires a minimum acceleration a such that it covers a distance $s = AB$ = 1000 m in time t = 40 s, travelling at a relative speed $u = v_{BA} = 5 \text{ ms}^{-1}$. Putting these values in relation

$$s = ut + \frac{1}{2} at^2$$

We get $\quad 1000 = 5 \times 40 + \frac{1}{2} \times a \times (40)^2$

which gives $a = 1 \text{ ms}^{-2}$ which in choice (a).

14. The relative speed of train A with respect to train B = 30 + 10 = 40 ms^{-1}. The minimum distance now is given by

$$0 - (40)^2 = 2 \times 4 \times s$$

which gives s = 200 m which is choice (b).

15. Since the bullet returns to its point of projection, its net displacement is zero.The bullet takes 5 s to reach the maximum height. Therefore, initial speed (u) of the bullet is ($\because$ final velocity = 0)
$u = gt = 10 \times 5 = 50 \text{ ms}^{-1}$ directed upwards. The maximum height (h) attained by the bullet is $h = \frac{1}{2} gt^2 = \frac{1}{2} \times 10 \times (5)^2 = 125$ m. Since the total time taken by the bullet to return to the point of firing is 10 s, it takes 5 s to attain the maximum height. In the next 2 seconds, the bullet falls a distance of $s_1 = \frac{1}{2} gt^2 = \frac{1}{2} \times 10 \times (2)^2 = 20$ m. Also the maximum height attained = 125 m. The location of the bullet after 7 s will be the same as that after t seconds, where t is the time taken by the bullet to rise to a height h = 125 – 20 = 105 m. This value of t is given by

$$h = ut - \frac{1}{2} gt^2 \text{ or } 105 = 50t - \frac{1}{2} \times 10 \times t^2$$

or $\quad t^2 - 10t + 21 = 0 \quad$ or $\quad (t - 3)(t - 7) = 0$

which gives t = 3 s or 7 s. Thus the correct choice is (c).

16. Since the initial velocity is zero, the distance travelled in the first time interval t is

$$s_1 = 0 + \frac{1}{2} at^2 = \frac{1}{2} at^2$$

The velocity of the body at the end of this time interval is $v = 0 + at = at$. This is the initial velocity for the next time interval t during which the body travels a distance.

$$s_2 = ut + \frac{1}{2} at^2 = at^2 + \frac{1}{2} at^2 = \frac{3}{2} at^2 \ (\because u = at)$$

$\therefore \quad s_2 = 3 s_1$. Thus the correct choice is (c).

17. Here $v_1 = 0 + at = at$ and $v_2 = v_1 + at = at + at = 2at$. Therefore, $v_2 = 2v_1$. Hence the correct choice is (b).

18. Let h be the height of the tower. Then $h = \frac{1}{2} gt^2 = \frac{1}{2} g(4)^2 = 8g$.

The time t taken to fall through $\frac{h}{2} = 4g$ is given by $\frac{h}{2} = \frac{1}{2} gt^2$ or $4g = \frac{1}{2} gt^2$ or $t^2 = 8$ or $t = 2\sqrt{2}$ s.

Thus the correct choice is (c).

19. It follows from Fig. 2.12 that from 0 to 5 s, displacement x increases linearly with time t. Therefore, velocity v is a positive constant between t = 0 and t = 5 s. Between t = 5 s and t = 15 s, displacement remains constant. Therefore, velocity (v) is zero between t = 5 s and t = 15 s. Between t = 15 s and t = 20 s, displacement (x) decreases linearly with time (t). Therefore, velocity (v) is constant but negative between t = 15 s and t = 20 s. Hence, the correct choice is graph (d) in Fig. 2.13.

20. Since the initial velocity of the stone is zero, the total time taken by the stone to hit the ground is given by

$$h = \frac{1}{2} gt^2 \text{ or } t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 125}{10}} = 5 \text{ s}$$

During the first second, the stone falls a distance h_1 given by

$$h_1 = \frac{1}{2} g(1)^2 = \frac{g}{2} = 5 \text{ m}$$

During the first four seconds, the stone falls a distance h given by

$$h = \frac{1}{2} g(4)^2 = 8g = 80 \text{ m}$$

$\therefore$ Distance h_2 through which the stone falls in the last (i.e. fifth) second

$$= 125 - 80 = 45 \text{ m}.$$

Now $h_1/h_2 = 5/45 = 1/9$. Hence the correct choice is (a).

21. The maximum height h attained by the bullet is given by

$$v^2 - u^2 = -2gh \text{ or } h = \frac{u^2}{2g} \quad (\because v = 0)$$

or $h = \dfrac{50 \times 50}{2 \times 10} = 125$ m. The total time taken by the stone to attain this height is given by $t = \dfrac{u}{g} = \dfrac{50}{10} = 5$ s. During the first second ($t = 1$ s), the stone covers a distance h_1 given by

$$h_1 = ut - \frac{1}{2} gt^2 = 50 \times 1 - \frac{1}{2} \times 10 \times (1)^2 = 45 \text{ m}$$

During the first four seconds ($t = 4s$), the stone covers a height h given by

$$h = 50 \times 4 - \frac{1}{2} \times 10 \times (4)^2 = 120 \text{ m}$$

$\therefore$ Distance travelled by the stone during the last (i.e. fifth) second of its upward motion is $h_2 = 125 - 120 = 5$ m. Hence $h_1/h_2 = 45/5 = 9$. Hence the correct choice is (a).

22. The distance covered in the nth second is given by

$$s_n = u + a\left(n - \frac{1}{2}\right)$$

$$\therefore \quad s_4 = u + a\left(4 - \frac{1}{2}\right) = u + \frac{7}{2}a$$

or $\quad 40 = u + \dfrac{7}{2} \times 10$ or $u = 40 - 35 = 5$ ms^{-1}

$$\therefore \quad s_6 = 5 + 10 \times \left(6 - \frac{1}{2}\right)$$ which gives $s_6 = 60$ m.

Thus the correct choice is (b).

23. Now, $s_n = u + a\left(n - \dfrac{1}{2}\right)$. Therefore,

$$30 = 5 + a\left(3 - \frac{1}{2}\right)$$ which gives $a = 10$ ms^{-2}.

$$\therefore \quad s_4 = 5 + 10\left(4 - \frac{1}{2}\right) = 40 \text{ m}$$

and $\quad s_5 = 5 + 10\left(5 - \dfrac{1}{2}\right) = 50$ m

$\therefore \quad s = s_4 + s_5 = 40 + 50 = 90$ m. Thus the correct choice is (c).

24. Given $40 = u + \dfrac{7a}{2}$ and $60 = u + \dfrac{11a}{2}$

These equations give $u = 5$ ms^{-1} and $a = 10$ ms^{-2}. Thus the correct choice is (d).

25. Since the initial velocity of the car is zero, its velocity at the end of the first time interval t_1 is $v = 0 + a_1 t_1 = a_1 t_1$. This is the initial velocity for the next time interval t_2. Since the final velocity is zero, we have, from $v = u + at$,

$$0 = a_1 t_1 - a_2 t_2 \qquad (\because u = a_1 t_1)$$

Now, the distance covered in the first time interval t_1 is given by

$$2a_1 s_1 = v^2 - u^2 = a_1^2 t_1^2 \quad (\because v = a_1 t_1 \text{ and } u = 0)$$

or $\quad s_1 = \dfrac{1}{2} a_1 t_1^2$ (i)

The distance covered in the next time interval t_2 is given by

$$-2a_2 s_2 = 0 - a_1^2 t_1^2 \quad (\because v = 0 \text{ and } u = a_1 t_1 \text{ now})$$

or $\quad s_2 = \dfrac{1}{2} \dfrac{a_1^2}{a_2} t_1^2 = \dfrac{1}{2} \dfrac{a_2^2 t_2^2}{a_2} \quad (\because a_1 t_1 = a_2 t_2)$

or $\quad s_2 = \dfrac{1}{2} a_2 t_2^2$ (ii)

From (i) and (ii) we get $\dfrac{s_1}{s_2} = \dfrac{a_1 t_1^2}{a_2 t_2^2} = \dfrac{a_2}{a_1} \dfrac{a_1^2 t_1^2}{a_2^2 t_2^2}$

$$= \frac{a_2}{a_1} \quad (\because a_1 t_1 = a_2 t_2)$$

Thus we have $\dfrac{s_2}{s_1} = \dfrac{a_1}{a_2} = \dfrac{t_2}{t_1}$ which is choice (d).

26. Average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{s_1 + s_2}{t_1 + t_2}$

As shown above, $s_1 + s_2 = \dfrac{1}{2} a_1 t_1^2 + \dfrac{1}{2} a_2 t_2^2$

$$= \frac{1}{2} a_1 t_1 (t_1 + t_2) \quad (\because a_1 t_1 = a_2 t_2)$$

$$\therefore \quad \text{Average speed} = \frac{\frac{1}{2} a_1 t_1 (t_1 + t_2)}{(t_1 + t_2)}$$

$$= \frac{1}{2} a_1 t_1 = \frac{1}{2} a_2 t_2$$

Hence the correct choice is (a).

27. The maximum speed v attained by the car = speed it attains at the end of time interval t_1 during which it is accelerated. As shown above, this speed is $v = a_1 t_1 = a_2 t_2$.

Now $s_1 = \dfrac{1}{2} a_1 t_1^2 = \dfrac{v^2}{2a_1} \quad (\because v = a_1 t_1)$

and $\quad s_2 = \dfrac{1}{2} a_2 t_2^2 = \dfrac{v^2}{2a_2} \quad (\because v = a_2 t_2)$

$$\therefore \quad s = s_1 + s_2 = \frac{v^2}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)$$

or $\quad v = \left[2s \cdot \dfrac{a_1 a_2}{a_1 + a_2}\right]^{1/2}$

Hence the correct choice is (a).

28. The distance s_1 covered by the car during the time it is accelerated is given by $2\alpha s_1 = v^2$, which gives $s_1 = v^2/2\alpha$. The distance s_2 covered during the time the car is decelerated is, similarly given by $s_2 = v^2/2\beta$. Therefore, the total distance covered is

$$s = s_1 + s_2 = \frac{v^2}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) \quad \text{(i)}$$

If t_1 is the time of acceleration and t_2 that of deceleration, then $v = \alpha t_1 = \beta t_2$ or $t_1 = v/\alpha$ and $t_2 = v/\beta$. Therefore, the total time taken is

$$t = t_1 + t_2 = v\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) \quad \text{(ii)}$$

From (i) and (ii), the average speed of the car is given by

$$\frac{\text{total distance}}{\text{total time}} = \frac{s}{t} = \frac{v}{2}$$

Hence the correct choice is (d).

29. Squaring both sides, we have

$$x = 12t + 4t^2 + 9$$

Since displacement x changes with time t, the body cannot be at rest. The velocity of the body is given by

$$v = \frac{dx}{dt} = 12 + 8t$$

Since the velocity v changes with time t, the body is not in uniform motion; it is accelerated because v increases with t. Hence the correct choice is (b).

30. We have seen that $v = 12 + 8t$. Comparing it with $v = u + at$ we find that $u = 12$ ms^{-1}. Hence the correct choice is (d).

31. Now $v = 12 + 8t$. Comparing it with $v = u + at$, we find that $a = 8$ ms^{-2}. Alternatively, acceleration a is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(12 + 8t) = 8$$

or $a = 8$ ms^{-2}. Hence the correct choice is (d).

32. Since the initial velocity is zero, the velocity at the end of the first time interval t is $v = at$. The distance covered during this time interval is $s_1 = \frac{1}{2}at^2$. Velocity $v = at$ is the initial velocity for the next time interval t. Therefore, the distance travelled in the next time interval t is

$$s_2 = at^2 + \frac{1}{2}at^2 = \frac{3}{2}at^2$$

Thus $s_2 = 3\,s_1$. Hence the correct choice is (c).

33. The distance over which the car can be stopped is given by $2as = v^2$ or $a = v^2/2s$. If v becomes nv, the value a^* of a to stop the car in the same distance is $a^* = (nv)^2/2s = n^2v^2/2s$. Thus $a^* = n^2a$. Hence the correct choice is (c).

34. The first ball falls for $t_1 = 4$ s. During this time it falls a distance $h_1 = \frac{1}{2}gt_1^2 = \frac{1}{2} \times 10 \times (4)^2 = 80$ m. The second ball falls for $t_2 = 3$ s. During this time it falls a distance $h_2 = \frac{1}{2}gt_2^2 = \frac{1}{2} \times 10 \times (3)^2 = 45$ m. Their separation $h_1 - h_2 = 80 - 45 = 35$ m. Hence the correct choice is (c).

35. The total time taken by the bullet to reach the highest point (where its velocity becomes zero) is given by $0 = u - gt$ or $t = u/g = 50/10 = 5$ s. The distance it covers in the first 1 second of its upward motion is $h_1 = ut - \frac{1}{2}gt^2 = 50 \times 1 - \frac{1}{2} \times 10 \times (1)^2 = 50 - 5 = 45$ m.

Now, the velocity of the bullet at the end of first 2 seconds is $v = u - gt = 50 - 10 \times 2 = 30$ ms^{-1}. This is the initial velocity for the last 3 seconds of upward motion. The distance covered in the last 3 seconds is $h_2 = 30 \times 3 - \frac{1}{2} \times 10 \times (3)^2 = 90 - 45 = 45$ m. Thus $h_1 = h_2$, which is choice (c).

36. From $v = u + gt$, we have $3u = u + gt$ or $2u = gt$ or $t = 2u/g$. Hence the correct choice is (b).

37. From $2gh = v^2 - u^2$, we have $2gh = (3u)^2 - u^2 = 8u^2$ or $h = 4u^2/g$. Hence the correct choice is (d).

38. The total distance covered in order to cross the platform is s = length of train + length of platform = 150 + 300 = 450 m. For this distance the initial speed is $u = 40$ ms^{-1} and the final speed is $v = 50$ ms^{-1}. Now, from $v^2 - u^2 = 2as$, we have $(50)^2 - (40)^2 = 2a \times 450$ which gives $a = 1.0$ ms^{-2} which is choice (c).

39. We have seen above that $a = 1.0$ ms^{-2}. Now, using the relation, $v = u + at$, we have $50 = 40 + 1 \times t$ or $t = 10$ s. Thus the correct choice is (c).

40. Given $\frac{dV(t)}{dt} = 6.0 - 3V(t)$ (i)

Since at $t = 0$, $V(0) = 0$ (given), the acceleration at $t = 0$ is

$$a(0) = \left.\frac{dV(t)}{dt}\right|_{t=0} = 6.0 - 0 = 6.0 \text{ ms}^{-2}$$

Thus the correct choice is (b).

41. Differentiating $V(t)$ with respect to t given in each choice, we find that choice (b) gives the given expression for $dV(t)/dt$.

$$\frac{dV(t)}{dt} = \frac{d}{dt}\left[2\left(1-e^{-3t}\right)\right] = 2\frac{d}{dt}(1 - e^{-3t}) = 6e^{-3t}$$

$$= 6 - 6\,(1 - e^{-3t}) = 6 - 3V(t)$$

$$\therefore\ V(t) = 2\,(1 - e^{-3t}) \qquad \text{(ii)}$$

42. The acceleration will be half the initial value, i.e. it will be equal to 6.0/2 = 3.0 ms^{-1} at time t^* given by [use Eq. (i)]

$$3.0 = 6.0 - 3V(t^*) \text{ or } V(t^*) = 1$$

or $\quad 2(1 - e^{-3t^*}) = 1$ or $e^{-3t^*} = 0.5$

or $\quad -3t^* \ln e^{-1} = \ln (0.5)$ or $-3t^* = -0.693$

or $\quad t^* = 0.231$ s

Putting this value of t in Eq. (ii) we have

$$V \text{ at } t = 0.231 \text{ s} = 2\,(1 - e^{-3 \times 0.231})$$

$$= 2(1 - e^{-0.693}) = 2(1 - 0.5) = 1.0 \text{ ms}^{-1}$$

Which is choice (a).

43. Up to the first 6 seconds, the positions of the two stones are given by

$$x_1 = u_1 t - \frac{1}{2}\, gt^2 \qquad (\because g \text{ is negative})$$

and $\quad x_2 = u_2 t - \frac{1}{2}\, gt^2$

Subtracting we get the separation between the two stones:

$$\Delta x = x_2 - x_1 = (u_2 - u_1)t$$

Since $(x_2 - x_1)$ varies linearly with t, the graph is a straight line upto t = 6 s.

At t = 6 s, the first stone hits the ground and stops there. Hence at 6 s, x_1 = 0; so we have for time between 6 s and 10 s.

$$\Delta x = x_2 - x_1 = u_2 t - \frac{1}{2}\, gt^2$$

Thus $(x_2 - x_1)$ versus t graph is not linear; it is a curve.

Now at t = 10 s, Δx = 0, therefore

$$0 = 10u_2 - \frac{1}{2} \times g \times (10)^2$$

or $\quad u_2 = 5g = 50 \text{ ms}^{-1}$

Hence $\Delta x = 50t - 5t^2$. Graph (a) represents this time variation.

44. Since the body is at rest at t = 0, u = 0. Now from t = 0 to t = 2 s, acceleration a = + 3 ms^{-2}. Therefore, during this time interval (i.e. from t = 0 to t = 2 s), the velocity of the body is $v = 0 + at$ or $v = 3t$. Thus at t = 1s, v = 3 ms^{-1} and at t = 2 s, v = 6 ms^{-1}. Hence from t = 0 to t = 2 s, the velocity–time graph is linear with a positive slope = + 3. Portion AB of graph (d) represents this.

For the next time interval from t = 2 s to t = 4 s, the acceleration a = – 3 ms^{-2}. For this time interval, the initial velocity u = velocity at the end of the first time interval = 6 ms^{-1}. Therefore, velocity in the time interval from t = 2 s to t = 4 s is given by

$$v = u + at = 6 - 3t$$

Therefore, velocity at t = 3 s (i.e. 1 s after t = 2 s) = 6 – 3 × 1 = 3 ms^{-1} and velocity at t = 4 s (i.e. 2 s after t = 2 s) = 6 – 3 × 2 = 0. The slope of the velocity time graph from t = 2 s to t = 4 s is negative = – 3.

The graph is again linear but its slope is negative = – 3. This is represented by the portion BC in Fig. 2.9(d). Hence the correct choice is (d).

45. As shown above, the velocity of the body in time interval t = 2 s to t = 4 s is given by $v = 6 - 3t$.

At t = 2.5 s (i.e. 0.5 s after 2.0 s), $v = 6 - 3 \times 0.5$ = 4.5 ms^{-1}. Hence the correct choice is (c).

46. The distance covered = area under the velocity time graph from t = 0 to t = 4 s = area of triangle ABC in Fig. 2.16 (d)

= 2 × area of triangle ABD

$$= 2 \times \frac{1}{2}\, BD \times AD = BD \times AD = 6 \text{ ms}^{-1} \times 2 \text{ s} = 12 \text{ m}$$

Hence the correct choice is (c).

47. Since the motion of the body is accelerated, the (x–t) graph of the motion cannot be linear; it is curved. Hence graph (a) in Fig. 2.17 is wrong. Between t = 0 s and t = 2 s, the motion is accelerated, i.e. the velocity is increasing with time. Therefore, from t = 0 s to t = 2 s, the slope of the graph must increase with t. Portion AB in Fig. 2.17 (d) is incorrect because the slope is decreasing with time. Hence graph (d) is also incorrect. From t = 2 s to t = 4 s, the motion of the body is decelerated. Therefore, the slope of the graph must decrease with time. In portion BC of graph (c), the slope is increasing with time. Thus graph (c) is also wrong. The correct choice is graph (b).

48. Let the total distance be 2 s and let t_1 and t_2 be the times taken to traverse the first half and the second half of the distance respectively. Then $t_1 = s/V$. For the second half of the distance, the distance s_1 covered in time $\frac{t_2}{2}$ with speed V' is $s_1 = V' \frac{t_2}{2}$ and the distance s_2 covered in time $\frac{t_2}{2}$ with speed V' is $s_2 = V'' \frac{t_2}{2}$, so that

$$s = s_1 + s_2 = V' \frac{t_2}{2} + V'' \frac{t_2}{2}$$

$$= (V' + V'')\, \frac{t_2}{2}, \text{ which gives}$$

$$t_2 = \frac{2s}{(V'+V'')}$$

$\therefore$ Total time taken $= t + t_2 = \frac{s}{V} + \frac{2s}{V'+V''}$

$$= \frac{s(2V+V'+V'')}{V(V'+V'')}$$

Hence, average speed $= \frac{\text{total distance}}{\text{total time}}$

$$= \frac{2s \times V(V'+V'')}{s(2V+V'+V'')} = \frac{2V(V'+V'')}{(2V+V'+V'')}$$

Thus, the correct choice is (a).

49. Let the total distance be S. The time taken to travel the first half, i.e. $\frac{S}{2}$ is

$$t_1 = \frac{S/2}{3} = \frac{S}{6}$$

Let t_2 be the time taken to cover a distance S_1 with speed 4.5 m/s and t_3 that to cover distance S_2 with speed 7.5 m/s.

Then $S_1 = 4.5\, t_2$ and $S_2 = 7.5\, t_3$

Now $S_1 + S_2 = \frac{S}{2}$ and $t_2 = t_3$ (given).

Therefore $\frac{S}{2} = S_1 + S_2 = (4.5 + 7.5)t_2$ or

$$t_2 = \frac{S}{24}.$$

$\therefore$ Total time taken $t = t_1 + t_2 = \frac{S}{6} + \frac{S}{24} = \frac{5S}{24}$

$\therefore$ Average speed $= \frac{\text{total distance}}{\text{total time}} = \frac{S}{5S/24}$

$$= \frac{24}{5} = 4.8 \text{ m/s}$$

Hence the correct choice is (d).

50. Motion corresponding to graph (*i*) cannot be realised because if we draw a line parallel to the v-axis, the body will have two different velocities at a given value of t, which is not possible. Graph (*iii*) is also not possible because at some values of t, the graph is parallel to the v-axis which correspond to infinite acceleration. Motions corresponding to graphs (*ii*) and (*iv*) are possible in nature. Hence the correct choice is (c).

51. For the stone dropped with zero initial velocity, we have

$$h = \frac{1}{2} gt^2 \qquad \text{(i)}$$

For the stone thrown upwards with velocity u, we have

$$h = -ut_1 + \frac{1}{2} g t_1^2 \qquad \text{(ii)}$$

For the stone thrown downwards with velocity u, we have

$$h = ut_2 + \frac{1}{2} gt^2 \qquad \text{(iii)}$$

Notice that the displacement of the stone in the three case is the same, equal to h. Using (i) in (ii) and (iii) we have

$$\frac{1}{2} gt^2 = -ut_1 + \frac{1}{2} g t_1^2 \Rightarrow ut_1 = \frac{1}{2} g t_1^2 - \frac{1}{2} gt^2$$

and $\frac{1}{2} gt^2 = ut_2 + \frac{1}{2} g t_2^2 \Rightarrow ut_2 = \frac{1}{2} gt^2 - \frac{1}{2} g t_2^2$

Dividing the two equations, we have

$$\frac{t_1}{t_2} = \frac{t_1^2 - t^2}{t^2 - t_2^2}$$

which gives $t = \sqrt{t_1 t_2}$. Hence the correct choice is (c).

52. Velocity is $v = \frac{dx}{dt} = a + 2bt - 3ct^2$ (i)

Acceleration is $a = \frac{d^2x}{dt^2} = 2b - 6ct$

Acceleration is zero at time t given by $0 = 2b - 6ct$ or $t = \frac{b}{3c}$. Putting this value of t in Eq. (i).

We have $v = a + 2b \cdot \frac{b}{3c} - 3c \times \frac{b^2}{9c^2} = a + \frac{b^2}{3c}$

Thus, the correct choice is (c).

53. The velocity at a height h is given by $v^2 = u^2 + 2gh$. For downward motion, $u = 0$ and the value of g is negative and h becomes more and more negative. Hence v^2 increases with h. Since the velocity vector is directed downwards, v becomes more and more negative. Since $v^2 \propto h$, the graph of v versus h is parabolic. Hence graphs (c) and (d) are wrong. For upward motion, $v^2 = u^2 + 2gh$. Here g is directed downwards and h is positive. Consequently, v^2 decreases with h. Since the direction of the velocity vector is positive, v becomes less and less positive. Here also the variation of v with h is parabolic. Since v becomes less and less positive, graph (b) is not correct. Hence the correct choice is (a).

54. Let t_1 be the time during which the car has an acceleration $a_1 = 5\ \text{ms}^{-2}$ and t_2 be the time during which the car has a deceleration $a_2 = -10\ \text{ms}^{-2}$. Since the car starts from rest, the maximum speed attained by the car $v = 0 + a_1 t_1 = a_1 t_1$. For time interval, t_2 this v is the initial speed and the final speed is zero.

Therefore, $0 = v - a_2 t_2$ or $v = a_2\, t_2$. Thus $a_1\, t_1 = a_2\, t_2$ or $t_1/t_2 = a_2/a_1 = 10/5 = 2$ or $t_1 = 2t_2$. Using $t_1 + t_2 = 6$, we get $t_1 = 4$ s and $t_2 = 2$ s.

55. The distance moved in $t_1 = 4$s is

$$s_1 = 0 \times t_1 + \frac{1}{2} a_1 t_1^2$$

The distance moved in $t_2 = 2$s is

$$s_2 = vt_2 - \frac{1}{2}\, a_2 t_2^2$$

56. Differentiating $2x^2 + 3x = t$ with respect to t we have

$$4x \frac{dx}{dt} + \frac{3dx}{dt} = 1 \qquad \text{(i)}$$

Now $\frac{dx}{dt} = v$. Therefore, $4xv + 3v = 1$ or $4x + 3 = 1/v$.

Differentiating Eq. (i) with respect to time t, we have

$$4\left(\frac{dx}{dt}\right)^2 + 4x \frac{d^2x}{dt^2} + 3\frac{d^2x}{dt^2} = 0$$

or $\quad 4v^2 + 4xa + 3a = 0$

or $\quad a = -\dfrac{4v^2}{4x+3} \qquad \text{(ii)}$

where $a = \dfrac{d^2x}{dt^2}$ is the acceleration. But $4x + 3 = 1/v$.

Using this in Eq. (ii) we get $a = -4v^3$.

57. Given, $x^2 = t^2 + 2t + 3$. Differentiating, we have

$$2x \frac{dx}{dt} = 2t + 2 \quad \text{or} \quad x \frac{dx}{dt} = t + 1 \qquad \text{(i)}$$

Differentiating again, we have

$$\left(\frac{dx}{dt}\right)^2 + x \frac{d^2x}{dt^2} = 1.$$

or $\quad x^2\left(\dfrac{dx}{dt}\right)^2 + x^3 a = x^2 \qquad \text{(ii)}$

Where $a = \dfrac{d^2x}{dt^2}$ is the acceleration. Using (i) in (ii) we have

$$(t+1)^2 + x^3 a = x^2$$

or $\quad t^2 + 2t + 1 + x^3 a = x^2 \qquad \text{(iii)}$

But it is given that $x^2 = t^2 + 2t + 3$. Using this in Eq. (iii) we get $a = 2/x^3$.

58. $-2g(AB) = \left(\dfrac{u}{3}\right)^2 - \left(\dfrac{u}{2}\right)^2 = \dfrac{u^2}{9} - \dfrac{u^2}{4} = -\dfrac{5u^2}{36}$ (i)

$-2g(BC) = \left(\dfrac{u}{4}\right)^2 - \left(\dfrac{u}{3}\right)^2 = \dfrac{u^2}{16} - \dfrac{u^2}{9} = -\dfrac{7u^2}{144}$ (ii)

Divide (i) by (ii).

59. We know that $\quad h = ut - \dfrac{1}{2}\, gt^2$

or $\quad t^2 - \dfrac{2u}{g} t + \dfrac{2h}{g} = 0$

The roots of this quadratic equation are t_1 and t_2. The sum of the roots is $t_1 + t_2 = \dfrac{2u}{g}$.

60. Product of roots is $t_1\, t_2 = \dfrac{2h}{g}$

61. The time t_1 taken by the body to strike the inclined plane is given by

$$t_1 = \sqrt{\frac{2(H-h)}{g}}$$

The time t_2 taken by the body to reach the ground after striking the plane is

$$t_2 = \sqrt{\frac{2h}{g}}$$

Total time $\quad t = t_1 + t_2 = \sqrt{\dfrac{2(H-h)}{g}} + \sqrt{\dfrac{2h}{g}}$

Time t will be maximum if $\frac{dt}{dh} = 0$, i.e. if

$$\frac{d}{dh}\left[\sqrt{\frac{2(H-h)}{g}} + \sqrt{\frac{2h}{g}}\right] = 0$$

or $\quad \sqrt{\dfrac{2}{g}}\left[-\dfrac{1}{2}(H-h)^{-1/2} + \dfrac{1}{2}h^{-1/2}\right] = 0$

or $\dfrac{1}{\sqrt{H-h}} = \dfrac{1}{\sqrt{h}}$ or $H - h = h$ or $h = \dfrac{H}{2}$.

62. Initial velocity on entering water is $u = \sqrt{2gh}$.

Effective retardation in water is $g_{\text{eff}} = \dfrac{g(\rho - \rho')}{\rho'}$.

For maximum depth, final velocity $v = 0$.

Find the maximum depth x from the relation

$$v^2 - u^2 = 2g_{\text{eff}} x$$

63. If h is the total distance travelled in n seconds, then

$$h = \frac{1}{2}\, gn^2$$

In the last second, i.e. $(n-1)$th second, the distance fallen is $h' = \dfrac{h}{2} = \dfrac{1}{4}\, gn^2$.

Hence $\quad \dfrac{1}{2}\, g(n-1)^2 = h' = \dfrac{1}{4}\, gn^2$

or $\quad n^2 - 4n + 2 = 0$

The positive root of this equation gives the required value of n.

64. Average velocity $= \dfrac{\text{not displacement}}{\text{time}} = \dfrac{AB}{t}$

65. For the body of mass m_1, we have

$$h = \frac{u^2}{2g}$$

For the body of mass m_2, if S is the maximum distance travelled along the incline then

$$v^2 - u^2 = 2aS$$

Now, when S is maximum, $v = 0$. Also $a = -g \sin\theta = -g \sin 30° = -\dfrac{g}{2}$.

66. Given $t = \sqrt{x} + 3$. Squaring, we have

$$x = t^2 - 6t + 9 \qquad \text{(i)}$$

$$\text{velocity} \quad v = \frac{dx}{dt} = \frac{d}{dt}(t^2 - 6t + 9) = 2t - 6 \qquad \text{(ii)}$$

Find t from Eq. (ii) when $v = 0$. Use this value of t is Eq. (i).

67. Given $a = -k\sqrt{v}$ or $\dfrac{dv}{dt} = -k\sqrt{v}$. Thus $v^{-1/2}\,dv = -k\,dt$

Integrating, we get $2v^{1/2} = -kt + c$. Using the given initial condition ($v = u$ at $t = 0$), we get $c = 2\sqrt{u}$. Thus, we have

$$2(v^{1/2} - u^{1/2}) = -kt$$

Now, use $t = T$ and $v = 0$.

68. The slope of the line is $m = -\dfrac{5}{6}$ ms^{-2} per second and its intercept is $c = 5$ ms^{-2}. Using $y = mx + c$, the acceleration a (in ms^{-2}) as a function of time t is given by

$$a = -\frac{5}{6}t + 5$$

or $$\frac{dv}{dt} = -\frac{5}{6}t + 5$$

or $$v = \int_0^t \left(-\frac{5}{6}t + 5\right)dt$$

or $$v = -\frac{5}{12}t^2 + 5t + k \qquad (1)$$

where k is the constant of integration. Since the particle starts from rest, $v = 0$ at $t = 0$. Using this in (1) we get $k = 0$. Hence

$$v = -\frac{5}{12}t^2 + 5t \qquad (2)$$

It follows from the graph that the deceleration becomes zero at $t = 6$ s. Hence, the speed of the particle will be maximum at $t = 6$ s. Putting $t = 6$ s in Eq. (2), we have

$$v_{\max} = -\frac{5}{12} \times (6)^2 + 5 \times 6$$
$$= -15 + 30 = 15 \text{ ms}^{-1}$$

Hence the correct choice is (b).

69. The slope of the given v versus x graph is $m = -\dfrac{v_0}{x_0}$ and intercept is $c = +v_0$. Hence v varies with x as

$$v = -\left(\frac{v_0}{x_0}\right)x + v_0 \qquad (1)$$

where v_0 and x_0 are constants of motion. Differentiating with respect to time t, we have

$$\frac{dv}{dt} = -\left(\frac{v_0}{x_0}\right)\frac{dx}{dt}$$

or $$a = -\left(\frac{v_0}{x_0}\right)v \qquad (2)$$

Using Eq. (1) in Eq. (2), we get

$$a = -\left(\frac{v_0}{x_0}\right)\left(-\frac{v_0}{x_0}x + v_0\right)$$

or $$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

Thus the graph of a versus x is a straight line having a positive slope $= \left(\dfrac{v_0}{x_0}\right)^2$ and negative intercept $= -\dfrac{v_0^2}{x_0}$. Hence the correct choice is (d).

70. Given $v = k\sqrt{x} \Rightarrow v^2 = k^2x$. Differentiating, we have

$$2v\frac{dv}{dt} = k^2\frac{dx}{dt} = k^2v \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{k^2}{2}$$

$$\Rightarrow \quad \int dv = \frac{k^2}{2}\int dt$$

$$\Rightarrow \quad v = \frac{k^2 t}{2}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{k^2 t}{2}$$

$$\Rightarrow \quad \int dx = \frac{k^2}{2}\int t\,dt$$

$$\Rightarrow \quad x = \frac{k^2}{4}t^2$$

Thus $x \propto t^2$. Hence the correct choice is (c)

71. Slope of $a - t$ graph is $m = \frac{-3}{-2} = \frac{3}{2}$ ms^{-2} per s.

Intercept $c = -3\ \text{ms}^{-2}$. Therefore, the equation of the line is

$$a = \frac{3}{2}t - 3$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{3}{2}t - 3$$

$$\Rightarrow \quad dv = \left(\frac{3}{2}t - 3\right)dt$$

Integrating

$$\int_0^v dv = \int_0^t \left(\frac{3}{2}t - 3\right)dt$$

$$\Rightarrow \quad v = \frac{3t^2}{4} - 3t = \frac{3t}{4}(t-4)$$

Thus $v = 0$ at $t = 4$s and $v - t$ graph is a parabola. So the correct choice is (c).

72.
$$\frac{dx}{dt} = \frac{3t^2}{4} - 3t$$

$$\Rightarrow \quad dx = \left(\frac{3t^2}{4} - 3t\right)dt$$

Integrating

$$\int_0^x dx = \frac{3}{4}\int_2^4 t^2\,dt - 3\int_2^4 t\,dt$$

$$\Rightarrow \quad x = \frac{3}{4}\left|\frac{t^3}{3}\right|_2^4 - 3\left|\frac{t^2}{2}\right|_2^4$$

$$= \frac{1}{4}[4^3 - 2^3] - \frac{3}{2}[4^2 - 2^2] = -4\text{m}$$

So the correct choice is (d).

73. Acceleration $a = -\frac{F}{m} = -\frac{kv}{m}$

$$\therefore \quad \frac{dv}{dt} = -\frac{k}{m}v$$

$$\Rightarrow \quad \frac{dv}{v} = -\frac{k}{m}dt$$

Integrating

$$\int_u^v \frac{dv}{v} = -\frac{k}{m}\int_0^t dt$$

$$\Rightarrow \quad |\ln|_u^v = -\frac{kt}{m}$$

$$\Rightarrow \quad \ln\left(\frac{v}{u}\right) = -\frac{kt}{m}$$

$$\Rightarrow \quad v = u\,e^{-kt/m} \qquad (1)$$

So the correct choice is (c).

74.
$$\frac{dx}{dt} = ue^{-kt/m}$$

$$\Rightarrow \quad dx = u\,e^{-kt/m}\,dt$$

Integrating

$$\int_0^x dx = u\int_0^t e^{-kt/m}\,dt$$

$$\Rightarrow \quad x = u\left|\frac{e^{-kt/m}}{-k/m}\right|_0^t$$

$$\Rightarrow \quad x = \frac{mu}{k}(1 - e^{-kt/m})$$

Hence the correct choice is (c).

75. Putting $v = \frac{u}{2}$ in Eq. (1) we get

$$\frac{u}{2} = u\,e^{-kt/m}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-kt/m} \quad \Rightarrow \quad 2 = e^{kt/m}$$

or $\quad \ln(2) = \frac{kt}{m} \quad \Rightarrow \quad t = \frac{m}{k}\ln(2)$,

which is choice (a).

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. At time $t = 0$, a bullet is fired vertically upwards with a speed of 98 ms^{-1}. At time $t = 5$ s (i.e. 5 seconds later) a second bullet is fired vertically upwards with the same speed. If the air resistance is neglected, which of the following statements will be true?
- (a) The two bullets will be at the same height above the ground at $t = 12.5$ s.
- (b) The two bullets will reach back their starting points at the same time.
- (c) The two bullets will have the same speed at $t = 20$ s.
- (d) The two bullets will attain the same maximum height.

2. The ratios of the distances covered by a freely falling particle, starting from rest, in the first, second, third, ..., n^{th} seconds of its motion
- (a) form an arithmetic progression
- (b) form the series corresponding to the squares of the first n natural numbers
- (c) do not form any well defined series
- (d) form a series corresponding to the differences of the squares of the successive natural numbers

3. The displacement x of a particle varies with time according to the relation $x = \frac{a}{b}\left(1 - e^{-bt}\right)$. Then
- (a) At $t = 1/b$, the displacement of the particle is nearly $(2/3)(a/b)$
- (b) The velocity and acceleration of the particle at $t = 0$ are a and $-ab$ respectively
- (c) The particle cannot reach a point at a distance x' from its starting position if $x' > a/b$
- (d) The particle will come back to its starting point as $t \to \infty$

4. A bullet is fired vertically upwards. After 10 seconds it returns to the point of firing. Which of the following statements are correct? Take $g = 10\ ms^{-2}$.
- (a) The net displacement of the bullet in 10 s is zero
- (b) The total distance travelled by the bullet in 10 s is 250 m
- (c) The rate of change of velocity with time is constant throughout the motion of the bullet
- (d) The bullet is fired at an initial velocity of 50 ms^{-1} directed vertically upwards.

5. Two bodies of masses m_1 and m_2 are dropped from heights h_1 and h_2 respectively. They reach the ground after time t_1 and t_2 and strike the ground with speeds v_1 and v_2 respectively. Choose the correct relation from the following:

(a) $\frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}}$ (b) $\frac{t_1}{t_2} = \sqrt{\frac{m_2 h_1}{m_1 h_2}}$

(c) $\frac{v_1}{v_2} = \sqrt{\frac{h_1}{h_2}}$ (d) $\frac{v_1}{v_2} = \sqrt{\frac{m_2 h_1}{m_1 h_2}}$

6. Which of the velocity-time (v-t) graphs shown in Fig. 2.28 can possibly represent one-dimensional motion of a particle?

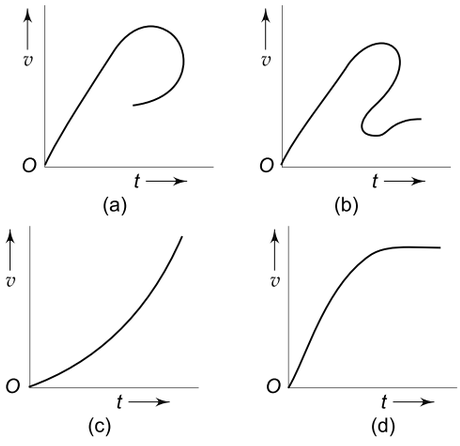

Fig. 2.28

7. Two balls of different masses are dropped from the same point at the same time. If the air resistance is neglected and the value of g remains constant, which of the following statements are true?
- (a) The heavier ball reaches the ground before the lighter one does.
- (b) Both balls reach the ground at the same time
- (c) The heavier ball hits the ground with a higher speed
- (d) Both balls reach the ground with the same speed

8. A body is dropped from the top of a tower of height h. Its covers a distance $h/3$ in the last second of its motion. If $g = 10\ \text{ms}^{-2}$, how long does it take to reach the ground?

(a) $(2 + \sqrt{3})$s (b) $(2 - \sqrt{3})$s

(c) $(3 + \sqrt{6})$s (d) $(3 - \sqrt{6})$s

9. A particle initially (i.e. at $t = 0$) located at $x = 0$ moves along the positive x-direction under the action of a force. The velocity of the particle varies with x as

$$v = k\sqrt{x}$$

where k is a constant. Then

(a) the displacement x varies with time t as $x = \frac{k^2t^2}{4}$.

(b) the velocity v varies with time t as $v = \frac{k^2t}{2}$.

(c) the acceleration of the particle is $\frac{k^2}{2}$.

(d) the distance s travelled by the particle in time T is $s = \frac{k^2T^2}{4}$.

IIT, 1993

10. The motion of a body is given by

$$\frac{dv}{dt} = 6 - 3v$$

where v is the velocity (in ms^{-1}) at time t (in seconds). The body is at rest at $t = 0$. Then

(a) the velocity of the body when its acceleration is zero is 2 ms^{-1}.

(b) the initial acceleration of the body is 6 ms^{-2}.

(c) the velocity of body when the acceleration is half the initial value is 1 ms^{-1}.

(d) the body has a uniform acceleration.

IIT, 2003

11. A stone falls freely from rest and the total distance covered by it in the last second of its motion is equal to the total distance covered by it in the first three seconds of its motion. If $g = 10\ \text{ms}^{-2}$,

(a) the stone remains in air for 5 s.

(b) the stone fell from a height of 125 m.

(c) the stone hits the ground with a speed of 50 ms^{-1}.

(d) the acceleration of the stone during the last 3 seconds of its motion is three times that during the first second.

12. From the top of a tower 40 m tall, a stone is projected vertically upwards with a speed of 10 ms^{-1} such that it falls in the sea below. If $g = 10\ \text{ms}^{-2}$,

(a) the stone passes the point from where it was projected after 2 s.

(b) the speed with which it passes the point of projection is 10 ms^{-1}.

(c) the stone falls in the sea 4 s after it was projected.

(d) the stone hits the water with a speed of 30 ms^{-1}.

13. A balloon is rising vertically upwards at a velocity of 10 ms^{-1}. When it is at a height of 45 m from the ground, a parachutist bails out from it. After 3 s he opens his parachute and decelerates at a constant rate of 5 ms^{-2}. Take $g = 10\ \text{ms}^{-2}$.

(a) He was 15 m above the ground when he opened his parachute.

(d) The velocity of the parachutist 3 s after he bails out is 5 ms^{-1} vertically upwards.

(c) He hits the ground with a speed of 10 ms^{-1}.

(d) He hits the ground 5 s after his exit from the balloon.

14. A particle moving in a straight line is subjected to a constant reterdation a which varies with instantaneous velocity v as

$$a = -kv$$

where k is a positive constant. If the initial velocity of the particle is u at time $t = 0$, then

(a) the velocity at time t is given by $v = u - at$.

(b) the velocity decreases exponentially with time.

(c) the velocity will decrease to $\frac{u}{2}$ in time $\frac{1}{k}$.

(d) the total distance covered by the particle before coming to rest is u/k.

15. A body moves from point A to point B with a velocity $\vec{v}_1$ and returns to point A with a velocity $\vec{v}_2$. Then, over the entire journey

(a) the average speed is $\frac{1}{2}(v_1 + v_2)$

(b) the average speed is $\frac{2v_1v_2}{(v_1 + v_2)}$

(c) the average velocity is zero.

(d) the average velocity is $\frac{1}{2}(\vec{v}_1 - \vec{v}_2)$

16. A body projected vertial upwards from the top of a tower of height h with a speed u reaches the ground after time t_1. If the body is projeced vertically downwards from the top of the tower with the same speed, it reaches the ground after time t_2.

Then

(a) $h = \frac{1}{2} g\left(t_1^2 + t_2^2\right)$ (b) $h = \frac{1}{2} g\ t_1 t_2$

(c) $u = \frac{1}{2} g(t_1 - t_2)$ (d) $u = \frac{1}{2} g(t_1 + t_2)$

17. A particle moving in a straight line is subjected to a constant ratardation a which varies with instantaneous velocity v as

$$a = -kv$$

where k is a positive constant. If the initial velocity of the particle is u at time $t = 0$, then

(a) the velocity at time t is given by $v = u - at$.

(b) the velocity decreases exponentially with time.

(c) the velocity will decrease to $\frac{u}{2}$ in time $\frac{1}{k}$.

(d) the total distance covered by the particle before coming to rest is u/k.

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ANSWERS AND SOLUTIONS

1. The two bullets will attain the same height at time $t = n$ seconds if $98\ n - \frac{1}{2} \times 9.8 \times n^2 = 98\ (n-5) - \frac{1}{2} \times 9.8\ (n-5)^2$ which gives $n = 12.5$ s. The time to reach the highest point is given by $0 = 98 - 9.8\ t$ or $t = 10$ s. Thus the time of flight of the first bullet is $2 \times 10 = 20$ s while that of the second bullet is $20 + 5 = 25$ s. Hence, the second bullet will reach the starting point 5 seconds later than the first bullet. At $t = 20$ s, the speed of the first bullet is 98 ms^{-1} while that of the second bullet is $98 - 9.8 \times 5 = 49$ ms^{-1}. The maximum height attained by each bullet is 490 m. Hence, height attained by each bullet is 490 m. Therefore the correct choices are (a) and (d).

2. The distance covered in the nth second (with $u = 0$) is given by

$$s_n = \frac{1}{2} g\ [n^2 - (n-1)^2] = \frac{1}{2} g(2n-1)$$

$\therefore\ s_1 = \frac{1}{2} g,\ s_2 = \frac{3}{2} g,\ s_3 = \frac{5}{2} g$, etc. Hence choices (a) and (d) are correct.

3. Velocity of the particle is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{\frac{a}{b}\left(1 - e^{-bt}\right)\right\} = ae^{-bt}$$

Acceleration of the particle is given by

$$\alpha = \frac{dv}{dt} = \frac{d}{dt}\left(ae^{-bt}\right) = -abe^{-bt}$$

At $t = 1/b$, the displacement of the particle is

$$x = \frac{a}{b}\left(1 - e^{-1}\right) \simeq \frac{a}{b}\left(1 - \frac{1}{3}\right) \simeq \frac{2}{3}\frac{a}{b}\left(\because e^{-1} \simeq \frac{1}{3}\right)$$

Hence choice (a) is correct. At $t = 0$, the values v and α respectively are $v = ae^{-0} = a$ and $\alpha = -abe^{-0} = -ab$. Hence choice (b) is also correct. The displacement x is maximum when $t \to \infty$, i.e. $x_{max} = \frac{a}{b}\left(1 - e^{-\infty}\right) = \frac{a}{b}$. Hence choice (c) is also correct. Thus the correct choices are (a), (b) and (c).

4. Since the bullet returns to its point of projection, its net displacement is zero, which is choice (a). The bullet takes 5 s to reach the maximum height. Therefore, initial speed (u) of the bullet is ($\because$ final velocity = 0)

$u = gt = 10 \times 5 = 50$ ms^{-1} directed upwards which is choice (d). The maximum height (h) attained by the bullet is $h = \frac{1}{2}\ gt^2 = \frac{1}{2} \times 10 \times (5)^2 = 125$ m. Therefore, the total distance travelled by the bullet in 10 s = 125 + 125 = 250 m, which is choice (b). For heights $h <<$ radius of the earth, the magnitude of g is constant, i.e. the rate of change of velocity is constant, which is choice (c). Thus, all four choices are correct.

5. Since the initial velocity is zero, we have

$$h_1 = 0 \times t_1 + \frac{1}{2} g t_1^2 = \frac{1}{2}\ gt_1^2$$

and $h_2 = 0 \times t_2 + \frac{1}{2}\ gt_2^2 = \frac{1}{2}\ gt_2^2$

Therefore

$\frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}}$, which is choice (a).

Also, we have $v_1 = 0 + gt_1 = gt_1$ and $v_2 = gt_2$.

Therefore $\frac{v_1}{v_2} = \frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}}$ which is choice (c).

Hence the correct choices are (a) and (c).

6. Draw a line perpendicular to t–axis. You will find that in graphs (a) and (b) shown in Fig. 2.28, the particle can have two different velocities at the same time, which is impossible. Hence the correct choices are graphs (c) and (d) in Fig. 2.28.

7. Since the time taken to reach the ground and the speed with which a ball reaches the ground are independent of mass, choices (b) and (c) are correct.

8. The total time t taken by the body to reach the ground is given by $h_1 = \frac{1}{2} gt^2 = \frac{1}{2} \times 10 \times t^2 = 5t^2$.

The body moves for $(t-1)$ second before the beginning of the last second. The velocity of the body at an instant $(t-1)$ second is $v = g(t-1) = 10\,(t-1)$. This is the initial velocity for the motion in the last one second. The distance covered in this one second is

$$h_2 = 10\,(t-1) \times 1 + \frac{1}{2} \times 10 \times (1)^2 = 10(t-1) + 5 = 10t - 5.$$

It is given that $h_2 = \frac{h_1}{3}$. Therefore

$$10t - 5 = \frac{5t^2}{3}$$

or $$t^2 - 6t + 3 = 0$$

The two roots of this quadratic equation are $t = \left(3 \pm \sqrt{6}\right)$ s. Hence the correct choices are (c) and (d).

9. All four choices are correct.

Given $v = k\sqrt{x}$ or $\frac{dx}{dt} = kx^{1/2}$ or $x^{-1/2}\,dx = k\,dt$. Integrating, we have $2x^{1/2} = kt + c$ where c is a constant of integration. Given that $x = 0$ at $t = 0$. This gives $c = 0$.

Use $v = \frac{dx}{dt}$ and $x = \frac{k^2t^2}{4}$. Hence $v = \frac{k^2t}{2}$.

Acceleration $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{k^2t}{2}\right) = \frac{k^2}{2}$ = constant

We have $\frac{dx}{dt} = \frac{k^2t}{2}$ or $dx = \frac{k^2}{2} tdt$.

Integrating, we have

$$\int_O^S dx = \frac{k^2}{2} \int_O^T tdt$$

which gives $s = \frac{k^2T^2}{4}$.

10. Given $a = \frac{dv}{dt} = 6 - 3v$ (1)

(a) Velocity v_0 when acceleration $a = 0$ is obtained from Eq.(1) by putting $a = 0$, we get $6 - 3v_0 = 0$ or $v_0 = 2$ ms^{-1}.

(b) At $t = 0$, the body is at rest, i.e. $v = 0$ at $t = 0$. Putting $v = 0$ in Eq.(1), initial acceleration $(a_0) = 6$ ms^{-2}.

(c) Putting $a = \frac{a_0}{2} = 3$ ms^{-2} in Eq. (1), we have

$$3 = 6 - 3v \Rightarrow v = 1 \text{ ms}^{-1}$$

Hence choices (a), (b) and (c) are correct. Choice (d) is wrong because acceleration a change with time because v changes with time.

11. Let h_1 be the distance covered during the first 3 seconds. Then

$$-h_1 = -\frac{1}{2} g(3)^2 \Rightarrow h_1 = \frac{9}{2} g$$

Let t be the time of flight. Then the velocity at $(t-1)$ is $g(t-1)$ and the distance covered in the last one second is

$$-h_2 = -g(t-1) \times 1 - \frac{1}{2} \times g \times (1)^2$$

$$\Rightarrow \quad h_2 = g(t-1) + \frac{g}{2} = gt - \frac{g}{2}$$

Given $h_1 = h_2$, i.e. $\frac{9}{2} g = gt - \frac{g}{2} \Rightarrow t = 5$ s.

The height H from which the stone falls is

$$-H = -\frac{1}{2} gt^2 = -\frac{1}{2} \times 10 \times (5)^2$$

$$= -125 \text{ m} \Rightarrow H = 125 \text{ m}$$

The speed with which the stone hits the ground is

$$-v = -gt = -10 \times 5 = -50 \text{ ms}^{-1} \Rightarrow v = 50 \text{ ms}^{-1}$$

Hence choices (a), (b) and (c) are correct. Choice (d) is wrong because for bodies not too far away from the earth, the acceleration due to gravity is contant = 9.8 ms^{-2}.

12. Let us assign a positive sign to quantities directed in the upward directon and a negative sign to those directed downwards. Now, the net displacement of the stone = final position – initial position = – 40 m directed downwards. Thus $s = -40$ m. The initial velocity is directed upwards. Therefore, $u = +10$ ms^{-1}. The acceleration due to gravity is directed downwards. Hence $g = -10$ ms^{-2}. Using these values in the relation $s = ut + \frac{1}{2} gt^2$, we get

$$-40 = 10t + \frac{1}{2}(-10)t^2 = 10t - 5t^2$$

or $t^2 - 2t - 8 = 0$ or $(t+2)(t-4) = 0$

which gives $t = -2$ s or 4 s. Since the negative sign of t is not possible, the correct answer is 4 s.

It is clear that the net displacement of the stone in coming back to the pointfrom where it was projected is zero, i.e. $s = 0$. Thus we have

$$0 = 10t + \frac{1}{2}(-10)t^2$$

which gives $t = 0$ or 2 s. Since $t = 0$ is not possible the correct answer is 2 s.

The final velocity of the stone is given by

$$-v = u + gt$$
$$= 10 + (-10) \times 4 = -30 \text{ ms}^{-1}$$
$$\Rightarrow \quad v = 30 \text{ ms}^{-1}$$

Hence all four choices are correct.

13. When the parachutist bails out, he shares the velocity of the balloon and has an upward velocity of 10 ms^{-1}, i.e. $u = +10 \text{ ms}^{-1}$. Also $g = -10 \text{ ms}^{-2}$ (acting downwards). The displacement in $t = 3$ s is given by

$$s = ut + \frac{1}{2}gt^2$$
$$= 10 \times 3 + \frac{1}{2} \times (-10) \times (3)^2 = -15 \text{ m}$$

Since the displacement is negative, it is directed downwards. So the height from the ground when he opened his parachute = 45 – 15 = 30 m.

In time 3 s, the balloon has risen through 30 m (as the velocity of the balloon is 10 ms^{-1} upwards).

Hence the parachutist is now 30 + 15 = 45 m away from the balloon. The velocity of the parachutist 3 s after he bails out is

$$v = u + gt$$
$$= 10 + (-10) \times 3 = -20 \text{ ms}^{-1}$$
(directed downwards)

At $t = 3$s, his initial velocity is $u = -20 \text{ ms}^{-1}$ and to hit the ground, his displacement $s = -30$ m (see solution of Q.18). Now $a = +5 \text{ ms}^{-2}$ (directed upwards). The time taken to hit the ground is given by

$$s = ut + \frac{1}{2}gt^2 \text{ or } -30 = -20t + \frac{1}{2}t^2$$

or $t^2 - 8t + 12 = 0$ or $(t - 6)(t - 2) = 0$

which gives $t = 6$ s or 2 s. If $t = 6$ s, then the velocity with which he hits the ground is $v = u + at = -20 + 5 \times 6 = 10 \text{ ms}^{-1}$. This is positive, i.e. v is directed upwards, which is not possible. Thus the correct answer is $t = 2$ s, in which case, the velocity with which he hits the ground is

$$v = -20 + 5 \times 2 = -10 \text{ ms}^{-1}$$

which is negative as it should be.

The total time the parachutist takes (after his exit from the balloon) to hit the ground is = 3 s + 2 s = 5 s.

Hence the correct choices are (c) and (d).

14. $\frac{dv}{dt} = -kv$

$$\therefore \quad \int_u^v \frac{dv}{v} = \int_0^t -k\,dt$$

$$\Rightarrow \quad \log_e\left(\frac{v}{u}\right) = -kt$$

$$\Rightarrow \quad v = u\,e^{-kt} \qquad (1)$$

Thus choice (a) is wrong and choice (b) is correct. It follows from Eq. (1) that $v = \frac{u}{2}$ in time t given by

$$\frac{u}{2} = ue^{-kt}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-kt} \Rightarrow 2 = e^{kt} \Rightarrow \log_e(2) = kt$$

$$\Rightarrow \quad t = \frac{\log_e(2)}{k} = \frac{0.693}{k}$$

Hence choice (c) is wrong.

From Eq.(1) we have

$$\frac{dx}{dt} = u\,e^{-kt}$$

$$\Rightarrow \quad \int_{x_0}^{x} dx = \int_0^t ue^{-kt}\,dt$$

$$\Rightarrow \quad x - x_0 = -\frac{u}{k}e^{-kt}$$

$$\Rightarrow \quad x = x_0 - \frac{u}{k}e^{-kt}$$

Now $x = 0$ at $t = 0$ which gives $x_0 = \frac{u}{k}$. Hence

$$x = \frac{u}{k}(1 - e^{-kt}) \qquad (2)$$

It follows from Eq. (1) that $v = 0$ at $t = \infty$. Hence distance travelled by the particle before coming to rest is obtained from Eq. (2) by putting $t = \infty$, which gives $x = u/k$. Hence the correct choices are (b) and (d).

15. Let s = distance between points A and B. It t_1 is the time taken to go from A to B and t_2 the time taken to return from B to A, then

$$t_1 + t_2 = \frac{s}{v_1} + \frac{s}{v_2}$$

$$\therefore \quad \text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{s+s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2v_1v_2}{(v_1+v_2)}$$

Since the net displacement is zero as the body returns to the strating point, the average velocity = 0. Hence the correct choices are (b) and (c).

16. The net displacement in both cases is $-h$ (vertically) downwards. For the body projected upwards, initial velocity is $+u$ and for the body projected downwards, the initial velocity is $-u$. Hence, we have (since $a = -g$ in both cases)

$$-h = ut_1 - \frac{1}{2}gt_1^2 \qquad (1)$$

and $$-h = ut_2 - \frac{1}{2}gt_2^2 \qquad (2)$$

Equations (1) and (2) give $u = \frac{1}{2}g\,(t_1 - t_2)$ and $h = \frac{1}{2}g\,t_1t_2$. Thus the correct choices are (b) and (c).

17. $\frac{dv}{dt} = -kv$

$$\therefore \quad \int_u^v \frac{dv}{v} = \int_0^t -kdt$$

$$\Rightarrow \quad \log_e\left(\frac{v}{u}\right) = -kt$$

$$\Rightarrow \quad v = u\,e^{-kt} \qquad (1)$$

Thus choice (a) is wrong and choice (b) is correct. It follows from Eq. (1) that $v = \frac{u}{2}$ in time t given by

$$\frac{u}{2} = ue^{-kt}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-kt} \Rightarrow 2 = e^{kt} \Rightarrow \log_e(2) = kt$$

$$\Rightarrow \quad t = \frac{\log_e(2)}{k} = \frac{0.693}{k}$$

Hence choice (c) is wrong.

From Eq.(1) we have

$$\frac{dx}{dt} = u\,e^{-kt}$$

$$\Rightarrow \quad \int_{x_0}^{x} dx = \int_0^t ue^{-kt}\,dt$$

$$\Rightarrow \quad x - x_0 = -\frac{u}{k}e^{-kt}$$

$$\Rightarrow \quad x = x_0 - \frac{u}{k}e^{-kt}$$

Now $x = 0$ at $t = 0$ which gives $x_0 = \frac{u}{k}$. Hence

$$x = \frac{u}{k}(1 - e^{-kt}) \qquad (2)$$

It follows from Eq. (1) that $v = 0$ at $t = \infty$. Hence distance travelled by the particle before coming to rest is obtained from Eq. (2) by putting $t = \infty$, which gives $x = u/k$. Hence the correct choices are (b) and (d).

SECTION III

Multiple Choice Question Based on Passage

Questions 1 to 4 are based on the following passage

Passage I

It must be clearly understood that distance is not the same as displacement. Distance is a scalar quantity and is given by the total length of the path travelled by the body in a certain interval of time. Displacement is a vector quantity and is given by the shortest distance (in a specified direction) between the initial and the final positions of the body. The direction of the displacement vector is from the initial position (starting point) to the final position (end point) of the motion. Speed is a scalar quantity. The average speed is defined as

$$v = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Velocity is a vector quantity. The average velocity is defind as

$$\vec{v} = \frac{\text{net displacement}}{\text{time taken}}$$

The direction of the velocity vector is the same as that of the displacement vector. Acceleration is defined as the rate of change of velocity and it is a vector quantity.

1. A cyclist starts from centre O of a circular track of radius $r = 1$ km, reaches edge P of the track and then cycles along the circumference and stops at point Q as shown in Fig. 2.29. the displacement of the cyclist is

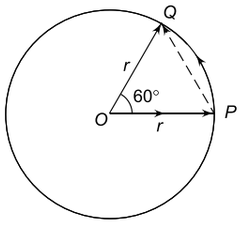

Fig. 2.29

(a) $r\left(1+\frac{\pi}{6}\right)$ (b) r

(c) $r\left(1+\frac{\pi}{3}\right)$ (d) $\frac{\pi r}{3}$

2. In Q.1 above the magnitude of the velocity in km h^{-1} of the cyclist is

(a) 12.3 (b) 12.0
(c) 3.0 (d) 6.0

3. In Q.1 the distance travelled by the cyclist is approximately

(a) 2 km (b) 2.01 km
(c) 2.05 km (d) 1 km

4. In Q.1, the average speed (in km h^{-1}) of the cyclist is

(a) 12.3 (b) 6.15
(c) 6.0 (d) 12.0

SOLUTION

1. Net displacement is OQ = shortest distance between the starting point O and end point Q = 1 km = r. Therefore, the correct choice (b).

2. Average velocity $= \frac{\text{displacement}}{\text{time}} = \frac{1\,\text{km}}{\frac{1}{6}\text{h}} = 6\ \text{km}^{-1}$

Hence the correct choice is (d).

3. Distance travelled = $OP + PQ$ (along the arc)

$$= r + \frac{2\pi r}{6} = r\left(1+\frac{\pi}{3}\right) = 1\ \text{km}\left(1+\frac{3.14}{3}\right) + 2.05\ \text{km}$$

Hence the correct choice is (c).

4. Average speed $= \frac{\text{distance}}{\text{time}} = \frac{2.05\,\text{km}}{\frac{1}{6}\text{h}}$

$= 12.3\ \text{km h}^{-1}$

Thus the correct choice is (a).

Question 5 to 7 are based on the following passage.

Passage II

Ball A is rolled along the positive x-direction with a speed of 5 ms^{-1} towards a bigger ball B 20 m away. After collision with ball B, ball A retraces the path and reaches its starting point with a speed of 4 ms^{-1}.

5. The average velocity of ball A during the time interval from 0 to 4 s is

(a) 5 ms^{-1} along positive x-direction.
(b) 5 ms^{-1} along negative x-direction.
(c) $\frac{40}{9}$ ms^{-1} along negative x-direction.
(d) $\frac{14}{3}$ ms^{-1} along positive x-direction.

6. The average velocity of ball A during the time interval 0 to 9 s is

(a) 4.5 ms^{-1} along positive x-direction.
(b) 4.5 ms^{-1} along negative x-direction.
(c) $\frac{40}{9}$ ms^{-1} along negative x-direction.
(d) zero

7. The average velocity of ball A during the time interval 0 to 6 s is

(a) 2 ms^{-1} along positive x-direction.
(b) 2 ms^{-1} along negative x-direction.
(c) $\frac{14}{3}$ ms^{-1} along positive x-direction.
(d) $\frac{14}{3}$ ms^{-1} along negative x-direction.

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5. Time taken by ball A to reach ball B is

$$t_1 = \frac{20}{5} = 4\ \text{s}$$

During the time interval 0 to 4s, the net displacement of ball A = 20 m along positive x-direction. Therefore, the velocity of ball A during 0 to 4s = 5 ms^{-1} along positive x- direction, which is choice (a).

6. Time taken by ball A to retrace its path and reach the starting point after collision with ball B is

$$t_2 = \frac{20}{4} = 5 \text{ s}$$

Therefore, the net displacement in time interval 0 to 9 s = 0 Hence velocity = 0, which is choice (d).

7. During the time interval 0 to 6 s, ball A covers a distance of 20 m along positive x-direction up to ball B (which takes 4 s) and in the next 2 s, it covers a distance of 4 $ms^{-1} \times 2$ s = 8 m along the negative x-direction. Therefore,
Net displacement of A from 0 to 6 s = 20 – 8 = 12 m along positive x-direction. Hence, average velocity of A during 0 and 6 s is

$$\frac{12}{6\text{s}} = 2 \text{ ms}^{-1} \text{ along positive } x\text{-direction}$$

Thus the correct choice is (a).

Questions 8 and 9 are based on the following passage

Passage III

A particle initially (i.e. at time $t = 0$) moving with a velocity u is subjected to a retarding force, as a result of which it decelerates at a rate

$$a = -k\sqrt{v}$$

where v is the instantaneous velocity and k is a positive constant.

8. The particle comes to rest in a time

(a) $\frac{2\sqrt{u}}{k}$ (b) $\frac{\sqrt{u}}{k}$ (c) $2k\sqrt{u}$ (d) $k\sqrt{u}$

9. The distance covered by the particle before coming to rest is

(a) $\frac{u^{3/2}}{k}$ (b) $\frac{2u^{3/2}}{k}$ (c) $\frac{3u^{3/2}}{2k}$ (d) $\frac{2u^{3/2}}{3k}$

SOLUTION

8. Given $a = -kv^{1/2}$ or $\frac{dv}{dt} = -kv^{1/2}$

Thus $v^{-\frac{1}{2}}\, dv = -k\, dt$

Integrating, we have

$$\int v^{-1/2} dv = -k \int dt$$

or $$2v^{1/2} = -kt + c \qquad \text{(i)}$$

where c is the constant of integration. Given that at $t = 0$, $v = u$. Using this in (i) we get $2u^{1/2} = c$. Using this value of c in (i),

we have $2(v^{1/2} - u^{1/2}) = -kt$ (ii)

Let τ be the time taken by the particle to come to rest. Then, $v = 0$ at $t = \tau$. Using this in (ii), we get

$$2(0 - u^{1/2}) = -k\tau \quad \text{or} \quad \tau = \frac{2u^{1/2}}{k} \qquad \text{(iii)}$$

Hence the correct choice is (a).

9. To find the distance s covered in this time, we use Eq. (i) to get

$$v^{1/2} = u^{1/2} - \frac{kt}{2}$$

Squaring, we have

$$v = u - ktu^{1/2} + \frac{k^2t^2}{4}$$

But $$v = \frac{ds}{dt}$$

Therefore, $$\frac{ds}{dt} = u - kt\,u^{1/2} + \frac{k^2t^2}{4}$$

Integrating from $t = 0$ to $t = \tau$, we have

$$s = \left| ut - \frac{ku^{1/2}t^2}{2} + \frac{k^2t^3}{12} \right|_0^{\tau}$$

or $$s = u\tau - \frac{1}{2}ku^{1/2}\tau^2 + \frac{1}{12}k^2\tau^3 \qquad \text{(iv)}$$

Substituting the value of t from (iii) in (iv), we get

$$s = \frac{2u^{3/2}}{k} - \frac{4u^{3/2}}{2k} + \frac{8u^{3/2}}{12k} \quad \text{or} \quad s = \frac{2u^{3/2}}{3k}$$

SECTION IV

Matching

1. Figure 2.30 shows the displacement-time ($x - t$) graph of the motion of a body.

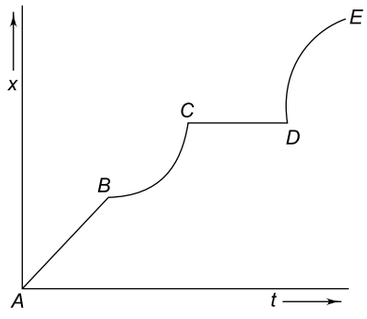

Fig. 2.30

Column I	***Column II***
(a) AB	(p) The body is a rest
(b) BC	(q) The body is not accelerating
(c) CD	(r) The velocity is decreasing with time
(d) DE	(s) The velocity is increasing with time.

ANSWER

1. (a) → (q) (c) → (p)
(b) → (s) (d) → (r)

Explanation: The slope of $x - t$ graph gives the velocity of the body. In AB, the slope is constant. In BC, the slope is increasing with time. In CD, the slope is zero and in DE the slope is decreasing with time.

2. The displacement x of a particle moving along the x-direction varies with time t according to the relation.

$$x = a + bt - ct^2$$

where a, b and c are constants of motion.

Column I	***Column II***
(a) Displacement when velocity = 0	(p) $(b - 7c)$
(b) Acceleration when velocity = 0	(q) $-2c$
(c) Initial displacement	(r) a
(d) Average velocity during the 4th second	(s) $a + \frac{b^2}{4c}$

SOLUTION

Displacement $x = a + bt - ct^2$

$\therefore$ x at $(t = 0) = a$

Velocity $$v = \frac{dx}{dt} = b - 2ct$$

$\therefore$ $$v = 0 \text{ at } t = \frac{b}{2c}$$

Acceleration = $\frac{dv}{dt} = -2c$ (which is constant)

Value of x at $\left(t = \frac{b}{2c}\right) = a + b \times \frac{b}{2c} - c\left(\frac{b}{2c}\right)^2 = a + \frac{b^2}{4c}$

Hence the solution is

(a) → (s) (c) → (r)

(b) → (q) (d) → (p)

SECTION V

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four options out of which only ONE choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is true but Statement-2 is NOT the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**
   A body moving in a straight line may have non-zero acceleration at an instant when its speed is zero.
   Statement-2
   If a body is thrown vertically upwards, its speed at the instant when it reaches the highest point is zero but its acceleration is 9.8 ms^{-2}.
2. **Statement-1**
   A body moving in a straight line with a constant speed must have a zero acceleration.
   Statement-2
   A body moving along a curve with a constant speed may have a zero acceleration.
3. **Statement-1**
   A wooden ball and a steel ball of the same mass, released from the same height in air, do not reach the ground at the same time.
   Statement-2
   The apparent weight of a body in a medium depends on the density of the body relative to that of the medium.
4. **Statement-1**
   If the displacement-time graph of the motion of a body is a straight line parallel to the time axis, then it follows that the body is at rest.
   Statement-2
   Velocity is equal to the rate of change of displacement.
5. **Statement-1**
   If the velocity-time graph of the motion of a body is a curve, then the body is either uniformly accelerated or uniformly retarded.
   Statement-2
   The slope of the velocity-time graph gives the acceleration.

SOLUTIONS

1. The correct choice is (b).
2. The correct choice is (c). The velocity of a body moving along a curve continuously changes because its direction of motion is changing. Hence a body moving along a curve with a constant speed has acceleration called centripetal acceleration.
3. The correct choice is (a). The effective acceleration due to gravity in a medium is given by

$$g_{\text{eff}} = g\left(1 - \frac{\rho}{\sigma}\right)$$

where ρ = density of the medium and σ = density of the body.

4. The correct choice is (a). If the displacement-time graph is parallel to the time axis, then, rate of change of displacement is zero.
5. The correct choice is (d). If the velocity-time graph is a curve, the slope of the graph is not constant.

SECTION VI

Integer Answer Type

1. The displacement x of a particle moving in one dimension, under the action of a constant force, is related to time t by the equation

$$t = \sqrt{x} + 3$$

where x is in metres and t in seconds. Find the displacement (in metre) of the particle when its velocity is zero.

IIT, 1979

2. The motion of a body is given by

$$\frac{dv(t)}{dt} = 6 - 3v(t)$$

where $v(t)$ is the velocity (in ms^{-1}) of the body at time t in second. If the body was at rest at $t = 0$, find its velocity (in ms^{-1}) when the acceleration is half the initial value.

IIT, 1995

SOLUTIONS

1. Given $\sqrt{x} = t - 3$. Squaring, we have

$$x = t^2 - 6t + 9 \qquad \text{(i)}$$

The instantaneous velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt}(t^2 - 6t + 9) = 2t - 6$$

Now, v will be zero at time t given by $2t - 6 = 0$ or $t = 3$ seconds. The displacement at $t = 3$ s is obtained from relation (i) by putting $t = 3$ s which gives x (at $t = 3$ s) $= (3)^2 - 6 \times 3 + 9 = 0$. Hence the net displacement of the particle is zero when its velocity is zero, i.e. the particle returns to its starting position, where its velocity is zero.

2. The acceleration of the body at time t is

$$a(t) = \frac{dv(t)}{dt} = 6 - 3v(t) \qquad (1)$$

Putting $t = 0$ in Eq. (1), the initial acceleration is

$$a(0) = 6 - 3v(0)$$

Since the body was at rest at $t = 0$, $v(0) = 0$. Hence

$$a(0) = 6 \text{ ms}^{-2}$$

When $a(t) = \frac{a(0)}{2} = \frac{6}{2} = 3 \text{ ms}^{-2}$, we have from Eq. (1)

$$3 = 6 - 3v \quad \Rightarrow \quad v = 1 \text{ ms}^{-1}$$

3 Chapter

Vectors

REVIEW OF BASIC CONCEPTS

3.1 EQUAL VECTORS

Two vectors **A** *and* **B** *of the same physical quantity are equal, if and only if they have the same magnitude and the same direction*. We can test the equality by shifting **B** parallel to itself until its tail coincides with the tail of **A**. If the tips of the two vectors also coincide, the vectors are equal. (Remember, the vector shifted parallel to itself is equal to the original vector since the magnitude and the direction of the vector do not change when it is shifted parallel to itself).

3.2 UNIT VECTOR

A vector having a unit modulus is called unit vector. If **A** is a vector, then unit vector $\hat{\mathbf{A}}$ along the direction of **A** is defined as

$$\hat{\mathbf{A}} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}$$

Conventionally, unit vectors along x, y and z directions are denoted by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ respectively.

3.3 NULL VECTOR

If two vectors **A** and **B** are equal, their difference (**A** – **B**) is defined as the *zero* or *null vector* and is represented by the symbol **0**. *It has zero magnitude* and no specific direction. A vector which is not null is called a *proper vector*. Thus, if

$$\mathbf{A} = \mathbf{B}$$

then $$\mathbf{A} - \mathbf{B} = \mathbf{0}$$

3.4 ADDITION OF VECTORS

The procedure of finding the resultant or sum of two vectors is known as the *parallelogram law of vector addition* and may be stated as follows.

If the two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.

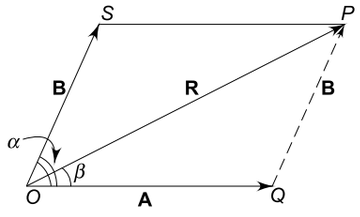

Fig. 3.1

Figure 3.1 show two vectors **A** and **B** of magnitudes A and B inclined at an angle α. The magnitude R of the resultant vector **R** is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\alpha}$$

The angle β which the resultant vector **R** subtends with vector **A** is given by

$$\tan\beta = \frac{B\sin\alpha}{A + B\cos\alpha}$$

or $$\sin\beta = \frac{B\sin\alpha}{R}$$

In vector notation the resultant vector is written as **R** = **A** + **B**.

Special Cases

(i) *When the two vectors are in the same direction*, i.e. $\alpha = 0°$, then $R = \sqrt{A^2 + B^2 + 2AB\cos 0°} = A + B$. Therefore, the magnitude of the resultant is equal to the sum of the magnitudes of the two vectors. Also $\tan\beta = 0$ or $\beta = 0$, i.e. the direction of the resultant is the same as that of either vector.

(ii) *When the two vectors are in opposite directions*, i.e., $\alpha = 180°$, then $R = \sqrt{A^2 + B^2 + 2AB\cos 180°} = A - B$. Also $\beta = 0$

(iii) When the two vectors are at right angles to each other, i.e. $\alpha = 90°$, then

$R = \sqrt{A^2 + B^2 + 2AB\cos 90°} = \sqrt{A^2 + B^2}$. Also $\tan\beta = \dfrac{B}{A}$.

Note: $R_{max} = A + B$ and $R_{min} = A - B$

EXAMPLE 3.1

The following pairs of forces act on a particle at an angle θ which can have any value

(a) 2 N and 3 N (b) 3 N and 3 N

(c) 2 N and 6 N and (d) 3 N and 8 N

The resultant of which pair cannot have a magnitude of 4 N?

SOLUTION

For pair (a) : $R_{min} = A - B = 3 - 2 = 1$ N and $R_{max} = A + B = 3 + 2 = 5$ N

For pair (b) : $R_{min} = 0$, $R_{max} = 6$ N

For pair (c) : $R_{min} = 4$ N, $R_{max} = 8$ N

For pair (d) : $R_{min} = 5$ N, $R_{max} = 11$ N

Hence the correct answer is (d).

EXAMPLE 3.2

The magnitude of the resultant of two vectors of the same magnitude is equal to the magnitude of either vector. Find the angle between the two vectors.

SOLUTION

Given $A = B$ and $R = A$ or B.

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\Rightarrow \quad A = \sqrt{A^2 + A^2 + 2A^2\cos\theta}$$

$$= \sqrt{2A^2(1 + \cos\theta)}$$

$$\Rightarrow \quad A^2 = 2A^2(1 + \cos\theta) \Rightarrow \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120°$$

3.5 TRIANGLE LAW OF VECTOR ADDITION

The parallelogram law of vector addition yields the triangle law of vector addition. In Fig. 3.1, vector QP = vector OS = **B**. In triangle OQP, vector OP = **R**. Hence, the triangle law of vector addition may be stated as follows:

If the two vectors are represented in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant is represented in magnitude and direction by the third side of the triangle taken in the opposite order.

3.6 SUBTRACTION OF VECTORS

Suppose we wish to subtract a vector **B** from a vector **A**. Since

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

the subtraction of vector **B** from vector **A** is equivalent to the addition of vector – **B** to vector **A**. Hence the procedure to find (**A** – **B**) is as follows:

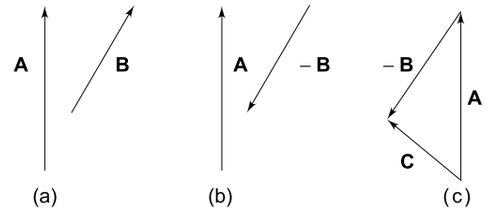

Fig. 3.2

Choose a convenient scale and draw the vectors **A** and **B** as shown in Fig. 3.2 (a). If **B** is to be subtracted from **A**, draw the vector negative of **B**, i.e. draw the vector – **B** [see Fig. 3.2 (b)]. Now shift the vector – **B** parallel to itself so that the tail of– **B** is at the head of **A**. Vector **C** is the sum of vectors **A** and – **B**, i.e. [see Fig. 3.2 (c)]

$$\mathbf{C} = \mathbf{A} + (-\mathbf{B}) = \mathbf{A} - \mathbf{B}$$

3.7 RESOLUTION OF A VECTOR INTO RECTANGULAR COMPONENTS

Consider a vector **A** in the x–y plane making an angle θ with the x-axis. The x and y components of **A** are $\mathbf{A}_x$ and $\mathbf{A}_y$. The magnitudes of $\mathbf{A}_x$ and $\mathbf{A}_y$ are (Fig. 3.3)

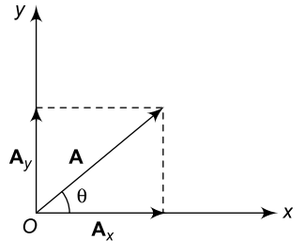

Fig. 3.3

$A_x = A\cos\theta$, along x-direction

and $A_y = A\sin\theta$, along y-direction

Thus $\mathbf{A}_x = A_x\,\hat{\mathbf{i}} = (A\cos\theta)\,\hat{\mathbf{i}}$

and $\mathbf{A}_y = A_y\ \hat{\mathbf{j}} = (A\ \sin\ \theta)\ \hat{\mathbf{j}}$

From parallelogram law

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y = (A\ \cos\ \theta)\,\hat{\mathbf{i}} + (A\ \sin\ \theta)\,\hat{\mathbf{j}}$$

The magnitude of **A** in terms of the magnitudes of its rectangular components is

$$A = \sqrt{A_x^2 + A_y^2}$$

Also $\tan\ \theta = \dfrac{A_y}{A_x}$

EXAMPLE 3.3

Resolve **A** into x and y components. The magnitude of **A** is 4 units.

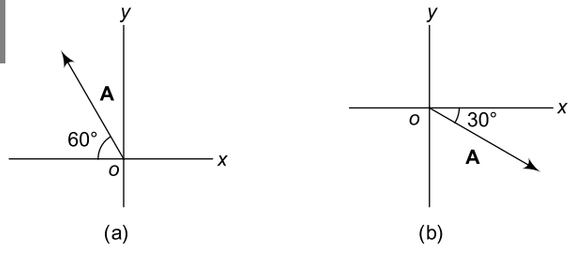

Fig. 3.4

SOLUTION

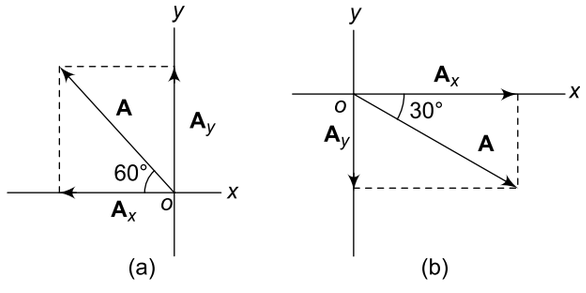

Fig. 3.5

In Fig. 3.5(a):

$$\mathbf{A}_x = -\ \mathbf{A}\ \cos\ 60° = -\ 4 \times \frac{1}{2} = -\ 2\ \text{units}$$

$$\mathbf{A}_y = +\ \mathbf{A}\ \sin\ 60° = +\ 4 \times \frac{\sqrt{3}}{2} = +\ 2\sqrt{3}\ \text{units}$$

In Fig. 3.5(b):

$$\mathbf{A}_x = +\ \mathbf{A}\ \cos 30° = +\ 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}\ \text{units}$$

$$\mathbf{A}_y = -\ \mathbf{A}\ \sin 30° = -\ 4 \times \frac{1}{2} = -\ 2\ \text{units}$$

NOTE

The magnitude of a component along + x direction or + y direction is taken to be positive while the magnitude of a component along – x direction or – y direction is taken to be negative.

3.8 ADDITION OF VECTORS USING COMPONENTS

Two or more vectors are added by using components as follows:

(a) Resolve each vector into its rectangular components.

(b) Add the magnitudes of all the x-components taking into account their signs.

R_x = sum of x-components of all the vectors

(c) Similarly R_y = sum of y-components of all the vectors

(d) Magnitude of resultant is

$$R = \sqrt{R_x^2 + R_y^2}$$

(e) The angle θ which the resultant subtends with the x-axis is given by

$$\tan\ \theta = \frac{R_y}{R_x}$$

EXAMPLE 3.4

Find the resultant of two vectors **A** = 4 units and **B** = 3 units shown in Fig. 3.6 by using

(a) parallelogram law and

(b) components method.

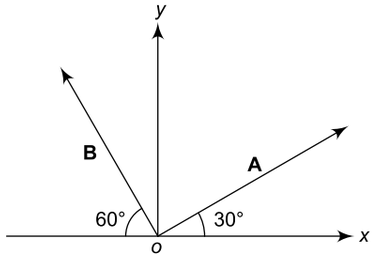

Fig. 3.6

SOLUTION

(a) Angle between the two vectors is (Fig. 3.7)

$$\theta = 90°$$

$$\therefore \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3\cos 90°}$$

$$= 5\ \text{units}$$

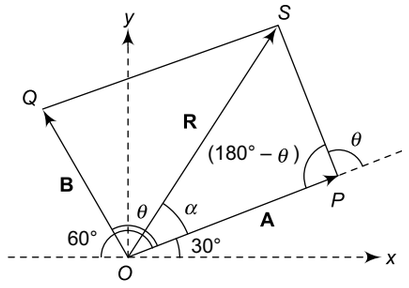

Fig. 3.7

$$\frac{R}{\sin(180^\circ-\theta)} = \frac{B}{\sin\alpha}$$

$$\Rightarrow \quad \sin\alpha = \frac{B\sin\theta}{R} = \frac{3\times\sin 90^\circ}{5} = \frac{3}{5}$$

$$\therefore \qquad \alpha = \sin^{-1}\left(\frac{3}{5}\right) \quad \Rightarrow \quad \alpha \simeq 37^\circ$$

(b) The x and y components of **A** and **B** are [see Fig. 3.8]

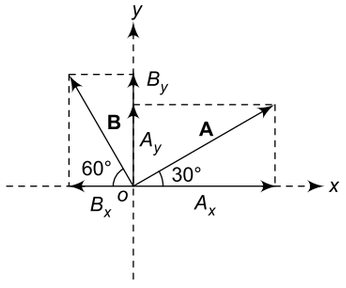

Fig. 3.8

$$A_x = A\cos 30^\circ = 4\times\frac{\sqrt{3}}{2}$$

$$= 2\sqrt{3} \text{ units along } +x \text{ direction}$$

$$A_y = A\sin 30^\circ = 4\times\frac{1}{2}$$

$$= 2 \text{ units along } +y \text{ direction}$$

$$\text{and } B_x = B\cos 60^\circ = -3\times\frac{1}{2}$$

$$= -3/2 \text{ units along } -x \text{ direction}$$

$$\text{and } B_y = B\sin 60^\circ$$

$$= 3\times\frac{\sqrt{3}}{2} \text{ units along } +y \text{ direction}$$

$$\therefore \quad R_x = A_x + B_x$$

$$= \left(2\sqrt{3} - 3/2\right) \text{ along } +x \text{ direction}$$

$$R_y = A_y + B_y$$

$$= \left(2+\frac{3}{2}\sqrt{3}\right) \text{along } +y \text{ direction}$$

$$R^2 = R_x^2 + R_y^2 = \left(2\sqrt{3}-\frac{3}{2}\right)^2 + \left(2+\frac{3\sqrt{3}}{2}\right)^2$$

$$= 25$$

$$\Rightarrow \quad R = 5 \text{ units}$$

The angle subtended by the resultant with the x-axis is given by

$$\tan\alpha = \frac{R_y}{R_x} = \frac{\left(2+\frac{3\sqrt{3}}{2}\right)}{\left(2\sqrt{3}-\frac{3}{2}\right)} = \frac{4.6}{1.96} = 2.347$$

giving $\alpha \simeq 67^\circ$ above the x-axis. Therefore, the angle which the resultant subtends with **A** is $67^\circ - 30^\circ = 37^\circ$.

NOTE

The components method is more useful if more than two vectors have to be added.

3.9 SCALAR OR DOT PRODUCT

The scalar (or dot) product of two vectors **A** and **B** is defined as the product of the magnitudes of **A** and **B** and the cosine of the angle between them, i.e.

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$

The scalar product is represented by putting a dot between the two vectors. The scalar product of two vectors is a scalar quantity.

Properties of the Scalar Product

(i) The scalar product is commutative, i.e. $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$

(ii) $\mathbf{A}\cdot\mathbf{A} = A^2$ or $A = \sqrt{\mathbf{A}\cdot\mathbf{A}}$. This is true because in this case $\theta = 0^\circ$.

(iii) If two vectors **A** and **B** are perpendicular to each other, then $\theta = 90^\circ$ and $\mathbf{A}\cdot\mathbf{B} = AB\cos 90^\circ = 0$. Note that $\mathbf{A}\cdot\mathbf{B}$ can be zero when neither **A** nor **B** is zero.

(iv) For unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ along the three rectangular axes x, y and z, we have

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$

and $$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$$

(v) $\mathbf{A}\cdot\mathbf{B} = (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}})\cdot(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})$

$$= A_xB_x + A_yB_y + A_zB_z$$

Some Examples of Scalar Product

(i) Work done is $W = \mathbf{F} \cdot \mathbf{s}$ where **F** is the force vector and **s** is the displacement vector.
(ii) Power consumed is $P = \mathbf{F} \cdot \mathbf{v}$ where **F** is the force vector and **v** is the velocity vector.
(iii) Electric current is $I = \mathbf{J} \cdot \mathbf{A}$ where **J** is the current density vector and **A** is the area vector.
(iv) Magnetic flux is $\Phi = \mathbf{B} \cdot \mathbf{A}$ where **B** is the magnetic induction vector and **A** is the area vector.

3.10 VECTOR OR CROSS PRODUCT

If the smaller angle between two vectors **A** and **B** is θ, then the vector or cross product of vectors **A** and **B** is defined as

$$\mathbf{A} \times \mathbf{B} = (AB \sin \theta)\ \hat{\mathbf{n}}$$

where A and B are the magnitudes of vectors **A** and **B** and $\hat{\mathbf{n}}$ is a unit vector perpendicular to the plane containing **A** and **B**. The vector product of two vectors **A** and **B** is equal to a vector **C**, i.e.

$$\mathbf{A} \times \mathbf{B} = \mathbf{C}$$

The magnitude of vector **C** is given by

$$C = AB \sin \theta$$

The direction of **C** is perpendicular to the plane formed by **A** and **B** and is given by the right hand screw rule.

Properties of a Vector Product

(i) Vector product is anticommutative, i.e.
$(\mathbf{A} \times \mathbf{B}) = -(\mathbf{B} \times \mathbf{A})$
(ii) $\mathbf{A} \times \mathbf{A} = 0$, i.e. the vector product of a vector by itself is zero. This is because, in this case, $\theta = 0$, and hence $\sin \theta = 0$.
Therefore $\mathbf{A} \times \mathbf{A} = AA \sin \theta = 0$
Hence, the condition for two vectors to be parallel ($\theta = 0°$) or antiparallel ($\theta = 180°$) is that their vector product should be zero.
If $\mathbf{A} \times \mathbf{B} = 0$, it means either (i) **A** is zero or, (ii) **B** = 0 or (iii) the angle θ between them is 0° or 180°.
(iii) The distributive law holds for both scalar and vector products, i.e.
$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$
$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$
(iv) $(\lambda\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (\lambda\mathbf{B}) = \lambda(\mathbf{A} \times \mathbf{B})$; λ a real number.
(v) $|\mathbf{A} \times \mathbf{B}|^2 = |\mathbf{A}|^2 |\mathbf{B}|^2 - (\mathbf{A} \cdot \mathbf{B})^2$.
(vi) $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are the three mutually perpendicular unit vectors at the origin O and along OX, OY and OZ respectively; the right-hand rule gives:

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}},$$
$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$
$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}},$$
$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

(vii) $$(\mathbf{A} \times \mathbf{B}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{\mathbf{i}}\ (A_yB_z - A_zB_y) + \hat{\mathbf{j}}\ (A_zB_x - A_xB_z) + \hat{\mathbf{k}}\ (A_xB_y - A_yB_x)$$

Some Examples of Vector Product

(i) Torque = $\mathbf{r} \times \mathbf{F}$, where **r** is the position vector and **F** is the force vector.
(ii) Linear velocity = $\boldsymbol{\omega} \times \mathbf{r}$ where $\boldsymbol{\omega}$ is the angular frequency vector and **r** is the position vector.
(iii) Angular momentum = $\mathbf{r} \times \mathbf{p}$ where **r** is the position vector and **p** is the linear momentum vector.

EXAMPLE 3.5

Find the angle between vectors $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$.

SOLUTION

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$$

Magnitude of **A** is $A = \sqrt{1^2 + 1^2} = \sqrt{2}$

Magnitude of **B** is $B = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$$\therefore \quad (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}}) = \sqrt{2} \times \sqrt{2} \cos\theta$$

$$\Rightarrow \quad \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} - \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = 2 \cos\theta \qquad (\because\ \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{i}} = 0)$$

$$\Rightarrow \quad 1 - 1 = 2 \cos\theta \text{ gives } \cos\theta = 0 \text{ or } \theta = 90°$$

EXAMPLE 3.6

Two vectors **A** and **B** are inclined at an angle θ. Using definition of scalar product, show that the magnitude of the resultant of vectors **A** and **B** is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

SOLUTION

$\mathbf{R} = \mathbf{A} + \mathbf{B}$. Therefore,

$$\mathbf{R} \cdot \mathbf{R} = (\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} + \mathbf{B})$$

$$= \mathbf{A} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{B} + \mathbf{B} \cdot \mathbf{A} + \mathbf{B} \cdot \mathbf{B}$$

$$\Rightarrow \quad R^2 = A^2 + 2\,\mathbf{A} \cdot \mathbf{B} + B^2$$

$$= A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

EXAMPLE 3.7

Given $\mathbf{A} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ and $\mathbf{B} = 5\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$. Find the magnitude and direction of $(\mathbf{A} \times \mathbf{B})$.

SOLUTION

$$(\mathbf{A} \times \mathbf{B}) = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}) \times (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$$

$$= 10\,\hat{\mathbf{i}} \times \hat{\mathbf{i}} + 8\,\hat{\mathbf{i}} \times \hat{\mathbf{j}} - 15\,\hat{\mathbf{j}} \times \hat{\mathbf{i}} - 12\,\hat{\mathbf{j}} \times \hat{\mathbf{j}}$$

$$(\because \hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = 0 \text{ and } \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}})$$

$$= 0 + 8\hat{\mathbf{k}} + 15\hat{\mathbf{k}} - 0$$

$$= 23\ \hat{\mathbf{k}}$$

Thus the magnitude of $(\mathbf{A} \times \mathbf{B})$ is 23 units and its direction is along + z axis.

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. The unit vector parallel to the resultant of vectors $\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is

(a) $\frac{1}{\sqrt{14}}(3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ (b) $\frac{1}{3}(2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

(c) $\frac{1}{\sqrt{17}}(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ (d) $\frac{1}{\sqrt{14}}(3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

2. The value of n so that vectors $\mathbf{A} = 4\hat{\mathbf{i}} + n\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ are perpendicular to each other is

(a) – 1 (b) – 2
(c) – 3 (d) – 4

3. The angle between vectors $\mathbf{A} = \hat{\mathbf{i}} - 5\hat{\mathbf{j}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} - 10\hat{\mathbf{j}}$ is

(a) 0° (b) 90°
(c) 120° (d) 150°

4. The angle between vectors $\mathbf{A}$ and $\mathbf{B}$ is 60°. The ratio $\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

5. The magnitude of the resultant of two equal vectors is equal to the magnitude of either vector. The angle between the two vectors is

(a) 60° (b) 90°
(c) 120° (d) 150°

6. Given $\mathbf{P} = \mathbf{A} + \mathbf{B}$ and $\mathbf{Q} = \mathbf{A} - \mathbf{B}$. If the magnitudes of $\mathbf{P}$ and $\mathbf{Q}$ are equal, the angle between vectors $\mathbf{A}$ and $\mathbf{B}$ is

(a) zero (b) 45°
(c) 90° (d) 180°

7. A vector $\mathbf{C}$ when added to vectors $\mathbf{A} = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ gives a unit vector along the y-axis as their resultant. Then $\mathbf{C}$ is

(a) $-5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ (b) $5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

(c) $-5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ (d) $5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

8. If the sum $\mathbf{A} + \mathbf{B}$ of two vectors $\mathbf{A}$ and $\mathbf{B}$ equals the difference $\mathbf{A} - \mathbf{B}$ between them, then

(a) $\mathbf{A}$ is a null vector
(b) $\mathbf{B}$ is a null vector
(c) both $\mathbf{A}$ and $\mathbf{B}$ are null vectors
(d) neither $\mathbf{A}$ nor $\mathbf{B}$ is a null vector

9. Given $\mathbf{A} \cdot \mathbf{B} = 0$ and $\mathbf{A} \times \mathbf{C} = 0$. What is the angle between $\mathbf{B}$ and $\mathbf{C}$?

(a) 45° (b) 90°
(c) 135° (d) 180°

10. The resultant of two vectors **A** and **B** subtends an angle of 45° with either of them. The magnitude of the resultant is
(a) zero (b) $\sqrt{2}\ A$
(c) A (d) $2\ A$

11. **A** and **B** are two vectors in a plane at an angle of 60° with each other. **C** is another vector perpendicular to the plane containing vectors **A** and **B**. Which of the following relations is possible?
(a) $\mathbf{A} + \mathbf{B} = \mathbf{C}$ (b) $\mathbf{A} + \mathbf{C} = \mathbf{B}$
(c) $\mathbf{A} \times \mathbf{B} = \mathbf{C}$ (d) $\mathbf{A} \times \mathbf{C} = \mathbf{B}$

12. Vector **C** is the sum of two vectors **A** and **B** and vector **D** is the cross product of vectors **A** and **B**. What is the angle between vectors **C** and **D**?
(a) zero (b) 60°
(c) 90° (d) 180°

13. The resultant of two vectors of magnitudes 3 units and 4 units is 1 unit. What is the value of their dot product?
(a) – 12 units (b) – 7 units
(c) – 1 unit (d) zero

14. The resultant of two vectors of magnitudes 3 units and 4 units is 1 unit. What is the magnitude of their cross product?
(a) 12 units (b) 7 units
(c) 1 unit (d) zero

15. Three vectors **A**, **B** and **C** are related as $\mathbf{A} + \mathbf{C} = \mathbf{B}$. If vector **C** is perpendicular to vector **A** and the magnitude of **C** is equal to the magnitude of **A**, what will be the angle between vectors **A** and **B**?
(a) 45° (b) 90°
(c) 135° (d) 180°

16. The magnitude of the resultant of $(\mathbf{A} + \mathbf{B})$ and $(\mathbf{A} - \mathbf{B})$ is
(a) $2A$ (b) $2B$
(c) $\sqrt{A^2 + B^2}$ (d) $\sqrt{A^2 - B^2}$

17. In Q.16, what is the angle between the resultant vector and vector **A**?
(a) zero (b) $\cos^{-1}\left(\frac{A}{B}\right)$
(c) $\cos^{-1}\left(\frac{B}{A}\right)$ (d) $\cos^{-1}\left(\frac{A-B}{A+B}\right)$

18. If $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along x-axis and y-axis respectively, the magnitude of vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ will be
(a) 1 (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) 2

19. In Q.18, the angle subtended by vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ with the x-axis is
(a) 30° (b) 45°
(c) 60° (d) 75°

20. If $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors along x, y and z-axes respectively, the angle θ between the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and vector $\hat{\mathbf{i}}$ is given by
(a) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(c) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

21. Given that $0.2\ \hat{\mathbf{i}} + 0.6\ \hat{\mathbf{j}} + a\hat{\mathbf{k}}$ is a unit vector. What is the value of a?
(a) $\sqrt{0.3}$ (b) $\sqrt{0.4}$
(c) $\sqrt{0.6}$ (d) $\sqrt{0.8}$

22. Given $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{k}}$. What is the value of the scalar product of **A** and **B**?
(a) 1 (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) 2

23. The cross product of vectors **A** and **B** in Q. 22 is
(a) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ (b) $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$
(c) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

24. Given $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$. The component of vector **A** along vector **B** is
(a) $\frac{1}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$ (b) $\frac{3}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$
(c) $\frac{5}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$ (d) $\frac{7}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$

25. In Q.24, above, what is the component of vector **A** perpendicular to vector **B** and in the same plane as **B**?
(a) $\frac{1}{\sqrt{2}}\left(\hat{\mathbf{j}} - \hat{\mathbf{i}}\right)$ (b) $\frac{3}{\sqrt{2}}\left(\hat{\mathbf{j}} - \hat{\mathbf{i}}\right)$
(c) $\frac{5}{\sqrt{2}}\left(\hat{\mathbf{j}} - \hat{\mathbf{i}}\right)$ (d) $\frac{7}{\sqrt{2}}\left(\hat{\mathbf{j}} - \hat{\mathbf{i}}\right)$

26. The magnitudes of vectors **A**, **B** and **C** are respectively 12, 5 and 13 units and $\mathbf{A} + \mathbf{B} = \mathbf{C}$. The angle between **A** and **B** is
(a) zero (b) π
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

27. The sum of two forces acting at a point is 16 N. If the resultant force is 8 N and its direction is perpen-

dicular to the smaller force, then the forces are

(a) 6 N and 10 N (b) 8 N and 8 N

(c) 4 N and 12 N (d) 2 N and 14 N

28. Vector **A** of magnitude 4 units is directed along the positive x-axis. Another vector **B** of magnitude 3 units lies in the x-y plane and is directed along 30º with the positive x-axis is as shown in Fig. 3.9. The $\mathbf{A} \cdot \mathbf{B}$ is

(a) 6 units (b) $6\sqrt{2}$ units

(c) $6\sqrt{3}$ units (d) 12 units

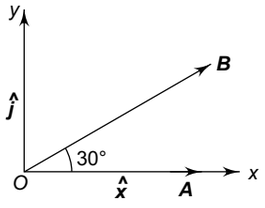

Fig. 3.9

29. In Q. 28, the magnitude of $\mathbf{A} \times \mathbf{B}$ is

(a) 6 units (b) $6\sqrt{2}$ units

(c) $6\sqrt{3}$ units (d) 12 units

30. The angle θ between vectors $\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ is given by

(a) $\cos\theta = \frac{3}{7}$ (b) $\cos\theta = \frac{2}{7}$

(c) $\cos\theta = \frac{1}{7}$ (d) zero

31. Two vectors $\mathbf{C} = \mathbf{A} + \mathbf{B}$ and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ are perpendicular to each other. Then

(a) **A** is parallel to **B**

(b) **A** is perpendicular to **B**

(c) **B** is a null vector

(d) **A** and **B** have equal magnitudes.

32. A body moves from a position $\mathbf{r}_1 = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}})$ metre to a position $\mathbf{r}_2 = (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ metre under the influence of a constant force $\mathbf{F} = (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 6\hat{\mathbf{k}})$ newton. The work done by the force is

(a) 60 J (b) 59 J

(c) 58 J (d) 57 J

33. A vector **A** is along the positive z-axis and its vector product with another vector **B** is zero, then vector **B** could be

(a) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ (b) $4\hat{\mathbf{i}}$

(c) $\hat{\mathbf{j}} + \hat{\mathbf{k}}$ (d) $-7\hat{\mathbf{k}}$

34. A body, initially at rest, is acted upon by four forces $\mathbf{F}_1 = \hat{\mathbf{i}} + \hat{\mathbf{k}}$, $\mathbf{F}_2 = 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $\mathbf{F}_3 = 3\hat{\mathbf{i}}$ and $\mathbf{F}_4 = 3\hat{\mathbf{j}} - 4\hat{\mathbf{i}}$. In which plane will the body move?

(a) $x - y$ plane (b) $x - z$ plane

(c) $y - z$ plane (b) none of these

35. **A** is a vector which when added to the resultant of vectors $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ and $(\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ yields a unit vector along the y-axis. Then vector **A** is

(a) $-3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ (b) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 6\hat{\mathbf{k}}$

(c) $3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ (d) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

36. The angle between two vectors **A** and **B** is θ. Vector **R** is the resultant of the two vectors. If **R** makes an angle $\frac{\theta}{2}$ with **A**, then

(a) $A = 2B$ (b) $A = \frac{B}{2}$

(c) $A = B$ (d) $AB = 1$

37. What is the torque of a force $\mathbf{F} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ newton acting at a point $\mathbf{r} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ metre about the origin?

(a) $6\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$ (b) $17\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 13\hat{\mathbf{k}}$

(c) $-6\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 12\hat{\mathbf{k}}$ (d) $-17\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 13\hat{\mathbf{k}}$

ANSWERS

1. (d)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (c)
7. (a)	**8.** (b)	**9.** (b)	**10.** (b)	**11.** (c)	**12.** (c)
13. (a)	**14.** (d)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (b)
19. (b)	**20.** (a)	**21.** (c)	**22.** (a)	**23.** (d)	**24.** (c)
25. (a)	**26.** (c)	**27.** (a)	**28.** (c)	**29.** (a)	**30.** (a)
31. (d)	**32.** (d)	**33.** (d)	**34.** (c)	**35.** (a)	**36.** (c)
37. (b)					

SOLUTIONS

1. The resultant of **A** and **B** is

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$= (3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$

$\therefore$ Magnitude of **R** is $\mathrm{R} = \sqrt{3^2 + 1^2 + (-2)^2} = \sqrt{14}$

$$\hat{\mathbf{R}} = \frac{\mathbf{R}}{\mathrm{R}} = \frac{1}{\sqrt{14}}(3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$

So the correct choice is (d).

2. **A** and **B** will be perpendicular to each other if $\mathbf{A} \cdot \mathbf{B} = 0$, i.e.

$$(4\hat{\mathbf{i}} + n\hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 0$$

$$\Rightarrow \quad 8 + 3n - 2 = 0 \quad \Rightarrow \quad n = -2$$

3. $\mathbf{B} = 2(\hat{\mathbf{i}} - 5\hat{\mathbf{j}}) = 2\mathbf{A}$. Hence the magnitude of **B** is twice that of **A** and the direction of **B** is the same as that of **A**. So the correct choice is (a).

4. $$\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|} = \frac{AB\cos\theta}{AB\sin\theta} = \cot\theta.$$

$$= \cot 60° \qquad (\because \theta = 60°)$$

$$= \frac{1}{\sqrt{3}}$$

5. $R^2 = A^2 + B^2 + 2AB\cos\theta$. It is given that $R = A = B$. Thus, we have

$$A^2 = A^2 + A^2 + 2A^2\cos\theta$$

$$\Rightarrow \qquad \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$$

6. If θ is the angle between **A** and **B**, the magnitudes of **P** and **Q** are given by

$$P = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

and $$Q = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Given $P = Q$. If follows that $\cos\theta = 0$ or $\theta = 90°$.

7. $$\hat{\mathbf{j}} = \mathbf{C} + \mathbf{A} + \mathbf{B} = \mathbf{C} + (3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + \hat{\mathbf{k}}) + (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}})$$

$$= \mathbf{C} + 5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

$$\Rightarrow \qquad \mathbf{C} = -5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

8. Given $\mathbf{A} + \mathbf{B} = \mathbf{A} - \mathbf{B}$ which gives $\mathbf{B} = -\mathbf{B}$. This is possible only if **B** is a null vector. Hence the correct choice is (b).

9. Since $\mathbf{A} \cdot \mathbf{B} = 0$, it follows that **A** is perpendicular to **B**. Also $\mathbf{A} \times \mathbf{C} = 0$. Therefore **A** is parallel to **C**. Hence **B** is perpendicular to **C**. Therefore, the correct choice is (b).

10. As shown in Fig. 3.10, the angle θ between vectors **A** and **B** is 90°. Also $A = B$. Therefore, the magnitude of the resultant is given by

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$= A^2 + A^2 + 2A^2\cos 90° = 2A^2$

or $\qquad R = \sqrt{2}\,A.$

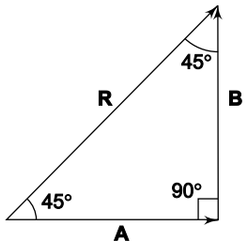

Fig. 3.10

Hence the correct choice is (b).

11. Since **C** is perpendicular to both **A** and **B**, the sum of any two cannot yield the third vector. Hence choices (a) and (b) are not possible. Choice (d) is also not possible because **B** is not perpendicular to **A**. Choice (c) is possible.

12. Vector **C** lies in the plane containing vectors **A** and **B**, and vector **D** is perpendicular to both **A** and **B**. Hence **D** must be perpendicular to **C**. Hence the correct choice is (c).

13. Let θ be the angle between the two vectors. The resultant is given by

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

Putting the values of R, A and B we get

$$(1)^2 = (3)^2 + (4)^2 + 2 \times 3 \times 4 \times \cos\theta$$

or $\qquad \cos\theta = -1$ or $\theta = 180°$

Now $\qquad \mathbf{A} \cdot \mathbf{B} = AB\cos\theta = 3 \times 4 \times \cos 180°$

$$= -12$$

Hence the correct choice is (a).

14. The magnitude of $\mathbf{A} \times \mathbf{B} = AB\sin\theta = 3 \times 4 \times \sin 180° = 0$. Hence the correct choice is (d).

15. Since $\mathbf{A} + \mathbf{C} = \mathbf{B}$, vector **B** is the resultant of vectors **A** and **C**. Using the triangle law of vector addition (see Fig. 3.11), we have $\theta = 45°$ ($\because A = C$)

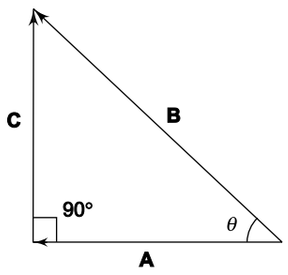

Fig. 3.11

Thus the correct choice is (a).

16. The resultant **R** of vectors (**A** + **B**) and (**A** – **B**) is
R = (**A** + **B**) + (**A** – **B**) = 2**A**
$\therefore$ The magnitude of the resultant = 2A. Hence the correct choice is (a)

17. Since **R** = 2**A**, **R** is parallel to **A**. Hence the correct choice is (a).

18. Let $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ represent the magnitudes of vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ respectively. Since $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors, $\hat{i} = 1$ and $\hat{j} = 1$. Therefore, the magnitude of vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} = \sqrt{\hat{i}^2 + \hat{j}^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$.
Thus the correct choice is (b).

19. The angle subtended by vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ with the x-axis is given by

$$\tan\theta = \frac{\hat{i}}{\hat{j}} = \frac{1}{1} = 1$$

or $\theta = 45°$ which is choice (b).

20. $\cos\theta = \dfrac{\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right).\hat{\mathbf{i}}}{\left(\hat{i}^2 + \hat{j}^2 + \hat{k}^2\right)^{1/2} \times \hat{i}} = \dfrac{\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + \hat{\mathbf{k}}\cdot\hat{\mathbf{i}}}{(1+1+1)^{1/2} \times 1}$

$$= \frac{1+0+0}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

($\because \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$ and $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = 1$)
Hence the correct choice is (a).

21. Here $(0.2)^2 + (0.6)^2 + a^2 = 1$ or $a^2 = 1 - 0.04 - 0.36$, $= 0.6$ or $a = \sqrt{0.6}$. So the correct choice is (c).

22. $\mathbf{A}\cdot\mathbf{B} = \left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}} + \hat{\mathbf{k}}\right) = \hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} + \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + \hat{\mathbf{j}}\cdot\hat{\mathbf{k}}$
$= 1 + 0 + 0 + 0 = 1$
Thus the correct choice is (a).

23. $\mathbf{A}\times\mathbf{B} = \left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)\times\left(\hat{\mathbf{i}} + \hat{\mathbf{k}}\right)$

$$= \hat{\mathbf{i}}\times\hat{\mathbf{i}} + \hat{\mathbf{i}}\times\hat{\mathbf{k}} + \hat{\mathbf{j}}\times\hat{\mathbf{i}} + \hat{\mathbf{j}}\times\hat{\mathbf{k}}$$

$= 0 - \hat{\mathbf{j}} - \hat{\mathbf{k}} + \hat{\mathbf{i}}$ which is choice (d).

24. The component of vector **A** along vector $\mathbf{B} = (\mathbf{A}\cdot\hat{\mathbf{B}})\hat{\mathbf{B}}$ where $\hat{\mathbf{B}} = \dfrac{\mathbf{B}}{B}$ where B is the magnitude of vector **B**. Now

$$(\mathbf{A}\cdot\mathbf{B}) = \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$$

$$= 2\,\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + 2\,\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} + 3\,\hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}$$

$$= 2 + 0 + 0 + 3 = 5$$

Also $\hat{\mathbf{B}} = \dfrac{\mathbf{B}}{B} = \dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\hat{i} + \hat{j}}$

$$= \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$$

Thus the answer is $\dfrac{5}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$ which is choice (c).

25. Since $\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right) = 0$, vector $\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)$ is perpendicular to vector $\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$. Let $\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right) = \mathbf{C}$. Now

$(\mathbf{A}\cdot\mathbf{C}) = \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)$

The required component is

$$(\mathbf{A}\cdot\mathbf{C})\frac{\mathbf{C}}{C} = \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)\cdot\frac{\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\left(\hat{i} - \hat{j}\right)}$$

$$= -\frac{1}{\sqrt{2}}\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right) = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{j}} - \hat{\mathbf{i}}\right)$$

$\because\ i - j = \sqrt{1+1} = \sqrt{2}$ and $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}} - \hat{\mathbf{j}}) = -1$
Thus the correct choice is (a).

26. Since **A** + **B** = **C**, vector **C** is the resultant of vectors **A** and **B**. If the angle between **A** and **B** is θ, the magnitude of the resultant is given by

$$C^2 = A^2 + B^2 + 2AB\cos\theta$$

or $(13)^2 = (12)^2 + (5)^2 + 2 \times 12 \times 5 \times \cos\theta$

which gives $\cos\theta = 0$ or $\theta = \pi/2$. Hence the correct choice is (c).

27. As shown in the figure, **F** is the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ and $\mathbf{F}_1$ is the smaller force. Now

$$F_2^2 = F_1^2 + F^2 = F_1^2 + (8)^2 \qquad \text{(i)}$$

and $F_1 + F_2 = 16$ or $F_2 = 16 - F_1$ (ii)

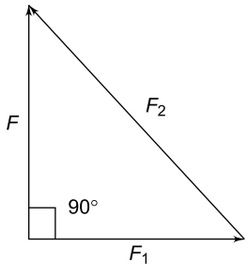

Fig. 3.12

Using (ii) in (i) we have $(16 - F_1)^2 = F_1^2 + 64$, which gives $F_1 = 6$ N.
Therefore $F_2 = 16 - 6 = 10$ N. Hence the correct choice is (a).

28. Let $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ be the unit vectors along positive x and y-axes respectively. Then

$$\mathbf{A} = 4\hat{\mathbf{i}} \text{ and } \mathbf{B} = 3\cos 30^\circ \hat{\mathbf{i}} + 3\sin 30^\circ \hat{\mathbf{j}}$$

$$= \frac{3\sqrt{3}}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}}$$

$$\mathbf{A} \cdot \mathbf{B} = 4\hat{\mathbf{i}} \cdot \left(\frac{3\sqrt{3}}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}}\right) = 6\sqrt{3}$$

$$(\because \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = 1; \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0)$$

29. $\mathbf{A} \times \mathbf{B} = 4\hat{\mathbf{i}} \times \left(\frac{3\sqrt{3}}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}}\right) = 0 + 6\hat{\mathbf{k}}$

$$(\because \hat{\mathbf{i}} \times \hat{\mathbf{i}} = 0; \hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}})$$

$$= 6\hat{\mathbf{k}}$$

Hence the correct choice is (a).

30. Angle θ between vectors **A** and **B** is given by

$$\cos\theta = \frac{\mathbf{A} \cdot \mathbf{B}}{AB}$$

$$\mathbf{A} \cdot \mathbf{B} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$
$$= (1)^2 + 2 \times -2 + (3)^2$$
$$= 1 - 4 + 9 = 6$$
$$A = \{(1)^2 + (2)^2 + (3)^2]^{1/2}$$
$$= (1 + 4 + 9)^{1/2} = \sqrt{14}$$
$$B = \left[(1)^2 + (-2)^2 + (3)^2\right]^{1/2} = \sqrt{14}$$
$$\cos\theta = \frac{6}{\sqrt{14} \times \sqrt{14}} = \frac{6}{14} = \frac{3}{7}$$

31. Since **C** and **D** are at right angles to each other, $\mathbf{C} \cdot \mathbf{D} = 0$ or $(\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B}) = 0$
or $A^2 - \mathbf{A} \cdot \mathbf{B} + \mathbf{B} \cdot \mathbf{A} - B^2 = 0$
or $A = B$ $(\because \mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A})$

32. Displacement $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$

$$= (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}})$$

$$= \left(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 9\hat{\mathbf{k}}\right) \text{ metre}$$

Work done $W = \mathbf{F} \cdot \mathbf{r}$

$= (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 6\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 9\hat{\mathbf{k}})$ newton metre

$= (4 - 1 + 54) = 57$ newton metre or 57 J.

33. The vector product of two non-zero vectors is zero if they are in the same direction. Hence, vector **B** must be parallel to vector **A**, i.e. along $\pm z$-axis.

34. Resultant force $= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$

$$= (\hat{\mathbf{i}} + \hat{\mathbf{k}}) + (2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + 3\hat{\mathbf{i}} + (3\hat{\mathbf{j}} - 4\hat{\mathbf{i}})$$
$$= 5\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

35. Given $\mathbf{A} + (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) + (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 1\hat{\mathbf{j}}$

36. The angle α which the resultant **R** makes with **A** is given by

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

given $\alpha = \frac{\theta}{2}$. Hence

$$\tan\left(\frac{\theta}{2}\right) = \frac{B\sin\theta}{A + B\cos\theta}$$

or $$\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{2B\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{A + B\cos\theta}$$

which gives $A = B$.

37. Torque $= \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$

$$= \hat{\mathbf{i}}\,[8 - (-9)] - \hat{\mathbf{j}}\,(12 - 6) + \hat{\mathbf{k}}\,(-9 - 4)$$

$$= 17\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 13\hat{\mathbf{k}}$$

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Two vectors of the same physical quantity are unequal if
   (a) they have the same magnitude and the same direction
   (b) they have different magnitudes but the same direction
   (c) they have the same magnitude but different directions
   (d) they have different magnitudes and different directions.

2. Given $\mathbf{A} = -\mathbf{B}$. This means that vectors **A** and **B**
   (a) have equal magnitudes
   (b) have unequal magnitudes
   (c) are in opposite directions
   (d) are in the same direction.

3. Which of the following is a null vector?
   (a) Velocity vector of a body moving in a circle with a uniform speed
   (b) Velocity vector of a body moving in a straight line with a uniform speed
   (c) Position vector of the origin of a rectangular coordinate system
   (d) Displacement vector of a stationary object

4. The magnitudes of four pairs of displacement vectors are given. Which pairs of displacement vectors cannot be added to give a resultant vector of magnitude 4 cm?
   (a) 1 cm, 1 cm (b) 1 cm, 3 cm
   (c) 1 cm, 5 cm (d) 1 cm, 7 cm

5. The dot product of two vectors **A** and **B** is zero if
   (a) **A** is a null vector and **B** a proper vector
   (b) **A** is a proper vector and **B** is a null vector
   (c) **A** and **B** are both null vectors
   (d) **A** and **B** are proper vectors perpendicular to each other.

6. The cross product of two vectors **A** and **B** is zero if
   (a) **A** is a null vector and **B** is a proper vector
   (b) **A** is a proper vector and **B** is a null vector
   (c) **A** and **B** are both null vectors
   (d) **A** and **B** are proper vectors parallel to each other.

7. If $\mathbf{A} \times \mathbf{B} = \mathbf{C}$, which of the following statements is/are correct?
   (a) **C** is perpendicular to **A**
   (b) **C** is perpendicular to **B**
   (c) **C** is perpendicular to both **A** and **B**
   (d) **C** is parallel to both **A** and **B**

8. Which of the following vector identities is/are false?
   (a) $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$ (b) $\mathbf{A} \cdot \mathbf{B} = -\mathbf{B} \cdot \mathbf{A}$
   (c) $\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{A}$ (d) $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$

9. **A** and **B** are two perpendicular vectors in a plane. **C** is another vector perpendicular to the plane containing vectors **A** and **B**. Which of the following relations is/are possible?
   (a) $\mathbf{A} + \mathbf{B} = \mathbf{C}$ (b) $\mathbf{A} + \mathbf{C} = \mathbf{B}$
   (c) $\mathbf{A} \times \mathbf{B} = \mathbf{C}$ (d) $\mathbf{A} \times \mathbf{C} = \mathbf{B}$

ANSWERS AND SOLUTIONS

1. The correct choices are (b), (c) and (d).

2. Given $\mathbf{A} = -\mathbf{B}$, i.e. $\mathbf{A} + \mathbf{B} = 0$. Two vectors add up to zero only if they have equal magnitudes and opposite directions. Hence the correct choices are (a) and (c).

3. The correct choices are (c) and (d)

4. The magnitude R of the resultant of two vectors **A** and **B** depends upon the magnitudes of **A** and **B** and the angle θ between them and is given by

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

When $\theta = 0$, R is maximum given by

$$R_{\max}^2 = A^2 + B^2 + 2AB$$

or $$R_{\max} = A + B$$

When $\theta = 180°$, R is minimum given by

$$R_{\min}^2 = A^2 + B^2 - 2AB$$

or $$R_{\min} = A - B$$

Thus, the magnitude of resultant will lie between $A - B$ and $A + B$. Hence the correct choices are (a) and (d).

5. $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta = 0$ if $A = 0$ or $B = 0$ or $\theta = 90°$. Hence all the four choices are correct.

6. $\mathbf{A} \times \mathbf{B} = AB\sin\theta = 0$ if $A = 0$ or $B = 0$ or $\theta = 0°$. Hence all the four choices are correct.

7. The correct choices are (a), (b) and (c).

8. The scalar product is commulative, i.e. $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$. Vector product is anti–commutative, i.e. $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$. Hence choices (b) and (c) are false.

9. Since **C** is perpendicular to both **A** and **B**, the sum of any two cannot yield the third vector. Hence choices (a) and (b) are not possible. Since **A** is perpendicular to **B**, the three vectors are mutually perpendicular. Hence choices (c) and (d) are possible.

SECTION III

Matching

1. Match the following.

Column I	*Column II*
(a) $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$.	(p) False
(b) $\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{A}$	(q) **A** and **B** are perpendicular to each other
(c) $\mathbf{A} \cdot \mathbf{B} = 0$	(r) **A** and **B** are parallel to each other
(d) $\mathbf{A} \times \mathbf{B} = 0$	(s) True

ANSWER

1. (a) → (s) (b) → (p)
(c) → (q) (d) → (r)

4 Chapter Motion in Two Dimensions

REVIEW OF BASIC CONCEPTS

4.1 PROJECTILE MOTION

Projectile is the name given to a body which, after having been given an initial velocity, is allowed to move under the influence of gravity alone.

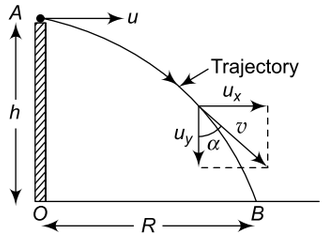

Fig. 4.1

(i) *A body projected horizontally with a velocity u from a height h*. [Fig. 4.1] Horizontal and vertical distances covered in time *t* are

$$x = ut \tag{4.1}$$

$$y = \frac{1}{2}gt^2 \tag{4.2}$$

Differentiating Eqs. (4.1) and (4.2) w.r.t time *t*, we get the horizontal and vertical velocities.

$$v_x = \frac{dx}{dt} = u \tag{4.3}$$

$$v_y = \frac{dy}{dt} = gt \tag{4.4}$$

Equation to trajectory

Eliminating *t* from Eqs. (4.1) and (4.2), we get

$$y = \left(\frac{g}{2u^2}\right)x^2$$

Since $y \propto x^2$, the trajectory of the body is parabolic.

Time of flight (t_f)

Putting $y = h$ and $t = t_f$ in Eq. (4.2), we get

$$t_f = \sqrt{\frac{2h}{g}}$$

Horizontal range (R)

Putting $t = t_f$ and $x = R$ in Eq. (4.1), we get

$$R = u\,t_f = u\sqrt{\frac{2h}{g}}$$

Magnitude of resultant velocity at time *t* is

$$v = \left(u^2 + g^2\,t^2\right)^{1/2}$$

The angle α which the resultant velocity vector subtends with the vertical is given by

$$\tan\,\alpha = \frac{u}{gt}$$

(ii) *A body projected from the ground with a velocity u at an angle θ with the horizontal*. (Fig. 4.2)

The horizontal and vertical distances covered in time *t* are

$$x = (u\,\cos\,\theta)t \tag{4.5}$$

and

$$y = (u\,\sin\,\theta)t - \frac{1}{2}gt^2 \tag{4.6}$$

Horizontal and vertical components of the velocity at time *t* are

$$v_x = u\,\cos\,\theta \tag{4.7}$$

and

$$u_y = u\,\sin\,\theta - gt \tag{4.8}$$

Magnitude of resultant velocity at time *t* is

$$v = \left(v_x^2 + v_y^2\right)^{1/2}$$

The angle α subtended by the resultant velocity vector with the horizontal is given by

$$\tan\ \alpha = \frac{v_y}{v_x}$$

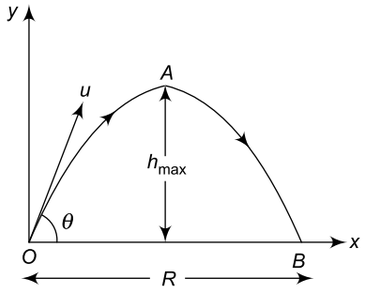

Fig. 4.2

Equation of trajectory

Eliminating t from Eqs. (4.5) and (4.6), we get

$$y = (\tan\ \theta)x - \frac{gx^2}{2u^2\ \cos^2\theta}$$

Time of flight (t_f)

Putting $y = 0$ and $t = t_f$ in Eq. (4.6), we get

$$t_f = \frac{2u\ \sin\theta}{g}$$

Maximum height attained (h_{max})

Put $t = t_f/2$ and $y = h_{max}$ in Eq. (4.6), we get

$$h_{max} = \frac{u^2\ \sin^2\theta}{2g}$$

Horizontal range (R)

Putting $t = t_f$ and $x = R$ in Eq. (4.5) we get

$$R = \frac{u^2\ \sin(2\theta)}{g}$$

(iii) *A body projected from a height h with a velocity u at an angle θ with the horizontal. (Fig. 4.3)*

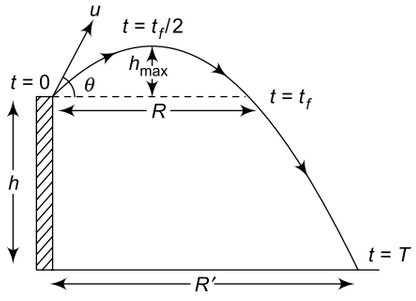

Fig. 4.3

If T is the total time of flight, then we have

$$h = u_y T - \frac{1}{2} g T^2 = (u \sin\theta) T - \frac{1}{2} g T^2$$

which gives $T = \dfrac{u \sin\theta}{g} + \left(\dfrac{u^2 \sin^2\theta}{g^2} + \dfrac{2h}{g} \right)^{1/2}$

The horizontal range is $R' = (u \cos\theta)T$

Applications

(i) The horizontal range is the same for angles θ and $(90° - \theta)$.

(ii) The horizontal range is maximum for $\theta = 45°$. $R_{max} = u^2/g$

(iii) When horizontal range is maximum, $h_{max} = \dfrac{R_{max}}{4}$

(iv) At the point of projection, KE $= \dfrac{1}{2} mu^2$, PE = 0. Total energy $E = \dfrac{1}{2} mu^2$.

(v) At the highest point, KE $= \dfrac{1}{2} mu^2\ \cos^2\theta$

and PE = total energy − KE $= \dfrac{1}{2} mu^2 - \dfrac{1}{2} mu^2 \cos^2\theta$

$= \dfrac{1}{2} mu^2 \sin^2\theta.$

(vi) To find R and h_{max} from the equation of trajectory

$$y = ax - bx^2$$

where a and b are constants, refer to Fig. 4.4.

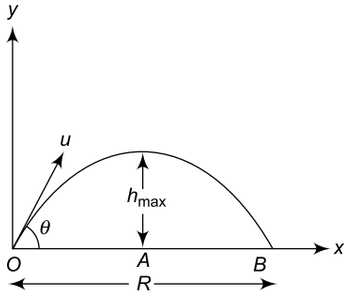

Fig. 4.4

(a) At O and B, $y = 0$. Putting $y = 0$ in the above equation, we have $0 = ax - bx^2 \Rightarrow x = 0$, $x = a/b$. Therefore $R = \dfrac{a}{b}$.

(b) At A, $y = h_{max}$ and $x = \dfrac{R}{2} = \dfrac{a}{2b}$. Using these values in $y = ax - bx^2$, we get $h_{max} = \dfrac{a^2}{4b}$.

(vii) If A and B are two points at the same horizontal level on a trajectory at a height h from the ground, (see Fig. 4.5), then

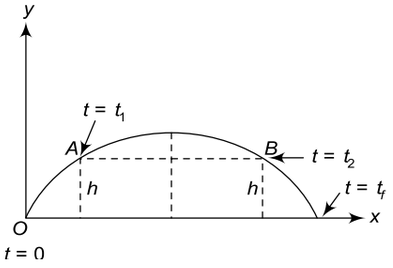

Fig. 4.5

(a) $t_f = \dfrac{2u\sin\theta}{g} = t_1 + t_2$

(b) $h = \dfrac{1}{2} g t_1 t_2$

(c) Average velocity during time interval $(t_2 - t_1)$ is $v_{av} = u \cos \theta$
($\because$ during this interval, the vertical displacement is zero)

(viii) *Velocity and Direction of Motion of Projectile at any Height.* Let P be a point on the trajectory of a projectile at a height h and let v be the velocity of the projectile at that height. If α is the angle which the velocity vector makes with the horizontal, then the horizontal and vertical components of the velocity are given by $v_x = u_x$ = constant

or $v \cos \alpha = u \cos \theta$ (i)

and $v_y^2 = u_y^2 - 2gh$

or $(v \sin \alpha)^2 = (u \sin \theta)^2 = -2gh$ (ii)

Squaring Eq. (i) and then adding to Eq. (ii), we get

$$v^2 = v_x^2 + v_y^2 = u_2 - 2gh$$

or $v = (u^2 - 2gh)^{1/2}$

This gives the speed of the projectile at height h. The direction of the velocity vector (i.e., direction of motion) is obtained by taking the square root of Eq. (ii) and then dividing by Eq. (i). We get

$$\tan \alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\left(u^2 \sin^2\theta - 2gh\right)^{1/2}}{u\cos\theta}$$

(ix) *Time of Flight and Range of a Projectile on an Inclined Plane*

Consider an inclined plane OAB making an angle α with the horizontal (Fig. 4.6). Let a body be projected with a velocity u at an angle θ with the horizontal. Let us choose the x-axis along the plane OA and y-axis perpendicular to the plane OA. Let the body hit the inclined plane at point P so that $R = OP$ is the range on the inclined plane. The x and y components of the velocity of the projectile are

$$v_x = u \cos (\theta - \alpha)$$

and $v_y = u \sin (\theta - \alpha)$

The x and y components of acceleration due to gravity are $-g \sin \alpha$ and $-g \cos \alpha$ respectively, as shown in Fig. 4.6. Let T_f be the time of flight on the inclined plane. Since the net vertical displacement in time T_f is zero (i.e., $h = 0$), we have

$$0 = v_y T_f - \frac{1}{2} g \cos\alpha T_f^2$$

or $0 = v_y - \dfrac{1}{2} g \cos\alpha T_f$

or $0 = u \sin (\theta - \alpha) - \dfrac{1}{2} g \cos\alpha T_f$

or $T_f = \dfrac{2u\sin(\theta - \alpha)}{g\cos\alpha}$ (iii)

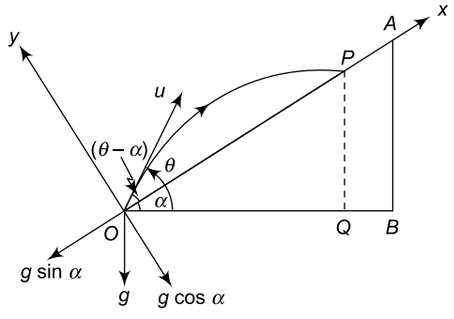

Fig. 4.6

During this time, the horizontal component of velocity $u \cos \theta$ remains constant. Hence, horizontal distance OQ is

$$OQ = (u \cos \theta) T_f$$

$\therefore$ Range of the projectile on the inclined plane is

$$R = OP = \frac{OQ}{\cos\alpha} = \frac{(u\cos\theta)\,T_f}{\cos\alpha} \quad \text{(iv)}$$

Using Eq. (iii) in Eq. (iv), we get

$$R = \frac{2u^2 \sin(\theta - \alpha)\cos\theta}{g\cos^2\alpha}$$

EXAMPLE 4.1

A body is projected horizontally with a velocity of 10 ms^{-1} from the top of building 20 m high. Find

(a) horizontal distance from the bottom of the building at which the body will strike the ground.

(b) the magnitude and direction of the velocity of the body 1 s after it is projected. Take $g = 10\ ms^{-2}$.

SOLUTION

(a) Refer to Fig. 4.1 on page 4.1. Given $h = -20$ m, $u = 10$ ms^{-1}, and $g = -10$ ms^{-2}. Time taken by the body to go from A to B is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times -20}{-10}} = 2 \text{ s}$$

Horizontal distance $OB = R = ut = 10 \times 2 = 20$ m

(b) Refer to Fig. 4.1 again. Horizontal velocity at $t = 1$ s is

$v_x = u = 10$ ms^{-1}. Vertical velocity at $t = 1$ s is (since initial vertical component of velocity $u_y = 0$)

$$v_y = u_y + at = 0 - 10 \times 1 \Rightarrow v_y = -10 \text{ ms}^{-1}$$

Magnitude of resultant velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{10^2 + (-10)^2}$$

$$= \sqrt{200} = 10\sqrt{2} \text{ ms}^{-1}$$

The angle α which the resultant velocity vector subtends with the vertical is given by

$$\tan\alpha = \frac{v_x}{|v_y|} = \frac{10}{10} = 1 \quad \Rightarrow \quad \alpha = 45°$$

EXAMPLE 4.2

A ball is thrown with a velocity of 20 ms^{-1} at an angle of 30° above the horizontal from the top of a building 15 m high. Find (take $g = 10$ ms^{-2})

(a) the time after which the ball hits the ground.

(b) the distance from the bottom of the building at which it hits the ground.

(c) the velocity with which the ball hits the ground.

(d) the maximum height attained by the ball above the ground.

SOLUTION

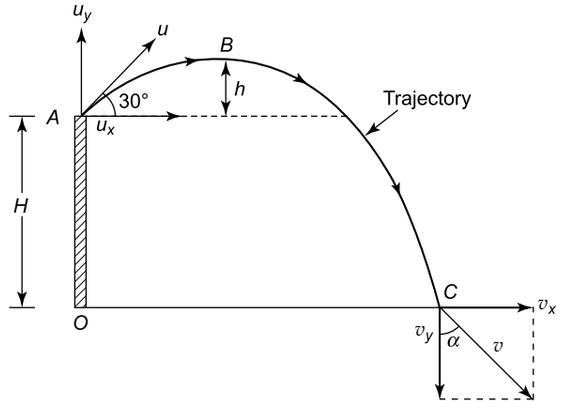

Fig. 4.7

Refer to Fig. 4.7. The horizontal and vertical components of initial velocity are

(a) $u_x = 20 \cos 30° = 10\sqrt{3}$ ms^{-1}

$u_y = 20 \sin 30° = 10$ ms^{-1} (vertically upwards)

Horizontal acceleration $a_x = 0$ and vertical acceleration $a_y = -10$ ms^{-2} (vertically downwards). Vertical displacement $S = -H = -15$ m.

Since the vertical and horizontal motions are independent of each other, we have, for vertical motion.

$$S = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad -15 = 10t + \frac{1}{2} \times (-10)t^2$$

$$\Rightarrow \quad -15 = 10t - 5t^2$$

$$\Rightarrow \quad t^2 - 2t - 3 = 0$$

$$\Rightarrow \quad t = -1 \text{ s or } 3 \text{ s.}$$

Since $t = -1$ s is not possible, the ball with strike ground at point C after 3 seconds.

(b) Horizontal range $R = OC = u_x t = 10\sqrt{3} \times 3$

$$= 30\sqrt{3} \text{ m}$$

(c) Horizontal velocity component at C is

$$v_x = u_x = 10\sqrt{3} \text{ ms}^{-1}$$

Vertical velocity component at C is

$$v_y = u_y + a_y t = 10 - 10 \times 3 = -20 \text{ ms}^{-1}$$

The negative sign shows that the ball is moving downwards.

Resultant velocity $v = \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{(10\sqrt{3})^2 + (20)^2} = 10\sqrt{7} \text{ ms}^{-1}$$

$$\tan\alpha = \frac{v_x}{|v_y|} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) \text{ with the vertical.}$$

(d) Maximum height attained above the ground is

$$h_{\max} = h + H$$

$$= \frac{u^2 \sin^2\theta}{2g} + H$$

$$= \frac{(20)^2 \sin^2(30°)}{2\times 10} + 15$$

$$= 5 + 15 = 20 \text{ m}$$

EXAMPLE 4.3

A stone thrown from the ground at an angle of 45° above the horizontal strikes a vertical wall at a point 10 m above the ground. If the wall is at a distance of 20 m from the point of projection, find (take $g = 10\ \text{ms}^{-2}$)

(a) the speed with which the stone is projected,

(b) the magnitude and direction of the velocity of the stone when it strikes the wall.

SOLUTION

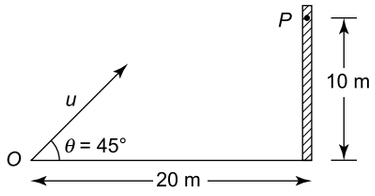

Fig. 4.8

(a) Let P be the point on the wall where the stone strikes it. Taking the point of projection O as the origin, the coordinates of P are (20 m, 10 m) [Fig. 4.8]

$$x = (u\ \cos\ \theta)t \qquad (1)$$

$$y = (u\ \sin\ \theta)t - \frac{1}{2}gt^2 \qquad (2)$$

Putting $x = 20$ m, $y = 10$ m, $g = 10\ \text{ms}^{-2}$ and $\theta = 45°$ in Eqs. (1) and (2), we have

$$20 = (u\ \cos\ 45°)t = \frac{u}{\sqrt{2}}\,t$$

$$\Rightarrow \qquad t = \frac{20\sqrt{2}}{u} \qquad (3)$$

and $$10 = (u\ \sin\ 45°)t - \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow \qquad 10 = \frac{ut}{\sqrt{2}} - 5\ t^2 \qquad (4)$$

Using (3) in (4)

$$10 = \frac{u}{\sqrt{2}} \times \frac{20\sqrt{2}}{u} - 5\left(\frac{20\sqrt{2}}{u}\right)^2$$

$$\Rightarrow \qquad 10 = 20 - \frac{4000}{u^2} \quad \Rightarrow \quad u^2 = 400$$

$$\Rightarrow \qquad u = 20\ \text{ms}^{-1}$$

(b) At point P, $v_x = 20 \cos 45° = \frac{20}{\sqrt{2}}\ \text{ms}^{-1}$

$$u_y = u\ \sin\theta = 20\ \sin\ 45° = \frac{20}{\sqrt{2}}\ \text{ms}^{-1}$$

$$v_y^2 = u_y^2 - 2g \times 10 \Rightarrow v_y^2 = \frac{400}{2} - 2 \times 10 \times 10$$

$$\Rightarrow \quad v_y^2 = 0 \quad \Rightarrow \quad v_y = 0$$

Hence point P is at the highest point on the trajectory where the velocity is only horizontal. Thus the stone strikes the wall at P with a velocity of $\frac{20}{\sqrt{2}} = 10\sqrt{2}\ \text{ms}^{-1}$ in the horizontal direction.

EXAMPLE 4.4

A ball projected with a velocity of $10\ \text{ms}^{-1}$ at an angle of 30° with the horizontal just clears two vertical poles, each of height 1.0 m. Find the separation between the poles. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

Refer to Fig. 4.9. Let us calculate the two values of t at which the ball passes just above P and R. For each pole $h = 1.0$ m

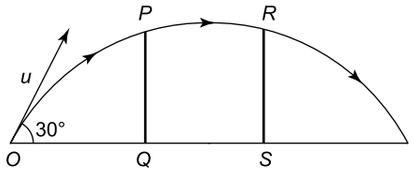

Fig. 4.9

y-component of velocity is $u_y = u\ \sin\theta = 10\ \sin\ 30° = 5\ \text{ms}^{-1}$.

$$h = u_y t + \frac{1}{2}gt^2$$

$$\Rightarrow \qquad 1.0 = 5t\ + \frac{1}{2}(-10)t^2$$

$$\Rightarrow \qquad 5t^2 - 5t + 1.0 = 0$$

The two roots of this quadratic equations are $t_1 = 0.72$ s and $t_2 = 2.76$ s. Therefore

$$OQ = u_x t_1 = 10\ \cos\ 30° \times 0.72 = 6.2\ \text{m}$$

and $$OS = u_x t_2 = 10\ \cos\ 30° \times 2.76 = 23.9\ \text{m}$$

$$\therefore \qquad QS = 23.9 - 6.2 = 17.7\ \text{m}$$

EXAMPLE 4.5

A projectile has the same range $R = 40$ m for two angles of projection. If T_1 and T_2 are the times of flight, find $T_1\ T_2$. Given $g = 10\ \text{ms}^{-2}$.

SOLUTION

For a given speed u of projection, a projectile has the same range for angles of projection θ and $(90° - \theta)$. Therefore

$$T_1 = \frac{2u \sin\theta}{g}$$

and $$T_2 = \frac{2u \sin(90° - \theta)}{g} = \frac{2u \cos\theta}{g}$$

$$\therefore \quad T_1 T_2 = \frac{4u^2 \sin\theta \cos\theta}{g^2}$$

$$= \frac{2u^2 \sin(2\theta)}{g^2} = \frac{2R}{g}$$

$$= \frac{2 \times 40}{10} = 8 \text{ s}^2$$

EXAMPLE 4.6

A ball is thrown from a point with a speed u at an angle θ with the horizontal. From the same point and at the same instant, a person starts running with a constant speed $u/2$ to catch the ball. Will he be able to catch the ball? If yes, what should be the value of θ?

SOLUTION

The person will catch the ball if the horizontal rane = the distance covered by him in the time of flight, i.e. if

$$R = \frac{u}{2} \times t_f$$

$$\Rightarrow \quad \frac{u^2 \sin(2\theta)}{g} = \frac{u}{2} \times \frac{2u \sin\theta}{g}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 60°$$

4.2 RELATIVE VELOCITY IN TWO DIMENSIONS

The relative velocity of a body B with respect to body A is defined as

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

If vectors $\mathbf{v}_A$ and $\mathbf{v}_B$ are inclined to each other at an angle θ as shown in Fig. 4.10, the relative veliocity v_{BA} is found as follows.

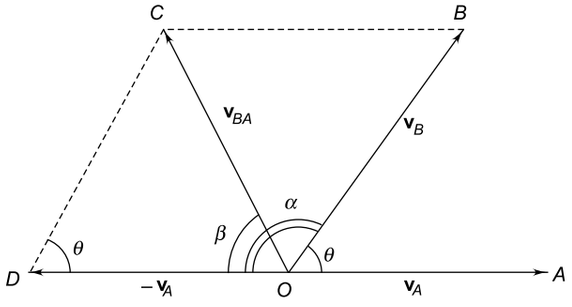

Fig. 4.10

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_B + (-\mathbf{v}_A)$$

Thus the magnitude and direction of vector $\mathbf{v}_{BA}$ can be found by finding the resultant of vectors $\mathbf{v}_B$ and $-\mathbf{v}_A$ which is vector **OC** as shown in Fig. 4.10.

Magnitude of vector $\mathbf{v}_{BA}$ is given by

$$v_{BA} = \left(v_A^2 + v_B^2 + 2v_A v_B \cos\alpha\right)^{1/2}$$

$$= \left(v_A^2 + v_B^2 - 2v_A v_B \cos\theta\right)^{1/2} \qquad (\because \alpha = 180° - \theta)$$

The angle β which the resultant vector **OC** subtends with vector **OD** is given by

$$\frac{OC}{\sin\theta} = \frac{CD}{\sin\beta}$$

$$\Rightarrow \quad \sin\beta = \frac{CD \sin\theta}{OC} = \frac{v_B \sin\theta}{v_{BA}}$$

Special Case

(i) If vector v_A and v_B are in the same direction, $\theta = 0°$, then $v_{BA} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B} = v_B - v_A$.

(ii) If vector v_A and v_B are in opposite direction, $\theta = 180°$, then $v_{BA} = v_B + v_A$.

Applications

(i) *To cross the river of width d along the shortest path which is PQ*, the boat must move along *PR* making an angle $(90° + \theta)$ with the direction of the stream such that the direction of the resultant velocity $\mathbf{v}$ is along *PQ*. Angle θ is given by (see Fig. 4.11)

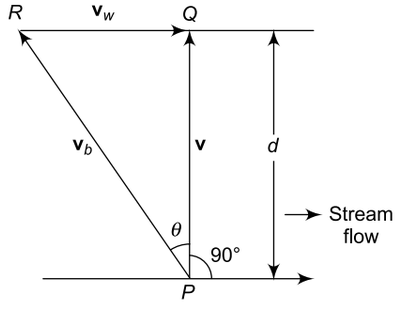

Fig. 4.11

$$\sin\theta = \frac{v_\omega}{v_b}$$

Also $$v = \sqrt{v_b^2 - v_\omega^2}$$

The time taken to cross the river along the shortest path is given by

$$t = \frac{d}{v} = \frac{d}{\sqrt{v_b^2 - v_\omega^2}}$$

(b) *To cross the river in the shortest time*, the boat should move along *PQ*. The shortest time is given by

$$t = \frac{d}{v_b}$$

At this time, the boat will reach the point *R* on the opposite bank of the river at a distance *x* from the point *Q* (Fig. 4.12). From the Figure, we have

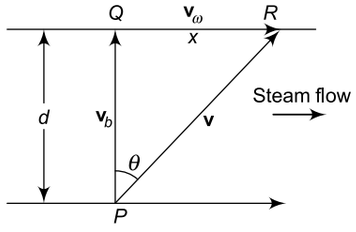

Fig. 4.12

$$x = d \tan \theta,$$

but $\tan \theta = \dfrac{\mathbf{v}_w}{\mathbf{v}_b}$ Therefore,

$$x = d\left(\frac{\mathbf{v}_w}{\mathbf{v}_b}\right)$$

(ii) *Holding an Umbrella to Project from Rain* Let $\mathbf{v}_r$ be the velocity of the rain falling vertically downward and $\mathbf{v}_m$ the velocity of a man walking from north to south direction (Fig. 4.13). In order to protect himself from rain, he must hold his umbrella in the direction of the resultant velocity $\mathbf{v}$, which is given by

$$v = \sqrt{v_r^2 + v_m^2}$$

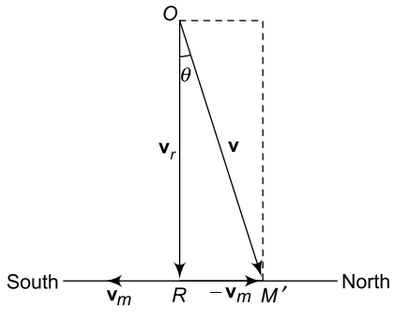

Fig. 4.13

This is the speed with which the rain strikes the umbrella. If θ is the angle subtended by the resultant velocity $\mathbf{v}$ with the vertical, then from triangle *ORM′*, we have

$$\tan \theta = \frac{RM'}{OR} = \frac{v_m}{v_r}$$

or $$\theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

Thus, the man must hold the umbrella at an angle θ with the vertical towards north.

4.3 UNIFORM CIRCULAR MOTION

(i) *For a body moving in a horizontal circle*
The centripetal acceleration of a body of mass *m* moving in a circle of radius *R* with a constant speed *v* (or angular speed ω) is

$$a_c = \omega\, v = \omega^2 R = \frac{v^2}{R}$$

Centripetal force is

$$f_c = ma_c = m\omega^2 R = \frac{mv^2}{R}$$

(ii) *For a body moving in a vertical circle*
The minimum speed to complete the circle when the body is at the top of the circle is $v = \sqrt{Rg}$. The minimum speed to complete the circle when the body is at the bottom of the circle is $v = \sqrt{5Rg}$.

NOTE

The magnitude of velocity (v) and the magnitude of acceleration (v^2/R) for a body in uniform circular motion are constant but the direction of velocity $\mathbf{v}$ (which is along the tangent and the direction of acceleration $\mathbf{a}_c$ (which is towards the centre keep on changing with time.

4.4 NON–UNIFORM CIRCULAR MOTION

If the speed of the body revolving in a circle changes with time, it is said to be in non-uniform circular motion. In this case, the acceleration of the body is the resultant of two components (Fig. 4.14).

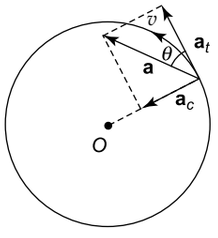

Fig. 4.14

(i) Radial (or centripetal) acceleration directed towards the centre *O* and has a magnitude $a_c = v^2/R$, where *v* is the instantaneous speed and *R* is the radius of the circle.

(ii) Tangential component directed along the tangent which causes the change in the magnitude of velocity. Its magnitude is given by

$$a_t = \frac{dv}{dt}$$

The net acceleration of the body is

$$a = \sqrt{a_c^2 + a_t^2}$$

and it makes an angle θ with the tangent given by

$$\tan\theta = \frac{a_c}{a_t}$$

EXAMPLE 4.7

A particle moves along a circle with a velocity $v = kt$, where k is a constant. Find the net acceleration of the particle at the instant when it has covered nth fraction of the circle after the beginning of motion.

SOLUTION

Let v be the speed of the particle at the instant when it has covered nth fraction of the circle of radius R.

Centripetal acceleration at that instant is

$$a_c = \frac{v^2}{R} \qquad (1)$$

Tangential acceleration at that instant is

$$a_t = \frac{dv}{dt} = \frac{dv}{dt}\frac{dx}{dx} = \frac{vdv}{dx}$$

$$\Rightarrow \qquad vdv = a_t\, dx \qquad (2)$$

Let x be the distance covered. Then $x = (2\pi R)n$. Integrating Eq. (2) we have

$$\int_0^v v\,dv = a_t\int_0^x dx$$

$$\Rightarrow \qquad \frac{v^2}{2} = a_t x = a_t\,(2\pi R)n$$

$$\Rightarrow \qquad v = \sqrt{2a_t\,(2\pi R)n} \qquad (3)$$

Using (3) in (1), we get

$$a_c = \frac{2a_t\,(2\pi R)n}{R} = 4\pi a_t\, n$$

Now $\qquad a_t = \dfrac{dv}{dt} = \dfrac{d}{dt}(kt) = k$

$$a_c = 4\pi kn$$

$\therefore$ Net acceleration $a = \sqrt{a_t^2 + a_c^2}$

$$= \sqrt{k^2 + (4\pi kn)^2}$$

$$= k\sqrt{1 + 16\pi^2 n^2}$$

EXAMPLE 4.8

A particle moves along a circle of radius R. Its speed v varies with distance x covered along the circle as $v = k\sqrt{x}$, where k is a constant. If $R = 1.0$ m and $x = 0.5$ m, find the angle between the net acceleration vector and the tangential acceleration vector.

SOLUTION

Tangential acceleration is

$$a_t = \frac{dv}{dt} = \frac{dv}{dt}\frac{dx}{dx} = \frac{vdv}{dx}$$

$$\Rightarrow \qquad a_t\, dx = v\, dv$$

Integrating

$$a_t\int_0^x dx = \int_0^v v\,dv$$

$$\Rightarrow \qquad a_t x = \frac{v^2}{2}$$

$$\Rightarrow \qquad a_t = \frac{v^2}{2x} = \frac{(k\sqrt{x})^2}{2x} = \frac{k^2}{2}$$

Centripetal acceleration is

$$a_c = \frac{v^2}{R} = \frac{k^2 x}{R}$$

Angle θ between net acceleration and tangential acceleration is given by

$$\tan\theta = \frac{a_c}{a_t} = \frac{k^2 x/R}{k^2/2} = \frac{x}{2R}$$

Putting $x = 0.5$ m and $R = 1.0$ m. We get

$$\tan\theta = \frac{2\times 0.5}{1.0} = 1 \quad \Rightarrow \quad \theta = 45°.$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. The maximum height attained by a projectile is $\sqrt{3}/4$ times its horizontal range. The angle of projection of the projectile with the horizontal is

(a) 30° (b) 45°

(c) 60° (d) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. A body is projected with a speed u at an angle θ with the horizontal. The speed of the body when it is at the highest point on its trajectory is $\sqrt{2/5}$ times its speed at half the maximum height. The value of θ is

(a) 30° (b) 45°

(c) 60° (d) $\tan^{-1}\left(\frac{3}{2}\right)$

3. A body of mass m is projected from the ground with linear momentum p such that it has the maximum horizontal range. The minimum kinetic energy of the body during its flight is

(a) zero (b) $\frac{p^2}{m}$

(c) $\frac{p^2}{2m}$ (d) $\frac{p^2}{4m}$

4. Two bodies are projected simultaneously from the same point on the ground with speeds 10 ms^{-1} and $10/\sqrt{3}$ ms^{-1} at angles 30° and 60° respectively with the horizontal. The separation between them when they hit the ground is (take $g = 10\ ms^{-2}$)

(a) $10\sqrt{3}$ m (b) $\frac{10}{\sqrt{3}}$ m

(c) $5\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)$ m (d) $5\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)$ m

5. A projectile is given an initial velocity $\mathbf{u} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})$ ms^{-1}. The cartesian equation of its trajectory is (take $g = 10\ ms^{-2}$)

(a) $y = 2x - 5x^2$ (b) $2y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = x - 5x^2$

6. A body is projected from the ground with a speed u at an angle θ with the horizontal. The magnitude of the average velocity of the body between the point of projection and the highest point of its trajectory is

(a) $\frac{u}{2}(\sin\theta + \cos\theta)$

(b) $\frac{u}{2}(1 + 2\cos^2\theta)^{1/2}$

(c) $\frac{u}{2}(1 + 3\cos^2\theta)^{1/2}$

(d) $\frac{u}{2}$

7. A body P is projected vertically upwards. Another body Q of the same mass is projected at an angle of 60° with the horizontal. If both attain the same maximum height, the ratio of the initial kinetic energy of P to that of Q is

(a) $\frac{3}{4}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

8. With what minimum speed must a body be projected from the origin in the x-y plane so that it can pass through a point whose x and y coordinates are 30 m and 40 m respectively? Take $g = 10\ ms^{-2}$.

(a) 10 ms^{-1} (b) 20 ms^{-1}

(c) 30 ms^{-1} (d) 40 ms^{-1}

9. It is possible to project a particle with a given velocity in two possible ways so as to make it pass through a point P at a distant r from the point of projection. The product of the times taken to reach this point in the two possible ways is then proportional to

(a) $1/r$ (b) r

(c) r^3 (d) $\frac{1}{r^2}$

10. A projectile has a maximum range of 200 m. What is the maximum height attained by it?

(a) 25 m (b) 50 m

(c) 75 m (d) 100 m

11. A body thrown along a frictionless inclined plane of angle of inclination 30° covers a distance of 40 m along the plane. If the body is projected with the same speed at angle of 30° with the ground, it will have a range of (take $g = 10\ ms^{-2}$)

(a) 20 m (b) $20\sqrt{2}$ m

(c) $20\sqrt{3}$ m (d) 40 m

12. Which of the following remains constant during the motion of a projectile fired from a planet?

(a) kinetic energy
(b) momentum
(c) vertical component of velocity
(d) horizontal component of velocity

13. A body is projected with kinetic energy K at an angle of 60° with the horizontal. Its kinetic energy at the highest point of its trajectory will be

(a) $2K$ (b) K

(c) $K/2$ (d) $K/4$

14. A body, projected with a certain kinetic energy, has a horizontal range R. The kinetic energy will be minimum at a position of the projectile when its horizontal range is

(a) R (b) $3R/4$

(c) $R/2$ (d) $R/4$

15. Four projectiles are projected with the same speed at angles 20°, 35°, 60° and 75° with the horizontal. The range will be the longest for the projectile whose angle of projection is
(a) 20° (b) 35°
(c) 60° (d) 75°

16. A player throws a ball which reaches the other player in 4 seconds. If the height of each player is 1.8 m, what is the maximum height attained by the ball above the ground?
(a) 19.4 m (b) 20.4 m
(c) 21.4 m (d) 22.4 m

17. The maximum height attained by a projectile is increased by 1% by changing the angle of projection, without changing the speed of projection. The percentage increase in the time of flight will be
(a) 20% (b) 15%
(c) 10% (d) 5%

18. A projectile has a range R and time of flight T. If the range is doubled (by increasing the speed of projection, without changing the angle of projection), the time of flight will become
(a) $\frac{T}{\sqrt{2}}$ (b) $\sqrt{2}\,T$
(c) $\frac{T}{2}$ (d) $2\,T$

19. A projectile has the same range R when the maximum height attained by it is either h_1 or h_2.Then R, h_1 and h_2 will be related as
(a) $R = \sqrt{h_1 h_2}$ (b) $R = 2\sqrt{h_1 h_2}$
(c) $R = 3\sqrt{h_1 h_2}$ (d) $R = 4\sqrt{h_1 h_2}$

20. A ball is projected vertically upwards with a certain initial speed. Another ball of the same mass is projected at an angle of 60° with the vertical with the same initial speed. At the highest point, the ratio of their potential energies will be
(a) 4 : 1 (b) 3 : 2
(c) 2 : 3 (d) 2 : 1

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21. A body of mass m_1, projected vertically upwards with an initial velocity u reaches a maximum height h. Another body of mass m_2 is projected along an inclined plane making an angle of 30° with the horizontal and with speed u. The maximum distance travelled along the incline is
(a) $2h$ (b) h
(c) $\frac{h}{2}$ (d) $\frac{h}{4}$

22. Two balls A and B are projected from the same location simultaneously. Ball A is projected vertically upwards and ball B at 30º to the vertical. They reach the ground simultaneously. The velocities of projection of A and B are in the ratio
(a) $\sqrt{3}$: 1 (b) 1 : $\sqrt{3}$
(c) $\sqrt{3}$: 2 (d) 2 : $\sqrt{3}$

23. A body is projected with a velocity $\boldsymbol{v} = (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})$ ms^{-1}. The maximum height attained by the body is (take $g = 10\ \text{ms}^{-2}$)
(a) 0.8 m (b) 8 m
(c) 80 m (d) 800 m

24. In Q. 23 above, the time of flight of the body is
(a) 0.8 s (b) 1.0 s
(c) 4.0 s (d) 8.0 s

25. A body is projected with a velocity **u** at an angle θ with the horizontal. The velocity of the body will become perpendicular to the velocity of projection after a time t given by
(a) $\frac{2u\sin\theta}{g}$ (b) $\frac{u\sin\theta}{g}$
(c) $\frac{2u}{g\sin\theta}$ (d) $\frac{u}{g\sin\theta}$

26. A body is projected at an angle θ with the horizontal. When it is at the highest point, the ratio of the potential and kinetic energies of the body is
(a) $\tan\theta$ (b) $\tan^2\theta$
(c) $\cot\theta$ (d) $\cot^2\theta$

27. At time $t = 0$ a body is projected horizontally from a certain height with a velocity u. The radius of curvature of its trajectory at time t is
(a) $\frac{u^2}{g}\left(1+\frac{gt}{u}\right)^{3/2}$ (b) $\frac{u^2}{g}\left(1+\frac{g^2t^2}{u^2}\right)^{1/2}$
(c) $\frac{u^2}{g}\left(1-\frac{g^2t^2}{u^2}\right)^{1/2}$ (d) $\frac{u^2}{g}\left(1+\frac{g^2t^2}{u^2}\right)^{3/2}$

28. A body is projected from the bottom of an inclined plane which has an inclination of 10° with the horizontal. At what angle θ from the horizontal should the body be projected so that its range on the inclined plane is maximum for a given velocity of projection?
(a) 45° (b) 50°
(c) 55° (d) 60°

29. From the top of a tower, two balls are thrown horizontally with velocities u_1 and u_2 in opposite directions. If their velocities are perpendicular to each other just before they strike the ground, the height of the tower is

(a) $\frac{(u_1+u_2)^2}{2g}$ (b) $\frac{(u_1-u_2)^2}{2g}$

(c) $\frac{(u_1^2+u_2^2)}{2g}$ (d) $\frac{u_1u_2}{2g}$

30. A body is projected at time $t = 0$ with a velocity u at an angle θ with the horizontal. It hits the ground at time $t = t_f$ where t_f is the time of flight. The average velocity of the body during the time interval $t = 0$ to $t = t_f$ is

(a) $u\cos\theta$ (b) $u\sin\theta$

(c) $\frac{u}{2}(1+\cos^2\theta)^{1/2}$ (d) $\frac{u}{2}(1+\sin^2\theta)^{1/2}$

31. A body is projected at time $t = 0$ with a velocity u at an angle θ with the horizontal. The horizontal and vertical components of its velocity will become equal at time $t \neq 0$,

(a) if $0 \le \theta \le \frac{\pi}{4}$ (b) if $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

(c) if $0 \le \theta \le \frac{\pi}{6}$ (d) for no value of θ

32. A boy whirls a stone in a horizontal circle 2 m above the ground by means of a string 1.25 m long. The string breaks and the stone flies off horizontally, striking the ground 10 m away. What is the magnitude of the centripetal acceleration during circular motion? Take g = 10 ms^{-2}.

(a) 100 ms^{-2} (b) 200 ms^{-2}

(c) 300 ms^{-2} (d) 400 ms^{-2}

33. A body is moving in a circle with a uniform speed v. What is the magnitude of the change in velocity when the radius vector describes an angle θ?

(a) zero (b) $v(1+\cos^2\theta)^{1/2}$

(c) $2v\cos\left(\frac{\theta}{2}\right)$ (d) $2v\sin\left(\frac{\theta}{2}\right)$

34. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follow that

(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line.

35. A particle of mass M is moving in a horizontal circle or radius R with uniform speed V. When it moves from one point to a diametrically opposite point, its

(a) kinetic energy changes by $MV^2/4$
(b) momentum does not change
(c) momentum changes by $2MV$
(d) kinetic energy changes by MV^2

36. A plumb line is hanging from the ceiling of a train. If the train moves along a horizontal track with a uniform acceleration a, the plumb line gets inclined to the vertical at a angle

(a) $\tan^{-1}\left(\frac{a}{g}\right)$ (b) $\tan^{-1}\left(\frac{g}{a}\right)$

(c) $\sin^{-1}\left(\frac{a}{g}\right)$ (d) $\cos^{-1}\left(\frac{g}{a}\right)$

37. A body moves along a circular track of radius 20 cm. It starts from one end of a diameter, moves along the circular track and reaches the other end of the diameter is 5 seconds. What is the angular speed of the body?

(a) $\frac{\pi}{2}$ rad s^{-1} (b) $\frac{\pi}{3}$ rad s^{-1}

(c) $\frac{\pi}{4}$ rad s^{-1} (d) $\frac{\pi}{5}$ rad s^{-1}

38. A cyclist is moving with a speed of 6 ms^{-1}. As the approaches a circular turn on the road of radius 120 m, he applies brakes and reduces his speed at a constant rate of 0.4 ms^{-2}. The magnitude of the net acceleration of the cyclist on the circular turn is

(a) 0.5 ms^{-2} (b) 1.0 ms^{-2}

(c) 2.0 ms^{-2} (d) 4.0 ms^{-2}

39. A car is travelling at a velocity of 10 km/h on a straight road. The driver of the car throws a parcel with a velocity of $10\sqrt{2}$ km/h when the car is passing by a man standing on the side of the road. If the parcel is to reach the man, the direction of throw makes the following angle with the direction of the car,

(a) 135° (b) 45°

(c) $\tan^{-1}(\sqrt{2})$ (d) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

40. Rain is falling vertically with a speed of 4 ms^{-1}. After some time, wind starts blowing with a speed of 3 ms^{-1} in the north to south direction. In order to protect himself from rain, a man standing on the ground should hold his umbrella at an angle θ given by

(a) $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ with the vertical towards south

(b) $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ with the vertical towards north

(c) $\theta = \cot^{-1}\left(\frac{3}{4}\right)$ with the vertical towards south

(d) $\theta = \cot^{-1}\left(\frac{3}{4}\right)$ with the vertical towards north

41. In Q.40 above, with what speed does the rain strike the umbrella?

(a) 3 ms^{-1} (b) 4 ms^{-1}

(c) 5 ms^{-1} (d) 6 ms^{-1}

42. A swimmer can swim in still water with a speed of 5 ms^{-1}. While crossing a river his average speed is 3 ms^{-1}. If he crosses the river in the shortest possible time, what is the speed of flow of water?

(a) 2 ms^{-1} (b) 4 ms^{-1}

(c) 6 ms^{-1} (d) 8 ms^{-1}

43. Water is flowing in a river of width 36 m with a speed of 2 ms^{-1}. A person in a boat at a point P on the bank of the river wants to cross the river by the shortest path to reach a point Q directly opposite on the other bank. If he can row his boat with a speed of 4 ms^{-1} in still water, he show row his boat at an angle of

(a) 30° upstream with the line PQ

(b) 30° downstream with the line PQ

(c) $\tan^{-1}(0.5)$ upstream with the line PQ

(d) $\tan^{-1}(2)$ downstream with the line PQ.

44. In Q.43 above, the time taken by him to cross the river by the shortest path is

(a) $\sqrt{3}$ s (b) $3\sqrt{3}$ s

(c) $6\sqrt{3}$ s (d) $18\sqrt{3}$ s

45. A body moving in a circular path with a constant speed has a

(a) constant velocity

(b) constant momentum

(c) constant kinetic energy

(d) constant acceleration

IIT, 1992

46. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

IIT, 1998

47. A projectile is projected with a velocity u at an angle θ with the horizontal. For a fixed θ, which of the graphs shown in Fig. 4.15 shows the variation of range R versus u?

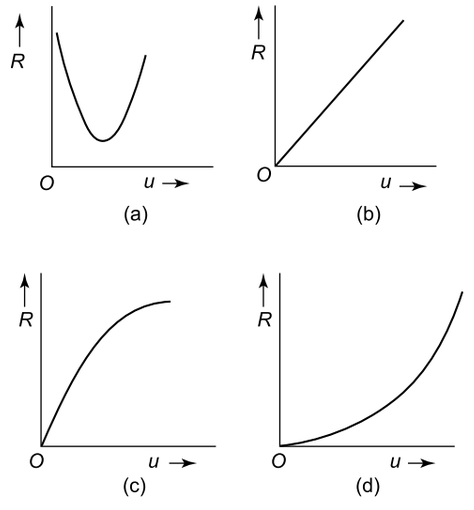


Fig. 4.15

IIT, 2008

ANSWERS

1. (c)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (c)
7. (a)	**8.** (c)	**9.** (b)	**10.** (b)	**11.** (c)	**12.** (d)
13. (d)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (b)
19. (d)	**20.** (a)	**21.** (a)	**22.** (c)	**23.** (a)	**24.** (a)
25. (d)	**26.** (b)	**27.** (d)	**28.** (c)	**29.** (d)	**30.** (a)
31. (b)	**32.** (b)	**33.** (d)	**34.** (c)	**35.** (c)	**36.** (a)
37. (d)	**38.** (a)	**39.** (a)	**40.** (b)	**41.** (c)	**42.** (b)
43. (a)	**44.** (c)	**45.** (c)	**46.** (d)	**47.** (d)	

SOLUTIONS

1. $h_{max} = \dfrac{u^2 \sin^2 \theta}{2g}$ and $R = \dfrac{u^2 \sin(2\theta)}{g}$

$= \dfrac{2u^2 \sin\theta \cos\theta}{g}$

Given $h_{max} = \dfrac{\sqrt{3}}{4} R$. Hence.

$\dfrac{u^2 \sin^2 \theta}{2g} = \dfrac{\sqrt{3}}{4} \times \dfrac{2u^2 \sin\theta \cos\theta}{g}$

$\Rightarrow \quad \tan\theta = \sqrt{3} \quad \Rightarrow \quad \theta = 60°$

2. Maximum height attained is

$$H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow gH = \frac{u^2 \sin^2\theta}{2}$$

Speed at maximum height is $u_x = u\cos\theta$

Speed v at $H/2$ is given by

$$v^2 = u^2 - 2g\left(\frac{H}{2}\right)$$
$$= u^2 - gH$$
$$= u^2 - \frac{u^2 \sin^2\theta}{2}$$

It is given that $u_x^2 = \frac{2}{5}v^2$, i.e.

$$u^2\cos^2\theta = \frac{2}{5}\left(u^2 - \frac{u^2\sin^2\theta}{2}\right)$$

$$u^2(1 - \sin^2\theta) = \frac{2}{5}\left(u^2 - \frac{u^2\sin^2\theta}{2}\right)$$

$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = 60°$

3. For maximum horizontal range, $\theta = 45°$. Also $p = mu$ which gives $u = p/m$.

The kinetic energy is minimum when the body is at the highest point of its trajectory. At this point the velocity is $u_x = u\cos\theta = u\cos 45° = \frac{u}{\sqrt{2}}$.

$$\therefore \text{Minimum K.E.} = \frac{1}{2}m \times \left(\frac{u}{\sqrt{2}}\right)^2 = \frac{(mu)^2}{4m} = \frac{p^2}{4m}.$$

4. The vertical components of velocity of the two bodies are $10\sin 30° = 5\ \text{ms}^{-1}$ and $(10/\sqrt{3})\sin 60° = 5\ \text{ms}^{-1}$. Since their vertical velocity components are equal, their times of flight are also equal. Hence the separation between them when they hit the ground is

x = difference in their horizontal ranges

$$= R_1 - R_2$$
$$= \frac{(10)^2\sin 60°}{10} - \frac{(10/\sqrt{3})^2\sin 120°}{10}$$
$$= 5\sqrt{3} - \frac{5}{\sqrt{3}}$$
$$= 5\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)\text{m}$$

So the correct choice is (d).

5. Given $u\cos\theta = 2\ \text{ms}^{-1}$ and $u\sin\theta = 1\ \text{ms}^{-1}$. These equations give $u = \sqrt{5}\ \text{ms}^{-1}$, $\tan\theta = \frac{1}{2}$ and $\cos\theta = \frac{2}{u} = \frac{2}{\sqrt{5}}$.

The equation of the trajectory is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$
$$= x \times \frac{1}{2} - \frac{10x^2}{2 \times (\sqrt{5})^2 \times \left(\frac{2}{\sqrt{5}}\right)^2}$$
$$= \frac{x}{2} - \frac{5x^2}{4}$$

$\Rightarrow \quad 4y = 2x - 5x^2$, which is choice (c).

6. Average velocity is

$$v_{av} = \frac{\text{net displacement}}{\text{time}}$$
$$= \frac{\sqrt{H^2 + \left(\frac{R}{2}\right)^2}}{T/2} \qquad (1)$$

where H = maximum height $= \frac{u^2\sin^2\theta}{2g}$

R = horizontal range $= \frac{u^2\sin(2\theta)}{g}$

T = time of flight $= \frac{2u\sin\theta}{g}$

Substituting in Eq. (1), we get

$$v_{av} = \frac{u}{2}(1 + 3\cos^2\theta)^{1/2}$$

The correct choice is (c).

7. For body P, $u_1^2 = 2gh$

For body Q, $h = \frac{u_2^2\sin^2(60°)}{2g} = \frac{3u_2^2}{8g}$

$$\Rightarrow \qquad u_2^2 = \frac{8gh}{3}$$

$$\therefore \frac{\text{K.E. of } P}{\text{K.E. of } Q} = \frac{\frac{1}{2}mu_1^2}{\frac{1}{2}mu_2^2} = \frac{u_1^2}{u_2^2} = \frac{3}{4}$$

So the correct choice is (a).

8. In projectile motion, the equation of the trajectory is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$= x \tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

Substituting the given values, we have

$$40 = 30 \tan\theta - \frac{10 \times (30)^2}{2u^2}(1 + \tan^2\theta)$$

or $900 \tan^2\theta - (6u^2 \tan\theta) + (900 + 8u^2) = 0$

The value of $\tan\theta$ will be real if

$$(6u^2)^2 \geq 4 \times 900 \times (900 + 8u^2)$$

or $u^4 \geq 100\,(900 + 8u^2)$

or $u^4 - 800u^2 \geq 90000$

or $(u^2 - 400)^2 - 160000 \geq 90000$

or $(u^2 - 400)^2 \geq 250000$

or $u^2 - 400 \geq 500 \Rightarrow u^2 = 900$

$\Rightarrow u = 30\ \text{ms}^{-1}$

9. The range of a projectile is $r = 2v_0^2 \cos\theta \sin\theta/g$, the two possible angles of projection are θ and $(90° - \theta)$. The times of flight corresponding to these two angles are

$$t_1 = \frac{2v_0 \sin\theta}{g}$$

and $$t_2 = \frac{2v_0 \sin(90° - \theta)}{g} = \frac{2v_0 \cos\theta}{g}$$

so that $t_1 t_2 = \frac{4v_0^2 \sin\theta \cos\theta}{g^2} = \frac{2r}{g}$.

Thus $t_1 t_2 \propto r$. Hence the correct choice is (b).

10. For maximum range $\theta = 45°$. Hence $R_{max} = v_0^2/g$ and $h_{max} = v_0^2/4g$. Thus $h_{max} = R_{max}/4 = 200/4 = 50$ m. Hence the correct choice is (b).

11. Let u be the initial speed with which the body is thrown along the inclined plane. As shown in Fig. 4.16, the effective deceleration is given by

$$a = g \sin\theta$$

$$= g \sin 30° = \frac{g}{2} = 5\ \text{ms}^{-2}$$

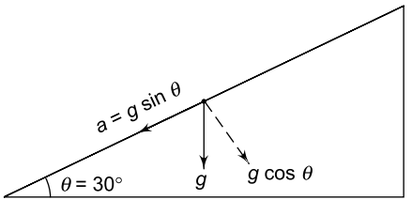


Fig. 4.16

The body stops after covering a distance, say, s along the plane, which is given by $-2as = 0 - u^2$ or $u = \sqrt{2as} = \sqrt{2 \times 5 \times 40} = 20\ \text{ms}^{-1}$. A projectile projected at angle $\theta = 30°$ with this speed will have a range of

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{20 \times 20 \times \sin 60°}{10} = 20\sqrt{3}\ \text{m}$$

Hence the correct choice is (c).

12. Since the velocity of the projectile changes continuously, both kinetic energy and momentum undergo a change with time. Only the vertical component of velocity changes due to gravity; the horizontal component always remains constant. Hence the correct choice is (d).

13. At the highest point, the velocity has only the horizontal component $v_x = v \cos\theta = v \cos 60° = v/2$. Now kinetic energy $\frac{1}{2}mv^2$ is proportional to v^2. Since the velocity is reduced to half, the kinetic energy becomes one–fourth, i.e. $K/4$. Hence the correct choice is (d).

14. Kinetic energy is minimum when the projectile is at the highest point of its trajectory. At the highest point, its range = half the horizontal range. Hence the correct choice is (c).

15. Range $R = v_0^2 \sin 2\theta/g$. For the same v_0, $R \propto \sin 2\theta$. Since $\sin 2\theta$ is the largest for $\theta = 35°$, the correct choice is (b).

16. The time of flight $t = 2v_0 \sin\theta/g$. Since $t = 4$ s, we have $v_0 \sin\theta = 2g$.

Now $$h_{max} = \frac{v_0^2 \sin^2\theta}{2g} = \frac{4g^2}{2g} = 2g$$

$$= 2 \times 9.8 = 19.6\ \text{m}$$

$\therefore$ Height above the ground = 19.6 + 1.8 = 21.4 m

Hence the correct choice is (c).

17. Given $h = \frac{v_0^2 \sin^2\theta}{2g}$. Differentiating partially we get ($\because v_0 = \text{constant}$) $\delta h = \frac{v_0^2}{2g} 2 \sin\theta \cos\theta\, \delta\theta$. Thus

$$\frac{\delta h}{h} = \frac{2\cos\theta\,\delta\theta}{\sin\theta} = 0.01 \text{ (given)}.$$

Therefore, $\frac{\cos\theta\,\delta\theta}{\sin\theta} = 0.005$. We also have $T = \frac{2v_0 \sin\theta}{g}$ which gives $\delta T = \frac{2v_0 \cos\theta\,\delta\theta}{g}$. Thus

$\frac{\delta T}{T} = \frac{\cos\theta\,\delta\theta}{\sin\theta}$. But $\frac{\cos\theta\,\delta\theta}{\sin\theta} = 0.005$. Therefore,

$\frac{\delta T}{T} = 0.05$ or $\delta T = 0.005\ T$.

Hence T increases by 0.5%. Thus the correct choice is (d).

18. Now $R = \dfrac{2v_0^2 \sin\theta\cos\theta}{g^2}$ and $T^2 = \dfrac{4v_0^2 \sin^2\theta}{g^2}$.

From these two equations we have $T^2 = 2R\tan\theta$ or $T \propto \sqrt{R}$. Hence the correct choice is (b).

19. The range of a projectile is the same for two angles of projection θ and $90° - \theta$. For these two angles of projection, the maximum heights are

$$h_1 = \frac{v_0^2 \sin^2\theta}{2g}$$

and $$h_2 = \frac{v_0^2 \sin^2(90° - \theta)}{2g} = \frac{v_0^2 \cos^2\theta}{2g}$$

$$\therefore \quad h_1 h_2 = \frac{v_0^4 \sin^2\theta\cos^2\theta}{4g^2}.$$

Also $$R^2 = \frac{4v_0^4 \sin^2\theta\cos^2\theta}{g^2}.$$

Which give $R^2 = 16\, h_1 h_2$ or $R = 4\sqrt{h_1 h_2}$. Hence the correct choice is (d).

20. The maximum height attained by the first ball is

$$h_1 = \frac{u^2}{2g}$$

where u is the initial speed of projection. The maximum height attained by the second ball is

($\because\ \theta = 90° - 60° = 30°$)

$$h_2 = \frac{u^2 \sin^2(30°)}{2g} = \frac{u^2}{8g}$$

Now, PE of ball 1 at height $h_1 = mgh_1$ and that of ball 2 at height $h_2 = mgh_2$. Therefore, the ratio of potential energies $= \dfrac{h_1}{h_2} = \dfrac{u^2}{2g} \times \dfrac{8g}{u^2} = 4$. Hence the correct choice is (a).

21. For the body of mass m_1, we have

$$h = \frac{u^2}{2g}$$

For the body of mass m_2, if S is the maximum distance travelled along the incline then

$$v^2 - u^2 = 2aS$$

Now, when S is maximum, $v = 0$. Also $a = -g\sin\theta$ $= -g\sin 30° = -\dfrac{g}{2}$. Hence

$$0 - u^2 = 2 \times \left(-\frac{g}{2}\right) S$$

or $S = \dfrac{u^2}{g} = 2h$, which is choice (a).

22. The correct choice is (c). Use

$$t_A = \frac{2u_A}{g} \quad \text{and} \quad t_B = \frac{2u_B \sin 60°}{g}$$

23. The correct choice is (a). The magnitude of velocity is

$$v = \sqrt{(3)^2 + (4)^2} = 5 \text{ ms}^{-1}$$

The angle subtended by the velocity vector with the horizontal (x-axis) is given by

$\tan\theta = \dfrac{4}{3}$ which gives $\sin\theta = \dfrac{4}{5}$

Now,

$$h_{\max} = \frac{v^2 \sin^2\theta}{2g} = \frac{(5)^2 \times \left(\frac{4}{5}\right)^2}{2 \times 10} = 0.8 \text{ m}$$

24. $t_f = \dfrac{2v\sin\theta}{g} = \dfrac{2 \times 5 \times \frac{4}{5}}{10} = 0.8$ s,

which is choice (a).

25. Velocity of projection is $\mathbf{v}_0 = (u\cos\theta)\,\hat{\mathbf{i}} + (u\sin\theta)\,\hat{\mathbf{j}}$. At time t, the velocity of the body is

$$\mathbf{v} = (u\cos\theta)\,\hat{\mathbf{i}} + (u\sin\theta - gt)\,\hat{\mathbf{j}}$$

The dot product of $\mathbf{v}_0$ and $\mathbf{v}$ is

$\mathbf{v}_0 \cdot \mathbf{v} = u^2\cos^2\theta + u\sin\theta\,(u\sin\theta - gt)$

or $\quad \mathbf{v}_0 \cdot \mathbf{v} = u^2 - (u\sin\theta)gt \qquad$ (i)

Since $\mathbf{v}$ is perpendicular to $\mathbf{v}_0$, $\mathbf{v}_0 \cdot \mathbf{v} = 0$. Using this in (i), we have

$$0 = u^2 - (u\sin\theta)gt \text{ or } t = \frac{u}{g\sin\theta}$$

Hence the correct choice is (d).

26. The correct choice is (b).

$$\text{PE} = mgh_{\max} = mg\left(\frac{u^2\sin^2\theta}{2g}\right) = \frac{mu^2\sin^2\theta}{2}$$

$\text{KE} = \dfrac{1}{2}\, m(u\cos\theta)^2$.

27. The horizontal and vertical distances travelled in time t are

$$x = ut$$

and $$y = \frac{1}{2}gt^2 = \frac{gx^2}{2u^2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{gx}{u^2} = \frac{gt}{u} \qquad (\because\ x = ut)$$

and $$\frac{d^2y}{dx^2} = \frac{g}{u^2}$$

The radius of curvature of the trajectory at time t is given by

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{u^2}{g}\left(1+\frac{g^2t^2}{u^2}\right)^{3/2}$$

Hence the correct choice is (d).

28. Refer to Fig. 4.6 on page 4.3. The range along the inclined plane is given by

$$R = \frac{2v_0^2 \sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$$

$$= \frac{v_0^2}{g\cos^2\alpha}[\sin(2\theta - \alpha) - \sin\alpha]$$

R is maximum when $\sin(2\theta - \alpha) = 1$ or $2\theta - \alpha = 90°$ or

$$\theta = \frac{1}{2}(90° + \alpha) = \frac{1}{2}(90° + 10°) = 55°.$$

Hence the correct choice is (c).

29. Let h be the height of the tower. Let u_1 be along positive x-direction u_2 along negative x-direction. The two balls hit after at time t given by

$$h = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

At time t, the respective vertical velocities (along the negative y directions) are gt each. Hence the velocities of the two balls at time t are

$$-\mathbf{v}_1 = u_1\hat{\mathbf{i}} - (gt)\hat{\mathbf{j}} \Rightarrow \mathbf{v}_1 = -u_1\hat{\mathbf{i}} + (gt)\hat{\mathbf{j}}$$

and $-\mathbf{v}_2 = -u_2\hat{\mathbf{i}} - (gt)\hat{\mathbf{j}} \Rightarrow \mathbf{v}_2 = u_2\hat{\mathbf{i}} + (gt)\hat{\mathbf{j}}$

Since $\mathbf{v}_1$ and $\mathbf{v}_2$ are perpendicular to each other,

$$\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$$

$$\Rightarrow [-u_1\hat{\mathbf{i}} + (gt)\hat{\mathbf{j}}] \cdot [u_2\hat{\mathbf{i}} + (gt)\hat{\mathbf{j}}] = 0$$

$$\Rightarrow -u_1u_2 + g^2t^2 = 0$$

$$\Rightarrow g^2t^2 = u_1u_2 \quad \Rightarrow \quad g^2 \times \frac{2h}{g} = u_1u_2$$

$\Rightarrow h = \frac{u_1u_2}{2g}$. Hence the correct choice is (d).

30. Average velocity $= \frac{\text{Displacement}}{\text{time}} = \frac{R}{t_f}$

where $R = \frac{2u^2\sin\theta\cos\theta}{g}$ is the horizontal range and $t_f = \frac{2u\sin\theta}{g}$ is the time of flight.

The correct choice is (a).

31. Horizontal component of velocity at a time t is

$$v_x = u\cos\theta$$

vertical component of velocity at that time t is

$$v_y = u\sin\theta - gt$$

They will become equal at time $t = t^*$ if

$$u\cos\theta = u\sin\theta - gt^*$$

which gives $t^* = u(\sin\theta - \cos\theta)$

Now t^* must be positive. Hence $\sin\theta > \cos\theta$ or $\theta > \pi/4$. Hence the correct choice is (b).

32. Given, $h = 2$ m, $R = 1.25$ m and horizontal distance $s = 10$ m. When the string breaks, the stone is projected in the horizontal direction, which means that there is no initial vertical velocity. From $s = ut + \frac{1}{2}gt^2$, we have ($\because u = 0$),

$$h = \frac{1}{2}gt^2 \qquad \text{(i)}$$

The horizontal distance travelled in time t is

$$s = vt \qquad \text{(ii)}$$

where v is the velocity of the stone in the horizontal direction which is the same as its velocity in circular motion.

Eliminating t from (i) and (ii) we get

$$v^2 = \frac{gs^2}{2h}$$

Now, centripetal acceleration is

$$a_c = \frac{v^2}{R} = \frac{gs^2}{2hR} = \frac{10\times100}{2\times2\times1.25} = 200 \text{ ms}^{-2}$$

Thus, the correct choice is (b).

33. Refer to Fig. 4.17. **AB** and **CD** represent the two velocity vectors. The change in velocity $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$. which can written as $\Delta\mathbf{v} = \mathbf{v}_2 + (-\mathbf{v}_1)$. Thus, to find $\Delta\mathbf{v}$, we reverse the direction of vector **AB** as shown in Fig. (b) and find the resultant of vectors $\mathbf{v}_2$ and $-\mathbf{v}_1$ by triangle or parallelogram law. This is shown in Fig. (c).

The magnitude of vector Δv is given by

$$\Delta v = [(v_1)^2 + v_2^2 + 2(v_1)(v_2)\cos(180° - \theta)]^{1/2}$$

$$= [2v^2(1 - \cos\theta)]^{1/2} \qquad (\because v_1 = v_2 = v)$$

$$= 2v\sin\left(\frac{\theta}{2}\right)$$

Hence the correct choice is (d).

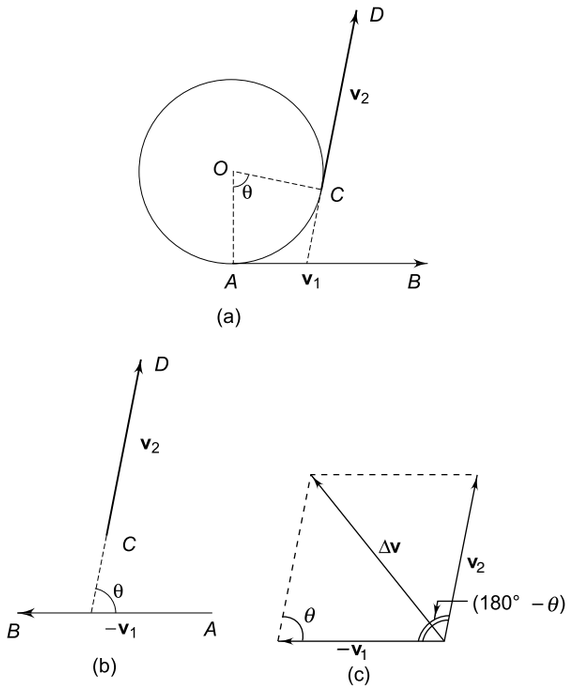


Fig. 4.17

34. Since the force is always perpendicular to the velocity (i.e. the direction of motion) of the particle, no work is done by the force on the particle. Hence the kinetic energy of the particle remains constant. The particle will move in a circle in a plane. Thus the correct choice is (c).

35. As shown in Fig. 4.18, at diametrically opposite points A and B, the magnitude of the velocity is same ($= V$) but the directions of the velocity are opposite. Hence the change in momentum is $MV - (-MV) = 2MV$. Thus the correct choice is (c).

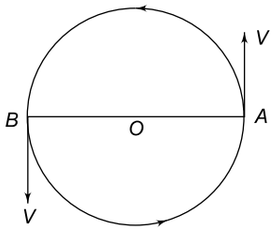


Fig. 4.18

36. When the train is at rest or moving with a uniform velocity, the plumb line hangs vertically along OB (Fig. 4.19). If the train moves with an acceleration a, the plumb line gets inclined along OC, the direction of the resultant of accelerations a and g. It is clear from the figure that $\tan\theta = a/g$. Hence the correct choice is (a).

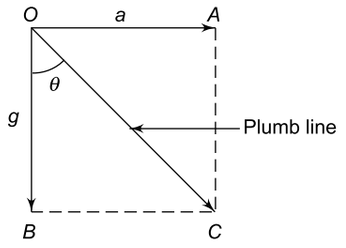


Fig. 4.19

37. If r is the radius of the track, then distance moved in 5s $= \pi r = \pi \times 20$ cm. Therefore, speed along the circle $(v) = \dfrac{20\pi}{5} = 4\pi$ cms^{-1}. Now, angular speed $= \dfrac{v}{r} = \dfrac{4\pi}{20} = \dfrac{\pi}{5}$ rad s^{-1}. Hence the correct choice is (d).

38. Referring to Fig. 4.20, the cyclist is moving on a straight road from A to B with a velocity $v = 6$ ms^{-1}. As he approaches the circular turn, he decelerates at rate a_t, represented by vector **BD**. The magnitude of deceleration is $a_t = 0.4$ ms^{-2}. At point B, two accelerations $\mathbf{a}_t$ and $\mathbf{a}_c$, the centripetal acceleration directed towards the centre C act on the cyclist.

Now $a_c = \dfrac{v^2}{R} = \dfrac{(6)^2}{120} = 0.3$ ms^{-2}. Using the law of parallelogram of vector addition, vector **BE** gives the resultant acceleration a whose magnitude is

($\because$ **DE** = $\mathbf{a}_c$)

$a = (a_t^2 + a_c^2)^{1/2} = \{(0.4)^2 + (0.3)^2\}^{1/2} = 0.5$ ms^{-2}

Hence the correct choice is (a).

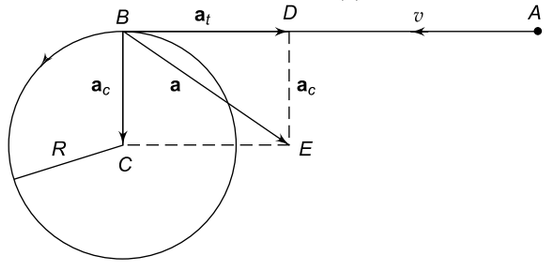


Fig. 4.20

39. In Fig. 4.21 v_c represents the velocity of the car and v_P that of the parcel. M is the position of the man. From parallelogram law, the direction of the resultant velocity v_r must be along the direction along which the man is standing. It follows from the figure that angle θ is given by

$$\sin\theta = \frac{v_c}{v_p} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ or } \theta = 45^\circ$$

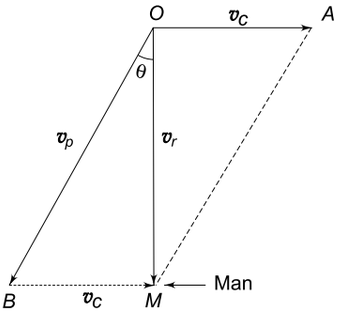


Fig. 4.21

Hence the correct choice is (a).

40. Velocity of rain (v_r) = 4 ms^{-1} vertically downwards. Velocity of wind (v_w) = 3 ms^{-1} from north to south direction. A rain drop is acted upon by two velocities $\vec{v}_r$ and $\vec{v}_w$ as shown in Fig. 4.22. From the triangle law, the resultant velocity of the rain drop is $\vec{v} = \vec{OW}$. In order to protect himself from rain, he must hold his umbrella at an angle θ with the vertical (towards north) given by

$$\tan\theta = \frac{RW}{OR} = \frac{v_w}{v_r} = \frac{3}{4}$$

Thus the correct choice is (b).

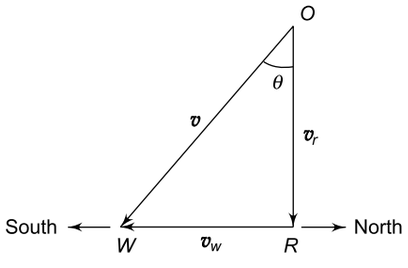


Fig. 4.22

41. The magnitude v of the resultant velocity gives the speed with which the rain strikes the umbrella, which is given by

$$v = [v_r^2 + v_w^2]^{1/2} = [16 + 9]^{1/2} = 5 \text{ ms}^{-1}$$

Hence the correct choice is (c).

42. In order to cross the river in the shortest time, the resultant velocity $\vec{v}$ of the swimmer must be perpendicular to the velocity $\vec{v}_w$ of water, as shown in Fig. 4.23. It follows from the figure that $v_s^2 = v^2 + v_w^2$ or $v_w^2 = v_s^2 - v^2$

$$= 25 - 9 = 16$$

or $v_w = 4$ ms^{-1} which is choice (b).

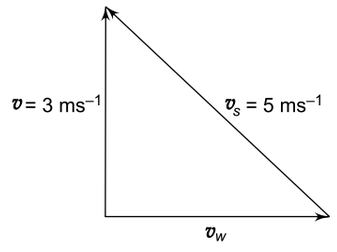


Fig. 4.23

43. Refer to Fig. 4.24. Let PR be the direction along which he should row his boat. The boat is acted upon by two velocities – boat velocity (v_b) and water velocity (v_w). The angle θ should be such that that the resultant velocity (v) is along PQ, i.e.

$$\sin\theta = \frac{v_w}{v_b} = \frac{2}{4} = 0.5$$

$$\Rightarrow \quad \theta = 30° \text{ (upstream).}$$

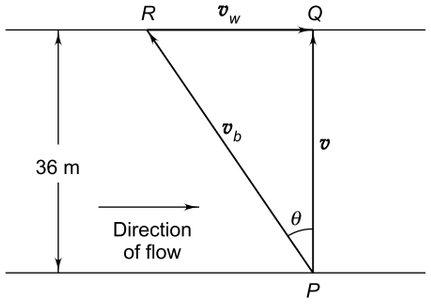


Fig. 4.24

Hence the correct choice is (a).

44. Time taken to cross the river by the shortest path PQ is

$$t = \frac{PQ}{v} = \frac{PQ}{\sqrt{v_b^2 - v_w^2}} = \frac{36}{\sqrt{(4)^2 - (2)^2}} = 6\sqrt{3} \text{ s}$$

Hence the correct choice is (c).

45. Since the direction of the velocity changes from point to point on the circle, choices (a), (b) and (d) are incorrect.

46. From energy conservation, [see Fig. 4.25]

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgL$$

$$\Rightarrow \quad v = \sqrt{u^2 - 2gL}$$

$$\Delta v = \sqrt{v^2 + (-u)^2} = \sqrt{2(u^2 - gL)}$$

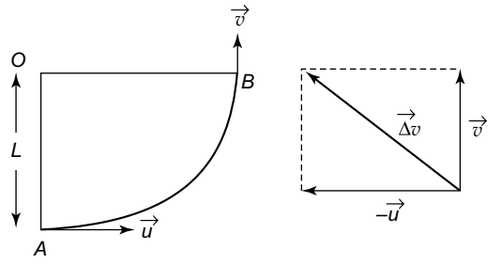


Fig. 4.25

47. For a fixed θ, $R \propto u^2$. So the correct graph is (d).

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. A projectile is fired with a constant speed at two different angles of projection, say, α and β, that give it the same range. Then, α and β are such that
(a) cosec α = sec β
(b) tan $(\alpha + \beta) \to \infty$
(c) $\sin^2 \alpha - \cos^2 \alpha = \sin^2\beta - \cos^2\beta$
(d) cot α = cos α sec β

2. A ball is projected upwards at a certain angle with the horizontal. Which of the following statements are correct? At the highest point
(a) the velocity of the projectile is zero
(b) the acceleration of the projectile is zero
(c) the velocity of the projectile is along the horizontal direction.
(d) the acceleration of the projectile is vertically downwards.

3. Choose the correct statements from the following. The range of a projectile depends upon
(a) the angle of projection
(b) the acceleration due to gravity
(c) the magnitude of the velocity of projection
(d) the mass of the projectile

4. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that:
(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a circular path

5. A simple pendulum of length r and bob mass m swings in a vertical circle with angular frequency ω. When the string makes an angle θ with the vertical, the speed of the bob is v. The radial acceleration of the bob at this instant is given by

(a) $\frac{v^2\omega}{r}$ (b) $\frac{v^2}{r\omega}$

(c) $\frac{v^2}{r}$ (d) $r\omega^2$

6. A body is moving in a circle of radius r with a uniform speed v, angular frequency ω, time period T and frequency ν. The centripetal acceleration is given by

(a) $\frac{v^2}{r}$ (b) $4\pi^2 r \nu^2$

(c) ωv (d) $\frac{4\pi^2 r}{T^2}$

7. Which of the following statements are true about a body moving in a circle with a uniform speed?
(a) The speed of the body is constant but its velocity is changing
(b) The acceleration is directed towards the centre
(c) The velocity and acceleration vector are perpendicular to each other.
(d) Elastic, frictional, gravitation and magnetic forces can cause a uniform circular motion.

8. A missile is fired for maximum range at your town from a place in the enemy country at a distance x from your town. The missile is first detected at its half-way point. Then
(a) the velocity with which the missile was projected is $\sqrt{gx}$

(b) you have a warning time of $\sqrt{\frac{x}{2g}}$

(c) the speed of the missile when it was detected is $\sqrt{\frac{gx}{2}}$

(d) the maximum height attained by the missible is $\frac{x}{4}$.

9. An enemy plane is flying horizontally with a speed v. An armyman with an anti-aircraft gun on the ground sights the enemy plane when it is directly overhead and fires a shell with a muzzle speed u. Then

(a) the angle with the vertical at which the gun should be fired in order to hit the plane is $\cos^{-1}\left(\frac{v}{u}\right)$

(b) the angle with the vertical at which the gun should be fired in order to hit the plane is $\sin^{-1}\left(\frac{v}{u}\right)$.

(c) the maximum height at which the enemy plane must fly to avoid being hit is $\frac{u^2 - v^2}{2g}$.

(d) the maximum height at which the enemy plane must fly to avoid being hit is $\frac{(u-v)^2}{2g}$.

10. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 ms^{-1} at an angle of elevation of 30º. The total time taken by the ball to hit the ground is T and the time taken to come back to the same elevation) is t. The horizontal distance covered by the ball is x. If $g = 10\ ms^{-2}$, then

(a) $\frac{T}{t} = 2$ (b) $\frac{T}{t} = \sqrt{2}$

(c) $x = 40\sqrt{2}$ m (d) $x = 40\sqrt{3}$ m

11. The horizontal distance x and the vertical height y of a projectile at time t are given by

$$x = at \quad \text{and} \quad y = bt^2 + ct$$

where, a, b and c are constants. Then

(a) the speed of the projectile 1 second after it is fired is $(a^2 + b^2 + c^2)^{1/2}$

(b) the angle with the horizontal at which the projectile is fired is $\tan^{-1}\left(\frac{c}{a}\right)$

(c) the acceleration due to gravity is $-2b$.

(d) the initial speed of the projectile is $(a^2 + c^2)^{1/2}$.

12. A projectle thrown at an angle of 30º with the horizontal has a range R_1 and attains a maximum height h_1. Another projectile, thrown with the same speed, at an angle of 30º with the vertical has a range R_2 and attains a maximum height h_2. Then

(a) $R_2 = 2R_1$ (b) $R_2 = R_1$

(c) $h_2 = 2h_1$ (d) $h_2 = 3h_1$

13. The maximum height attained by a projectile is increased by 1% by increasing its speed of projection without changing the angle of projection. Then the percentage increase in the

(a) horizontal range will be 2%

(b) horizontal range will be 1%

(c) time of flight will be 0.5%

(d) time of flight will be 2%

14. The speed of projection of a projectile is increased by 1% without changing the angle of projection. Then, the percentage increase in the

(a) horizontal range will be 1%.

(b) maximum height attained will be 2%

(c) time of flight will be 2%

(d) time of flight will be 0.5%

15. A body is projected at time $t = 0$ from a certain point on a planet's surface with a certain velocity at a certain angle with the planet's surface (assumed horizontal). The horizontal and vertical displacements x and y (in meters) respectively vary with time t (in seconds) as

$$x = 10\sqrt{3}\ t$$

$$y = 10\ t - t^2$$

(a) The acceleration due to gravity on the surface of the planet is 10 ms^{-2}.

(b) The maximum height attained by the body is 25 m.

(c) The time of flight is 10 s.

(d) The horizontal range is 100 m.

16. For a particle moving in a circle with a constant speed,

(a) the velocity vector is always along the tangent to the circle.

(b) the acceleration vector points towards the centre of the circle.

(c) the velocity and acceleration vectors are perpendicular to each other.

(d) the velocity and acceleration vectors are parallel to each other.

17. A particle is acted upon by a constant force which is always perpendicular to its velocity. The motion of the particle takes place in a plane. It follows that
(a) its speed is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) its momentum is constant.

18. A stone of mass 250 g is tied to the end of a string of length 1.0 m. It is whirled in a horizontal circle with a frequency of 30 rev./min.
(a) The tension in the string changes as the stone moves in the circle.
(b) The tension in the string is constant equal to $\frac{\pi^2}{4}$ newton.
(c) The speed of the stone is π ms^{-1}.
(d) The maximum speed with which the stone can be whirled is 20 ms^{-1}.

19. A uniform disc of radius R is rotating about its axis with angular speed ω. It is gently placed on a horizontal surface which is perfectly frictionless (Fig. 4.26). If v_A, v_B and v_C are the linear speeds of points A, B and C respectively, then

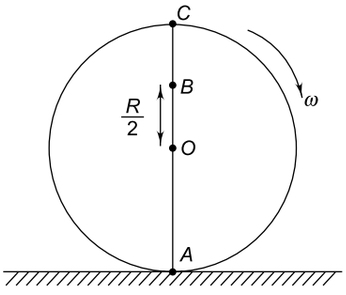


Fig. 4.26

(a) $v_A = v_B = v_C$ (b) $v_A = v_B > v_C$
(c) $v_A = v_C > v_B$ (d) $v_A < v_B < v_C$

20. A uniform disc of radius R is rolling (without slipping) on a horizontal surface with an angular speed ω as shown in Fig. 4.27. O is the centre of the disc, points A and C are located on its rim and point B is at a distance $\frac{R}{2}$ from O. During rolling, the points A, B and C lie on the vertical diameter at a certain instant of time. If v_A, v_B and v_C are the linear speeds of points A, B and C respectively at that instant, then
(a) $v_A = v_B = v_C$ (b) $v_A > v_B > v_C$
(c) $v_A = 0$, $v_B = \frac{3R\omega}{2}$ (d) $\frac{v_B}{v_C} = \frac{3}{4}$

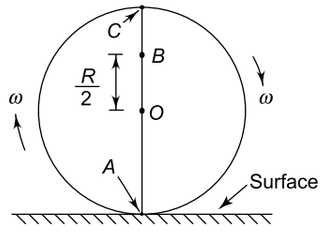


Fig. 4.27

IIT, 2004

21. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a and b are constants and x and y are respectively the horizontal and vertical distances of the projectile from the point of projection.
(a) The horizontal range of the projectile is $\frac{a}{2b}$.
(b) The maximum height attained by the projectile is $\frac{a^2}{4b}$.
(c) The time of flight is $\frac{2a}{\sqrt{bg}}$, where g is the acceleration due to gravity.
(d) The angle of projection from the horizontal is $\theta = \tan^{-1}(a - 2bx)$.

IIT, 1990

22. The coordinates of a particle moving in a plane are given by

$$x = a \cos(pt)$$

and $$y = b \sin(pt)$$

where a, b and p are positive constants and $b < a$. Then
(a) the path of the particle is an ellipse
(b) the velocity and acceleration of the particle are perpendicular to each other at $t = \pi/2p$.
(c) the acceleration of the particle is always directed towards a focus.
(d) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/2p$ is a.

IIT, 1999

ANSWERS AND SOLUTIONS

1. For the same range $\alpha + \beta = 90^\circ$ or $\beta = 90^\circ - \alpha$. Choices (a), (b) and (d) satisfy this relation between β and α but choice (c) does not.

2. The correct choices are (c) and (d).

3. Range $R = \dfrac{u^2 \sin 2\theta}{g}$, is independent of the mass of the projectile. Hence choices (a), (b) and (c) are correct.

4. The magnitudes of velocity and acceleration remain constant but their directions are changing continuously. In uniform circular motion the force is radial (centripetal) and is always perpendicular to the velocity which is tangential. Thus, choice (c) and (d) are correct.

5. The radial component of acceleration is a_r = centripetal acceleration

$$= \frac{v^2}{r} = r\omega^2 \quad (\because v = r\omega)$$

Hence, the correct choices are (c) and (d).

6. Since $v = r\omega = 2\pi\nu r = \dfrac{2\pi r}{T}$, all the four choices are correct.

7. All the four choices are correct.

8. For maximum range $\theta = 45^\circ$ for which $R_{\max} = v_0^2/g$
Hence $v_0 = \sqrt{g \times R_{\max}} = \sqrt{gx}$ which is choice (a).

The warning time is half the time of flight (since the missile is first detected at half-way point. Hence, warning time is

$$t = \frac{t_f}{2} = \frac{v_0 \sin\theta}{g} = \frac{\sqrt{gx}\sin 45^\circ}{g} = \sqrt{\frac{x}{2g}}$$

Hence choice (b) is also correct.

At half-way point, the missile is at its maximum height. Therefore, the vertical component of velocity is zero at this point. Hence the velocity is given only by the horizontal component which is $v_x = v_0 \cos\theta = \sqrt{gx}\ \cos 45^\circ = \sqrt{\dfrac{gx}{2}}$, which is choice (c). The maximum height attained is

$$h_{\max} = \frac{v_0^2 \sin^2\theta}{2g} = \frac{gx \times \sin^2 45^\circ}{2g} = \frac{x}{4}$$

Thus, all the four choices are correct.

9. Let G be the position of the gun and E that of the enemy plane flying horizontally with speed v, when the shell is fired with a speed u in a direction θ with the horizontal (Fig. 4.28). The horizontal component of u is

$$v_x = u\cos\theta$$

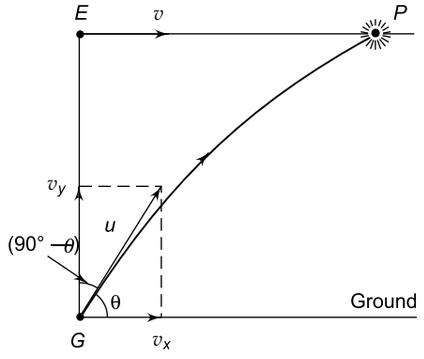


Fig. 4.28

Let the shell hit the plane at point P and let t be the time taken for the shell to hit the plane. It is clear that the shell will hit the plane, if the horizontal distance EP travelled by the plane in time t = the distance travelled by the shell in the horizontal direction in the same time, i.e. $v \times t = v_x \times t$ or $v = v_x = u\cos\theta$ or $\cos\theta = \dfrac{v}{u}$.

To avoid being hit, the plane should have a minimum altitude = maximum height attained by the shell which is

$$h_{\max} = \frac{u^2\sin^2\theta}{2g} = \frac{u^2(1-\cos^2\theta)}{2g}$$

$$= \frac{u^2\left(1-\dfrac{v^2}{u^2}\right)}{2g} = \frac{(u^2 - v^2)}{2g}$$

Hence the correct choices are (a) and (c).

10. $$t = \frac{2u\sin\theta}{g} = \frac{2\times 20\times \sin 30^\circ}{10} = 2 \text{ s}$$

Initial downward velocity = $u\sin\theta = 20 \times \sin 30^\circ = 10\ \text{ms}^{-1}$. The time taken to fall through a height of 40 m is given by

$40 = 10t_1 + \dfrac{1}{2} \times 10 \times t^2{}_1$ which gives $t_1 = 2$ s. Hence, the total time taken to hit the ground is $T = 2 + 2 = 4$ s. Therefore $T/t = 2$. Also, the horizontal distance

travelled in 4 s = $(u \cos \theta) \times T = 20 \times \cos 30° \times 4 = 40\sqrt{3}$ m.

Hence the correct choices are (a) and (c).

11. The horizontal component of velocity is

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(at) = a \qquad \text{(i)}$$

The vertical component of velocity is

$$v_y = \frac{dy}{dt} = \frac{d}{dt}\left(bt^2 + ct\right) = 2\,bt + c \qquad \text{(ii)}$$

The value of v_y at $t = 1$ s is $(2b + c)$. Therefore, the magnitude of velocity at $t = 1$ s is

$$v = (v_x^2 + v_y^2)^{1/2} = [a^2 + (2b + c)^2]^{1/2}$$

If a projectile is projected with an initial velocity v_0 at an angle θ with the horizontal, the horizontal and vertical components of its velocity at time t are given by

$$v_x = v_0 \cos \theta \qquad \text{(iii)}$$

and $$v_y = v_0 \sin \theta - gt \qquad \text{(iv)}$$

Comparing (iii) and (iv) with (i) and (ii) above we have $v_0 \cos \theta = a$ and $v_0 \sin \theta = c$. Dividing, we get, $\tan \theta = c/a$.

Compairing (iv) with (ii) we have $g = -2b$

We have seen above that $v_0 \cos \theta = a$ and $v_0 \sin \theta = c$. Squaring and adding we get: $v_0^2 = a^2 + c^2$ or $v_0 = (a^2 + c^2)^{1/2}$.

Hence the correct choice are (b), (c) and (d).

12. The range is the same for θ and $(90° - \theta)$. Hence $R_1 = R_2$ for $\theta = 30°$ or $60°$.

Since $h_{\max} = \dfrac{v_0^2 \sin^2 \theta}{2g}$ and v_0 is the same, we have

$$\frac{h_1}{h_2} = \frac{\sin^2 30°}{\sin^2 60°} = \frac{1}{3} \qquad \text{or} \qquad h_1 = \frac{h_2}{3}.$$

Thus the correct choice are (b) and (d).

13. We know that $h = \dfrac{v_0^2 \sin^2 \theta}{2g}$. The increase δh in h when v_0 changes by δv_0 can be obtained by partially differenting this expression. Thus

$$\delta h = \frac{2v_0\,\delta v_0 \sin^2 \theta}{2g}, \quad \therefore \quad \frac{\delta h}{h} = \frac{2\delta v_0}{v_0}$$

$$\therefore \quad \frac{\delta v_0}{v_0} = \frac{1}{2}\frac{\delta h}{h} = \frac{1}{2} \times 0.01 = 0.005$$

Now range $R = \dfrac{v_0^2 \sin^2 \theta}{g}$

$$\therefore \quad \frac{\delta R}{R} = \frac{2\delta v_0}{v_0} = 2 \times 0.005 = 0.01 = 1\ \%$$

Time of flight is $T = \dfrac{2v_0 \sin \theta}{g}$

$$\therefore \quad \frac{\delta T}{T} = \frac{\delta v_0}{v_0} = 0.005 = 0.5\ \%$$

Hence the correct choices are (b) and (c).

14. The correct choices are (a) and (d). Use $\dfrac{\delta R}{R} = \dfrac{2\delta v_0}{v_0}$, $\dfrac{\delta h}{h} = \dfrac{2\delta v_0}{v_0}$ and $\dfrac{\delta T}{T} = \dfrac{\delta v_0}{v_0}$.

15. The horizontal and vertical displacements are given by

$$x = (v_0 \cos \theta)t \qquad (1)$$

and $$y = (v_0 \sin \theta)t - \frac{1}{2}gt^2 \qquad (2)$$

Given $$x = 10\sqrt{3}\ t \qquad (3)$$

and $$y = 10\ t - t^2 \qquad (4)$$

Comparing (3) with (1) and (4) with (2), we have

$$v_0 \cos \theta = 10\sqrt{3} \qquad (5)$$

$$v_0 \sin \theta = 10 \qquad (6)$$

and $$\frac{1}{2}g = 1 \Rightarrow g = 2 \text{ m s}^{-2}$$

Equations (5) and (6) give $v_0 = 20$ ms^{-1} and $\theta = 30°$.

$$\therefore h_{\max} = \frac{v_0^2 \sin^2 \theta}{2g} = \frac{(20)^2 \times \sin^2 30°}{2 \times 2} = 25 \text{ m}$$

$$t_f = \frac{2v_0 \sin \theta}{g} = \frac{2 \times 20 \times \sin 30°}{2} = 10 \text{ s}$$

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{(20)^2 \times \sin 60°}{2} = 100\sqrt{3} \text{ m}$$

Hence the correct choices are (b) and (c).

16. The correct choices are (a), (b) and (c).

17. Since the force is always perpendicular to the velocity (i.e. the direction of motion) of the particle, no work is done by the force on a particle. The particle will move in a circle in a plane. Its speed and hence its kinetic energy will remain constant. Since the direction of the velocity is along the tangent to the circle, it will keep changing with time. Hence, the momentum will not remain constant. Since velocity is changing with time, the acceleration (which is perpendicular to velocity) will also keep changing. Hence the correct choices are (a) and (c).

18. Since the stone is whirled in a horizontal plane, the weight mg of the stone (which acts vertically down-

wards) is perpendicular to the plane of the circular motion and, therefore, has no component along this plane. Hence the tension in the string is constant.

Given $m = 250$ g $= 0.25$ kg, $R = 1.0$ m, frequency (ν) $= 30$ rev./min $= \frac{30}{60} = 0.5$ Hz. Angular frequency $(\omega) = 2\pi\nu = 2\pi \times 0.5 = \pi$ rad s^{-1}. The necessary centripetal force is provided by the tension in the string. Therefore,

$$T = \frac{mv^2}{R} = \frac{m(\omega R)^2}{R} = m\omega^2 R$$

$$= 0.25 \times \pi^2 \times 1.0 = \frac{\pi^2}{4} \text{ N}$$

Speed of the stone is $v = R\omega = 1.0 \times \pi = \pi$ ms^{-1}. The maximum speed is given by

$$\frac{mv_{\max}^2}{R} = 100$$

or $$v_{\max} = \sqrt{\frac{100 \times R}{m}} = \sqrt{\frac{100 \times 1.0}{0.25}}$$

$$= 20 \text{ ms}^{-1}$$

Hence the correct choices are (b), (c) and (d)

19. Since the surface is perfectly frictionless, the disc will not roll on the surface; it will simply keep on rotating at point A where it is placed. Now, linear speed = distance from centre × angular speed. Therefore,

$$v_A = \omega R,\ v_B = \frac{\omega R}{2} \quad \text{and} \quad v_C = \omega R$$

Hence the only correct choice is (c).

20. The disc is rolling about the point O. Thus the axis of rotation passes through the point A and is perpendicular to the plane of the disc. From the relation $v = r\omega$ where r is the distance of the point on the rim about the axis of rotation, we have

$$v_A = 0,\ v_B = (AB)\ \omega = \frac{3R\omega}{2}$$

and $$v_C = (AC)\ \omega = 2R\omega$$

Hence $\frac{v_B}{v_c} = \frac{3R\omega}{2} \times \frac{1}{2R\omega} = \frac{3}{4}$.

Hence the correct choices are (c) and (d).

21. Given $y = ax - bx^2$ (1)

(a) The value of y is zero at $x = 0$ and $x = R$ (horizontal range). Putting $y = 0$ and $x = R$ in Eq. (1), we get $R = a/b$.

(b) Differentiating Eq. (1) with respect to time t, we have

$$\frac{dy}{dt} = a\frac{dx}{dt} - 2bx\frac{dx}{dt}$$

$$\Rightarrow \quad v_y = av_x - 2bx\ v_x = (a - 2bx)v_x \qquad (2)$$

At the maximum height $v_y = 0$. Using this in Eq. (2), we get $(a - 2bx) = 0$ or $x = a/2b$. Putting this value of x in Eq. (1), we have (since $y = h_{\max}$ at this value of x)

$$h_{\max} = a\left(\frac{a}{2b}\right) - b\left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$$

(c) The time t to reach the maximum height is given by

$$h_{\max} = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2h_{\max}}{g}} = \sqrt{\frac{2a^2}{4bg}} = \frac{a}{\sqrt{2bg}}$$

Therefore, the time of flight is

$$t_f = 2t = a\sqrt{\frac{2}{bg}}$$

(d) $\tan\theta = \frac{dy}{dx} = \frac{d}{dx}(ax - bx^2) = a - 2bx$

Hence the correct choices are (b) and (d).

22. Given $x = a\cos(pt)$ (1)

$y = b\sin(pt)$ (2)

From (1) and (2) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2(pt) + \sin^2(pt) = 1$

Hence the path of the particle is an ellipse.

Let the position vector of the particle at time t be

$$\vec{r} = x\hat{i} + y\hat{j}$$

$\therefore$ Velocity $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{x}}{dt}\hat{i} + \frac{d\vec{y}}{dt}\hat{j}$

$\Rightarrow \quad \vec{v} = -ap\sin(pt)\,\hat{i} + bp\cos(pt)\,\hat{j}$

At $t = \pi/2p$, $\vec{v} = -ap\sin\left(\frac{\pi}{2}\right)\hat{i} + bp\cos\left(\frac{\pi}{2}\right)\hat{j}$

$$= -ap\,\hat{i} \qquad (3)$$

Acceleration $\vec{a} = -ap^2\cos(pt)\,\hat{i} - bp^2\sin(pt)\,\hat{j}$

At $t = \pi/2p$, $\vec{a} = -bp^2\,\hat{j}$

$\therefore$ At $t = \pi/2p$, $\vec{v}\cdot\vec{a} = abp^3\,(\hat{i}\cdot\hat{j}) = 0$.

Hence $\vec{v} \perp \vec{a}$.

It is easy to see that choices (c) and (d) are incorrect.

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage

Passage I

Two objects are projected from the same point with the same speed u at angles of projection α and β with the horizontal respectively. They strike the ground at the same point at a distance R from the point of projection. The respective maximum heights attained by the objects are h_1 and h_2 and t_1 and t_2 are the respective times of flight.

1. R, h_1 and h_2 are related as

(a) $R = \sqrt{h_1 h_2}$ (b) $R = \sqrt{2h_1 h_2}$

(c) $R = 2\sqrt{2h_1 h_2}$ (d) $R = 4\sqrt{2h_1 h_2}$

2. The ratio $\frac{t_1}{t_2}$ is equal to

(a) $\sin \alpha$ (b) $\cos \alpha$

(c) $\tan \alpha$ (d) $\cot \alpha$

3. The ration h_1/h_2 is equal to

(a) $\sin^2 \alpha$ (b) $\cos^2 \alpha$

(c) $\tan^2 \alpha$ (d) $\cot^2 \alpha$

4. The sum $(h_1 + h_2)$ is equal to

(a) $\frac{u^2}{g} \sin^2 \alpha$ (b) $\frac{u^2}{g} \cos^2 \alpha$

(c) $\frac{u^2}{g}$ (d) $\frac{u^2}{2g}$

SOLUTIONS

Since the horizontal range in the same $\alpha + \beta = 90°$. Therefore $\beta = 90° - \alpha$ and we have

$$h_1 = \frac{u^2}{2g} \sin^2 \alpha \quad (1)$$

$$h_2 = \frac{u^2}{2g} \sin^2 \beta = \frac{u^2}{2g} \cos^2 \alpha \quad (2)$$

$$R = \frac{u^2}{2g} \times 2 \sin \alpha \cos \alpha \quad (3)$$

$$t_1 = \frac{2u \sin \alpha}{g} \quad (4)$$

$$t_2 = \frac{2u \sin \beta}{g} = \frac{2u}{g} \cos \alpha \quad (5)$$

1. From Eqs. (1), (2) and (3), we have

$$R = \frac{u^2}{g} \times 2 \times \sqrt{\frac{2gh_1}{u^2} \times \frac{2gh_2}{u^2}} = 4\sqrt{h_1 h_2}$$

Hence the correct choice is (d)

2. From Eqs. (4) and (5) it follows that the correct choice is (c).

3. From Eqs. (1), (2), (4) and (5), we have

$$\frac{h_1}{h_2} = \tan^2 \alpha$$

Hence the correct choice is (c).

4. From Eqs. (1) and (2), we find that

$h_1 + h_2 = \frac{u^2}{2g}$, which is choice (d).

Questions 5 to 11 are based on the following passage

Passage II

The position vector **r** with respect to the origin of a particle varies with time t as

$$\mathbf{r} = (at)\,\hat{\mathbf{i}} + (bt - ct^2)\,\hat{\mathbf{j}}$$

where a, b and c are constants.

5. The trajectory of the particle is a

(a) straight line (b) circle

(c) parabola (d) none of these

6. The magnitude of the initial velocity of the particle is

(a) $\sqrt{a^2 + c^2}$ (b) $\sqrt{b^2 + c^2}$

(c) $\sqrt{a^2 + b^2}$ (d) $(a + b - 2c)$

7. The angle θ with the horizontal along which the particle is projected is given by

(a) $\sin\theta = \frac{b}{c}$ (b) $\cos\theta = \frac{a}{c}$

(c) $\tan\theta = \frac{b}{a}$ (d) $\tan\theta = \frac{2c}{\sqrt{a^2+b^2}}$

8. The time of flight of the particle is

(a) $\frac{b}{c}$ (b) $\frac{ab}{c}$

(c) $\frac{c}{a}$ (d) $\frac{b}{ac}$

9. The acceleration due to gravity at that place is

(a) 2 a (b) 2 b

(c) 2 c (d) none of these

10. The maximum height to which the particle rises is

(a) $\frac{2b^2}{c}$ (b) $\frac{b^2}{2c}$

(c) $\frac{b^2}{4c}$ (d) $\frac{4b^2}{c}$

11. The horizontal range of the particle is

(a) $\frac{ab}{c}$ (b) $\frac{ac}{b}$

(c) $\frac{bc}{a}$ (d) abc

SOLUTIONS

5. Comparing Eq. $\mathbf{r} = (at)\,\hat{\mathbf{i}} + (bt - ct^2)\,\hat{\mathbf{j}}$ with Eq. $r = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$, we get

$$x = at \quad (1)$$

and $$y = bt - ct^2 \quad (2)$$

Eliminating t from (1) and (2) we get $y = \left(\frac{b}{a}\right)x - \left(\frac{c}{a^2}\right)x^2$ which is the equation of a parabola. Hence the correct choice is (c).

6. From Eqs. (1) and (2), we have

$$v_x = \frac{dx}{dt} = a \quad (3)$$

$$v_y = \frac{dy}{dt} = b - 2ct \quad (4)$$

Putting $t = 0$ in Eqs. (3) and (4), the initial values of v_x and v_y are a and b respectively. The initial speed of the particle is

$u = \sqrt{a^2+b^2}$, which is choice (c).

7. If the particle is projected with an initial velocity u at an angle θ with the horizontal, then the horizontal displacement x and vertical displacement y at time t are

$$x = (u\cos\theta)t \quad (5)$$

and $$y = (u\sin\theta)t - \frac{1}{2}gt^2 \quad (6)$$

Comparing Eqs. (5) and (6) with Eqs. (3) and (4) we have $u\cos\theta = a$ and $u\sin\theta = b$ which give $\tan\theta = b/a$, which is choice (c).

8. When $t = t_f$, $y = 0$. Putting $y = 0$ and $t = t_f$ in Eq. (2) we get $0 = t_f(b - ct_f)$ gives $t_f = 0$ and $t_f = b/c$. But $t_f = 0$ is not possible. Hence the correct choice is (a).

9. Comparing Eq. (6) with Eq. (2), we get $g = 2c$, which is choice is (c).

10. Now $y = h_{\max}$ when $t = \frac{1}{2}t_f = \frac{b}{2c}$. Putting $y = h_{\max}$ and $t = b/2c$ in Eq. (2) we get

$$h_{\max} = b \times \frac{b}{2c} - c\left(\frac{b}{2c}\right)^2 = \frac{b^2}{4c}$$

11. $$R = (u\cos\theta)t_f \quad (7)$$

Now $u = \sqrt{a^2+b^2}$, $\tan\theta = \frac{b}{a}$ which gives $\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$ and $t_f = \frac{b}{c}$. Putting these values in Eq. (7), we get $R = \frac{ab}{c}$. Hence the correct choice is (a).

Questions 12 to 16 are based on the following passage

Passage III

An object of mass m is whirled with a constant speed v in a vertical circle with centre O and radius R. T_1, T_2, T_3 and T_4 are the tensions in the string when the object is at A (top of the circle), B, C (the lowermost point of the circle) and D respectively (Fig. 4.29)

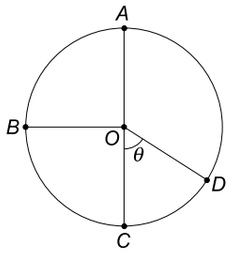


Fig. 4.29

12. Tensions T_1, T_2 and T_3 are related as

(a) $T_1 = T_2 = T_3$ (b) $T_1 < T_2 < T_3$

(c) $T_1 > T_2 > T_3$ (d) $T_1 = T_3 < T_2$

13. Tension T_4 is given by

(a) $T_4 = \dfrac{mv^2}{R} + mg \cos \theta$

(b) $T_4 = \dfrac{mv^2}{R} - mg \cos \theta$

(c) $T_4 = \dfrac{mv^2}{R} + mg \sin \theta$

(d) $T_4 = \dfrac{mv^2}{R} - mg \sin \theta$

14. The minimum speed the object must have at the highest point A to complete the circle is

(a) $\sqrt{\dfrac{Rg}{2}}$ (b) $\sqrt{Rg}$

(c) $\sqrt{2Rg}$ (d) $2\sqrt{Rg}$

Here g is the acceleration due to gravity.

15. The minimum speed the object must have at the lowest point C to complete the circle is

(a) $\sqrt{Rg}$ (b) $\sqrt{2Rg}$

(c) $2\sqrt{2Rg}$ (d) $\sqrt{5Rg}$

16. At the minimum speed of Q.15, the tension in the string is

(a) 4 mg (b) 5 mg

(c) 6 mg (d) mg

SOLUTIONS

12. Refer to Fig. 4.30

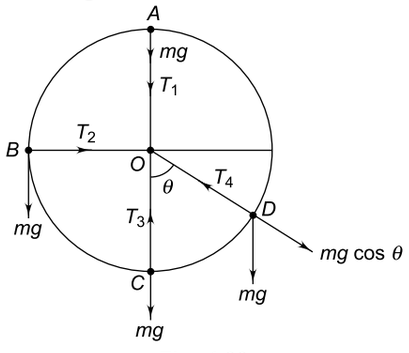


Fig. 4.30

When the object is at A, since the weight mg acts vertically downwards, the force towards centre O is.

$$F_1 = T_1 + mg$$

$$\Rightarrow \quad \frac{mv^2}{R} = T_1 + mg$$

$$\Rightarrow \quad T_1 = \frac{mv^2}{R} - mg$$

At point B, weight mg has no component along BO. Hence the force towards O is

$$F_2 = T_2 \quad \Rightarrow \quad \frac{mv^2}{R} = T_2$$

At point C, the weight mg acts in opposite direction to tension T_3. Thus at C, the force towards centre O is

$$F_3 = T_3 - mg$$

$$\Rightarrow \quad \frac{mv^2}{R} = T_3 - mg$$

$$\Rightarrow \quad T_3 = \frac{mv^2}{R} + mg$$

Hence the correct choice is (b).

13. At point D, the force towards the centre is

$$F_4 = T_4 - mg \cos \theta$$

$$\Rightarrow \quad \frac{mv^2}{R} = T_4 - mg \cos \theta$$

$$\Rightarrow \quad T_4 = \frac{mv^2}{R} + mg \cos \theta, \text{ which is choice (a).}$$

14. In order to keep a body of mass m in a circular path, the centripetal force, at the highest point A, must at least be equal to the weight of the body. Thus

$$\frac{mv_A^2}{R} = mg \quad \text{or} \quad v_A = \sqrt{Rg}$$

gives the minimum speed the body must have at the highest point so that it can complete the circle. Hence the correct choice is (b).

15. The minimum speed v_C of the body must have at the lowest point C is given by $v_C^2 = v_A^2 + 2 \times 2\ Rg$ where we have used $v^2 = u^2 + 2gh$, with $h = 2\ R$. Thus

$$v_C^2 = Rg + 4\ Rg = 5\ Rg$$

or $v_C = \sqrt{5\,Rg}$, which is choice (d).

16. The tension at this point is given by

$$T_3 = m\left(\frac{v_C^2}{R} + g\right) = m(5\ g + g) = 6\ mg$$

Hence the correct choice is (c).

Questions 17 to 20 are based on the following passage

Passage IV

The kinetic energy of a particle moving along a circle of radius R depends on distance (s) as $K = as^2$ where a is a constant.

17. The centripetal force is given by

(a) $\frac{as^2}{2R}$ (b) $\frac{as^2}{R}$

(c) $\frac{2as^2}{R}$ (d) $\frac{4as^2}{R}$

18. The speed of the particle around the circle is

(a) $2s\left(\frac{a}{m}\right)^{1/2}$ (b) $s\left(\frac{a}{m}\right)^{1/2}$

(c) $s\left(\frac{2a}{m}\right)^{1/2}$ (d) $s\left(\frac{a}{2m}\right)^{1/2}$

19. The tangential force acting on the particle is

(a) mas (b) $2mas$

(b) as (d) $2as$

20. The net force acting on the particle is

(a) $2as\left(1+\frac{s}{R}\right)$ (b) $as\left(1+\frac{s^2}{R^2}\right)^{1/2}$

(c) $2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$ (d) zero

SOLUTIONS

17. Given KE $= \frac{1}{2}mv^2 = as^2$. Therefore, the centripetal force is

$$f_c = \frac{mv^2}{R} = \frac{2\times\left(\frac{1}{2}mv^2\right)}{R} = \frac{2as^2}{R},$$

which is choice (c).

18. The speed v of the particle around the circle is given by

$$\frac{1}{2}mv^2 = as^2 \quad \text{or} \quad v = s\left(\frac{2a}{m}\right)^{1/2}$$

Hence the correct choice is (c).

19. The tangential acceleration is

$$a_t = \frac{dv}{dt} = \frac{d}{dt}\left[s\left(\frac{2a}{m}\right)^{1/2}\right] = \left(\frac{2a}{m}\right)^{1/2}\frac{ds}{dt}$$

But $\frac{ds}{dt} = v = s\left(\frac{2a}{m}\right)^{1/2}$. Therefore,

$$a_t = \left(\frac{2a}{m}\right)^{1/2}\times s\left(\frac{2a}{m}\right)^{1/2} = \frac{2as}{m}$$

$\therefore$ Tangential force is $f_t = ma_t$

$= m\times\frac{2as}{m} = 2as$, which is choice (d).

20. Net force acting on the particle is

$$f = (f_c^2 + f_t^2)^{1/2}$$

$$= \left[\left(\frac{2as^2}{R}\right)^2 + (2as)^2\right]^{1/2}$$

$$= 2\ as\left[1+\frac{s^2}{R^2}\right]^{1/2}$$

Thus the correct choice is (c).

Questions 21 to 23 are based on the following passage

Passage V

A conical pendulum consists of a string of length L fixed at one end carrying a body of mass m at the other end. The mass is revolved in a circle in the horizontal plane about-a vertical axis passing through the fixed end of the string. The angular frequency of revolution of the body is ω. The string makes an angle θ with the vertical axis.

21. The tension in the string is

(a) $\frac{m\omega^2}{L}$ (b) $\frac{L\omega^2}{m}$

(c) $m\omega^2 L$ (d) $m\omega L^2$

22. The angle of inclination of the string with the vertical is given by

(a) $\cos\theta = \frac{g}{\omega^2 L}$ (b) $\sin\theta = \frac{g}{\omega^2 L}$

(c) $\cos\theta = \frac{\omega^2 L}{g}$ (d) $\sin\theta = \frac{\omega^2 L}{g}$

23. The linear speed of the body is

(a) ωL (b) $\omega L\sin\theta$

(c) $\omega L\cos\theta$ (d) $\omega L\tan\theta$

SOLUTIONS

21. Let T be tension in the string. Figure 4.31 shows the forces acting on the system. Tension T can be resolved into two mutually perpendicular components. The horizontal component $T \sin \theta$ provides the centripetal force for circular motion and the vertical component $T \cos \theta$ balances the weight mg.

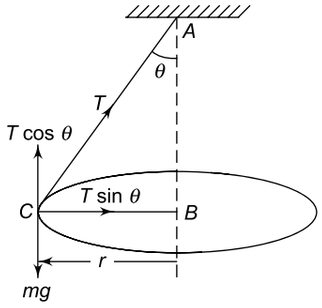


Fig. 4.31

Thus

$$T \cos \theta = mg \quad (1)$$

and $$T \sin \theta = \frac{mv^2}{r} = m\omega^2 r$$

But $r = L \sin \theta$. Therefore,

$$T \sin \theta = m\omega^2 L \sin \theta \text{ or } T = m\omega^2 L \quad (2)$$

Hence the correct choice is (c).

22. From (1), we have $\cos \theta = \frac{mg}{T}$ (3)

Using (2) in (3), we get

$\cos \theta = \frac{mg}{m\omega^2 L} = \frac{g}{\omega^2 L}$, which is choice (a).

23. Linear velocity is $v = \omega r = \omega L \sin \theta$, which is choice (b).

SECTION IV

Matching

1. For a body projected at time $t = 0$ horizontally with velocity u from a height h.

Column I	***Column II***
(a) Horizontal displacement	(p) $\sqrt{\frac{2h}{g}}$
(b) Vertical displacement	(q) $u\sqrt{\frac{2h}{g}}$
(c) Time taken to hit the ground	(r) is proportional to t
(d) Horizontal range	(d) is proportional to t^2

ANSWER

(a) → (r) (b) → (s)
(c) → (p) (d) → (q)

2. A body is projected from the ground with velocity u such that its range is maximum.

Column I	***Column II***
(a) Maximum height attained	(p) $\frac{\sqrt{2}u}{g}$
(b) Horizontal range	(q) $\frac{u}{g\sqrt{2}}$
(c) Time of flight	(r) $\frac{u^2}{g}$
(d) Time to reach maximum height	(s) $\frac{u^2}{4g}$

ANSWER

For maximum range $\theta = 45°$.

(a) $\rightarrow$ (s) (b) $\rightarrow$ (r)
(c) $\rightarrow$ (p) (d) $\rightarrow$ (q)

3. A body is projected with velocity u at angle $\theta = 30°$ with the horizontal.

Column I	***Column II***
(a) Velocity at maximum height	(p) $\frac{\sqrt{13}u}{4}$
(b) Velocity at half the maximum height	(q) $\frac{2\sqrt{7}}{3}u$
(c) Average velocity between the point of projection and highest point	(r) $\frac{\sqrt{3}u}{2}$
(d) Velocity at $t = \frac{2}{3}$ (time of flight)	(s) $\sqrt{\frac{7}{8}}\,u$

ANSWER

Answer with Explanation

Maximum height $h = \dfrac{u^2 \sin^2 30°}{2g} = \dfrac{u^2}{8g}$

(a) At maximum height, $v_x = u\cos\theta$ and $v_y = 0$. Therefore, $v = u\cos\theta = u\cos 30° = \dfrac{\sqrt{3}u}{2}$.

(b) For half the maximum height, $h' = \dfrac{h}{2}$.

$$v_y^2 = (u\sin\theta)^2 - 2gh' = u^2\sin^2\theta - gh$$

$$= u^2 \sin^2 30° - g \times \frac{u^2}{8g} = \frac{u^2}{4} - \frac{u^2}{8} = \frac{u^2}{8}$$

$$v_x = u\cos 30° = \frac{\sqrt{3}u}{2}$$

$\therefore$ $$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\frac{3u^2}{4} + \frac{u^2}{8}} = \sqrt{\frac{7}{8}}\,u$$

(c) Average velocity $= \dfrac{\text{net displcemrnt}}{\text{time taken}} = \dfrac{OA}{\frac{1}{2}t_f}$ (see Fig. 4.32)

$$OA = \sqrt{h^2 + \frac{R^2}{4}}$$

Now $$R = \frac{u^2}{g} \times \sin 60° = \frac{\sqrt{3}u^2}{2g}$$

$\therefore$ $$OA = \left(\frac{u^4}{64g^2} + \frac{3u^4}{16g^2}\right)^{1/2} = \frac{\sqrt{13}u^2}{8g}$$

Now $$\frac{1}{2}t_f = \frac{u\sin\theta}{g} = \frac{u}{2g}$$

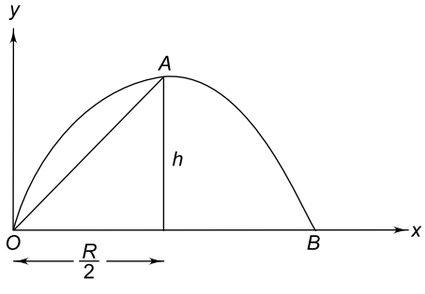


Fig. 4.32

$$\text{Average velocity} = \frac{OA}{\frac{1}{2}t_f} = \frac{\sqrt{13}u^2 \times 2g}{8g \times u} = \frac{\sqrt{13}u}{4}$$

(d) $$\text{At } t = \frac{2}{3}t_f,$$

$$v_y = u \sin\theta - g \times \frac{2}{3}t_f = u \sin 30^\circ - g \times \frac{2}{3} \times \frac{u}{g}$$

$$= \frac{u}{2} - \frac{2u}{3} = -\frac{u}{6}$$

$$v_x = u \cos\theta = u \cos 60^\circ = \frac{\sqrt{3}u}{2}$$

$\therefore$ $$v = \sqrt{v_x^2 + v_y^2} = \left(\frac{3u^2}{4} + \frac{u^2}{36}\right)^{1/2} = \frac{2\sqrt{7}}{3}u$$

Thus the answer is as follows:

(a) → (r) (b) → (s)
(c) → (p) (d) → (q)

4. Match objects in circular motion listed in column I with the sources that provide the necessary centripetal force listed in column II.

Column I	***Column II***
(a) A boy whirling a stone tied to a string in a circle	(p) Frictional force
(b) The moon revolving around the earth	(q) Muscular force
(c) The electron revolving around the proton in a hydrogen atom	(r) Gravitational force
(d) A car negotiating a curved road	(s) Electrostatic force

ANSWER

(a) → (q) (b) → (r)
(c) → (s) (d) → (p)

SECTION V

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each questions has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. Statement-1
A body is projected horizontally with a velocity u from the top of a building of height h. It hits the ground after a time $t = \sqrt{2h/g}$.
Statement-2
The vertical and horizontal motions can be treated independently.

2. **Statement-1**
A body is projected from the ground with kinetic energy K at an angle of 60° with the horizontal. If air resistance is neglected, its kinetic energy when it is at the highest point of its trajectory will be $K/4$.
Statement-2
At the highest point of the trajectory, the directions of the velocity and acceleration of the body are perpendular to each other.

3. **Statement-1**
One end of a string of length R is tied to stone of mass m and the other end to a small pivot on a frictionless vertical board. The stone is whirled in a vertical circle with the pivot as the centre. The minimum speed the stone must have, when it is at the topmost point on the circle, so that the string does not slack is $\sqrt{gR}$.
Statement-2
At the topmost point on the circle, the centripetal force is provided partly by tension in the string and partly by the weight of the stone.

4. **Statement-1**
The maximum range on an inclined plane when a body is projected upwards from the base of the plane is less than that when it is projected downwards from the top of the same plane with the same speed.
Statement-2
The maximum range along an inclined plane is independent of the angle of inclination of the plane.

5. **Statement-1**
In projectile motion, the velocity of the body at a point on it trajectory is equal to the slope at that point.
Statement-2
The velocity vector at a point is always along the tangent to the trajectory at that point.

6. **Statement-1**
In a uniform circular motion, the centripetal force is always perpendicular to the velocity vector.
Statement-2
Then the force does no work on the body and its kinetic energy remains constant.

7. **Statement-1**
In a non-uniform circular motion, the particle has two acceleration-one along the tangent to the circle and the other towards the centre of the circle.
Statement-2
In a non-uniform circular motion, the magnitude and the direction of the velocity vector both change with time.

8. **Statement-1**
In a non-uniform circular motion, the acceleration of the particle is equal to sum of the tangential acceleration and the centripetal acceleration.
Statement-2
The two accelerations are perpendicular to each other.

9. **Statement-1**
In a uniform circular motion, the kinetic energy of the body remains constant.
Statement-2
The momentum of the body does not change with time.

10. **Statement-1**
In a uniform circular motion, the acceleration is always directed towards the centre of the circle.
Statement-2
Otherwise the speed of the body moving along the circle will change with time.

SOLUTIONS

1. The correct choice is (a). The time taken by the body to hit the ground is the same as if it was dropped from that height and fell freely under gravity.
2. The correct choice is (b). If m is the mass of the body and u its velocity of projection, the initial kinetic energy is

$$K = \frac{1}{2}mu^2$$

At the highest point, the horizontal velocity is ($u \cos 60°$) and vertical velocity is zero. Hence the kinetic energy at the highest point is

$$K' = \frac{1}{2}m(u\cos 60°)^2 = \frac{1}{4} \times \frac{1}{2}mu^2 = \frac{K}{4}$$

At the highest point of the trajectory, the velocity of the body is horizontal (parallel to the ground) but its acceleration is g directed vertically downwards.

3. The correct choice is (a). When the stone is at the topmost point A on the circle, the centripetal force is provided by ($mg + T$) as shown in Fig. 4.33.

Thus $$\frac{mv^2}{R} = mg + T$$

When the stone is at A, the string will not slack if tension $T = 0$, which gives $\frac{mv^2}{R} = mg \Rightarrow v = \sqrt{Rg}$

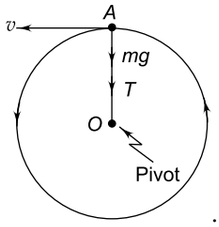


Fig. 4.33

4. The range along the inclined plane when a body is projected with velocity u at an angle θ with the horizontal is given by

$$R = \frac{2u^2 \sin(\theta - \alpha)\cos\theta}{g\cos^2\alpha}$$

$$= \frac{u^2}{g\cos^2\alpha}[\sin(2\theta - \alpha) - \sin\alpha]$$

where α is the angle of inclination of the plane. Range R will be maximum of $\sin(2\theta - \alpha) = 1$ or $2\theta - \alpha = 90°$ in which case

$$R_{max} = \frac{u^2}{g\cos^2\alpha}[1 - \sin\alpha] = \frac{u^2}{g(1+\sin\alpha)}$$

If the body is projected downwards from the top of the same inclined plane, the maximum range will be

$$R'_{max} = \frac{u^2}{g(1-\sin\alpha)}$$

Thus $R'_{max} > R_{max}$. Since the range R depends on angle α, Statement is false. Hence the correct choice is (c).

5. The correct choice is (d). At the highest point on the trajectory, the slope is zero but velocity is $u\cos\theta$.

6. The correct choice is (a).

7. The correct choice is (a).

8. The correct choice is (d). The acceleration of the particle is given by

$$a = \sqrt{a_c^2 + a_t^2}$$

where a_c = centripetal acceleration and a_t = tangential acceleration.

9. The correct choice is (c). The speed of the body remains constant but the momentum changes with time because the direction of the velocity vector changes with time.

10. The correct choices is (a). If the acceleration vector is directed towards the centre of the circle, it will have a component along the tangent, as a result the speed of the body will change and the motion no longer remains uniform.

SECTION VI

Integer Answer Type

1. A body falling freely from a given height H hits an inclined plane in its path at a height h. As a result of this impact, the direction of the velocity of the body becomes horizontal. For what value of H/h, will the body take the maximum time to reach the ground?

IIT, 1986

2. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis with a constant velocity $v_T = (\sqrt{3} - 1)$ ms^{-1} as shown Fig. 4.34. At a particular instant when OA makes an angle of 45° with the x-axis, a ball is thrown from origin O. Its velocity makes an angle of 60° with the x-axis and its velocity is such that it hits the trolley. Find the magnitude of the veolcity of the ball with respect to the horizontal surface.

IIT, 2002

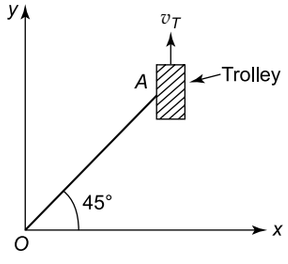


Fig. 4.34

3. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

IIT, 2011

SOLUTIONS

1. A body falling freely from point P at a height H hits the inclined plane at point Q at a height h. As a result, the velocity becomes horizontal and the body then follows a parabolic path and finally hits the ground at point R. The motion of the body from P to R via Q is shown in Fig. 4.35.

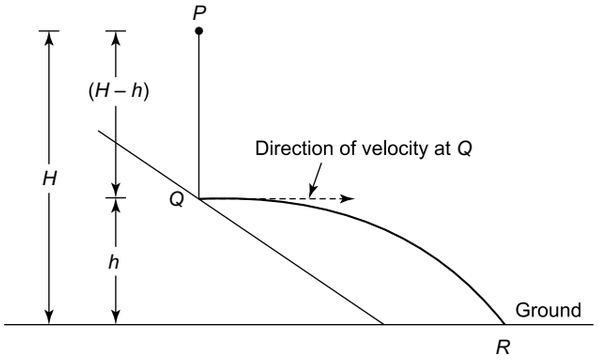


Fig. 4.35

If t' is the time taken by the body to fall from P to Q through a height $(H - h)$, then we have

$$(H - h) = \frac{1}{2}gt'^2 \text{ or } t' = \left(\frac{2(H-h)}{g}\right)^{1/2}$$

The time t'' taken by the body to fall from Q to R is given by ($\because$ the initial velocity at O is zero)

$$h = \frac{1}{2}gt''^2 \text{ or } t'' = \left(\frac{2h}{g}\right)^{1/2}$$

The total time taken by the body to reach the ground is

$$t = t' + t'' = \left(\frac{2(H-h)}{g}\right)^{1/2} + \left(\frac{2h}{g}\right)^{1/2} \quad (1)$$

Time t will be maximum if $\frac{dt}{dh} = 0$ and $\frac{d^2t}{dh^2}$ is negative. Therefore, differenting Eq. (1) w.r.t. h and setting $\frac{dt}{dh} = 0$, we have

$$\frac{dt}{dh} = 0 = \frac{d}{dh}\left[\left(\frac{2(H-h)}{g}\right)^{1/2} + \left(\frac{2h}{g}\right)^{1/2}\right]$$

$$= \left[\frac{2}{g}\left\{\frac{1}{2}(H-h)^{-1/2} \times (-1) + \frac{1}{2}h^{-1/2}\right\}\right]$$

$$= \frac{1}{g}\left[-\frac{1}{(H-h)^{1/2}} + \frac{1}{h^{1/2}}\right]$$

which gives $\frac{1}{(H-h)^{1/2}} = \frac{1}{h^{1/2}} \Rightarrow \frac{H}{h} = 2$

It is easy to check that when $\frac{H}{h} = 2$, $\frac{d^2t}{dh^2}$ is negative.

2. Refer to Fig. 4.36. It follows that $\vec{v}_B$ is the resultant of $\vec{v}$ and $\vec{v}_T$. In triangle OAB

$$\frac{v_B}{\sin 135°} = \frac{v_T}{\sin 15°}$$

$$\Rightarrow \quad v_B = \frac{v_T \sin(90° + 45°)}{\sin(60° - 45°)}$$

$$= \frac{(\sqrt{3}-1) \times \cos 45°}{\sin 60° \cos 45° - \cos 60° \sin 45°}$$

$$= \frac{(\sqrt{3}-1) \times 1/\sqrt{2}}{\sqrt{3}/2\sqrt{2} - 1/2\sqrt{2}} = 2 \text{ ms}^{-1}$$

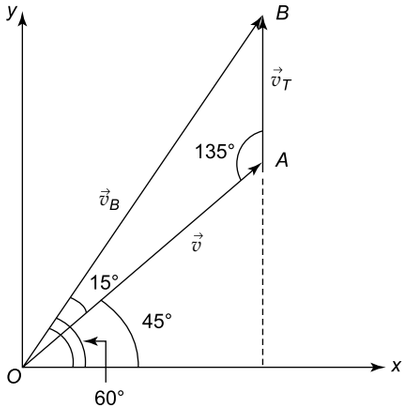


Fig. 4.36

3. $u = 10 \text{ ms}^{-1}$, $\theta = 60°$

Time of flight is

$$t = \frac{2u\sin\theta}{g} = \frac{2 \times 10 \times \sin 60°}{10} = \sqrt{3} \text{ s}$$

Let v be the velocity of the train. The horizontal velocity of ball at the instant it is thrown = $(v + u_x)$ = $(v + u\cos\theta)$. Therefore, the horizontal range of the ball with respect to the ground is

$$R = (v + u\cos\theta)t, \text{ where } t = \sqrt{3} \text{ s}$$

It is clear that

Distance travelled by ball in time $t + 1.15 = R$

i.e. $\quad vt + \frac{1}{2}at^2 + 1.15 = (v + u\cos\theta)t$

$$\Rightarrow \quad \frac{1}{2}at^2 + 1.15 = (u\cos\theta)t$$

$$\Rightarrow \quad \frac{1}{2}a \times (\sqrt{3})^2 + 1.15 = (10 \cos 60°) \times \sqrt{3}$$

$$\Rightarrow \quad a = 5 \text{ ms}^{-2}$$

5 Chapter Laws of Motion and Friction

REVIEW OF BASIC CONCEPTS

5.1 NEWTON'S FIRST LAW OF MOTION

Newton's first law of motion states that "*every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by an external unbalanced force*."

5.2 NEWTON'S SECOND LAW OF MOTION

Newton's second law of motion states that *the rate of change of linear momentum of a body is directly proportional to the applied force and the change takes place in the direction in which the force acts*.

Linear Momentum Newton defined linear momentum as the product of the mass and the velocity of a body.

$$\mathbf{p} = m\mathbf{v}$$

Differentiating this equation with respect to time, we get

$$\frac{d\mathbf{p}}{dt} = m\frac{d\mathbf{v}}{dt} \qquad (\because m \text{ is constant})$$

$$= m\mathbf{a}$$

where $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ is the acceleration produced.

NOTE

(1) Force = slope of momentum–time ($p - t$) graph.
(2) Change in momentum = area under the force–time ($F - t$) graph.

EXAMPLE 5.1

A constant force acts for 0.5s on a body of mass 1.5 kg initially at rest. When the force ceases to act, the body is found to travel a distance of 5.0 m in 2.0 s in the direction of the force. Find the magnitude of the force applied.

SOLUTION

According to Newton's first law, when the force ceases to act, the body will move with a uniform velocity given by

$$v = \frac{5}{2} = 2.5 \text{ ms}^{-1}$$

Using $u = 0$, $v = 2.5$ ms^{-2} and $t = 0.5$ s in $v = u + at$, we get $a = 5$ ms^{-2}. From Newton's second law,

Force $F = ma = 1.5 \times 5 = 7.5$ N.

EXAMPLE 5.2

Two forces each of magnitude 10 N act on a body of mass 5 kg at an angle of 120°. Find the magnitude of acceleration produced.

SOLUTION

$F_1 = F_2 = 10$ N and $\theta = 120°$. Resultant force is

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{10^2 + 10^2 + 2\times10\times10\times\left(-\frac{1}{2}\right)}$$

$$= 10 \text{ N}$$

$$\therefore \quad a = \frac{F}{m} = \frac{10}{5} = 2 \text{ ms}^{-2}$$

EXAMPLE 5.3

The velocity of a body of mass 2 kg changes from $\mathbf{v}_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})\text{ms}^{-1}$ to $\mathbf{v}_2 = (-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\text{ms}^{-1}$ in 3 s. Find (a) the magnitude of the change in momentum of the body and (b) the magnitude of the force applied.

SOLUTION

(a) Change in momentum = final momentum – initial momentum

or $\Delta\mathbf{p} = m\mathbf{v}_2 - m\mathbf{v}_1$

$= m[(-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})]$

$= m(-5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$\therefore\ |\Delta\mathbf{p}| = 2\sqrt{(-5)^2 + (-1)^2 + (4)^2}$

$= 2 \times \sqrt{42} = 12.96 \text{ kg ms}^{-1}$

(b) $|\mathbf{F}| = \dfrac{|\Delta\mathbf{p}|}{\Delta t} = \dfrac{12.96}{3} = 4.32 \simeq 4.3 \text{ N}$

EXAMPLE 5.4

A particle of mass 1 g is moving along the positive x-axis under the influence of a force.

$$F = -\frac{k}{x^2}$$

where $k = 10^{-3}$ Nm^2. When the particle is at $x = 1.0$ m, its velocity $v = 0$. Find (a) the magnitude of its velocity when, it reaches $x = 0.5$ m and (b) its position when its speed is 1 ms^{-1}.

SOLUTION

$$F = ma = m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt} = mv\frac{dv}{dx}$$

Given, $F = -\dfrac{k}{x^2}$. Therefore

$$-\frac{k}{x^2} = mv\frac{dv}{dx} \Rightarrow v\,dv = -\frac{k}{mx^2}dx$$

Integrating, we have

$$\int v\,dv = -\frac{k}{m}\int x^{-2}\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{k}{mx} + c \qquad \text{(i)}$$

where c is the constant of integration. Given $v = 0$ when $x = 1.0$ m. Using this in eq. (i), we get $c = -\dfrac{k}{m}$. Equation (i) becomes

$$\frac{v^2}{2} = \frac{k}{mx} - \frac{k}{m}$$

$$\Rightarrow \quad v = \left[\frac{2k}{m}\left(\frac{1}{x} - 1\right)\right]^{1/2}$$

$$v = \left[\frac{2\times10^{-3}}{10^{-3}}\left(\frac{1}{x} - 1\right)\right]^{1/2}$$

[$\because\ k = 10^{-3}$ Nm^2 and $m = 10^{-3}$ kg]

$$\Rightarrow \quad v = \left[2\left(\frac{1}{x} - 1\right)\right]^{1/2}$$

(a) when $x = 0.5$ m, $v = \left[2\left(\dfrac{1}{0.5} - 1\right)\right]^{1/2}$

$= \sqrt{2}$ ms^{-1}

(b) when $v = 1$ ms^{-1}, $1 = \left[2\left(\dfrac{1}{x} - 1\right)\right]^{1/2} \Rightarrow x = 0.67$ m

5.3 NEWTON'S THIRD LAW OF MOTION

Newton's third law of motion states that *whenever one body exerts a force on a second body, the second body exerts an equal and opposite force on the first, or, to every action there is an equal and opposite reaction*. The action and reaction forces act on different bodies.

5.4 LAW OF CONSERVATION OF LINEAR MOMENTUM

The *law of conservation of linear momentum* may be stated as '*when no net external force acts on a system consisting of several particles, the total linear momentum of the system is conserved, the total linear momentum being the vector sum of the linear momentum of each particle in the system*'.

Recoil of a Gun

The gun and the bullet constitute a two-body system. Before the gun is fired, both the gun and the bullet are at rest. Therefore, the total momentum of the gun-bullet system is zero. After the gun is fired, the bullet moves forward and the gun recoils backwards. Let m_b and m_g be the masses of the bullet and the gun. If $\mathbf{v}_b$ and $\mathbf{v}_g$ are their respective velocities after firing, the total momentum of the gun-bullet system after firing is ($m_b\,\mathbf{v}_b + m_g\,\mathbf{v}_g$). From the law of conservation of momentum, the total momentum after and before the gun is fired must be the same, i.e.

$$m_b\,\mathbf{v}_b + m_g\,\mathbf{v}_g = 0$$

or $$\mathbf{v}_g = -\frac{m_b\,\mathbf{v}_b}{m_g}$$

The negative sign indicates that the gun recoils in a direction opposite to that of the bullet. In terms of magnitudes, we have

$$v_g = \frac{m_b v_b}{m_g}$$

5.5 IMPULSE

Consider a collision between two bodies A and B moving in the same straight line. Let Δt be the duration of the collision, i.e. the time for which the bodies were in contact during which time the transfer of momentum took place. We assume that the bodies continue moving in the same straight line after the collision with velocities different from their initial velocities.

Impulse of a force is the product of the average force and the time for which the force acts and it is equal to the change in momentum of the body during that time. Impulse is a vector and is measured in kg m s^{-1} or N s.

$$\mathbf{I} = \mathbf{F}_{av}\,\Delta t = \Delta \mathbf{p}$$

EXAMPLE 5.5

A ball of mass m is moving with a velocity $\mathbf{v}$ towards a rigid vertical wall. After striking the wall, the ball deflects through an angle θ without change in its speed. Obtain the expression for the impulse imparted to the ball.

SOLUTION

Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the initial and final velocities of the ball [Fig. 5.1(a)]

Impulse = change in momentum

$$= m\mathbf{v}_2 - m\mathbf{v}_1 = m(\mathbf{v}_2 - \mathbf{v}_1)$$

$$= m[\mathbf{v}_2 + (-\mathbf{v}_1)]$$

or Impulse $= m\Delta\mathbf{v}$

where $\Delta\mathbf{v} = \mathbf{v}_2 + (-\mathbf{v}_1)$ is the resultant of $\mathbf{v}_2$ and $-\mathbf{v}_1$ [Fig. 5.1(b)].

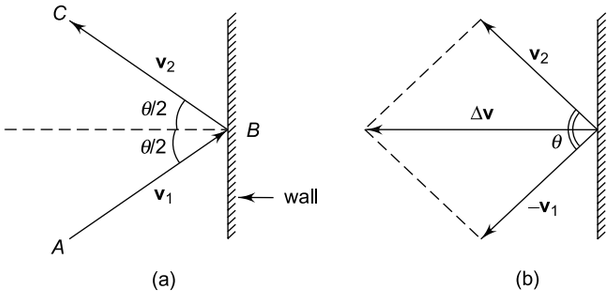


Fig. 5.1

Magnitude of $\Delta\mathbf{v}$ is [$\because$ magnitude of $\mathbf{v}_1$ = magnitude of $\mathbf{v}_2 = v$]

$$\Delta v = \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos\theta}$$

$$= \sqrt{v^2 + v^2 + 2v^2\cos\theta}$$

$$= \sqrt{2v^2(1+\cos\theta)} = 2v\cos\left(\frac{\theta}{2}\right)$$

The direction of impulse is perpendicular to the wall and away from it.

5.6 CONTACT FORCES

(1) Normal Reaction

The force exerted by one body when placed on the surface of another body is known as contact force. If the two surfaces in contact are perfectly smooth (i.e., frictionless), then the contact force acts only perpendicular (normal) to their surface of contact and is known as *normal reaction* (R).

If a block of mass m is placed on a horizontal frictionless surface [Fig. 5.2 (a)], the normal reaction $R = mg$. If the block is placed on an inclined plane of inclination α [Fig. 5.2 (b)], the normal reaction $R = mg\cos\alpha$

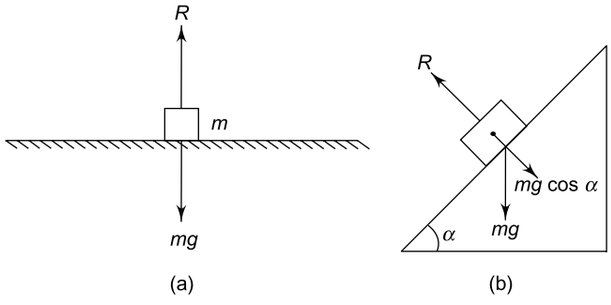


Fig. 5.2

If there is friction between the surfaces of contact, then the component of the contact force perpendicular to their surface gives the normal reaction and the other component which acts along the tangent to the surface of contact gives the force of friction. The normal reaction, tension and friction are examples of contact force.

(2) Tension

The force in a string is called tension (T). If the string is massless, the tension has the same magnitude at all points of the string. Tension in the string always acts away from the body to which it is attached. If the string passes over a frictionless pulley and its ends are attached to two bodies, the tension in the entire string has the same magnitude and its direction is towards its point of contact with the pulley.

5.7 FRICTION

Friction is the force which comes into play when one body slides or rolls over the surface of another body and

acts in a direction tangential to the surfaces in contact and opposite to the direction of motion of the body.

The maximum (or limiting) force of friction when a body just begins to slide over the surface of another body is called the *limiting friction*. The force of friction just before one body begins to slide over another is called the *limiting static friction* (f_s). The coefficient of limiting static friction (μ_s) is defined as

$$\mu_s = \frac{f_s}{R}$$

where R is the normal reaction, i.e. the normal force pressing the two surfaces together.

The force necessary to maintain a body in uniform motion over the surface of another body, after motion has started, is called the *kinetic* or *sliding friction* (f_k). The *coefficient of kinetic friction* (μ_k) is defined as

$$\mu_k = \frac{f_k}{R}$$

Note that μ_k is always less than μ_s.

Angle of Friction Angle of friction is the angle between the resultant of the force of limiting friction (f) and the normal reaction (R). In Fig. 5.3, θ is the angle of friction, which is given by

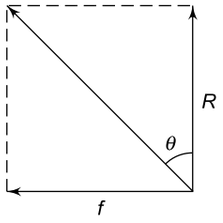


Fig. 5.3

$$\tan\theta = \frac{f}{R} = \mu$$

$$\theta = \tan^{-1}(\mu)$$

Angle of Repose Suppose a body is placed on an inclined plane. The angle of inclination is gradually increased until the body just begins to slide along the plane. When this happens the angle of inclination α of the inclined surface with the horizontal is called the angle of repose (See Fig. 5.4). It follows from the figure that

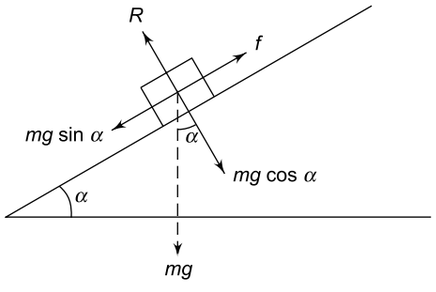


Fig. 5.4

Force of limiting friction (f) = $mg \sin\alpha$

Force of normal reaction (R) = $mg \cos\alpha$

Therefore, $\tan\alpha = \frac{f}{R} = \mu = \tan\theta$

or $\alpha = \theta = \tan^{-1}(\mu)$

5.8 BANKING OF ROUND TRACKS

When a car (or some other vehicle) negotiates a curved level road, the centripetal force required to keep the car in motion around the curve is provided by the friction between the road and the types. The weight of the car is supported by the normal reaction due to the earth. If the surface of the road is very rough, it provides a large amount of friction and hence the car can successfully negotiate the bend with a fairly high speed. If F is the total frictional force between the tyres and the road, then

$$F = \frac{mv^2}{R}$$

when m is the mass of the car, v its speed around the curve and R is the radius of the curved track. The higher the value of F, the faster is the speed at which the bend can be negotiated. If μ is the coefficient of friction between the tyres and the road, then $F \le \mu N$; N = normal reaction = mg. The maximum speed which friction can sustain is $v \le v_{max} = (\mu Rg)^{1/2}$

BANKING OF CURVES

The large amount of friction between the tyres and the road would damage the tyres. To minimize the wearing out of tyres of road bed is banked, i.e. the outer part of the road is raised a little so that that road slopes towards the centre of the curved track. Suppose a car of mass m is moving around a banked track in a circular path of radius R as shown in Fig. 5.5. Let N_1 and N_2 be the reaction at each tyre due to the road. Then the total reaction is $N = N_1 + N_2$ acting in the middle of the car. If θ is the angle of the banking, the vertical component $N \cos\theta$ supports the weight mg of the car while the horizontal component $N \sin\theta$ provides the necessary centripetal force.

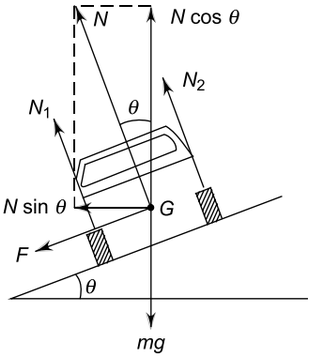


Fig. 5.5 Car on a banked curved road

Thus

$$N \sin\theta = \frac{mv^2}{R} - F\cos\theta$$

and $N \cos\theta = mg + F \sin\theta$

Also $F = \mu N$

where F is the force of friction acting radially inwards on the car. These equations give

$$\tan\theta = \frac{v^2 - \mu Rg}{Rg + \mu v^2} \text{ and } v^2 = Rg\,\frac{\mu + \tan\theta}{1 - \mu\tan\theta}$$

The first equation determines the proper banking angle for given v, R and μ, and the second equation the maximum speed at which the car can successfully negotiate the curve for given R, μ and θ.

For given θ and R, there is an optimum (best) speed for negotiating a banked curve at which there will be the least wear and tear, i.e. when friction is not needed at all ($\mu = 0$). If $\mu = 0$, this speed is $v = (Rg\tan\theta)^{1/2}$

The car will not skid if the angle of banking of the track satisfies the relation

$$\tan\theta = \frac{v^2}{Rg}$$

5.9 A CYCLIST NEGOTIATING A CURVED LEVEL ROAD

While negotiating a curved level (unbanked) road, a cyclist has to lean inwards which provides the necessary centripetal force which prevents him from falling down. Figure 5.6 shows a cyclist leaning at an angle θ with the vertical. N is the normal reaction which is given by

$$N = mg$$

where m is the mass of the cyclist plus the bicycle. The force of friction between the road and the tyres is

$$F = \mu N = \mu mg$$

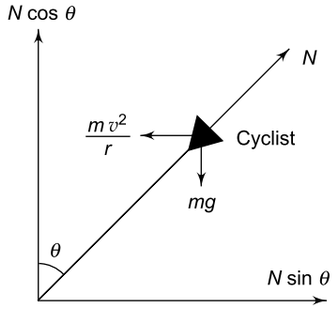


Fig. 5.6

The cyclist will skid if the centripetal force mv^2/R exceeds the frictional force F, i.e. if

$$\frac{mv^2}{R} > \mu mg$$

where R is the radius of the curved road. Thus skidding occurs if

$$v = \sqrt{\mu gR}$$

5.10 MOTION IN A VERTICAL CIRCLE

Figure 5.7 shows an object of mass m whirled with a constant speed v in a vertical circle of centre O with a string of length R. When the object is at top A of the circle, let us say that the tension (force) in the string is T_1. Since the weight mg acts vertically downwards towards the centre O, we have,

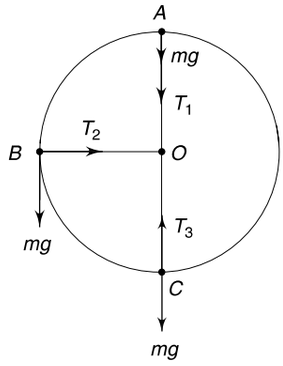


Fig. 5.7

Force towards centre, $F = T_1 + mg = \dfrac{mv^2}{R}$

or $$T_1 = \frac{mv^2}{R} - mg \qquad \text{(i)}$$

At the point B, where OB is horizontal, the weight mg has no component along OB. Thus, if the tension in the string is T_2 at B, we have

Force towards centre,

$$F = T_2 = \frac{mv^2}{R} \qquad \text{(ii)}$$

At C, the lowest point of motion, the weight mg acts in the opposite direction to the tension T_3 in the string. Thus at C we have,

Force towards centre, $F = T_3 - mg = \dfrac{mv^2}{R}$

or $$T_3 = \frac{mv^2}{R} + mg$$

From (i), (ii) and (iii), we see that the maximum tension occurs at lowest point C of the motion. Here the tension T_3 must be greater than mg by $\dfrac{mv^2}{R}$ to keep the object in a circular path. The minimum tension is given by (i) when the object is at the highest point A of the motion. Here part of the required centripetal force is provided by the weight and the rest by T_1.

In order to keep a body of mass m in a circular path, the centripetal force, at the highest point A, must at least be equal to the weight of the body. Thus

$$\frac{mv_A^2}{R} = mg \quad \text{or} \quad v_A^2 = Rg$$

gives the minimum speed the body must have at the highest point so that it can complete the circle. Then the minimum speed v_C the body must have at the lowest point C is given by

$$v_C^2 = v_A^2 + 2 \times 2\ Rg$$

where we have used $v^2 = u^2 + 2gh$, with $h = 2R$. Thus

$$\therefore \qquad v_C^2 = Rg + 4\ Rg = 5\ Rg$$

or $$v_C = \sqrt{5Rg}$$

The tension at this point is given by

$$T_1 = m\left(\frac{v_C^2}{R} + g\right) = m\ (5g + g) = 6\ mg$$

EXAMPLE 5.6

A body of mass m = 20 g is attached to an elastic spring of length L = 50 cm and spring constant k = 2 Nm^{-1}. The system is revolved in a horizontal plane with a frequency ν = 30 rev/min. Find the radius of the circular motion and the tension in the spring.

SOLUTION

Angular velocity $\omega = 2\pi\nu = 2\pi \times \frac{30}{60} = \pi$ rad s^{-1}.

For an elastic spring force $F = kx$ where x is the extension.

Radius of circular motion $r = L + x$.

Centripetal force = $mr\omega^2 = F$

Or $\quad m(L + x)\omega^2 = kx$

$$\Rightarrow \qquad x = \frac{mL\omega^2}{k - m\omega^2}$$

$$= \frac{0.02 \times 0.5 \times (3.14)^2}{2 - 0.02 \times (3.14)^2}$$

$$\simeq 0.05\text{m}$$

$$\therefore \qquad r = L + x = 0.5 + 0.05 = 0.55 \text{ m}$$

Tension $\qquad F = kx = 2 \times 0.05 \simeq 0.1$ N

EXAMPLE 5.7

A liquid of mass M and density ρ is filled in a tube AB of length L. The tube is rotated about end A with angular velocity ω. Obtain the expression for the force exerted at the other end B.

SOLUTION

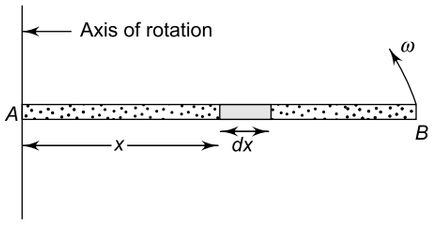


Fig. 5.8

Mass of element of length dx is $m = \frac{M}{L}dx$

Centripetal force on element = $\int_o^L m\omega^2 x$

$$= \frac{M\omega^2}{L}\int_o^L x dx$$

$$= \frac{1}{2}ML\omega^2$$

EXAMPLE 5.8

A conical pendulum has a string of length l = 50 cm and bob of mass m = 200 g. The bob is revolved in a horizontal circle of radius r = 20 cm. If the string makes an angle θ = 60° with the vertical, find (a) the tension in the string and (b) the speed of the bob around the circle. Take g = 10 ms^{-2}.

SOLUTION

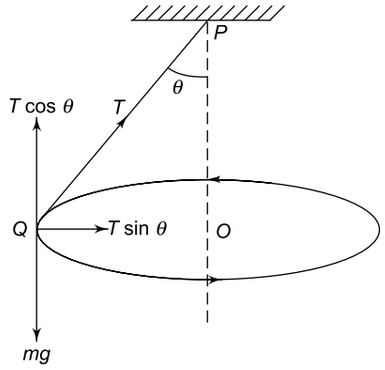


Fig. 5.9

Refer to Fig. 5.9,

$$OQ = r \text{ and } PQ = l$$

Force towards centre $O = T \sin \theta$

i.e. $$\frac{mv^2}{r} = T \sin \theta \qquad \text{(i)}$$

Also $mg = T\cos\theta$ (ii)

(a) From (ii) $T = \dfrac{mg}{\cos\theta} = \dfrac{0.2\times10}{\cos60°} = 4$ N

(b) Dividing (i) by (ii) we have

$$\frac{v^2}{rg} = \tan\theta$$

$$\Rightarrow \quad v = \sqrt{rg\tan\theta} = \sqrt{0.2\times10\times\tan60°}$$

$$= 1.86 \text{ ms}^{-1}$$

EXAMPLE 5.9

A coin of mass m = 10 g is placed at a distance of 30 cm from the centre of a disc. The disc is rotated at 30 rev/min about a vertical axis passing through its centre. What should be the minimum value of the coefficient of friction between the coin and the disc so that the coin does not skid off the disc?

SOLUTION

For no slipping, frictional force > centripetal force, i.e.

$$\mu mg > m\omega^2 r$$

$$\Rightarrow \quad \mu > \frac{\omega^2 r}{g}$$

$$\Rightarrow \quad \mu > \frac{(2\pi\nu)^2 r}{g}$$

$\nu = 30$ rev/min $= \dfrac{30}{60} = 0.5$ Hz,

$r = 0.3$ m and $g = 9.8$ ms^{-2}.

$$\therefore \quad \mu_{\min} = \frac{4\pi^2\nu^2 r}{g} = \frac{4\times(3.14)^2\times(0.5)^2\times0.3}{9.8} = 0.3$$

EXAMPLE 5.10

A small sphere of mass m = 500 g moving on the inner surface of a large hemispherical bowl of radius R = 5m describes a horizontal circle at a distance OC = 2.5 m below the centre O of the bowl as shown in Fig. 5.10. Find the force exerted by the sphere on the bowl and the time period of revolution of the sphree around the circle. Take g = 10 ms^{-2}.

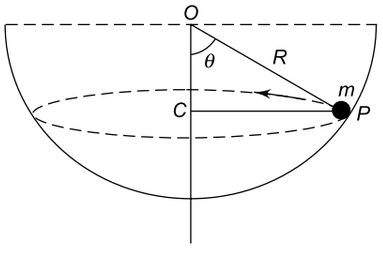


Fig. 5.10

SOLUTION

Given OP = 5 m and OC = 2.5 m. Therefore $\cos\theta = \dfrac{OC}{OP} = \dfrac{1}{2} \Rightarrow \theta = 60°$.

Radius of circle is $r = CP = OP\sin\theta = 5\sin60°$

$$= \frac{5\times\sqrt{3}}{2}\text{m}$$

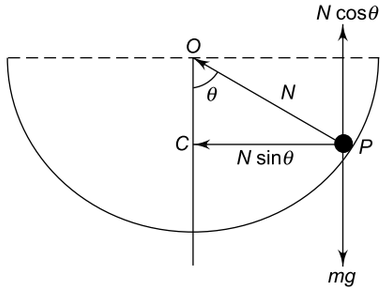


Fig. 5.11

Figure 5.11 shows the forces acting on the sphere, N is the normal reaction.

Net force towards centre C of the circle = $N\sin\theta$

$$\Rightarrow \quad mr\omega^2 = N\sin\theta \quad \text{(i)}$$

Also $mg = N\cos\theta$ (ii)

From (ii), $N = \dfrac{mg}{\cos\theta} = \dfrac{0.5\times10}{\cos60°} = 10$ N

From (i), $m\omega^2 R\sin\theta = N\sin\theta$

$$\Rightarrow \quad \omega = \sqrt{\frac{N}{mR}} = \sqrt{\frac{10}{0.5\times5}} = 2 \text{ rad s}^{-1}$$

$\therefore$ Time period $T = \dfrac{2\pi}{\omega} = \pi$ second = 3.14 s

5.11 SOLVING PROBLEMS IN MECHANICS BY FREE BODY DIAGRAM METHOD

In mechanics, we often have to handle problems which involve a group of bodies connected to one another by strings, pulleys, springs, etc. They exert forces on one another. Furthermore, there are frictional forces and the force of gravity acting on each body in the group. To solve such complicated problems, it is always convenient to choose one body in the group, find the magnitude and the direction of the forces acting on this body by all the remaining bodies in the group. Then we find the resultant of all the forces acting on the body to obtain the net force exerted on it. We then use the laws of motion to determine

the dynamics of the body. We apply this procedure to all other bodies in the group one by one. It is useful to draw a separate diagram for each body, showing the directions of the different forces acting on it. Such a diagram is called the *free body diagram* (F.B.D.) of the body.

(1) Two masses tied to a string going over a frictionless pulley Consider two bodies of masses m_1 and m_2 ($m_1 > m_2$) connected by a string which passes over a pulley, as shown in Fig. 5.12(a). When the bodies are released, the heavier one moves downwards and the lighter one moves up.

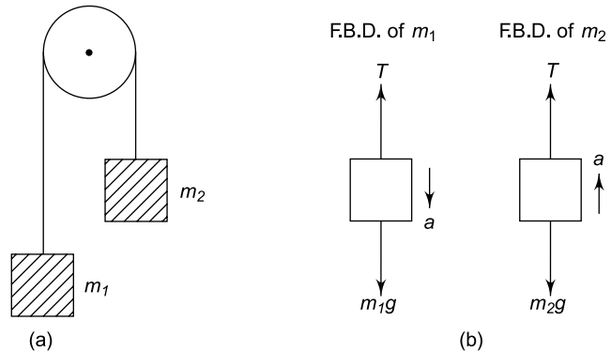


Fig. 5.12

Net force in the direction of motion is m_1 is $m_1g - T$. Therefore, the equation of motion of m_1 is [Fig. 5.12(b)]

$$m_1g - T = m_1a \qquad \text{(i)}$$

Net force in the direction of motion of m_2 is $(T - m_2g)$. Therefore, the equation of motion of m_2 is

$$T - m_2g = m_2a \qquad \text{(ii)}$$

From (i) and (ii) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

and

$$T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

(2) Two masses in contact Figure 5.13(a) shows two blocks of masses m_1 and m_2 placed in contact on a horizontal frictionless surface. A force F is applied to mass m_1. As a result, the masses move with a common acceleration a. To find a and the contact force on m_2, we draw the free body diagrams as shown in Figs. 5.13(b) and (c).

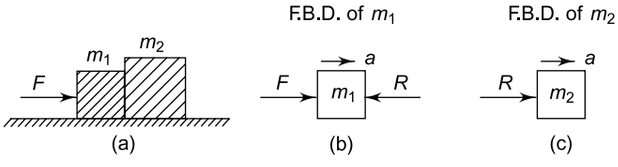


Fig. 5.13

R = normal reaction force between the blocks. From Figs. 5.13(b)and (c), we get

$$F - R = m_1a \qquad \text{(i)}$$

and

$$R = m_2a \qquad \text{(ii)}$$

Adding (i) and (ii) we get

$$F = (m_1 + m_2)a$$

$$\Rightarrow \quad a = \frac{F}{(m_1 + m_2)}$$

Contact force on m_2 is

$$F_2 = m_2\ a = \frac{m_2F}{(m_1 + m_2)}$$

(3) Three masses in contact Figure 5.14(a) shows three blocks of masses m_1, m_2, and m_3 placed in contact on a horizontal frictionless surface. A force F is applied to m_1. As a result, the three masses move with a common acceleration a. To find a and the contact forces on m_2 and m_3, we draw the free body diagrams as shown in Figs. 5.14(b) (c) and (d).

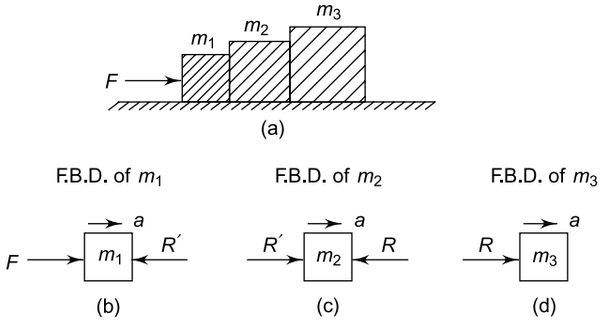


Fig. 5.14

R' = contact force on m_2 = reaction force between m_1 and m_2

R = contact force on m_3 = reaction force between m_2 and m_3

It follows from Figs. 5.14(b), (c) and (d) that

$$F - R' = m_1a \qquad \text{(i)}$$

$$R' - R = m_2a \qquad \text{(ii)}$$

and

$$R = m_3a \qquad \text{(iii)}$$

Adding (i), (ii) and (iii) we get

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

Contact force on m_2 is $F_2 = R'$ which from (ii) is given by

$$F_2 = R' = R + m_2a$$

Using (iii) we have

$$F_2 = m_3a + m_2a$$

$$= (m_2 + m_3)a$$

$$\Rightarrow \quad F_2 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

Contact force on m_3 is

$$F_3 = R = m_3 a = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

(4) Two masses connected with a string Figure 5.15(a) shows two blocks of masses m_1 and m_2 connected with a string and lying on a horizontal frictionless surface. A force F is applied to m_2. As a result, the masses move with a common acceleration a. To find a and force exerted on m_1, we draw the free body diagrams as shown in Figs. 5.15(b) and (c). T is the tension in the string.

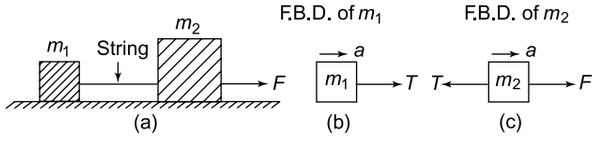


Fig. 5.15

It follows from Figs. 5.15(b) and (c) that

$$T = m_1 a \qquad \text{(i)}$$

and $$F - T = m_2 a \qquad \text{(ii)}$$

Adding (i) and (ii) we get

$$a = \frac{F}{m_1 + m_2}$$

Tension in the string is

$$T = m_1 a = \frac{m_1 F}{m_1 + m_2}$$

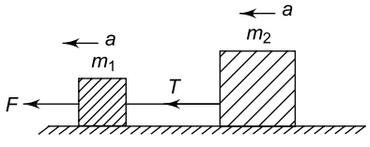


Fig. 5.16

If force F is applied on mass m_1 as shown in Fig. 5.16, then

$$a = \frac{F}{m_1 + m_2}$$

Tension in the string is

$$T = \frac{m_2 F}{m_1 + m_2}$$

(5) Three masses connected by strings Figure 5.17 (a) shows three blocks of masses m_1, m_2 and m_3 connected by two strings and placed on a horizontal frictionless surface. A force F is applied to m_1. As a result, the blocks move with a common acceleration a. To find a and the forces acting on m_2 and m_3, we draw free body diagrams as shown in Fig. 5.17(b) and (c) and (d). T is the tension in the string between m_1 and m_2 and T' is the tension in the string between m_2 and m_3.

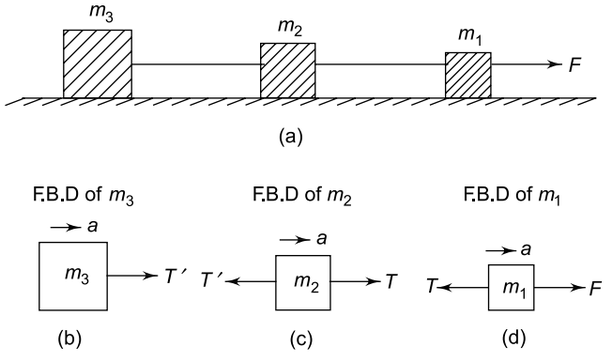


Fig. 5.17

It follows from Figs. 5.17(b), (c) and (d) that

$$T' = m_3 a \qquad \text{(i)}$$

$$T - T' = m_2 a \qquad \text{(ii)}$$

and $$F - T = m_1 a \qquad \text{(iii)}$$

Adding (i), (ii) and (iii), we get

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

The tension in the string between m_1 and m_2 is T, which is obtained by adding (i) and (ii).

$$T = (m_2 + m_3)a = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

The tension in the string between m_2 and m_3 is T', which from (i) is given by

$$T' = m_3 a = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

(6) Two masses connected by a string and suspended from a support Two blocks of masses m_1 and m_2 are connected by two strings and suspended from a support as shown in Fig. 5.18(a). Mass m_2 is pulled down by a force F. The tension T is the string between m_1 m_2 and tension T' in the string between m_1 and the support can be found from the free body diagrams as shown in Fig. 5.18(b) and (c).

$$T' = T + m_1 g \qquad \text{(i)}$$

$$T = F + m_2 g \qquad \text{(ii)}$$

Using (ii) and (i), we get

$$T' = F + (m_1 + m_2)g$$

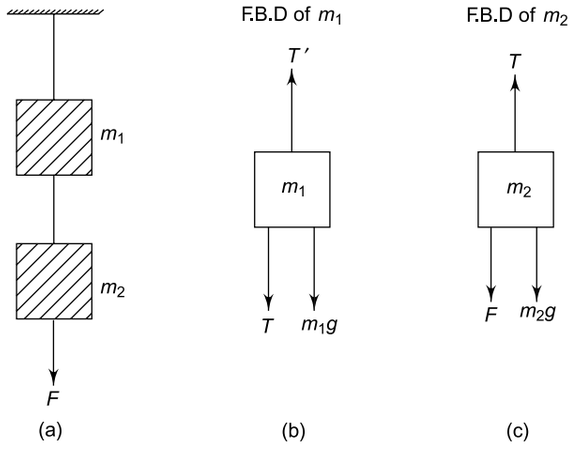


Fig. 5.18

(7) Two blocks connected by a string passing over a frictionless pulley fixed at the edge of a horizontal table Consider a block of mass m_1 lying on a frictionless table connected through a pulley to another block of mass m_2 hanging vertically (Fig. 5.19). When the system is released, let acceleration of the blocks be a.

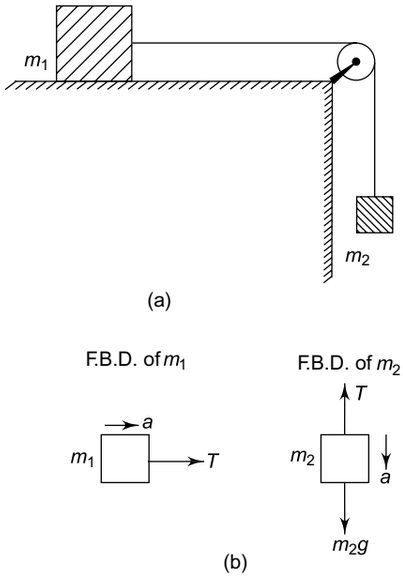


Fig. 5.19

From free body diagrams, the equations of motion of m_1 and m_2 are

$$T = m_1a \quad \text{(i)}$$

and $$m_2g - T = m_2a \quad \text{(ii)}$$

Adding (i) and (ii), we get

$$a = \frac{m_2g}{(m_1 + m_2)}$$

Also $$T = m_1a = \frac{m_1m_2g}{(m_1 + m_2)}$$

If the table top is frictionless, the blocks will move even if $m_2 < m_1$.

If μ is the coefficient of friction between block m_1 and the table, the force of friction is

$$f = \mu R = \mu m_1g$$

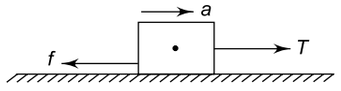


Fig. 5.20

From F.B.D. of m_1 (Fig. 5.20), Eq. (i) becomes

$$T - f = m_1a \quad \Rightarrow \quad T - \mu m_1g = m_1a$$

(8) Two blocks connected by a string passing over a frictionless pulley fixed at the top of an inclined plane Let T be the tension in the string. Since the pulley is frictionless, the tension is the same throughout the string (Fig. 5.21). There are the following two cases:

(a) Mass m_1 moving up along the incline with acceleration a [Fig. 5.21]

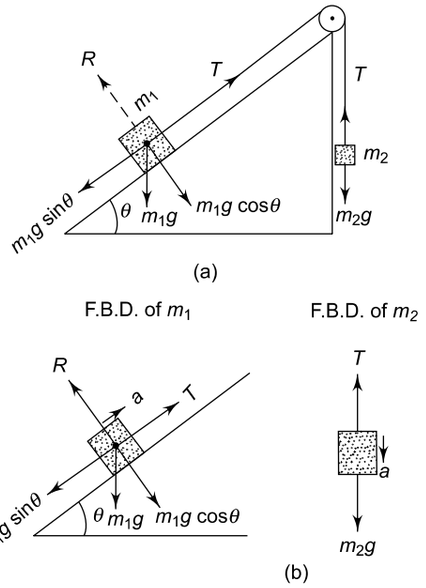


Fig. 5.21

The equations of motion of m_1 and m_2 [see Fig. 5.21(b)]

$$T - mg \sin\theta = m_1a \quad \text{(i)}$$

$$m_2g - T = m_2a \quad \text{(ii)}$$

which give $$a = \frac{(m_2 - m_1 \sin\theta)g}{(m_1 + m_2)}$$

and $T = m_2\ (g - a)$

If μ is the coefficient of friction between m_1 and the inclined plane, the frictional force $f = \mu R = \mu m_1 g \cos\theta$ will act down the plane because the block m_1 is moving up the plane. In this case, Eq. (i) is replaced by

$$T - m_1 g \sin\theta - f = m_1 a$$

$$\Rightarrow \quad T - m_1 g \sin\theta - \mu m_1 g \cos\theta = m_1 a$$

(*b*) *Mass m_1 moves down the incline with acceleration a*
In this case, we get $m_1 g \sin\theta - T = m_1 a$ and $T - m_2 g = m_2 a$ which give

$$a = \frac{(m_1 \sin\theta - m_2)\, g}{(m_1 + m_2)}$$

and $T = m_2\ (g + a)$

(9) Two blocks connected by a string passing over a frictionless pulley fixed at the top of a double inclined plane Let the block of mass m_1 move up along the inclined plane of angle of inclination θ_1, and the block of mass m_2 move down the inclined plane of angle of inclination θ_2 (Fig. 5.22). Let T be the tension in the string. Then, for m_1 and m_2, we have

$$T - m_1 g \sin\theta_1 = m_1 a$$

and $$m_2 g \sin\theta_2 - T = m_2 a$$

Eliminating T, we get

$$a = \frac{(m_2 \sin\theta_2 - m_1 \sin\theta_1)\, g}{(m_1 + m_2)}$$

Also $$T = m_1(a + g \sin\theta_1) = m_2\ (g \sin\theta_2 - a)$$

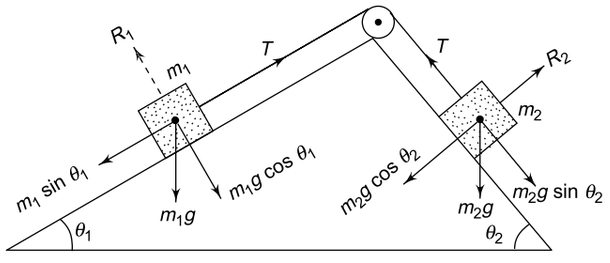


Fig. 5.22

(10) One block placed on top of the another A block of mass m_1 is placed on another block of mass m_2, which is lying on a horizontal frictionless surface. The coefficient of friction between the blocks is μ.

Case 1: The maximum force that can be applied on the lower block so that the upper block does not slip [Fig. 5.23(a)]
F_{max} = maximum value of force F so that block m_1 does not slip of block m_2

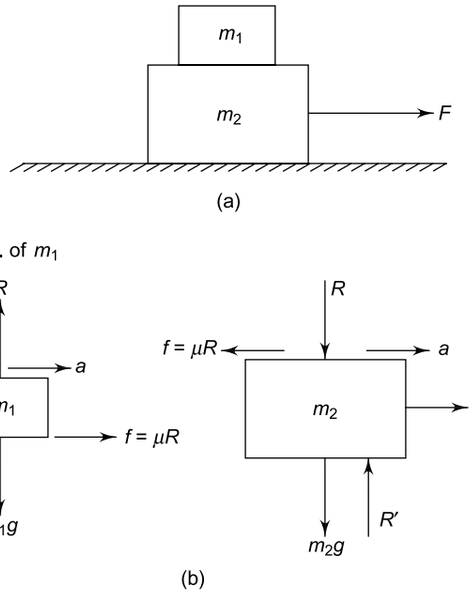


Fig. 5.23

f = frictional force on m_1 due to m_2

$$= \mu R = \mu m_1 g$$

Due to friction, m_2 will try to drag m_1 to the right. Hence frictional force f acts towards left. From F.B.D. of m_1,

$$f = m_1 a \Rightarrow \mu m_1 g \quad \Rightarrow \mu g = a \qquad \text{(i)}$$

Here, a is the acceleration of each block.
R' = normal reaction on m_2 by the horizontal surface.
From F.B.D. of m_2, we have

$$R + m_2 g = R' \qquad \text{(ii)}$$

and $$F_{max} - f = m_2 a$$

$$\Rightarrow \quad F_{max} - \mu R = m_2 a \qquad \text{(iii)}$$

$$\Rightarrow \quad F_{max} - \mu m_1 g = m_2 a$$

Using (i) in (iii), we get

$$F_{max} = (m_1 + m_2)\ \mu g$$

(a) If $F > F_{max}$, m_1 will begin to slide on m_2 and then their accelerations will be different.
(b) If $F < F_{max}$, m_1 and m_2 move together without any relative motion between them.

Case 2: The maximum force that can be applied on the upper block so that it does not slip on the lower block. [Fig. 5.24(a)]

F_{max} = maximum value of force F so that block m_1 just begins to slide on block m_2

Block m_1 tries to drag block m_2 toward right due to frictional force $f = \mu R = \mu m_1 g$. The frictional force exerted by m_1 on m_2 will be towards right.

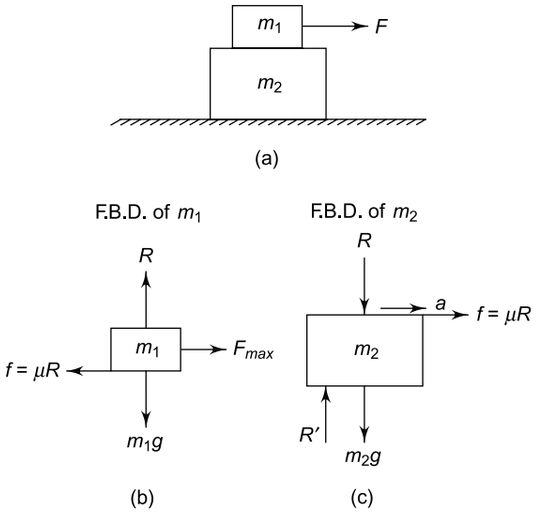


Fig. 5.24

R' = normal reaction on m_2 by the horizontal surface. If a is the acceleration of blocks towards right, from F.B.D. of m_1 we have

$$F_{max} = \mu R = m_1 a \text{ and } R = m_1 g$$

or $$F_{max} - \mu m_1 g = m_1 a \quad \text{(i)}$$

From F.B.D. of m_2,

$$f = m_2 a \Rightarrow \mu_1 m_1 g = m_2 a$$

$$\Rightarrow \quad a = \frac{\mu m_1 g}{m_2} \quad \text{(ii)}$$

Using (ii) in (i), we get

$$F_{max} = \frac{\mu(m_1 + m_2) m_1 g}{m_2}$$

(a) If $F < F_{max}$, the blocks move together without any relative motion.

(b) If $F > F_{max}$, the blocks slide relative to each other and then their accelerations are different.

EXAMPLE 5.11

A block of mass $m = 1$ kg is pulled by a force $F = 10$ N at an angle $\theta = 60°$ with a horizontal surface as shown in Fig. 5.25. Find the acceleration of the block if

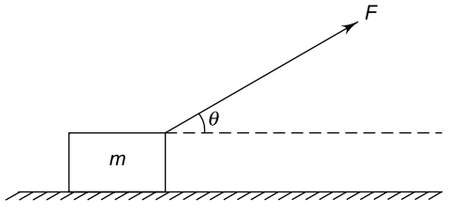


Fig. 5.25

(a) the surface is frictionless and

(b) the coefficient of kinetic friction between the surface and the block is $\mu = 0.2$. Take $g = 10$ ms^{-2}.

SOLUTION

The free body diagrams of the block in the two cases are shown in Fig. 5.26.

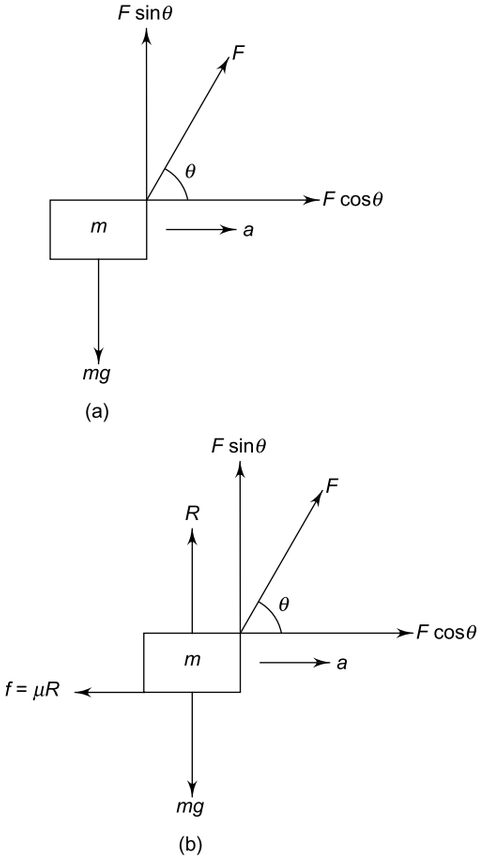


Fig. 5.26

(a) From Fig. 5.26(a)

$$F\cos\theta = ma$$

$$\Rightarrow \quad a = \frac{F\cos\theta}{m} = \frac{10 \times \cos 60°}{1} = 5 \text{ ms}^{-2}$$

(b) From Fig. 5.26(b)

$$F\cos\theta - f = ma$$

$$\Rightarrow \quad F\cos\theta - \mu mg = ma$$

$$\Rightarrow \quad a = \frac{F\cos\theta - \mu mg}{m}$$

$$= \frac{10\cos 60° - 0.2 \times 1 \times 10}{1}$$

$$= 3 \text{ ms}^{-2}$$

From Fig. 5.26(b) we also have $F \sin \theta + R = mg$ or $F\sin \theta = mg - R$. Since $F \sin \theta < mg$, the block does not move upwards.

EXAMPLE 5.12

Two blocks of masses $m_1 = 2$ kg and $m_2 = 3$ kg are suspended from a rigid support by means of strings AB and CD as shown in Fig. 5.27. String AB has negligible mass and string CD has mass 0.5 kg/m. Each string has length 50 cm. Find the tension (a) at mid-point P of string AB and (b) at point Q of string CD where CQ = 20 cm. Take $g = 10$ ms^{-2}.

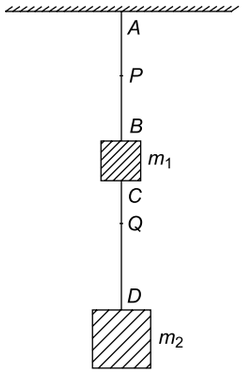


Fig. 5.27

SOLUTION

Mass of string CD is $m = 0.5 \times 0.5 = 0.25$ kg

Since string AB is massless, the tension in AB is the same at every point.

(a) Total mass below point $P = m_1 + m + m_2$

$= 2 + 0.25 + 3 = 5.25$ kg

$\therefore$ Tension at $P = 5.25 \times 10 = 52.5$ N

(b) Total mass below point Q = mass of length QD + m_2

$= 0.5 \times 0.8 + 3 = 3.4$ kg

$\therefore$ Tension at $Q = 3.4 \times 10 = 34$ N

EXAMPLE 5.13

A block of mass m = 100 g is placed on an inclined plane of inclination $\theta = 30°$ as shown in Fig. 5.28. There is no friction between the block and the inclined plane. What minimum acceleration a should be given to the system to the left so that the block does not slide down the plane?

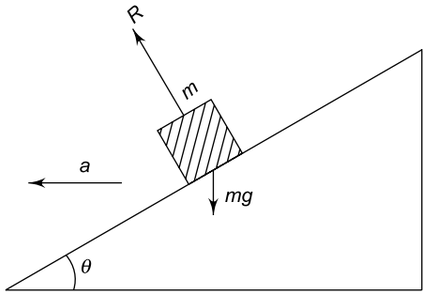


Fig. 5.28

SOLUTION

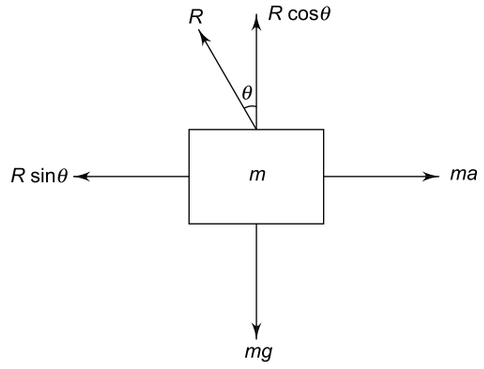


Fig. 5.29

Figure 5.29 shows the forces acting on the block

$$R \cos \theta = mg$$

$$R \sin \theta = ma$$

$$\therefore \quad a = g \tan \theta$$

$$= g \tan 30° = \frac{g}{\sqrt{3}}$$

EXAMPLE 5.14

A pendulum of bob of mass m = 100 g is suspended from the ceiling of the compartment of a train. If the train has the acceleration a as shown in Fig. 5.30, the string makes an angle $\theta = 60°$ with the vertical. Find the value of a.

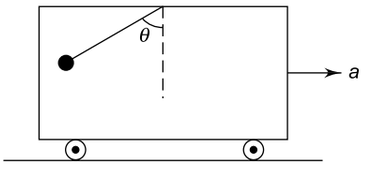


Fig. 5.30

SOLUTION

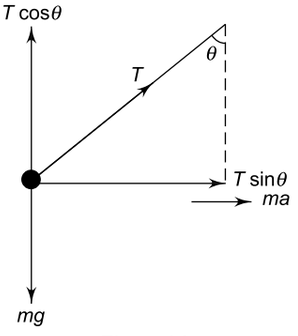


Fig. 5.31

Figure 5.31 shows the free body diagram of the bob.

Force on the bob in the direction of motion of the train = $T \sin \theta$. Hence the equation for horizontal direction is

$$T \sin \theta = ma \quad \text{(i)}$$

For equilibrium along the vertical direction

$$T \cos \theta = mg \quad \text{(ii)}$$

Dividing (i) and (ii), we get

$$\tan \theta = \frac{a}{g}$$

$$\Rightarrow \quad a = g \tan \theta = 9.8 \times \tan 60°$$

$$= 9.8 \times \sqrt{3} \simeq 17 \text{ ms}^{-2}$$

EXAMPLE 5.15

Two blocks of masses $m_1 = 1.5$ kg and $m_2 = 2$ kg are attached to each other by strings and pulleys as shown in Fig. 5.32. Assume that pulleys are massless and frictionless and strings are massless. The system is released. If the table is frictionless, find the accelerations of m_1 and m_2 and tensions in the strings. Take $g = 10$ ms^{-2}.

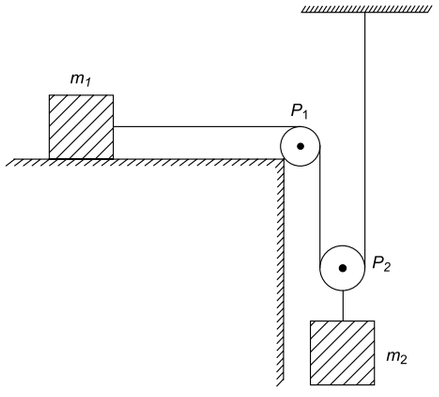


Fig. 5.32

SOLUTION

Let a_1 and a_2 be the acceleration of m_1 and m_2 respectively. Let x_1 and x_2 be the distances moved by m_1 and m_2 in a time t. Since the total length of the string remains unchanged, it follows that if m_1 moves a distance x_1 to the right, m_2 will descend by a distance $x_2 = \frac{x_1}{2}$ or $x_1 = 2x_2$. Differentiating twice w.r.t. time we get

$$\frac{d^2x_1}{dt^2} = 2\frac{d^2x_2}{dt^2} \Rightarrow a_1 = 2a_2 \quad \text{(i)}$$

Figure 5.33 shows the free body diagrams of m_1 and m_2. Here T_1 = tension in string attached to m_1 and T_2 = tension in string attached to m_2.

For block m_1

$$T_1 = m_1a_1 \quad \text{(i)}$$

For block m_2

$$m_2g - T_2 = m_2a_2 \quad \text{(ii)}$$

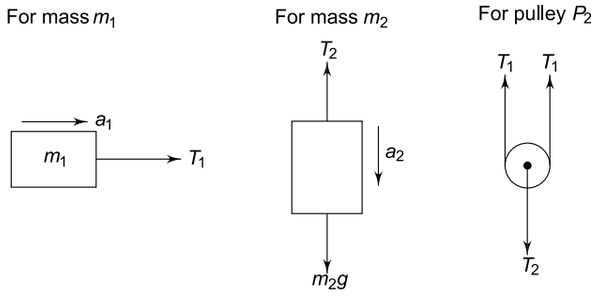


Fig. 5.33

Since the pulley is massless and frictionless

$$T_2 = 2T_1 \quad \text{(iii)}$$

Also $\quad a_1 = 2a_2. \quad \text{(iv)}$

Using (iii) and (iv) in (ii), we have

$$m_2g - 2T_1 = \frac{m_2\, a_1}{2} \quad \text{(v)}$$

Eliminating T_1 from (i) and (v), we get

$$a_1 = \frac{2m_2g}{m_2 + 4m_1} = \frac{2 \times 2 \times 10}{2 + 4 \times 1.5} = 5 \text{ ms}^{-2}$$

$$\therefore \quad a_2 = \frac{a_1}{2} = \frac{5}{2} = 2.5 \text{ ms}^{-2}$$

From eq. (i),

$$T_1 = m_1a_1 = 1.5 \times 5 = 7.5 \text{ N}$$

$$\therefore \quad T_2 = 2T_1 = 2 \times 7.5 = 15 \text{ N}$$

EXAMPLE 5.16

Two blocks of masses $m_1 = 100$ g and $m_2 = 5$ kg with m_1 placed on m_2 are connected to a frictionless and massless pulley as shown in Fig. 5.34. The string connecting them is also massless. The coefficient of static friction between m_1 and m_2 is $\mu = 0.5$. There is no frictionless between m_2 and the horizontal surface. Find the maximum horizontal force F that can be applied on m_1 so that it does not slide on m_2.

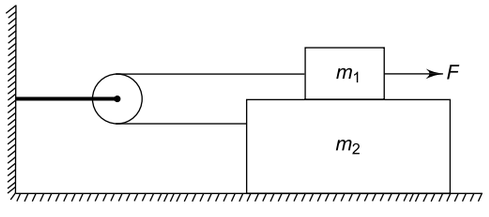


Fig. 5.34

SOLUTION

Frictional force between m_1 and m_2 is $f = \mu R_1 = \mu m_1 g$. If T is the tension in the string, the free body diagrams of m_1 and m_2 are as shown in Fig. 5.35.

It follows from Fig. 5.35(a) that

$$R_1 = m_1 g \qquad \text{(i)}$$

and $$F - f - T = 0 \qquad \text{(ii)}$$

From Fig. 5.35(b), we have

$$T - f = 0 \Rightarrow T = f \qquad \text{(iii)}$$

and $$R_2 = R_1 + m_2 g \qquad \text{(iv)}$$

Using (iii) in (ii)

$$F - f - f = 0 \Rightarrow F = 2f = 2\mu m_1 g$$

Block m_1 will not slide on block m_2 if F is less than a maximum value F_{max} given by

$$F_{max} = 2\mu m_1 g$$
$$= 2 \times 0.5 \times 0.1 \times 10 = 1 \text{ N}$$

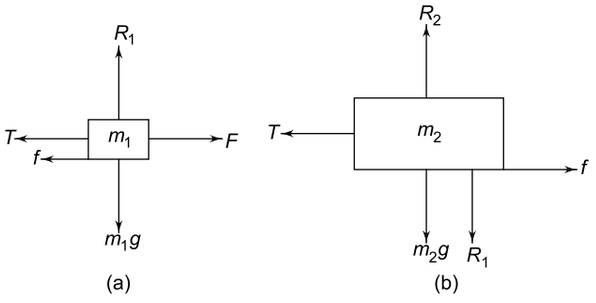


Fig. 5.35

EXAMPLE 5.17

A boy of mass $m = 50$ kg is standing on a weighing machine placed on the floor of an elevator. What is the weight of the boy when the elevator is (a) at rest, (b) moving up with an acceleration $a = 2.2$ ms^{-1} and (c) moving down with an acceleration 2.2 ms^{-2}.

SOLUTION

The weighing machine reads the reaction R exerted by it on the boy. Fig. 5.36 shows the free body diagrams in the three cases.

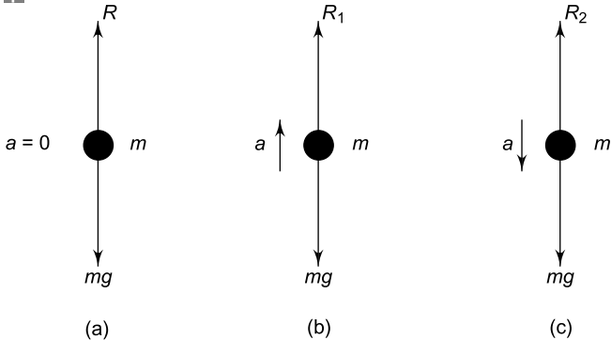


Fig. 5.36

(a) $R = mg = 50 \times 9.8 = 490$ N, the true weight of the boy.

(b) $R_1 - mg = ma$

$\Rightarrow R_1 = m(g + a) = 50 \times (9.8 + 2.2) = 600$ N

(c) $mg - R_2 = ma$

$\Rightarrow R_2 = m(g - a) = 50 \times (9.8 - 2.2) = 380$ N

NOTE

If the elevator is moving up or down with a uniform velocity $a = 0$ then the reading of the machine gives the actual weight.

EXAMPLE 5.18

A uniform cord AB of mass $M = 2$ kg and length $L = 100$ cm is pulled at ends A and B will forces $F_1 = 4$ N and $F_2 = 3$ N as shown in Fig. 5.37. Find the tension at point P at a distance $x = 20$ cm from end A.

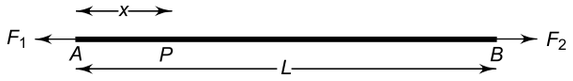


Fig. 5.37

SOLUTION

Since $F_1 > F_2$, the cord will accelerate to the left. Let a be the acceleration. To find tension T at P we consider the sections AP and PB of the cord.

Mass of part AP is $m_1 = \dfrac{Mx}{L}$

Mass of part PB is $m_2 = \dfrac{M}{L}(L - x)$

Figure 5.38 shows the free body diagrams of parts AP and PB.

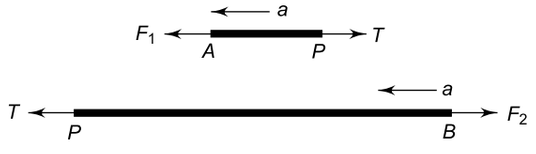


Fig. 5.38

For part AP: $F_1 - T = m_1 a$ (i)

For part PB: $T - F_2 = m_2 a$ (ii)

Dividing (i) and (ii) we get

$$\frac{F_1 - T}{T - F_2} = \frac{m_1}{m_2} = \frac{\frac{Mx}{L}}{\frac{M}{L}(L - x)} = \frac{x}{L - x}$$

which gives $T = \dfrac{F_1(L-x)+F_2x}{L}$

$= \dfrac{4(1-0.2)+3\times 0.2}{1}$

$= 3.8$ N

EXAMPLE 5.19

A monkey of mass $m = 30$ kg is climbing up a rope with an acceleration $a = 5$ ms^{-2} relative to the rope. The rope passes over a frictionless fixed pulley and has a block of mass $M = 15$ kg at the other end as shown in Fig. 5.39. Find (a) acceleration of the rope, (b) acceleration of monkey and (c) tension in the rope. Take $g = 10$ ms^{-2}.

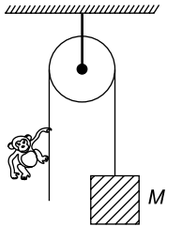


Fig. 5.39

SOLUTION

Let A be the acceleration of the block in the upward direction and T be the tension in the rope. The rope will have acceleration A in the downward direction. Hence the monkey will have a net acceleration $(A - a)$ in the downward direction. Figure 5.40 shows the free body diagrams of the monkey and the block.

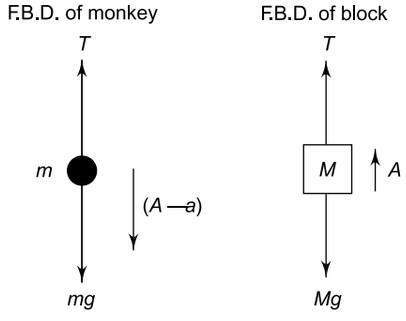


Fig. 5.40

For monkey: $mg - T = m(A - a)$ (i)

For block: $T - Mg = MA$ (ii)

Adding (i) and (ii), we get

$mg - Mg = m(A - a) + MA$

$\Rightarrow \quad A = \dfrac{m(g+a)-Mg}{M+m}$

$= \dfrac{30(10+5)-15\times 10}{15+30}$

$= 6.7$ ms^{-2}

Acceleration of monkey $= A - a = 6.7 - 5 = 1.7$ ms^{-2}

Tension in the rope $T = M(g + A) = 15 \times (10 + 6.7) \simeq 250$ N

EXAMPLE 5.20

A block of mass $m = 2$ kg is held stationary against a wall by applying a horizontal force F on it as shown in Fig. 5.41. If the coefficient of friction between the block and the wall is $\mu = 0.25$, find the minimum value of F required to hold the block against the wall. Take $g = 10$ ms^{-2}.

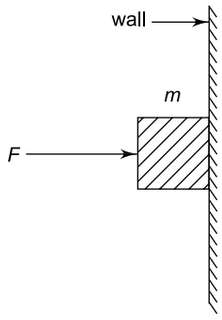


Fig. 5.41

SOLUTION

Let R be the normal reaction exerted on the block by the wall and f be the frictrorial force. Figure 5.42 shows the forces on the block.

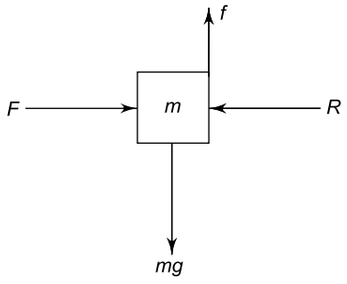


Fig. 5.42

Since the block is held stationary, no net force acts on it. Hence

$f = mg$

and $F = R$

For no slipping, $f \le \mu R$ or $mg \le \mu F$ or $F \ge \dfrac{mg}{\mu}$

$\therefore \quad F_{min} = \dfrac{mg}{\mu} = \dfrac{2\times 10}{0.25} = 80$ N

EXAMPLE 5.21

A block of mass $m = 2$ kg is held in contact with a block of mass $M = 10$ kg by applying a horizontal force F on it as shown in Fig. 5.43. Block M is lying on a horizontal frictionless surface. The coefficient of

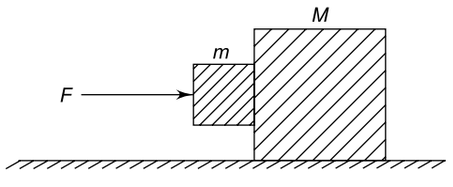


Fig. 5.43

friction between the block is $\mu = 0.4$. Find the minimum value of F required to hold m against M. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

Since the two blocks are always in contact, they will have the same acceleration, say a. Figure 5.44 show the free body diagram of the blocks.

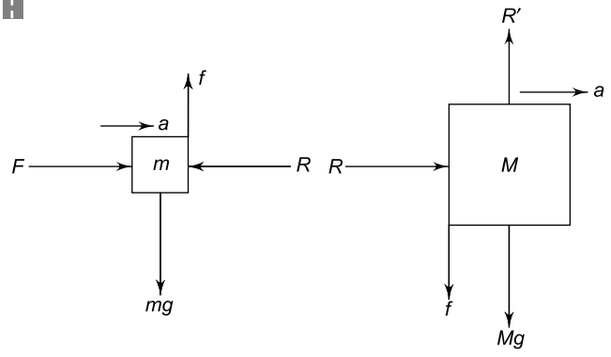


Fig. 5.44

For block m : $F - R = ma$ (i)

and $f = mg$ (ii)

For block M : $R = Ma$ (iii)

and $Mg + f = R'$ (iv)

Eliminating a from (i) and (iii), we get

$$R = \frac{MF}{M+m}$$

For no slipping, $f \le \mu R$

or $$mg \le \frac{\mu MF}{M+m}$$

or $$F \ge \frac{mg(M+m)}{\mu M}$$

$\therefore$ $$F_{\min} = \frac{mg(M+m)}{\mu M}$$

$$= \frac{2\times 10(10+2)}{0.4\times 10} = 60\ \text{N}$$

EXAMPLE 5.22

A block P of mass $m = 1$ kg is placed over a plank Q of mass $M = 6$ kg, placed over a smooth horizontal surface as shown in Fig. 5.45. Block P is given a velocity $v = 2\ \text{ms}^{-1}$ to the right. If the coefficient of friction between P and Q is $\mu = 0.3$, find the acceleration of Q relative to P.

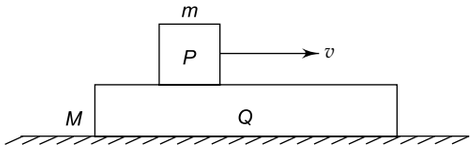


Fig. 5.45

SOLUTION

Frictional force between P and Q is $f = \mu mg$ which will retard P and accelerate Q.

Retardation of P is $a_P = -\frac{f}{m} = -\frac{\mu mg}{m} = -\mu g$

Acceleration of Q is $a_Q = \frac{f}{M} = \frac{\mu mg}{M}$

$\therefore$ Acceleration of Q relative to P is

$$a_{QP} = a_Q - a_P = \frac{\mu mg}{M} - (-\mu g)$$

$$= \mu g\left(1 + \frac{m}{M}\right)$$

$$= 0.3 \times 10\left(1 + \frac{1}{6}\right)$$

$$= 3.5\ \text{ms}^{-2}$$

EXAMPLE 5.23

A block of mass $m = 500\ g$ is placed on the top of an inclined of inclination $\theta = 30°$ kept on the floor of a lift which is moving up with an acceleration $a = 2\ \text{ms}^{-2}$. Find the coefficient of friction between the block and the incline so that the block moves down with a constant velocity.

SOLUTION

$$g_{\text{eff}} = g + a$$

The block will move down the plane with a constant velocity if no net force acts on it, i.e.

Force down the plane = frictional force

$$mg_{\text{eff}} \sin\theta = \mu mg_{\text{eff}} \cos\theta$$

$\Rightarrow$ $$\mu = \tan\theta = \tan 30° \simeq 0.58$$

EXAMPLE 5.24

A cube of mass $m = 1$ kg is placed on a wedge of mass $M = 2$ kg as shown in Fig. 5.46. There is no friction between the cube and the wedge. Find the minimum coefficient of friction between the wedge and the horizontal surface so that the wedge does not move.

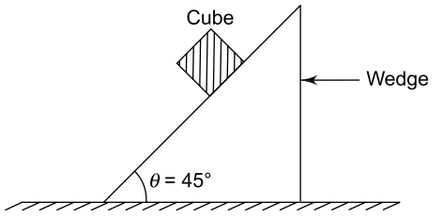


Fig. 5.46

SOLUTION

Figure 5.47 shows the horizontal force F_x and the vertical force F_y exerted by the cube on the wedge.

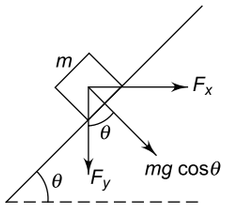


Fig. 5.47

$F_x = (mg \cos \theta) \sin \theta$ and $F_y = (mg \cos \theta) \cos \theta$

Weight of the wedge = Mg acting vertically down wards.

Hence Net horizontal force on the wedge is

$$F = mg \cos\theta \sin\theta$$

Net vertical force on the wedge is

$$N = Mg + mg \cos^2\theta$$

$$\therefore \quad \mu_{\min} = \frac{F}{N} = \frac{m\cos\theta \sin\theta}{M + m\cos^2\theta}$$

$$= \frac{1 \times \cos 45° \times \sin 45°}{2 + 1 \times \cos^2 45°} = 0.2$$

EXAMPLE 5.25

In Example 24 above there is no friction between the cube and the wedge and between the wedge and the horizontal surface below. If the wedge moves towards the right with an acceleration $a = \frac{1}{\sqrt{2}}$ ms^{-2}, find the acceleration of the cube relative to the wedge when the cube is released.

SOLUTION

Let A be the acceleration of the cube relative to the wedge as the cube moves down the plane. Its acceleration when the wedge moves to the right with acceleration a is $(A \cos\theta - a)$ directed towards the left. For dynamic equilibrium,

$$m\,(A \cos\theta - a) = Ma$$

$$\Rightarrow \quad A = \frac{(M+m)a}{m\cos\theta}$$

$$= \frac{(2+1) \times 1/\sqrt{2}}{1 \times \cos 45°}$$

$$= 3 \text{ ms}^{-2}$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. In order to raise a mass m a man ties it to a rope and passes the rope over a frictionless pulley. He climbs the rope with an acceleration $3g/2$ relative to the rope. If the mass of the man is $m/2$ and the mass of the rope is negligible, the tension in the rope is

(a) $\frac{3mg}{2}$ (b) $\frac{5mg}{3}$

(c) $\frac{7mg}{6}$ (d) $\frac{9mg}{7}$

2. In the arrangement shown in Fig. 5.48, the ends P and Q of a string move downwards with a uniform speed u. Pulleys B and C are frictionless and fixed. The mass M will move upwards with a speed

(a) $2u \cos\theta$ (b) $\frac{u}{\cos\theta}$

(c) $\frac{2u}{\cos\theta}$ (d) $u \cos\theta$

IIT, 1982

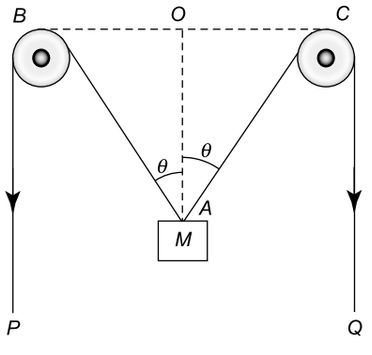


Fig. 5.48

3. A block P of mass m is placed over a plank of mass M. Plank Q is placed over a smooth horizontal surface as shown in Fig. 5.49. Block P is given a velocity v to the right. If μ is the coefficient of friction between P and Q, the acceleration of Q relative to P is

(a) μg (b) $\dfrac{\mu mg}{M}$

(c) $\mu g\left(1-\dfrac{m}{M}\right)$ (d) $\mu g\left(1+\dfrac{m}{M}\right)$

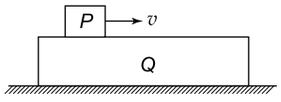


Fig. 5.49

4. A block is placed on the top of an inclined plane of inclination θ kept on the floor of a lift which is moving down with an acceleration a. The coefficient of friction between the block and the incline so that the block moves down with a constant velocity is

(a) $\mu = \tan\theta$ (b) $\mu = \dfrac{a}{g}\tan\theta$

(c) $\mu = \left(1-\dfrac{a}{g}\right)\tan\theta$ (d) $\mu = \left(1+\dfrac{a}{g}\right)\tan\theta$

5. A cube of mass m is placed on top of a wedge of mass 2 m as shown in Fig. 5.50. There is no friction between the cube and the wedge. The minimum coefficient of friction between the wedge and the horizontal surface so that the wedge does not move is

(a) 0.1 (b) 0.2

(c) 0.3 (d) $\dfrac{1}{\sqrt{2}}$

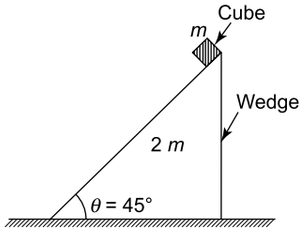


Fig. 5.50

6. In Q.5 above, all surfaces are frictionless. If the wedge moves towards right with acceleration a, the acceleration of the cube relative to the wedge when it is released is

(a) $\left(\sqrt{2}\right)a$ (b) $\left(2\sqrt{2}\right)a$

(c) $\left(3\sqrt{2}\right)a$ (d) $\left(4\sqrt{2}\right)a$

7. A block of mass m is held against a wall by applying a horizontal force F on the block. If the coefficient of friction between the block and the wall is μ, the magnitude of the frictional force acting on the block is

(a) mg (b) μmg

(c) μF (d) zero

8. A particle of mass $m = 10^{-2}$ kg is moving along the positive x-axis under the influence of a force

$$F = -\frac{k}{2x^2}$$

where $k = 10^{-2}$ Nm^2. When the particle is at $x = 1.0$ m, its velocity $v = 0$. When it reaches $x = 0.5$ m, the magnitude of its velocity is

(a) 0.5 ms^{-1} (b) 1.0 ms^{-1}

(c) 1.5 ms^{-1} (d) 2.0 ms^{-1}

IIT, 1998

9. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A block starting from rest at the top of the plane will come to rest at the bottom if the coefficient of friction between the block and the lower half of the plane is given by

(a) $\mu = 2\tan\theta$ (b) $\mu = \tan\theta$

(c) $\mu = \dfrac{2}{\tan\theta}$ (d) $\mu = \dfrac{1}{\tan\theta}$

10. Two skaters A and B of mass 50 and 70 kg, respectively, stand facing each other, 6 metres apart on a horizontal smooth surface. They pull a rope stretched between them. How far has each moved when they meet?

(a) Both have moved 3 m.
(b) A moves 4 m and B moves 2 m.
(c) A moves 2.5 m and B moves 3.5 m.
(d) A moves 3.5 m and B moves 2.5 m.

11. A person is sitting facing the engine in a moving train. He tosses a coin. The coin falls behind him. This shows that the train is

(a) moving forward with a finite acceleration
(b) moving forward with a finite retardation
(c) moving backward with a uniform speed
(d) moving forward with a uniform speed.

12. N bullets each of mass m kg are fired with a velocity v ms^{-1}, at the rate of n bullets per second, upon a wall. The reaction offered by the wall to the bullets is given by

(a) $nNmv$ (b) $\dfrac{Nmv}{n}$

(c) $\frac{nNm}{v}$ (d) $\frac{nNv}{m}$

13. A stretching force of 10 N is applied at one end of a spring balance and an equal stretching force is applied at the other end at the same time. What will be reading of the balance?

(a) zero (b) 5 N
(c) 10 N (d) 20 N

14. A block A is released from the top of smooth inclined plane and slides down the plane. Another block B is dropped from the same point and falls vertically downwards. Which one of the following statements will be true if the friction offered by air is negligible?

(a) Both blocks will reach the ground at the same time.
(b) Block A reaches the ground earlier than block B.
(c) Both blocks will reach the ground with the same speed.
(d) Block B reaches the ground with a higher speed than block A.

15. A block is released from the top of an inclined plane of height h and angle of inclination θ. The time taken by the block to reach the bottom of the plane is given by

(a) $\sqrt{\frac{2h}{g}}$ (b) $\sin\theta\sqrt{\frac{2h}{g}}$
(c) $\frac{1}{\sin\theta}\cdot\sqrt{\frac{2h}{g}}$ (d) $\frac{1}{\cos\theta}\cdot\sqrt{\frac{2h}{g}}$

16. A block of mass M is resting on an inclined plane as shown in Fig. 5.51. The inclination of the plane to the horizontal is gradually increased. It is found that when the angle of inclination is θ the block just begins to the slide down the plane. What is the minimum force F applied parallel to the plane that would just make the block move up the plane?

(a) $Mg\sin\theta$ (b) $Mg\cos\theta$
(c) $2Mg\cos\theta$ (d) $2Mg\sin\theta$

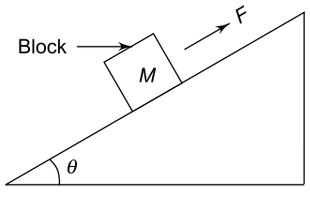


Fig. 5.51

17. A stream of a liquid of density ρ flowing horizontally with a speed v gushes out of a tube of radius r and hits at a vertically wall nearly normally. Assuming that the liquid does not rebound from the wall, the force exerted on the wall by the impact of liquid is given by

(a) $\pi r\rho v$ (b) $\pi r\rho v^2$
(c) $\pi r^2\rho v$ (d) $\pi r^2\rho v^2$

18. A ball of mass m is moving towards a batsman at a speed v. The batsman strikes the ball and deflects it by an angle θ without changing its speed. The impulse imparted to the ball is given by

(a) $mv\cos(\theta)$ (b) $mv\sin(\theta)$
(c) $2mv\cos\left(\frac{\theta}{2}\right)$ (d) $2mv\sin\left(\frac{\theta}{2}\right)$

19. Figure 5.52 shows the position-time (x–t) graph of one-dimensional motion of a body of mass 0.4 kg. What is the time interval between consecutive impulses received by the body?

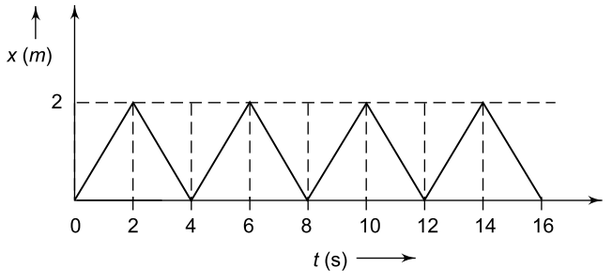


Fig. 5.52

(a) 2 s (b) 4 s
(c) 8 s (d) 16 s

20. In Q.19, what is the magnitude of each impulse?

(a) 0.2 Ns (b) 0.4 Ns
(c) 0.8 Ns (d) 1.6 Ns

21. An aeroplane of mass M requires a speed v for take-off. The length of the runway is s and the coefficient of friction between the tyres and the ground is μ. Assuming that the plane accelerates uniformly during the take-off, the minimum force required by the engine of the plane for take-off is given by

(a) $M\left(\frac{v^2}{2s}+\mu g\right)$ (b) $M\left(\frac{v^2}{2s}-\mu g\right)$
(c) $M\left(\frac{2v^2}{s}+2\mu g\right)$ (d) $M\left(\frac{2v^2}{s}-2\mu g\right)$

22. A block of mass m is projected up an inclined plane of inclination θ with an initial velocity u. If the coefficient of kinetic friction between the block and the plane is μ, the distance up to which the block will rise up the plane, before coming to rest, is given by

(a) $\frac{u^2\mu}{2g\sin\theta}$ (b) $\frac{u^2\mu}{2g\cos\theta}$

(c) $\frac{u^2}{4g\sin\theta}$ (d) $\frac{u^2}{4g\cos\theta}$

23. A man of mass 60 kg is standing on a horizontal conveyer belt (Fig. 5.53). When the belt is given an acceleration of 1 ms^{-2}, the man remains stationary with respect to the moving belt. If $g = 10\ ms^{-2}$, the net force acting on the man is

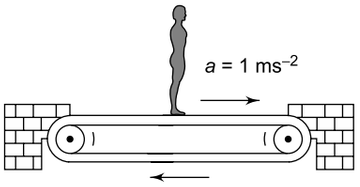


Fig. 5.53

(a) 0.6 N (b) 6 N
(c) 60 N (d) 600 N

24. In Q.23, if the coefficient of static friction between his shoes and the belt is 0.2 to what value can the acceleration of the belt be increased so that the man continues to remain stationary relative to the belt?

(a) 1 ms^{-2} (b) 2 ms^{-2}
(c) 3 ms^{-2} (d) 4 ms^{-2}

25. A block of mass 10 kg is placed at a distance of 5 m from the rear end of a long trolley as shown in Fig. 5.54. The coefficient of friction between the block and the surface below is 0.2. Starting from rest, the trolley is given a uniform acceleration of 3 ms^{-2}. At what distance from the starting point will the block fall off the trolley? Take $g = 10\ ms^{-2}$.

(a) 15 m (b) 20 m
(c) 25 m (d) 30 m

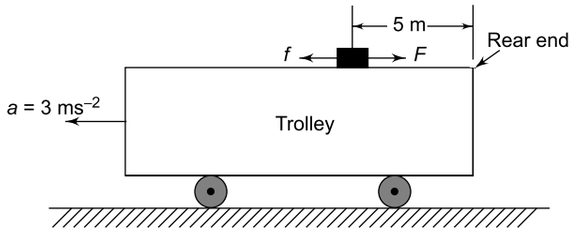


Fig. 5.54

26. Two blocks of masses $m_1 = 5$ kg and $m_2 = 6$ kg are connected by a light string passing over a light frictionless pulley as shown in Fig. 5.55. The mass m_1 is at rest on the inclined plane and mass m_2 hangs vertically. If the angle of incline $\theta = 30°$, what is the magnitude and direction of the force of friction on the 5 kg block? Take $g = 10\ ms^{-2}$.

(a) 35 N up the plane
(b) 35 N down the plane
(c) 85 N up the plane
(d) 85 N down the plane

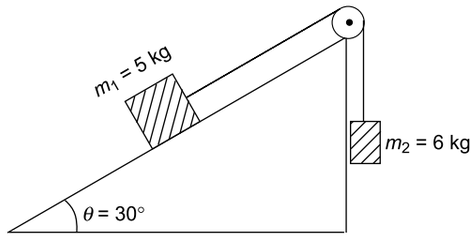


Fig. 5.55

27. A given object takes n times as much time to slide down a 45° rough incline as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is given by

(a) $\mu_k = 1/(1 - n^2)$ (b) $\mu_k = 1 - 1/n^2$
(c) $\mu_k = \sqrt{1/\left(1-n^2\right)}$ (d) $\mu_k = \sqrt{\left(1-1/n^2\right)}$

28. A body is sliding down a rough inclined plane of angle of inclination θ for which the coefficient of friction varies with distance x as $\mu(x) = kx$, where k is a constant. Here x is the distance moved by the body down the plane. The net force on the body will be zero at a distance x_0 given by

(a) $\frac{\tan\theta}{k}$ (b) $k\tan\theta$

(c) $\frac{\cot\theta}{k}$ (d) $k\cot\theta$

29. A body is moving down a long inclined plane of angle of inclination θ. The coefficient of friction between the body and the plane varies as $\mu = 0.5x$, where x is the distance moved down the plane. The body will have the maximum velocity when it has travelled a distance x given by

(a) $x = 2\tan\theta$ (b) $x = \frac{2}{\tan\theta}$

(c) $x = \sqrt{2}\cot\theta$ (d) $x = \frac{\sqrt{2}}{\cot\theta}$

30. An object is kept on a smooth inclined plane of 1 in l. The horizontal acceleration to be imparted to the inclined plane so that the object is stationary relative to the incline is given by

(a) $g\sqrt{l^2-1}$ (b) $g\ (l^2 - 1)$

(c) $\frac{g}{\sqrt{l^2-1}}$ (d) $\frac{g}{l^2-1}$

31. An insect is crawling up a hemispherical bowl of radius R. If the coefficient of friction is 1/3, the insect will be able to go up to height h equal to (take $3/\sqrt{10} = 0.95$)

(a) $\frac{R}{5}$ (b) $\frac{R}{10}$

(c) $\frac{R}{20}$ (d) $\frac{R}{30}$

32. A ball of mass m is connected to a ball of mass M by means of a massless spring. The balls are pressed so that the spring is compressed. When released, ball of mass m moves with acceleration a. The magnitude acceleration of mass M will be

(a) $\frac{ma}{(M+m)}$ (b) $\frac{Ma}{(M+m)}$

(c) $\frac{ma}{M}$ (d) $\frac{Ma}{m}$

33. Two blocks of masses m and M are placed on a horizontal frictionless table connected by a spring as shown in Fig. 5.56. Mass M is pulled to the right with a force F. If the acceleration of mass m is a, the acceleration of mass M will be

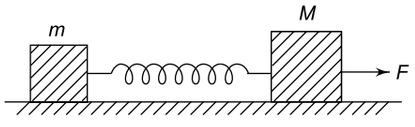


Fig. 5.56

(a) $\frac{(F-ma)}{M}$ (b) $\frac{(F+ma)}{M}$

(c) $\frac{F}{M}$ (d) $\frac{am}{M}$

34. A boy wants to climb down a rope. The rope can withstand a maximum tension equal to two-thirds the weight of the boy. If g is the acceleration due to gravity, the minimum acceleration with which the boy should climb down the rope should be

(a) $\frac{g}{3}$ (b) $\frac{2g}{3}$

(c) g (d) zero

35. A block, released from rest from the top of a smooth inclined plane of angle of inclination θ_1, reaches the bottom in time t_1. The same block, released from rest from the top of another smooth inclined plane of angle of inclination θ_2, reaches the bottom in time t_2. If the two inclined planes have the same height, the relation between t_1 and t_2 is

(a) $\frac{t_2}{t_1} = \left(\frac{\sin\theta_1}{\sin\theta_2}\right)^{1/2}$ (b) $\frac{t_2}{t_1} = \frac{\sin^2\theta_1}{\sin^2\theta_2}$

(c) $\frac{t_2}{t_1} = \frac{\sin\theta_1}{\sin\theta_2}$ (d) $\frac{t_2}{t_1} = 1$

36. A thick uniform rope of mass 6 kg and length 3 m is hanging vertically from a rigid support. The tension in the rope at a point 1 m from the support will be (Take $g = 10\ \text{ms}^{-2}$)

(a) 20 N (b) 30 N

(c) 40 N (d) 60 N

37. Two blocks of masses $M = 5$ kg and $m = 3$ kg are placed on a horizontal surface as shown in Fig. 5.57. The coefficient of friction between the blocks is 0.5 and that between the block M and the horizontal surface is 0.7. What is the maximum horizontal force F that can be applied to block M so that the two blocks move without slipping? Take $g = 10\ \text{ms}^{-2}$.

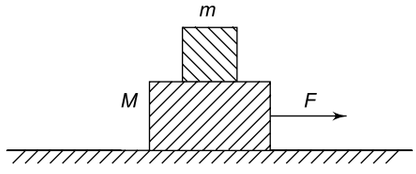


Fig. 5.57

(a) 4 N (b) 16 N

(c) 24 N (d) 96 N

38. Two blocks of masses $m_1 = 4$ kg and $m_2 = 6$ kg are connected by a string of negligible mass passing over a frictionless pulley as shown in Fig. 5.58 The coefficient of friction between block m_1 and the horizontal surface is 0.4. When the system is released, the masses m_1 and m_2 start accelerating. What additional mass m should be placed over mass m_1 so that the masses $(m_1 + m)$ slide with a uniform speed?

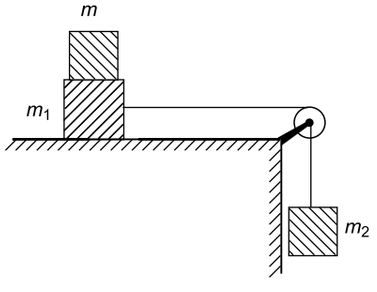


Fig. 5.58

(a) 9 kg (b) 10 kg

(c) 11 kg (d) 12 kg

39. A block of mass 5 kg is lying on a rough horizontal surface. The coefficients of static and kinetic fric-

tion between the block and the surface respectively are 0.7 and 0.5. A horizontal force just sufficient to move the block is applied to it. If the force continues to act even after the block has started moving, the acceleration of block will be (take $g = 10\ \text{ms}^{-2}$)

(a) $1\ \text{ms}^{-2}$ (b) $2\ \text{ms}^{-2}$

(c) $3\ \text{ms}^{-2}$ (d) $4\ \text{ms}^{-2}$

40. A body projected along an inclined plane of angle of inclination 30° stops after covering a distance x_1. The same body projected with the same speed stops after covering a distance x_2, if the angle of inclination of the inclined plane is increased to 60°. The ratio x_1/x_2 is

(a) 1 (b) $\sqrt{2}$

(c) $\sqrt{3}$ (d) 2

41. A smooth inclined plane of angle of inclination 30° is placed on the floor of a compartment of a train moving with a constant acceleration a. When a block is placed on the inclined plane, it does not slide down or up the plane. The acceleration a must be

(a) g (b) $\frac{g}{2}$

(c) $\frac{g}{\sqrt{2}}$ (d) $\frac{g}{\sqrt{3}}$

42. A block of mass m placed on a rough inclined plane of inclination $\theta = 30°$ can be just prevented from sliding down by applying a force F_1 up the plane and it can be made to just side up the plane by applying a force F_2 up the plane. If the coefficient of friction between the block and the inclined plane is $1/2\sqrt{3}$, the relation between F_1 and F_2 is

(a) $F_2 = F_1$ (b) $F_2 = 2F_1$

(c) $F_2 = 3F_1$ (d) $F_2 = 4F_1$

43. A uniform iron chain of length 120 cm is placed on a rough horizontal table. If the coefficient of friction between the chain and the table is 0.5, how much length of the chain can hang from the edge of the table?

(a) 20 cm (b) 40 cm

(c) 60 cm (d) 80 cm

44. A person standing in a stationary lift drops a coin from a certain height h. It takes time t to reach the floor of the lift. If the lift is rising up with a uniform acceleration a, the time taken by the coin, dropped from the same height h, to reach the floor will be

(a) t (b) $t\sqrt{\frac{a}{g}}$

(c) $t\left(1+\frac{a}{g}\right)^{1/2}$ (d) $t\left(1-\frac{a}{g}\right)^{1/2}$

45. A block is lying on a horizontal frictionless surface. One end of a uniform rope is fixed to the block which is pulled in the horizontal direction by applying a force F at the other end. If the mass of the rope is half the mass of the block, the tension in the middle of the rope will be

(a) F (b) $\frac{2F}{3}$

(c) $\frac{3F}{5}$ (d) $\frac{5F}{6}$

46. When a force F acts on a body of mass m, the acceleration produced in the body is a. If three equal forces $F_1 = F_2 = F_3 = F$ act on the same body as shown in Fig. 5.59 the acceleration produced is

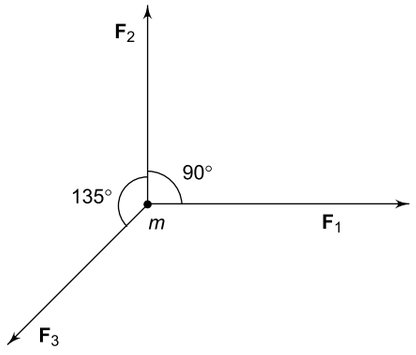


Fig. 5.59

(a) $\left(\sqrt{2}-1\right)a$ (b) $\left(\sqrt{2}+1\right)a$

(c) $\sqrt{2}\ a$ (d) a

47. An elastic spring has a length l_1 when it is stretched with a force of 2 N and a length l_2 when it is stretched with a force of 3 N. What will be the length of the spring if it is stretched with force of 5 N?

(a) $l_1 + l_2$

(b) $\frac{1}{2}(l_1 + l_2)$

(c) $3l_2 - 2l_1$

(d) $3l_1 - 2l_2$

48. A block of mass 4 kg is suspended through two light spring balances A and B as shown in Fig. 5.60. Then balances A and B will respectively read

(a) 4 kg and zero kg

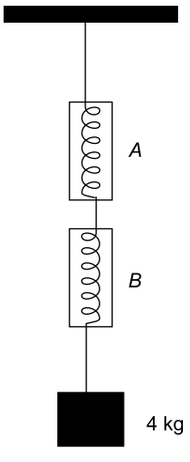


Fig. 5.60

(b) zero kg and 4 kg
(c) 4 kg and 4 kg
(d) 2 kg and 2 kg

49. A mass $M = 100$ kg is suspended with the use of strings A, B and C as shown in Fig. 5.61 W is a vertical wall and R is a rigid horizontal rod. The tension in string B is

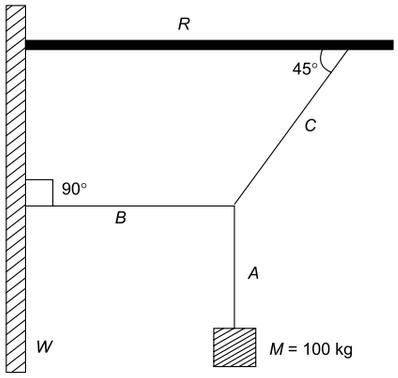


Fig. 5.61

(a) 100 g newton (b) zero
(c) $100\sqrt{2}$ g newton (d) $\frac{100}{\sqrt{2}}$ g newton

50. A bullet is fired from a gun. The force on the bullet is given by

$$F = 600 - 2 \times 10^5 t$$

where F is in newton and t in seconds. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet?

(a) 9 Ns (b) zero
(c) 0.9 Ns (d) 1.8 Ns

51. A long horizontal rod has a bead which can slide along its length, and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

(a) $\sqrt{\frac{\mu}{\alpha}}$ (b) $\frac{\mu}{\sqrt{\alpha}}$
(c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

IIT, 2000

52. A force vector $\mathbf{F} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ newton applied to a body accelerates it by 1 ms^{-2}. What is the mass of the body?

(a) $10\sqrt{2}$ kg (b) $2\sqrt{10}$ kg
(c) 10 kg (d) 20 kg

53. A block of weight 200 N is pulled along a rough horizontal surface at a constant speed by a force of 100 N acting at an angle of 30° above the horizontal. The coefficient of friction between the block and the surface is

(a) 0.43 (b) 0.58
(c) 0.75 (d) 0.85

54. Bullets of mass 0.03 kg each hit a plate at the rate of 200 bullets per second with a velocity of 50 ms^{-1} and reflect back with a velocity of 30 ms^{-1}. The average force (in newton) acting on the plate is

(a) 120 (b) 180
(c) 300 (d) 480

55. A body of mass M kg is on the top point of a smooth hemisphere of radius 5 m. It is released to slide down the surface of the hemisphere. It leaves the surface when its velocity is 5 m/s. At this instant the angle made by the radius vector of the body with the vertical is: (Acceleration due to gravity = 10 ms^{-2})

(a) 30° (b) 45°
(c) 60° (d) 90°

56. A stationary body of mass 3 kg explodes into three equal pieces. Two of the pieces fly off at right angles to each other, one with a velocity $2\hat{\mathbf{i}}$ m/s and the other with a velocity $3\hat{\mathbf{j}}$ m/s. If the explosion takes place in 10^{-5} sec, the average force acting on the third piece in newton is:

(a) $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times 10^{-5}$ (b) $-(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times 10^{5}$
(c) $(3\hat{\mathbf{j}} - 2\hat{\mathbf{i}}) \times 10^{5}$ (d) $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}) \times 10^{-5}$

57. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?

(a) One (b) Four
(c) Two (d) Three

58. A horizontal force, just sufficient to move a body of mass 4 kg lying on a rough horizontal surface, is applied on it. The coefficients of static and kinetic friction between the body and the surface are 0.8 and 0.6 respectively. If the force continues to act even after the body has started moving, the acceleration of the body (in ms^{-2}) is (take $g = 10$ ms^{-2})

(a) 2 (b) 4
(c) 6 (d) 8

59. A block is placed on an inclined plane. The angle of inclination (θ) of the plane is such that the block

slides down the plane at a constant speed. The coefficient of kinetic friction between the block and the inclined plane is equal to

(a) $\sin\theta$ (b) $\cos\theta$
(c) $\tan\theta$ (d) $\cot\theta$

60. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are:

(a) up the incline while ascending and down the incline while descending
(b) up the incline while ascending as well as descending
(c) down the incline while ascending and up the incline while descending
(d) down the incline while ascending as well as descending

IIT, 2002

61. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time $t = 0$. Their velocities become $\vec{v}_1'$ and $\vec{v}_2'$ at time $2t_0$ while still moving in air. The value of $|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)|$ is

(a) zero (b) $(m_1 + m_2)gt_0$
(c) $2(m_1 + m_2)gt_0$ (d) $\frac{1}{2}(m_1 + m_2)gt_0$

IIT, 2001

62. What is the maximum value of the force F such that the block shown in the arrangement does not move? The coefficient of friction between the block and the horizontal surface is 0.5. (Take $g = 10\ \text{ms}^{-2}$) (See Fig. 5.62)

(a) 20 N (b) 10 N
(c) 12 N (d) 15 N

IIT, 2003

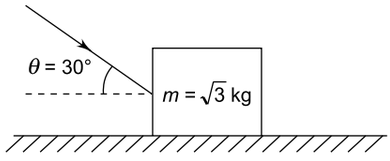


Fig. 5.62

63. An insect crawls up a hemispherical surface very slowly (see Fig. 5.63). The coefficient of friction between the insect and the surface is 1/3. If the line joining the center of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by

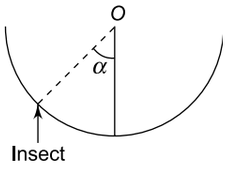


Fig. 5.63

(a) $\cot\alpha = 3$ (b) $\tan\alpha = 3$
(c) $\sec\alpha = 3$ (d) $\operatorname{cosec}\alpha = 3$

IIT, 2001

64. The pulleys and strings shown in Fig. 5.64 are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be

(a) 0° (b) 30°
(c) 45° (d) 60°

IIT, 2001

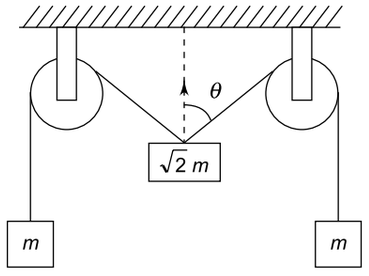


Fig. 5.64

65. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown Fig. 5.65. The force on the pulley by the clamp is given by

(a) $\sqrt{2}\ Mg$
(b) $\sqrt{2}\ mg$
(c) $\sqrt{(M+m)^2 + m^2}\ g$
(d) $\sqrt{(M+m)^2 + M^2}\ g$

IIT, 2001

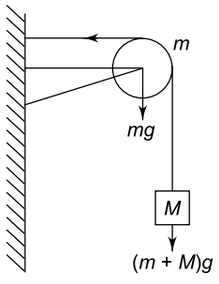


Fig. 5.65

66. A boy of mass m stands on one end of a wooden plank of length L and mass M. The plank is floating on water. If the boy walks from one end of the plank to the other end at a constant speed, the resulting displacement of the plank is given by

(a) $\frac{mL}{M}$ (b) $\frac{ML}{m}$

(c) $\frac{mL}{(M+m)}$ (d) $\frac{mL}{(M-m)}$

67. A shell of mass $2m$ fired with a speed u at an angle θ to the horizontal explodes at the highest point of its trajectory into two fragments of mass m each. If one fragment falls vertically, the distance at which the other fragment falls from the gun is given by

(a) $\frac{u^2 \sin 2\theta}{g}$ (b) $\frac{3u^2 \sin 2\theta}{2g}$

(c) $\frac{2u^2 \sin 2\theta}{g}$ (d) $\frac{3u^2 \sin 2\theta}{g}$

68. A jet of water with a cross-sectional area a is striking against a wall at an angle θ to the normal and rebounds elastically. If the velocity of water in the jet is v, the normal force acting on the wall is,

(a) $2\ av^2\ \rho \cos\ \theta$ (b) $av^2\ \rho \cos\ \theta$

(c) $2\ av^2\ \rho \sin\ \theta$ (d) $av^2\ \rho \sin\ \theta$

69. A block released from rest from the top of a smooth inclined plane of inclination 45° takes t seconds to reach the bottom. The same block released from rest from top of a rough inclined plane of the same inclination of 45° takes $2t$ seconds to reach the bottom. The coefficient of friction is

(a) $\sqrt{0.5}$ (b) $\sqrt{0.75}$

(c) 0.5 (d) 0.75

70. A block, released from rest from the top of a smooth inclined plane of inclination θ, has a speed v when it reaches the bottom. The same block, released from the top of a rough inclined plane of the same inclination θ, has a speed v/n on reaching the bottom, where n is a number greater than unity. The coefficient of friction is given by

(a) $\mu = \left(1-\frac{1}{n^2}\right) \tan\ \theta$

(b) $\mu = \left(1-\frac{1}{n^2}\right) \cot\ \theta$

(c) $\mu = \left(1-\frac{1}{n^2}\right)^{1/2} \tan\ \theta$

(d) $\mu = \left(1-\frac{1}{n^2}\right)^{1/2} \cot\ \theta$

71. A object is gently placed on a long conveyer belt moving with a speed of 5 ms^{-1}. If the coefficient of friction between the block and the belt is 0.5, the block will slide on the belt up to a distance (take g = 10 ms^{-2})

(a) 2.0 m (b) 2.5 m

(c) 3.0 m (d) 3.5 m

72. A boy of mass m is sliding down a vertical pole by pressing it with a horizontal force f. If μ is the coefficient of friction between his palms and the pole, the acceleration with which he slides down will be

(a) g (b) $\frac{\mu f}{m}$

(c) $g + \frac{\mu f}{m}$ (d) $g - \frac{\mu f}{m}$

73. A boy of mass 40 kg is climbing a vertical pole at a constant speed. If the coefficient of friction between his palms and the pole is 0.8 and g = 10 ms^{-2}, the horizontal force that he is applying on the pole is

(a) 300 N (b) 400 N

(c) 500 N (d) 600 N

74. A spring is compressed between two blocks of masses m_1 and m_2 placed on a horizontal frictionless surface as shown in Fig. 5.66. When the blocks are released, they have initial velocity of v_1 and v_2 as shown. The blocks travel distances x_1 and x_2 respectively before coming to rest. The ratio x_1/x_2 is

(a) $\frac{m_1}{m_2}$ (b) $\frac{m_2}{m_1}$

(c) $\sqrt{\frac{m_1}{m_2}}$ (d) $\sqrt{\frac{m_2}{m_1}}$

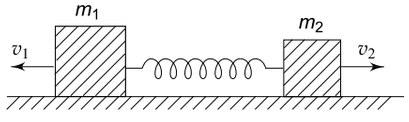


Fig. 5.66

75. A balloon of mass M is rising up with an acceleration a. If a mass m is removed from the balloon, its acceleration becomes

(a) $\frac{Ma+mg}{M-m}$ (b) $\frac{Ma+mg}{M+m}$

(c) $\frac{ma+Mg}{M-m}$ (d) $\frac{ma+Mg}{M+m}$

76. A uniform rope is hanging vertically from the ceiling such that its free end just touches the horizontal floor of a room. The upper end of the rope is then released. At any instant during the fall of the rope, the total force exerted by it on the floor is n times the weight of that part of the rope which is on the floor at that time. What is the value of n?

(a) 1 (b) 2

(c) 3 (d) 4

77. A block of mass m is lying a horizontal surface of coefficient of friction μ. A force F is applied to the block at an angle θ with the horozontal. The block will move with a minimum force F if

(a) $\mu = \tan\theta$ (b) $\mu = \cot\theta$

(c) $\mu = \sin\theta$ (d) $\mu = \cos\theta$

78. In Q. 77 above, the minimum F is given by

(a) $\dfrac{\mu mg}{\mu - 1}$ (b) $\dfrac{\mu mg}{\mu + 1}$

(c) $\dfrac{\mu^2 mg}{\sqrt{1-\mu^2}}$ (d) $\dfrac{\mu mg}{\sqrt{1+\mu^2}}$

79. The coefficients of static and kinetic friction between a body and the surface are 0.75 and 0.5 respectively. A force is applied to the body to make it just slide with a constant acceleration which is

(a) $\dfrac{g}{4}$ (b) $\dfrac{g}{2}$

(c) $\dfrac{3g}{4}$ (d) g

80. A block P of mass m is placed on a horizontal frictionless surface. Another block Q of the same mass is kept on P and connected to a rigid wall by means of a spring of spring constant k as shown in Fig. 5.67. The two blocks move together, without slipping, performing simple harmonic motion of amplitude A. If μ is the coefficient of static fric-tion between blocks P and Q, the maximum value of the force of friction between P and Q is

(a) μmg (b) $\dfrac{kA}{2}$

(c) kA (d) zero

IIT, 2004

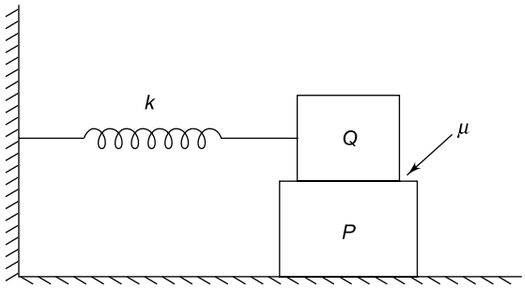


Fig. 5.67

81. A block of mass m is held stationary against a wall by applying a horizontal force F on the block Fig. 5.68. Which of the following statements is false?

(a) The frictional force acting on the block is $f = mg$

(b) The normal reaction force acting on the block is $N = F$

(c) No net torque acts on the block

(d) N does not produce any torque.

IIT, 2005

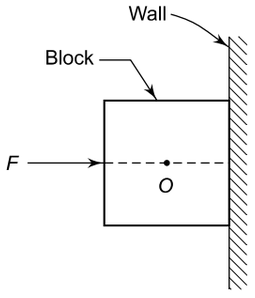


Fig. 5.68

82. A car moves at a speed of 36 km h^{-1} on a level road. The coefficient of friction between the tyres and the road is 0.8. The car negotiates a curve of radius R. If $g = 10\ ms^{-2}$, the car will skid (or slip) while negotiating the curve if the value of R is

(a) 20 m (b) 12 m

(c) 14 m (d) 16 m

83. A train has to negotiate a curve of radius 200 m. By how much should the outer rails be raised with respect to the inner rails for a speed of 36 km h^{-1}? The distance between the rails is 1.5 m. Take $g = 10\ ms^{-2}$.

(a) 7.5 cm (b) 10 cm

(c) 12.5 cm (d) 15 cm

84. A train rounds an unbanked circular bend of radius 50 m at a speed of 54 km h^{-1}. If $g = 10\ ms^{-2}$, the angle of banking required to prevent wearing out of rails is given by

(a) $\theta = \tan^{-1}(0.15)$ (b) $\theta = \tan^{-1}(0.25)$

(c) $\theta = \tan^{-1}(0.35)$ (d) $\theta = \tan^{-1}(0.45)$

85. A body is resting on top of a hemispherical mound of ice of radius R. If ice is frictionless, what minimum horizontal velocity must be imparted to the body so that it leaves the mound without sliding over it?

(a) $\sqrt{\dfrac{gR}{2}}$ (b) $\sqrt{gR}$

(c) $\sqrt{2gR}$ (d) $2\sqrt{gR}$

86. The over-bridge of a river is in the form of a circular arc of radius of curvature 10 m. If $g = 10\ ms^{-2}$, what is the highest speed at which a motor cyclist can cross the bridge without leaving the ground?

(a) 10 ms^{-1} (b) $10\sqrt{2}$ ms^{-1}

(c) $10\sqrt{3}$ ms^{-1} (d) 20 ms^{-1}

87. The blocks A and B of masses 2 m and m are connected as shown in Fig. 5.69. The spring has negligible mass. The string is suddenly cut. The magnitudes of accelerations of masses 2 m and m at that instant are

(a) g, g

(b) $g, \frac{g}{2}$

(c) $\frac{g}{2}, g$

(d) $\frac{g}{2}, \frac{g}{2}$

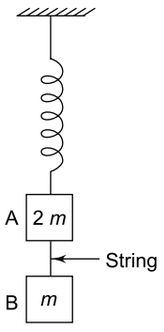


Fig. 5.69

88. A particle moves in the $x - y$ plane under the influence of a force such that its linear momentum is

$$\vec{p}\,(t) = A[\hat{i}\,\cos(kt) - \hat{j}\,\sin(kt)]$$

where A and k are constants. The angle between the force and momentum is

(a) 0° (b) 30°
(c) 45° (d) 90°

IIT, 2007

89. Two particles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a frictionless horizontal surface with the string held taut so that each mass is at a distance 'a' from the center P as shown in Fig. 5.70. Now, the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$, is

(a) $\frac{F}{2m}\frac{a}{\sqrt{a^2 - x^2}}$ (b) $\frac{F}{2m}\frac{x}{\sqrt{a^2 - x^2}}$

(c) $\frac{F}{2m}\frac{x}{a}$ (d) $\frac{F}{2m}\frac{\sqrt{a^2 - x^2}}{x}$

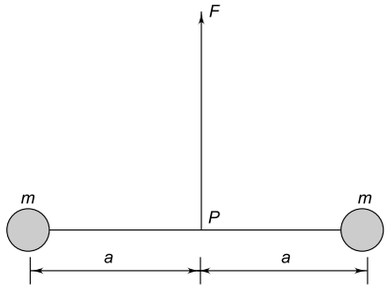


Fig. 5.70

IIT, 2007

90. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

(a) at $\theta = 30°$, the block will start sliding down the plane
(b) the block will remain at rest on the plane up to certain θ and then it will topple
(c) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles
(d) at $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ.

IIT, 2009

91. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is

(a) $\frac{a}{gk}$ (b) $\frac{a}{2gk}$

(c) $\frac{2a}{gk}$ (d) $\frac{a}{4gk}$

IIT, 2009

92. A ball of mass (m) 0.5 kg is attached to the end of string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is [Fig. 5.71]

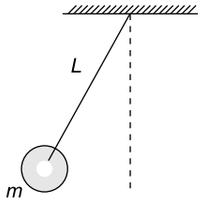


Fig. 5.71

(a) 9 (b) 18
(c) 27 (d) 36

IIT, 2011

ANSWERS

1. (c)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (c)
7. (a)	**8.** (b)	**9.** (a)	**10.** (d)	**11.** (a)	**12.** (a)
13. (c)	**14.** (c)	**15.** (c)	**16.** (d)	**17.** (d)	**18.** (c)
19. (a)	**20.** (c)	**21.** (a)	**22.** (c)	**23.** (c)	**24.** (b)
25. (a)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (c)
31. (c)	**32.** (c)	**33.** (a)	**34.** (a)	**35.** (c)	**36.** (c)
37. (d)	**38.** (c)	**39.** (b)	**40.** (c)	**41.** (d)	**42.** (c)
43. (b)	**44.** (c)	**45.** (d)	**46.** (a)	**47.** (c)	**48.** (c)
49. (a)	**50.** (c)	**51.** (a)	**52.** (a)	**53.** (b)	**54.** (d)
55. (c)	**56.** (b)	**57.** (d)	**58.** (a)	**59.** (c)	**60.** (b)
61. (c)	**62.** (b)	**63.** (a)	**64.** (c)	**65.** (d)	**66.** (c)
67. (b)	**68.** (a)	**69.** (d)	**70.** (a)	**71.** (b)	**72.** (d)
73. (c)	**74.** (b)	**75.** (a)	**76.** (c)	**77.** (a)	**78.** (d)
79. (a)	**80.** (b)	**81.** (d)	**82.** (b)	**83.** (a)	**84.** (d)
85. (b)	**86.** (a)	**87.** (c)	**88.** (d)	**89.** (b)	**90.** (b)
91. (b)	**92.** (d)				

SOLUTIONS

1. Let T be the tension in the rope and a its acceleration. Therefore, the aceeleration of the man = $\left(\frac{3g}{2} - a\right)$.

The equations of motion of the mass and the man are

$$T - mg = ma \qquad (1)$$

and $$T - \frac{mg}{2} = \frac{m}{2}\left(\frac{3g}{2} - a\right) \qquad (2)$$

Eliminating a from Eqs. (1) and (2), we get $T = \frac{7mg}{6}$, which is choice (c).

2. Let $AB = AC = r$, $OB = OC = x$ and $OA = y$. In ΔAOB

$$r^2 = x^2 + y^2$$

Differentiating with respect to t, we get

$$2r\frac{dr}{dt} = 0 + 2y\frac{dy}{dt}$$

$$\Rightarrow \qquad \frac{dy}{dt} = \frac{r}{y}\frac{dr}{dt}$$

Now $\frac{dr}{dt} = u$, $y = r\cos\theta$ and $\frac{dy}{dt}$ = speed with which M moves up. Hence the required speed = $\frac{u}{\cos\theta}$, which is choice (b).

3. Frictional force between P and Q is $f = \mu mg$, which will retard P and accelerate Q.

Retardation of P is $a_P = \frac{f}{m} = \frac{\mu mg}{m} = \mu g$

Acceleration of Q is $a_Q = \frac{f}{M} = \frac{\mu mg}{M}$

$\therefore$ Acceleration of Q relative to $P = a_P + a_Q$

$$= \mu g\left(1 + \frac{m}{M}\right)$$

4. $g_{\text{eff}} = (g - a)$. The block will move down the plane with a constant velocity if no net force acts on it, i.e. force down the plane = frictional force. Hence

$$m(g - a)\sin\theta = \mu m(g - a)\cos\theta$$

$$\Rightarrow \qquad \tan\theta = \mu$$

5. Net horizontal force on the wedge is

$$F = mg\cos\theta\sin\theta$$

Normal reaction $N = 2mg + mg\cos^2\theta$

Now $F = \mu N$ gives

$$\mu = \frac{F}{N} = \frac{mg\cos\theta\sin\theta}{2mg + mg\cos^2\theta}$$

$$= \frac{\cos\theta\sin\theta}{2 + \cos^2\theta}$$

For $\theta = 45°$, $\mu = 0.2$, which is choice (b).

6. Let A be the acceleration of the cube relative to the wedge moving down the plane. Its acceleration when the wedge moves to the right with acceleration a is $(A\cos\theta - a)$ directed towards the left. If the normal reaction between the cube and the wedge is N. Then

$$N\sin\theta = (2m)\,a = m\,(A\cos\theta - a)$$

which gives $$A = \frac{3ma}{m\cos\theta}$$

$$= \frac{3a}{\cos 45°} = \left(3\sqrt{2}\right)a$$

7. For vertical equilibrium of the block, the frictional force is $f = mg$ which must be less than $(f)_{max} = \mu N = \mu F$. So the correct choice is (a).

8. $F = ma = m\,\frac{dv}{dt} = m\,\frac{dv}{dx}\cdot\frac{dx}{dt} = mv\,\frac{dv}{dx}$

Given $F = -\frac{k}{2x^2}$. Hence

$$-\frac{k}{2x^2} = mv\,\frac{dv}{dx} \quad \Rightarrow \quad v\,dv = -\frac{k}{2m}\,x^{-2}\,dx$$

Integrating, we have

$$\int v\,dv = -\frac{k}{2m}\int x^{-2}\,dx$$

$$\Rightarrow \qquad \frac{v^2}{2} = \frac{k}{2mx} + C \qquad (1)$$

where C is the constant of integration.
Given $v = 0$ when $x = 1.0$ m. Using this in Eq. (1), we get $C = -k/2m$. Equation (1) becomes

$$\frac{v^2}{2} = \frac{k}{2m}\left(\frac{1}{x} - 1\right) \quad \Rightarrow \quad v = \left[\frac{k}{m}\left(\frac{1}{x} - 1\right)\right]^{1/2}$$

Substituting $k = 10^{-2}$ N m^2, $m = 10^{-2}$ kg and $x = 0.5$ m and simplifying we get $v = 1.0$ ms^{-1}.

9. The acceleration of the block while it is sliding down the upper half of the inclined plane is $g \sin\theta$. If μ is the coefficient of kinetic friction between the block and the lower half of the plane, the retardation of the block while it is sliding down the lower half $= -(g \sin\theta - \mu g \cos\theta)$. For the block to come to rest at the bottom of the inclined plane, the acceleration in the first half must be equal to the retardation in the second half, i.e.

$$g \sin\theta = -(g \sin\theta - \mu g \cos\theta)$$

or $$\mu \cos\theta = 2 \sin\theta$$

$$\mu = 2 \tan\theta$$

Hence the correct choice is (a).

10. Let m_A and m_B be the masses of skaters A and B and a_A and a_B their respective accelerations, when they pull at each other. From Newton's third law, action and reaction forces are equal in magnitude, i.e.

$$m_A a_A = m_B a_B \text{ or } m_A\,\frac{v_A}{t} = m_B\,\frac{v_B}{t}$$

or $$m_A v_A = m_B v_B \text{ or } m_A{}^2 v_A{}^2 = m^2{}_B v^2{}_B \qquad (i)$$

where v_A and v_B are their respective speeds and t is the time taken for them to meet. Let s_A and s_B be the distances travelled by them when they meet, we have,

$$2a_A s_A = v^2{}_A \quad \text{and} \quad 2a_B s_B = v^2{}_B$$

Using these equations in Eq. (i), noting that $m_A a_A = m_B a_B$, we get $\frac{s_A}{s_B} = \frac{m_B}{m_A} = \frac{70}{50} = \frac{7}{5}$. Since $s_A + s_B = 6$ m; $s_A = 3.5$ and $s_B = 2.5$ m. Hence, the correct choice is (d).

11. As long as the coin is in the hand of the person, it shares the acceleration of the train; it has the inertia of motion. When he tosses the coin, it falls behind him opposite to the direction of accelerated motion but now it no longer shares the acceleration of the train. Hence the correct choice is (a).

12. The reaction force offered by the wall to the bul-lets = the force exerted by bullets on the wall (third law of motion) = the rate of change of momentum of bullets (second law of motion). Now, total mass of n bullets = Nm. Momentum of n bullets = Nmv. If n bullets are fired per second, the change of momentum per second = $nNmv$. Hence, the correct choice is (a).

13. When a spring is hung on a support and load is attached at its lower end the weight of the load exerts a force on the support and the support, in turn, exerts an equal and opposite force on the load, and the spring remains stretched. Hence the correct choice is (c).

14. Consider a block of mass m lying on a frictionless inclined plane of length $AB = L$, height $AC = h$ and angle of inclination θ. (See Fig. 5.72). The acceleration due to gravity acting vertically downwards is resolved into two rectangular components $g \cos\theta$ and $g \sin\theta$. The component $mg \cos\theta$ balances with the normal reaction R. When the block is released, it moves down the plane under a force $mg \sin\theta$. Hence the acceleration of the block down the plane is

$$a = g \sin\theta$$

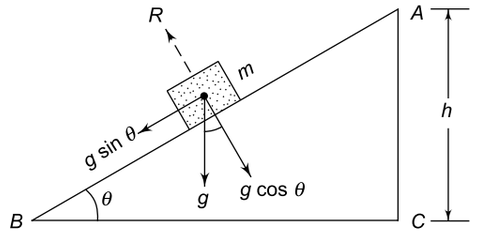


Fig. 5.72

If the block starts from rest from point A, then its velocity when it reaches the bottom B is given by

$$v^2 - u^2 = 2as$$

or $v^2 - 0 = 2aL$

or $v = \sqrt{2aL} = \sqrt{2g\sin\theta L} = \sqrt{2gh}$

So the correct choice is (c).

$\because h = L \sin\theta$. The time taken by the block to reach the bottom is given by $v = u + at$ or $v = 0 + at$. Thus

$$t = \frac{v}{a} = \frac{\sqrt{2gh}}{g\sin\theta} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$$

15. Refer to Fig. 5.72. The correct choice is (c).

16. The component of weight Mg of the block along the inclined plane = $Mg \sin\theta$. The minimum frictional force to be overcome is also $Mg \sin\theta$. To make the block just move up the plane the minimum force applied must overcome the component $Mg \sin\theta$ of gravitational force as well as the frictional force $Mg \sin\theta = 2\ Mg \sin\theta$. Hence the correct choice is (d).

17. Crosssectional area of tube (A) = πr^2. Since the speed of the liquid is v, the volume of liquid flowing out per second = $Av = \pi r^2 v$. Mass of liquid flowing out per second = $\pi r^2 v\rho$. Therefore,

Initial momentum of liquid per second

= mass of liquid flowing per second × speed of liquid

$= \pi r^2 \rho v^2$

This is the rate at which momentum is imparted to wall on impact. Since the liquid does not rebound after impact, the momentum after impact is zero. Hence, the rate of change of momentum = $\pi r^2 \rho v^2$. From Newton's second law, the force exerted on the wall = rate of change of momentum = $\pi r^2 \rho v^2$. Hence, the correct choice is (d).

18. Refer to Example 5.5 on page 5.3. The correct choice is (c).

19. The slope of the graph between $t = 0$ and $t = 2$s is constant and positive. Therefore, the body moves from position $x = 0$ to $x = 2$ m during the time interval from $t = 0$ to 2s. Between $t = 2$s and $t = 4$s, the slope of the graph is constant but negative. This implies that at $t = 2$s, the velocity of the body is reversed and it retraces its path and returns to $x = 0$ at $t = 4$s; and so on. Thus, the body receives impulses at $t = 0$, 2s, 4s, ..., etc. Therefore, the interval between two consecutive impulses is 2s. Hence, the correct choice is (a).

20. Between $t = 0$ and $t = 2$s, the speed of the body is v = slope of the (x–t) graph between $t = 0$ and $t = 2$s, i.e.

$$v = \frac{(2-0)\text{m}}{(2-0)\text{s}} = 1\ \text{ms}^{-1}$$

At $t = 2$s, the velocity of the body is reversed and it moves in the opposite direction with a speed = $-1\ \text{ms}^{-1}$. Therefore,

Impulse = change in momentum

$= mv - (-mv) = 2\ mv$

$= 2 \times 0.4\ \text{kg} \times 1\ \text{ms}^{-1}$

$= 0.8\ \text{kg ms}^{-1} = 0.8\ \text{Ns}$

Hence, the correct choice is (c).

21. The required force is (i) to accelerate the plane from rest to a speed v over a distance s and (ii) to overcome the force of friction (= $\mu R = \mu Mg$). The acceleration a required to impart a speed v in a distance s is given by $v^2 - u^2 = 2as$. Since, $u = 0$, we have $v^2 = 2\ as$ or $a = v^2/2s$. The force needed to produce this acceleration is

F_1 = mass × acceleration

$$= \frac{Mv^2}{2s}$$

The force needed to overcome the force of friction is

$$F_2 = \mu Mg$$

$\therefore$ Total force needed = $F_1 + F_2 = M\left(\frac{v^2}{2s} + \mu g\right)$

Hence, the correct choice is (a).

22. Refer to Fig. 5.73. Since the block is projected upwards, it rises after overcoming two forces: (i) the component $mg \sin\theta$ of the weight mg and (ii) the force of friction $F = mg \sin\theta$, both acting downwards. Therefore, the total downward acceleration is

$a = -g \sin\theta - g \sin\theta$

$= -2g \sin\theta$

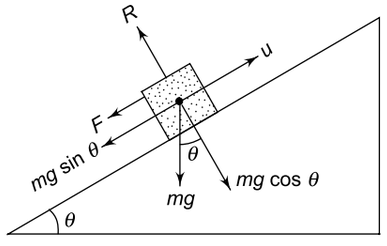


Fig. 5.73

Let s be the distance moved up the plane before the block comes to rest. Then, from $v^2 - u^2 = 2as$, we have $0 - u^2 = 2 \times (-2g \sin\theta) \times s$ or

$$s = \frac{u^2}{4g\sin\theta}$$

Hence, the correct choice is (c).

23. Since the man remains stationary relative to the belt, when the belt is given an acceleration $a = 1.0\ \text{ms}^{-2}$, the acceleration of the man = $a = 1.0\ \text{ms}^{-2}$. The downward force mg exerted by him on the belt is balanced by an equal upward reaction of the belt on him. Therefore, the only force acting on the man is due to his acceleration a. This force is given by

$$F = ma = 60 \times 1.0 = 60\ \text{N}$$

Thus, the correct choice is (c).

24. We know that $\mu_s = \dfrac{\text{limiting friction}}{\text{normal reaction}} = \dfrac{f}{mg}$

Since $\mu_s = 0.2$, the force of limiting friction between the man's shoes and belt is $f = \mu_s \times mg = 0.2 \times 60 \times 10 = 120\ \text{N}$

The man will continue to remain stationary relative to the belt and will just start slipping on the belt if the acceleration of the belt (which equals his acceleration) is increased to a critical value a_c such that the force ma_c acting on him happens to equal the force of limiting friction, f, i.e. if,

$$ma_c = f \text{ or } a_c = \frac{f}{m} = \frac{120}{60} = 2\ \text{ms}^{-2}$$

Hence, the correct choice is (b).

25. Since the block is placed on the trolley, the acceleration of the block = acceleration of the trolley = $a = 3\ \text{ms}^{-2}$. Therefore, the force acting on the block is $F = ma = 10 \times 3 = 30\ \text{N}$

The weight mg of the block is balanced by the normal reaction R. As the trolley accelerates in the forward direction, it exerts a reaction force $F = 30$ N on the block in the backward direction, as shown in the figure. The force of friction will oppose this force and will act in a direction opposite to that of F. The force of limiting friction f is given by

$$\mu = \frac{f}{R} = \frac{f}{mg}$$

or $$f = \mu mg = 0.2 \times 10 \times 10$$
$$= 20\ \text{N}$$

Thus, the block is acted upon by two forces—force $F = 30$ N towards the right and frictional force $f = 20$ N towards the left (see Fig. 5.54). The net force on the block towards the right, i.e. towards the rear end of the trolley is $F' = F - f = 30 - 20 = 10$ N

Due to this force, the block experiences an acceleration towards the rear end which is given by

$$a' = \frac{F'}{m} = \frac{10}{10} = 1\ \text{ms}^{-2}$$

Let t be the time taken for the block to fall from the rear end of the trolley. Clearly, the block has to travel a distance $s = 5$ m to fall off the trolley. Since the trolley starts from rest, initial velocity $u = 0$. Now t can be obtained from the relation

$$s = ut + \frac{1}{2}\ at^2$$

Putting $s = 5$ m, $u = 0$ and $a = a' = 1\ \text{ms}^{-2}$, we get $t = \sqrt{10}$ s.

The distance covered by the trolley in time $t = \sqrt{10}$ s is ($\because u = 0$)

$$s' = ut + \frac{1}{2}\ at^2$$
$$= 0 + \frac{1}{2} \times 3 \times 10 = 15\ \text{m}$$

26. Weight of mass $m_2 = 6 \times 10 = 60$ N. The weight of m_2 provides the tension. Thus $T = 60$ N

Opposing this force along the plane is the component $F_1 = m_1 g \sin\theta$ of the force $m_1 g$. Now $F_1 = m_1 g \sin\theta = 5 \times 10 \times \sin 30° = 25$ N. Since F_1 is less than T and is, therefore, insufficient to balance T (see Fig. 5.74), the force of friction (F) down the plane is necessary to keep block m_1 at rest. Thus, f must act down the plane. Since mass m_1 is at rest, the net force on m_1 along the plane must be zero.

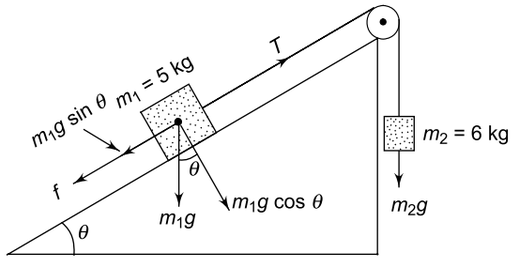


Fig. 5.74

Thus $T - m_1 g \sin 30° - f = 0$

or $f = T - m_1 g \sin 30°$

$= 60 - 5 \times 10 \times \sin 30°$

$= 60 - 25 = 35$ N

Hence, the correct choice is (b).

27. The square of the time of slide is inversely proportional to the acceleration. The accelerations in the two cases are

$a_1 = g \sin 45° = \dfrac{g}{\sqrt{2}}$ and

$a_2 = (g \sin 45° - \mu_k g \cos 45°)$

$= \dfrac{g}{\sqrt{2}}(1 - \mu_k)$

$$\therefore \quad \frac{t_2^2}{t_1^2} = n^2 = \frac{a_1}{a_2} = \frac{1}{1-\mu_k} \quad \text{or} \quad \mu_k = 1 - \frac{1}{n^2}.$$

Hence, the correct choice is (b).

28. Refer to Fig. 5.75. The net downward force on the body at a distance x is

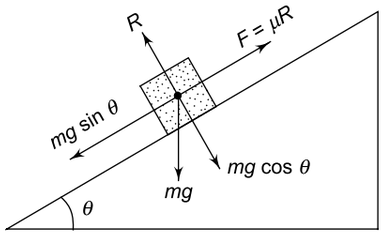


Fig. 5.75

$$f(x) = mg \sin\theta - \mu mg \cos\theta$$
$$= mg(\sin\theta - \mu\cos\theta)$$
$$= mg(\sin\theta - kx\cos\theta)$$

$\therefore f(x) = 0$ at a value of $x = x_0$ given by

$$\sin\theta - kx_0\cos\theta = 0$$

which gives $x_0 = \dfrac{\tan\theta}{k}$

Thus, the correct choice is (a).

29. The acceleration of the body down the plane is $g\sin\theta - \mu g\cos\theta = g(\sin\theta - \mu\cos\theta) = g(\sin\theta - 0.5x\cos\theta)$. Therefore, the body will first accelerate up to $x < 2\tan\theta$. The velocity will be maximum at $x = 2\tan\theta$, because for $x > 2\tan\theta$, the body starts decelerating. Hence, the correct choice is (a).

30. Refer to Fig. 5.76. Given $AB = 1$, $AC = l$, so that $BC = \sqrt{l^2 - 1}$. Thus $\tan\theta = AB/BC = 1/\sqrt{l^2-1}$. A horizontal acceleration a imparted to the inclined plane has a component $a\cos\theta$ down the plane. If this equals the component $g\sin\theta$ of the g along the plane, the object will appear stationary relative to the incline, i.e. if

$$a\cos\theta = g\sin\theta$$

or $$a = g\tan\theta = \frac{g}{\sqrt{l^2-1}}$$

Hence, the correct choice is (c).

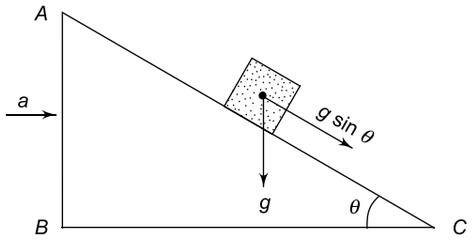


Fig. 5.76

31. Refer to Fig. 5.77. Let A be the position of the insect when it has reached the maximum height h. Now $OA = OC = R$. The insect will crawl up the bowl until the component $mg\sin\theta$ of his weight down the plane equals the force $F = \mu mg\cos\theta$ of limiting friction (the insect will slip down if $mg\sin\theta$ exceeds $\mu mg\cos\theta$). Thus $mg\sin\theta = \mu mg\cos\theta$ or $\tan\theta = \mu = 1/3$. Now in triangle OAB, $\tan\left(\dfrac{\pi}{2} - \theta\right) = \dfrac{OB}{AB}$. Let $OB = x$, then $AB = \sqrt{R^2 - x^2}$.

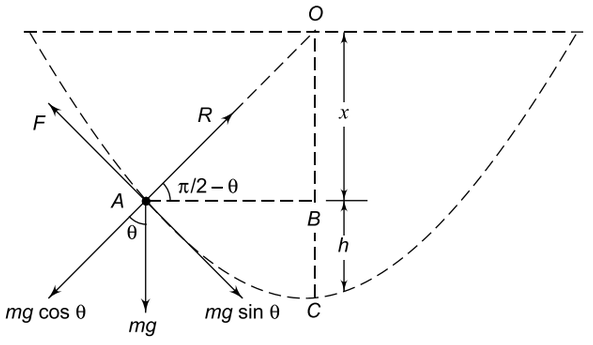


Fig. 5.77

Thus $$\cot\theta = \frac{x}{\sqrt{R^2 - x^2}}$$

or $$3 = \frac{x}{\sqrt{R^2 - x^2}} \qquad \left(\because \tan\theta = \frac{1}{3}\right)$$

which gives $x = \dfrac{3R}{\sqrt{10}} = 0.95\,R$. Therefore, $h = R - x = R - 0.95\,R = 0.05\,R$. Hence, the correct choice is (c).

32. When the balls are released, the force experienced by mass m is F = its mass × its acceleration = ma. This is the force exerted by mass M on mass m. From Newton's third law, the mass m will exert an equal force F on mass M. Thus, force on $m_2 = F = ma$. Therefore, the acceleration of M is

$$a' = \frac{F}{M} = \frac{ma}{M}$$

Hence the correct choice is (c).

33. Force acting on mass m is $f = ma$. Mass m will pull mass M to the left with a force $f = ma$.
Hence, the net acting on mass
$M = F - f = F - ma$. Therefore, acceleration of mass $M = \dfrac{F - ma}{M}$. Hence the correct choice is (a).

34. Let a be the minimum acceleration with which the boy must climb down the rope. Then $mg - T = ma$ or $T = mg - ma$ is the maximum tension. Now,

$$T = \frac{2}{3}\, mg. \text{ Therefore,}$$

$$\frac{2}{3}\, mg = mg - ma$$

which gives $a = g/3$. Hence the correct choice is (a).

35. Let h be the height of each inclined plane. Then, the distances along the plane are $s_1 = \dfrac{h}{\sin\theta_1}$ and $s_2 = \dfrac{h}{\sin\theta_2}$ respectively. The accelerations of the block are $a_1 = g \sin \theta_1$ and $a_2 = g \sin \theta_2$ respectively. Now, since the block is released from rest, the velocity of the block when it reaches the bottom of the planes is $v_1^2 = 2a_1 s_1$ and $v_2^2 = 2a_2 s_2$ respectively. But $v_1 = a_1 t_1$ and $v_2 = a_2 t_2$ or $a_1^2 t_1^1 = 2a_1 s_1$ and $a_2^2 t_2^2 = 2a_2 s_2$. These equations give

$$\frac{t_2^2}{t_1^2} = \frac{a_1}{a_2}\cdot\frac{s_2}{s_1}$$

$$= \frac{g\sin\theta_1}{g\sin\theta_2}\cdot\frac{h}{\sin\theta_2}\cdot\frac{\sin\theta_1}{h}$$

$$= \frac{\sin^2\theta_1}{\sin^2\theta_2}$$

Hence, the correct choice is (c).

36. Let m be the mass of the rope and l its length. The tension T at a distance x from the support = weight of length $(l - x)$ of the rope $= \dfrac{mg}{l} \times (l - x)$ or

$$T = \frac{6\times 10}{3}\times(3-1) = 40 \text{ N}$$

Hence the correct choice is (c).

37. The force of friction between block m and block $M = \mu_1 mg$, where μ_1 is the coefficient of friction between the two blocks. Now, the force of friction between block M (with block m on top of it) and the horizontal surface $= \mu_2 (M + m)g$, where μ_2 is the coefficient of friction between block M and the surface. The maximum force F applied to block M must be enough to overcome this force of friction and the force due to acceleration of the system. If the acceleration of the system is a then this force $= (M + m)a$. Thus

$$F = (M + m)a + \mu_2 (M + m)g \qquad \text{(i)}$$

Now, since the force on block m is $\mu_1 mg$, its acceleration is

$$a = \frac{\text{force on mass } m}{\text{mass } m}$$

$$= \frac{\mu_1 mg}{m} = \mu_1 g \qquad \text{(ii)}$$

Using (ii) in (i) we get

$$F = \mu_1 (M + m)g + \mu_2 (M + m)g$$

$$= (\mu_1 + \mu_2)(M + m)\, g$$

$$= (0.5 + 0.7) \times (5 + 3) \times 10$$

$$= 96 \text{ N}$$

Hence the correct choice is (d).

38 . When the masses are accelerating, there is a tension in the string. When a mass m is added to m_1 such that the acceleration is zero, the system of masses $(m_1 + m)$ will slide on the surface with a uniform speed and then there is no tension in the string. This will happen if the downward force $m_2 g$ equals the force of friction $\mu(m_1 + m)g$ on blocks m_1 and m, i.e. if $\mu(m_1 + m)g = m_2 g$ or $m = \dfrac{m_2}{\mu} - m_1 = \dfrac{6}{0.4} - 4$ = 11 kg. Hence the correct choice is (c).

39. Given $m = 5$ kg, $\mu_s = 0.7$ and $\mu_k = 0.5$. The force applied to the block sufficient to move it = force of static friction, i.e. $F = \mu_s mg = 0.7 \times 5 \times 10 = 35$ N. Force responsible for producing acceleration of the block is

$f =$ applied force – force of dynamic friction

$$= F - m_k mg = 35 - 0.5 \times 5 \times 10 = 35 - 25$$

$$= 10 \text{ N}$$

$\therefore$ Acceleration $a = \dfrac{f}{m} = \dfrac{10}{5} = 2 \text{ ms}^{-2}$

Hence the correct choice is (b).

40. Let the initial speed be u. Final speed $v = 0$ in both cases. The retardation for $\theta_1 = 30°$ is $a_1 = g \sin \theta_1$ and for $\theta_2 = 60°$ is $a_2 = g \sin \theta_2$. Now, using $v^2 - u^2 = 2ax$, we have $u^2 = 2a_1 x_1 = 2a_2 x_2$

Thus $$\frac{x_1}{x_2} = \frac{a_2}{a_1}$$

$$= \frac{g\sin\theta_2}{g\sin\theta_1} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

Hence the correct choice is (c).

41. Refer to Fig. 5.78. The component of acceleration vector **a** along the plane is $a\cos\theta$. The component of accele ration due to gravity **g** along the plane is $g \sin \theta$. The block will stay at rest if $a \cos \theta = g \sin \theta$ or $a = g \tan \theta$

Now $\theta = 30°$. Therefore, $a = g \tan 30° = \dfrac{g}{\sqrt{3}}$. Hence the correct choice is (d).

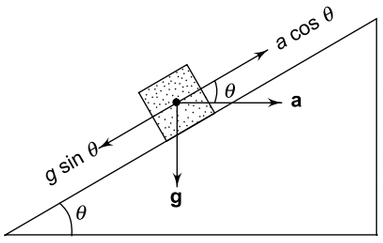


Fig. 5.78

42. The force F_1 required to prevent the block from sliding down is

$$F_1 = mg \sin\theta - \mu\, mg \cos\theta \qquad \text{(i)}$$

The force F_2 required to make the block move up the plane is

$$F_2 = mg \sin\theta + \mu\, mg \cos\theta \qquad \text{(ii)}$$

From Eqs. (i) and (ii) we get

$$F_2 + F_1 = 2\, mg \sin\theta$$

and $$F_2 - F_1 = 2\, \mu mg \cos\theta$$

Dividing the two equations, we get

$$\frac{F_2 + F_1}{F_2 - F_1} = \frac{\tan\theta}{\mu} = \frac{\tan 30^\circ}{1/2\sqrt{3}} = 2$$

or $F_2 = 3\, F_1$. Hence the correct choice is (c).

43. Let m be the mass per unit length of the chain and suppose a length l of the chain hangs from the edge of the table. If L is the total length of the chain, then a length $(L - l)$ of the chain remains on the table. Now, mass of length $l = ml$ and that of length $(L - l) = m(L - l)$. It is clear that the downward force $= mlg$ of the hanging part of the chain balances with the frictional force $m(L - l)\mu g$ of the part of the chain left on the table. Thus

$$mlg = m(L - l)\,\mu g$$

or $$l = \frac{\mu L}{(\mu + 1)} = \frac{0.5 \times 120}{(0.5 + 1)}$$

$$= 40 \text{ cm}$$

Hence the correct choice is (b).

44. We have, $h = \frac{1}{2}\, gt^2$. When the lift is rising up with an acceleration a, the effective acceleration is $g' = g + a$ and t' is given by $h = \frac{1}{2}\, g't'^2$. Thus

$$\frac{1}{2}\, g't'^2 = \frac{1}{2}\, gt^2 \text{ or } (g + a)t'^2 = gt^2$$

or $$t' = t\left(1 + \frac{a}{g}\right)^{1/2}$$ which is choice (c).

45. Let M be the mass of the block and m that of the rope. The acceleration of the block-rope system is

$$a = \frac{F}{(M + m)}$$

Therefore, the tension at the middle point of the rope will be

$$T = \left(M + \frac{m}{2}\right)a$$

$$= \frac{\left(M + \frac{m}{2}\right)F}{(M + m)}$$

Given, $m = \frac{M}{2}$. Therefore, $T = \frac{5F}{6}$. Hence the correct choice is (d).

46. The acceleration $a = \frac{F}{m}$. The resultant of F_1 and F_2 has magnitude F' given by (see Fig. 5.79).

$$F' = (F_1^{\,2} + F_2^{\,2})^{1/2} = \sqrt{2}\, F \; (\because F_1 = F_2 = F)$$

The direction $\mathbf{F}'$ is opposite to that of $\mathbf{F}_3$.

$\therefore$ Net force on body $= F' - F_3$

$$= \sqrt{2}\; F - F$$

$$= (\sqrt{2}\; - 1)\, F$$

$\therefore$ Acceleration $= (\sqrt{2}\; - 1)\, \frac{F}{m} = (\sqrt{2}\; - 1)a.$

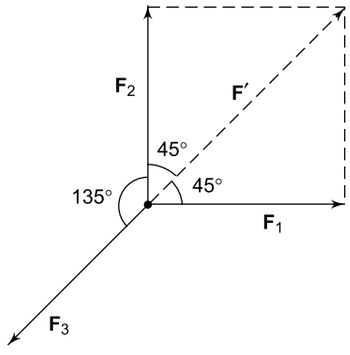


Fig. 5.79

Hence the correct choice is (a).

47. For an elastic spring, the relation between force F and extension x is

$$F = kx$$

where k is the force constant of the spring. Let l_0 be the original length of the spring, then $F = k(l - l_0)$ where l is the spring length when stretched by a force F. We are given that

$$2 = k(l_1 - l_0) \qquad \text{(i)}$$

and $$3 = k(l_2 - l_0) \qquad \text{(ii)}$$

Dividing (ii) by (i) we have

$$\frac{3}{2}=\frac{l_2-l_0}{l_1-l_0}$$

Which gives $l_0 = 3l_1 - 2l_2$. Using this value of l_0 in either (i) or (ii) we get $k = \dfrac{1}{l_2 - l_1}$.

When a stretching force of 5 N is applied, let l_3 be the length of the spring. Then

$$5 = k(l_3 - l_0)$$

Substituting the values of l_0 and k, and solving we get

$$l_3 = 3l_2 - 2l_1$$

Hence the correct choice is (c).

48. If the springs A and B are massless or their mass is negligible compared to the mass with which they are loaded, the tension is the same everywhere on the spring. Hence each spring balance will read 4 kg. Thus the correct choice is (c).

49. Let T be the tension in string C and T' in string B. The y-component $T\cos\theta$ balance with the weight Mg and the x-component $T\sin\theta$ balances with tension T'. Thus (see Fig. 5.80)

$$T' = T\sin\theta$$

and $$Mg = T\cos\theta$$

Dividing the two we get

$$T' = Mg\tan\theta$$
$$= 100\,g\tan 45°$$
$$= 100\,g \text{ newton}$$

Hence the correct choice is (a).

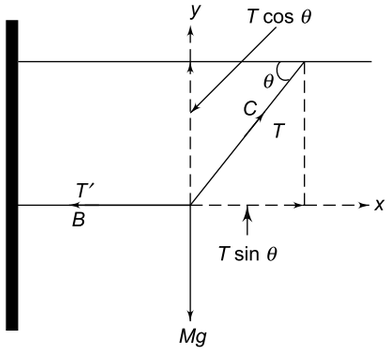


Fig. 5.80

50. Now $F = 600 - 2\times10^5\,t$. F will be zero at time t given by

$$600 - 2\times10^5\,t = 0 \text{ or } t = 3\times10^{-3}\text{ s}$$

Therefore, impulse $= \int_0^t F\,dt$

$$= \int_0^t (600 - 2\times10^5\,t)dt$$

$$= \left|(600t - 10^5\,t^2\right|_0^{t=3\times10^{-3}\text{ s}}$$
$$= 600\times3\times10^{-3} - 10^5\times(3\times10^{-3})^2$$
$$= 1.8 - 0.9 = 0.9 \text{ Ns}$$

Hence the correct choice is (c).

51. The radius of the circular motion of the bead is $r = L$. The linear acceleration of the bead is $a = \alpha r = \alpha L$. If m is the mass of the bead, then

Force acting on the bead $= ma = m\,\alpha\,L$

$\therefore$ Reaction force acting on the bead is $R = m\,\alpha\,L$

The bead starts slipping when frictional force between the bead and the rod becomes equal to centrifugal force acting on the bead, i.e.

$$\mu R = \frac{mv^2}{r}$$

or $\mu\, m\,\alpha\, L = mr\omega^2 = mL\omega^2$ $(\because v = r\omega)$

or $\mu\alpha = \omega^2 = (\alpha t)^2$ $(\because \omega = \alpha t)$

or $\mu\alpha = (\alpha t)^2$ or $t = \sqrt{\dfrac{\mu}{\alpha}}$

52. The magnitude of the force is

$$F = \sqrt{\mathbf{F}\cdot\mathbf{F}} = [(6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}})\cdot(6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}})]^{1/2}$$
$$= \{(6)^2 + (8)^2 + (10)^2]^{1/2} = (200)^{1/2}$$
$$= 10\sqrt{2}\text{ N}$$

$\therefore$ Mass $= \dfrac{F}{a} = \dfrac{10\sqrt{2}\text{ N}}{1\text{ ms}^{-2}} = 10\sqrt{2}$ kg. Hence the correct choice is (b).

53. Refer to Fig. 5.81. Since the block moves with a constant velocity, no net force acts on it. Therefore, the horizontal component $F\cos\theta$ of force F must balance with the frictional force, i.e. $f_r = F\cos\theta$.

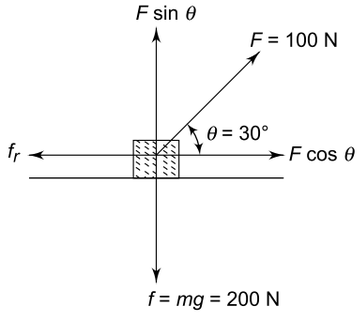


Fig. 5.81

Also $$f_r = \mu\,(mg - F\sin\theta)$$
$$= \mu\,(f - F\sin\theta)$$

$\therefore$ $\mu\,(f - F\sin\theta) = F\cos\theta$

or $\mu\,(200 - 100\sin 30°) = 100\cos 30°$

or $\mu\left(200-100\times\frac{1}{2}\right) = 100 \times 0.866$

$= 86.6$

or $\mu = \frac{86.6}{150} = 0.58,$

which is choice (b).

54. Change of momentum of one bullet = $m\,(v-u)$

$= 0.03 \times \{50-(-30)\}$

$= 2.4 \text{ kg ms}^{-1}$

Average force = rate of change of momentum of 200 bullets

$= 200 \times 2.4 = 480$ N, which is choice (d).

55. Let the body leave the surface at point B as shown in Fig. 5.82. When the body is between points A and B, we have

$$Mg\cos\theta - N = \frac{Mv^2}{r}$$

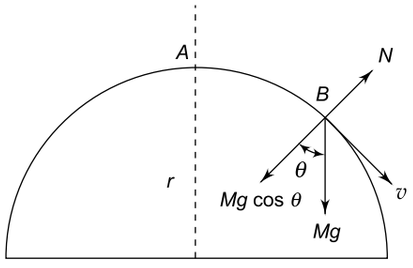


Fig. 5.82

When the body leaves the surface at point B, the normal reaction N becomes zero. Thus

$$Mg\cos\theta = \frac{Mv^2}{r}$$

or $\cos\theta = \frac{v^2}{rg} = \frac{(5)^2}{5\times10}$

$= \frac{1}{2}$ or $\theta = 60°$

Hence the correct choice is (c).

56. Mass of each piece (m) = 1 kg. Initial momentum = 0. Final momentum = $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$. From the principle of conservation of momentum, we have

$$\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$$

or $\mathbf{p}_3 = -(\mathbf{p}_1 + \mathbf{p}_2)$

$= -(mv_1 + mv_2)$

$= -m\,(v_1 + v_2)$

$= -1 \text{ kg} \times (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \text{ ms}^{-1}$

$= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \text{ kg ms}^{-1}$

Force $\mathbf{F} = \frac{\mathbf{p}_3}{t}$

$= \frac{-(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \text{ kgms}^{-1}}{10^{-5} \text{ s}}$

$= -(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times 10^5$ newton

Hence the correct choice is (b).

57. Magnitude of recoil momentum of the gun = forward momentum of the bullet = $m_b v_b = (40 \times 10^{-3} \text{ kg}) \times 1200 \text{ ms}^{-1} = 48 \text{ kg ms}^{-1}$

If the man can fire a maximum of n bullets per second, the maximum force exerted by him on the gun is

$F_{max} = n \times$ force exerted per bullet

$= n \times \frac{\text{change in momentum}}{\text{time}}$

$= \frac{n \times 48 \text{ kg ms}^{-1}}{1 \text{ s}}$

$= 48\,n$ newton

Given $F_{max} = 144$ N. Thus $144 = 48\,n$ which gives $n = 3$. Hence the correct choice is (d).

58. Force

$F = (\mu_s - \mu_k)\,mg$

$= (0.8 - 0.6) \times 4 \times 10 = 8$ N

$\therefore$ Acceleration $= \frac{F}{m} = \frac{8 \text{ N}}{4 \text{ kg}} = 2\text{ms}^{-2}$, which is choice (a).

59. No net force acts on the block as it moves at a constant velocity. Therefore, downward force = upward force or $mg \sin\theta = \mu\, mg \cos\theta$ or $\mu = \tan\theta$, which is choice (c).

60. When a cylinder rolls up or down an inclined plane, its angular acceleration is always directed down the plane. Hence the frictional force acts up the inclined plane when the cylinder rolls up or down the plane. Thus, the correct choice is (b).

61. Linear momentum of the system at time $t = 0$ is $\vec{p}_1 = (m_1\vec{v}_1 + m_2\vec{v}_2)$ and at time $t = 2t_0$ it is $\vec{p}_2 = (m_1\vec{v}_1' + m_2\vec{v}_2')$. Change in linear momentum in time $2t_0 = \vec{p}_2 - \vec{p}_1$. The rate of change of linear momentum is $\vec{p}_2 - \vec{p}_1/2t_0$. From Newton's second law of motion, the rate of change of momentum equals the force acting on the two particles, which is $(m_1\vec{g} + m_2\vec{g})$. Hence

$$\frac{(\vec{p}_2 - \vec{p}_1)}{2t_0} = (m_1 + m_2)\,\vec{g}$$

or $(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)$

$= (m_1 + m_2)\,\vec{g}\,(2t_0)$

Hence the correct choice is (c).

62. The horizontal component of F parallel to the surface is $F \sin \theta$. Hence maximum value of F is given by

$$F \sin \theta = \mu mg$$

or $$F \sin 60° = 0.5 \times \sqrt{3} \times 10$$

or $$F \frac{\sqrt{3}}{2} = 0.5 \times \sqrt{3} \times 10$$

which gives $F = 10$ N. Hence the correct choice is (b).

63. As shown in Fig. 5.83, the insect will crawl without slipping if the value of α is not greater than that given by the condition: force of friction $f = mg \sin \alpha$. Now $f = \mu N$, where N is the normal reaction. Thus

$$\mu N = mg \sin \alpha$$

or $$\mu\, mg \cos \alpha = mg \sin \alpha \quad \text{or}$$

$$\cot \alpha = \frac{1}{\mu} = 3, \text{ which is choice (a).}$$

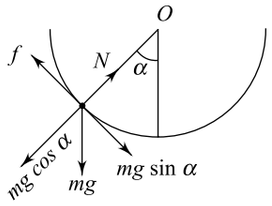


Fig. 5.83

64. Let T be the tension in the string. When the system is in equilibrium, then for the two equal masses m, we have

$$T = mg \qquad (1)$$

and for the mass $\sqrt{2}\, m$, we have

$$2T \cos \theta = \sqrt{2}\ mg \qquad (2)$$

Dividing (2) by (1), we get $\cos \theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$, which is choice (c).

65. The force F on the pulley by the clamp is given by the resultant of two forces: tension $T = Mg$ acting horizontally and a force $(m + M)g$ acting vertically downwards. Thus

$$F = \sqrt{(Mg)^2 + \{(m + M)\, g\}^2}$$
$$= [M^2 + (m + M)^2]^{1/2}\, g$$

which is choice (d).

66. Before the boy starts walking on the plank, both the boy and the plank are at rest. Therefore, the total momentum of the boy–plank system is zero. If the boy walks with a speed v on the plank and as a result if the speed of the plank in the opposite direction is V, then the total momentum of the system is $mv - (M + m)V$. From the principle of conservation of momentum, we have

$$mv - (M + m)V = 0$$

or $$\frac{V}{v} = \frac{m}{(M + m)}$$

Since the distance moved is proportional to speed, the displacement L' of the plank is given by

$$\frac{L'}{L} = \frac{V}{v} = \frac{m}{(M + m)}$$

or $$L' = \frac{mL}{(M + m)}$$

Hence the correct choice is (c).

67. At the highest point of trajectory, the projectile has only a horizontal velocity which is $u \cos \theta$. After explosion, the fragment falling downwards has no horizontal velocity. If u' is the horizontal velocity of the other fragment, the law of conservation of momentum gives

$$(2m)\, u \cos \theta = m \times 0 + mu'$$

which gives $$u' = 2u \cos \theta$$

Now, the time taken to reach the highest point (as well as the time taken to fall down from this point) is $\frac{u \sin \theta}{g}$. Therefore, the horizontal distance travelled by the other fragment is

$$u \cos \theta \times \frac{u \sin \theta}{g} + 2\, u \cos \theta \times \frac{u \sin \theta}{g}$$

$$= \frac{u^2 \sin 2\theta}{2g} + \frac{u^2 \sin 2\theta}{g} = \frac{3u^2 \sin 2\theta}{2g}$$

Hence the correct choice is (b).

68. The mass of water stream striking against the wall in 1 second $= av\rho$. Hence, the change in its momentum per second is $(av\rho)v - (-av\rho)v = 2a\rho v^2$. The normal component of the rate of change of momentum and, therefore, force is $2a\rho v^2 \cos \theta$. Hence the correct choice is (a).

69. The acceleration of the block sliding down the smooth inclined plane is $a_1 = g \sin \theta$ and down the rough inclined plane is $a_2 = g \sin \theta - \mu g \cos \theta$. Given $t_1 = t$ and $t_2 = 2t$. If the length of the inclined plane is s, we have

$$s = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_2 t_2^2$$

or $$a_1 t_1^2 = a_2 t_2^2$$

or $$g \sin \theta \times t^2 = (g \sin \theta - \mu g \cos \theta) \times (2t)^2$$

or $$\sin \theta = 4\, (\sin \theta - \mu \cos \theta)$$

which gives $\mu = \frac{3}{4} \tan \theta = \frac{3}{4}$ $(\because\ \theta = 45°)$

Hence the correct choice is (d).

70. We use the relation $v^2 - u^2 = 2as$. Since $u = 0$, we have $v^2 = 2as$.
Now $v_1^2 = 2a_1 s$ or $v^2 = 2g \sin\theta \times s$ $(\because v_1 = v)$
and $v_2^2 = 2\, a_2 s$ or $\dfrac{v^2}{n^2} = 2(g \sin\theta - \mu g \cos\theta) \times s$
Dividing, we get or $n^2 (\sin\theta - \mu \cos\theta) = \sin\theta$
which gives $\mu = \left(1 - \dfrac{1}{n^2}\right) \tan\theta$, which is choice (a).

71. The force of friction between the block and the belt is $f = \mu mg$, where m is the mass of the object. This force produces an acceleration of the block which is given by

$$a = \frac{\text{force}}{\text{mass}} = \frac{\mu mg}{m} = \mu g$$

The block will slide on the belt without slipping until its speed (v) becomes equal to the speed of the belt. Since $u = 0$, we have

$$v^2 = 2\ as$$

or $s = \dfrac{v^2}{2a} = \dfrac{v^2}{2\mu g} = \dfrac{(5)^2}{2 \times 0.5 \times 10} = 2.5$ m

Hence the correct choice is (b).

72. Normal reaction $R = f$. Therefore, force of friction $= \mu R = \mu f$. The net downward force $F = mg - \mu f$. Hence, the acceleration $a = \dfrac{F}{m} = \dfrac{mg - \mu f}{m} = g - \dfrac{\mu f}{m}$. Hence the correct choice is (d).

73. As the boy is climbing the pole at a constant speed (no acceleration), the force of friction must be just balanced by his weight, i.e. $\mu R = mg$ or $R = \dfrac{mg}{\mu} = \dfrac{40 \times 10}{0.8} = 500$ N. Hence the correct choice is (c).

74. From the principle of conservation of momentum, we have

$$m_1 v_1 = m_2 v_2 \text{ or } \frac{v_1}{v_2} = \frac{m_2}{m_1} \qquad \text{(i)}$$

When the spring is released, it exerts an equal and opposite force F on each block. Let a_1 and a_2 be the accelerations of blocks m_1 and m_2 respectively. Then

$$F = m_1 a_1 = m_2 a_2 \text{ or } \frac{a_2}{a_1} = \frac{m_1}{m_2} \qquad \text{(ii)}$$

Also $v_1^2 = 2a_1 x_1$ and $v_2^2 = 2a_2 x_2$, which give

$$\frac{x_1}{x_2} = \frac{v_1^2}{v_2^2} \cdot \frac{a_2}{a_1} = \left(\frac{m_2}{m_1}\right)^2 \times \left(\frac{m_1}{m_2}\right) = \frac{m_2}{m_1}$$

[Use Eqs. (i) and (ii)]
Hence the correct choice is (b).

75. The forces acting on the balloon are its weight acting downwards and upthrust F acting upwards. Thus

$$F - Mg = Ma \qquad \text{(i)}$$

When mass m is removed, we have

$$F - (M - m)\ g = (M - m)a' \qquad \text{(ii)}$$

where a' is the new acceleration. Eliminating F from (i) and (ii) and simplifying we get

$$a' = \frac{Ma + mg}{M - m}$$

which is choice (a).

76. Let m be the mass per unit length of the rope. Let x be the part of the rope on the floor at time t. The weight of this part is

$$F_1 = mgx$$

Now, if a small part dx falls on the floor in time dt, the force exerted by it is

$$F_2 = \text{rate of change of momentum} = \frac{(mdx)v}{dt} = mv^2$$

Now $\dfrac{dx}{dt} = v$, where v is the velocity of that part of the rope at that instant. But $v^2 = 2gx$. Hence $F_2 = mv^2 = m \times (2gx) = 2mgx$. Total force $F = F_1 + F_2 = mgx + 2mgx = 3mgx = 3F_1$

Hence the correct choice is (c).

77. Refer to Fig. 5.84. Vertical component of F is $F \sin\theta$ and the horizontal component is $F \cos\theta$.

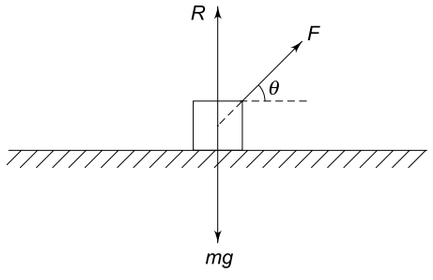


Fig. 5.84

Thus

$$R + F \sin\theta = mg$$

or $$R = mg - F \sin\theta$$

Frictional force $\mu R = \mu\,(mg - F \sin\theta)$. Also

$$\mu\,(mg - F \sin\theta) = F \cos\theta$$

or $$F = \frac{\mu mg}{(\mu \sin\theta + \cos\theta)} \qquad \text{(i)}$$

F will be minimum if the denominator is maximum, i.e. if

$$\frac{d}{d\theta}(\mu \sin\theta + \cos\theta) = 0$$

or $\mu \cos\theta - \sin\theta = 0$

or $\mu = \tan\theta$, which is choice (a).

78. Now $\tan\theta = \mu$. Therefore, $\cos\theta = \dfrac{1}{\sqrt{1+\mu^2}}$ and $\sin\theta = \dfrac{\mu}{\sqrt{1+\mu^2}}$

Using these in Eq. (i) above and simplifying, we get

$$F = \frac{\mu m g}{\sqrt{1+\mu^2}}$$

Hence the correct choice is (d).

79. Force required to accelerate the body of mass m is $F = (\mu_s - \mu_k)\, mg = (0.75 - 0.5)\, mg = 0.25\, mg$

$\therefore$ Acceleration $= \dfrac{F}{m} = \dfrac{0.25\, mg}{m} = 0.25\, g$, which is choice (a).

80. Let a be the acceleration at a time t of the blocks executing SHM. The force on the blocks due to acceleration is

$$F = (m + m)\, a = 2\, ma$$

$$\therefore \quad F_{max} = 2\, m\, a_{max} \qquad (1)$$

Now, the acceleration is maximum when the blocks are at the extreme position of maximum displacement, i.e.

$$F_{max} = kA \qquad (2)$$

Equating (1) and (2), we get

$$a_{max} = \frac{kA}{2m}$$

$\therefore$ Maximum force of friction $= ma_{max}$

$$= m \times \frac{kA}{2m} = \frac{kA}{2}$$

Hence the correct choice is (b).

81. Since the block is held stationary, it is in translational as well as rotational equilibrium. Hence no net force and no net torque acts on the block. No net force will act on the block if $f = mg$ and $N = F$. No net torque will act on the block, if torque by frictional force f about centre O = counter torque by normal reaction N about centre O. Hence choice (d) is false.

82. Speed of car (v) = 36 km h^{-1} = 10 ms^{-1}. The maximum centripetal force that friction can provide is

$$f_{max} = \mu\, mg = \frac{mv^2}{R}$$

or $$R_{min} = \frac{v^2}{\mu g} = \frac{10 \times 10}{0.8 \times 10} = 12.5 \text{ m}$$

This is the minimum radius the curve must have for the car to negotiate it without sliding at a speed of 10 ms^{-1}. Hence the correct choices is (b).

83. Speed of train (v) = 36 km h^{-1} = 10 ms^{-1}

Radius of the curve (R) = 200 m

Distance between rails (x) = 1.5 m

Let the outer rails be raised by a height h with respect to the inner rails so that the angle of banking is θ (Fig. 5.85).

Then $$\tan\theta = \frac{h}{x} = \frac{v^2}{Rg}$$

or $$h = \frac{xv^2}{Rg} = \frac{1.5 \times (10)^2}{200 \times 10}$$

$$= 0.075 \text{ m} = 7.5 \text{ cm}$$

Thus, the correct choice is (a).

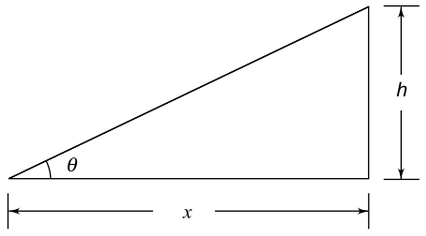


Fig. 5.85

84. Now v = 54 km h^{-1} = 15 ms^{-1}, R = 50 m. The required angle of banking is given by

$$\tan\theta = \frac{v^2}{Rg} = \frac{15 \times 15}{50 \times 10} = 0.45$$

Thus, the correct choice is (d).

85. The horizontal velocity v must be such that the centripetal force equals the weigth of the body, i.e. $\dfrac{mv^2}{R} = mg$ or $v = \sqrt{gR}$, which is choice (b).

86. The motor cyclist can leave the ground only at the highest point on the bridge. At this point, the centripetal force is mv^2/R. He will not leave the ground if the centripetal force equals the weight mg. Thus $\dfrac{mv^2}{R} = mg$ or $v = \sqrt{gR} = \sqrt{10 \times 10} = 10$ ms^{-1}. Hence, the correct choice is (a).

87. When the system is in equilibrium, the spring force = 3 mg. When the string is cut, the net force on block A = 3 mg – 2 mg = mg. Hence the acceleration of this block at this instant is

$$a = \frac{\text{force on block A}}{\text{mass of block A}} = \frac{mg}{2m} = \frac{g}{2}$$

When the string is cut, the block B falls freely with an acceleration equal to g. Hence the correct choice is (c).

88. Force $\vec{F} = \dfrac{d\vec{p}}{dt}$

$$= \frac{d}{dt}[A\{\hat{i}\cos(kt) - \hat{j}\sin(kt)]$$

$$= Ak[-\hat{i}\sin(kt) - \hat{j}\cos(kt)]$$

Now $\vec{F}\cdot\vec{p} = Ak[-\hat{i}\sin(kt) - \hat{j}\cos kt]\cdot$

$$A[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$$

$= A^2k[-\sin(kt)\cos(kt) + \cos(kt)\sin kt]$

$= 0$ ($\because \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = 1$ and $\hat{i}\cdot\hat{j} = 0$)

Hence the angle between $\vec{F}$ and $\vec{p}$ is 90° which is choice (d).

89. Refer to Fig. 5.86. Let f be the force producing the acceleration of each mass. It follows from the figure that

$$F = T\sin\theta + T\sin\theta = 2T\sin\theta$$

$$\Rightarrow \quad T = \frac{F}{2\sin\theta} \qquad (1)$$

Also $T\cos\theta = mf$ (2)

Using (1) and (2), we get

$$f = \frac{F\cos\theta}{2m\sin\theta} = \frac{F}{2m\tan\theta} = \frac{Fx}{2m\sqrt{a^2 - x^2}}$$

Hence the correct choice is (b).

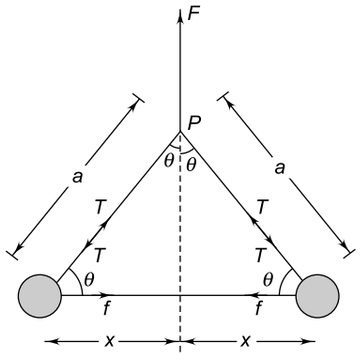


Fig. 5.86

90. The block will just begin to slide if the downward force $mg\sin\theta$ just overcomes the frictional force, i.e. if $mg\sin\theta = \mu N = \mu\, mg\cos\theta \Rightarrow \tan\theta = \mu = \sqrt{3} \Rightarrow \theta = 60°$ (see Fig. 5.87)

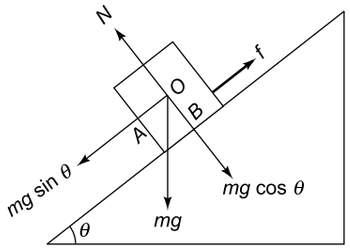


Fig. 5.87

The block will topple if the torque due to normal reaction N about O just exceeds the torque due to $mg\sin\theta$ about O, i.e.

$$N \times OA = mg\sin\theta \times OB$$

$$\Rightarrow mg\cos\theta \times 5\text{ cm} = mg\sin\theta \times \frac{15}{2}\text{ cm}$$

$$\Rightarrow \tan\theta = \frac{2}{3} \Rightarrow \theta \simeq 34°.$$

Since θ for toppling is less than θ for sliding, the correct choice is (b).

91.

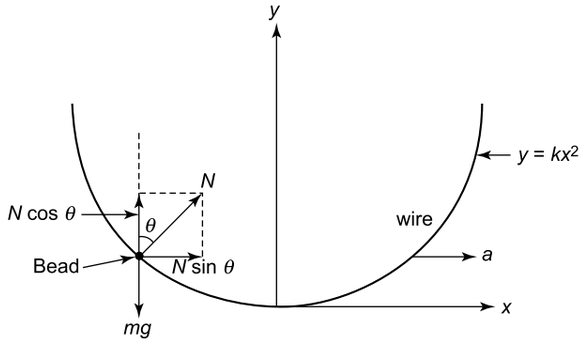


Fig. 5.88

For the bead to stay at rest, (see Fig. 5.88)

$$N\cos\theta = mg$$

$$N\sin\theta = ma$$

which give $\tan\theta = \dfrac{a}{g}$. Now

$$\tan\theta = \text{slope of the curve} = \frac{dy}{dx} = \frac{d}{dx}(kx^2) = 2kx$$

$$\therefore \qquad 2kx = \frac{a}{g} \Rightarrow x = \frac{a}{2gk}$$

92. Radius of the circular path is $BC = r = L\sin\theta$, where $L = AB$ is the length of the string. The vertical component $T\cos\theta$ of tension T balances with the weight mg and the horizontal component $T\sin\theta$ provides the necessary centripetal force for circular motion. Hence

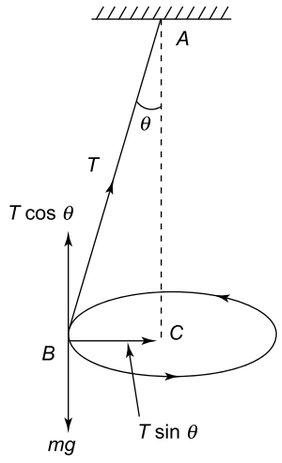


Fig. 5.89

$$T\sin\theta = mr\omega^2 = m(L\sin\theta)\,\omega^2$$

$$\Rightarrow \quad T = mL\omega^2$$

$$\therefore \quad T_{max} = mL\,\omega^2_{max}$$

$$\Rightarrow \quad 324 = 0.5 \times 0.5 \times \omega^2_{max}$$

$$\Rightarrow \quad \omega_{max} = 36 \text{ rad s}^{-1}$$

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Which of the following statements are true? No net force acts on
- (a) a drop of rain falling vertically with a constant speed
- (b) a cork floating on water
- (c) a car moving with a constant velocity on a rough road
- (d) a body moving in a circular path at constant speed.

2. In which of the following situations would a force of 9.8 N act on a stone of mass 1 kg? Neglect air
- (a) Just after it is dropped from the window of stationary train
- (b) Just after it is dropped from the window of a train running at a constant speed of 36 km h^{-1}.
- (c) Just after it is dropped from the window of a train accelerating at 1 ms^{-2}.
- (d) When it is lying at rest on the floor of a train which is accelerating at 1 ms^{-2}.

3. The coefficient of friction between the wheels of a car and the ground is 0.5. The car starts from rest and moves along a perfectly horizontal road. If $g = 10$ ms^{-2}, the car
- (a) can acquire a maximum acceleration of 5 ms^{-2} without slipping.
- (b) can attain a speed of 20 ms^{-1} in a minimum distance of 40 m.
- (c) can go up to a speed of 100 ms^{-1} in 10 s.
- (d) after acquiring a speed of 50 ms^{-1}, can come to rest, with the engine shut off and brakes not applied, in a time of 10 s.

4. A stream of water flowing horizontally with a speed of 15 ms^{-1} gushes out of a tube of cross sectional area 1 cm^2, strikes against a *kachha* vertical wall, and flows down without any re–bound. The wall can withstand a maximum force of 2000 N on it. The impact of the water stream on the wall will
- (a) not damage it as the maximum force on it due to the water stream is only about 1500 N.
- (b) will break it.
- (c) will exert a force of 2250 N on it.
- (d) will exert a pressure of 2.250 10^5 pascal on the wall.

5. A block is released from the top of a smooth inclined plane. Another block of the same mass is allowed to fall freely from the top of the inclined plane. Both blocks reach the bottom of the plane
- (a) in equal time
- (b) with equal speed
- (c) with equal momentum
- (d) with equal kinetic energy

6. A body of mass 200 g is moving with a velocity of 5 ms^{-1} along the posotive x- direction. At time $t = 0$ when the body is at $x = 0$, a constant forse of 0.4 N

directed along the negative x-direction is applied to the body for 10s.

(a) At time $t = 2.5$ s, the body will be at $x = 1.25$ m

(b) At time $t = 2.5$ s, the speed of the body will be zero.

(c) At time $t = 30$ s, the body will return to $x = 0$.

(d) At time $t = 30$ s, the speed of the body will be 15 ms^{-1}

7. A train starts from rest with a constant acceleration $a = 2$ ms^{-2}. After 5 seconds, a stone is dropped from the window of the train. If $g = 10$ ms^{-2}.

(a) the magnitude of the valocity of the stone 0.2 second after it is dropped is 2 ms^{-1}.

(b) the angle between the resultant velocity vector of the stone and the horizontal 0.2 second after it is dropped is $\theta = \tan^{-1}(0.2)$.

(c) the acceleration of the stone after it is dropped is $a = 2$ ms^{-2}.

(d) the acceleration of the stone after it is dropped is $g = 10$ ms^{-2}.

8. A man of mass m is standing on the floor of a lift. His weight when the lift is

(a) stationary is mg

(b) moving up with a uniform speed of 2 ms^{-1} is 5 mg.

(c) moving up with a uniform acceleraiton a ($< g$) is $m(g + a)$.

(d) moving down with a uniform acceleration a ($< g$) is $m(g - a)$.

9. Two identical blocks, each of mass m, connected by a light string, are placed on a rough horizontal surface. When a force F is applied on a block in the horizontal direction, each block moves with an acceleration a. Assuming that the frictional forces on the two blocks are equal,

(a) the tension in the string will be F.

(b) the tension in the string will be $F/2$.

(c) the frictional force on each block will be $(F - ma)$.

(d) the frictional force on each block will be $\left(\frac{F}{2} - ma\right)$.

10. A body of weight W is suspended from a rigid support P by means of a massless string as shown in Fig. 5.90. A horizontal force F is applied at point O of the rope. The system is in equilibrium when the string makes an angle θ with the vertical. If the tension in the string is T,

(a) $F = T \sin \theta$ (b) $W = T \sin \theta$

(c) $T = \sqrt{W^2 + F^2}$ (d) $F = W \tan \theta$

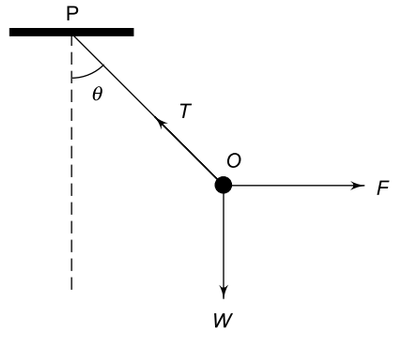


Fig. 5.90

11. Two blocks of masses m_1 and m_2 are connected by a string of negligible mass which passes over a frictionless pulley fixed on the top of an inclined plane as shown in Fig. 5.91.

The coefficient of friction between mass m_1 and the plane is μ.

(a) If $m_1 = m_2$, the mass m_1 first begins to move up the inclined plane when the angle of inclination is θ, then $\mu = \tan \theta$.

(b) If $m_1 = m_2$, the mass m_1 first begins to move up the inclined plane when the angle of inclination is θ, then $\mu = \sec \theta - \tan \theta$.

(c) If $m_1 = 2\ m_2$, the mass m_1 first begins to slide down the plane if $\mu = 2 \tan \theta$.

(d) If $m_1 = 2\ m_2$, the mass m_1 first begins to slide down the plane if $\mu = \tan \theta - \frac{1}{2} \sec \theta$.

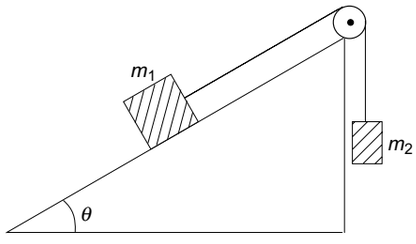


Fig. 5.91

12. Two blocks A and B are connected to each other by a string and a spring of force constant k, as shown in Fig. 5.92. The string passes over a frictionless pulley as shown. The block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C both with the same uniform speed. The coefficient of friction between the surfaces of the blocks is μ. If the mass of block A is m,

(a) the mass of block B is μm.
(b) the mass of block B is m/μ.
(c) the energy stored in the spring is $m^2g^2/2k$.
(d) the energy stored in the spring is $\mu m^2g^2/k$.

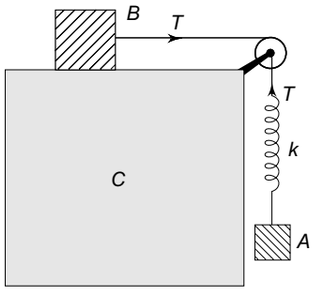


Fig. 5.92

13. Two blocks A and B of equal masses m each are connected to each other by a string passing over a frictionless pulley as shown in Fig. 5.93. The coefficient of friction between block A and the surface below is 0.5. When the system is released,
(a) the acceleration of the blocks is $3g/4$.
(b) the acceleration of the blocks is $g/4$.
(c) the tension in the string is $3mg/4$.
(d) the tension in the string is $mg/4$.

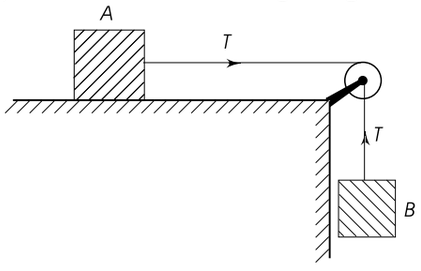


Fig. 5.93

14. A lift is going up. The variation of the speed of the lift with time is shown in Fig. 5.94. The total mass of the lift and passengers is 1000 kg. If $g = 10\ \text{ms}^{-2}$,

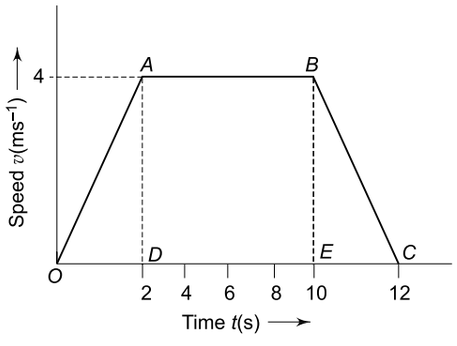


Fig. 5.94

(a) the tension in the rope of the lift at $t = 1$ s will be 12,000 N.
(b) the tension in the rope at $t = 6$ s will be 10,000 N.
(c) the tension in the rope at $t = 11$ s will be 8,000 N.
(d) the height upto which the lift takes the passengers is 40 m.

15. Two blocks of masses $m_1 = 2\ m$ and $m_2 = m$ are connected by a light string passing over a friction less pulley as shown in Fig. 5.95. When they are released,
(a) the acceleration of m_1 is $\frac{g}{3}$ vertically downwards.
(b) the acceleration of m_2 is $\frac{2g}{3}$ vertically upwards.
(c) the tension in the string is $\frac{4g}{3}$.

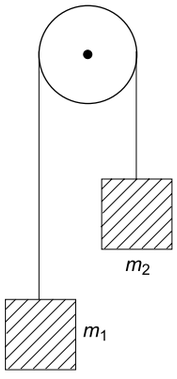


Fig. 5.95

(d) the distance through which m_2 rises in t seconds after the blocks are released is $\frac{gt^2}{6}$.

16. Two blocks of masses m_1 and m_2 connected by a light inextensible string are lying on a horizontal frictionless surface as shown in Fig. 5.96. A force F is applied to m_2 in the horizontal directon as shown.
(a) the acceleration of each block is $\frac{F}{(m_1 + m_2)}$.
(b) the tension in the string is $\frac{m_1F}{(m_1 + m_2)}$.
(c) the tension in the string is F.
(d) the force on mass m_1 is $\frac{m_2F}{(m_1 + m_2)}$.

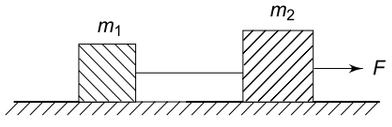


Fig. 5.96

17. Ten coins, each of mass m, are placed on top of each other on a horizontal table.
(a) The force on the 7th coin (counted from the bottom) due to all the coins above it is $3mg$ vertically downwards.
(b) The force on the 7th coin by the 8th coin (both counted from the bottom) is $3mg$ vertically downwards.

(c) The force on the 7th coin by the 8th coin (both counted from the bottom) is $2mg$ vertically upwards.

(d) The reaction force of the 6th coin from the bottom on the 7th coin from the bottom is $4mg$ vertically upwards.

18. Two blocks of masses m_1 and m_2 are placed in contact on a horizontal frictionless surface as shown in Fig. 5.97. A force F is applied to mass m_1 as shown.

(a) The acceleration of mass m_2 is F/m_2

(b) The force exerted on mass m_2 is $\frac{m_2F}{(m_1+m_2)}$.

(c) The force exerted on mass m_2 is F.

(d) The acceleration of mass m_1 is $\frac{F}{(m_1+m_2)}$.

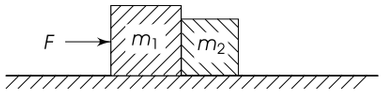


Fig. 5.97

19. Two blocks of masses m_1 and m_2 ($m_2 < m_1$) are placed on an inclined plane of inclination θ and joined by a string as shown in Fig. 5.98.

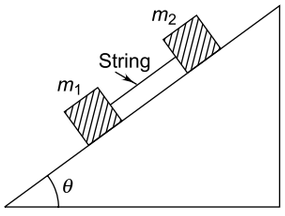


Fig. 5.98

If the coefficient of friction between the blocks and the plane is μ and the blocks are released,

(a) the acceleration of the blocks is $g(\sin\theta - \mu\cos\theta)$.

(b) the acceleration of the blocks is zero.

(c) the tension in the string is zero.

(d) the tension in the string is $(m_1 + m_2)g(\sin\theta - \mu\cos\theta)$.

20. A uniform chain of length L is placed on a rough horizontal table. The coefficient of friction between the chain and the table is μ. The maximum length of the chain that can hang from the edge of the table is l. Then

(a) $l = \frac{\mu L}{(1+\mu)}$

(b) $l = \frac{L}{(1+\mu)}$

(c) If $\mu = 0.25$, $\frac{l}{L} = 20\%$

(d) If $\mu = 0.25$, $\frac{l}{L} = 25\%$

21. A block starts sliding from the top of an inclined plane of inclination θ. The coefficient of friction between the block and the plane varies as $\mu = kx$ where x is the distance moved down the plane and k is a positive constant.

(a) The block has a uniform acceleration along the plane.

(b) The acceleration of the block increases for $x < \frac{\tan\theta}{k}$.

(c) The velocity of the block is maximum at $x = \frac{\tan\theta}{k}$.

(d) The block starts decelerating for $x > \frac{\tan\theta}{k}$.

22. A block of mass m is lying at $x = 0$ on a smooth horizontal surface. A variable force $F = kx$ is applied to it as shown in Fig. 5.99 where k is a constant. Then

(a) the block will move on the surface with a uniform acceleration.

(b) the block will move on the surface with a variable acceleration.

(c) the block will lose contact with the surface after travelling a distance $x_0 = \frac{mg}{k\sin\theta}$.

(d) the block will always remain in contact with the surface.

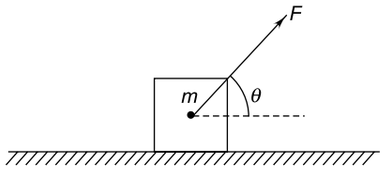


Fig. 5.99

23. A pendulum of length L and bob mass M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement $|\theta| < \phi$, the tension in the string is T and the velocity of the bob is V. Then

(a) $T\cos\theta = Mg$

(b) $T - Mg\cos\theta = \frac{MV^2}{L}$

(c) The magnitude of the tangential acceleration of the bob is $|a_t| = g\sin\theta$

(d) $T = Mg\cos\theta$

IIT, 1986

24. When a bicycle is in motion, the force of friction exerted by the ground on the two wheels is such that it acts
(a) in the backward direction on the front wheel and in the forward direction on the rear wheel.
(b) in the forward direction on the front wheel and in the backward direction on the rear wheel.
(c) in the backward direction on both the front and the rear wheels.
(d) in the forward direction on both the front and the rear wheels.

IIT, 2007

ANSWERS AND SOLUTIONS

1. Choice (a) is true. Since the direction of motion as well as the speed of the drop of rain are constant, its velocity is constant and hence no net force acts on the drop. The downward gravitational force (i.e. weight) of the drop is completely cancelled by the upward buoyant force on it due to air and the viscous force due to its motion. Choice (b) is also true. The weight of the cork is balanced by the upward buoyant force due to water. Choice (c) is also true. The force due to the car's acceleration is balanced by the backward force of friction between its tyres and the rough road. Choice (d) is false. Although the speed of the body is constant, its velocity is changing all the time because the direction of motion keeps changing. Hence, the motion of the body is being accelerated. A force (called centripetal force) must act on the body to produce the acceleration.
Thus, choices (a), (b) and (c) are correct.

2. In choice (a), a force $F = mg = 1 \times 9.8 = 9.8$ N acts on the stone vertically downwards. In choice (b), the velocity of the train is constant. Hence, there is no acceleration (and therefore, force) along the direction of motion. When the stone is dropped, the only force acting on it is $F = mg = 9.8$ N vertically downwards. In choice (c), before the stone is dropped, a force $= ma = 1$ kg $\times$ 1 ms^{-2} = 1 N acts on it in the horizontal direction. But, after it is dropped, this force ceases to act because the stone is no longer located in the accelerating system (i.e. train). Hence, in this case also, the net force on the stone is 9.8 N vertically downwards. In choice (d), the weight of the stone is balanced by the normal reaction of the floor of the train. But the stone is accelerated in the forward direction along with the train. Hence, the force acting on the stone $= ma = 1$ kg $\times$ 1 ms^{-2} = 1 N in the direction of motion of the train. Thus, the correct choices are (a), (b) and (c).

3. The maximum acceleration acquired without slipping is given by

$$ma = \mu mg \text{ or } a = \mu g = 0.5 \times 10 = 5 \text{ ms}^{-2}$$

With this maximum acceleration, the minimum distance s covered to acquire a speed $v = 20$ ms^{-1} is given by $v^2 - u^2 = 2as$ or $(20)^2 - 0 = 2 \times 5 \times s$ or $s = 40$ m. Further, since the frictional retardation is 5 ms^{-2}, the time needed to come to rest from a speed of 50 ms^{-1} is $t = v/a = 50/5 = 10$ s. Hence, choice (a), (b) and (d) are correct.

4. The rate of change of momentum of water stream $= av^2\rho - 0 = av^2\rho$. This is the force F exerted by the water stream on the wall.

$\therefore F = av^2\rho = (1 \times 10^{-2}) \times (15)^2 \times 1000 = 2250$ N

The pressure on the wall is

$$P = \frac{F}{a} = \frac{2250}{1 \times 10^{-2}} = 2.25 \times 10^5 \text{ pascal}$$

Hence, the correct choices are (b), (c) and (d).

5. Since the acceleration along the inclined plane ($g \sin \theta$) is less than g, the blocks take different times to reach the bottom. The speed of each block on reaching the bottom is $v = \sqrt{2gh}$, where h is the height of the inclined plane. Thus choice (b) is correct. The directions of the velocity are different, hence their momenta are not the same. Their kinetic energy $\frac{1}{2} mv^2$ is the same. Hence the correct choices are (b) and (d).

6. Given $u = +5$ ms^{-1} along positive x-direction
$F = -0.4$ N along negative x-direction

$$m = 200\ g = 0.2 \text{ kg}$$

The acceleration $a = \frac{F}{m} = \frac{-0.4}{0.2} = -2$ ms^{-2}. The negative sign shows that the motion is retarded. The position of the body at time t is given by

$$x = x_0 + ut + \frac{1}{2} at^2$$

At $t = 0$, the body is at $x = 0$. Therefore, $x_0 = 0$. Hence

$$x = ut + \frac{1}{2} at^2$$

Since the force acts during the time interval from $t = 0$ to $t = 10$ s, the motion is accelerated only between $t = 0$ and $t = 10$ s. The position of the body $t = 2.5$ s is given by

$$x = 5 \times 2.5 + \frac{1}{2} \times (-2) \times (2.5)^2 = 1.25 \text{ m}$$

The velocity of the body at $t = 2.5$ s is

$$v = u + at = 5 + (-2) \times (2.5) = 5 - 5 = 0$$

During the first ten seconds (i.e. from $t = 0$ to $t = 10$ s) the motion is accelerated. During this time $a = -2$ ms^{-2}. Putting $u = 5$ ms^{-1}, $a = -2$ ms^{-2} and $t = 10$ s in equation $x = ut + \frac{1}{2}at^2$. We have

$$x_1 = 5 \times 10 + \frac{1}{2} \times (-2) \times (10)2 = -50 \text{ m} \quad \text{(i)}$$

The velocity of the body at $t = 10$ s is

$v = u + at = 5 + (-2) \times 10 = -15$ ms^{-1}

During the remaining 20 seconds, i.e. from $t = 10$ s to $t = 30$ s, the acceleration $a = 0$, because the force ceases to act after $t = 10$ s. The velocity of the body remains constant at -15 ms^{-1} during the last 20 seconds. The distance covered by the body during the last 20 seconds is

$$x_2 = -15 \times 20 = -300 \text{ m}$$

$\therefore$ Position of the body at $t = 30$ s is

$$x = x_1 + x_2 = -50 - 300 = -350 \text{ m}$$

The magnitude of the velocity (i.e. speed) of the body at $t = 30$ s is 15 ms^{-1}.

Hence the correct choices are (a), (b) and (c).

7. Given $u = 0$, $a = 2$ ms^{-2}. Since the stone is located in the train, the acceleration of the stone is $a = 2$ ms^{-2}. At time $t = 5$ s, the velocity of the stone is $v = u + at = 0 + 2 \times 5 = 10$ ms^{-1}. Before the stone is dropped, its motion is accelerated with the train. But, the moment it is dropped, its acceleration due to the motion of the train ceases. Therefore, after the stone is dropped, it has the following two motions:

(a) a uniform motion with velocity 10 ms^{-1} parallel to the ground, i.e. $v_x = 10$ ms^{-1} (the horizontal velocity)

(b) an accelerated motion vertically downwards due to gravity. In time $t = 0.2$ s, the vertical velocity of the stone is $v_y = 0 + gt = 10 \times 0.2 = 2$ ms^{-2}.

The resultant velocity of stone at $t = 0.2$ s is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10)^2 + (2)^2} = \sqrt{104} = 2\sqrt{26} \text{ ms}^{-1}$$

The angle, which the resultant velocity vector, makes with the horizontal is given by

$$\tan\theta = \frac{v_y}{v_x} = \frac{2}{10} = 0.2$$

After the stone is dropped, the horizontal velocity v_x remains unchanged because the acceleration is zero along the horizontal direction. The only acceleration of the stone is the acceleration due to gravity.

Hence the correct choices are (b) and (d).

8. The man exerts a downward force (mg) on the weighing machine. The machine, in turn, exerts on him an upward reaction for (R) which it measures. When the lift is stationary, $R = mg$.

When the lift is moving up or down with a uniform speed, it has no acceleration of its own. Hence, the reading of the machine will still be mg.

When the machine is moving downwards with an acceleration a, a force $F = ma$ acts downwards. But the reaction $R = mg$ acts upwards. Hence, the effecting reading will be

$$R_{\text{eff}} = F - R = mg - ma = m(g - a)$$

In case (c), R and mg both act in the same direction (upwards). Hence, the effective reading will be

$$R_{\text{eff}} = mg + ma = m(g + a)$$

Hence the correct choices are (a), (c) and (d).

9. Refer to Sec. 5.11 and Fig. 5.15 on page 5.9

Equations of motion of m_1 and m_2 are

$$m_1 a = T - f \quad \text{(i)}$$

and

$$m^2 a = F - T - f \quad \text{(ii)}$$

Subtracting the two equations, we have

$$(m_1 - m_2)a = 2T - F$$

Since $m_1 = m_2$, we get, we get $0 = 2T - F$

or

$$T = \frac{F}{2}$$

Putting $T = 10$ N in Eq. (i) above, we have

$$f = T - m_1 a = \frac{F}{2} - ma$$

Hence the correct choices are (b) and (d).

10. The mass is in equilibrium at point O under the action of the concurrent forces F, T and $W = mg$. Therefore, as shown in Fig. 5.100, the horizontal component $T \sin\theta$ of tension T must balance with force F and the vertical component $T \cos\theta$ must balance with weight $W = mg$.

Thus

$$F = T \sin\theta \quad \text{(i)}$$

and

$$W = T \cos\theta \quad \text{(ii)}$$

Squaring Eqs. (i) and (ii) and adding we get

$$T^2 = W^2 + F^2.$$

Dividing (i) by (ii) we get $F = W \tan\theta$.

Hence the correct choices are (a), (c) and (d).

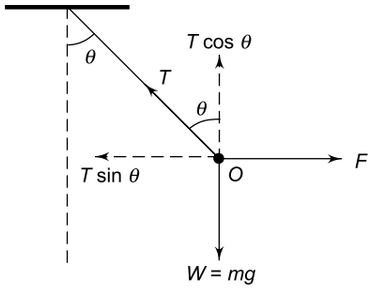


Fig. 5.100

11. The block m_1 will just begin to move up the plane if the downward force m_2g due to mass m_2 trying to pull the mass m_1 up the plane just equals the force ($m_1g \sin\theta + \mu m_1 g \cos\theta$) trying to push the mass m_1 down the plane, i.e. when

$m_2g = m_1g(\sin\theta + \mu\cos\theta)$

Now, it is given that $m_1 = m_2 = m$.

Therefore, we have

$$1 = \sin\theta + \mu\cos\theta$$

$$\Rightarrow \quad \mu = \sec\theta - \tan\theta$$

The block m_1 will just begin to move down the plane if the downward force ($m_1g \sin\theta - \mu m_1 g \cos\theta$) on m_1 just equals the upwards force m_2g acting on m_1 due to m_2, i.e. if

$$m_2g = m_1g(\sin\theta - \mu\cos\theta)$$

or $$\frac{m_1}{m_2} = \frac{1}{\sin\theta - \mu\cos\theta}$$

If $m_1 = 2m_2$, then we have

$$2 = \frac{1}{\sin\theta - \mu\cos\theta}$$

$$\Rightarrow \quad \mu = \tan\theta - \frac{1}{2}\sec\theta$$

Hence the correct choices are (b) and (d).

12. Since the blocks slide at the same uniform speed, no net force acts on them. If M is the mass of block B, then the tension in the string is $T = \mu Mg$. Also $T = mg$. Equating the two, we get $\mu M = m$ or $M = \frac{m}{\mu}$

Extension in the spring $x = \frac{F}{k} = \frac{mg}{k}$. Therefore, potential energy stored in the spring is

$$\text{PE} = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{2k}$$

Hence the correct choices are (b) and (c).

13. If the acceleration of the blocks is a, then we have

$$mg - T = ma \qquad \text{(i)}$$

and $$T - \mu mg = ma \qquad \text{(ii)}$$

Adding (i) and (ii) we get

$$a = \left(\frac{1-\mu}{2}\right)g = \left(\frac{1-0.5}{2}\right)g = \frac{g}{4}$$

Using $a = g/4$ in Eq. (i) gives $T = 3mg/4$

Hence the correct choices are (b) and (c).

14. Between $t = 0$ and $t = 2$ s, the acceleration of the lift is

$$a = \frac{\text{AD}}{\text{OD}} = \frac{4\ \text{ms}^{-1}}{2\,\text{s}} = 2\ \text{ms}^{-2}$$

Since the lift is accelerating upward, the tension in the rope at $t = 1$ s (between $t = 0$ and $t = 2$ s) is

$T = m(g + a) = 1000 \times (10 + 2) = 12{,}000$ N

Between $t = 2$ s and $t = 10$ s, the speed of the lift is constant. Hence $a = 0$ and $T = mg = 1000 \times 10 = 10{,}000$ N

Between $t = 10$ s and $t = 12$ s, the lift is decelerating. Its deceleration is given by

$$a = \frac{\text{BE}}{\text{EC}} = \frac{4\ \text{ms}^{-1}}{2\,\text{s}} = 2\ \text{ms}^{-2}$$

$\therefore$ Tension $= m(g - a) = 1000 \times (10 - 2) = 8{,}000$ N

The height to which the lift rises = area of OABC = 40 m. Hence all the four choices are correct.

15. The correct choices are (a), (c) and (d). The acceleration of each mass is the same and is given by

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$$

The tension in the string is $T = \frac{2m_1 m_2 g}{(m_1 + m_2)}$

Since the masses start from rest, the distance moved by m_1 in time t is

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2 = \frac{1}{2}at^2$$

16. The correct choices are (a), (b) and (d). The acceleration of each mass is given by

$$a = \frac{F}{(m_1 + m_2)}$$

The tension in the string is $T = m_1 a$. The force on mass m_1 is

$$F_1 = \frac{m_2 F}{(m_1 + m_2)}$$

17. The 7th coin from the bottom has 3 coins above it. Hence, the force on the 7th coin = weight of 3 coins = 3 mg, vertically downwards.

Since the 8th coin has 2 coins above, it supports the weight of two coins. Hence the force on the 7th

coin by the 8th coin = weight of 8th coin + weight of two coins above it = weight of three coins = 3 *mg* vertically downwards.

From Newton's third law, the reaction force exerted by the 6th coin on the 7th coin is equal and opposite to the action force exerted by the 7th coin on the 6th coin. Now, the force exerted by the 7th coin on the 6th coin = weight of 7th coin + weight of 3 coins above it = weight of 4 coins = 4 *mg* vertically downwards. Hence, the reaction of the 6th coin on the 7th coin = 4 *mg* vertically upwards.

Hence the correct choices are (a), (b) and (d).

18. The correct choices are (b) and (d). The acceleration of both the masses is the same and is given by

$$a = \frac{\text{net force}}{\text{total mass}} = \frac{F}{(m_1 + m_2)}$$

Force on mass $m_2 = m_2\, a$.

19. Figures 5.101 (a) and 5.101 (b) show the free body diagram of the two blocks.

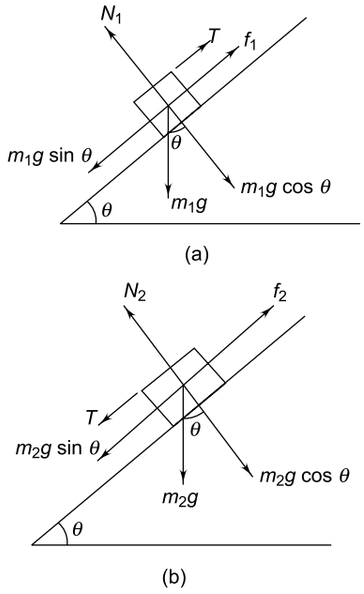


Fig. 5.101

T is the tension in the string and f_1 and f_2 are the frictional forces. It follows from the diagrams that

$$N_1 = m_1 g \cos\theta \text{ and } f_1 = \mu m_1 g \cos\theta$$

$$N_2 = m_2 g \cos\theta \text{ and } f_2 = \mu m_2 g \cos\theta$$

If a is the acceleration of the blocks down the plane, the equations of motion are

$$m_1 a = m_1 g \sin\theta - T - f_1$$

$$= m_1 g \sin\theta - T - \mu m_1 g \cos\theta \quad \text{(i)}$$

and $$m_2 a = m_2 g \sin\theta + T - f_2$$

$$= m_2 g \sin\theta + T - \mu m_2 g \cos\theta \quad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get $T = 0$ and

$$a = g(\sin\theta - \mu \cos\theta)$$

Hence the correct choices are (a) and (c).

20. Let M be the mass of the chain and L its length. If a length l hangs over the edge of the table, the force pulling the chain down is $\frac{Ml}{L}g$. The force of friction between the rest of the chain of length $(L - l)$ and the table is $\frac{\mu M(L-l)}{L}g$.

For equilibrium, the two forces must be equal, i.e.

$$\frac{Ml}{L}g = \frac{\mu M(L-l)}{L}g$$

or $$l = \mu\,(L - l)$$

or $$l = \frac{\mu L}{1+\mu}$$

$$\frac{l}{L} = \frac{\mu}{1+\mu} = \frac{0.25}{1+0.25} = \frac{1}{5} \text{ or } 20\%.$$

Hence the correct choices are (a) and (c).

21. The acceleration of the block is given by

$$a = g(\sin\theta - \mu\cos\theta)$$

$$= g(\sin\theta - kx\cos\theta)$$

Acceleration a is not uniform; it varies with x. For $\sin\theta > kx\cos\theta$, a is positive, i.e. a is positive for $x < \tan\theta/k$. For $x > \tan\theta/k$, a is negative. For $x = \tan\theta/k$, $a = 0$ and velocity is maximum after which it begins to decrease as the block is decelerated. Hence the correct choices are (b), (c) and (d).

22. The horizontal and vertical components of F are $F\cos\theta$ and $F\sin\theta$ (see Fig. 5.102).

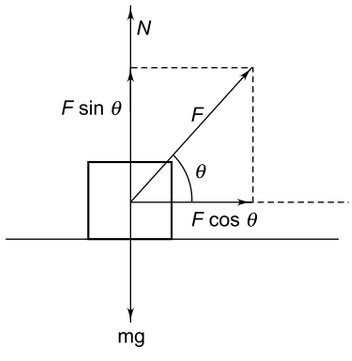


Fig. 5.102

$$N + F\sin\theta = mg$$

$$\Rightarrow \quad N = mg - F\sin\theta$$

$$= mg - kx\sin\theta$$

where N is the normal reaction.

The block will lose contact with surface at $x = x_0$ for which $N = 0$. Putting $N = 0$ and $x = x_0$, we have $0 = mg - kx_0 \sin\theta \Rightarrow x_0 = \dfrac{mg}{k\sin\theta}$

The acceleration a of block along the horizontal surface is given by

$$ma = F\cos\theta = kx\cos\theta$$

$$\Rightarrow \quad a = \frac{kx\cos\theta}{m}$$

which depends upon x. Hence the correct choices are (b) and (c).

23.

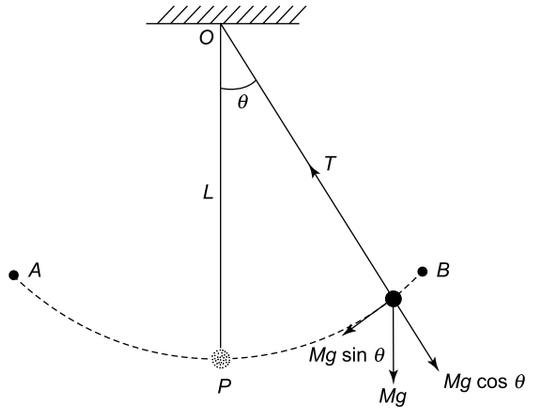


Fig. 5.103

It follows from Fig. 5.103, that

$$\frac{MV^2}{L} = T - Mg\cos\theta$$

Tangential force $f_t = Mg\sin\theta$

$\therefore$ Tangential acceleration $a_t = g\sin\theta$.

So the correct choices are (b) and (c).

24. When an external force is applied to move a body, the force of friction acts in the opposite direction. But when a body itself applies a force in order to move, the force of friction acts in the direction of motion. While pedalling, the external force is applied to the rear wheel and as a result the front wheel moves by itself. So, while pedalling, the choice (a) is correct. When pedalling is stopped, the choice (c) is correct as long as bicycle remains in motion.

SECTION III

Multiple Choice Questions Based On Passage

Question 1 to 3 are based on the following passage

Passage I

Reference Frames

We define the position of a body in terms of a frame of reference. Rest and motion are relative; there is nothing like absolute rest or absolute motion. The position or state of motion of a body may appear different from different frames of reference. For example, the passengers and everything else in a moving train are at rest in a reference frame situated in the train but they are in motion in a reference frame situated on the platform. Similarly, a stone dropped by a passenger from the window of a railway carriage in uniform motion appears to him to fall vertically downwards but to a person outside the carriage, it appears to follow a parabolic path.

Newton's first law of motion does not hold in all frames of reference; it holds only for inertial frames of reference. An inertial reference frame is a frame which moves with a constant velocity.

1. A reference frame attached to the earth
- (a) is an inertial frame by definition
- (b) cannot be an inertial frame because the earth is revolving round the sun
- (c) is an inertial frame because Newton's laws of motion are applicable in this frame
- (d) cannot be an inertial frame because the earth is rotating about its own axis.

2. Which of the following observers are inertial?
- (a) A child revolving in a merry-go-round
- (b) A driver in a car moving with a constant velocity

(c) A pilot in an aircraft which is taking off
(d) A passenger in a train which is slowing down to a stop.

3. Choose the correct statements from the following.
(a) An inertial frame is non-accelerating.
(b) An inertial frame is non-rotating.
(c) A reference frame moving at a constant velocity with respect to an inertial frame is also an inertial frame.
(d) Newton's laws of motion hold for both inertial and non-inertia frames.

SOLUTION

1. The velocity of the earth changes with time (due to a change in its direction) as it revolves round the sun. Therefore, a frame attached to the earth is accelerated. Accelerated frames and rotating frames of reference are not inertial frames. Hence the correct choices are (b) and (d).

2. The observers in (a), (c) amd (d) are all accelerating. Hence, they are non-inertial. Only the driver in (b) is inertial since his motion is not accelerated.

3. The correct choices are (a), (b) and (c).

Question 4 to 6 are based on the following passage

Passage II

Two bodies A and B of masses m and 2 m respectively are moving with equal linear momenta. They are subjected to the same retarding force.

4. If x_1 and x_2 are the respective distances moved by them before stopping, then x_1/x_2 is
(a) $\frac{1}{2}$ (b) 1
(c) $\sqrt{2}$ (d) 2

5. If t_1 and t_2 are the respective times taken by them to stop, then t_1/t_2 is
(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

6. If a_1 and a_2 are their respective decelerations, then a_1/a_2 is
(a) 1 (b) 2
(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

SOLUTION

4. $2\,ax = u^2 \Rightarrow 2\,max = mu^2 \Rightarrow 2\,Fx = \frac{(mu)^2}{m} = \frac{p^2}{m}$
where $F = ma$ is the retarding force $p = mu$ is linear momentum. Thus

$$x = \frac{p^2}{2Fm}$$

Since p and F are constants, $x \propto \frac{1}{m}$. Hence the correct choice is (d).

5. $0 = u + at \Rightarrow t = -\frac{u}{a} = -\frac{mu}{ma} = -\frac{p}{F}$. Hence $t_1 = t_2$. Thus the correct choice is (a).

6. $a_1 = F/m$ and $a_2 = F/2m$. Hence the correct choice is (b).

Questions 7 to 9 are based on the following passage

Passage III

Two bodies of masses m and 2 m respectively are moving with equal kinetic energies. They are subjected to the same retarding force.

7. If x_1 and x_2 are the respective distances moved by them before stopping, then x_1/x_2 is
(a) 2 (b) $\sqrt{2}$
(c) $\frac{1}{2}$ (d) 1

8. If t_1 and t_2 are the respective times taken by them to stop, then t_1/t_2 is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$
(c) $\sqrt{2}$ (d) 2

9. If a_1 and a_2 are their respective decelerations, then a_1/a_2 is
(a) 4 (b) 2
(c) $\frac{1}{2}$ (d) 1

SOLUTION

7. $2ax = u^2 \Rightarrow 2\,max = mu^2 \Rightarrow Fx = \frac{1}{2}\,mu^2$. If K is the kinetic energy, then $Fx = K \Rightarrow x = K/F$.

Since K and F are constants, the correct choice is (d).

8. $0 = u + at \Rightarrow t = -\frac{u}{a} = -\frac{mu}{ma} = -\frac{p}{F}$. Now

$p = mu \Rightarrow p^2 = m^2u^2 = (2m)\left(\frac{1}{2}mu^2\right) = 2mK.$

Hence $t = -\frac{\sqrt{2mK}}{F}$. Thus $t \propto \sqrt{m}$. Hence the correct choice is (a).

9. $a = \frac{F}{m}$, i.e. $a \propto \frac{1}{m}$ Hence the correct choice is (b).

Questions 10 to 13 are based on the following passage

Passage IV

A block of masses m is initially at rest on a frictionless horizontal surface. A time-dependent force $F = at - bt^2$ acts on the body, where a and b are positive constants.

10. The magnitude of the force is maximum at time t_1 given by

(a) $\frac{a}{b}$ (b) $\frac{2a}{b}$

(c) $\frac{a}{2b}$ (d) $\frac{a}{\sqrt{2}b}$

11. The maximum force F_{max} is given by

(a) $\frac{a^2}{2b}$ (b) $\frac{a^2}{4b}$

(c) $\frac{2a^2}{b}$ (d) $\frac{4a^2}{b}$

12. The maximum impulse I_{max} imparted to the block is given by

(a) $\frac{a^3}{3b^2}$ (b) $\frac{a^3}{6b^2}$

(c) $\frac{a^3}{9b^2}$ (d) $\frac{a^3}{12b^2}$

13. The maximum velocity v_{max} attained by the block is

(a) $\frac{a^3}{4mb^2}$ (b) $\frac{a^3}{8mb^2}$

(c) $\frac{a^3}{12mb^2}$ (d) $\frac{a^3}{16mb^2}$

SOLUTION

10. The force is maximum when $\frac{dF}{dt} = 0$ and $\frac{d^2F}{dt^2} < 0$.

Now $\frac{dF}{dt} = \frac{d}{dt}(at - bt^2) = a - 2bt$

Putting $\frac{dF}{dt} = 0$ and $t = t_1$, we get

$0 = a - 2bt_1 \Rightarrow t_1 = \frac{a}{2b}$

Also $\frac{d^2F}{dt^2} = \frac{d}{dt}(a - 2bt) = -2b$, which is negative.

Hence the correct choice is (c).

11. $F_{max} = at_1 - bt_1^2 = a \times \frac{a}{2b} - b \times \left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$.

Hence the correct choice is (b).

12. Maximum impulse is given by

$$I_{max} = \int_0^{t_1} F dt = \int_0^{t_1} (at - bt^2) dt = \frac{at_1^2}{2} - \frac{bt_1^3}{3} = \frac{a}{2}\left(\frac{a}{2b}\right)^2 - \frac{b}{3}\left(\frac{a}{2b}\right)^3 = \frac{a^3}{12b^2}$$

Hence the correct choice is (d).

13. Now impulse = change in momentum

$= mv - 0 = mv$

$\therefore v_{max} = \frac{I_{max}}{m} = \frac{a^3}{12mb^2}$, which is choice (c).

Questions 14 to 17 are based on the following passage

Passage V

A body of mass m is initially at rest. A periodic force $F = a\cos(bt + c)$ is applied to it, where a, b and c are constants.

14. The time period T of the force is

(a) $\frac{1}{b}$ (b) $\frac{2\pi}{b}$

(c) $2\pi\sqrt{\frac{a}{b}}$ (d) $2\pi\sqrt{\frac{b}{a}}$

15. The maximum velocity of the body is

(a) $\frac{a}{mb}$ (b) $\frac{b}{mc}$

(c) $\frac{c}{ma}$ (d) $\frac{b+c}{ma}$

16. The smallest value of t after $t = 0$ when the velocity of the body becomes zero is given by

(a) $t_1 = \dfrac{\pi}{a}$ (b) $t_1 = \dfrac{\pi - a}{c}$

(c) $t_1 = \dfrac{\pi - c}{b}$ (d) $t_1 = \dfrac{\pi}{b}$

17. The distance travelled by the body from time $t = 0$ to $t = t_1$ is given by

(a) $\dfrac{a}{mb^2}\cos c$ (b) $\dfrac{2a}{mb^2}\sin c$

(c) $\dfrac{a^2}{mb}\cos c$ (d) $\dfrac{2a^2}{mb}\sin c$

SOLUTION

14. F will repeat itself at values of t given by $\cos(bt + c) = +1$, i.e.
$bt + c = 0, 2\pi, 4\pi, \ldots$

$\Rightarrow t = -\dfrac{c}{b}, \dfrac{2\pi - c}{b}, \dfrac{4\pi - c}{b}, \ldots$

The smallest time interval is $T = \dfrac{2\pi}{b}$. Hence the correct choice is (b).

15. From Newton's second law of motion, $F = \dfrac{dp}{dt} = m\dfrac{dv}{dt}$

Thus $m\dfrac{dv}{dt} = a\cos(bt + c)$

$\Rightarrow dv = \dfrac{a}{m}\cos(bt + c)dt$

$\therefore v = \dfrac{a}{m}\int_0^t \cos(bt + c)dt = \dfrac{a}{mb}\sin(bt + c)$ (i)

Since the maximum value of $\sin(bt + c) = 1$, $v_{max} = \dfrac{a}{mb}$.

Hence the correct choice is (a).

16. From Eq (i) it follows that $v = 0$ at values of t given by $\sin(bt + c) = 0$ or $(bt + c) = 0, \pi, 2\pi, \ldots$ or $t = -\dfrac{c}{b}, \dfrac{\pi - c}{b}, \dfrac{2\pi - c}{b}$. Therefore,

$t_1 = \dfrac{\pi - c}{b} - \left(-\dfrac{c}{b}\right) = \dfrac{\pi}{b}$, which is choice (d).

17. Now $v = \dfrac{dx}{dt} \Rightarrow dx = v\,dt$. Therefore, the distance moved between $t = 0$ and $t = t_1$ is

$$x = \int_0^{t_1} v dt = \frac{a}{mb}\int_0^{t_1}\sin(bt + c)dt$$

$$= -\frac{a}{mb^2}\cos(bt_1 + c)$$

$$= -\frac{a}{mb^2}\cos\left[b \times \frac{\pi}{b} + c\right]$$

$$= -\frac{a}{mb^2}\cos(\pi + c) = \frac{a\cos c}{mb^2}$$

Hence the correct choice is (a).

Questions 18 to 20 are based on the following passage

Passage VI

Three masses $m_1 = m$, $m_2 = 2\,m$ and $m_3 = 3\,m$ are hung on a string passing over a frictionless pulley as shown in Fig. 5.104. The mass of the string is negligible. The system is then released.

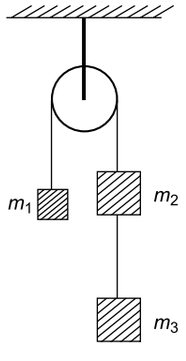


Fig. 5.104

18. If a_1, a_2 and a_3 are the accelerations of masses m_1, m_2 and m_3 respectively, then

(a) $a_1 < a_2 < a_3$

(b) $a_1 > a_2 > a_3$

(c) $a_1 > a_2 = a_3$ (d) $a_1 = a_2 = a_3$

19. The tension in the string between masses m_2 and m_3 is

(a) mg (b) $3\,mg$

(c) $4\,mg$ (d) $\dfrac{5mg}{3}$

20. The tension in the string between masses m_1 and m_2 is

(a) $4\,mg$ (b) $\dfrac{2mg}{3}$

(c) $\dfrac{5mg}{3}$ (d) $2\,mg$

SOLUTION

18. When the masses are released, mass m_1 moves upward and masses m_2 and m_3 move downward with a common acceleration given by

$$a = \frac{(m_2 + m_3 - m_1)g}{(m_1 + m_2 + m_3)} = \frac{(2m + 3m - m)g}{(m + 2m + 3m)} = \frac{2g}{3}$$

The correct choice is (d).

19. The let T be the tension in the string between m_1 and m_2 and T' be the tension in the string between m_2 and m_3 [see Fig. 5.105 (a)]. Figure 5.105 (b) shows the free-body diagram of mass m_3.

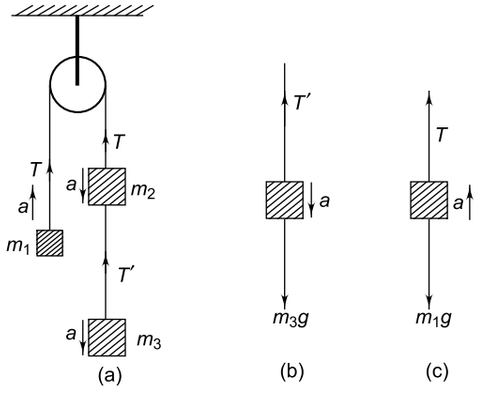


Fig. 5.105

$$m_3 g - T' = m_3 a$$

$$\Rightarrow T' = m_3(g - a) = 3\ m \times \left(g - \frac{2g}{3}\right) = mg$$

Hence the correct choice is (a).

20. Figure 5.105(c) shows the free-body diagram of mass m_1.

$$T - m_1 g = m_1 a$$

$$\Rightarrow T = m_1(g + a) = m \times \left(g + \frac{2g}{3}\right) = \frac{5mg}{3}$$

Hence the correct choice is (c).

Questions 21 to 23 are based on the following passage

Passage VII

Three blocks of masses $m_1 = m$, $m_2 = 2m$ and $m_3 = 3m$ connected by two strings are placed on a horizontal frictionless surface as shown in Fig. 5.106. A horizontal force F is applied to mass m_1 as shown.

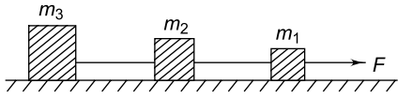


Fig. 5.106

21. The common acceleration of the blocks is

(a) $\frac{F}{m}$ (b) $\frac{F}{3m}$

(c) $\frac{F}{5m}$ (d) $\frac{F}{6m}$

22. The force on mass m_2 is

(a) $\frac{5F}{6}$ (b) $2\ F$

(c) $\frac{2F}{3}$ (d) F

23. The force on mass m_3 is

(a) F (b) $\frac{F}{2}$

(c) $\frac{F}{3}$ (d) $\frac{F}{6}$

SOLUTION

21. If F_2 and F_3 are the forces on masses m_2 and m_3 respectively, then the free-body diagrams of m_1, m_2 and m_3 are as shown in Fig. 5.107 where a is the common acceleration of the system.

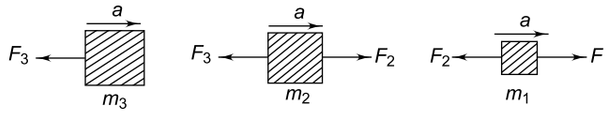


Fig. 5.107

$$F - F_2 = m_1 a \quad (1)$$

$$F_2 - F_3 = m_2 a \quad (2)$$

$$F_3 = m_3 a \quad (3)$$

Adding Eqs. (1), (2) and (3) we get

$$a = \frac{F}{(m_1 + m_2 + m_3)} = \frac{F}{(m + 2m + 3m)} = \frac{F}{6m}$$

Hence the correct choice is (d).

22. Adding Eqs. (2) and (3) we have

$$F_2 = (m_2 + m_3)a = (2m + 3m) \times \frac{F}{6m} = \frac{5F}{6},$$

which is choice (a).

23. From Eq. (3), we have

$$F_3 = 3ma = 3m \times \frac{F}{6m} = \frac{F}{2}$$

Hence the correct choice is (b).

Questions 24 to 26 are based on the following passage

Passage VIII

Two blocks of masses $m_1 = m$ and $m_2 = 2\ m$ are connected by a light string passing over a frictionless pulley. The mass m_1 is placed on a smooth inclined plane of inclination $\theta = 30°$ and mass m_2 hangs vertically as shown in Fig. 5.108.

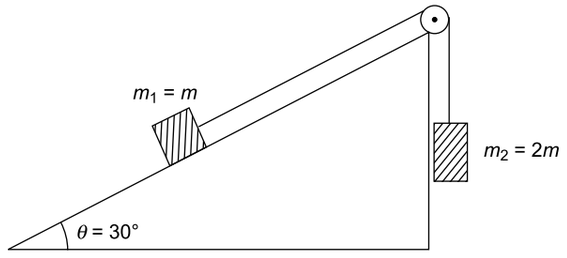


Fig. 5.108

24. If the system is released, the blocks move with an acceleration equal to

(a) $\frac{g}{4}$ (b) $\frac{g}{3}$

(c) $\frac{g}{2}$ (d) g

25. If the system is released the tension in the string is

(a) mg (b) $\frac{3mg}{2}$

(c) $2mg$ (d) $\frac{2mg}{3}$

26. If the inclined plane was rough, it was found that when the system was released, block m_1 remained at rest. The frictional force between block m_1 and the inclined plane is

(a) $\frac{3mg}{2}$ (b) $3\ mg$

(c) $\frac{4mg}{3}$ (d) $\frac{2mg}{3}$

SOLUTION

24. Since the inclined plane is smooth and $m_2 > m_1$, block m_1 will up the plane and block m_2 will move vertically with a common acceleration a. If T is the tension in the string, the free-body diagrams of masses m_1 and m_2 are as shown in Fig. 5.109

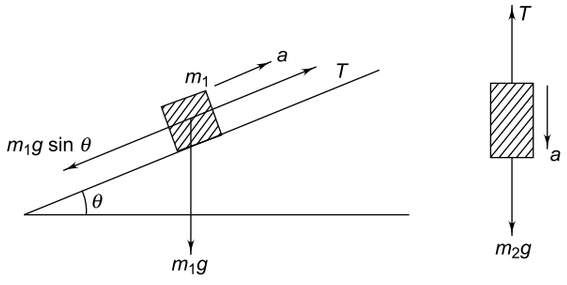


Fig. 5.109

The equations of motion of the blocks are

$$T - m_1g \sin \theta = m_1a \quad (1)$$

and $$m_2g - T = m_2a \quad (2)$$

Equations (1) and (2) give

$$a = \frac{(m_2 - m_1 \sin\theta)g}{(m_1 + m_2)} = \frac{(2m - m \times \sin 30°)g}{(m + 2m)} = \frac{g}{2}$$

Hence the correct choice is (c).

25. From Eqs. (1) and (2), we get

$$T = m_2(g - a) = 2m \times \left(g - \frac{g}{2}\right) = mg$$

Hence the correct choice is (a).

26. Since the blocks remain at rest, the equations of motions of blocks m_1 and m_2 are (here f is the frictional force on m_1)

$T - m_1g \sin \theta - f = 0$

and $T = m_2g$

These equations give

$$f = m_2g - m_1g \sin \theta$$
$$= 2\ m \times g - m \times g \times \sin 30°$$
$$= 2\ mg - \frac{mg}{2} = \frac{3mg}{2},$$

which is choice (b).

Questions 27 to 29 are based on the following passage

Passage IX

Two blocks of masses $m_1 = 3\ m$ and $m_2 = 2\ m$ are suspended from a rigid support by two inextensible uniform wires A and B. Wire A has negligible mass and wire B has a mass $m_3 = m$, as shown in Fig. 5.110. The whole system of blocks, wires and the support have an upward acceleration a.

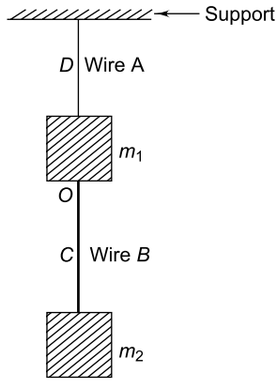


Fig. 5.110

27. The tension at the mid-point C of wire B is

(a) $\frac{1}{2}m(g+a)$ (b) $\frac{3}{2}m(g-a)$

(c) $\frac{3}{2}m(g+a)$ (d) $\frac{5}{2}m(g+a)$

28. The tension at point O of wire B is

(a) $3m(g+a)$ (b) $3m(g-a)$

(c) $2m(g+a)$ (d) $2m(g-a)$

29. The tension at the mid-point D of wire A is

(a) $2m(g+a)$ (b) $4m(g-a)$

(c) $6m(g+a)$ (d) $8m(g-a)$

SOLUTION

27. Refer to Fig. 5.111. Let T be the tension at the mid-point C of wire B. Then

$$T - \left(m_2 + \frac{m_3}{2}\right)g = \left(m_2 + \frac{m_3}{2}\right)a$$

$$\Rightarrow \quad T = \left(m_2 + \frac{m_3}{2}\right)(g+a) = \left(2m + \frac{m}{2}\right)(g+a)$$

$$= \frac{5}{2}\,m(g+a),$$

which is choice (d).

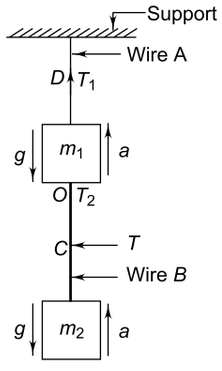


Fig. 5.111

28. Let T_1 be the tension in wire A. Since this wire has negligible mass, the tension is the same ($= T_1$) at every point on this wire. Let T_2 be the tension at point O of wire B. Then, we have for wire A.

$$T_1 - T_2 - m_1 g = m_1 a \qquad (1)$$

where T_2 is given by

$$T_2 - (m_2 + m_3)g = (m_2 + m_3)a$$

$$\Rightarrow \quad T_2 = (m_2 + m_3)(g + a)$$

$$= (2m + m)(g + a) = 3m(g + a)$$

Hence the correct choice is (a).

29. Putting $T_2 = 3m(g + a)$ in Eq. (1), we get

$$T_1 = 6\,m(g + a).$$

Hence the correct choice is (c).

SECTION IV

Matching

1. Three blocks of masses $m_1 = 3\,m$, $m_2 = 2\,m$ and $m_3 = m$ are placed in contact on a horizontal frictionless surface as shown in Fig. 5.112. A horizontal force F is applied to m_1 as shown. Match items in column I with those in column II

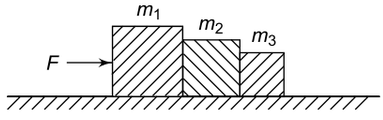


Fig. 5.112

Column I	***Column II***
(a) Force acting on m_2 if $F = 12$ N	(p) 1 N
(b) Force acting on m_2 if $F = 6$ N	(q) 3 N
(c) Force acting on m_3 if $F = 12$ N	(r) 2 N
(d) Forcer acting on m_3 if $F = 6$ N	(s) 6 N

SOLUTION

The contact forces acting on m_2 and m_3 respectively are

$$F_2 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

and

$$F_3 = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

Hence the correct matching is as follows

(a) → (s) (b) → (q)

(c) → (r) (d) → (p)

2. Two blocks of masses M = 5 kg and m = 3 kg are placed on a horizontal surface as shown in Fig. 5.113. The coefficient of friction between the blocks is $\mu_1 = 0.5$ and that between the blocks M and the horizontal surface is $\mu_2 = 0.7$. Taking $g = 10$ ms^{-2}, match items in column I with those in column II

Column I	***Column II***
(a) Frictional force between the blocks	(p) 5 ms^{-2}
(b) Acceleration of the upper block	(q) 96 N
(c) The maximum horizontal force F_{max} applied to M so that the two blocks move together without slipping.	(r) 4 ms^{-2}
(d) The common acceleration of the blocks if F = 32 N.	(s) 15 N

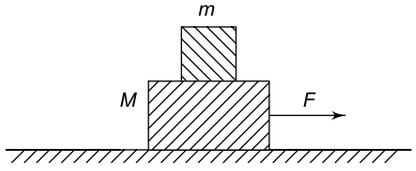


Fig. 5.113

SOLUTION

(a) Force of frictional between blocks = $\mu_1 mg = 0.5 \times 3 \times 10 = 15$ N.

(b) Acceleration of the upper block of mass m is a = $\mu_1 mg/m = \mu_1 g = 0.5 \times 10 = 5$ ms^{-2}.

(c) The force of friction between block M (with block m placed on top of it) and the horizontal surface = $(M + m)\mu_2 g = (5 + 3) \times 0.7 \times 10 = 56$ N

The two blocks will move together if the acceleration of the lower block does not exceed $a = 5$ ms^{-2}. The force due to this acceleration = $(M + m)a = (5 + 3) \times 5 = 40$ N

$\therefore$ Maximum horizontal force $F_{max} = 56 + 40 = 96$ N

(d) If F = 32 N, the common acceleration of the blocks is

$$a' = \frac{F}{(M+m)} = \frac{32}{(5+3)} = 4 \text{ ms}^{-2}$$

Hence the correct matching is as follows:

(a) → (s) (b) → (p)

(c) → (q) (d) → (r)

SECTION V

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has four choices out of which only one choice is correct

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

1. Statement-1

A block is pulled along a horizontal frictionless surface by a thick rope. The tension in the rope will not always be the same at all points on it.

Statement-2

The tension in the rope depends on the acceleration of the block-rope system and the mass of the rope.

2. Statement-1

A truck moving on a horizontal surface with a uniform speed u is carrying sand. If a mass Δm of the sand 'leaks' from the truck in a time Δt, the force needed to keep the truck moving at its uniform speed is $u\,\Delta m/\Delta t$.

Statement-2

Force = rate of change of momentum.

3. Statement-1

Two blocks of masses m and M are placed on a horizontal surface as shown in Fig. 5.114. The coefficient of friction between the two blocks is μ_1 and that between the block M and the horizontal surface is μ_2. The maximum force that can be applied to block M so that the two blocks move without slipping is $F = (\mu_1 + \mu_2)(M + m)g$.

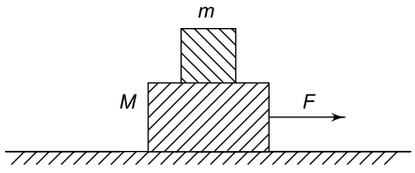


Fig. 5.114

Statement-2

Maximum force = total mass × maximum acceleration.

4. Statement-1

A shell of mass m is at rest initially. It explodes into three fragments having masses in the ratio 2:2:1. The fragments having equal masses fly off along mutually perpendicular directions with speed v. The speed of the third (lighter) fragment will be $2\sqrt{2}\,v$.

Statement-2

The momentum of a system of particles is conserved if no external force acts on it.

5. Statement-1

The maximum value of force F such that the block shown in Fig. 5.115 does not move is $\mu\, mg/\cos\theta$, where μ is the coefficient of friction between the block and the horizontal surface.

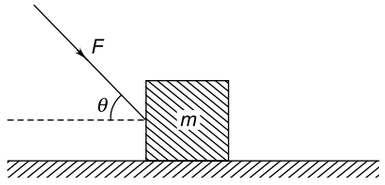


Fig. 5.115

Statement-2

Frictional force = coefficient of friction × normal reaction.

6. Statement-1

A ball of mass m is moving towards a batsman at a speed v. The batsman strikes the ball and deflects it by an angle θ without changing its speed. The impulse imparted to the ball is zero.

Statement-2

Impulse = change in momentum

7. Statement-1

A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

Statement-2

For every action there is an equal and opposite reaction.

IIT, 2007

8. Statement-1

When a ball dropped from a certain height hits the floor, it exerts a force equal to the rate of change of momentum.

Statement-2

The floor does not move because the action and reaction forces, being equal and opposite, cancel each other.

ANSWERS/SOLUTIONS

1. The correct choice is (a).

2. The correct choice is (d). The force exerted by the leaking sand on the truck = rate of change of momentum = $u\,\Delta m/\Delta t$. The sand falling vertically downwards will exert this force on the truck in the vertically upward direction. This perpendicular force can do no work on the truck. Since the truck moves with a uniform velocity, the force exerted just overcomes the frictional force.

3. The correct choice is (c). The force of friction between block m and block $M = \mu_1 mg$, where μ_1 is the coefficient of friction between the two blocks. Now, the force of friction between block M (with block m on top of it) and the horizontal surface $= \mu_2(M + m)g$, where μ_2 is the coefficient of friction between block M and surface. The maximum force F applied to block M must be enough to overcome this force of friction and the force due to acceleration of the system. If the acceleration of the system is a then this force $= (M + m)a$. Thus

$$F = (M + m)a + m_2(M + m)g \qquad \text{(i)}$$

Now, since the force on block m is $\mu_1 mg$, its acceleration a is

$$a = \frac{\text{force on mass } m}{\text{mass } m} = \frac{\mu_1 mg}{m} = \mu_1 g \qquad \text{(ii)}$$

Using (ii) in (i) we get

$$F = \mu_1(M + m)g + \mu_2(M + m)g$$
$$= (\mu_1 + \mu_2)\,(M + m)g$$

4. The correct choice is (a). The mass of two fragments of equal masses $= \frac{2m}{5}$ each. The mass of the lighter fragment $= \frac{m}{5}$. The momenta of heavier fragments are $p = \frac{2mv}{5}$. The resultant of momenta p and p is

$$p' = (p^2 + p^2)^{1/2} = \sqrt{2}\,p$$

From the principle of conservation of momentum, the momentum of the third (lighter) fragment of mass $\frac{m}{5}$ must be $\sqrt{2}\,p$ but opposite in direction. Thus, if V is the speed of the lighter fragment, we have

$$\frac{mV}{5} = \sqrt{2}\,p = \sqrt{2}\,\frac{2mv}{5}$$

or $$V = 2\sqrt{2}\,v$$

5. The correct choice is (a). The component of F parallel to the horizontal surface is $F \cos \theta$. F will be maximum when $F \cos \theta$ just overcomes the frictional force $f = \mu mg$. Thus

$$F_{\max} \cos \theta = \mu mg \Rightarrow F_{\max} = \frac{\mu mg}{\cos\theta}$$

6. The correct choice is (d). Refer to the solution of Q.27 of section I.

7. Statement-1 follows the Newton's first law of motion also called the law of inertia. The dishes are not dislodged even when the cloth is suddenly pulled because the dishes have the inertia of rest. Statement-2 is Newton's third law of motion, it does not explain statement-1. Hence the correct choice is (b).

8. The assertion is true but the reason is not correct because action and reaction forces do not act on the same body and hence do not cancel each other. Hence the correct choice is (c).

SECTION VI

Integer Answer Type

1. The magnitude of force f (in newton) acting on a body varies with time t (in millisecond) as shown in Fig. 5.116. Find the magnitude of the total impulse (in Ns) of the force on the body from $t = 4$ ms to $t = 16$ ms.

IIT, 1994

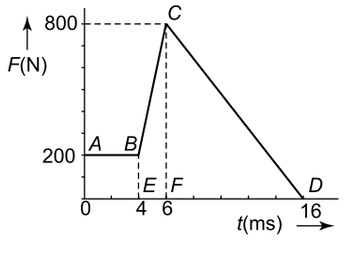


Fig. 5.116

2. A block of mass 0.2 kg is held against a wall by applying a horizontal force of 5 N on the block. The coefficient of friction between wall and block is 0.5. Find the magnitude (in newton) of the frictional force acting on the block. Take g = 10 ms^{-2}.

IIT, 1995

3. Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of a massless string and a frictionless pulley as shown in Fig. 5.117. The coefficient of friction between block A and the wedge is 2/3 and that between block B and the wedge is 1/3. If the blocks are released from rest, find the acceleration of block A (in ms^{-2}).

IIT, 1997

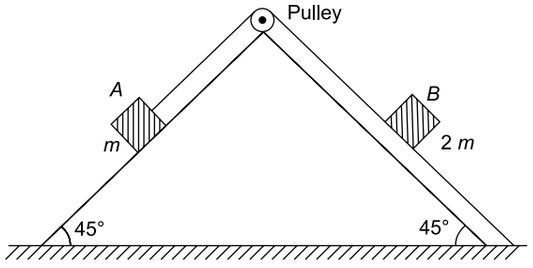


Fig. 5.117

4. A piece of uniform string hangs vertically so that its free end just touches the horizontal surface of a table. The upper end of the string is now released. At any time during the falling of the string, the total force on the surface of the table is n times the weight of the part of the string lying on the surface. Find the value of n.

IIT, 1989

5. A uniform rope of mass M and length L is pulled by a constant force of 10 N. Find the tension (in newton) in the rope at a point at a distance $L/5$ from the end where the force is applied.

IIT, 1978

SOLUTIONS

1. Impulse from $t = 4$ ms to $t = 16$ ms = area under the $F - t$ graph = area of $EBCD$

$$= \text{area of trapezium } EBCF + \text{area of } \Delta CDF$$

$$= \frac{1}{2} \times (200 + 800)\ \text{N} \times (2 \times 10^{-3}\ \text{s}) + \frac{1}{2} \times 800\ \text{N} \times (10 \times 10^{-3}\ \text{s})$$

$$= 1 + 4 = 5\ \text{Ns}$$

2. Normal reaction $R = 5$ N.
At equilibrium, the force of friction = weight of the block (see Fig. 5.118)

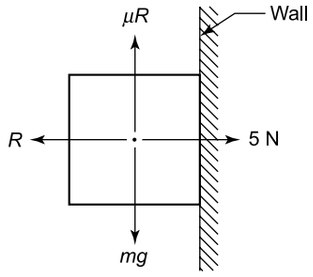
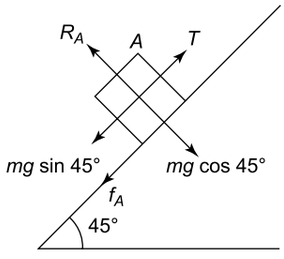


Fig. 5.118

$$= mg = 0.2 \times 10 = 2\ \text{N}$$

3. Case (a): Let us assume that block A moves up the plane and block B moves down the plane. The free body diagrams of the blocks are as follows (See Fig. 5.119)



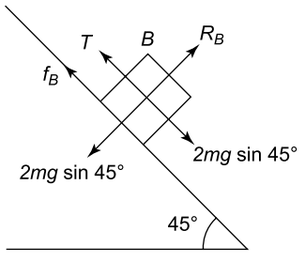


Fig. 5.119

The equations of motion of blocks A and B are $T - mg \sin 45° - \mu_A\ mg \cos 45° = ma$, where $\mu_A = 2/3$ and $2\ \mu g \sin 45° - \mu_B\ 2\ mg \cos 45° - T = 2\ ma$, where $\mu_B = 1/3$.
Adding these equations and solving we get

$$a = -\frac{g}{9\sqrt{2}}$$

Case (b): If we assume that block A moves down and block B moves up, we would get $a = -\frac{7g}{9\sqrt{2}}$.

Thus in both cases, the acceleration has a negative value which implies that the blocks will decelerate. This is not possible because the blocks start from rest. Hence when the blocks are released, they move with zero acceleration. Thus acceleration of block $A = 0$.

4. Let x be the length of the string lying on the surface of the table at an instant of time t. If an additional length dx of the string falls on the surface in time dt, the velocity v of this element when it strikes the surface is given by ($\because u = 0$)

$$v^2 = u^2 + 2gx = 0 + 2gx$$

or $$v^2 = 2gx \qquad (1)$$

The total force on the surface is
F = rate of change of momentum of element of length dx + weight of a length x of the string lying on the table.
If m is the mass per unit length of the string, then

$$F = \frac{d}{dt}(mdxv) + mxg = mv\frac{dx}{dt} + mxg = mv^2 + mxg \qquad (2)$$

$$\left(\because v = \frac{dx}{dt}\right)$$

Using (1) in (2) we get

$$F = 2\ mgx + mgx = 3\ mgx$$

But $mx = M$, the mass of the string lying on the table. Hence

$$F = 3\ Mg$$

Thus $n = 3$.

5. Mass per unit length of the rope is $m = \frac{M}{L}$. Let us find the tension at point P at a distance x from the end $x = 0$. Let T be the tension in the rope at point P.

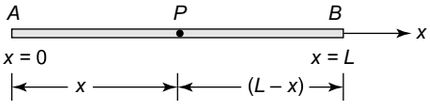


Fig. 5.120

For part AP the tension is towards the positive x-direction and for part BP the tension is towards the negative x-direction. If a is the acceleration produced in the rope by the constant force F, then for part AP,

$$T = (\text{mass of } AP) \times a \text{ or } T = mxa \qquad (1)$$

For part BP, we have

$$F - T = (\text{mass of } BP) \times a = m(L - x)a \qquad (2)$$

From (1), we have $a = \frac{T}{mx}$

Using this in (2), we get

$$F - T = m(L - x) \times \frac{T}{mx} = \frac{(L - x)T}{x}$$

or $$F = T\left[\frac{(L - x)}{x} + 1\right] = T\frac{L}{x} \text{ or } T = \frac{Fx}{L}$$

At $$x = L - \frac{L}{5} = \frac{4L}{5},$$

$$T = \frac{F}{L} \times \frac{4L}{5} = \frac{4F}{5} = \frac{4 \times 10}{5} = 8 \text{ N}$$

6 Chapter Work, Energy and Power

REVIEW OF BASIC CONCEPTS

6.1 WORK

1. Work done by a Force

(a) Work done by a constant force

When a constant force **F** acting on a body produces a displacement **S**, then the work done by the force is given by

$$W = \mathbf{F}\cdot\mathbf{S} = FS\cos\theta$$

where θ is the angle between the force vector **F** and the displacement vector **S** [see Fig. 6.1]. F and S are the magnitudes of **F** and **S** respectively.

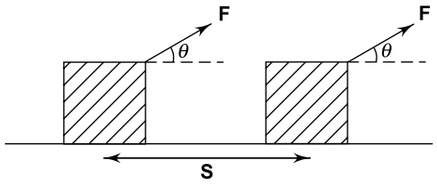


Fig. 6.1

(i) If θ is acute, $\cos\theta$ is positive. Hence work is positive for acute θ. In this case the force increases the speed of the body.

(ii) If $\theta = 90°$, $W = 0$, i.e. if the force is perpendicular to displacement work done by the force is zero.

(iii) If θ is obtuse, W is negative. In this case the force decreases the speed of the body.

(iv) If $\theta = 0$, i.e. force **F** is in the same direction as displacement **S**, then $W = FS$

(v) If $\theta = 180°$, force **F** is opposite to **S** (example frictional force), $W = -FS$. Work done by frictional and viscous force is always negative.

(b) Work done by a variable force

Suppose a force **F** is not constant but depends on the position vector **r** of the body, then the work done by the force **F** in moving the body from a position r_1 to a position r_2 is given by

$$W = \int_{r_1}^{r_2} \mathbf{F}\cdot\mathbf{dr} = \text{area under the } (F - r) \text{ graph}$$

EXAMPLE 6.1

A box is dragged on a horizontal floor by a rope which makes an angle of 60° with the horizontal. Find the work done if a force of 150 N is applied to drag the box through a distance of 10 m.

SOLUTION

$$W = FS\cos\theta$$
$$= 150 \times 10 \times \cos 60° = 750 \text{ J}$$

EXAMPLE 6.2

A horizontal force F pulls a 20 kg box at a constant velocity along a horizontal floor. If the coefficient of friction between the box and the floor is 0.25, find the work done by force F in moving the box through a distance of 2 m.

SOLUTION

Since the box is moved at a constant velocity, the applied force F just overcomes the frictional force f, i.e.

$$F = f = \mu mg$$

$\therefore$ Work done $W = FS\cos\theta = \mu mgS\cos 0°$

$$= 0.25 \times 20 \times 9.8 \times 2 = 98 \text{ J}$$

EXAMPLE 6.3

A block of mass $m = 5$ kg slides down from the top of an inclined plane of inclination $\theta = 30°$ with the horizontal. The coefficient of sliding friction between

the block and the plane is 0.25. The length of the plane is 2 m. Find the work done by the (a) gravitational force, (b) frictional force and (c) normal reaction if the block slides to the the bottom of the plane.

SOLUTION

Refer to Fig. 6.2.

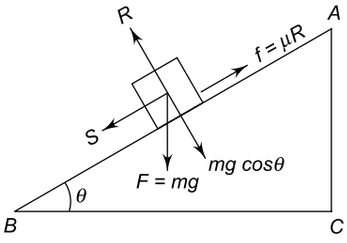


Fig. 6.2

Displacement $S = AB = 2$ m from A to B.

(a) Angle between **F** and **S** is $(90° - \theta) = 90° - 30° = 60°$

$\therefore$ Work done by gravitational force is

$$W_1 = FS \cos 60° = mgS \cos 60°$$

$$= 5 \times 9.8 \times 2 \times \frac{1}{2} = 49 \text{ J}$$

(b) Work done by frictional force is

$$W_2 = fS \cos 180°$$
$$= -fS = -\mu RS$$
$$= -\mu mg (\cos\theta) S$$
$$= -0.25 \times 5 \times 9.8 \times \cos 30° \times 2$$
$$= -21.2 \text{ J}$$

(c) Since the normal reaction R is perpendicular to displacement S, work done by normal reaction is

$$W_3 = RS \cos 90° = 0$$

EXAMPLE 6.4

A block of mass $m = 2$ kg is raised vertically upwards by means of a massless string through a distance of $S = 4$ m with a constant acceleration $a = 2.2$ ms^{-2}. Find the work done by (a) tension and (b) gravity. Also find the net work done on the block.

SOLUTION

(a) From the free body diagram (Fig. 6.3)

$$T - mg = ma$$

$$\Rightarrow \quad T = m(a + g)$$

$$= 2 \times (2.2 + 9.8) = 24 \text{ N}$$

$\therefore$ Work done by tension is ($\because$ T and S are in the same direction)

$$W_1 = TS \cos 0°$$
$$= 24 \times 4 \times 1 = 96 \text{ J}$$

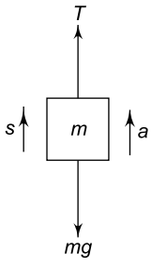


Fig. 6.3

(b) Since the gravitational force mg and displacement S are in opposite directions, work done by gravity is

$$W_2 = mgS \cos 180°$$
$$= -2 \times 9.8 \times 4 = -78.4 \text{ J}$$

(c) Net work done $W = W_1 + W_2 = 96 - 78.4 = 17.6$ J

EXAMPLE 6.5

A block of mass $m = 2$ kg in suspended by a light string from the ceiling of a lift. The lift starts moving down with an acceleration $a = 1.8$ ms^{-2}. Find the work done by the tension in the string during the first 5 seconds.

SOLUTION

Tension $T = m(g - a) = 2 \times (9.8 - 1.8) = 16$ N

Distance moved in $t = 5$ s is

$$S = \frac{1}{2} at^2 = \frac{1}{2} \times 1.8 \times (5)^2$$
$$= 22.5 \text{ m}$$

Since the tension and displacement are in opposite directions, the work done by tension is

$$W = TS \cos 180°$$
$$= -TS = -16 \times 22.5 = -360 \text{ J}$$

EXAMPLE 6.6

A constant force $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ newton displaces a body from position $\mathbf{r}_1 = (4\hat{\mathbf{i}} - 5\hat{\mathbf{j}})$ metre to $\mathbf{r}_2 = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ metre. Find the work done by the force.

SOLUTION

Displacement $\mathbf{S} = \mathbf{r}_2 - \mathbf{r}_1$

$$= (\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) - (4\hat{\mathbf{i}} - 5\hat{\mathbf{j}}) = -3\hat{\mathbf{i}} + 8\hat{\mathbf{j}}$$

$\therefore$ $W = \mathbf{F} \cdot \mathbf{S} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (-3\hat{\mathbf{i}} + 8\hat{\mathbf{j}})$

$$= -6 + 24 = 18 \text{ J}$$

$$[\because \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = 1 \text{ and } \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0]$$

EXAMPLE 6.7

A body of mass $m = 0.5$ kg travels in a straight line with a velocity $v = 5x^{3/2}$ where v is in ms^{-1} and x is in metre. Find the work done in displacing the body from $x = 0$ to $x = 2$ m.

SOLUTION

Acceleration $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(5x^{3/2}\right)$

$$= 5 \times \frac{3}{2} x^{1/2} \frac{dx}{dt}$$

$$= \frac{15}{2} x^{1/2} \times \left(5x^{3/2}\right) \quad \left[\because \frac{dx}{dt} = v\right]$$

$$= \frac{75}{2} x^2$$

$\therefore$ Work done $W = \int_{x=0}^{x=2} F dx = \int_0^2 ma dx$

$$= 0.5 \times \frac{75}{2} \int_0^2 x^2 dx$$

$$= 0.5 \times \frac{75}{2} \times \left|\frac{x^3}{3}\right|_0^2$$

$$= \frac{0.5 \times 75}{2 \times 3}(8 - 0)$$

$$= 50 \text{ J}$$

EXAMPLE 6.8

A block of mass 5 kg slides down from the top of an inclined plane of angle of inclination 30°. The coefficient of sliding friction between the block and the plane is 0.3. The length of the plane is 2 m. Find work done by (a) by gravity, (b) frictional force and (c) normal reaction.

SOLUTION

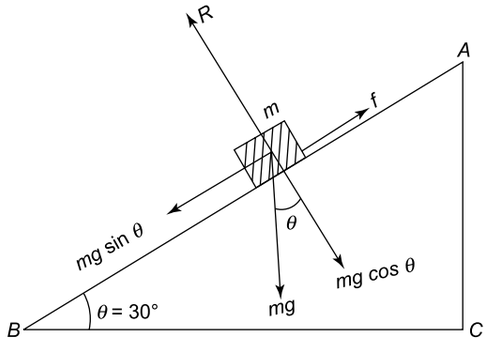


Fig. 6.4

Displacement $S = 2$ m down the inclined plane.

(a) Work done by gravity $= mg \sin\theta \times AB = (mg \sin\theta) S$

$$= 5 \times 9.8 \times \sin 30° \times 2$$

$$= 49 \text{ J}$$

(b) Work done by friction $= f S \cos 180°$

$$= -fS$$

$$= -\mu RS$$

$$= -\mu mg \cos\theta \times S$$

$$= -0.3 \times 5 \times 9.8 \times \cos 30° \times 2$$

$$= -14.7\sqrt{3} = -25.5 \text{ J}$$

(c) Work done by normal reaction $= RS \cos 90° = 0$ ($\because \mathbf{R} \perp \mathbf{S}$)

6.2 ENERGY

Energy can be defined as the capacity or ability to do work and is measured by the amount of work a body can do. So, energy is measured in the same units as work, namely, joule. Like work, energy is a scalar quantity.

Energy can exist in various forms, such as heat energy, electrical energy, sound energy, light energy, chemical energy, nuclear energy, mechanical energy, etc. Mechanical energy is of two types, *kinetic* and *potential.*

Kinetic Energy: Energy due to Motion A moving object can do work on another object when it strikes it. In other words, an object in motion has the ability to do work and, by definition, has energy. *The energy possessed by a body by virtue of its motion is called kinetic energy.*

An initially motionless body can move and acquire a velocity only if a force acts on it. The work done by the force in causing the body to move measures the kinetic energy (written as KE) of the moving body, i.e.

$$\text{KE} = W$$

The kinetic energy of a body of mass m, moving with a velocity v is given by

$$\text{KE} = \frac{1}{2} mv^2$$

This relation holds even if the force is variable, i.e. if the force varies both in magnitude and direction.

Work-Energy Principle Suppose a body of mass m moves with an initial velocity u. A force F acts on it, as a result of which it acquires a final velocity v. The work done by the force is given by

$$W = \int F dx = \int ma\, dx$$

$$= m \int_u^v \frac{dv}{dt} dx = \frac{1}{2} m(v^2 - u^2)$$

$$= \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

= final KE – initial KE = change in KE

Thus, *the work done by a force in displacing a body measures the change in its kinetic energy.* This is the work-energy principle.

Thus, when a force does work on a body, its kinetic energy increases; the increase in kinetic energy being equal to the amount of work done. The converse of this is also true. When the kinetic energy of a body is decreased by a retarding force, the decrease is equal to the work done by the body against the retarding force. Thus kinetic energy and work are equivalent quantities and are, therefore, measured in the same units, namely, joule.

Potential Energy: Energy due to Position or Configuration An object can have energy not only by virtue of its motion, but also because of its position or configuration. *The energy possessed by a body owing to its position or configuration is called potential energy.*

Gravitational Potential Energy An object held at a position above the surface of the earth has potential energy by virtue of its position.

Consider a body of mass m. It is lifted vertically to a height h above the earth by applying a force F vertically upward. The force F must be just enough to overcome the gravitational attraction, i.e.

$$F = mg$$

where g is the acceleration due to gravity at that place. For bodies not too far above the surface of the earth, the value of g is practically constant. Hence the work done by a constant force F in displacing a body by a height h can be calculated by the product $F \times h = mgh$. Thus gravitational potential energy of a body of mass m at a height h above the surface of the earth is mgh.

Gravitational PE = mgh

Potential Energy of a Spring. Consider a perfectly elastic spring. One end of the spring is fixed to a rigid wall and other end is fixed to a block which is placed on a frictionless horizontal surface as shown in Fig. 6.5. We assume that the mass of the spring is negligible compared to the mass of the block.

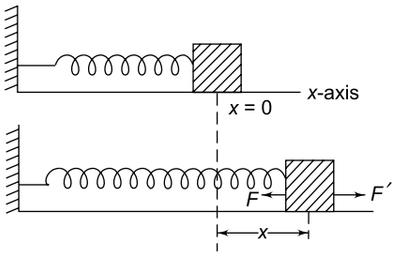


Fig. 6.5

If we stretch the spring by a distance x, the spring will exert a force on us during stretching. This force is due to the reaction of the spring and is called the *restoring force* which is proportional to the displacement x and acts in a direction opposite to the displacement, i.e.

$$F \propto -x \text{ or } F = -kx$$

where k is the force constant of the spring. The negative sign indicates that the force acts in a direction opposite to displacement.

To stretch a spring by a displacement x, we must exert a force F' on it, equal but opposite to the force F exerted by the spring on us. Therefore, the applied force is

$$F' = -F = kx$$

Notice that F' is a variable force as it depends on x. Therefore, the work done by the applied force in stretching the spring through a distance x is given by

$$W = \int_0^x F'\,dx = \int_0^x (kx)dx$$

$$= k\int_0^x x\,dx = k\left|\frac{x^2}{2}\right|_0^x = \frac{1}{2}kx^2$$

It is evident that the work done in compressing the spring by an amount x is also given by $W = \frac{1}{2}kx^2$.

Law of Conservation of Energy

The total energy of an isolated system remains constant, the energy can only change from one form to another.

EXAMPLE 6.9

A block of mass 0.5 kg is taken from the bottom of an inclined plane to its top and then allowed to slide down to the bottom. The length of the inclined plane is 2.5 m and its height is 1.5 m. The coefficient of friction between the block and the plane is 0.2. Find

(a) work done by the gravitational force over the round trip,

(b) work done by the applied force over the upward journey,

(c) work done by the frictional force over the round trip and

(d) the kinetic energy of the block when it reaches the bottom of the plane. What conclusion will you draw from your answers to (b), (c) and (d)? Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

Refer to Fig. 6.6.

$m = 0.5$ kg, $\mu = 0.2$, $l\,(= AC) = 2.5$ m and $h\,(= AB) = 1.5$ m

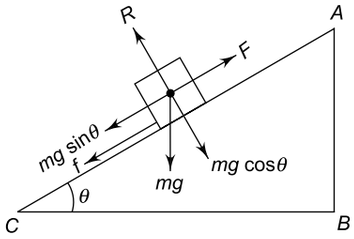


Fig. 6.6

$$\sin\theta = \frac{AB}{AC} = \frac{1.5}{2.5} = 0.6$$

$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta} = 0.8$$

(a) Work done by gravitational force to take the block from C to A is

$$W_1 = (-mg\sin\theta)l$$

The negative sign indicates that the gravitational force $mg\sin\theta$ is opposite to the displacement which is from C to A. Work done by the gravitational force to move the block from A to C is (because now the gravitational force is in the direction of the displacement)

$$W_2 = (+mg\sin\theta)l$$

$\therefore$ Total work done by the gravitational force over the round trip is

$$W_G = W_1 + W_2 = 0$$

This shows that gravitational force is conservative.

(b) Frictional force $f = \mu R = \mu mg\cos\theta$. The applied force to move the block from C to A is

$$F = mg\sin\theta + f = mg\sin\theta + \mu mg\cos\theta$$
$$= mg(\sin\theta + \mu\cos\theta)$$

Work done by applied force to move the block from C to A is

$$W_a = F \times AC$$
$$= mg(\sin\theta + \mu\cos\theta) \times l$$
$$= 0.5 \times 10\,(0.6 + 0.2 \times 0.8) \times 2.5$$
$$= 9.5 \text{ J}$$

(c) Friction always opposes motion. When the block is moved from C to A, frictional force f is opposite to direction. Hence work done by frictional force in the upward journey is

$$W_3 = -fl$$

When the block slides from A to C, f acts upwards along the plane and is opposite to the displacement. Hence work done by frictional force in the downward journey is

$$W_4 = -fl$$

$\therefore$ Total work done by frictional force over the round trip is

$$W_f = W_3 + W_4 = -2fl$$
$$= -2\,\mu mg\cos\theta \times l$$
$$= -2 \times 0.2 \times 0.5 \times 10 \times 0.8 \times 2.5$$
$$= -4 \text{ J}$$

(d) When the block is at A, its initial velocity $u = 0$. Let v be the velocity when it reaches C. Since f acts upwards, the net force on the block when it slides down is

$$F' = mg\sin\theta - f = mg(\sin\theta - \mu\cos\theta)$$

$$\therefore \text{ Acceleration } a = \frac{F'}{m} = g(\sin\theta - \mu\cos\theta)$$
$$= 10\,(0.6 - 0.2 \times 0.8)$$
$$= 4.4 \text{ ms}^{-2}$$

From $v^2 - u^2 = 2as$, we have

$$v^2 - 0 = 2 \times 4.4 \times 2.5$$

$$\Rightarrow \qquad v^2 = 22 \text{ m}^2\text{s}^{-2}$$

$$\therefore \text{ Kinetic energy at } C = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 0.5 \times 22$$
$$= 5.5 \text{ J}$$

Conclusion

Initial kinetic energy at $A = 0$. Therefore, change in K.E.$= 5.5 - 0 = 5.5$ J. Now total work done is

$$W = W_G + W_a + W_f$$
$$= 0 + 9.5 - 4 = 5.5 \text{ J}$$

Thus, work done = change in kinetic energy. This is the work-energy principle.

EXAMPLE 6.10

A block of mass $m = 500$ g is placed at the top of an inclined plane of inclination $\theta = 60°$. The length of the plane is 2 m. The block is released from rest. Find its speed when it reaches the bottom of the plane if

(a) the inclined plane is smooth

(b) the coefficient of friction between the block and the plane is 0.4.

Take $g = 10$ ms^{-2}.

SOLUTION

Refer to Fig. 6.6 of Example 6.9 above. Height of the inclined plane is $h = AB = AC\sin\theta = 2 \times \sin 60° = \sqrt{3}$ m $= 1.73$ m

(a) As the block slides down the plane, it loses potential energy and gains kinetic energy. From the principle of conservation of energy,

Gain in K.E = loss in P.E.

or $\frac{1}{2}mv^2 = mgh$

$\Rightarrow v = \sqrt{2gh} = \sqrt{2\times 10\times 1.73} \simeq 5.9 \text{ ms}^{-1}$

(b) As the block slides down, loss of P.E. = gain in K.E. + work done against friction, i.e.,

$$mgh = \frac{1}{2}mv^2 + (\mu mg\cos\theta)\times AC$$

$$\Rightarrow 1.73 \times 10 = \frac{1}{2}v^2 + 0.4\times 10\cos 60^\circ \times 2$$

$$\Rightarrow \quad v \simeq 5.2 \text{ ms}^{-1}$$

EXAMPLE 6.11

An elastic spring of negligible mass has a force constant $k = 4$ Nm^{-1}. One end of the spring is fixed to the wall and the other end touches a block of mass $m = 250$ g placed on a horizontal surface. The spring is compressed by an amount $x = 5$ cm as shown in Fig. 6.7. The coefficient of friction between the block and the horizontal surface is $\mu = 0.2$. If the system is released, find the speed of the block when it leaves the spring.

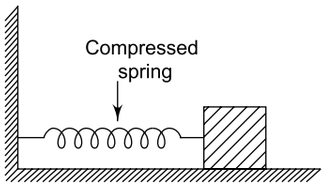


Fig. 6.7

SOLUTION

Loss in P.E. of spring + work done against friction = gain in K.E.

or $\frac{1}{2}kx^2 + \mu mgx = \frac{1}{2}mv^2$

$$\Rightarrow \quad \frac{1}{2}\times 4\times (0.05)^2 + 0.2\times 0.250\times 10\times 0.05$$

$$= \frac{1}{2}\times 0.250\times v^2$$

which gives $v = 0.5$ ms^{-1}

EXAMPLE 6.12

A bullet moving with a speed of 100 ms^{-1} travels a distance of 2 cm in a plank of wood before coming to rest. How much distance will the same bullet travel in the same plank before coming to rest if it were moving with a speed of 200 ms^{-1}?

SOLUTION

Since the bullet and the plank are the same, the resistive force F exerted on the bullet is the same in the two cases. The kinetic energy is spent in doing work against friction. Hence

$$\frac{1}{2}mv_1^2 = Fx_1$$

and $\frac{1}{2}mv_2^2 = Fx_2$

These equations give

$$x_2 = x_1 \times \frac{v_2^2}{v_1^2} = 2 \text{ cm} \times \left(\frac{200}{100}\right)^2$$

$$= 8 \text{ cm}$$

EXAMPLE 6.13

A uniform chain of length L and mass M lies on a frictionless horizontal table with a very small part hanging from the edge of the table. The chain begins to fall under the weight of the hanging part. Obtain the expression for the velocity of the chain at the instant when the length of the hanging part becomes L/n where $n > 1$.

SOLUTION

Mass per unit length of chain = $\frac{M}{L}$. The mass of length $\frac{L}{n}$ of the chain is

$$m = \frac{M}{L}\times\frac{L}{n} = \frac{M}{n}$$

As the chain slips down from the table, gravitational potential energy of the length $\frac{L}{n}$ (hanging part) decreases while the part of the chain left on the table does not lose any gravitational potential energy. The loss of P.E. of the hanging part gets converted into K.E. of the entire chain, i.e.

Gain of K.E. of the complete chain = loss of P.E. of the hanging part. The mass m of the hanging part can be assumed to be concentrated at its centre of mass which is at a height $h = \frac{L}{2n}$ below the edge of the table. If v is the velocity of the slipping chain, then

$$\frac{1}{2}Mv^2 = mgh = \frac{M}{n}\times g\times\frac{L}{2n} = \frac{MgL}{2n^2}$$

$$\Rightarrow \qquad v^2 = \frac{gL}{n^2}$$

$$\Rightarrow \qquad v = \frac{\sqrt{gL}}{n}$$

6.3 CONSERVATIVE AND NON-CONSERVATIVE FORCES

(a) Conservative force

A force is conservative if

(i) the work done by it on a body in moving it from one position to another depends only on the initial and final positions of the body and not on the path followed by it between the two positions.

or

(ii) the net work done by the force on a body that moves through any closed path is zero.

The above two conditions are equivalent. Examples of conservative forces are gravitational force, electrostatic force and spring force.

(b) Non-conservative force

A force is non-conservative if

(i) The work done by it on a body in moving it from one position to another depends on the path followed by the body between the two positions.

or

(ii) The work done by the force on a body that moves through a closed path is non zero.

Examples of non-conservative forces are frictional and viscous forces.

Conservative Force and Potential Energy

For a conservative force F that depends upon position r, there is a potential energy function U which also depends on r. When a conservative force does positive work, the potential of the system decreases, i.e.

Work done = decrease in potential energy

or $$Fdr = -dU$$

or $$F = -\frac{dU}{dr}$$

Hence *the negative derivative of the potential energy function with respect to position gives the conservative force acting on the system.*

The change in potential energy when the body is displaced from $r = a$ to $r = b$ is

$$U_b - U_a = \int_a^b F dr$$

NOTE

F is negative if **r** is opposite to **F** and positive if **r** is in the same direction as **F**.

EXAMPLE 6.14

The force between two point charges q_1 and q_2 separated by a distance r is $F = \frac{kq_1q_2}{r^2}$ where k is a constant. Find the potential energy of the system of charges.

SOLUTION

$$F = -\frac{dU}{dr} \Rightarrow dU = -Fdr. \text{ Integrating}$$

$$U = -\int_0^r F dr = -kq_1q_2 \int_0^r r^{-2} dr$$

$$\Rightarrow \qquad U = \frac{kq_1q_2}{r}$$

EXAMPLE 6.15

The potential energy U of a particle in a field varies with position r as

$$U = \frac{a}{r^2} - \frac{b}{r}$$

where a and b are positive constants. Find the position r_0 where the particle will be in stable equilibrium.

SOLUTION

If no force acts on a particle, it will be in stable equilibrium, i.e. $F = 0$ or

$$F = -\frac{dU}{dr} = 0$$

$$\Rightarrow \quad -\frac{d}{dr}\left(\frac{a}{r^2} - \frac{b}{r}\right) = 0$$

$$\Rightarrow \qquad \frac{2a}{r^3} - \frac{b}{r^2} = 0 \quad \Rightarrow \quad r = r_0 = \frac{2a}{b}$$

Stable equilibrium corresponds to minimum potential energy, i.e. $\frac{dU}{dr} = 0$ and $\frac{d^2U}{dr^2} > 0$. If $r = r_0 = \frac{2a}{b}$, $\frac{dU}{dr} = 0$, then U can be minimum or maximum. If $\frac{d^2U}{dr^2} > 0$ at $r = r_0$, U will be minimum.

Now $$\frac{dU}{dr} = -\frac{2a}{r^3} + \frac{b}{r^2}$$

$$\therefore \quad \frac{d^2U}{dr^2} = \frac{6a}{r^4} - \frac{2b}{r^3}$$

At $r = r_0 = \frac{2a}{b}$,

$$\left(\frac{d^2U}{dr^2}\right)_{\text{at } r = r_0} = 6a \times \left(\frac{b}{2a}\right)^4 - 2b\left(\frac{b}{2a}\right)^3$$

$$= \frac{3}{8}\frac{b^4}{a^3} - \frac{b^4}{4a^3} = \frac{b^4}{8a^3}, \text{ which is positive.}$$

Hence $r = r_0 = \frac{2a}{b}$ corresponds to stable equilibrium.

NOTE

For stable equilibrium U is minimum and for unstable equilibrium, U is maximum.

For stable equilibrium; $\frac{dU}{dr} = 0$ and $\frac{d^2U}{dr^2} > 0$

For unstable equilibrium; $\frac{dU}{dr} = 0$ and $\frac{d^2U}{dr^2} < 0$.

6.4 MOTION IN A VERTICAL CIRCLE

Consider a body of mass tied to a string of length r revolved in a vertical circle with centre O at the other end of the string as shown in Fig. 6.8. At all positions of the body, there are two forces acting on it: its own weight and the tension in the string.

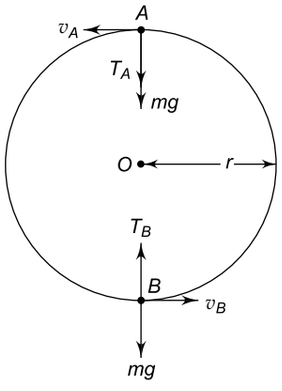


Fig. 6.8

Case (a): When the body is at top *A* of the circle

When the body is at top A of the circle, the net force towards centre O is $T_A + mg$. Hence [see Fig. 6.8]

$$T_A + mg = \frac{mv_A^2}{r}$$

where v_A = speed of the body at top A and T_A = tension in the string when the body is at A.

$$T_A = \frac{mv_A^2}{r} - mg \qquad \text{(i)}$$

The body will revolve in the circle if the string does not sag, i.e. $T_A \geq 0$. From Eq. (i) it follows that

$$\frac{mv_A^2}{r} - mg \geq 0$$

$$\Rightarrow \quad v_A^2 \geq \sqrt{rg}$$

Therefore, the minimum speed at the top that the body must have so that it can complete the circle is given by

$$(v_A)_{\min} = \sqrt{rg} \qquad \text{(ii)}$$

Case (b): When the body is at bottom B of the circle

When the body is at B, the net force on the body towards the centre is $(T_B - mg)$. Hence

$$T_B - mg = \frac{mv_B^2}{r}$$

where v_B = speed of the body at bottom B
and T_B = tension in the string when the body is at B.

$$T_B = \frac{mv_B^2}{r} + mg \qquad \text{(iii)}$$

The minimum speed at B that the body must have so that it can complete the circle is obtained from the conservation of energy. As the body goes up from B to A, it K.E. decreases and P.E. increases.

Loss in K.E. = gain in P.E.

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = mg \times AB = mg \times 2r$$

$$\Rightarrow \quad v_B = \sqrt{v_A^2 + 4gr}$$

$$\therefore \quad (v_B)_{\min} = \sqrt{(v_A)_{\min}^2 + 4gr} \qquad \text{(iv)}$$

Using (ii) in (iv) we get

$$(v_B)_{\min} = \sqrt{5gr} \qquad \text{(v)}$$

The net force towards the centre acting on the body is obtained from (iii) by using (v).

$$T_B = \frac{m}{r} \times 5gr + mg = 6\ mg$$

NOTE

1. When a body moves in a vertical circle, the speed decreases as it goes up and increases as it goes down. Hence the body has a non-uniform circular motion.
2. The tension in the string is different positions of the body on the circle. The tension is minimum when the body is at the top of the circle and maximum when it is at the bottom of the circle.

EXAMPLE 6.16

A small block of mass m, starts from rest at A and slides on a frictionless track which ends in a circular loop of radius r. If $h = 6r$, find the speed of the block when it reaches C as shown in Fig. 6.9. What is the force exerted on the block by the track when it is at C? Also find the minimum height h so that the block is able to complete the circle.

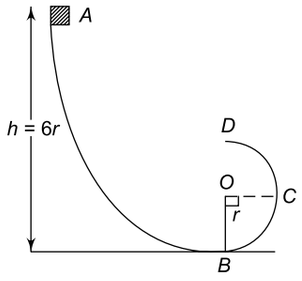


Fig. 6.9

SOLUTION

Let v be the speed of the block when it reaches C. From conservation of energy, gain in K.E. = loss in P.E., i.e.

$$\frac{1}{2}mv^2 - 0 = mgh - mg \times OB = mgh - mgr$$

$$\Rightarrow \quad \frac{1}{2}v^2 = g \times 6r - gr = 5gr$$

$$\Rightarrow \quad v = \sqrt{10gr}$$

When the block is at C, the track exerts a normal reaction N on the block. Since the block is moving in a circular path, the necessary centripetal force for circular motion is provided by the normal reaction (Fig. 6.10).

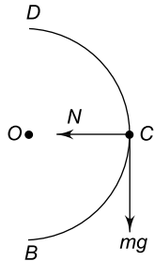


Fig. 6.10

$$\therefore \quad N = \frac{mv^2}{r} = \frac{m \times 10gr}{r} = 10\ mg$$

Thus the track exerts a force on the block equal to 10 times the weight of the block.

To complete the circle, the minimum speed at D must be $v_{min} = \sqrt{5gr}$. Hence

$$mgh_{min} = \frac{1}{2}mv_{min}^2 \Rightarrow h_{min} = 2.5r.$$

6.5 POWER

The rate of doing work is called power, i.e.

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

The faster a given amount of work is done, the greater is the power of the agent that does the work.

In the SI system, the unit of power is the watt (symbol W). *Power is said to be 1 watt when 1 joule of work is done in 1 second,* i.e.

$$1\ \text{W} = 1\ \text{Js}^{-1}$$

Since the watt is a small unit for the measurement of power, larger units, namely kilowatt (kW) and megawatt (MW) are often used.

$$1\ \text{kW} = 1000\ \text{W} = 10^3\ \text{W}$$

$$1\ \text{MW} = 1{,}000{,}000\ \text{W} = 10^6\ \text{W}$$

The power of an agent can also be expressed in terms of the force applied and the velocity of the object on which the force is applied. Now, power P is given by

$$P = \frac{W}{t} = \frac{\mathbf{F} \cdot \mathbf{S}}{t} = \mathbf{F} \cdot \mathbf{v}$$

$$\left(\because \frac{\mathbf{S}}{t} = \text{rate of change of displacement} = \mathbf{v}\right)$$

Power is a scalar quantity as it is the ratio of two scalars W and t, or a scalar product of two vectors **F** and **v**.

EXAMPLE 6.17

An engine pulls a car of mass 1000 kg on a level road at a constant velocity of 5 ms^{-1} . If the frictional force is 500 N, what power does the engine generate? What extra power must the engine supply to maintain the same speed up an inclined plane having a gradient of 1 in 10?

SOLUTION

Since the car moves at a constant velocity, its acceleration is zero. Hence the engine has to do work only to overcome the frictional force f .

$$\therefore \quad \text{Power} = f \times v = 500 \times 5 = 2500\ \text{W} = 2.5\ \text{kW}$$

For an inclined plane having a gradient of 1 in 10, $\sin\theta = \frac{1}{10}$. To maintain the same speed up the inclined plane, the engine has to do extra work against the force $mg \sin\theta$.

Therefore,

$$\text{Extra power} = mg \sin\theta \times v$$

$$= 100 \times 9.8 \times \frac{1}{10} \times 5 = 4900 \text{ W}$$

$$= 4.9 \text{ kW}$$

EXAMPLE 6.18

An electric pump on the ground floor of a building takes 10 minutes to fill a tank of volume 2000 litre with water. If the tank is 40 m above the ground and the efficiency of the pump is 40%, how much electric power is consumed by the pump in filling the tank? Take $g = 10 \text{ ms}^{-2}$.

SOLUTION

Volume of tank $V = 2000 \text{ litre} = 2000 \times 10^{-3} \text{ m}^3 = 2 \text{ m}^3$

Mass of water $m = \rho V = 1000 \times 2 = 2 \times 10^3$ kg

Work done to lift this mass to a height $h = 40$ m is

$$W = mgh = 2 \times 10^3 \times 10 \times 40 = 8 \times 10^5 \text{ J}$$

$$\text{Power needed} = \frac{W}{t} = \frac{8 \times 10^5}{10 \times 60} = \frac{4}{3} \times 10^3 \text{ W}$$

If P is the total power consumed, the useful power available = 40% if $P = 0.4\,P$. Hence

$$0.4\,P = \frac{4}{3} \times 10^3$$

$$\Rightarrow \quad P = 3.33 \times 10^3 \text{ W} = 3.33 \text{ kW}$$

EXAMPLE 6.19

A constant power P is supplied to a car of mass $m = 3000$ kg. The velocity of the car increases from $u = 2 \text{ ms}^{-1}$ to $v = 5 \text{ ms}^{-1}$ when the car travels a distance $x = 117$ m. Find the value of P. Neglect friction.

SOLUTION

$$P = Fv = mav \quad \Rightarrow \quad a = \frac{P}{mv}$$

Now $$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{v\,dv}{dx}$$

$$\therefore \quad v\,\frac{dv}{dx} = \frac{P}{mv}$$

$$\Rightarrow \quad v^2 dv = \frac{P}{m}\,dx$$

$$\therefore \quad \int_u^v v^2\,dv = \frac{P}{m}\int_0^x dx$$

$$\Rightarrow \quad \frac{1}{3}\left(v^3 - u^3\right) = \frac{Px}{m}$$

$$\Rightarrow \quad P = \frac{m\left(v^3 - u^3\right)}{3x}$$

$$= \frac{3000 \times \left[(5)^3 - (2)^3\right]}{3 \times 117}$$

$$= 1000 \text{ W} = 1 \text{ kW}$$

EXAMPLE 6.20

A car of mass m starts from rest at time $t = 0$ and is driven on a straight horizontal road by the engine which exerts a constant force F. If friction is negligible, the car acquires kinetic energy E at time t and develops a power P. Which of the following is/are correct ?

(a) $E \propto t$ (b) $E \propto t^2$

(c) $P \propto t$ (d) $P \propto t^2$

SOLUTION

Since F is constant, acceleration $a = \frac{F}{m}$ is constant.

At time t, the velocity of the car is

$$v = u + at = 0 + at = at = \frac{Ft}{m}.$$

$\therefore$ Kinetic energy E at time $t = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Ft}{m}\right)^2$

$$= \left(\frac{F^2}{m}\right)t^2.$$

Since F is constant, $E \propto t^2$.

Power P at time $t = Fv = F \times \frac{Ft}{m} = \left(\frac{F^2}{m}\right)t$

Thus $P \propto t$. So the correct choices are (b) and (c).

EXAMPLE 6.21

A body, initially at rest, moves in a straight line under the influence of a source of constant power. Its displacement in time t is proportional to

(a) $t^{1/2}$ (b) t

(c) $t^{3/2}$ (d) t^2

SOLUTION

$$P = Fv = mav = m\frac{dv}{dt} \times v$$

$$\Rightarrow \quad v\,dv = \frac{P}{m}\,dt$$

Integrating $\int_0^v v dv = \frac{P}{m}\int_0^t dt \quad (\because P = \text{constant})$

$\Rightarrow \quad \frac{v^2}{2} = \frac{p}{m}t$

$\Rightarrow \quad v = \sqrt{\frac{2p}{m}}t^{1/2}$

$\Rightarrow \quad \frac{dx}{dt} = \sqrt{\frac{2p}{m}}t^{1/2}$

$\Rightarrow \quad dx = \sqrt{\frac{2p}{m}}t^{1/2}dt$

$\therefore \quad \int_0^x dx = \frac{2P}{m}\int_0^t t^{1/2}dt$

$\Rightarrow \quad x = \frac{4P}{3m}t^{3/2}$

So the correct choice is (c).

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. A particle of mass m is moving in a circular path of a constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2 r t^2$ where k is a constant. The power delivered to the particle by the force acting on it is

(a) $2\pi m k^2 r^2 t$ (b) $m k^2 r^2 t$

(c) $\frac{m k^4 r^2 t^5}{3}$ (d) zero

IIT, 1994

2. A stone is tied to a string of length L and whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at the lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

IIT, 1998

3. The power P supplied to a body initially at rest varies with time t as $P = kt^2$ where k is a constant. The velocity of the body at an instant of time t will be proportional to

(a) t (b) $t^{3/2}$

(c) t^2 (d) t^3

4. A force $\mathbf{F} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ newton acts on a particle moving along a line $4y + kx = 3$. The work done by the force is zero if the value of k is

(a) 1 (b) 2

(c) 3 (d) 4

5. Force F acting on a body moving in a straight line varies with the velocity v of the body as $F = k/v$ where k is a constant. The work done by the force in time t is proportional to

(a) t (b) $t^{3/2}$

(c) $t^{-1/2}$ (d) $t^{-3/2}$

6. The force F acting on a body varies with its displacement x as $F = kx^{-2/3}$. The power delivered by the force will be proportional to

(a) $x^{-3/2}$ (b) $x^{-1/2}$

(c) $x^{1/2}$ (d) $x^{3/2}$

7. A bullet is fired at a plank of wood with a speed of 200 ms^{-1}. After passing through the plank, its speed reduces to 180 ms^{-1}. Another bullet, of the same mass and size but moving with a speed of 100 ms^{-1} is fired at the same plank. What would be the speed of this bullet after passing through the plank? Assume that the resistance offered by the plank is the same for both the bullets?

(a) 48 ms^{-1} (b) 49 ms^{-1}

(c) 50 ms^{-1} (d) 51 ms^{-1}

8. If the mass of either bullet in Q. 10 is 7 g and the thickness of the wooden plank is 1 m, what is the average resistance offered by the plank?

(a) 36 N (b) 38 N

(c) 40 N (d) 42 N

9. An engine pulls a car of mass 1500 kg on a level road at a constant speed of 5 ms^{-1}. If the frictional force is 1500 N, what power does the engine generate?

(a) 5.0 kW (b) 7.5 kW
(c) 10 kW (d) 12.5 kW

10. In Q. 9, what extra power must the engine develop to maintain the same speed up an inclined plane having a gradient of 1 in 10? Take $g = 10\ \text{ms}^{-2}$.
(a) 2.5 kW (b) 5.0 kW
(c) 7.5 kW (d) 10 kW

11. Two identical cylindrical vessels, with their bases at the same level, each contain a liquid of density ρ. The height of the liquid in one vessel is h_1 and that in the other is h_2. The area of either base is A. What is the work done by gravity in equalizing the levels when the vessels are interconnected?
(a) $A\rho g\ (h_1 - h_2)^2$ (b) $A\rho g\ (h_1 + h_2)^2$
(c) $A\rho g \left(\frac{h_1 - h_2}{2}\right)^2$ (d) $A\rho g \left(\frac{h_1 + h_2}{2}\right)^2$

12. An electric pump on the ground floor of a building takes 10 minutes to fill a tank of volume 30 m^3 with water. If the tank is 60 m above the ground and the efficiency of the pump is 30% how much electric power is consumed by the pump in filling the tank? Take $g = 10\ \text{ms}^{-2}$.
(a) 100 kW (b) 150 kW
(c) 200 kW (d) 250 kW

13. The distance x moved by a body of mass 0.5 kg by a force varies with time t as

$$x = 3t^2 + 4t + 5$$

where x is expressed in metre and t in second. What is the work done by the force in the first 2 seconds?
(a) 25 J (b) 50 J
(c) 75 J (d) 100 J

14. In a hydroelectric power station, the height of the dam is 10 m. How many kg of water must fall per second on the blades of a turbine in order to generate 1 MW of electrical power? Take $g = 10\ \text{ms}^{-2}$.
(a) $10^3\ \text{kgs}^{-1}$ (b) $10^4\ \text{kgs}^{-1}$
(c) $10^5\ \text{kgs}^{-1}$ (d) $10^6\ \text{kgs}^{-1}$

15. A uniform steel rod of mass m and length l is pivoted at one end. If it is inclined with the horizontal at an angle θ its potential energy will be
(a) $\frac{1}{2}\ mgl \cos\theta$ (b) $\frac{1}{2}\ mgl \sin\theta$
(c) $mgl \cos\theta$ (d) $mgl \sin\theta$

16. A bullet, incident normally on a wooden plank, loses one-tenth of its speed in passing through the plank. The least number of such planks required to stop the bullet is
(a) 5 (b) 6
(c) 7 (d) 8

17. A bullet is fired normally on an immovable wooden plank. It loses 25% of its momentum in penetrating a thickness of 3.5 cm. The total thickness penetrated by the bullet is
(a) 8 cm (b) 10 cm
(c) 12 cm (d) 14 cm

18. A bullet is fired normally on an immovable wooden plank. It loses 25% of its kinetic energy in penetrating a thickness x of the plank. What is the total thickness penetrated by the bullet?
(a) $2x$ (b) $4x$
(c) $6x$ (d) $8x$

19. A body of mass m, having momentum p, is moving on a rough horizontal surface. If it is stopped in a distance x, the coefficient of friction between the body and the surface is given by
(a) $\mu = \frac{p^2}{2gm^2x}$ (b) $\mu = \frac{p^2}{2mgx}$
(c) $\mu = \frac{p}{2mgx}$ (d) $\mu = \frac{p}{2gm^2x}$

20. A uniform chain of mass M and length L is held on a horizontal frictionless table with $\frac{1}{n}$ th of its length hanging over the edge of the table. The work done is pulling the chain up on the table is
(a) $\frac{Mgl}{n}$ (b) $\frac{Mgl}{2n}$
(c) $\frac{Mgl}{n^2}$ (d) $\frac{Mgl}{2n^2}$

21. A body of mass $m = 1$ kg is dropped from a height $h = 40$ cm on a horizontal platform fixed to one end of an elastic spring, the other being fixed to a base, as shown in Fig. 6.11. As a result the spring is compressed by an amount $x = 10$ cm. What is the force constant of the spring. Take $g = 10\ \text{ms}^{-2}$.
(a) $600\ \text{Nm}^{-1}$ (b) $800\ \text{Nm}^{-1}$
(c) $1000\ \text{Nm}^{-1}$ (d) $1200\ \text{Nm}^{-1}$

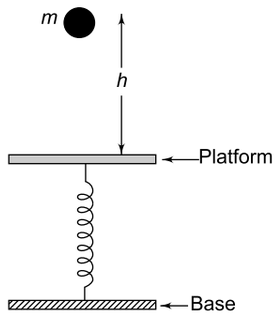


Fig. 6.11

22. A force acts on a particle of mass 3 g in such a way that the position of the particle as a function of time is given by

$$x = 3t - 4t^2 + t^3$$

where x is in metres and t is in seconds. The work done during the first 4 s is

(a) 570 mJ (b) 450 mJ
(c) 490 mJ (d) 530 mJ

23. A sphere of mass m is tied to one end of a string of length l and rotated through the other end along a horizontal circular path with speed v. The work done in one full horizontal circle is

(a) zero (b) $mg \cdot 2\pi l$
(c) $\left(\frac{mv^2}{l}\right) \cdot 2\pi l$ (d) $\left(\frac{mv^2}{l}\right) \cdot l$

24. The kinetic energy acquired by a mass m in travelling a certain distance d, starting from rest, under the action of a constant force is

(a) directly proportional to $\sqrt{m}$
(b) independent of m
(c) directly proportional to $\frac{1}{\sqrt{m}}$
(d) directly proportional to m

25. A position dependent force $F = 7 - 2x + 3x^2$ newton acts on a body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. The work done in joules is

(a) 70 (b) 270
(c) 35 (d) 135

26. A particle is moved from a position $\mathbf{r}_1 = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 6\hat{\mathbf{k}})$ metre to a position $\mathbf{r}_2 = (14\hat{\mathbf{i}} + 13\hat{\mathbf{j}} + 9\hat{\mathbf{k}})$ metre under the action of a force $\mathbf{F} = (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$. What is the work done?

(a) 10 J (b) 100 J
(c) 0.01 J (d) 1 J

27. If momentum is increased by 20%, then kinetic energy increases by

(a) 44% (b) 55%
(c) 66% (d) 77%

28. A particle of mass 0.1 kg is subjected to a force which varies with distance as shown in Fig. 6.12. If it starts its journey from rest at $x = 0$, its velocity at $x = 12$ m is

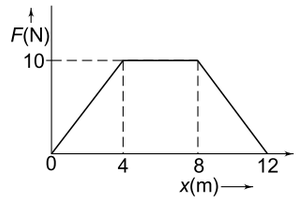


Fig. 6.12

(a) zero (b) $20\sqrt{2}$ m/s
(c) $20\sqrt{3}$ m/s (d) 40 m/s

29. A force $\mathbf{F} = -K(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$ where K is a positive constant, acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$ and then parallel to the y-axis to the point (a, a). The total work done by the force $\mathbf{F}$ on the particle is

(a) $-2Ka^2$ (b) $2Ka^2$
(c) $-Ka^2$ (d) Ka^2

IIT, 1998

30. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

(a) v (b) v^2
(c) v^3 (d) v^4

IIT, 2000

31. A body of mass 6 kg is acted upon by a force which causes a displacement in it given by $x = \frac{t^2}{4}$ metre where t is the time in second. The work done by the force is 2 seconds is

(a) 12 J (b) 9 J
(c) 6 J (d) 3 J

32. A ladder 2.5 m long and of weight 150 N has its centre of gravity 1 m from its bottom. A weight of 40 N is attached to the top end. The work required to raise the ladder from the horizontal position to the vertical position is

(a) 190 J (b) 250 J
(c) 285 J (d) 475 J

33. A body of mass 5 kg rests on a rough horizontal surface of coefficient of friction 0.2. The body is pulled through a distance of 10 m by a horizontal force of 25 N. The kinetic energy acquired by it is (take $g = 10\ \text{ms}^{-2}$)

(a) 200 J (b) 150 J
(c) 100 J (d) 50 J

34. A body is moving up an inclined plane of angle θ with an initial kinetic energy E. The coefficient of friction between the plane and the body is μ. The work done against friction before the body comes to rest is:

(a) $\frac{\mu E \cos\theta}{\cos\theta + \sin\theta}$ (b) $\mu E \cos\theta$
(c) $\frac{\mu E \cos\theta}{\mu\cos\theta - \sin\theta}$ (d) $\frac{\mu E \cos\theta}{\mu\cos\theta + \sin\theta}$

35. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to

(a) x^2 (b) e^x

(c) x (d) $\log_e x$

36. A body of mass m accelerates uniformly from rest to velocity v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is

(a) $\dfrac{mv_1t}{t_1}$ (b) $\dfrac{mv_1^2t}{t_1^2}$

(c) $\dfrac{mv_1^2t^2}{t_1^2}$ (d) $\dfrac{mv_1t^2}{t_1}$

37. A force $\mathbf{F}=\left(5\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}\right)$ newton is applied to a particle which displaces it from its origin to the point $\mathbf{r}=(2\hat{\mathbf{i}}-\hat{\mathbf{j}})$ metre. The work done on the particle (in joule) is

(a) – 7 (b) + 7

(c) + 10 (d) + 13

38. Two masses M and m (with $M > m$) are connected by means of a pulley as shown in Fig. 6.13. The system is released. At the instant when mass M has fallen through a distance h, the velocity of mass m will be

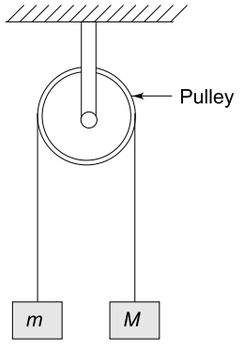


Fig. 6.13

(a) $\sqrt{2gh}$

(b) $\sqrt{\dfrac{2gh\,M}{m}}$

(c) $\sqrt{\dfrac{2gh\,(M-m)}{(M+m)}}$

(d) $\sqrt{\dfrac{2gh\,(M+m)}{(M-m)}}$

39. A mass m, lying on a horizontal frictionless sur-face is connected to mass M as shown in Fig. 6.14. The system is now released. The velocity of mass m when mass M as descended a distance h is

(a) $\sqrt{\dfrac{2\,Mgh}{m}}$ (b) $\sqrt{\dfrac{2\,mgh}{M}}$

(c) $\sqrt{\dfrac{2\,Mgh}{(M+m)}}$ (d) $\sqrt{2gh}$

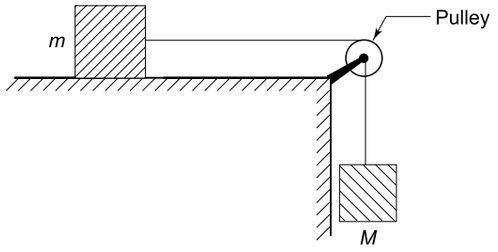


Fig. 6.14

40. An escalator is moving downwards with a uniform speed u. A man of mass m is running upwards on it at a uniform speed v. If the height of the escalator is h, the work done by the man in going up the escalator is

(a) zero (b) mgh

(c) $\dfrac{mghu}{(v-u)}$ (d) $\dfrac{mghv}{(v-u)}$

41. The potential energy (in joule) of a body of mass 2 kg moving in the $x-y$ plane is given by

$$U = 6x + 8y$$

where the position coordinates x and y are measured in metre. If the body is at rest at point (6 m, 4 m) at time $t = 0$, it will cross the y-axis at time t equal to

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s

42. In Q. 41 above, the speed of the body when it crosses the y-axis is

(a) zero (b) 5 ms^{-1}

(c) 10 ms^{-1} (d) 20 ms^{-1}

43. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 (as shown in Fig. 6.15) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.

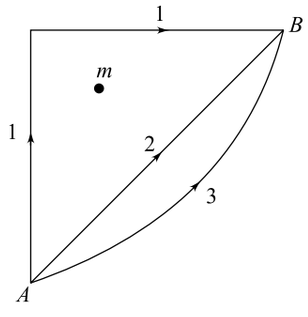


Fig. 6.15

(a) $W_1 > W_3 > W_2$ (b) $W_1 = W_2 = W_3$

(c) $W_1 < W_3 < W_2$ (d) $W_1 < W_2 < W_3$

IIT, 2003

44. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is (see Fig. 6.16)

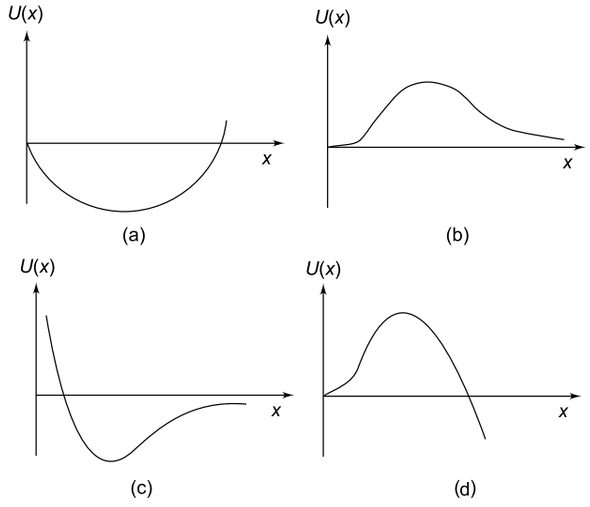


Fig. 6.16

IIT, 2002

45. A raindrop of radius r falls from a certain height h above the ground. The work done by the gravitational force is proportional to

(a) r (b) r^2

(c) r^3 (d) r^4

46. A smooth steel ball is moving to and fro about the lowest position O of a frictionless hemispherical bowl. The ball attains a maximum height of 20 cm on either side of O. If $g = 10\ \text{ms}^{-2}$, the speed of the ball when it passes through O will be

(a) $\sqrt{2}\ \text{ms}^{-1}$

(b) $2\ \text{ms}^{-1}$

(c) $0.2\ \text{ms}^{-1}$

(d) $0.02\ \text{ms}^{-1}$

47. The bob of a pendulum is released from a horizontal position A as shown in Fig. 6.17. The length of the pendulum is 2 m. If 10% of the initial energy of the bob is dissipated as heat due to the friction of air, what would be the speed of the bob when it reaches the lowermost point B? Take $g = 10\ \text{ms}^{-2}$.

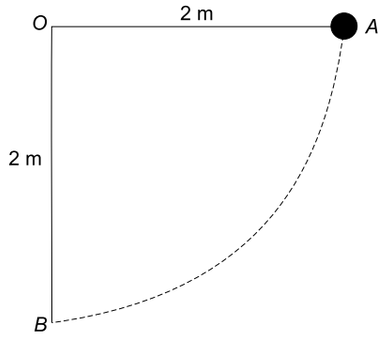


Fig. 6.17

(a) $3\ \text{ms}^{-1}$ (b) $4\ \text{ms}^{-1}$

(c) $5\ \text{ms}^{-1}$ (d) $6\ \text{ms}^{-1}$

48. A body of mass m is dropped from a height h above the ground. The velocity v of the body when it has lost half its initial potential energy is given by

(a) $v = \sqrt{gh}$ (b) $v = \sqrt{2gh}$

(c) $v = \sqrt{\dfrac{gh}{2}}$ (d) $v = 2\sqrt{gh}$

49. A body of mass m is thrown vertically upwards with a velocity v. The height h at which the kinetic energy of the body is half its initial value is given by

(a) $h = \dfrac{v^2}{g}$ (b) $h = \dfrac{v^2}{2g}$

(c) $h = \dfrac{v^2}{3g}$ (d) $h = \dfrac{v^2}{4g}$

50. A car of mass m moving at a speed v is stopped in a distance x by the friction between the tyres and the road. If the kinetic energy of the car is doubled, its stopping distance will be

(a) $8x$ (b) $4x$

(c) $2x$ (d) x

51. A body is allowed to fall freely under gravity from a height of 10 m. If it loses 25% of its energy on impact with the ground, to what height will it rise after one impact?

(a) 2.5 m (b) 5.0 m

(c) 7.5 m (d) none of these

52. In Q. 51, to what height will the body rise after two such impacts with the ground?

(a) 2.5 m (b) 5.0 m

(c) 7.5 m (d) none of these

53. A body of mass m thrown vertically upwards attains a maximum height h. At what height will its kinetic energy be 75% of its initial value?

(a) $\frac{h}{6}$ (b) $\frac{h}{5}$

(c) $\frac{h}{4}$ (d) $\frac{h}{3}$

54. A body, having kinetic energy k, moving on a rough horizontal surface, is stopped in a distance x. The force of friction exerted on the body is

(a) $\frac{k}{x}$ (b) $\frac{\sqrt{k}}{x}$

(c) $\frac{k}{\sqrt{x}}$ (d) kx

55. A particle at the origin is under the influence of a force $F = kx$, where k is a positive constant. If the potential energy U is zero at $x = 0$, the variation of potential energy with the coordinate x is represented by [see Fig. 6.18]

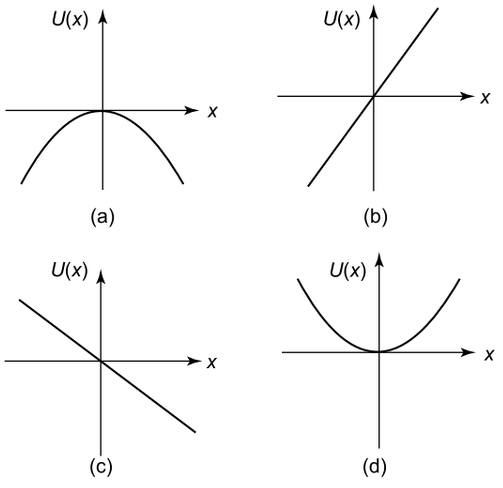


Fig. 6.18

IIT, 2004

56. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and 4 k, respectively (see Figure 6.19). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{x}{y}$ is

(a) 4 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

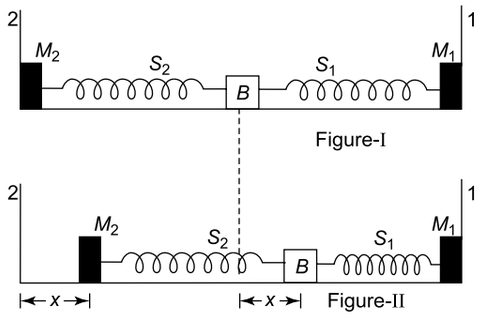


Fig. 6.19

IIT, 2008

57. A bob of mass m is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A satisfies [see Fig. 6.20]

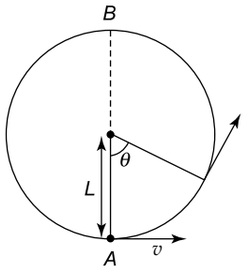


Fig. 6.20

(a) $\theta = \frac{\pi}{4}$ (b) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(c) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (d) $\frac{3\pi}{4} < \theta < \pi$

IIT, 2008

ANSWERS

1. (b)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (b)
7. (b)	**8.** (b)	**9.** (b)	**10.** (c)	**11.** (c)	**12.** (a)
13. (c)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (b)
19. (a)	**20.** (d)	**21.** (c)	**22.** (d)	**23.** (a)	**24.** (b)
25. (d)	**26.** (b)	**27.** (a)	**28.** (d)	**29.** (c)	**30.** (c)
31. (d)	**32.** (b)	**33.** (b)	**34.** (d)	**35.** (a)	**36.** (b)
37. (b)	**38.** (c)	**39.** (c)	**40.** (d)	**41.** (b)	**42.** (c)

43. (b) **44.** (d) **45.** (c) **46.** (b) **47.** (d) **48.** (a)
49. (d) **50.** (c) **51.** (c) **52.** (d) **53.** (c) **54.** (a)
55. (a) **56.** (b) **57.** (d)

SOLUTIONS

1. $a_c = k^2 r t^2 \Rightarrow \dfrac{v^2}{r} = k^2 r t^2 \Rightarrow v = krt$

Tangential acceleration $a_t = \dfrac{dv}{dt} = kr$

$\therefore$ Tangential force $F = m a_t = mkr$

$\therefore$ Power $= Fv = mkr \times krt = mk^2r^2t$

2. Refer to Fig. 6.21.

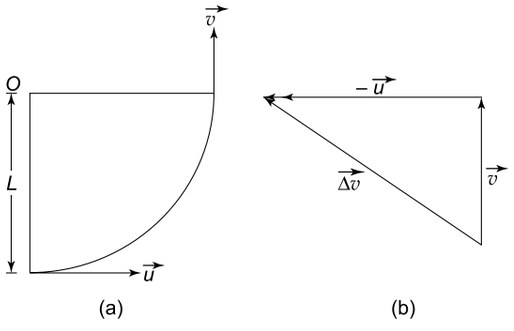


Fig. 6.21

From conservation of energy,

$$\frac{1}{2} mu^2 = \frac{1}{2} mv^2 + mgL$$

$$\Rightarrow \quad v = \sqrt{u^2 - 2gL}$$

Change in velocity $\Delta\vec{v} = \vec{v} - \vec{u} = \vec{v} + (-\vec{u})$. Thus $\Delta\vec{v}$ is the resultant of $\vec{v}$ and $-\vec{u}$. It follows from Fig. 6.21 (b) that

$$|\Delta\vec{v}| = \sqrt{v^2 + (-u)^2}$$

$$= \sqrt{u^2 - 2gL + u^2} = \sqrt{2(u^2 - gL)}$$

3. From work-energy principle, change in K.E. = work done.

$$\frac{1}{2} mv^2 = \int P dt = k \int t^2 dt = \frac{kt^3}{3}$$

$$\Rightarrow \quad v = \sqrt{\frac{2}{3}\frac{k}{m} t^3}$$

Hence the correct choice is (b).

4. The force **F** is parallel to the line

$$y = \frac{4}{3}x + C \qquad (1)$$

The particle moves along the line

$$y = -\frac{kx}{4} + \frac{3}{4} \qquad (2)$$

Work done is zero if the force is perpendicular to the displacement, i.e. if lines (1) and (2) are perpendicular each other. Thus the product of their slopes = – 1, i.e.

$$\frac{4}{3} \times \left(-\frac{k}{4}\right) = -1 \quad \Rightarrow \quad k = 3$$

5. Power $P = F \times v = \dfrac{k}{v} \times v = k$. Therefore, work done in time t is

$$W = \int_0^t P dt = kt \qquad (\because P = \text{constant})$$

Hence the correct choice is (a).

6. Given $F \propto x^{-2/3}$. Therefore, acceleration $\propto x^{-2/3}$, i.e.

$$v \frac{dv}{dx} = Kx^{-2/3} \qquad (K = \text{constant})$$

$$\therefore \quad \int v dv = K \int x^{-2/3} dx$$

$$\Rightarrow \quad v^2 \propto x^{1/3} \quad \text{or} \quad v \propto x^{1/6}$$

$$P = F \cdot v \propto x^{-2/3} \times x^{1/6} \propto x^{-1/2}$$

7. Let m be the mass of each bullet. Since the resistance offered by the plank is the same for the two bullets, the amount of work done by the plank is the same for the two bullets. From work-energy principle, the decrease in the kinetic energy is the same for the two bullets.

Decrease in KE of first bullet

$$= \frac{1}{2} mu_1^2 - \frac{1}{2} mv_1^2$$

$$= \frac{1}{2} m(200)^2 - \frac{1}{2} m(180)^2 \qquad \text{(i)}$$

If v_2 is the speed of the second bullet after passing through the plank, then

Decrease in KE of second bullet

$$= \frac{1}{2} mu_2^2 - \frac{1}{2} mv_2^2$$

$$= \frac{1}{2} m(100)^2 - \frac{1}{2} mv_2^2 \qquad \text{(ii)}$$

Equations (i) and (ii) we have,

$$\frac{1}{2} m(200)^2 - \frac{1}{2} m(180)^2$$

$$= \frac{1}{2} m(100)^2 - \frac{1}{2} mv_2^2$$

which gives $v_2^2 = 2400$ or $v \simeq 49$ ms $^{-1}$

Hence the correct choice is (b).

8. Thickness of plank (S) = 1.0 m
Mass of bullet (m) = 10 g = 10^{-2} kg
If F is the average resistive force exerted by the plank on the bullet, the work done by the plank on the bullet is

$$W = FS$$

Work done = decrease in KE of the bullet.

Hence $FS = \frac{1}{2}m(200)^2 - \frac{1}{2}m(180)^2 = 3800$ m

Putting $S = 1$ m and $m = 10^{-2}$ kg, we get $F = 38$ N.
Hence the correct choice is (b).

9. Since the car moves at a constant velocity, its acceleration is zero. Hence the engine has to do work only to overcome the frictional force (f). Since the distance moved in 1 second is v metres, the work done per second or the power of the engine is

$$P = f \times v = 1500 \times 5 = 7500$$

W = 7.5 kW

Hence the correct choice is (b).

10. When the car is being pulled along an inclined plane of gradient 1 in 10 $\left(\text{i.e. } \sin\theta = \frac{1}{10}\right)$, the engine has to do extra work against the component $Mg \sin\theta$ of the weight Mg of the car. The extra work per second or extra power the engine has to develop to maintain the same speed v is

$$= Mg \sin\theta \times v$$

$$= 1500 \times 10 \times \frac{1}{10} \times 5$$

$$= 7500 \text{ W} = 7.5 \text{ kW}$$

Hence the correct choice is (c).

11. The work done by gravity equals the change in the potential energy of the system after the vessels are interconnected. We may regard the liquid in each vessel as equivalent to a point mass kept at their respective centres of gravity. Remembering that the mass of the liquid is given by ($Ah\rho$) and that the PE of a mass at a height h in earth's gravity is mgh, we have

$$\text{Total PE at start} = (Ah_1\rho)g\frac{h_1}{2} + (Ah_2\rho)g\frac{h_2}{2}$$

$$= \frac{A\rho g}{2}(h_1^2 + h_2^2)$$

After the vessels are connected, the height of liquid in each vessel is $(h_1 + h_2)/2$.

Hence

$$\text{PE after connection} = \left\{A\rho\left(\frac{h_1+h_2}{2}\right)g\left(\frac{h_1+h_2}{2}\right)\right\}$$

$$= \frac{A\rho g}{4}(h_1 + h_2)^2$$

$$\text{Change in PE} = \frac{A\rho g}{4}\{(h_1 + h_2)^2 - 2(h_1^2 + h_2^2)\}$$

$$= -\frac{A\rho g}{4}(h_1 - h_2)^2$$

$$= -A\rho g\left(\frac{h_1 - h_2}{2}\right)^2$$

This must be equal to the work done 'by' gravity on the liquid. Thus the work done 'by' gravity is

$$A\rho g\left(\frac{h_1 - h_2}{2}\right)^2$$

Hence the correct choice is (c).

12. Volume (V) = 30 m^3, density of water (ρ) = 1000 kg m^{-3}. Therefore, mass of water to be lifted is

$$m = \rho V = 1000 \times 30 = 3 \times 10^4 \text{ kg}$$

Work done to lift this mass of water to a height $h = 60$ m is

$$W = mgh = 3 \times 10^4 \times 10 \times 60$$

$$= 1.8 \times 10^7 \text{ J}$$

Since the efficiency of the engine is 30%, the actual work done by the pump is

$$W' = \frac{W \times 100}{30} = \frac{1.8 \times 10^7 \times 100}{30}$$

$$= 6 \times 10^7 \text{ J}$$

Time taken $t = 10$ min = 600 s. Therefore, power consumed is

$$P = \frac{W'}{t} = \frac{6 \times 10^7}{600}$$

$$= 100{,}000 \text{ W} = 100 \text{ kW}$$

Hence the correct choice is (a).

13. Velocity $(v) = \frac{dx}{dt} = \frac{d}{dt}(3t^2 + 4t + 5) = 6t + 4$.

Acceleration is $a = \frac{dv}{dt} = \frac{d}{dt}(6t + 4) = 6$ ms^{-2}.

Therefore, applied force is $F = ma = 0.5 \times 6 = 3$ N.
Now $t = 2$s, the distance moved is

$$x = 3 \times (2)^2 + 4 \times 2 + 5 = 25 \text{ m}$$

$\therefore$ Work done $W = Fx = 3 \times 25 = 75$ J. Hence the correct choice is (c).

14. Let M kg of water fall per second. The power is P = rate at which work is done = mass per second $\times g \times h = Mgh$

But $P = 1$ MW $= 10^6$ W, $h = 10$ m. Therefore

$$M = \frac{P}{g\,h} = \frac{10^6}{10 \times 10} = 10^4 \text{ kg s}^{-1},$$

which is choice (b).

15. The weight of the rod acts at the centre of gravity which is at a distance of $l/2$ from the pivoted end. When the rod makes an angle θ with the horizontal, the vertical height of the centre of gravity is

$$h = \frac{l}{2} \sin\,\theta$$

i.e. the centre of gravity rises by an amount h. Therefore

$$\text{PE} = mgh = \frac{1}{2}\, mgl \sin\,\theta$$

Hence the correct choice is (b).

16. Let v be the speed of the bullet incident on the first plank. Its speed after it passes the plank $= \frac{9v}{10}$. If x is the thickness of the plank, the deceleration a due to the resistance of the plank is given by

$$2ax = v^2 - \left(\frac{9v}{10}\right)^2 = \frac{19v^2}{100} \qquad \text{(i)}$$

Suppose the bullet is stopped after passing through n such planks. Then the distance covered by the bullet is $s = nx$. Thus, we have

$$v^2 - 0 = 2as = 2anx$$

or $$n = \frac{v^2}{2ax} = \frac{v^2 \times 100}{19v^2} = \frac{100}{19} = 5.26$$

Thus the minimum number of planks required is 6. Hence the correct choice is (b).

17. Let u cms^{-1} be the speed of the bullet. Since the mass of the bullet remains unchanged, its speeds becomes $v = \frac{3u}{4}$ cms^{-1} after it penetrates a distance $x = 3.5$ cm. The retardation a due to the resistance of the wooden plank is given by

$$u^2 - v^2 = 2ax \text{ or } u^2 - \left(\frac{3u}{4}\right)^2 = 2a \times 3.5$$

which gives $a = \frac{u^2}{16}$ cms^{-2}. The bullet will come to rest when its velocity $v' = 0$. If x' is the thickness penetrated by the bullet, then

$$u^2 - v'^2 = 2ax'$$

or $$x' = \frac{u^2}{2a}. \text{ But } a = \frac{u^2}{16} \text{ cms}^{-2}.$$

Therefore $$x' = \frac{u^2 \times 16}{2u^2} = 8 \text{ cm}$$

Hence the correct choice is (a).

18. Since the wood offers a constant deceleration and hence a constant retardation force, the bullet will lose the remaining 75% of its kinetic energy after penetrating a further distance of $3x$. Therefore, the total distance penetrated by the bullet before it comes to rest $= x + 3x = 4x$. Hence the correct choice is (b).

19. Force of friction $= \mu mg$. Therefore, retardation $a = \mu\, mg/m = \mu g$.

Also $2ax = v^2$ or $2am^2x = m^2v^2$. But $p = mv$. Therefore,

$$2\, am^2x = p^2$$

But $a = \mu g$. Therefore, $2\, \mu g\, m^2x = p^2$

or $$\mu = \frac{p^2}{2gm^2x}.$$ Hence the correct choice is (a).

20. The mass per unit length of the chain $m = \frac{M}{L}$. The mass of the hanging portion of the chain is $m' = \frac{mL}{n}$. This mass can be assumed to be concentrated at the centre of the hanging portion of the chain which is a distance of $x = \frac{L}{2n}$ from the edge of the table. Therefore, the work done in pulling the hanging portion of the chain on to the table top is

$$W = m'gx = \frac{mL}{n} \times g \times \frac{L}{2n}$$

$$= \frac{mgL^2}{2n^2} = \frac{MgL}{2n^2}$$

Hence the correct choice is (d).

21. Since the platform is depressed by an amount x, the total work done on the spring is $mg\ (h + x)$. This work is stored in the spring in the form of potential energy $\frac{1}{2}\ kx^2$. Equating the two, we have

$$\frac{1}{2}\ kx^2 = mg\ (h + x) \text{ or } k = \frac{2m\,g(h+x)}{x^2}$$

Given, $h = 0.4$ m, $x = 0.1$ m, $m = 1$ kg and $g = 10\ \text{ms}^{-2}$. Substituting these values, we get $k = 1000\ \text{Nm}^{-1}$. Hence the correct choice is (c).

22. The instantaneous velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt}(3t - 4t^2 + t^3) = 3 - 8t + 3t^2$$

The instantaneous acceleration of the particle is

$$a = \frac{dv}{dt} = \frac{d}{dt}(3 - 8t + 3t^2) = -8 + 6t$$

Work done in first 4 seconds is

$$W = \int_0^4 F\,dx = \int_0^4 ma\,\frac{dx}{dt}\cdot dt$$

$$= m\int_0^4 (-8 + 6t)(3 - 8t + 3t^2)\,dt$$

$$= m\int_0^4 (-24 + 82t - 72t^2 + 18t^3)\,dt$$

$$= m\left|-24t + 41t^2 - 24t^3 + \frac{9}{2}t^4\right|_0^4$$

$$= m\,(-96 + 656 - 1536 + 1152)$$

$$= 176\,m = 176 \times 3 \times 10^{-3}$$

$$(\because\ m = 3 \times 10^{-3}\ \text{kg})$$

$$= 528 \times 10^{-3} = 528\ \text{mJ}$$

$$(\because\ 1\ \text{mJ} = 10^{-3}\ \text{J})$$

The closest choice is (d).

23. The centripetal force, being directed towards the centre is always perpendicular to the direction of displacement which is the direction of the velocity. Thus the dot product $\mathbf{F}\cdot\mathbf{S} = 0$, i.e. work done is zero, which is choice (a).

24. Kinetic energy $K = \frac{1}{2}mv^2$. If a is the acceleration, then $v^2 = 2ad$. But a = force/mass = F/m. Therefore, $v^2 = \frac{2Fd}{m}$. Hence $K = \frac{1}{2}m \times \frac{2Fd}{m}$ = Fd, which is independent of m. Thus the correct choice is (b).

25. Work done is

$$W = \int_0^5 F\,dx = \int_0^5 \left(7 - 2x + 3x^2\right)dx$$

$$= \left|7x - x^2 + x^3\right|_0^5$$

$$= 7 \times 5 - (5)^2 + (5)^3 = 135\ \text{J}$$

Hence the correct choice is (d).

26. The displacement of the particle is

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = (14\hat{\mathbf{i}} + 13\hat{\mathbf{j}} + 9\hat{\mathbf{k}}) - (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 6\hat{\mathbf{k}})$$

$$= (11\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 15\hat{\mathbf{k}})$$

Work done = $\mathbf{F}\cdot\mathbf{r}$

$$= (11\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 15\hat{\mathbf{k}})\cdot(4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$= 4 \times 11 + 11 + 3 \times 15 = 100\ \text{J}$$

Hence the correct choice is (b).

27. Momentum $p = mv$ or $p^2 = m^2v^2$ or $\frac{1}{2}\frac{p^2}{m} = \frac{1}{2}mv^2$. Thus the kinetic energy is

$$K = \frac{p^2}{2m}$$

If p increases by 20%, the new momentum is $p' = 1.2\,p$. Therefore the new kinetic energy will be

$$K' = \frac{(1.2p)^2}{2m} = 1.44\,\frac{p^2}{2m} = 1.44\,K$$

i.e K increases by 0.44 K. The percentage increase in K is $\frac{0.44\,K}{K} \times 100 = 44\%$, which is choice (a).

28. Work done = area under the $(F - x)$ graph

$$= \frac{1}{2} \times 10 \times 4 + 10 \times 4 + \frac{1}{2} \times 10 \times 4 = 80\ \text{J}$$

Now, work done = increase in kinetic energy. If v is the velocity at $x = 2$ m, then increase in K.E. = $\frac{1}{2}mv^2$. Therefore

$$\frac{1}{2}mv^2 = 80 \text{ or } v^2 = \frac{80 \times 2}{m} = \frac{80 \times 2}{0.1} = 1600$$

or $v = 40\ \text{ms}^{-1}$, which is choice (d).

29. In going from (0, 0) to (a, 0), the x-coordinate varies from 0 to a while the y-coordinate remains zero.

$\therefore$ Work done by force $\mathbf{F}$ along this path is ($\because\ y = 0$)

$$W_1 = \int_0^a \mathbf{F}\cdot\mathbf{dx} = \int_0^a -(Kx\hat{\mathbf{j}})\cdot dx\,\hat{\mathbf{i}} = 0$$

$$(\because\ \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} = 0)$$

In going from (a, 0) to (a, a), the x-coordinate remains constant at $x = a$ while the y-coordinate changes from 0 to a.

$\therefore$ Work done by force **F** along this path is ($\because x = a$)

$$W_2 = \int_0^a \mathbf{F} \cdot \mathbf{dy} = \int_0^a -K\left(y\,\hat{\mathbf{i}} + a\,\hat{\mathbf{j}}\right) \cdot dy\,\hat{\mathbf{j}}$$

$$= -Ka \int_0^a dy = -Ka^2$$

$$(\because \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0,\ \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = 1)$$

Since work is a scalar quantity, the total work done is

$$W = W_1 + W_2 = 0 - Ka^2 = -Ka^2$$

Hence the correct choice is (c).

30. The power output of the generator is directly proportional to (i) the velocity v of the air molecules in wind and (ii) the average kinetic energy KE = $\frac{1}{2}mv^2$ of the striking molecules (which is proportional v^2). Hence

Power output $\propto v \times v^2 \propto v^3$

which is choice (c).

31. The velocity of the body at time t is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{t^2}{4}\right) = \frac{t}{2}$$

$\therefore$ At $t = 0$, $v = u = 0$ and $t = 2$ s, $v = 1$ ms^{-1}, Now, work done = increase in KE

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}mv^2 - 0$$

$$= \frac{1}{2}mv^2 = \frac{1}{2} \times 6 \times (1)^2$$

= 3 J. Hence the correct choice is (d).

32. Work done = increase in potential energy in (i) raising the weight 150 N of the ladder through a height 1 m and (ii) raising a weight 40 N through 2.5 m

$$= 150 \text{ N} \times 1 \text{ m} + 40 \text{ N} \times 2.5 \text{ m}$$

$$= 250 \text{ Nm} = 250 \text{ J}$$

Hence the correct choice is (b).

33. Friction force = $\mu mg = 0.2 \times 5 \times 10 = 10$ N. Effective force F = applied force – frictional force = 25 – 10 = 15 N. Kinetic energy = work done by force F in pulling the body through a distance S (= 10 m) = $15 \times 10 = 150$ J, which is choice (b).

34. The retardation is given by [see Fig. 6.22]

$$a = g(\mu \cos\theta + \sin\theta) \qquad \text{(i)}$$

Let u be the initial velocity of the body. If it is stopped after moving a distance s up the plane, then

$$u^2 = 2as$$

$\therefore$ Kinetic energy = $E = \frac{1}{2}mu^2$

$$= \frac{1}{2} m \times 2as = mas \qquad \text{(ii)}$$

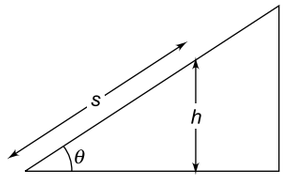


Fig. 6.22

Now, work done is

$$W = \text{gain in PE} = mgh$$

It is clear from the figure that $h = s \sin\theta$. Therefore,

$$W = mgs \sin\theta \qquad \text{(iii)}$$

From (i), we have

$$g = \frac{a}{(\mu\cos\theta + \sin\theta)} \qquad \text{(iv)}$$

Also $\mu = \tan\theta = \dfrac{\sin\theta}{\cos\theta}$

or $\sin\theta = \mu\cos\theta \qquad \text{(v)}$

Using (iv) and (v) in (iii), we have

$$W = \frac{mas(\mu\cos\theta)}{(\mu\cos\theta + \sin\theta)} \qquad \text{(vi)}$$

Using (ii) in (vi), we get

$$W = \frac{\mu E \cos\theta}{(\mu\cos\theta + \sin\theta)}, \text{ which is choice (d).}$$

35. Retardation ($-a$) is proportional to displacement (x), i.e. $-a \propto x$ or $a \propto -x$. Hence the motion of the particle is simple harmonic. When the displacement is x, the kinetic energy = $\frac{1}{2}m\omega^2(A^2 - x^2)$, where m, ω and A are the mass, angular frequency and amplitude respectively. When displacement $x = 0$, the kinetic energy = $\frac{1}{2}m\omega^2A^2$. Therefore, the loss of kinetic energy for a displacement x is

$$\frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2x^2$$

which is proportional to x^2 (since m and ω are constants of the motion. Hence the correct choice is (a).

36. Power delivered in time t_1 is $P_1 = \mathbf{F} \cdot \mathbf{v}_1 = m\,\mathbf{a} \cdot \mathbf{v}_1$

Now, acceleration vector is $\mathbf{a} = \dfrac{\mathbf{v}_1}{t_1}$. Therefore

$$P_1 = \frac{m\,v_1 \cdot v_1}{t_1} = \frac{mv_1^2}{t_1} \qquad (\because\ \mathbf{v}_1 \cdot \mathbf{v}_1 = v_1^2)$$

$\therefore$ Power delivered per unit time $= \dfrac{P_1}{t_1}$

Power delivered at time $t = \dfrac{P_1}{t_1} \times t = \dfrac{mv_1^2 t}{t_1^2}$

Hence the correct choice is (b). Notice that choices (a), (c) and (d) do not have the dimensions of power.

37. $W = \mathbf{F} \cdot \mathbf{r} = \left(5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) \cdot \left(2\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)$

$= 10\,\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} \cdot \hat{\mathbf{j}} \quad \left(\because\ \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{j}} = 0\right)$

$= 10 - 3 = 7$

Hence the correct choice is (b).

38. If mass m falls through a distance h, mass m rises up through the same distance h. Let v be the common velocity of the masses when this happens. Now, loss in PE = gain in KE, i.e.

$$Mgh - mgh = \frac{1}{2}\,(M + m)\,v^2$$

which gives $v = \sqrt{\dfrac{2gh\,(M-m)}{(M+m)}}$, which is choice (c).

39. When M has descended a distance h, loss of PE = Mgh. If v is the common velocity of the masses, gain in KE $= \dfrac{1}{2}\,(M+m)\,v^2$. Hence

$$\frac{1}{2}(M + m)\,v^2 = Mgh$$

or $\quad v = \sqrt{\dfrac{2Mgh}{(M+m)}}$, which is choice (c).

40. Relative speed of man with respect to escalator = $(v - u)$.

$\therefore$ Actual displacement of man per second = $(v - u)$.

Hence, the actual displacement of man in going up the escalator of hight h is $\dfrac{v\,h}{(v-u)}$. Therefore,

Work done $= mg \times \dfrac{vh}{(v-v)}$, which is choice (d)

41. Given $U = 6x + 8y$ joule and mass $m = 2$ kg. Force along x-axis is

$$|F_x| = \frac{dU}{dx} = \frac{d}{dx}(6x + 8y) = 6 \text{ newton}$$

Force along y-axis is

$$|F_y| = \frac{dU}{dy} = \frac{d}{dy}(6x + 8y) = 8 \text{ newton}$$

Therefore, the x and y components of acceleration are

$$a_x = \frac{|F_x|}{m} = \frac{6}{2} = 3 \text{ ms}^{-2}$$

and $\quad a_y = \dfrac{|F_y|}{m} = \dfrac{8}{2} = 4 \text{ ms}^{-2}$

$\therefore$ Resultant acceleration $a = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{(3)^2 + (4)^2} = 5 \text{ ms}^{-2}$

The x and y coordinates of the body at time t are

$$x = x_0 - \frac{1}{2}\,a_x\,t^2 = 6 - \frac{1}{2} \times 3 \times t^2$$

$$= \left(6 - \frac{3}{2}t^2\right) \text{ metre}$$

and $\quad y = y_0 - \dfrac{1}{2}\,a_y\,t^2 = 4 - \dfrac{1}{2} \times 4 \times t^2$

$= \left(4 - 2t^2\right)$ metre

The body will cross the y-axis when $x = 0$, i.e. at time t given by $\left(6 - \dfrac{3}{2}t^2\right) = 0$ or $t = 2$ s. Hence the correct choice is (b).

42. $v_x = a_x\,t = 3 \text{ ms}^{-2} \times 2 \text{ s} = 6 \text{ ms}^{-1}$

$v_y = a_y\,t = 4 \text{ ms}^{-2} \times 2 \text{ s} = 8 \text{ ms}^{-1}$

$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{(6)^2 + (8)^2} = 10 \text{ ms}^{-1}$

Hence the correct choice is (c).

43. Gravitational force is conservative. The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle. Hence the correct choice is (b).

44. The potential energy of the particle is given by

$$U = -\int F dx = -\int (-kx + ax^3)\,dx$$

or $\quad U = k\dfrac{x^2}{2} - a\dfrac{x^4}{4} = \dfrac{x^2}{4}\,(2k - ax^2) \qquad (1)$

From Eq. (1) it follows that $U = 0$ at two values of x which are $x = 0$ and $x = \sqrt{2k/a}$. Hence graphs (b) and (c) are not possible. Also U is maximum or minimum at a value of x given by $\dfrac{dU}{dx} = 0$, i.e.

$$0 = \frac{d}{dx}\left(\frac{kx^2}{2} - \frac{ax^4}{4}\right) = kx - ax^3$$

$$= x(k - ax^2)$$

or $\quad x = \sqrt{k/a}$. At this value of x,

U is maximum if $\frac{d^2U}{dx^2} < 0$,

Now $\frac{d^2U}{dx^2} = \frac{d}{dx}(kx - ax^3) = k - 3ax^2$.

At $x = \sqrt{k/a}$,

$$\frac{d^2U}{dx^2} = k - 3a\ \frac{k}{a}$$

$$= k - 3k = -2k, \text{ which is negative.}$$

Hence U is maximum at $x = \sqrt{k/a}$.
Hence graph (a) is also not possible. Also U is negative for $x > \sqrt{2k/a}$. Therefore, the correct graph is (d).

45. Mass of the drop m = volume × density of water = $\frac{4\pi}{3}r^3\rho$, where ρ is the density of water. Work done by gravitational force is

$$W = mgh = \frac{4\pi}{3}r^3\rho\, gh$$

Thus $W \propto r^3$. Hence the correct choice is (c).

46. At the highest point, the energy of the ball is entirely potential = mgh and at the lowest point, the energy is entirely kinetic = $\frac{1}{2}\ mv^2$. Since friction is absent, the principle of conservation of energy requires

$$\frac{1}{2}\ mv^2 = mgh$$

or $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = 2\ \text{ms}^{-1}$. Hence the correct choice is (b).

47. PE at $A = mgh$. Since 10% of this energy is lost, KE at point $B = mgh \times \frac{90}{100} = 0.9\ mgh$. Therefore,

$$\frac{1}{2}\ mv^2 = 0.9\ mgh$$

or $v^2 = 1.8\ gh = 1.8 \times 10 \times 2 = 36$

which gives $v = 6\ \text{ms}^{-1}$. Hence the correct choice is (d).

48. Initial PE = mgh. Now, gain in KE = loss in PE. Thus

$$\frac{1}{2}\ mv^2 = \frac{1}{2}\ mgh$$

or $$v = \sqrt{gh}$$

Hence the correct choice is (a).

49. Initial KE = $\frac{1}{2}\ mv^2$. Now, gain in PE = loss in KE. Thus

$$mgh = \frac{1}{4}\ mv^2$$

or $$h = \frac{v^2}{4g}$$

Hence the correct choice is (d).

50. If a is the deceleration due to the force of friction f, then $2ax = v^2$

or $$\frac{1}{2}\ mv^2 = max$$

or $\text{KE} = fx \qquad (\because f = ma)$

Thus if KE is doubled, x is also doubled. Hence the correct choice is (c).

51. Height h = 10 m. PE at this height = mgh. On reaching the ground, KE = mgh. Since the body loses 25% of energy due to impact, KE of the body after one impact = 0.75 mgh. If v_1 is the initial upward velocity after the impact, we have

$$\frac{1}{2}\ mv_1^2 = 0.75\ mgh = \frac{3}{4}\ mgh$$

or $$v_1^2 = 1.5\ gh$$

The height h_1 to which the body will rise is

$$h_1 = \frac{v_1^2}{2g} = \frac{1.5gh}{2g} = 0.75\ h$$

$$= 0.75 \times 10 = 7.5 \text{ m } (\because h = 10 \text{ m})$$

Hence the correct choice is (c).

52. After the second impact, the initial KE of body = 75% of $\frac{3}{4}\ mgh = \left(\frac{3}{4}\right)^2 mgh$, i.e.

$$\frac{1}{2}\ mv_2^2 = \left(\frac{3}{4}\right)^2 mgh$$

or $$v_2^2 = \frac{9}{8}\ gh$$

The height h_2 to which the body will rise after the second impact is

$$h_2 = \frac{v_2^2}{2g} = \frac{9gh}{8 \times 2g} = \frac{9}{16}h = \frac{9 \times 10}{16} = \frac{45}{8}\ \text{m}$$

Hence the correct choice is (d).

53. As the body rises, the initial kinetic energy is converted into potential energy. At the maximum height h, the energy is entirely potential = mgh, which is equal to the initial kinetic energy. Let h' be the height where the kinetic energy is 75% of its initial value. At this height, the potential energy

must be 25% of its maximum value, i.e. at height h', PE = 0.25 mgh. Thus $mgh' = 0.25\ mgh$ or $h' = \frac{h}{4}$. Hence the correct choice is (c).

54. Let f be the force of friction and m be the mass of the body. The retardation $a = f/m$. If v is the initial speed of the body, then

$$2ax = v^2$$

or
$$max = \frac{1}{2}mv^2 = k$$

But $ma = f$. Therefore $fx = k$ or $f = k/x$. Hence the correct choice is (a).

55. Potential energy function is

$$U(x) = -\int_0^x F\,dx = -k\int_0^x x\,dx = -\frac{1}{2}kx^2$$

The value of $U(x)$ is always negative for both positive and negative values of x. Thus the variation of potential energy with x is an inverted parabola as shown in choice (a).

56. Potential energy stored in spring S_1 when the block B is moved through a distance x is $U_1 = \frac{1}{2}k_1x^2 = \frac{1}{2}kx^2$. When the block is released, it moves to the left, compressing the spring S_2 through a distance y. The potential energy stored in spring S_2 when its compression is y is $U_2 = \frac{1}{2}k_2y^2 = \frac{1}{2}(4k)y^2 = 2ky^2$. Since y is the maximum compression of spring S_2, from conservation of energy, we have $U_1 = U_2$, i.e.

$$\frac{1}{2}kx^2 = 2ky^2$$

which givens $\frac{y}{x} = \frac{1}{2}$, which is choice (c)

57. Refer to the Fig. 6.23. Here $OA = OB = OC = L$ and $OD = OC\cos\theta = L\cos\theta$. Therefore $h = OA - OD = L - L\cos\theta$

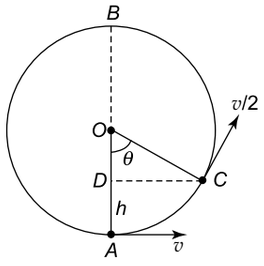


Fig. 6.23

or $h = L(1 - \cos\theta)$. From conservation of energy, total energy at A = total energy at C, i.e.

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + mgL(1 - \cos\theta)$$

$$v^2 = \frac{8gL}{3}(1 - \cos\theta) \qquad (1)$$

The minimum velocity the bob must have at A so as to reach B is $v = \sqrt{5gL}$. Putting this in Eq. (1), we get $\cos\theta = -\frac{7}{8}$. Therefore θ lies between $\frac{3\pi}{4}$ and π.

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. In which of the following is no work done by the force?
- (a) A man carrying a bucket of water, walking on a level road with a uniform velocity.
- (b) A drop of rain falling vertically with a constant velocity.
- (c) A man whirling a stone tied to a string in a circle with a constant speed
- (d) A man walking up on a staircase.

2. The work done by a force on a body does not depend upon
- (a) the mass of the body
- (b) the displacement of the body
- (c) the initial velocity of the body
- (d) the angle between the force vector and the displacement vector

3. A simple pendulum of length L and having a bob of mass M is oscillating in a plane about a vertical line between angular limits $-\alpha$ and $+\alpha$. At a time when the angular displacement is θ ($< \alpha$), the tension in the string is T and the velocity of the bob is v. Which of the following relations will hold?
- (a) $T\cos\theta = Mg$
- (b) $T - Mg\cos\theta = Mv^2/L$

(c) The magnitude of the tangential acceleration of the bob is $|a_T| = g \sin\theta$
(d) $T = Mg \cos\theta$

4. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that
(a) the velocity of the particle is constant
(b) the acceleration of the particle is constant
(c) the kinetic energy of the particle is constant
(d) the particle moves in a circular path.

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5. Two inclined frictionless tracks of different inclinations meet at A from where two blocks P and Q of different masses are allowed to slide down from rest at the same time, one on each track, as shown in Fig. 6.24.

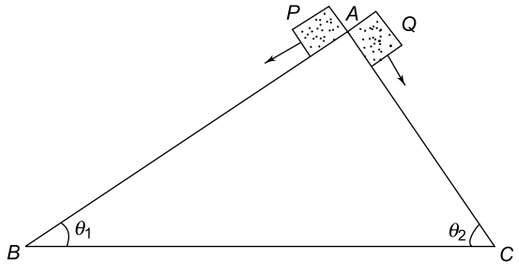

Fig. 6.24

(a) Both blocks will reach the bottom at the same time
(b) Block Q will reach the bottom earlier than block P
(c) Both blocks reach the bottom with the same speed
(d) Block Q will reach the bottom with a higher speed than block P

6. Choose the correct statements from the following:
(a) When a conservative force does positive work on a body, its potential energy increases.
(b) When a body does work against friction, its kinetic energy decreases.
(c) The rate of change of total momentum of a many–particle system is proportional to the net external force acting on the system.
(d) The rate of change of total momentum of many–particle system is proportional to the net internal force acting on the system.

7. Which of the following forces are conservative?
(a) Coulomb force between charged particles at rest
(b) Force of a compressed elastic spring
(c) Gravitational force between two masses
(d) Frictional force.

8. A body of m is moving in a straight line at a constant speed v. Its kinetic energy is k and the magnitude of its momentum is p. Which of the following relations is/are correct?

(a) $p = \sqrt{2mk}$ (b) $p = \sqrt{\frac{2k}{m}}$

(c) $2k = pv$ (d) $v = \sqrt{\frac{2k}{p}}$

9. A block of mass m is taken from the bottom of an inclined plane to its top and then allowed to slide down to the bottom again. The length of the inclined plane is L and the coefficient of friction between the block and the plane is μ. The inclination of the plane is θ.
(a) The work done by the gravitational force over the round trip is zero.
(b) The work done by the applied force over the upward journey is $mgL\,(\sin\theta + \mu\cos\theta)$.
(c) The work done by the frictional force over the round trip is zero.
(d) The kinetic energy of the block when it reaches the bottom is $mgL\,(\sin\theta - \mu\cos\theta)$.

10. A uniform rod has a mass m and a length l. The potential energy of the rod when
(a) it stands vertically is zero.
(b) it stands vertically is $mgl/2$
(c) it is inclined at an angle θ with vertical is $\frac{1}{2} mgl \cos\theta$.
(d) it is inclined at an angle θ with the vertical is $\frac{1}{2} mgl \sin\theta$.

11. A body is subjected to a constant force $\mathbf{F}$ in newton given by

$$\mathbf{F} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors along x, y and z axes respectively. The work done by this force in moving the body through a distance of
(a) 4 m along the z-axis is 12 J.
(b) 3 m along the y-axis is 6 J.
(c) 4 m along the z-axis and then 3 m along the y-axis is 18 J.
(d) 4 m along the z-axis and then 3 m along the y-axis is $\sqrt{(12)^2 + (6)^2}$ J.

12. Figure 6.25 shows the force F (in newton) acting on a body as a function of x. The work done in moving the body
(a) from $x = 0$ to $x = 1$ m is 2.5 J.
(b) from $x = 1$ m to $x = 3$ m is 10 J.
(c) from $x = 0$ to $x = 4$ m is 15 J.
(d) from $x = 0$ to $x = 4$ m is 12.5 J.

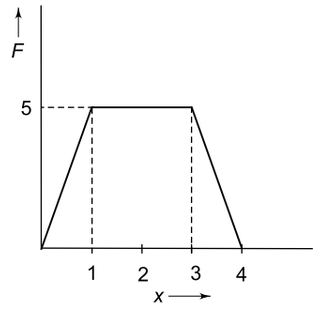

Fig. 6.25

13. A block of mass 2 kg, initially at rest on a horizontal floor, moves under the action of a force of 10 N. The coefficient of friction between the block and the floor is 0.2. If $g = 10\ \text{ms}^{-2}$

(a) the work done by the applied force in 4 s is 240 J.

(b) the work done by the frictional force in 4 s is 96 J.

(c) the work done by the net force in 4 s is 336 J.

(d) the change in kinetic energy of the block in 4 s is 144 J.

14. In which of the following cases is no work done by the force?

(a) A satellite revolving around the earth in a circular orbit.

(b) The electron revolving around the proton in a hydrogen atom.

(c) A charged particle moving in a circle in a uniform magnetic field.

(d) A man carrying a load on his head and walking on a level road at a uniform velocity.

15. A raindrop falls from a certain height above the ground. Due to the resistance of air, its acceleration gradually decreases until it becomes zero when the drop is at half its original height. If W_1 and W_2 are the amounts of work done by the gravitational force during the first and second half of the journey, then

(a) $W_1 > W_2$ (b) $W_1 < W_2$

(c) $W_1 = W_2 \neq 0$ (d) $W_1 = W_2 = 0$

16. A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force $F = k/r^2$ where k is a constant.

(a) The kinetic energy of the particle is $k/2r$.

(b) The potential energy of the particle is $-k/2r$.

(c) The total energy of the particle is $-k/2r$.

(d) The total energy of the particle is zero.

17. The displacement x (in metres) of a particle of mass 100 g moving in a straight line under the action of a constant force is related to time t (in seconds) as

$$\sqrt{x} = t - 2$$

(a) The acceleration of the particle is 1 ms^{-2}.

(b) The acceleration of the particle is 2 ms^{-2}.

(c) The velocity of the particle at $t = 3$ s is 2 ms^{-1}.

(d) The work done by the force in 5 s is 1.8 J.

18. Two springs 1 and 2 have spring constants k_1 and k_2 ($k_1 > k_2$). W_1 and W_2 are the amounts of work done to increase the lengths of springs 1 and 2 by the same amount and W_3 and W_4 are the amounts of work done when springs 1 and 2 are stretched with the same force. Then

(a) $W_1 > W_2$ (b) $W_1 < W_2$

(c) $W_3 > W_4$ (d) $W_3 < W_4$

19. A box of mass m is dragged along a horizontal surface with a uniform speed with a force F directed at an angle θ with the horizontal as shown in Fig. 6.26. The coefficient of kinetic friction between the box and the surface is μ.

(a) The normal reaction on the box is

$$R = \frac{F\cos\theta}{\mu}.$$

(b) The normal reaction on the box is

$$R = (mg - F\sin\theta)$$

(c) The work done on the box in dragging it through a distance x is

$$W = \frac{\mu mgx}{(\sin\theta + \mu\cos\theta)}.$$

(d) The work done on the box in dragging it through a distance x is

$$W = \frac{\mu mgx}{(\cos\theta + \mu\sin\theta)}.$$

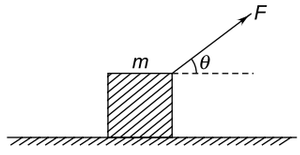

Fig. 6.26

20. A force $\mathbf{F} = (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})$ newton acts on a particle located at the origin O.

(a) The work done in taking the particle from O along the x-axis to a point A (2 m, 0) is 6 J.

(b) The work done in taking the particle from A parallel to the y-axis to a point B (3 m, 2 m) is 8 J.

(c) The work done in taking the particle from O to A and then to B is 14 J.

(d) The work done in taking particle from O directly to B is 10 J.

21. The potential energy of a system varies with distance x as

$$U = ax^2 - bx$$

where a and b are positive constants. Then

(a) the potential energy is minimum at $x = b/2a$ and $U_{min} = -\frac{b^2}{4a}$.

(b) the potential energy is maximum at $x = b/2a$ and $U_{max} = \frac{b^2}{4a}$.

(c) The force acting on the system decreases linearly with x.

(d) The force acting on the system is proportional to x^2.

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22. A car is of mass m moving along a circular track of radius r with a speed which increases linearly with time t as $v = kt$, where k is a constant. Then

(a) the instantaneous power delivered by the centripetal force is mk^3t^3/r.

(b) the power delivered by the centripetal force is zero.

(c) the instantaneous power delivered by the tangential force is mk^2t.

(d) the power delivered by the tangential force is zero.

23. A body of weight mg is suspended from a rigid support by two light strings AB and AC as shown in Fig. 6.27. The tension in string AB is T_1 and in string AC the tension is T_2. Then

(a) $T_1 = \sqrt{3}\, T_2$

(b) $T_2 = \sqrt{3}\, T_1$

(c) $T_1 = \frac{mg}{2}$, $T_2 = \frac{\sqrt{3}mg}{2}$

(d) $T_1 = \frac{\sqrt{3}mg}{2}$, $T_1 = \frac{mg}{2}$

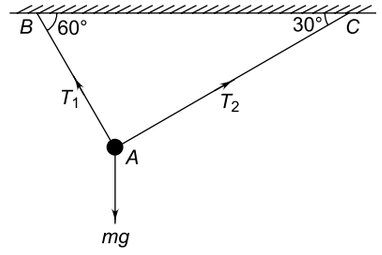


Fig. 6.27

24. The potential energy function of a particle executing linear simple harmonic motion is given by

$$U(x) = \frac{1}{2}kx^2$$

where x is the displacement of the particle from the equilibrium position $x = 0$ and k is the force constant of the oscillator. Figure 6.28 shows the graph of $U(x)$ against x for $k = 0.5$ Nm^{-1}. If the total energy of the particle is 1 J, it will turn back when it reaches the position

(a) $x = -1$ m (b) $x = -2$ m

(c) $x = +2$ m (d) $x = +1$ m

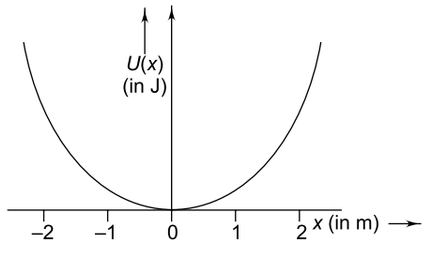


Fig. 6.28

ANSWERS AND SOLUTIONS

1. In choices (a) and (c), no work is done because the force of gravity in choice (a), and the centripetal force in choice (c) are perpendicular to the direction of the displacement. In choice (b), no net force acts on the raindrop since it is falling with a uniform velocity, hence no work is done. In choice (d), the man has to do work against gravity. Hence the correct choices are (a), (b) and (c).

2. The correct choices are (a) and (c).

3. Referring to Fig. 6.29, the net force along the string is $F = T - Mg \cos\theta$ and this force provides the centripetal force Mv^2/L necessary for circular motion of the bob. Hence choice (b) is correct. Relation (a) i.e. $T \cos\theta = Mg$ cannot hold for all values of θ because if $\theta = 0$, then $T = Mg$ and the net force along the string is

$$F = T - Mg \cos 0 = T - Mg = 0$$

Since $F = 0$, there would be no centripetal force and the bob would not oscillate. Hence choice (a) is incorrect.

The tangential acceleration a_T of the bob is caused by the tangential component $Mg \sin\theta$. Therefore, $a_T = Mg \sin\theta/M = g \sin\theta$. Hence choice (c) is correct. The relation (d) cannot hold because if $T = Mg \cos\theta$, the net force along the string will be zero. Therefore, there will be no centripetal force.

Hence the correct choices are (b) and (c).

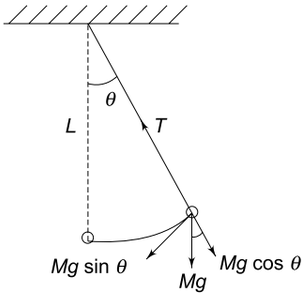


Fig. 6.29

4. The particle moves in a circular path with a uniform speed, because the force has a constant magnitude and is perpendicular to the velocity of the particle. The velocity and acceleration of the particle are not constant because their directions are changing as the particle moves in a circular path. The magnitude of the velocity (i.e. speed) is constant. Therefore, the kinetic energy $\frac{1}{2}\ mv^2$ is constant. Hence choices (c) and (d) are correct and choices (a) and (b) are incorrect.

5. Refer to Fig. 6.30.

The accelerations of blocks P and Q are

$$a_1 = \frac{m_1 g \sin\theta_1}{m_1} = g\ \sin\ \theta_1$$

and $$a_2 = \frac{m_2 g \sin\theta_2}{m_2} = g\ \sin\ \theta_2$$

Since $\theta_2 > \theta_1$; $a_2 > a_1$. Now PE of block P at A = $m_1\ gh$. Its KE on reaching the bottom = $\frac{1}{2}\ m_1 v_1^2$.

Equating the two we get

$$\frac{1}{2}\ m_1 v_1^2 = m_1 gh$$

or $$v_1 = \sqrt{2gh}$$

Similarly, for block Q, $v_2 = \sqrt{2gh}$. Since $v_1 = v_2$, both blocks will reach the bottom with the same speed. Now, $v_1 = a_1 t_1$ ($\because\ u = 0$) and $v_2 = a_2 t_2$. But $v_1 = v_2$. Therefore

$$a_1 t_1 = a_2 t_2$$

$$\frac{t_1}{t_2} = \frac{a_2}{a_1}$$

Since $a_2 > a_1$; $t_1 > t_2$; i.e. block P takes a longer time to reach the bottom. Hence the correct choices are (b) and (c).

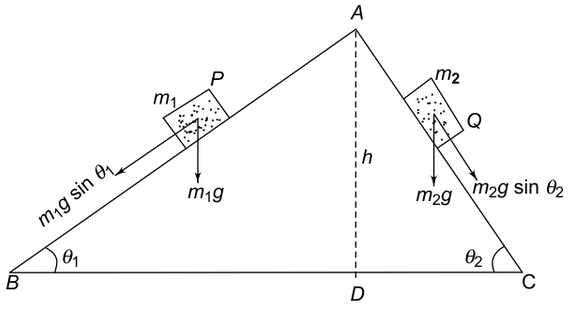


Fig. 6.30

6. The work done by a conservative force is equal to the negative of the potential energy. When the work done is positive, the potential energy decreases. Thus choice (a) is incorrect. Friction always opposes motion. Hence, when a body does work against friction, its kinetic energy decreases. Thus choice (b) is correct. The rate of change of total momentum of a many–particle system is proportional to the net force external to the system; the internal forces between particles cannot change the momentum of the system. Hence the correct choices are (b) and (c).

7. The correct choices are (a), (b) and (c).

8. Now $p = mv$ or $p^2 = m^2 v^2$ or $\frac{p^2}{2m} = \frac{1}{2}\ mv^2 = k$ or $p = \sqrt{2mk}$. Also $p = mv$ and $k = \frac{1}{2}\ mv^2$.

Dividing the two we get $2k = pv$. Hence the correct relations are (a) and (c).

9. The gravitational force is conservative. Therefore, the work done by the gravitational force over the round trip is zero.

Refer to Fig. 6.31.

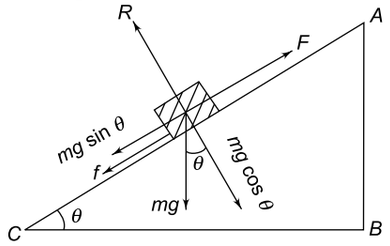


Fig. 6.31

When the block is moved from C to A, the force of friction f acts along the plane in the downward direction which is in the direction of the component $mg \sin\theta$ of the gravitational force. Hence the applied force F is

$$F = mg\ \sin\ \theta + f$$

Now $\mu = \dfrac{\text{frictional force}}{\text{normal reaction}} = \dfrac{f}{R} = \dfrac{f}{mg\cos\theta}$

$\therefore \quad f = \mu mg \cos\theta$

Therefore $F = mg \sin\theta + \mu mg \cos\theta$

$= mg(\sin\theta + \mu \cos\theta)$

$\therefore$ Work done by the applied force over the upward journey is

$W_a = F \times L = mg(\sin\theta + m\cos\theta) \times L$

The frictional force is non-conservative. Hence the work done by the frictional force over the round trip is not zero.

When the block is at a point A, it is at rest and its initial velocity $u = 0$. It is allowed to slide down the plane. Let v be the velocity when it reaches the bottom C of the plane. Since the frictional force now acts upwards, the net force acting on the block when it slides down is

$F_n = mg \sin\theta - f = mg \sin\theta - \mu mg \cos\theta$

$= mg(\sin\theta - \mu \cos\theta)$

$\therefore$ Acceleration of the block, $a = \dfrac{f_n}{m}$

$= g(\sin\theta - \mu \cos\theta)$. The velocity v is given by $v^2 - u^2 = 2aL$ (where $L = AC$) or $v^2 = 2aL$ $(\because u = 0)$

$\therefore$ Kinetic energy,

$$\text{KE} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} m \times 2AL = maL$$

or $\text{KE} = mg(\sin\theta - \mu\cos\theta)L$

Hence the correct choices are (a), (b) and (d).

10. The potential energy in the vertical position = work done in raising it from horizontal position to vertical position. In doing so, the mid-point of the rod is raised through a height $h = l/2$. Since the entire mass of the rod can be assumed to be concentrated at the mid-point (centre of gravity), the work done $= mgh = mgl/2$.

Refer to Fig. 6.32. $AD = AB = l$. In the inclined position, let the centre of gravity C of the rod be at a height h above the ground, so that $AC = l/2$. In triangle ACE, we have

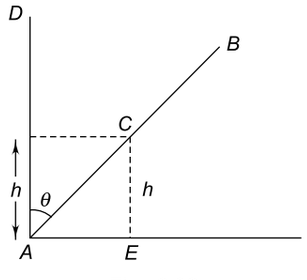


Fig. 6.32

$$h = AC \sin(90° - \theta) = \frac{l}{2} \cos\theta$$

$\therefore$ PE $= mgh = \dfrac{1}{2} mgl \cos\theta$. The correct choices are (b) and (c).

11. The correct choices are (a), (b) and (c).

Displacement along the z-axis is $\mathbf{S} = 4\hat{\mathbf{k}}$ metres. Therefore, work done is

$$W_1 = \mathbf{F} \cdot \mathbf{S}$$

$$= (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (4\hat{\mathbf{k}})$$

$$= -4\hat{\mathbf{i}} \cdot \hat{\mathbf{k}} + 8\hat{\mathbf{j}} \cdot \hat{\mathbf{k}} + 12\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}$$

Now $\hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = 0$ and $\hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$ because $\hat{\mathbf{j}}$ and $\hat{\mathbf{j}}$ are perpendicular to $\hat{\mathbf{k}}$. But $\hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$. Therefore, $W_1 = 12$ J.

Displacement along the y-axis is $\mathbf{S} = 3\hat{\mathbf{j}}$ metres. Therefore, the work done is

$W_2 = \mathbf{F} \cdot \mathbf{S} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (3\hat{\mathbf{j}}) = 6\hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = 6$ J

Since work is a scalar, the total work done is just the algebraic sum of W_1 and W_2, i.e. $W = W_1 + W_2 = 12 + 6 = 18$ J.

12. The correct choices are (a), (b) and (c).

Work done = area under the F–x graph. Work done by the force in moving the body from $x = 0$ to $x = 1$ m is (see Fig. 6.33)

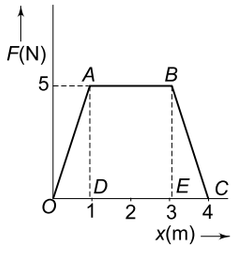


Fig. 6.33

W_1 = area of triangle OAD

$$= \frac{1}{2} \times AD \times OD$$

$$= \frac{1}{2} \times 5\text{N} \times 1\text{m} = 2.5 \text{ J}$$

Hence the correct choice is (a).

Work done in moving the body from $x = 1$ m to $x = 3$ m is

W_2 = area of rectangle $ABDE$

$= AD \times DE = 5 \text{ N} \times 2 \text{ m} = 10$ J,

which is choice (c)

Work done in moving the body from $x = 0$ to $x = 4$ m is

W_3 = area of (triangle OAD + rectangle $ABDE$ + triangle BEC)

$= 2.5 + 10 + 2.5 = 15$ J, which is choice (d).

13. The correct choices are (a), (b) and (d).

Force of friction (f) $= \mu mg = 0.2 \times 2 \times 10 = 4$N. Applied force ($F$) = 10 N. Since friction opposes motion, the net force acting on the body when it is moving is

$F' = F - f = 10 - 4 = 6$ N

$\therefore$ Acceleration $a = \dfrac{F'}{M} = \dfrac{6\text{ N}}{2\text{ kg}} = 3\text{ ms}^{-2}$

The distance travelled by block in 4s is

$S = ut + \dfrac{1}{2}\,at^2 = 0 + \dfrac{1}{2} \times 3 \times (4)^2 = 24$ m

$\therefore$ Work done by applied force in 4 s is

W = applied force × distance moved in 4 s

$= 10\text{ N} \times 24\text{ m} = 240$ J

Work done by the force of friction in 4 s is

$W = fS = 4\text{ N} \times 24\text{ m} = 96$ J.

Work done by the net force in 4 s is

$W = F's = 6\text{ N} \times 24\text{ m} = 144$ J

Velocity acquired by the block in 4 s is

$v = u + at = 0 + 3 \times 4 = 12\text{ ms}^{-1}$

Kinetic energy of the block at $t = 4$ s is

$$\text{KE} = \frac{1}{2}\,mv^2 = \frac{1}{2} \times 2 \times (12)^2 = 144\text{ J}$$

Since the initial KE = 0, the change in KE = 144 J.

14. All the four choices are correct. In choices (a), (b) and (c), the centripetal force (being radial) is perpendicular to the velocity (and hence displacement) vector. In choice (d), the gravitational force (being vertically downwards) is perpendicular to the displacement.

15. The correct choice is (c). Work done is each half of the journey = mgh where $h = H/2$; H being the original height from which the drop fell.

16. The correct choices are (a) and (c).

$$F = \frac{mv^2}{r} = \frac{k}{r^2} \text{ or } mv^2 = \frac{k}{r}\text{. Therefore,}$$

$$\text{KE} = \frac{1}{2}\,mv^2 = \frac{k}{2r}$$

Now
$$\text{PE} = \int_{\infty}^{r} F\,dr = \int_{\infty}^{r} \frac{k}{r^2}\,dr = k\int_{\infty}^{r} \frac{dr}{r^2}$$

$$= -k\left|\frac{1}{r}\right|_{\infty}^{r} = -\frac{k}{r}$$

$\therefore$ Total energy = KE + PE

$$= \frac{k}{2r} - \frac{k}{r} = -\frac{k}{2r}$$

17. The correct choices are (b), (c) and (d).

$x = (t-2)^2 = t^2 - 4t + 4$

$$v = \frac{dx}{dt} = \frac{d}{dt}(t^2 - 4t + 4) = 2t - 4$$

$\therefore$ Acceleration $a = \dfrac{dv}{dt} = \dfrac{d}{dt}(2t - 4) = 2\text{ ms}^{-2}$

Now $F = ma = 0.1\text{ kg} \times 2\text{ ms}^{-2}$

$= 0.2$ N

Now distance moved in $t = 5$ s

$= (5)^2 - 4 \times 5 + 4 = 9$ m.

$\therefore$ Work done $W = 0.2\text{ N} \times 9\text{ m}$

$= 1.8$ Nm or J

18. If the increase in length is x, $W_1 = \dfrac{1}{2}\,k_1x^2$ and $W_2 = \dfrac{1}{2}\,k_2x^2$. Therefore, $\dfrac{W_1}{W_2} = \dfrac{k_1}{k_2}$. Since $k_1 > k_2$; W_1 is greater than W_2.

Let a force F extend the first spring by x_1 and the second by x_2. Then $F = k_1x_1 = k_2\,x_2$ or $\dfrac{x_1}{x_2} = \dfrac{k_2}{k_1}$.

Now $W_3 = \dfrac{1}{2}\,k_1x_1^2$ and $W_4 = \dfrac{1}{2}\,k_2\,x_2^2$. Therefore,

$$\frac{W_3}{W_4} = \frac{k_1}{k_2}\left(\frac{x_1}{x_2}\right)^2 = \frac{k_1}{k_2} \times \left(\frac{k_2}{k_1}\right)^2 = \frac{k_2}{k_1} < 1.$$

Thus the correct choices are (a) and (d).

19. The different forces acting on the block are shown in Fig. 6.34. It follows that

$$\mu R = F\cos\theta \text{ and } R + F\sin\theta = mg$$

Eliminating R we get

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

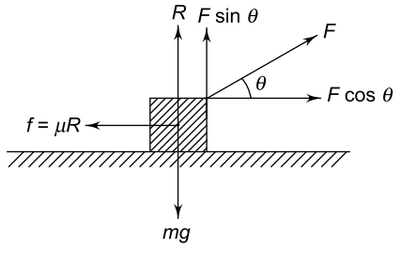


Fig. 6.34

Work done $W = Fx$

Thus the correct choices are (a), (b) and (d).

20. $W_{O \to A} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \cdot (2\hat{\mathbf{i}}) = 6$ J

$W_{A \to B} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \cdot (2\hat{\mathbf{j}}) = 8$ J

$W_{O \to A \to B} = 6 + 8 = 14$ J

$W_{O \to B} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \cdot (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = 6 + 8 = 14$ J

Hence the correct choices are (a), (b) and (c)

21. U is minimum if $\dfrac{dU}{dx} = 0$ and $\dfrac{d^2U}{dx^2} > 0$. Now

$\frac{dU}{dx} = \frac{d}{dx}(ax^2 - bx) = 2ax - b$. So $\frac{dU}{dx} = 0$ at $x = b/2a$. Also $\frac{d^2U}{dx^2} = \frac{d}{dx}(2ax - b) = 2a$, which is positive. Hence choice (a) is correct and choice (b) is wrong. For $x = b/2a$,

$$U_{\min} = a\left(\frac{b}{2a}\right)^2 - b\left(\frac{b}{2a}\right) = -\frac{b^2}{4a}.$$

Force acting on the system is

$$F = -\frac{dU}{dx} = b - 2ax$$

Hence choice (c) is correct and choice (d) is wrong.

22. Centripetal force $F_c = \frac{mv^2}{r} = \frac{mk^2t^2}{r}$. Since $\mathbf{F}_c$ is perpendicular to $\boldsymbol{v}$, $P = \mathbf{F}_c \cdot \boldsymbol{v} = 0$. Tangential force $F_t = \frac{mdv}{dt} = m\frac{d}{dt}(kt) = mk$. Since $\mathbf{F}_t$ is parallel to $\boldsymbol{v}$, $P = \mathbf{F}_t \cdot \boldsymbol{v} = mk \cdot kt = mk^2t$.

Hence the correct choices are (b) and (c).

23. Refer to Fig. 6.35. It follows from the figure that

$$T_1 \sin 60° = T_2 \sin 30° \quad (1)$$

and $$T_1 \cos 60° + T_2 \cos 30° = mg \quad (2)$$

Equations (1) and (2) give $T_1 = \frac{mg}{2}$ and $T_2 = \sqrt{3}\,T_1 = \frac{\sqrt{3}mg}{2}$. Thus the correct choices are (b) and (c).

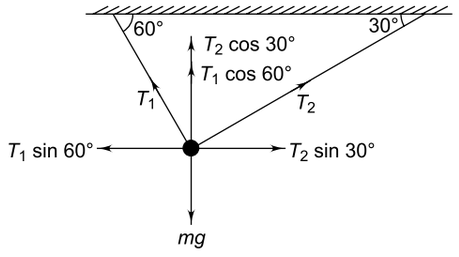


Fig. 6.35

24. At any instant during its motion, the energy of the particle is partly kinetic and partly potential, but the total energy remains constant. If v is the velocity of the particle and x its displacement from the mean position at an instant of time, then at that instant the total energy is

$$E = \text{K.E.} + \text{P.E.} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where m is the mass of the particle. The particle will turn back when $v = 0$ momentarily. At that moment $E = \frac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2\times1}{0.5}} = \pm 2$ m. Hence the correct choices are (b) and (c).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage

Passage I

Invariance

Newton's laws of motion are applicable in all inertial reference frames. Some physical quantities, when measured by observers in different reference frames, have exactly the same value. Such physical quantities are called *invariant*. In Newtonian mechanics mass, time, and force are invariant quantities. On the other hand, some physical quantities, when measured by observers in different reference frames, do not have the same value. Such physical quantities are called not invariant. In Newtonian mechanics displacement, velocity and work (which is the dot product of force and displacement) are not invariant. Also kinetic energy $\left(=\frac{1}{2}mv^2\right)$ is not invariant.

Physicists believe that all laws of physics are invariant in all inertial reference frames. For example, the work-energy principle states that the change in the kinetic energy of a particle is equal to the work done on it by the force. Although, work and kinetic energy are not invariant in all reference frames, the work-energy principle remains invariant. Thus even though different observers measuring the motion of the same particle find different values of work and change in kinetic energy, they all find that the work-energy principle holds in their respective frames.

1. Choose the invariant quantities from the following
(a) mass (b) time
(c) velocity (d) displacement

2. Which of the following quantities is/are not invariant
(a) Work (b) Kinetic energy
(c) Torque (d) Displacement

3. Choose the correct statements form the following.
   (a) Kinetic energy is not invariant.
   (b) Potential energy is not invariant.
   (c) Laws of conservation of energy and momentum are invariant.
   (d) All laws of physics are invariant.

ANSWERS

1. The correct choices are (a) and (b).
2. All choices are correct.
3. The correct choices are (c) and (d).

Questions 4 to 8 are based on the following passage

Passage II

Mechanical energy exists in two forms: kinetic energy and potential energy. Kinetic energy is the energy possessed by a body by virtue of motion. Potential energy is the energy possessed by a body by virtue of its position or configuration. These two forms of energy are inter-convertible. If no other form of energy is involved in a process, the sum of kinetic energy and potential energy always remains constant.

4. Two particles of masses m_1 and m_2 have equal linear momenta. The ratio of their kinetic energies is
   (a) 1 (b) $\sqrt{\frac{m_2}{m_1}}$
   (c) $\frac{m_2}{m_1}$ (d) $\left(\frac{m_2}{m_1}\right)^2$
5. Two particles of masses m_1 and m_2 have equal kinetic energies. The ratio of their linear momenta is
   (a) 1 (b) $\sqrt{\frac{m_1}{m_2}}$
   (c) $\frac{m_1}{m_2}$ (d) $\left(\frac{m_1}{m_2}\right)^2$
6. A particle of mass m has half the kinetic energy of another particle of mass $m/2$. If the speed of the heavier particle is increased by 2 ms^{-1}, its new kinetic energy equals the original kinetic energy of the lighter particle. The ratio of the original speeds of the lighter and heavier particles is
   (a) 1 : 1 (b) 1 : 2
   (c) 1 : 3 (d) 1 : 4
7. In Q.6, what is the original speed of the heavier particle?
   (a) $2(1 + \sqrt{2})$ ms^{-1} (b) $2(1 - \sqrt{2})$ ms^{-1}
   (c) $(2\sqrt{2} + 1)$ ms^{-1} (d) $(2\sqrt{2} - 1)$ ms^{-1}
8. A uniform rod of mass m and length l is made to stand vertically on one end. The potential energy of the rod in this position is
   (a) $\frac{mgl}{4}$ (b) $\frac{mgl}{3}$
   (c) $\frac{mgl}{2}$ (d) mgl

SOLUTION

4. Kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2m}(mv)^2 = \frac{p^2}{2m}$, where $p = mv$ is the linear momentum. Thus

$$K_1 = \frac{p^2}{2m_1} \text{ and } K_2 = \frac{p^2}{2m_2}$$

$\therefore \frac{K_1}{K_2} = \frac{m_2}{m_1}$, which is choice (c).

5. $K = p^2/2m \rightarrow p = \sqrt{2mK}$. Hence the correct choice is (b).
6. Let v and v' be the original speeds of the heavier and the lighter particles respectively. We then have

$$\frac{1}{2}mv^2 = \frac{1}{2} \times \left\{\frac{1}{2}\left(\frac{m}{2}\right)v'^2\right\}$$

$$\therefore \quad v^2 = \frac{1}{4}v'^2 \text{ or } v' = 2v$$

Hence the correct choice is (b).

7. When the heavier particle is speeded up by 2.0 ms^{-1}, its kinetic energy becomes $\frac{1}{2}m(v + 2)^2$. Since this equals the original kinetic energy of the lighter particle, we have

$$\frac{1}{2}m(v + 2)^2 = \frac{1}{2}(m/2)(4v^2)$$

$$v^2 + 4 + 4v = 2v^2 \text{ or } v^2 - 4v - 4 = 0$$

$$\therefore \quad v = \frac{4 \pm \sqrt{16 + 16}}{2}$$

$$= \frac{4 \pm 2\sqrt{8}}{2} = 2 \pm 2\sqrt{2}$$

The positive root is $v = 2 + 2\sqrt{2} = 2(1 + \sqrt{2})$.

Hence the correct choice is (a).

8. The potential energy in the vertical position = work done in raising it from horizontal position to vertical position. In doing so, the mid-point of the rod is raised through a height $h = l/2$. Since the entire mass of the rod can be assumed to be concentrated at the mid-point (centre of gravity), the work done $= mgh = mgl/2$. Hence the correct choice is (c).

Questions 9 to 11 are based on the following passage

Passage III

A light rod of length L having a body of mass M attached to its end hangs vertically. It is turned through 90° so that it is horizontal and then released.

9. The centripetal acceleration when the rod makes an angle θ with the vertical is

(a) $g \cos \theta$ (b) $2g \cos \theta$

(c) $g \sin \theta$ (d) $2g \sin \theta$

10. The tension in the rod when it makes an angle θ with the vertical is

(a) $Mg \cos \theta$ (b) $2\ Mg \cos \theta$

(c) $3\ Mg \cos \theta$ (d) zero

11. The value of θ when the tension in the rod equals the weight of the body is given by

(a) $\theta = \cos^{-1}\left(\frac{1}{2}\right)$ (b) $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

(c) $\theta = \sin^{-1}\left(\frac{1}{2}\right)$ (d) $\theta = \sin^{-1}\left(\frac{1}{3}\right)$

SOLUTION

9. The rod is released from the horizontal position OA. Let OB be the position of the rod when tension in the rod is T (Fig. 6.36).

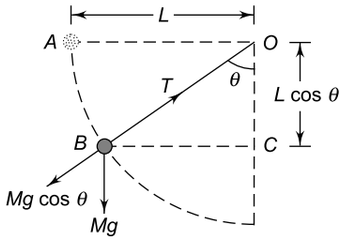


Fig. 6.36

Let θ be the angle with the vertical at this position. The loss of PE when the body falls from A to $B = Mg \times OC = MgL \cos \theta$. If v is the velocity of the body at B, then

$$\frac{1}{2} Mv^2 = MgL \cos \theta \text{ or } v^2 = 2gL \cos \theta \quad (1)$$

$$\text{centripetal acceleration} = \frac{v^2}{L} = \frac{2gL\cos\theta}{L}$$
$$= 2g \cos \theta,$$

which is choice (b).

10. The centripetal force when the body is at B is

$$F_c = \frac{Mv^2}{L}$$

Thus, we have

$$T - Mg \cos \theta = \frac{Mv^2}{L} \quad (2)$$

Using (1) in (2), we get

$$T - Mg \cos \theta = \frac{M}{L} \times 2gL \cos \theta = 2\ Mg \cos \theta$$

or $T = 3\ Mg \cos \theta$

Thus the correct choice is (c).

11. $T = Mg$. Therefore,

$Mg = 3\ Mg \cos \theta$ or $\cos \theta = \frac{1}{3}$, which is choice (b).

Questions 12 to 14 are based on the following passage

Passage IV

A small roller coaster starts at point A with a speed u on a curved track as shown in Fig. 6.37. The friction between the roller coaster and the track is negligible and it always remains in contact with the track.

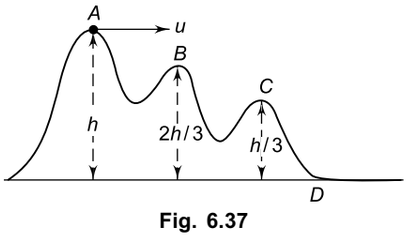


Fig. 6.37

12. The speed of the roller coaster at point B on the track will be

(a) $(u^2 + gh)^{1/2}$ (b) $\left(u^2 + \frac{2gh}{3}\right)^{1/2}$

(c) $(u^2 + 2gh)^{1/2}$ (d) $\left(u^2 + \frac{3gh}{2}\right)^{1/2}$

13. The speed of the roller coaster at point C on the track will be

(a) $\left(u^2 + \frac{gh}{3}\right)^{1/2}$ (b) $\left(u^2 + \frac{2gh}{3}\right)^{1/2}$

(c) $\left(u^2 + \frac{4gh}{3}\right)^{1/2}$ (d) $(u^2 + 2gh)^{1/2}$

14. The speed of the roller coaster at point D on the track will be

(a) $(u^2 + gh)^{1/2}$ (b) $(u^2 + 2gh)^{1/2}$

(c) $(u^2 + 3gh)^{1/2}$ (d) $(u^2 + 4gh)^{1/2}$

SOLUTION

12. Total energy at A = KE + PE = $\frac{1}{2}mu^2 + mgh$. If v_b is the speed at point B, the total energy at $B = \frac{1}{2}mv^2{}_b + mg(2h/3)$. From the principle of conservation of energy, we have

$$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv_b^2 + \frac{2mgh}{3}$$

which gives $v_b = \left(u^2 + \frac{2gh}{3}\right)^{1/2}$,

which is choice (b).

13. Similarly, the speed at point C is given by

$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv_c^2 + \frac{mgh}{3}$ which gives

$v_c = \left(u^2 + \frac{4gh}{3}\right)^{1/2}$, which is choice (c).

14. At point D, the energy is entirely kinetic. If the speed of the roller coaster at point D is v_d, then we have

$$\frac{1}{2}mv_d^2 = mgh + \frac{1}{2}mu^2$$

or $v_d = (u^2 + 2gh)^{1/2}$, which is choice (b).

Questions 15 to 17 are based on the following passage

Passage V

The displacement x of a particle moving in one dimension, under the action of a constant force is related to time t by the equation

$$t = \sqrt{x} + 3$$

where x is in metre and t is in second.

15. The displacement of the particle when its velocity is zero is

(a) zero (b) 1 m (c) 2 m (d) 3 m

16. The acceleration of the particle

(a) increases with time

(b) decreases with time

(c) increases with time up to t = 3 s and then decreases with time.

(d) remains constant at 2 ms^{-2}.

17. The work done by the force in first 6 s is

(a) 1 J (b) 3 J (c) 6 J (d) zero

SOLUTION

15. Given $t = \sqrt{x} + 3$ or $\sqrt{x} = t - 3$ or

$$x = (t-3)^2 \quad (1)$$

Differentiating (1) with respect to t, we get

$$\frac{dx}{dt} = 2(t-3)$$

or $v = 2(t-3)$ (2)

From (2) it follows that $v = 0$ at t = 3 s. Using t = 3 s in (1), we get x = 0. Thus, the displacement of the particle is zero when its velocity is zero. Thus the correct choice is (a).

16. From Eq. (2), we have

$$a = \frac{dv}{dt} = \frac{d}{dt}[2(t-3)] = 2\ ms^{-2}.$$

Hence the correct choice is (d).

17. From Eq. (2), the initial velocity, i.e., velocity at t = 0 is

$$v_0 = 2(0-3) = -6\ ms^{-1}$$

Final velocity, i.e., velocity at t = 6 s is

$$v = 2(6-3) = 6\ ms^{-1}$$

Work done = final KE – initial KE

$$= \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m(v^2 - v_0^2)$$

$$= \frac{1}{2}m[(6)^2 - (-6)^2] = 0,$$ which is choice (d).

Questions 18 to 20 are based on the following passage

Passage VI

The work done by a constant force acting on a body is given by

$$W = \mathbf{F} \cdot \mathbf{r}$$

where $\mathbf{F}$ is the force vector and $\mathbf{r}$ is displacement vector. The displacement vector $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ where $\mathbf{r}_1$ is the initial position vector and $\mathbf{r}_2$ is the final position vector.

If the force is variable, the work done in moving a body from a position $\mathbf{r}_1$ to a position $\mathbf{r}_2$ is given by

$$W = \int_{r_1}^{r_2} \mathbf{F} \cdot \mathbf{dr}$$

where $\mathbf{dr}$ is an infinitesimally small displacement.

18. A particle is moved from a position $\mathbf{r}_1 = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}})$ metre to a position $\mathbf{r}_2 = (5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 9\hat{\mathbf{k}})$ metre under the action of a force $\mathbf{F} = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})$ newton. The work done is

(a) zero (b) 13 J

(c) 27 J (d) 35 J

19. A body of mass m is projected from a tower of height h at an angle θ above the horizontal. The work done by the gravitational force during the time it takes to hit the ground is

(a) $mgh(1 + \cos\theta)$ (b) $mgh(1 + \sin\theta)$

(c) mgh (d) zero

20. A body of mass m is projected from the ground with a velocity u at an angle θ above the horizontal. The work done by the gravitational force in time $t = \dfrac{u\sin\theta}{g}$ is

(a) $2mu^2\sin^2\theta$ (b) $mu^2\sin^2\theta$

(c) $\dfrac{mu^2\sin^2\theta}{2}$ (d) zero

21. If $\theta = 45°$, the work by the gravitational force in Q.20 above in time $t = \dfrac{2u\sin\theta}{g}$ is

(a) $\dfrac{mu^2}{4}$ (b) $\dfrac{mu^2}{2}$

(c) mu^2 (d) zero

SOLUTION

18. $W = \mathbf{F}\cdot\mathbf{r} = \mathbf{F}\cdot(\mathbf{r}_2 - \mathbf{r}_1)$

$= (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot[(5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 9\hat{\mathbf{k}}) - (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}})]$

$= (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 13\hat{\mathbf{k}})$

$= 2 + 12 + 13 = 27$ J, which is choice (c)

19. Net displacement from the time the body is projected to the time it hits the ground is = h vertically downwards.

$\therefore$ Work done $W = mgh$, which is choice (c).

20. $t = \dfrac{u\sin\theta}{g}$ = half the time of flight. During this time the body attains maximum height $h_{max} = \dfrac{u^2\sin^2\theta}{2g}$.

$\therefore$ Work done = $mgh_{max} = \dfrac{mu^2\sin^2\theta}{2}$ which is choice (c).

21. Net displacement = 0. Hence $W = 0$. Thus the correct choice is (d).

SECTION IV

Matching

1.

Column I	*Column II*
(a) Force	(p) Slope of force-extension graph
(b) Impulse	(q) Area under force-time graph
(c) Energy stored in a spring	(r) Area under force-extension graph
(d) Force constant of a spring	(s) Slope of linear momentum-time graph

ANSWER

(a) → (s) (b) → (q)

(c) → (r) (d) → (p)

SECTION V

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each questions has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1
A simple pendulum of length l is displayed from its mean position O to position A so that the string makes an angle θ_1 with the vertical and then released. If air resistances is neglected, the speed of the bob when the string makes an angle θ_2 with the vertical is $v = \sqrt{2gl(\cos\theta_2 - \cos\theta_1)}$.

Statement-2
The total momentum of a system is conserved if no external force acts on it.

2. Statement-1
A uniform rod of mass m and length l is held at an angle θ with the vertical. The potential energy of the rod in this position is $\frac{1}{2}\ mg\ l \cos\ \theta$.

Statement-2
The entire mass of the rod can be assumed to be concentrated at its centre of mass.

3. Statement-1
A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

Statement-2
The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

IIT, 2007

4. Statement-1
A man carrying a bucket of water and walking on a rough level road with a uniform velocity does no work while carrying the bucket.

Statement-2
The work done on a body by a force **F** in giving it a displacement **S** is defined as

$$W = \mathbf{F} \cdot \mathbf{S} = FS \cos\ \theta$$

where θ is the angle between vectors **F** and **S**.

5. Statement-1
A crane P lifts a car up to a certain height in 1 min. Another crane Q lifts the same car up to the same height in 2 min. Then crane P consumes two times more fuel than crane Q.

Statement-2
Crane P supplies two times more power than crane Q.

6. Statement-1
Two inclined frictionless tracks of different inclinations θ_1 and θ_2 meet at A from where two blocks P and Q of different masses m_1 and m_2 are allowed to slide down from rest, one on each track as shown in Fig. 6.38. Then blocks P and Q will reach the bottom with the same speed.

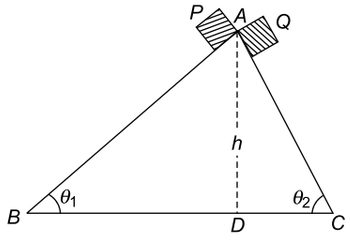


Fig. 6.38

Statement-2
Blocks P and Q have equal accelerations down their respective tracks.

7. Statement-1
In Q.6 above, block P will take a longer time to reach the bottom than block Q.

Statement-2
Block Q has a greater acceleration down the track than block P.

8. Statement-1
Comets move around the sun in highly elliptical orbits. The work done by the gravitational force of the sun on a comet over a complete orbit is zero.

Statement-2
The gravitational force is conservative.

9. Statement-1
The total energy of a system is always conserved irrespective of whether external forces act on the system.
Statement-2
If external forces act on a system, the total momentum and energy will increase.

10. Statement-1
The rate of change of the total linear momentum of a system consisting of many particles is proportional to the vector sum of all the internal forces due to inter-particle interactions.
Statement-2
The internal forces can change the kinetic energy of the system of particles but not the linear momentum of the system.

11. Statement-1
An elastic spring of force constant k is stretched by a small length x. The work done in extending the spring by a further length x is $2\,kx^2$.
Statement-2
The work done in extending an elastic spring by a length x is proportional to x^2.

SOLUTIONS

1. The correct choice is (b). It is clear from Fig. 6.39 that $PQ = l\cos\theta_1$ and $PR = l\cos\theta_2$. Therefore, $h_1 = l - l\cos\theta_1 = l(1-\cos\theta_1)$ and $h_2 = l(1-\cos\theta_2)$. Let m be the mass of the bob and v be its speed when it reaches position B. Then, from the principle of conservation of energy, K.E. at B = loss of P.E. as the bob moves from A to B.
Hence

$$\frac{1}{2}mv^2 = mgh_1 - mgh_2$$
$$= mg[l(1-\cos\theta_1) - l(1-\cos\theta_2)]$$
$$= mg\,l\,(\cos\theta_2 - \cos\theta_1)$$
$$\Rightarrow \quad v = \sqrt{2gl(\cos\theta_2 - \cos\theta_1)}$$

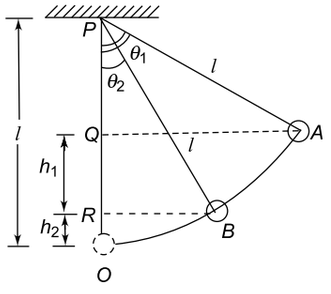


Fig. 6.39

2. The correct choice is (a). Let C be the centre of mass of the rod AB so that $AC = l/2$. Let h be the height of C above the ground. In triangle ACD, we have $CD = AC\sin(90° - \theta)$ (see Fig. 6.40).
Or $h = \frac{l}{2}\cos\theta$. Since the entire mass of the rod can be assumed to be concentrated at the centre of mass, therefore, potential energy = work done to raise the rod from horizontal position on the ground to the position shown in the figure = $mgh = \frac{1}{2}mgl\cos\theta$.

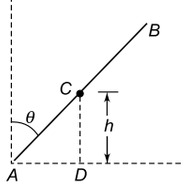


Fig. 6.40

3. Statement-1 is true. The decrease in mechanical energy is smaller when the block made to go up on the inclined surface because some part of the kinetic energy is converted into gravitational potential energy. Statement-2 is false. The coefficient of friction does not depend on the angle of inclination of the plane. Hence the correct choice is (c).

4. The correct choice is (a). Since the velocity is uniform, the man exerts no net force on the bucket in the direction of motion. The only force he exerts on the bucket is against gravity (to overcome) the weight mg of the bucket) and this force is perpendicular to the displacement (i.e. $\theta = 90°$).
Hence $W = FS\cos 90° = 0$.

5. The two cranes do the same amount of work = mgh. Hence they consume the same amount of fuel. Crane P does the same amount of work in half the time. Hence crane P supplies two times more power than crane Q. Thus the correct choice is (d).

6. The acceleration of blocks P and Q respectively are

$$a_1 = \frac{m_1 g\sin\theta_1}{m_1} = g\sin\theta_1$$

and

$$a_2 = \frac{m_2 g\sin\theta_2}{m_2} = g\sin\theta_2$$

Since $\theta_2 > \theta_1$; $a_2 > a_1$. The potential energy of block P at $A = m_1gh$. When it reaches the bottom B, its kinetic energy is $\frac{1}{2}m_1v_1^2$ where $v_1$is its speed when it reaches B. Now P.E. at A = K.E. at B. Hence

$$m_1gh = \frac{1}{2}m_1v_1^2 \Rightarrow v_1 = \sqrt{2gh}\,.$$

Similarly $m_2gh = \frac{1}{2} m_2 v_2^2 \Rightarrow v_2 = \sqrt{2gh} = v_1$.

Hence the correct choice is (c).

7. The correct choice is (a). If t_1 and t_2 are the times taken by P and Q to reach the bottom, then

$$v_1 = u_1 + a_1 t_1 = a_1 t_1 \qquad (\because u_1 = 0)$$

and $$v_2 = u_2 + a_2 t_2 = a_2 t_2 \qquad (\because u_2 = 0)$$

Now $v_1 = v_2$. Hence $a_1 t_1 = a_2 t_2$. Thus

$$\frac{t_1}{t_2} = \frac{a_2}{a_1}$$

Since $a_2 > a_1$; $t_1 > t_2$.

8. The correct choice is (a). For a conservation force, the work done in moving a body from one point to another does not depend on the nature of the path and the work done over a closed path is zero, irrespective of the nature of the path.

9. Statement-1 is false; the total energy of an isolated system is conserved. Statement-2 is true. Hence the correct choice is (d).

10. Statement-1 is false and Statement-2 is true. The rate of change of momentum is proportional to the net external force acting on the system. Hence the correct choice is (d).

11. The correct choice is (d). Potential energy stored in the spring when it is extended by x is $U_1 = \frac{1}{2} kx^2$

Potential energy stored in the spring when it is further extended by x is

$$U_2 = \frac{1}{2} k(x + x)^2 = 2kx^2$$

$\therefore$ Work done = gain in potential energy = $U_2 - U_1$

$$= 2\,kx^2 - \frac{1}{2} kx^2 = \frac{3}{2}\, kx^2$$

SECTION VI

Integer Answer Type

1. A particle of mass 1g executes an oscillatory motion on the concave surface of a spherical dish of radius 2 m placed on a horizontal plane. If the motion of the particle begins from a point on the dish at a height of 2 cm from the horizontal plane and the coefficient of friction is 0.01, find the total distance in metre covered by the particle before it comes to rest.

IIT, 1990

2. A light inextensible string that goes over a smooth fixed pulley as shown in Fig. 6.41 connects two blocks of masses 0.36 kg and 0.72 kg. Taking g = 10 m/s^2, find the work done (**in joules**) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.

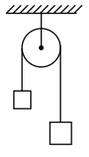


Fig. 6.41

IIT, 2009

3. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block as shown in Fig. 6.42. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $V = N/10$. Then N is

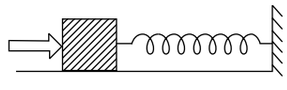


Fig. 6.42

IIT, 2011

SOLUTION

1. Refer to Fig. 6.43 in which θ is the angle which the normal to the surface at the location of the particle makes with the vertical. The particle will keep on oscillating about O till its initial potential energy mgh is completely used up in doing work against the frictional force f and the particle comes to rest. It follows from the figure that

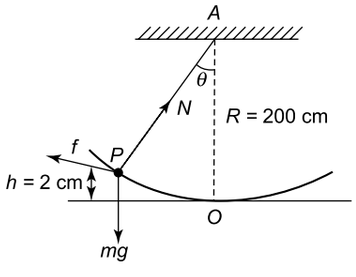


Fig. 6.43

$$f = \mu\, mg \cos\theta$$

Since $h << R$, $\theta \simeq 0$. Therefore, $\cos\theta \simeq 1$. Hence

$$f = \mu\, mg$$

If s is the distance travelled by the particle before it comes to rest, then the work done against friction is

$$W = fs = \mu\, mgs$$

Now, $mgh = \mu\, mgs$

or $$s = \frac{h}{\mu} = \frac{2\text{ cm}}{0.01} = 200\text{ cm} = 2\text{m}$$

2. Refer to Fig. 6.44.

$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$

which give

$$a = \frac{(m_2 - m_1)g}{(m_1 - m_2)} = \frac{(0.72 - 0.36)\times 10}{(0.72 + 0.36)} = \frac{10}{3}\text{ m s}^{-2}$$

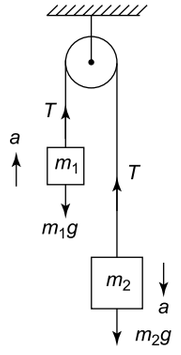


Fig. 6.44

and $$T = m_1(a + g)$$

$$= 0.36 \times \left(\frac{10}{3} + 10\right) = 4.8\text{ N}$$

Distance moved in $t = 1$ s is

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times \frac{10}{3} \times (1)^2 = \frac{5}{3}\text{ m}$$

$\therefore$ Work done $= T \times S = 4.8 \times \frac{5}{3} = 8$ J

3. Refer to Fig. 6.45.

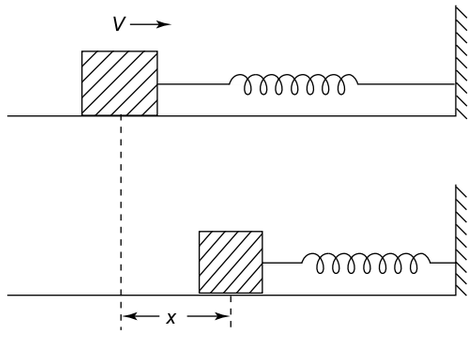


Fig. 6.45

Loss of kinetic energy $= \frac{1}{2}mV^2$

Work done against friction $= \mu mgx$

Gain in potential energy $= \frac{1}{2}kx^2$

From work-energy principle,

$$\frac{1}{2}\text{m}V^2 = \mu mgx + \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2} \times 0.18 \times V^2 = 0.1 \times 0.18 \times 10 \times 0.06 + \frac{1}{2} \times 2 \times (0.06)^2$$

$$\Rightarrow \quad V = 0.4 = \frac{4}{10}\text{ms}^{-1}.\text{ Hence } N = 4.$$

7 Chapter

Conservation of Linear Momentum and Collisions

REVIEW OF BASIC CONCEPTS

7.1 LAW OF CONSERVATION OF LINEAR MOMENTUM

The *law of conservation of linear momentum* may be stated as '*when no net external force acts on a system consisting of several particles, the total linear momentum of the system is conserved, the total linear momentum being the vector sum of the linear momentum of each particle in the system*'.

According to Newton's second law of motion, we have for a system of particles,

$$\mathbf{F}_{\text{ext}} = \frac{\mathbf{dp}}{\mathbf{dt}}$$

where $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \ldots$

$= m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 + m_3 \mathbf{v}_3 + \ldots$

In a system consisting of many particles, there can be two kinds of forces: (i) internal forces and (ii) external forces. *Internal* forces are the forces that the particles exert on each other during their interactions (e.g. collisions). From Newton's third law, these forces always occur in pairs of action and reaction. Since these forces are equal and opposite, they bring about equal and opposite changes in the momentum of the particles. Thus, internal forces cannot bring about any net change in the momentum of a particle. The external forces, on the other hand, are the forces exerted from outside the system.

If $\mathbf{F}_{\text{ext}} = 0$, we have

$$\frac{\mathbf{dp}}{\mathbf{dt}} = 0$$

or $\mathbf{p} = \text{constant}$

Thus, the vector sum $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \ldots$ of the linear momenta of the particles remains constant if the net external force is zero.

7.2 RECOIL OF A GUN

The gun and the bullet constitute a two-body system. Before the gun is fired, both the gun and the bullet are at rest. Therefore, the total momentum of the gun-bullet system is zero. After the gun is fired, the bullet moves forward and the gun recoils backwards. Let m_b and m_g be the masses of the bullet and the gun. If $\mathbf{v}_b$ and $\mathbf{v}_g$ are their respective velocities after firing, the total momentum of the gun-bullet system after firing is $(m_b \mathbf{v}_b + m_g \mathbf{v}_g)$. From the law of conservation of momentum, the total momentum after and before the gun is fired must be the same, i.e.

$$m_b \mathbf{v}_b + m_g \mathbf{v}_g = 0$$

or $$\mathbf{v}_g = -\frac{m_b \mathbf{v}_b}{m_g}$$

The negative sign indicates that the gun recoils in a direction opposite to that of the bullet. In terms of magnitudes, we have

$$v_g = \frac{m_b v_b}{m_g}$$

7.3 COLLISIONS

Elastic Collisions: *If there is no change of kinetic energy during a collision it is called an elastic collision.* The collision between subatomic particles is generally elastic. The collision between two steel or glass balls is nearly elastic.

Inelastic Collisions: *If there is a loss of kinetic energy during a collision, it is called an inelastic collision.* Since there is always some loss of kinetic energy in any collision, collisions are generally inelastic. If the loss is negligibly small, the collision is very nearly elastic. Perfectly elastic collisions are not possible. If two bodies stick together, after colliding, the collision is perfectly inelastic, e.g. a bullet striking a block of wood and being embedded

in it. The loss of kinetic energy usually results in heat or sound energy.

In may be remembered that *the total momentum remains conserved in both elastic and inelastic collisions.* Further, since the interacting forces become effectively zero after the collision, *the potential energy remains the same both before and after the collision.*

One-dimensional or Head-on Collision

Consider two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 in the same straight line (with $u_1 > u_2$) colliding with each other. Let v_1 and v_2 be their respective velocities after the collision. If velocities u_1, u_2, v_1 and v_2 are all along the same straight line, the collision is known as one-dimensional or head-on collision (Fig. 7.1)

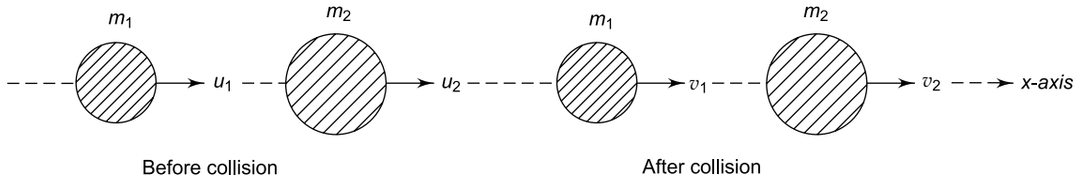


Fig. 7.1

From the law of conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

If momentum along positive x-axis is taken to be positive, the momentum along the negative x-axis is taken to be negative.

Two-dimensional or Oblique Collision

If the velocities of the colliding bodies are not along the same straight line, the collision is known as two-dimensional or oblique collision (Fig. 7.2)

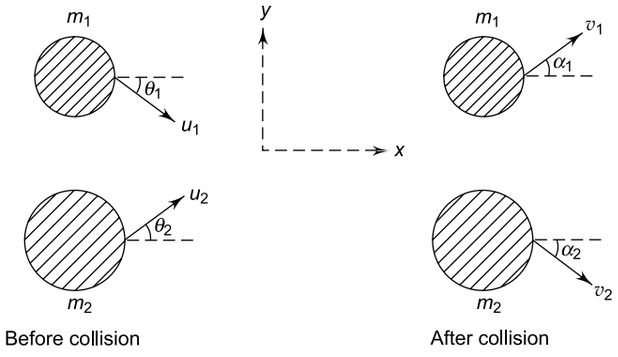


Fig. 7.2

In this case, we apply the law of conservation of momentum separately for x and y components of momenta. The components of momentum along the positive x-axis and positive y-axis are taken to be positive and components of momentum along negative x-axis and negative y-axis are taken to be negative.

Momentum conservation of x-components gives

$$m_1 u_1 \cos\theta_1 + m_2 u_2 \cos\theta_2 = m_1 v_1 \cos\alpha_1 + m_2 v_2 \cos\alpha_2$$

Momentum conservation of y-components gives

$$-m_1 u_1 \sin\theta_1 + m_2 u_2 \sin\theta_2 = m_1 v_1 \sin\alpha_1 - m_2 v_2 \sin\alpha_2$$

Coefficient of Restitution

Newton proved experimentally that, when two bodies collide, the ratio of the relative velocity after collision to the relative velocity before collision is constant for the two bodies. This constant is known as *coefficient of restitution* and is denoted by letter e.

$$e = -\frac{\text{velocity of separation after collision}}{\text{velocity of approach before collision}}$$

or $$e = -\frac{\mathbf{v}_2 - \mathbf{v}_1}{\mathbf{u}_2 - \mathbf{u}_1}$$

(i) For a perfectly elastice collision, $e = 1$.

(ii) For a perfectly in inelastic collision, $e = 0$, because the two bodies stick together and hence $\mathbf{v}_2 = \mathbf{v}_1$.

(iii) Perfectly elastic or perfectly inelastic collisions do not occur in nature. Hence, for any collision, e lies between 0 and 1.

(iv) For a head-on collision (Fig. 7.1)

$$e = -\frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$= -\frac{v_2 - v_1}{u_2 - u_1}$$

(v) For an oblique collision (Fig. 7.2)

Velocity of approach $= u_1 \cos\theta_1 - u_2 \cos\theta_2$

Velocity of separation $= v_2 \cos\alpha_2 - v_1 \cos\alpha_1$

$$\therefore \quad e = -\frac{v_2 \cos\alpha_2 - v_1 \cos\alpha_1}{u_1 \cos\theta_1 - u_2 \cos\theta_2}$$

Velocities after Head-on Elastic Collision

Refer to Fig. 7.1 again. From the law of conservation of momentum, we have

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \qquad (1)$$

The coefficient of restitution is defined as

$$e = \frac{v_1 - v_2}{u_2 - u_1}$$

$$v_1 - v_2 = e\ (u_2 - u_1) \qquad (2)$$

Eliminating v_2 from (1) and (2), we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{m_2\,(1+e)}{m_1 + m_2}\right)u_2 \qquad (3)$$

Using (3) in (2), we get

$$v_2 = \left(\frac{m_1\,(1+e)}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 \qquad (4)$$

Perfectly Elastic Collision

For perfectly elastic collision, $e = 1$. Putting $e = 1$ in Eqs. (3) and (4) we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \qquad (5)$$

and $$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \qquad (6)$$

Special Cases

(i) *If both bodies have the same mass, then*

$$m_1 = m_2 = m$$

In this case,

$$v_1 = u_2$$

and $$v_2 = u_1$$

This means that *in a one-dimensional elastic collision between two bodies of equal mass, the bodies merely exchange their velocities after the collision.*

(ii) *If one of the bodies, say m_2, is initially at rest*, then

$$u_2 = 0$$

In this case,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

and $$v_2 = \left(\frac{2m_1u_1}{m_1 + m_2}\right)$$

If, in addition, $m_1 = m_2 = m$, these equations give

$$v_1 = 0 \quad v_2 = u_1$$

Thus, *if a body suffers a one-dimensional elastic collision with another body of the same mass at rest, the first body is stopped dead, but the second begins in move with the velocity of the first.*

However, if the body at rest, namely B, is much more massive than the colliding body A, i.e. $m_2 >> m_1$, such that m_1 is negligibly small, then

$$v_1 = -u_1$$

and $$v_2 \to 0$$

Thus, *if a very light body suffers an elastic collision with a very heavy body at rest, the velocity of the lighter body is reversed on collision, while the heavier body remains practically at rest.*

A common example of this type of collision is the dropping of a hard steel ball on a hard concrete floor. The ball rebounds and regains its original height from where it was dropped while the much more massive ground remains at rest.

Finally, if the body at rest is much lighter than the colliding body, i.e. if $m_2 << m_1$, we have

$$v_1 \simeq u_1 \quad v_2 \simeq 2u_1$$

i.e. *the velocity of the massive body remains practically unchanged on collision with the lighter body at rest and the lighter body acquires nearly twice the initial velocity of the massive body.*

(iii) *Kinetic energy delivered by incident body to a stationary body in perfectly elastic head-on collision.*

Kinetic energy of m_1 before collision is $K_i = \frac{1}{2}m_1u_1^2$ and after collision is $K_f = \frac{1}{2}m_1v_1^2$. Therefore

$$\frac{K_f}{K_i} = \frac{v_1^2}{u_1^2}$$

or $$\frac{K_i - K_f}{K_i} = 1 - \frac{v_1^2}{u_1^2}$$

The fractional decrease in kinetic energy of m_1 is

$$\frac{\Delta K}{K_i} = 1 - \frac{v_1^2}{u_1^2}$$

If $u_2 = 0$, $\frac{v_1}{u_1} = \frac{m_1 - m_2}{m_1 + m_2}$. Therefore,

$$\frac{\Delta K}{K_i} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

NOTE

The fraction of kinetic energy lost by mass m_1 is maximum if $m_1 = m_2$ and minimum if $m_2 \to \infty$.

(iv) *Change in kinetic energy of a system in a perfectly inelastic head-on collision.*

In a perfectly inelastic collision, the two bodies stick together after the collision. Hence $v_1 = v_2$ and $e = 0$.

Putting $e = 0$ in Eqs. (3) and (4), we get

$$v_1 = \left(\frac{m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2}{m_1 + m_2}\right)u_2$$

and $$v_2 = \left(\frac{m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2}{m_1 + m_2}\right)u_2$$

If mass m_2 is stationary, $u_2 = 0$. Then

$$v_1 = \left(\frac{m_1}{m_1 + m_2}\right)u_1 \quad (7)$$

and $$v_2 = \left(\frac{m_1}{m_1 + m_2}\right)u_1$$

Notice that $v_1 = v_2 = v$ (say)

Total K.E. of the system before collision is

$$K_i = \frac{1}{2}m_1u_1^2$$

and after the collision is

$$K_f = \frac{1}{2}(m_1 + m_2)v^2$$

$\therefore$ Loss in K.E. of the system is

$$K_i - K_f = \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2 \quad (8)$$

From Eq. (7) $\frac{v}{u_1} = \left(\frac{m_1}{m_1 + m_2}\right)$. Using this in Eq. (8) we get

$$K_i - K_f = \frac{m_1m_2u_1^2}{2(m_1 + m_2)}$$

In general, if $u_2 \neq 0$, we have

$$K_i - K_f = \left(\frac{m_1m_2}{2(m_1 + m_2)}\right)(u_1 - u_2)^2 \quad (9)$$

Oblique Impact on a Fixed Horizontal Plane

Consider a body of mass m moving with a velocity u making an angle α with the normal ON to a fixed horizontal floor as shown in Fig. 7.3. After collision with the horizontal plane, the body is deflected with a velocity v making an angle β with the normal. Since the horizontal plane is fixed, it remains at rest. Hence the impact takes place along the normal. The normal component of u is $u\cos\alpha$ along $-y$ direction and the normal component of v is $v\cos\beta$ along the $+y$ directon. Now

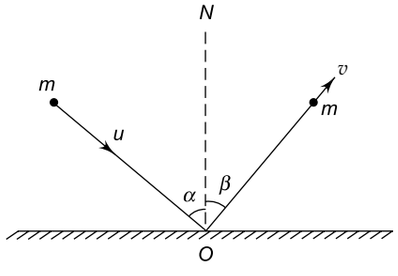


Fig. 7.3

$$e = -\frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$e = -\frac{(v\cos\beta)\hat{\mathbf{j}}}{(u\cos\alpha)(-\hat{\mathbf{j}})} = \frac{v\cos\beta}{u\cos\alpha}$$

$$\Rightarrow v\cos\beta = eu\cos\alpha \quad (10)$$

Since the impulsive force acts along the normal, the momentum along the normal is not conservved. Since the component of the impulsive force along the horizontal is zero, the momentum along the horizontal is conserved. Hence

$$u\sin\alpha = v\sin\beta \quad (11)$$

From Eqs. (10) and (11), we get

$$v = (e^2\cos^2\alpha + \sin^2\alpha)^{1/2}u \quad (12)$$

and $$\tan\beta = \frac{\tan\alpha}{e} \quad (13)$$

For a perfectly elastic collision, $e = 1$ and Eqs. (12) and (13) give

$$v = u$$

and $$\beta = \alpha$$

i.e. for a perfectly elastic collision, the body rebounds from a fixed surface with the same speed and at the same angle on the other side of the normal.

Direct Impact on a Fixed Plane

If the body falling normally on a fixed plane rebounds after impact, then, in this case $\alpha = \beta = 0$. Using this in Eq. (10) we get [Fig. 7.4]

$$v = eu$$

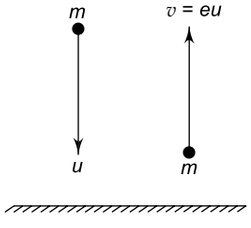


Fig. 7.4

EXAMPLE 7.1

A body is dropped from rest from a height $h = 5.0$ m. After rebounding twice from a horizontal floor, to what height will it rise if the coefficient of restitution is 0.8?

SOLUTION

Speed of the body just before first impact with floor $= \sqrt{2gh}$

Speed just after first impact $= e\sqrt{2gh}$. This is also the speed just before the second impact. Therefore, speed just after second impact $= e^2\sqrt{2gh}$. This is the initial speed for the upward motion of the body after the second impact, i.e. $u = e^2\sqrt{2gh}$. Therefore, height attained after two impacts is

$$h_2 = \frac{u^2}{2g} = \frac{1}{2g}\left(e^2\sqrt{2gh}\right)^2 = e^4 h$$

$$= (0.8)^4 \times 5 = 2.05 \text{ m}$$

NOTES

(1) Height attained after n impacts is $h_n = e^{2n}h$

(2) Speed of rebound after nth impact is $v_n = e^n\sqrt{2gh}$

(3) Total distance travelled before the body comes to rest

$$= h\left(\frac{1+e^2}{1-e^2}\right).$$

EXAMPLE 7.2

A steel ball of mass m moving with velocity u_1 undergoes a perfectly elastic head-on collision with another identical steel ball moving with velocity u_2. Show that, after the collision, they merely exchange their velocities.

SOLUTION

Refer to Fig. 7.1 again. From conservation of momentum,

$$mu_1 + mu_2 = mv_1 + mv_2$$

$$u_1 + u_2 = v_1 + v_2 \quad \text{(i)}$$

From the definition of coefficient of restitution,

$$v_2 - v_1 = e\,(u_1 - u_2)$$

For a perfectly elastic collision, $e = 1$. Hence

$$v_2 - v_1 = u_1 - u_2 \quad \text{(ii)}$$

From (i) and (ii) we get

$$v_1 = u_2 \text{ and } v_2 = u_1$$

EXAMPLE 7.3

A steel ball of mass m moving with a velocity u undergoes a perfectly elastic oblique collision with another indentical steel ball initially at rest. Show that, after the collision, they move at right angles to each other.

SOLUTION

Refer to Fig. 7.5.

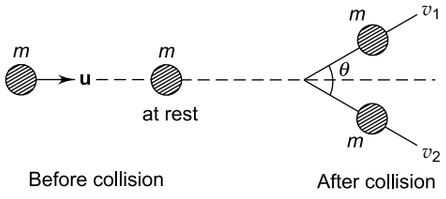


Fig. 7.5

From conservation of momentum,

$$m\mathbf{u} = m\mathbf{v}_1 + m\mathbf{v}_2$$

$$u = \mathbf{v}_1 + \mathbf{v}_2 \quad \text{(i)}$$

Taking the scalar product of u with itself, we have

$$\mathbf{u} \cdot \mathbf{u} = (\mathbf{v}_1 + \mathbf{v}_2) \cdot (\mathbf{v}_1 + \mathbf{v}_2)$$

$$u^2 = v_1^2 + 2\mathbf{v}_1 \cdot \mathbf{v}_2 + v_2^2 \quad \text{(ii)}$$

Since kinetic energy is also conserved in an elastic collision, we have

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$u^2 = v_1^2 + v_2^2 \quad \text{(iii)}$$

Using (iii) in (ii), we get

$$2\,\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$$

$$\Rightarrow \quad \mathbf{v}_1 \cdot \mathbf{v}_2 = 0$$

$$\Rightarrow \quad v_1 v_2 \cos\theta = 0$$

$$\Rightarrow \quad \cos\theta = 0 \Rightarrow \theta = 90°$$

EXAMPLE 7.4

Two steel balls of the same mass m moving in opposite directions with the same speed u collide head-on. If the collision is perfectly elastic, predict the result of the collision.

SOLUTION

$m_1 = m_2 = m$, $u_1 = u$ and $u_2 = -u$. Let v_1 and v_2 be their velocities after collision.

Total momentum before collision $= m_1u_1 + m_2u_2 = m\,(u - u) = 0$

Total momentum after collision $= mv_1 + mv_2 = m\,(v_1 + v_2)$

From conservation of momentum,

$$0 = m(v_1 + v_2) \Rightarrow v_2 = -v_1$$

Since $e = 1$, we have

$$v_2 - v_1 = u_1 - u_2 = u - (-u) = 2u$$

Putting $v_1 = -v_2$, we get $v_2 = u$. Also $v_1 = -u$. Thus, after the collision, the two balls move in opposite directions with equal speeds, each equal to u but their directions are reversed.

EXAMPLE 7.5

A ball of mass 2 kg moving with a velocity of 8 ms^{-1} collides head-on with another ball of mass of 4 kg moving with a velocity of 2 ms^{-1} moving in the same direction. The collision is elastic and the coefficient restitution is $e = 0.5$.

(a) Find the velocities of the balls after the collision.

(b) Calculate the loss of kinetic energy due to collision.

SOLUTION

Refer to Fig 7.1 again.

(a) Given $m_1 = 2$ kg, $m_2 = 4$ kg, $u_1 = 8$ ms^{-1}, $u_2 = 2$ ms^{-1} and $e = 0.5$

From conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow \quad 2 \times 8 + 4 \times 2 = 2v_1 + 4v_2$$

$$\Rightarrow \quad 12 = v_1 + 2v_2 \qquad \text{(i)}$$

Since $e = 0.5$, we have

$$v_2 - v_1 = e\,(u_1 - u_2)$$

$$= 0.5 \times (8 - 2) = 3 \qquad \text{(ii)}$$

Eliminating v_2 from (i) and (ii) we get $v_1 = 2$ ms^{-1}. Using this in (i) or (ii), we get $v_2 = 5$ ms^{-1}

(b) Kinetic energy before collision is

$$K_i = \frac{1}{2}\, m_1u_1^2 + \frac{1}{2}\, m_2u_2^2$$

$$= \frac{1}{2} \times 2 \times (8)^2 + \frac{1}{2} \times 4 \times (2)^2$$

$$= 72 \text{ J}$$

Kinetic energy after collision is

$$K_f = \frac{1}{2}\, m_1v_1^2 + \frac{1}{2}\, m_2v_2^2$$

$$= \frac{1}{2} \times 2 \times (2)^2 + \frac{1}{2} \times 4 \times (5)^2 = 54 \text{ J}$$

$\therefore$ Loss of K.E. $= K_i - K_f = 72 - 54 = 18$ J

EXAMPLE 7.6

In Example 7.5, what is the loss of kinetic energy if the ball of mass 4 kg is moving towards the mass of mass 2 kg, their speeds being the same?

SOLUTION

In this case $u_2 = -2$ ms^{-1}. Equations (i) and (ii) become

$$4 = v_1 + 2v_2 \qquad \text{(iii)}$$

and $$v_2 - v_1 = 5 \qquad \text{(iv)}$$

Equations (iii) and (iv) give $v_1 = -2$ ms^{-1} and $v_2 = 3$ ms^{-1}

$$K_i = \frac{1}{2} \times 2\,(8)^2 + \frac{1}{2} \times 4 \times (-2)^2 = 72 \text{ J}$$

$$K_f = \frac{1}{2} \times 2 \times (-2)^2 \;\; \frac{1}{2} \times (3)^2 = 22\text{J}$$

$\therefore$ Loss of K.E = 72 – 22 = 50 J

EXAMPLE 7.7

Two blocks B and C of masses 1 kg and 2 kg respectively are connected by a massless elastic spring of spring constant 150 Nm^{-1} and placed on a horizontal frictionless surface as shown in Fig. 7.6. A third block A of mass 1 kg moves with a velocity of 3 ms^{-1} along the line joining B and C and collides with B. If the collision is perfectly elastic and the natural length of the spring is 80 cm, find the minimum separation between blocks B and C.

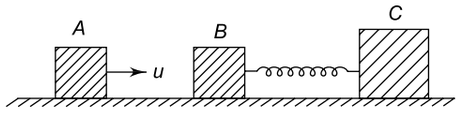


Fig. 7.6

SOLUTION

Given $m_A = m_B = 1$ kg, $m_C = 2$ kg and $u = 3$ ms^{-1}. Block A will collide with block B. Since they have equal masses and the collision is perfectly elastic, A will come to rest and B will move to the right with a velocity u. Block B will compress the spring. Hence block C will accelarate and block B will retard until both B and C move with the same velocity. Let this common velocity be v. Since no external force acts, the momentum of B and C is conserved, i.e.

$$m_B\, u = (m_B + m_C)\, v$$

$$\Rightarrow \quad 1 \times 3 = (1 + 2)\, v \Rightarrow v = 1 \text{ } ms^{-1}$$

If x is the maximum compression, then from the principle of conservation of energy,

$$\frac{1}{2} m_A u^2 = \frac{1}{2} (m_B + m_C)\, v^2 + \frac{1}{2} kx^2$$

$$\Rightarrow \quad \frac{1}{2} \times 1 \times (3)^2 = \frac{1}{2} \times (1 + 2) \times (1)^2 + \frac{1}{2} \times 150 \times x^2$$

which gives $x = 0.2$ m = 20 cm

$\therefore$ Minimum separation between B and C = 80 cm – 20 cm = 60 cm

EXAMPLE 7.8

A block of $m_1 = m$ is moving on a frictionless horizontal surface with velocity $u_1 = 2u$ towards another block of mass $m_2 = 3m$ moving on the same surface with velocity $u_2 = u$ in the same direction. A massless spring of force constant k is attached to m_2 as shown in Fig. 7.7. When block m_1 collides with the spring, show that the maximum compression of the spring is given by $x = \frac{u}{2}\sqrt{\frac{3m}{k}}$.

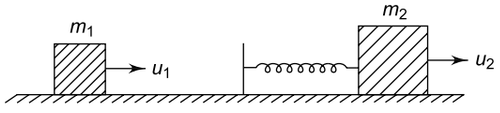


Fig. 7.7

SOLUTION

When block m_1 collides with spring, it begins to get compressed. As a result m_2 gains speed. The compression of the spring is maximum at the instant when the relative velocity of m_1 with respect to m_2 is zero, i.e. when both m_1 and m_2 have equal velocities. Let v be the common velocity of the blocks. From conservation of momentum,

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$$

$$\Rightarrow \quad 2mu + 3mu = (m + 3m)v$$

$$\Rightarrow \quad v = \frac{5u}{4}$$

From the law of conservation of energy

Loss in K.E. = gain in P.E of spring

If x is the maximum compression, then

$$\left(\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2\right) - \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2} kx^2$$

$$\Rightarrow \frac{1}{2} \times m \times (2u)^2 + \frac{1}{2} \times 3m \times u^2 - \frac{1}{2} (m + 3m) \times \left(\frac{5u}{4}\right)^2 = \frac{1}{2} kx^2$$

$$\Rightarrow \quad \frac{3}{4} mu^2 = kx^2 \Rightarrow x = \frac{u}{2}\sqrt{\frac{3m}{k}}$$

7. Useful Formulae and Tips

1. A body of mass m is dropped from a height h. Due to the friction of air, it will hit the ground with a speed less than $\sqrt{2gh}$. If v is the speed with which it hits the ground, the work done by friction is

$$W_f = \frac{1}{2} mv^2 - mgh = \frac{1}{2} m(v^2 - 2gh)$$

If friction is absent, $W_f = 0$, then $v = \sqrt{2gh}$.

2. Two block A and B of masses m_1 and m_2 are released from the same height at the same time. Block A slides along an inclined plane of inclination θ and block B falls vertically downwards (Fig. 7.8)

If the inclined plane is frictionless, gain in KE = loss in PE, i.e.

$$\frac{1}{2} m_1 v_1^2 = m_1 gh \Rightarrow v_1 = \sqrt{2gh}$$

If the air friction is neglected.

$$\frac{1}{2} m_2 v_2^2 = m_2 gh \Rightarrow v_2 = \sqrt{2gh}$$

Thus both block will hit the ground with the same speed independent of the mass. But the times taken to reach the ground will be different.

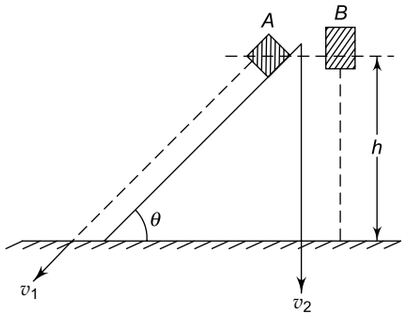


Fig. 7.8

For block A, $t_1 = \sqrt{\frac{2h}{g \sin^2 \theta}}$

For block B, $t_2 = \sqrt{\frac{2h}{g}}$

3. If a block of mass m in contact with a spring compressed by a distance x is released, the block will leave the spring with a velocity v determined from

$$\frac{1}{2} kx^2 = \frac{1}{2} mv^2$$

which gives $v = \sqrt{\frac{k}{m}}\, x$, where k is the spring constant.

4. If a block of mass m moving with speed u comes in contact with a relaxed spring of spring constant k, its velocity v when the spring is compressed by an amount x is obtained from.

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

which gives $v = \pm\left(\frac{k\,x^2}{m} - u^2\right)^{1/2}$

5. If two springs of spring constants k_1 and k_2 are stretched by the same force F, then $F = k_1x_1 = k_2\,x_2$. Potential energy stored is

$$U_1 = \frac{1}{2}\,k_1x^2{}_1 = \frac{1}{2}\,Fx_1$$

and $$U_2 = \frac{1}{2}k_2x^2{}_2 = \frac{1}{2}\,Fx_2$$

which give $\frac{U_1}{U_2} = \frac{x_1}{x_2} = \frac{k_2}{k_1}$

6. If the two springs are stretched by the same amount x, then $F_1 = k_1x$ and $F_2 = k_2x$.

$$U_1 = \frac{1}{2}k_1x^2 \text{ and } U_2 = \frac{1}{2}\,k_2x^2.$$

$$\frac{U_1}{U_2} = \frac{k_1}{k_2} = \frac{F_1}{F_2}$$

7. A chain has a length L and mass M. A part L/n is hanging at the edge of the table. The length of the chain lying on the table is $(L - L/n)$. Then work done against gravity to pull the hanging part on the table $= \frac{MgL}{2n^2}$

8. If a body of mass m moving with velocity v is stopped in a distance x by a retarding force F, then

$$\frac{1}{2}\,mv^2 = Fx$$

(a) If two bodies of masses m_1 and m_2 moving with the same velocity are subjected to the same retarding force, the ratio of the stopping distance is

$$\frac{x_1}{x_2} = \frac{m_1}{m_2}$$

(b) If the two bodies are moving with equal kinetic energy and are stopped by the same retarding force, then

$$x_1 = x_2$$

(c) If the two bodies are moving with equal liner momentum and are stopped by the same force, then.

$$\frac{p^2}{2m} = Fx$$

and $$\frac{x_1}{x_2} = \frac{m_2}{m_1}$$

SECTION I

Multiple Choice Questions with only One Choice Correct

1. A ball P moving with a velocity u suffers a one-dimensional collision with another ball Q of the same mass but at rest. After the collision the velocity of Q is found to be three times that of P. The coefficient of restitution is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

2. A ball strikes a horizontal floor at an angle $\theta = 45°$. If the coefficient of restitution between the ball and the floor is 1/2, the fraction of kinetic energy lost in the collision is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{3}{8}$ (d) $\frac{5}{8}$

3. Two particles of the same mass m moving in different directions with the same speed v collide and stick together. After the collision, the speed of the composite particle is $v/2$. The angle between the velocities of the two particles before collision is

(a) 60° (b) 90°

(c) 120° (d) 150°

4. Two blocks of masses $m_1 = m$ and $m_2 = 3\ m$ are connected by a spring of force constant k and placed on a horizontal frictionless surface as shown in Fig. 7.9. The spring is stretched by an amount x and released. The system executes simple harmonic motion. The relative velocity of the blocks when the spring is at its natural length is

(a) $x\sqrt{\frac{3k}{2m}}$ (b) $2x\sqrt{\frac{k}{m}}$

(c) $\frac{x}{2}\sqrt{\frac{k}{3m}}$ (d) $2x\sqrt{\frac{k}{3m}}$

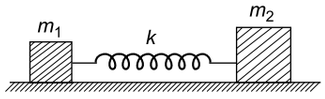


Fig. 7.9

5. A block P moves with an initial velocity of 4 ms^{-1} towards a block Q of the same mass at rest at a distance of 2 m on a rough horizontal surface. The coefficient of friction between the blocks and surface is 0.2. An elastic one-dimensional collision occurs between the two blocks. If $g = 10\ ms^{-2}$, the separation between the blocks when both have come to rest is

(a) 1 m (b) 2 m

(c) 3 m (d) 4 m

6. A perfectly elastic oblique collision occurs between a ball A moving along the x-axis and a ball B at rest and of the same mass as ball A. After the collision, ball A moves at an angle of 30° with the x-direction and ball B at an angle θ with the x-axis. The value of θ is

(a) 15° (b) 30°

(c) 45° (d) 60°

7. A particle A moving with momentum p suffers a one-dimensional collision with a particle B of the same mass but at rest. During the collision B imparts an impulse I to A. The coefficient of restitution between A and B is

(a) $\frac{2I}{p}$ (b) $\frac{2I}{p} - 1$

(c) $\frac{2I}{p} + 1$ (d) $2\left(\frac{I}{p} - 1\right)$

8. A man of mass m stands on one end of a wooden plank of length L and mass M kept initially at rest on a horizontal frictional surface. If the man walks from one end of the plank to the other end at a constant speed, the resulting displacement of the plank is

(a) $\frac{mL}{M}$ (b) $\frac{ML}{m}$

(c) $\frac{mL}{(M+m)}$ (d) $\frac{mL}{(M-m)}$

9. A body P of mass m moving with kinetic energy k and momentum p undergoes a one-dimensional elastic collision with a body of mass 2 m at rest. After the collision, the ratio of the kinetic energy of P to that of Q is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{9}$ (d) $\frac{8}{9}$

10. A shell of mass m initially at rest explodes into three fragments of masses in the ratio 2 : 2 : 1. The fragments having equal masses fly off along mutually perpendicular directions with speed v. The speed of the third (lighter) fragment will be

(a) v (b) $\sqrt{2}\,v$

(c) $2\sqrt{2}\,v$ (d) $3\sqrt{2}\,v$

11. A shell is fired with a speed of 100 ms^{-1} at an angle of 30° with the vertical (y-direction). At the highest point of its trajectory, the shell explodes into two fragments of masses in the ratio 1 : 2. The lighter fragment moves vertically upwards with an initial speed of 200 ms^{-1}. The speed of the heavier fragment immediately after the explosion is

(a) 125 ms^{-1} (b) 150 ms^{-1}

(c) 175 ms^{-1} (d) 200 ms^{-1}

12. A body of mass m moving with a certain speed suffers an inelastic collision with a body of mass M at rest. The ratio of the final kinetic energy of the system to the initial kinetic energy is

(a) $\frac{m}{m+M}$ (b) $\frac{M}{m+M}$

(c) $\frac{m+M}{m}$ (d) $\frac{m+M}{M}$

13. A neutron moving at a speed v undergoes a head-on elastic collision with a nucleus of mass number A at rest. The ratio of the kinetic energies of the neutron after and before collision is

(a) $\left(\frac{A-1}{A+1}\right)^2$ (b) $\left(\frac{A+1}{A-1}\right)^2$

(c) $\left(\frac{A}{A+1}\right)^2$ (d) $\left(\frac{A}{A-1}\right)^2$

14. A radioactive nucleus of mass number A, initially at rest, emits an α-particle with a speed v. What will be the recoil speed of the daughter nucleus?

(a) $\frac{2v}{A-4}$ (b) $\frac{2v}{A+4}$

(c) $\frac{4v}{A-4}$ (d) $\frac{4v}{A+4}$

15. A ball is dropped from a height of 10 m. It is embedded 1 m in sand. In this process
(a) only momentum is conserved
(b) only kinetic energy is conserved
(c) both momentum and kinetic energy are conserved
(d) neither momentum nor kinetic energy is conserved.

16. n small balls, each of mass m, impinge elastically each second on a surface with velocity u. The force experienced by the surface will be
(a) mnu (b) $2\ mnu$
(c) $4\ mnu$ (d) $\frac{1}{2}\ mnu$

17. A rubber ball is dropped from a height of 5 m on a planet where the acceleration due to gravity is not known. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of
(a) $\frac{16}{25}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) $\frac{9}{2}$

18. An isolated particle of mass m is moving in a horizontal plane ($x - y$), along the x-axis, at a certain height above the ground. It suddenly explodes into two fragments of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at
(a) $y = -5$ cm (b) $y = +20$ cm
(c) $y = +5$ cm (d) $y = -20$ cm
IIT, 1997

19. A body P strikes another body Q of mass that is p times that of body P and moving with a velocity that is $\frac{1}{q}$ of the velocity of body P. If body P comes to rest, the coefficient of restitution is
(a) $\frac{p+q}{p-q}$ (b) $\frac{p-q}{q(p-1)}$
(c) $\frac{p-q}{p(q-1)}$ (d) $\frac{p+q}{p(q-1)}$

20. Two equal spheres A and B lie on a smoot horizontal circular groove at opposite ends of a diameter. Sphere A is projected along the groove and at the end of time T impinges on sphere B. If e is the coefficient of restitution, the second impact will occur after a time equal to
(a) T (b) eT
(c) $\frac{2T}{e}$ (d) $2\ eT$
IIT, 1997

21. A nucleus of mass M amu emits an α-particles with a energy K MeV. The total energy of disintegration (in MeV) is
(a) K (b) $\frac{KM}{(M-4)}$
(c) $\frac{K(1+M)}{M}$ (d) $\frac{KM}{(M+4)}$

22. A particle falls from a height h on a fixed horizontal plate and rebounds. If e is the coefficient of restitution, the total distance travelled by the particle before it stops rebounding is
(a) $\frac{h(1+e^2)}{(1-e^2)}$ (b) $\frac{h(1-e^2)}{(1+e^2)}$
(c) $\frac{h(1-e^2)}{2(1+e^2)}$ (d) $\frac{h(1+e^2)}{2(1-e^2)}$

23. A bullet of mass m is fired horizontally with a velocity v on a wooden block of mass M suspended from a support and gets embedded in it. The kinetic energy of the bullet + block system is
(a) $\frac{1}{2}\ mv^2$ (b) $\frac{1}{2}\ (M+m)v^2$
(c) $\frac{Mmv^2}{2(M+m)}$ (d) $\frac{m^2v^2}{2(M+m)}$

24. A body of mass m moving with a speed v suffers an inelastic collision and sticks with another body of mass $M = 2m$ at rest. The speed of the composite body will be
(a) $3v$ (b) $\frac{v}{3}$
(c) $\frac{2v}{3}$ (d) $\frac{3v}{2}$

25. In Q. 24 above, the ratio of the final kinetic energy of the system to the initial kinetic energy is
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) 3 (d) $\frac{3}{2}$

26. A body of mass 5 kg is moving along the x-axis with a velocity 2ms^{-1}. Another body of mass 10 kg is moving along the y-axis with a velocity $\sqrt{3}\ \text{ms}^{-1}$. They collide at the origin and stick together. The final velocity of the combined mass is
(a) $\sqrt{3}\ \text{ms}^{-1}$ (b) $\left(\sqrt{3}+1\right)\ \text{ms}^{-1}$
(c) $\frac{4}{3}\ \text{ms}^{-1}$ (d) none

27. A block of wood of mass M is suspended by means of a thread. A bullet of mass m is fired horizontally into the block with a velocity v. As a result of the impact, the bullet is embedded in the block. The block will rise to vertical height given by

(a) $\frac{1}{2g}\left(\frac{mv}{M+m}\right)^2$ (b) $\frac{1}{2g}\left(\frac{mv}{M-m}\right)^2$

(c) $\frac{1}{2g}\frac{mv^2}{(M+m)}$ (d) $\frac{1}{2g}\frac{mv^2}{(M-m)}$

28. A moving particle of mass m makes a head-on collision with a particle of mass $2m$ initially at rest. If the collision is perfectly elastic, the percentage loss of energy of the colliding particle is

(a) 50% (b) 66.7%
(c) 88.9% (d) 100%

29. A body of mass m moving with a velocity v in the x-direction collides with a body of mass M moving with a velocity V in the y-direction. They stick together during collision. Then

(a) the magnitude of the momentum of the composite body is $\sqrt{(mv)^2+(MV)^2}$

(b) the composite body moves in a direction making a angle $\theta = \tan^{-1}\left(\frac{MV}{mv}\right)$ with the x-axis.

(c) the loss of kinetic energy as a result of collision is $\frac{1}{2}\frac{Mm}{(M+m)}(V^2+v^2)$

(d) all the above choices are correct.

30. A body falls from a height h on a horizontal surface and rebounds. Then it falls again and again rebounds and so on. If the restitution coefficient is $\frac{1}{3}$, the total distance covered by the body before it comes to rest is

(a) $\frac{h}{4}$ (b) $\frac{5h}{4}$

(c) $2h$ (d) $3h$

31. In Q. 30 above, the total time taken by the body to come to rest is

(a) $\sqrt{\frac{2h}{g}}$ (b) $2\sqrt{\frac{2h}{g}}$

(c) $3\sqrt{\frac{2h}{g}}$ (d) $4\sqrt{\frac{2h}{g}}$

32. A body of mass m moving with a velocity v in the x-direction collides and sticks with another body of mass M moving with a velocity V in the y-direction. The magnitude of the momentum of the composite body is

(a) $(mv+MV)$ (b) $(m+M)(v+V)$

(c) $[(mv)^2+(MV)^2]^{1/2}$ (d) $(Mv+mV)$

33. In Q. 32 above, the angle θ subtended by the velocity vector of the composite body with the x-axis is given by

(a) $\theta = \tan^{-1}\left(\frac{MV}{mv}\right)$ (b) $\theta = \tan^{-1}\left(\frac{mv}{MV}\right)$

(c) $\theta = \tan^{-1}\left(\frac{mV}{Mv}\right)$ (d) $\theta = \tan^{-1}\left(\frac{Mv}{mV}\right)$

34. A body P of mass m_1 moving with a certain velocity collides head-on with a stationary body Q of mass m_2. It the collision is elastic, the fraction of kinetic energy transferred from body P to body Q is

(a) $\frac{2(m_1m_2)}{(m_1+m_2)^2}$ (b) $\frac{4(m_1m_2)}{(m_1+m_2)^2}$

(c) $\frac{2m_1^2}{(m_1+m_2)^2}$ (d) $\frac{2m_2^2}{(m_1+m_2)^2}$

35. Two particles, each of mass m, moving along different directions with a velocity u making the same angle with the x-axis collide and stick together. The composite particle moves along the x-axis with a velocity $u/2$. The angle between their directions of motion before collision is

(a) 60° (b) 90°
(c) 120° (d) 150°

36. A bullet of mass m moving with a horizontal velocity u strikes a stationary wooden block of mass M suspended by a string of length $L = 50$ cm. The bullet emerges out of the block with speed $u/4$. If $M = 6\ m$, the minimum value of u so that the block can complete the vertical circle is (take $g = 10\ \text{ms}^{-2}$)

(a) 10 ms^{-1} (b) 20 ms^{-1}
(c) 30 ms^{-1} (d) 40 ms^{-1}

37. A compound pendulum consists of a uniform rod of length L of negligible mass. A body of mass $m_1 = m$ is fixed at the lower end and a body of mass $m_2 = 2\ m$ is fixed exactly in the middle of the rod as shown in Fig. 7.10

The horizontal velocity v that must be given to mass m_1 to rotate the pendnlum to the horizontal position OC is

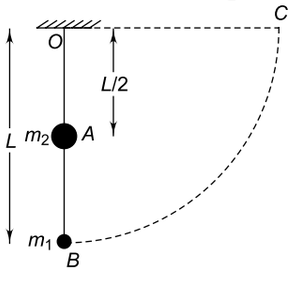


Fig. 7.10

(a) $2\sqrt{\frac{2gL}{3}}$ (b) $2\sqrt{gL}$

(c) $\sqrt{2gL}$ (d) $\sqrt{gL}$

38. A ball is thrown will a velocity v_1 towards a vertical wall at an angle α with the wall. It rebounds with a velocity v_2 making an angle β with the wall as shown in Fig. 7.11. If the coefficient of restitution between the ball and the wall is e, then v_2 is given by

(a) $v_2 = v_1(\cos\ \alpha + e\ \sin\ \alpha)$

(b) $v_2 = v_1(\sin\ \alpha + e\ \cos\ \alpha)$

(c) $v_2 = v_1\sqrt{\sin^2\alpha + e^2\cos^2\alpha}$

(d) $v_2 = v_1\sqrt{\cos^2\alpha + e^2\sin^2\alpha}$

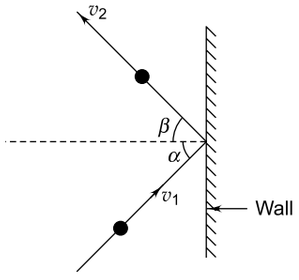


Fig. 7.11

39. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in Fig. 7.12. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, will these two particles again reach the point A?

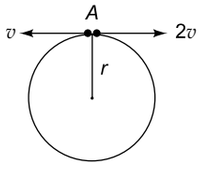


Fig. 7.12

(a) 4 (b) 3

(c) 2 (d) 1

IIT, 2009

40. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity V of the bullet is (see Fig.7.13)

(a) 250 m/s (b) 250 m/s

(c) 400 m/s (d) 500 m/s

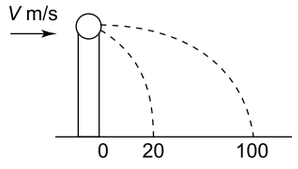


Fig. 7.13

IIT, 2011

41. A ball is thrown from a point O with a velocity $u = 20\ \text{ms}^{-1}$ at an angle $\theta = 30°$ with the horizontal. It hits a vertical wall which is at a distance x from O as shown in Fig. 7.14. After rebounding from the wall, the ball returns to O without retracing its path. If $g = 10\ \text{ms}^{-2}$ and the coefficient restitution $e = 0.5$, the value of x is

(a) 9.6 m (b) 10.3 m

(c) 11.5 m (d) 12.8 m

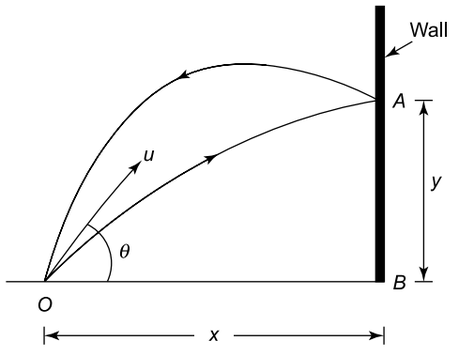


Fig 7.14

ANSWERS

1. (a)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (d)
7. (b)	**8.** (c)	**9.** (a)	**10.** (c)	**11.** (a)	**12.** (a)
13. (a)	**14.** (c)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (a)
19. (d)	**20.** (c)	**21.** (b)	**22.** (a)	**23.** (d)	**24.** (b)
25. (a)	**26.** (c)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (b)
31. (b)	**32.** (c)	**33.** (a)	**34.** (b)	**35.** (c)	**36.** (d)
37. (a)	**38.** (d)	**39.** (c)	**40.** (d)	**41.** (c)	

SOLUTIONS

1. Let v_1 and v_2 be the velocities of P and Q after the collision.

Coefficient of restitution $= \dfrac{v_2 - v_1}{u - 0} = \dfrac{v_2 - v_1}{u}$ (1)

From conservation of momentum, we have

$$mu + 0 = mv_1 + mv_2 \quad (2)$$

$$\Rightarrow \quad u = v_1 + v_2$$

From Eqs. (1) and (2), we get

$$v_1 = \frac{u}{2}(1 - e) \text{ and } v_2 = \frac{u}{2}(1 + e)$$

$$\therefore \quad \frac{v_2}{v_1} = \frac{1+e}{1-e}$$

Given $v_2 = 3v_1$. Hence $3 = \dfrac{1+e}{1-e} \Rightarrow e = \dfrac{1}{2}$

2. Let u be the speed of the ball before the collision. After the collision, its speed will be

$$v = \sqrt{\left(\frac{u}{\sqrt{2}}\right)^2 + \left(\frac{eu}{\sqrt{2}}\right)^2}$$

$$= \sqrt{\frac{u^2}{2} + \frac{u^2}{8}} = \sqrt{\frac{5}{8}}\, u \quad \left(\because e = \frac{1}{2}\right)$$

$$\therefore \text{ Fraction of K.E. lost} = \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2}$$

$$= 1 - \frac{v^2}{u^2} = 1 - \frac{5}{8} = \frac{3}{8}$$

3. Let θ be the angle between the velocities of the two particles before collision. If p_1 and p_2 are the momenta of the particles before collision and p is the momentum of the composite particle, then the conservation of momentum gives

$$p^2 = p_1^2 + p_2^2 + 2p_1p_2 \cos\theta$$

$$\Rightarrow \left(2m\frac{v}{2}\right)^2 = (mv)^2 + (mv)^2 + 2(mv)(mv)\cos\theta$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2} \quad \text{or} \quad \theta = 120°$$

4. If v is the relative velocity of the two blocks when the spring is at its natural length, then from the conservation of energy, we have

$$\frac{1}{2}\mu v^2 = \frac{1}{2}kx^2 \quad (1)$$

where μ is the reduced mass of the system and is given by

$$\mu = \frac{m_1 m_2}{(m_1 + m_2)} = \frac{m \times 3m}{(m + 3m)} = \frac{3m}{4}$$

Substituting in Eq. (1) we get

$$kx^2 = \frac{3}{4}mv^2 \Rightarrow v = 2x\sqrt{\frac{k}{3m}}$$

5. Frictional force $f = \mu mg$. The retardation due to friction is

$$a = \frac{f}{m} = \mu g = 0.2 \times 10 = 2 \text{ ms}^{-2}$$

Since the blocks have the same mass and the collision is elastic, after the collision, block P will come to rest at the position previously occupied by block Q and Q will begin to move with the velocity at which P strikes Q which is given by

$$v^2 = 2as = 2 \times 2 \times 2 = 8 \Rightarrow v = \sqrt{8} \text{ ms}^{-1}$$

Moving with velocity of $\sqrt{8}$ ms^{-1}, the block Q will come to rest after travelling a distance x given by

$$x = \frac{v^2}{2a} = \frac{8}{2 \times 2} = 2 \text{ m}$$

Hence the correct choice is (b).

6. In an oblique elastic collision between two body of the same mass, they move at right angles to each other after the collision. Hence the correct choice is (d).

7. Let p_1 and p_2 be the linear momenta of A and B after the collision. Now, impulse = change in momentum.

For particle B : $I = p_1$

For particle A : $I = p - p_2 \quad \Rightarrow \quad p_2 = p - I$

$$\text{coefficient of restitution } e = \frac{v_1 - v_2}{u} = \frac{mv_1 - mv_2}{mu}$$

$$= \frac{p_1 - p_2}{p}$$

where m is the mass of each particle and u is the velocity A before collision and v_1 and v_2 are the velocities of A and B after the collision. Hence

$$e = \frac{p_1 - p + I}{p} = \frac{I - p + I}{p} = \frac{2I}{p} - 1$$

So the correct choice is (b).

8. Total initial momentum of the man-plank system is zero. If he walks with a speed v on the plank, as a result, the plank moves with a speed, say, v' in the opposite direction. The total final momentum of system $= mv - (M + m)v'$. From conservation of momentum,

$$0 = mv - (M + m)v'$$

$$\Rightarrow \quad \frac{v'}{v} = \frac{m}{(M+m)}$$

Since the distance moved in proportional to speed (since there is no acceleration), the displacement L' of the plank is given by choice (c).

9. Total momentum before collision is $p = mu + 0 = mu$ and kinetic energy is $K = \frac{1}{2}\ mu^2$. If v_1 and v_2 are the velocities of P and Q after the collision, then, from momentum conservation,

$$mu = mv_1 + (2\,m)\,v_2$$

$$\Rightarrow \quad p = p_1 + p_2 \qquad (1)$$

From conservation of kinetic energy,

$$K = K_1 + K_2$$

where $K_1 = \frac{1}{2}\ mv_1^2$ and $K_2 = \frac{1}{2}\ (2\,m)\,v_2^2$

$$\text{Thus } \frac{1}{2}\ mu^2 = \frac{1}{2}\ mv_1^2 + mv_2^2$$

$$\Rightarrow \quad \frac{p^2}{2\,m} = \frac{p_1^2}{2\,m} + \frac{p_2^2}{4\,m}$$

$$\Rightarrow \quad 2p^2 = 2p_1^2 + p_2^2 \qquad (3)$$

From Eq. (1), $p_2 = p - p_1$. Using this in Eq. (3) and solving we get

$$p_1 = -\frac{p}{3}. \text{ Hence } p_2 = \frac{4p}{3}$$

$$\text{Now } K_1 = \frac{1}{2}\ mv_1^2 = \frac{p_1^2}{2\,m} = \frac{p^2}{18\,m}$$

$$\text{and} \quad K_2 = \frac{1}{2}\ mv_2^2 = \frac{p_2^2}{4\,m} = \frac{4p^2}{9\,m}$$

$$\therefore \quad \frac{K_1}{K_2} = \frac{1}{8}, \text{ which is choice (a).}$$

10. Mass of each heavier fragment = 2 m/5 and of lighter fragment = m/5. Momentum of each heavier fragment is $p = 2\,mv/5$.Since they fly off along mutually perpendicular directions, their resultant momentum $= \sqrt{p^2 + p^2} = \sqrt{2}\,p$. If V is the speed of the lighter fragment, from the conservation of momentum, we have

$$\frac{mV}{5} = \sqrt{2}\,p = \sqrt{2} \times \frac{2\,mv}{5}$$

$$\Rightarrow \quad V = 2\sqrt{2}\ v.$$

11. Velocity of the shell at the highest point is $v = 100 \times \sin 30° = 50\ \text{ms}^{-1}$ parallel to x-direction. Its momentum is $p = mv$ (Fig. 7.15).

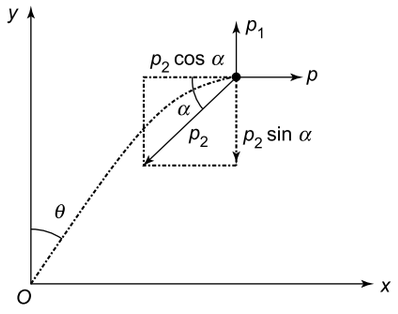


Fig. 7.15

$p_1 = \frac{m}{3}\ v_1$, where $v_1 = 200\ \text{ms}^{-1}$. If v_2 is the velocity of the heavier fragment, its momentum is $2\,mv_2/3$. Conservation of momentum along x and y-directions gives

$$p = p_2 \cos \alpha$$

$$\Rightarrow \quad mv = \frac{2\,mv_2}{3} \cos \alpha$$

$$\Rightarrow \quad 3v = 2v_2 \cos \alpha \qquad (1)$$

$$\text{and} \quad p_1 = p_2 \sin \alpha$$

$$\Rightarrow \quad \frac{m}{3}\ v_1 = \frac{2\,mv_2}{3} \sin \alpha$$

$$\Rightarrow \quad v_1 = 2v_2 \sin \alpha \qquad (2)$$

From (1) and (2), we get $v_2 = \frac{1}{2}\ (v_1^2 + 9v^2)^{1/2}$.

$$= \frac{1}{2}\ [(200)^2 + 9(50)^2]^{1/2}$$

$$= 125\ \text{ms}^{-1}$$

12. Initial momentum of the system = mv, since body of mass M is at rest. After the inelastic collision, the bodies stick together and the mass of the composite body is $(m + M)$. If V is the speed of the composite body, its momentum will be $(m + M)V$. From the principle of conservation of momentum, we have

$$mv = (m + M)V$$

$$\text{or } V = \left(\frac{m}{m+M}\right)v$$

$$\text{Initial KE} = \frac{1}{2}\ mv^2. \text{ Final KE} = \frac{1}{2}\ (m + M)V^2.$$

Therefore,

$$\frac{\text{Final KE}}{\text{Initial KE}} = \left(\frac{m+M}{m}\right)\frac{V^2}{v^2} = \frac{m}{m+M}$$

Hence the correct choice is (a).

13. Mass of neutron (m_1) = 1 unit. Mass of nucleus (m_2) = A units. Refer to page 7.2. Here $u_1 = u$ and $u_2 = 0$. Therefore the velocity of the neutron after the collision is

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u = \left(\frac{1-A}{1+A}\right)u$$

KE of neutron after collision = $\frac{1}{2}\ m_1 v_1^2$

$$= \frac{1}{2} \times 1 \times \left(\frac{1-A}{1+A}\right)^2 u^2$$

KE of neutron before collision = $\frac{1}{2}\ m\ u^2$

$$= \frac{1}{2} \times 1 \times u^2 = \frac{1}{2}\ u^2.$$

Their ratio is $\left(\frac{1-A}{1+A}\right)^2$, which is choice (a).

14. The total number of nucleus (i.e. protons + neutrons) in a nucleus is called its mass number. An α-particle is a helium nucleus having 2 protons and 2 neutrons. So the mass number of an α-particle = 4. When a nucleus of mass number A emits an α-particle, the mass number of the daughter nucleus reduces to $(A - 4)$. If V is the recoil speed of the daughter nucleus, we have, from the law of conservation of momentum,

$$(A - 4)V - 4v = 0$$

or $$V = \frac{4v}{A-4}$$

Hence the correct choice is (c).

15. The collision is inelastic because the two bodies stick to each other after collision. In an inelastic collision, only the momentum is conserved; there being a loss in kinetic energy. Hence the correct choice is (a).

16. Since the collision (impact) is elastic, the ball rebounds with the same speed. Therefore, the change in momentum of each ball = 2 mu. The change in momentum per second due to n balls = 2 mnu. But the change in momentum per second is the force. Hence the correct choice is (b).

17. A ball dropped from a height h_1 on reaching the planet's surface will have a velocity given by

$$v_1 = \sqrt{2gh_1}$$

Let v_2 be the velocity with which the ball bounces. It will attain a height h_2 given by

$$v_2^2 = \sqrt{2gh_2}$$

$$\therefore \quad \frac{v_2}{v_1} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{1.8}{5}} = 0.6$$

$$\text{or} \quad 1 - \frac{v_2}{v_1} = 1 - 0.6$$

$$\text{or} \quad \frac{v_1 - v_2}{v_1} = 0.4 = \frac{2}{5}$$

Hence the correct choice is (b).

18. Let m and M be the masses of the lighter and heaviers fragments respectively. Since the particle is moving along the x-axis, the y-component of momentum will be zero immediately after and before explosion, i.e.

$$mv_y + MV_y = 0$$

where v_y and V_y are the velocities of the lighter and heavier fragments respectively immediately after explosion. Thus

$$V_y = -\left(\frac{m}{M}\right)\ v_y = -\left(\frac{m/4}{3m/4}\right)\ v_y$$

$$= -\frac{1}{3}\ v_y$$

Since y = + 15 cm, the direction of v_y is along the positive y-axis and that of V_y will be along the negative y-axis. An instant later (say, at time t), it is given that

$$y = 15 \text{ cm} = v_y t$$

$$\therefore \quad Y = V_y t = -\frac{1}{3}\ v_y t = -\frac{1}{3}\ y$$

$$= -\frac{1}{3} \times 15 \text{ cm} = -5 \text{ cm}$$

19. Given $m_Q = p\ m_P$ and $v_Q = v_P/q$. From the principle of conservation of momentum, we have (since body P comes to rest after collision)

$$m_P\ v_P + m_Q\ v_Q = m_Q\ v$$

where v is the velocity of body Q after collision. Thus

$$m_P\ v_P + p\ m_P\ \frac{v_P}{q} = p\ m_P v.$$

which gives $$\frac{v}{v_P} = \frac{p+q}{pq} \qquad \text{(i)}$$

Now, the coefficient of restitution is given by

$$e = \frac{v}{v_p - v_Q} = \frac{v}{v_p - \frac{v_p}{q}}$$

which gives $\frac{v}{v_P} = \frac{e}{q}(q-1)$ (ii)

Equating (i) and (ii), we get $e = \frac{p+q}{p(q-1)}$ which is choice (d).

20. Refer to Fig. 7.16. If sphere A is projected with velocity v, the time taken by it to strike B is equal to $\frac{\pi r}{v} = T$ or $\pi r = Tv$. Now, the coefficient of restitution is given by

$$e = \frac{v_B - v_A}{v}$$

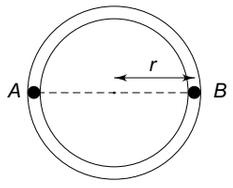


Fig. 7.16

where v_A and v_B are the velocities of A and B after the collision. Thus, $v_B - v_A = ev$. The spheres travel with this relative velocity. It is clear that one will overtake the other after travelling a distance $= 2\pi r$.

$$\therefore \text{ Time taken } = \frac{2\pi r}{v_B - v_A} = \frac{2\pi r}{ev} = \frac{2Tv}{ev} = \frac{2T}{e}$$

(since $\pi r = Tv$). Hence the correct choice is (c).

21. Let V be the velocity of the nucleus and v that of the α-particle after disintegration, then from the principle of conservation of momentum, we have (since the mass of an α-particle is 4 amu)

$$(M-4)\ V = 4v \text{ or } V = \frac{4v}{(M-4)} \quad \text{(i)}$$

$$\text{Total KE} = \frac{1}{2}(M-4)\ V^2 + \frac{1}{2} \times 4 \times v^2 \quad \text{(ii)}$$

Now $\frac{1}{2} \times 4 \times v^2 = K$. Using (i) in (ii), we have

$$\text{Total KE} = \frac{1}{2}(M-4) \times \frac{16v^2}{(M-4)^2} + 2v^2$$

$$= \frac{8v^2}{M-4} + 2v^2$$

$$= \frac{4K}{M-4} + K = \frac{MK}{M-4} \quad (\because 2v^2 = K)$$

Hence the correct choice is (b).

22. The total distance travelled is

$$S = h + e^2h + 2e^4h + 2e^6h + \ldots$$

$$= h + 2h(e^2 + e^4 + e^6 + \ldots)$$

$$= h + 2h\left(\frac{e}{1-e^2}\right)$$

$$= \left[1 + \frac{2e^2}{1-e^2}\right] = \frac{h(1+e^2)}{1-e^2}$$

23. Initial momentum (p) = momentum of bullet + momentum of block = $mv + 0 = mv$. From the principle of conservation of momentum, final momentum of bullet + block system of mass $(M + m)$ = Initial momentum p. Now

$$\text{KE} = \frac{p^2}{2 \times (M+m)} = \frac{m^2v^2}{2(M+m)}$$

Hence the correct choice is (d).

24. The correct choice is (b). In an inelastic collision, the bodies stick together. To find the speed of the composite body, use the principle of conservation of linear momentum.

25. The correct choice is (a). Find kinetic energies before and after the collision.

26. Momentum of 5 kg mass (p_1) = $5 \times 2 = 10$ kg ms^{-1} along the x-axis. Momentum of 10 kg mass (p_2) = $10\sqrt{3}$ kg ms^{-1} along the y-axis. These two momenta are perpendicular to each other. Therefore, the resultant initial momentum is

$$p = \sqrt{p_1^2 + p_2^2} = \sqrt{(10)^2 + \left(10\sqrt{3}\right)^2} = 20 \text{ kg ms}^{-1}$$

If v ms^{-1} is the velocity of the combined mass, then the final momentum = $(10 + 5)\ v = 15\ v$ kg ms^{-1}. Now, from the principle of conservation of momentum, we have $15\ v = 20$ or $v = \frac{4}{3}$ ms^{-1}, which is choice (c).

27. Let V be the velocity of the block with the bullet embedded in it at the time of impact. Then from the principle of conservation of momentum, we have

$$mv = (M + m)\ V$$

$$\text{or} \qquad V = \frac{mv}{(M+m)} \quad \text{(i)}$$

If the block, with the bullet embedded in it, rises to a vertical height h, then from the principle of conservation of energy, we have

$$\frac{1}{2}(M + m)\ V^2 = (M + m)\ gh$$

$$\text{or} \qquad V = \sqrt{2gh} \quad \text{(ii)}$$

Using (ii) in (i), we get

$$\sqrt{2gh} = \frac{mv}{(M+m)}$$

Squaring this equation, we find that h is given correctly by choice (a).

28. Percentage loss of energy $= \dfrac{4mM}{(M+m)^2} \times 100$

$$= \frac{4m \times 2m}{(2m+m)^2} \times 100 = \frac{800}{9} = 88.9\%$$

Hence the correct choice is (c).

29. Refer to Fig. 7.17. Here $p = mv$ and $P = MV$. The resultant of p and P is

$$p_r = \sqrt{p^2 + P^2} = \sqrt{(mv)^2 + (MV)^2}$$

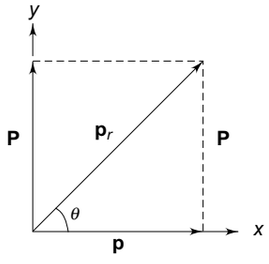


Fig. 7.17

which is choice (a). The angle which the resultant momentum p_r subtends with the x-axis is given by

$\tan\theta = \dfrac{P}{p} = \dfrac{MV}{mv}$, which is choice (b).

Loss of KE

$$= \left(\frac{1}{2}mv^2 + \frac{1}{2}MV^2\right) - \frac{1}{2}\left[\frac{m^2v^2 + M^2V^2}{(M+m)}\right]$$

$$= \frac{1}{2}\frac{Mm}{(M+m)}(V^2 + v^2)$$, which is choice (c).

30. Total distance

$$= h + 2e^2 h + 2e^4 h + \ldots$$

$$= h + 2e^2 h (1 + e^2 + \ldots)$$

$$= h + \frac{2e^2h}{1-e^2} = h\left(\frac{1+e^2}{1-e^2}\right)$$

$$= h\left[\frac{1+\left(\frac{1}{3}\right)^2}{1-\left(\frac{1}{3}\right)^2}\right] = \frac{5h}{4},$$

which is choice (b).

31. Total time $= \sqrt{\dfrac{2h}{g}} + 2e\sqrt{\dfrac{2h}{g}} + 2e^2\sqrt{\dfrac{2h}{g}} + \cdots$

$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h}{g}}\left(e + e^2 + \cdots\right)$$

$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h}{g}}\frac{e}{(1-e)} = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$

$$= \sqrt{\frac{2h}{g}}\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right) = 2\sqrt{\frac{2h}{g}},$$

which is choice (b).

32. Refer to Fig. 7.18.

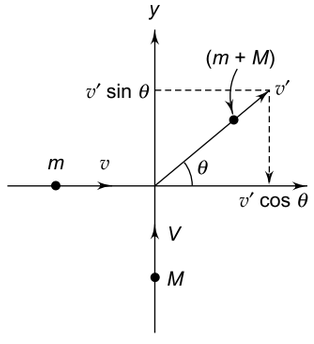


Fig. 7.18

Let v' be the velocity of the composite body at an angle θ with the x-axis (Fig. 7.11). Equating the initial and the final momentum along x and y-axes, we have

$$mv = (m + M)\, v' \cos\theta = p' \cos\theta \qquad \text{(i)}$$

and $$MV = (m + M)\, v' \sin\theta = p' \sin\theta \qquad \text{(ii)}$$

where $p' = (m + M)\, v'$ is the momentum of the composite body. Find p' by squaring and adding (i) and (ii). The correct choice is (c).

33. Divide (ii) by (i). The correct choice is (a).

34. Let u be the velocity of P before collision and v_1 and v_2 the velocities of P and Q after collision. From conservation of momentum, we have $m_1u + 0 = m_1v_1 + m_2v_2$ which gives

$$m_1(u - v_1) = m_2v_2 \qquad (1)$$

From the conservation of kinetic energy we have

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \quad m_1(u^2 - v_1^2) = m_2v_2^2$$

$$\Rightarrow \quad m_1(u - v_1)(u + v_1) = m_2v_2^2 \qquad (2)$$

Dividing Eq. (2) by Eq. (1) we get

$$u + v_1 = v_2 \qquad (3)$$

From Eqs. (1) and (3) we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u \qquad (4)$$

Initial K.E. of $P(K) = \frac{1}{2} m_1 u^2$,

final K.E of $P = \frac{1}{2} m_1 v_1^2$

$\therefore$ Decrease in K.E. of $P(\Delta K) = \frac{1}{2} m_1 u^2 - \frac{1}{2} m_1 v_1^2$.

$\therefore$ Fractional decreases in K.E. of P is

$$\frac{\Delta K}{K} = \frac{\frac{1}{2} m_1 u^2 - \frac{1}{2} m_1 v_1^2}{\frac{1}{2} m_1 u^2}$$

$$= \frac{u^2 - v_1^2}{u^2} = 1 - \frac{v_1^2}{u^2}$$

$$= 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \qquad \text{[use Eq. (4)]}$$

$$= \frac{4 m_1 m_2}{(m_1 + m_2)^2}$$

Thus the correct choice is (b).

35. Refer to Fig. 7.19.

From conservation of x-component of momentum, we have

$$mu\cos\left(\frac{\theta}{2}\right) + mu\cos\left(\frac{\theta}{2}\right) = 2mv$$

$$= 2\ m \times \frac{u}{2} = mu \quad (\because\ v = u/2)$$

$$\Rightarrow \qquad 2\cos\left(\frac{\theta}{2}\right) = 1$$

which gives $\cos\left(\frac{\theta}{2}\right) = \frac{1}{2} \Rightarrow \frac{\theta}{2}\ 60° \Rightarrow \theta = 120°$, which is choice (c)

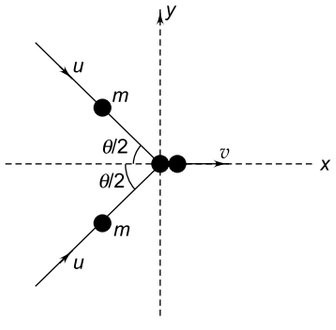


Fig. 7.19

36. Let V be the speed of the block after the bullet emerges out of it. From conservation of momentum we have

$$mu = MV + m\left(\frac{u}{4}\right)$$

which gives $V = \frac{3mu}{4M}$

Now refer to Q. 27 of Section III of Chapter 4. The minimum speed the block must have to complete the vertical circle is

$$V = \sqrt{5gL}$$

$$\Rightarrow \qquad \frac{3mu}{4M} = \sqrt{5gL}$$

$$\Rightarrow \qquad u = \frac{4M}{3m} \times \sqrt{5gL}$$

$$= \frac{4}{3} \times 6 \times \sqrt{5 \times 10 \times 0.5} = 40\ \text{ms}^{-1}.$$

Hence the correct choice is (d).

37. Since mass m_2 is at a distance $L/2$ from the axis of rotation, it speed will be $v/2$ (half that of mass m_1). From the principle of conservation of energy we have

$$\frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 \left(\frac{v}{2}\right)^2 = m_1 gOB + m_2 gOA$$

$$\Rightarrow \quad \frac{1}{2} mv^2 + \frac{1}{2} \times 2m \times \frac{v^2}{4} = mgL + 2mg\frac{L}{2}$$

which gives $v = 2\sqrt{\frac{2gL}{3}}$, which is choice (a).

38. The component of velocity parallel to the wall remains unchanged and the component of velocity perpendicular to the wall reduces by e times its value before collision. Thus we have

$$v_2 \cos\beta = v_1 \cos\alpha$$

and $v_2 \sin\beta = ev_1 \sin\alpha$

Squaring and adding we get

$$v_2 = v_1\sqrt{(\cos^2\alpha + e^2 \sin^2\alpha)}$$

Hence the correct choice is (d).

39. Refer to Fig. 7.20. First collision will occur when angle θ satisfies the equation

$$\frac{\theta r}{v} = \frac{(2\pi - \theta) r}{2v}$$

which gives $\theta = 120°$.

After the first collision at B, m_2 will move back with a speed v and will make a second collision with m_1

at C. After the second collision at C, m_1 will move back with a speed v and meet m_2 at A. If the third collision at A is neglected, the particles will make two collisions before they reach A. Hence the correct choice is (c).

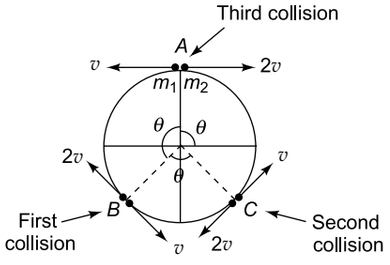


Fig. 7.20

40. Time of flight $(t_f) = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 5}{10}} = 1\text{s}$

Horizontal range (R) = horizontal velocity × time of flight

$\therefore$ Horizontal velocities of the bullet and of the ball after the collision respectively are

$$(v)_{\text{bullet}} = \frac{100}{1} = 100 \text{ ms}^{-1}$$

$$(v)_{\text{ball}} = \frac{20}{1} = 20 \text{ ms}^{-1}$$

From conservation of momentum,

Total initial momentum = total final momentum

$$\Rightarrow \quad (m)_{\text{bullet}} \times V = (m)_{\text{bullet}} \times (v)_{\text{bullet}} + (m)_{\text{ball}} \times (v)_{\text{ball}}$$

$$\Rightarrow \quad 0.01\ V = 0.01 \times 100 + 0.2 \times 20$$

$$\Rightarrow \quad V = 500 \text{ ms}^{-1}$$

41. Refer to Fig 7.14 on page 7.12.

Horizontal velocity before hitting the wall is $u_x = u \cos \theta$.

Horizontal velocity after rebounding from the wall is

$$u'_x = e\ u_x = e\ u \cos\ \theta\ .$$

Horizontal displacement from O to A or from A to $O = x$.

$\therefore$ Time taken to go from O to A is

$$t_1 = \frac{x}{u\cos\theta} \qquad \text{(i)}$$

Time taken to return from A to O is

$$t_2 = \frac{x}{eu\cos\theta} \qquad \text{(ii)}$$

Since the horizontal and vertical motions are independent of each other, the net vertical displacement $S_y = 0$ since the ball returns to O. If t is the total time taken by the ball to go from O to A and return to O, then from

$$S_y = u_y t - \frac{1}{2}\ gt^2 \text{ we have}$$

$$0 = (u\ \sin\theta)\ t - \frac{1}{2}\ gt^2$$

$$\Rightarrow \quad t = \frac{2u\sin\theta}{g} \qquad \text{(iii)}$$

Now $t = t_1 + t_2$. Using (i) and (ii) in (iii), we have

$$\frac{2u\sin\theta}{g} = \frac{x}{u\cos\theta} + \frac{x}{eu\cos\theta}$$

$$\Rightarrow \quad x = \frac{u^2 \sin(2\theta)}{\left(1+\frac{1}{e}\right)}$$

$$= \frac{(20)^2 \sin 60^\circ}{\left(1+\frac{1}{0.5}\right)}$$

$$= \frac{20}{\sqrt{3}} \simeq 11.5 \text{ m, which is choice (c)}$$

SECTION II

Multiple Choice Questions with one or More Choices Correct

1. In an inelastic collision of two bodies, which of the following do not change after the collision?
- (a) total kinetic energy
- (b) total linear momentum
- (c) total energy
- (d) total angular momentum

2. Which of the following statements are true?
- (a) In a elastic collision of two bodies, the momentum and energy of each body is conserved.
- (b) The total energy of a system is always conserved irrespective of whether external forces act on the system.
- (c) The work done by a force in nature on a body, over a closed loop, is not always zero.

(d) In an inelastic collision of two bodies, the final kinetic energy is less than the initial kinetic energy of the system.

3. A molecule in a gas container hits the wall with speed v at an angle θ with the normal and rebounds with the same speed as shown in Fig. 7.21. Which of the following statements are true?
(a) The momentum of the system is conserved in the collision.
(b) The momentum of the molecule before collision with the wall is equal to the momentum of the molecule after collision.
(c) The collision is elastic.
(d) The collision is inelastic.

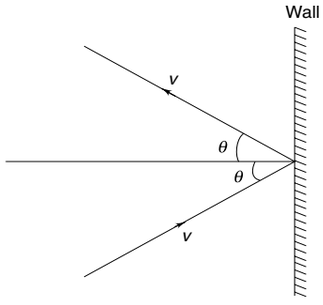


Fig. 7.21

4. A particle A suffers an oblique elastic collision with a particle B that is at rest initially. If their masses are the same, then, after the collision
(a) they will move in the opposite directions
(b) A continues move in the original direction while B remains at rest
(c) they will move in the mutually perpendicular directions
(d) A comes to rest and B starts moving in the direction of the original motion of A

5. Choose the correct statements from the following:
(a) The general form of Newton's second law of motion is $\mathbf{F}_{ext} = m\mathbf{a}$.
(b) A body can have energy and yet no momentum.
(c) A body having momentum must necessarily have kinetic energy.
(d) The relative velocity of two bodies in a head–on collision remains unchanged in magnitude and direction

6. A ball of mass m moving horizontally at a speed v collides with the bob of a simple pendulum at rest. The mass of the bob is also m.
(a) If the balls stick together, the height to which the two balls rise after the collision is $\frac{v^2}{8g}$.
(b) If the balls stick together, the kinetic energy of the system immediately after the collision becomes half of that before collision.
(c) If the collision is perfectly elastic, the bob of the pendulum will rise to a height of $\frac{v^2}{2g}$.
(d) If the collision is perfectly elastic, the kinetic energy of the system immediately after the collision is equal to that before collision.

7. A body of mass 1 kg, initially at rest explodes into three fragments of masses in the ratio of 1 : 1 : 3. The two pieces of equal masses fly off perpendicular to each other with a speed of 30 ms^{-1}, one along the + x direction and the other along the + y direction. Then
(a) the speed of the heavier fragment will be $10\sqrt{2}$ ms^{-1}.
(b) the speed of the heavier fragment will be $15\sqrt{2}$ ms^{-1}.
(c) the direction of motion of the heavier fragment will be at angle of 135° with the + y direction
(d) the direction of motion of the heavier fragment will be at an angle of 45° with the + x direction.

8. Two bodies A and B of masses m and $2m$ respectively are placed on a smooth floor. They are connected by a spring of spring constant k. A third body C of mass m moves with a velocity v_0 along the line joining A and B and collides elastically with A as shown in Fig. 7.22. At a certain instant of time t_0 after the collision, it is found that A and B have the same velocity v and at this instant, the compression of the spring is x_0. Then

(a) $v = \frac{v_0}{2}$ (b) $v = \frac{v_0}{3}$

(c) $k = \frac{2}{3}\frac{mv_0^2}{x_0^2}$ (d) $k = \frac{3}{2}\frac{mv_0^2}{x_0^2}$

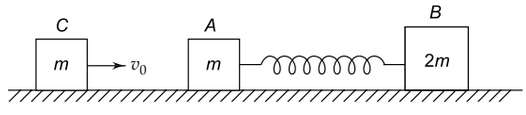


Fig. 7.22

9. A block of mass M attached to a light spring of force constant k rests on a horizontal frictionless surface as shown in Fig. 7.23. A bullet of mass m moving with a horizontal velocity v strikes the block and gets embedded in it. The velocity of the block with the bullet in it just after impact is V. If the impact compresses the spring by an amount x, then

(a) $v = [k(M + m)]^{1/2} \dfrac{x}{m}$

(b) $v = \left(\dfrac{2k}{M+m}\right)^{1/2} x$

(c) $V = [2k\ (M + m)]^{1/2} \dfrac{x}{m}$

(d) $V = \left(\dfrac{k}{M+m}\right)^{1/2} x$

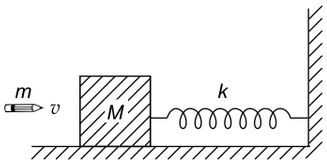


Fig. 7.23

10. A body P of mass 1 kg moving with a velocity of 3 ms^{-1} along the $+x$ direction collides head-on with a body Q of mass 2 kg at rest. The collision is elastic. After the collision
(a) P moves along the $+x$ direction with a velocity of 1 ms^{-1}.
(b) P moves along the $-x$ direction with a velocity of 1 ms^{-1}.
(c) Q moves along the $+x$ direction with a velocity of 2 ms^{-1}.
(d) Q moves along the $-x$ direction with a velocity of 2 ms^{-1}.

11. A block of mass M with a massless spring of force constant k is resting on a horizontal frictionless surface (Fig. 7.24). A block of mass m projected horizontally with a speed u collides and sticks to the spring at the point of maximum compression of the spring.

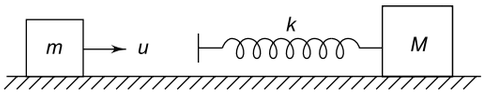


Fig. 7.24

If v is the velocity of the system after mass m sticks to the spring and n is the fraction of the initial kinetic energy of mass m that is stored in the spring, then

(a) $\dfrac{v}{u} = \dfrac{M}{(M+m)}$ (b) $\dfrac{v}{u} = \dfrac{m}{(M+m)}$

(c) $n = \dfrac{M}{(M+m)}$ (d) $n = \dfrac{m}{(M+m)}$

12. A small ball A slides down the quadrant of a circle as shown in Fig. 7.25 and hits the ball B of equal mass which is initially at rest.

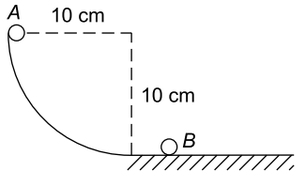


Fig. 7.25

The track is frictionless and the collision is elastic. If v_A and v_B are the velocities of A and B after the collision, then
(a) $v_A = 0$, $v_B = 1.4$ ms^{-1}
(b) $v_A = 1$ ms^{-1}, $v_B = 1$ ms^{-1}
(c) $v_A = -1.4$ ms^{-1}, $v_B = 1.4$ ms^{-1}
(d) $v_A = -1$ ms^{-1}, $v_B = 1$ ms^{-1}

13. Two blocks, each of mass m, moving in opposite directions with the same speed u, on a horizontal frictionless surface, collide with each other, stick together and come to rest. Then
(a) work done by external force on the system is zero.
(b) work done by the external force on the system is mu^2.
(c) work done by the internal force on the system is zero.
(d) work done by the internal force on the system is $-mu^2$.

14. A ball P of mass m_1 moving with velocity u collides head-on with a stationary ball Q of mass m_2. The collision is perfectly elastic. After the collision
(a) if $m_1 = 2\ m_2$, balls P and Q move in the same direction with speeds in the ratio of 1 : 4.
(b) if $m_1 = 3\ m_2$, balls P and Q move in the same direction with speeds in the ratio of 1 : 3.
(c) if $m_2 = 2\ m_1$, balls P and Q move in opposite directions with speeds in the ratio of 1 : 2.
(d) if $m_2 = 3\ m_1$, balls P and Q move in opposite directions with equal speeds.

15. A ball P of mass m_1 moving with a velocity u collides obliquely with a stationary ball Q of mass m_2. The collision is perfectly elastic. After the collision, they fly off making the same angle with the original direction of ball P as shown in Fig. 7.26.

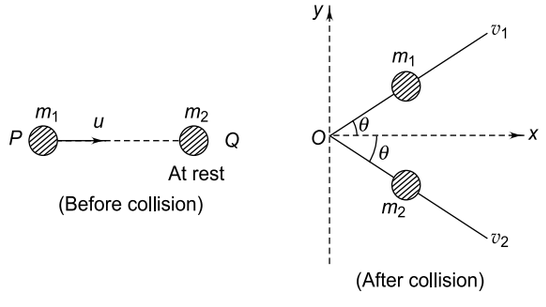


Fig. 7.26

(a) if $m_1 = m_2$, balls P and Q fly off at right angles to each other with the same speed.
(b) if $m_1 = m_2$, balls P and Q fly off at an angle of 60° with each other with the same speed.
(c) if $m_1 = 2\ m_2$, balls P and Q fly off at right angles to each other with speeds in the ratio of 1 : 2.
(d) if $m_1 = 2\ m_2$, balls P and Q fly off at an angle of 60° with each other with speeds in the ratio of 1 : 2.

16. Which of the following statements is/are incorrect in the case of an elastic collision between two bodies?
(a) If two balls of the same mass moving with the same speed in opposite directions, collide head-on, then after the collision they move in opposite direction with the speed each ball had before collision.
(b) If a body suffers a head-on collision with another body of the same mass but at rest, then after the collision, the first body is stopped dead and the second body moves with the velocity of the first.
(c) The coefficient of restitution $e = 1$ for a perfectly elastic collision.
(d) If a body P collides head-on with a body Q of the same mass but at rest, then the percentage fraction of kinetic energy transferred from P to Q is 50%.

17. Which of the following statements is/are true in the case of an inelastic collision between two bodies?
(a) The vector sum of the linear momenta of the two bodies before collision is equal to the vector sum of the linear momenta after the collision in the case of both one-dimensional and two-dimensional collisions.
(b) The total energy of the system is conserved.
(c) The two bodies stick together after inelastic collision.
(d) If a body collides with another body of the same mass but at rest and two bodies stick together, the ratio of the total kinetic energy before and after collision is 2 : 1.

18. A body P of mass 1 kg moving a velocity of 15 ms^{-1} collides head-on with a stationary body Q of the same mass. If the coefficient of restitution is 1/3, then
(a) velocity of P after collision will be 5 ms^{-1}.
(b) velocity of Q after collision will be 10 ms^{-1}.
(c) the loss of kinetic energy of the system is 50 J.
(d) the percentage fractional decrease in the kinetic energy of body P is 50%.

19. A body P of mass m_1 moving with a velocity $\boldsymbol{u}_1 = (a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}})$ collides with a stationary body Q of m_2. After the collision body P is found to move with velocity $\boldsymbol{v}_1 = (c\,\hat{\mathbf{i}} + d\,\hat{\mathbf{j}})$ where a, b, c and d are constants. Then
(a) Impulse received by P is $m_1[(a - c)\,\hat{\mathbf{i}} + (b - d)\,\hat{\mathbf{j}}]$
(b) Impulse received by P is $m_1[(c - a)\,\hat{\mathbf{i}} + (d - b)\,\hat{\mathbf{j}}]$
(c) Impulse imparted to Q is $m_1[(a - c)\,\hat{\mathbf{i}} + (b - d)\,\hat{\mathbf{j}}]$
(d) Impulse imparted to Q is $m_2[(a - c)\,\hat{\mathbf{i}} + (b - d)\,\hat{\mathbf{j}}]$

20. A billiards ball C of mass m moving with velocity u collides two identical balls A and B in contact and at rest. After the collision, ball C is stopped dead and balls A and B move along directions shown in Fig. 7.27 with the same speed v. Then

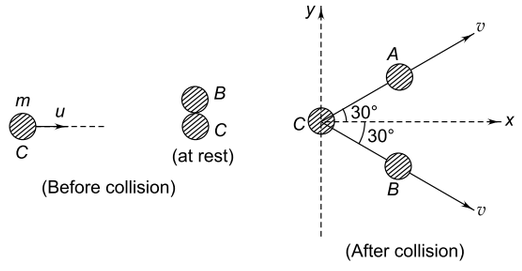


Fig. 7.27

(a) $v = \dfrac{u}{\sqrt{3}}$
(b) $v = \dfrac{u}{\sqrt{2}}$
(c) Loss of kinetic energy = $\dfrac{1}{3}mu^2$
(d) Loss of kinetic energy = $\dfrac{1}{6}mu^2$

21. A U-238 nucleus emits an alpla particle and changes into Th-234. In this process
(a) the momentum of Th-234 is equal and opposite to that of the alpha particle.
(b) the magnitude of the momentum of Th-234 is greater than that of the alpha particle.
(c) the kinetic energy of the alpha particle is equal to that of Th-234.
(d) the kinetic energy of the alpha particle is greater than that of Th-234.

22. A ball hits the floor and rebounds after an inelastic collision. In this case
(a) the momentum of the ball just after the collision is the same as that just before collision,
(b) the mechanicial energy of the ball remains the same in the collision.
(c) the total momentum of the ball and the earth is conserved.
(d) the total energy of the ball and the earth is conserved.

IIT, 1986

23. Two balls having linear momenta $\vec{p}_1 = p\hat{i}$ and $p_2 = -p\hat{i}$ undergo a collision in free space. There is no external force acting on the balls. If $\vec{p}'_1$ and $\vec{p}'_2$ are their final momenta, which of the following option (s) is/are not allowed for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2

(a) $\vec{p}'_1 = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}}; \vec{p}'_2 = a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}}$

(b) $\vec{p}'_1 = c_1\hat{\mathbf{k}}; \vec{p}'_1 = c_2\hat{\mathbf{k}};$

(c) $\vec{p}'_1 = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}}; \vec{p}'_1 = a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{i}} - c_1\hat{\mathbf{k}}$

(d) $\vec{p}'_1 = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}}; \vec{p}'_2 = a_2\hat{\mathbf{i}} + b_1\hat{\mathbf{i}}$

IIT, 2008

SOLUTIONS

1. The correct choices are (b) and (c).

2. Choice (a) is false. In an elastic collision of two bodies, the speeds of the bodies change due to collision. Therefore, the momentum and energy of each body will change but the total momentum and total energy of the system of two bodies are conserved. Choice (b) is also false. The total energy of an isolated system is conserved. If external forces act on the system, the total momentum and energy will change. Choice (c) is true. For a non-conservative force such as friction, the work done over a closed loop is not zero. Choice (d) is also true. In an inelastic collision, the two bodies stick together after colliding. This results in heat or sound energy which is dissipated at the expense of kinetic energy. Hence choices (c) and (d) are correct.

3. The system consists of the molecule and the wall. Let us assume that initially the wall is stationary so that its momentum and kinetic energy are both zero before the collision. Therefore, the total momentum of the wall + molecule system before the collision is $\mathbf{P} = 0 + mv$, where m is the mass of the molecule. After the collision the wall acquires a recoil velocity, say, $\mathbf{V}$ and a recoil momentum $M\mathbf{V}$ where M is the mass of the wall. After the collision, the recoil momentum of the wall + momentum of the outgoing molecule = momentum of the incoming molecule so that the total momentum of the system is conserved. Notice that the momenta of outgoing and incoming molecules are not the same, their directions are different.
Since the wall is infinitely massive, the recoil momentum produces a negligible velocity so that the kinetic energy of the wall is negligible after the collision. Since the speed v of the molecule is the same before and after collision, its kinetic energy remains unchanged. Hence the total kinetic energy is also conserved. Therefore, the collision is elastic. Remember, in an inelastic collision, although the total momentum is conserved, the total kinetic energy is not conserved, it decreases. Hence the correct choices are (a) and (c).

4. From the principle of conservation of momentum, we have total final momentum = total initial momentum.
Momentum conservation is possible in cases (c) and (d). In case (c), the two masses should move in mutually perpendicular directions with velocity $v/\sqrt{2}$ each inclined at 45° with the original direction of motion of particle A. In case (d), particle B must move with velocity v in the original direction of motion of A. Hence the correct choices are (c) and (d).

5. The general form of Newton's second law is

$$\mathbf{F}_{\text{ext}} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{dm}{dt}$$

The form $\mathbf{F}_{\text{ext}} = m\mathbf{a}$ is valid only if $\frac{dm}{dt} = 0$, i.e. if mass does not change with time. Hence choice (a) is incorrect. Choice (b) is correct because a body at rest may have potential energy and yet no momentum. Choice (c) is also correct. A body has momentum if it has mass and velocity and a body having a mass and velocity must have kinetic energy. Choice (d) is incorrect because the relative velocity remains unchanged in magnitude and gets reversed in direction; $(v_2 - v_1) = -(u_2 - u_1)$. Hence the correct choices are (b) and (c).

6. In the inelastic collision, two bodies stick together. After the collision, the speed of the ball and the bob (sticking together) is $v' = v/2$. The height to which they will rise is given by

$$v' = \sqrt{2gh'}$$

or $$h' = \frac{v'^2}{2g} = \frac{v^2}{8g}$$

Mass of the ball and the bob sticking together is $m' = 2\,m$. KE after collision $= \frac{1}{2}\,m'v'^2 = \frac{1}{2} \times 2\,m \times \left(\frac{v}{2}\right)^2 = \frac{1}{4}\,mv^2$. KE before collision $= \frac{1}{2}\,mv^2$.

Therefore, their ratio is 1:2. In an elastic collision between two bodies of the same mass with one of them initially at rest, the moving body is brought to rest and the other moves in the same direction with the same speed. Thus the ball will come to rest and the bob of the pendulum acquires a speed v. At this speed, it will rise to height h given by $h = v^2/2\,g$. Thus all four choices are correct.

7. Let m_1, m_2 and m_3 be the masses of the three fragments. As the total mass is 1 kg and $m_1 : m_2 : m_3 = 1:1:3$, we have $m_1 = m_2 = 0.2$ kg and $m_3 = 0.6$ kg. The linear momentum of m_1 is

$$p_1 = m_1\,v_1 = 0.2 \times 30 = 6 \text{ kg ms}^{-1}$$

and let it be directed along the x-axis (Fig. 7.28).

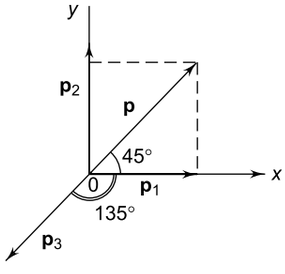


Fig. 7.28

The linear momentum of m_2 is

$$p_2 = m_2\,v_2 = 0.2 \times 30 = 6 \text{ kg ms}^{-1}$$

and let it be directed along the y-axis. The magnitude of the resultant momentum is

$$p = (p_1^2 + p_2^2)^{1/2} = [(6)^2 + (6)^2]^{1/2}$$

$$= 6\sqrt{2} \text{ kg ms}^{-1}$$

The direction of the resultant momentum is given by

$$\tan\theta = \frac{p_2}{p_1} = 1$$

or $\theta = 45°$ with the x or y axes. From the principle of conservation of linear momentum, the magnitude of the momentum of the third fragment is (here v_3 is the speed of the heavier fragment)

$$p_3 = m_3\,v_3 = 6\sqrt{2}$$

But $m_3 = 0.6$ kg. Therefore,

$$v_3 = \frac{6\sqrt{2}}{0.6} = 10\sqrt{2} = 14.1 \text{ ms}^{-1}$$

The direction of the velocity of the heavier fragment is inclined with x or y axes at an angle of 135° (see Fig. 7.28). The correct choices are (a) and (c).

8. Initially (i.e. before collision) bodies A and B are at rest and C is moving to the right (towards A) with a velocity v_0. At a certain instant, say $t = 0$, C collides with A. Since the collision is elastic and A and C have equal masses, the entire momentum (mv_0) and kinetic energy $\left(\frac{1}{2}mv_0^2\right)$ of C are transferred to A and hence C comes to rest. Thus at $t = 0$, A moves to the right with a velocity v_0 and at this instant the spring is uncompressed and B is at rest. Hence the momentum of the system at $t = 0$ is (mv_0). When A moves to the right, it compresses the spring and as a result body B begins to move to the right. It is given that at time $t = t_0$, the compression of the spring is x_0. Let v be the common velocity of A and B at this instant. From the principle of conservation of linear momentum, we have momentum of C before collision = momentum of A after collision + momentum of B after collision

or $$mv_0 = mv + (2m)v \text{ or } v = \frac{v_0}{3} \qquad (1)$$

From the principle of conservation of energy, we have

KE of C before collision =(KE of A + KE of B) after collision + PE in stored spring

or $$\frac{1}{2}\,mv_0^2 = \frac{1}{2}\,mv^2 + \frac{1}{2}\,(2m)v^2 + \frac{1}{2}\,kx_0^2$$

where k is the spring constant. Thus

$$mv_0^2 = 3mv^2 + kx_0^2 \qquad (2)$$

Using (1) in (2), we get

$$mv_0^2 = 3m \times \left(\frac{v_0}{3}\right)^2 + kx_0^2$$

or $$\frac{2mv_0^2}{3} = kx_0^2 \text{ or } k = \frac{2}{3}\frac{mv_0^2}{x_0^2}$$

Thus the correct choices are (b) and (c).

9. PE stored in the spring = $\frac{1}{2}\ kx^2$.

$$mv = (M + m)V$$

$$\text{or } V = \frac{mv}{(M+m)} \qquad (1)$$

After collision, KE of block + bullet in it = PE of the spring. Thus

$$\frac{1}{2}\ (M + m)V^2 = \frac{1}{2}\,kx^2$$

which gives $V = \sqrt{\frac{k}{(M+m)}}\cdot x$ (2)

Using Eq. (2) in (1), we have

$$v = \frac{(M+m)\,V}{m}$$

$$= \frac{(M+m)}{m}\left(\frac{k}{M+m}\right)^{1/2} x$$

$$= [k\ (M + m)]^{1/2}\ \frac{x}{m}$$

Thus the correct choices are (a) and (d).

10. From conservation of momentum, we have

$$m_1u_1 + m_2u = m_1v_1 + m_2v_2$$

$$\Rightarrow \quad 1 \times u_1 + 0 = 1 \times v_1 + 2 \times v_2$$

$$\Rightarrow \qquad v_2 = \frac{1}{2}\,(u_1 - v_1) \qquad (1)$$

From conservation of kinetic energy, we have

$$\frac{1}{2}\,m_1u_1^2 + \frac{1}{2}\,m_2u_2^2 = \frac{1}{2}\,m_1{v_1}^2 + \frac{1}{2}\,m_2v_2^2$$

$$\Rightarrow \qquad u_1^2 = v_1^2 + 2v_2^2$$

$$\Rightarrow \qquad v_1^2 = u_1^2 - 2{v^2}_2 \qquad (2)$$

Solving Eqs. (1) and (2) we get $v_1 = -1$ ms^{-1} and $v_2 = +2$ ms^{-1}.

Hence the correct choices are (b) and (c).

11. For conservation of momentum and conservation of total energy, we have

$$mu = (M + m)\ v \qquad (1)$$

Also $\frac{1}{2}\,mu^2 = \frac{1}{2}\ (M + m)v^2 + \frac{1}{2}\,kx^2$ (2)

Dividing Eq. (2) bv $\frac{1}{2}\ mu^2$, we get

$$1 = \frac{(M+m)v^2}{mu^2} + \frac{\frac{1}{2}kx^2}{\frac{1}{2}mu^2}$$

or $$\frac{\frac{1}{2}kx^2}{\frac{1}{2}mu^2} = 1 - \frac{(M+m)v^2}{mu^2} \qquad (3)$$

From Eq. (1), we have $\frac{v}{u} = \frac{m}{(M+m)}$

Using this in Eq. (3), we get

$$n = \frac{\frac{1}{2}kx^2}{\frac{1}{2}mu^2} = \frac{M}{(M+m)}$$

Thus the correct choices are (b) and (c).

12. If u_A is the velocity with which A strikes B, then

$$\frac{1}{2}\ mu_A^2 = mgh$$

$$\Rightarrow \qquad u_A = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.1} = 1.4 \text{ ms}^{-1}.$$

Since the masses of the balls are equal and the collision is elastic, they exchange their velocities after collision. Hence the only correct choice is (a).

13. Total initial momentum before collision = mu + ($-mu$) = 0. Total final momentum after collision = 0, as the blocks come to rest. Since the change in momentum is zero, no external force acts on the system. Hence no work is done by the external force on the system. From work-energy principle, the work done by the internal force = final K.E – initial K.E = $0 - \left(\frac{1}{2}mu^2 + \frac{1}{2}mu^2\right) = -mu^2$. Hence the correct choices are (a) and (d).

14. Let v_1 and v_2 be the velocities of balls P and Q respectively after the collision. Then we have

$$m_1u = m_1v_1 + m_2v_2 \qquad (1)$$

and $$\frac{1}{2}\,m_1u_1^2 = \frac{1}{2}\,m_1v_1^2 + \frac{1}{2}\,m_2{v^2}_2$$

$$\Rightarrow \qquad m_1u_1^2 = m_1v_1^2 + m_2{v^2}_2 \qquad (2)$$

(a) Putting $m_1 = 2m_2$ in Eqs. (1) and (2) and solving them, we get $v_1 = \frac{u}{3}$ and $v_2 = \frac{4u}{3}$. Thus $v_1 = v_2/4$.

(b) For $m_1 = 3m_2$, we get $v_1 = \frac{u}{2}$ and $v_2 = \frac{3u}{2}$. Thus $v_1 = v_2/3$.

(c) For $m_2 = 2\ m_1$, we get $v_1 = -\frac{u}{2}$ and $v_2 = \frac{2u}{3}$ giving $v_1 = -\ v_2/3$

(d) For $m_2 = 3m_1$, we get $v_1 = -\frac{u}{2}$ and $v_2 = \frac{u}{2}$ giving $v_1 = -\ v_2$

Hence all the four choices are correct.

15. From conservation of x and y components of momentum we have

$$m_1u = m_1v_1 \cos\theta + m_2v_2 \cos\theta$$

and $$0 = m_1v_1 \sin\theta - m_2v_2 \sin\theta$$

which give $$m_1u = (m_1v_1 + m_2v_2)\cos\theta \quad (1)$$

and $$m_1v_1 = m_2v_2 \quad (2)$$

From conservation of kinetic energy, we have

$$\frac{1}{2}m_1u^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \quad m_1u^2 = m_1v_1^2 + m_2v_2^2 \quad (3)$$

If $m_1 = m_2$, then from Eqs. (1) and (2), we get

$$v_1 = v_2 \text{ and } u = 2v_1\cos\theta$$

Using these in Eq. (3), we get $\cos^2\theta = \frac{1}{2}$

$$\Rightarrow \quad \theta = 45°.$$

Hence choice (a) is correct and choice (b) is wrong.

Similarly putting $m_1 = 2m_2$ in Eqs. (1), (2) and (3), we get

$$v_1 = \frac{v_2}{2} \text{ and } \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°.$$

Hence choice (d) is correct and choice (c) is wrong.

16. The only incorrect statement is (d). Refer to the solution of Q.34 of Section I.

$$\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1+m_2)^2} = \frac{4m\times m}{(m+m)^2} = 1 \text{ or } 100\ \%$$

17. The only incorrect statement is (c). Statements (a), (b) and (d) are true.

18. Given $m = 1$ kg, $u = 15$ ms^{-1} and $e = 1/3$. Let v_1 and v_2 be the velocities of P and Q after the collision.

$$mu + 0 = m\,v_1 + m\,v_2$$

$$\Rightarrow \quad u = v_1 + v_2 \quad (1)$$

$$e = \frac{v_2 - v_1}{u} \quad (2)$$

From Eqs. (1) and (2), we get

$$v_1 = \frac{u}{2}(1-e) = \frac{15}{2}\times\left(1-\frac{1}{3}\right) = 5 \text{ ms}^{-1}$$

$$\text{and } v_2 = \frac{u}{2}(1+e) = \frac{15}{2}\times\left(1+\frac{1}{3}\right) = 10 \text{ ms}^{-1}$$

Total K.E. before collision $= \frac{1}{2}mu^2$

$$= \frac{1}{2}\times 1\times(15)^2 = 112.5 \text{ J}$$

Total K.E. after collision $= \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$$= \frac{1}{2}\times 1\times(5^2+10^2) = 62.5 \text{ J}$$

$\therefore$ Loss in K.E. $= 112.5 - 62.5 = 50$ J.

Percentage fractional decrease in K.E. of P is

$$\frac{\Delta K}{K} = \frac{\frac{1}{2}mu^2 - \frac{1}{2}v_1^2}{\frac{1}{2}mu^2}\times 100$$

$$= \frac{u^2 - v_1^2}{u^2}\times 100$$

$$= \frac{(15)^2 - (5)^2}{(15)^2}\times 100 \approx 89\%$$

Hence the correct choices are (a), (b) and (c).

19. Impulse = final momentum – initial momentum. If $\boldsymbol{v}_2$ is the velocity of ball Q after the collision, then from the conservation of momentum, we have

$$m_1\boldsymbol{u} + 0 = m_1\,\boldsymbol{v}_1 + m_2\,\boldsymbol{v}_2$$

$$\Rightarrow \quad m_2\,\boldsymbol{v}_2 = m_1(\boldsymbol{u} - \boldsymbol{v}_1)$$

$$= m_1[(a\hat{i} + b\hat{j}) - (c\hat{i} + d\hat{j})]$$

$$= m_1[(a-c)\hat{i} + (b-d)\hat{j}]$$

Impulse received by P is

$$I_P = m_1\,\boldsymbol{v}_1 - m_1\,\boldsymbol{u}$$
$$= m_1(\boldsymbol{v}_1 - \boldsymbol{u})$$

$$= m_1[(c-a)\hat{i} + (d-b)\hat{j}]$$

Impulse imparted to Q is

$$I_Q = m_2\,\boldsymbol{v}_2 - 0$$

$$= m_1[(a-c)\hat{i} + (b-d)\hat{j}]$$

Hence the correct choices are (b) and (c).

20. Conservation of x-component of momentum gives

$$mu = mv\cos 30° + mv\cos 30°$$

$$= 2\,mv\times\frac{\sqrt{3}}{2} = \sqrt{3}\,mv$$

$$\Rightarrow \quad v = \frac{u}{\sqrt{3}}. \text{ Hence choice (c) is wrong.}$$

K.E. before collision $= \frac{1}{2}mu^2$

K.E. after collision $= \frac{1}{2} mv^2 + \frac{1}{2} mv^2 = mv^2$

$= \frac{mu^2}{3}$

$\therefore$ Loss of K.E. $= \frac{1}{2} mu^2 - \frac{1}{3} mu^2 = \frac{1}{6} mu^2$.

Hence the correct choices are (a) and (d).

21. Initially U-238 is at rest and has zero momentum. When it emits an α-particle, the sum of the momenta of α-particle and Th-234 nucleus must be zero, i.e.

$m_\alpha v_\alpha + m_{Th} v_{Th} = 0 \Rightarrow \frac{v_\alpha}{v_{Th}} = - \frac{m_{Th}}{m_\alpha}$

Now $K_{Th} = \frac{1}{2} m_{Th} v_{Th}^2$ and $K_\alpha = \frac{1}{2} m_\alpha v^2$

Hence $\frac{K_\alpha}{K_{TH}} = \frac{m_\alpha}{m_{TH}} \times \left(\frac{v_\alpha}{v_{TH}}\right)^2$

$= \frac{m_\alpha}{m_{TH}} \times \left(-\frac{m_{TH}}{m_\alpha}\right)^2 = \frac{m_{TH}}{m_\alpha}$

Since $m_{TH} > m_\alpha$; $K_\alpha > K_{Th}$. Hence the correct choices are (a) and (d).

22. In a collision, the momentum of indivdual bodies is not conserved. In an inelastic collision, there is a loss of kinetic energy. Hence choices (a) and (b) are incorrect. The correct choices are (c) and (d).

23. Since no external force acts, the total final initial momentum is $\vec{p}_i = \vec{p}_1 + \vec{p}_2 = p\hat{i} - p\hat{i} = 0$. Therefore, the total final momentum must be zero. This condition can be satisfied in choice (a) if $c_1 = 0$ and in choice (b) and (c) for non-zero values of the coefficients of $\hat{i}$ and $\hat{j}$. Hence the choices (a) and (d) are not allowed.

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage

Passage I

Collisions:

In physics we come across many examples of collisions. The molecules of a gas collide with one another and with the walls of the container. The collision of a neutron with an atom is well known. In a nuclear reactor fast neutrons produced in the fission of uranium atom have to be slowed down. They are, therefore, made to collide with hydrogen atoms. The term collision does not necessarily mean that a particle or a body must actually strike another. In fact, two particles may not even touch each other and yet they are said to collide if one particle influences the motion of the other. When two bodies collide, each body exerts an equal and opposite force on the other. The fundamental conservation laws of physics are used to determine the velocities of the bodies after the collision. Collision may be *elastic* or *inelastic*. Thus a *collision may be defined as an event in which two or more bodies exert relatively strong forces on each other for a relatively short time*. The forces that the bodies exert on each other are internal to the system.

Almost all the knowledge about the sub-atomic particles such as electrons, protons, neutrons, muons, quarks, etc. is obtained from the experiments involving collisions.

There are certain collisions called nuclear reactions in which new particles are formed. For example, when a slow neutron collides with a uranium-235 nucleus, new nuclei baruim-141 and krypton-92 are formed. This collision is called nuclear fission. In nuclear fusion, two nuclei deuterium and trituim collide (or fuse) to form a helium nucleus with the emission of a neutron.

1. Which one of the following collisions is NOT elastic?
- (a) A hard steel ball dropped on a hard concrete floor and rebounding to its original height.
- (b) Two balls moving in the same direction collide and stick to each other.
- (c) Collisions between molecules of an ideal gas
- (d) Collisions of fast neutrons with hydrogen atoms in a fission reactor.

2. Which one of the following statements is true about inelastic collisions?
- (a) The total kinetic energy of the particles after collision is equal to that before collision.
- (b) The total kinetic energy of the particles after collision is less than that before collision.
- (c) The total momentum of the particles after collision is less than that before collision.

(d) Kinetic energy and momentum are both conserved in the collision.

3. In elastic collisions
(a) only energy is conserved
(b) only momentum is conserved
(c) neither energy nor momentum is conserved
(d) both energy and momentum are conserved.

ANSWERS

1. The correct choice is (b)
2. The correct choice is (b)
3. The correct choice is (d)

Questions 4 and 5 are based on the following passage

Passage II

Two balls marked 1 and 2 of the same mass m and a third ball marked 3 of mass M are arranged over a smooth horizontal surface as shown in Fig. 7.29. Ball 1 moves with a velocity v_1 towards balls 2 and 3. All collisions are assumed to be elastic.

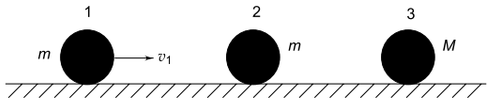


Fig. 7.29

4. If $M < m$, the number of collision between the balls will be
(a) one (b) two
(c) three (d) four

5. If $M > m$, the number of collisions between the balls will be
(a) one (b) two
(c) three (d) four

SOLUTION

4. The first collision will be between balls 1 and 2. Since both have the same mass, after the collision ball 1 with come to rest and ball 2 will move with speed v_1. The ball will collide with the stationary ball 3. After this second collision, let v_2 and v_3 be the speeds of balls 2 and 3 respectively. Since the collision are elastic, v_2 and v_3 are given by (see page 7.2)

$$v_2 = \left(\frac{m-M}{m+M}\right) v_1 \qquad \text{(i)}$$

and $$v_3 = \left(\frac{2m}{m+M}\right) v_1 \qquad \text{(ii)}$$

If $M < m$, it follows from (i) and (ii) that $v_2 < v_3$ and both have the same direction. Therefore, ball 2 cannot collide with ball 3 again. Hence there are only two collisions. Thus the correct choice is (b).

5. If $M > m$, we have from Eq. (i)

$$v_2 = -\left(\frac{M-m}{M+m}\right) v_1$$

The negative sign indicates that, after the second collision, ball 2 will move in opposite direction towards the ball 1 which is at rest after the first collision. Therefore, ball 2 will make another collision with ball 1.

Hence, in this case, there are three collisions in all between the balls. Thus the correct choice is (c).

Questions 6 to 8 are based on the following passage

Passage-III

A body of mass $m_1 = m$ moving with a velocity $v_1 = v$ in the x-direction collides with another body of the same mass $m_2 = m$ moving in the y-direction with the same speed $v_2 = v$. They coalesce into one body during the collision.

6. The magnitude of the momentum of the composite body is
(a) mv (b) $\sqrt{2}\, mv$
(c) $2mv$ (d) $2\sqrt{2}\, mv$

7. The angle which the direction of the momentum vector of the composite body makes with the x-axis is
(a) 30° (b) 45°
(c) less than 30° (d) greater than 45°

8. The fraction of initial kinetic energy transformed into heat during the collision is
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

SOLUTION

6. Refer to Fig. 7.30. Let v' be the velocity of the composite body and let θ be the angle which the velocity vector v' makes with the x-axis.

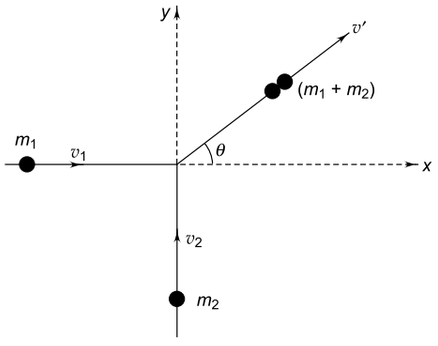


Fig. 7.30

Conservation of x and y components of momentum gives

$$m_1v_1 = (m_1 + m_2)v'\cos\theta \Rightarrow mv = 2mv'\cos\theta \quad (1)$$

$$m_2v_2 = (m_1 + m_2)v'\sin\theta \Rightarrow mv = 2mv'\sin\theta \quad (2)$$

Squaring and adding Eqs. (1) and (2), we get

$$2(mv)^2 = (2mv')^2 \Rightarrow 2mv' = \sqrt{2}\,mv \quad (3)$$

Momentum of composite body is $2mv'$. Hence the correct choice is (b).

7. Diving Eq. (2) by Eq. (1), we have $\tan\theta = 1 \Rightarrow \theta = 45°$, which is choice is (b).

8. Initial kinetic energy is

$$K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

From Eq. (3), we have $v' = v/\sqrt{2}$. Final kinetic energy is

$$K_f = \frac{1}{2}(m_1 + m_2)(v')^2$$

$$= \frac{1}{2} \times 2m \times \frac{v^2}{2} = \frac{1}{2}mv^2$$

$\therefore$ Loss in K.E. $= K_i - K_f$

$$= mv^2 - \frac{1}{2}mv^2$$

$$= \frac{1}{2}mv^2$$

Fraction of initial K.E. transformed into heat is

$$\frac{\Delta K}{K_i} = \frac{\frac{1}{2}mv^2}{mv^2} = \frac{1}{2}.$$

Hence the correct choice is (a).

Questions 9 to 11 are based on the following passage

Passage IV

A ball P moving with a velocity u strikes an identical stationary ball Q such that after the collision, the direction of motion of balls P and Q make an angle of 30° with the original direction of motion of ball P, as shown in Fig. 7.31.

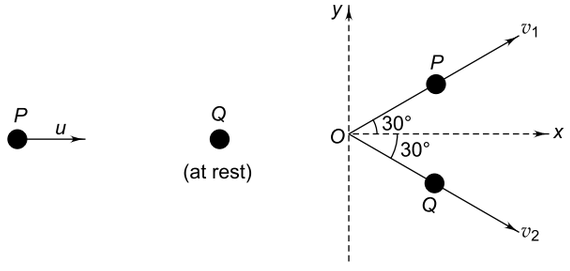


Fig. 7.31

9. The speed v_1 of ball P after the collision is

(a) $\frac{u}{2}$ (b) $\frac{u}{3}$

(c) $\frac{u}{\sqrt{2}}$ (d) $\frac{u}{\sqrt{3}}$

10. The speed v_2 of ball Q after the collision is

(a) $\frac{u}{\sqrt{3}}$ (b) $\frac{u}{3}$

(c) $\frac{2u}{\sqrt{3}}$ (d) $\frac{2u}{3}$

11. The ratio of the total kinetic energy of the balls after collision to that before collision is

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{2}{3}$ (d) $\frac{1}{\sqrt{2}}$

SOLUTION

From conservation of x and y components of momentum,

$$mu = mv_1\cos 30° + mv_2\cos 30°$$

$$\Rightarrow \quad u = (v_1 + v_2)\frac{\sqrt{3}}{2} \quad (1)$$

and $$0 = mv_1\sin 30° - mv_2\sin 30°$$

$\Rightarrow \quad v_1 = v_2 \qquad (2)$

9. Eqs. (1) and (2) give $v_1 = \frac{u}{\sqrt{3}}$, which is choice (d).

10. $v_2 = v_1 = \frac{u}{\sqrt{3}}$. Hence the correct choice is (a).

11. Total kinetic energy before collision is

$$K_i = \frac{1}{2} mu^2$$

and after collision

$$K_f = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2$$

$$= \frac{1}{2} m \left(\frac{u^2}{3}\right) + \frac{1}{2} m \left(\frac{u^2}{3}\right) = \frac{mu^2}{3}$$

$\therefore \quad \frac{K_f}{K_i} = \frac{2}{3}$, which is choice (c).

Questions 12 to 14 are based on the following passage

Passage V

A body P of mass m moving with a velocity u along the + x-direction makes a head-on elastic collision a body Q of mass 2 m at rest.

12. After the collision, body P moves along the
(a) positive x-direction with speed $u/3$.
(b) negative x-direction with speed $u/3$.
(c) positive x-direction with speed $2u/3$.
(d) negative x-direction with speed $2u/3$.

13. After the collision, body Q moves along the
(a) positive x-direction with speed $u/3$.
(b) negative x-direction with speed $u/3$.
(c) positive x-direction with speed $2u/3$.
(d) negative x-direction with speed $2u/3$.

14. What fraction of its kinetic energy does body P lose after the collision?

(a) $\frac{8}{9}$ (b) $\frac{7}{8}$

(c) $\frac{6}{7}$ (d) $\frac{5}{6}$

SOLUTION

Let V be the velocity of body Q after the collisions.

From the principle of conservation of linear momentum, we have

$$mu = mv + (2m)V$$

or $\quad u - v = 2\ V \qquad (1)$

The conservation of kinetic energy gives

$$\frac{1}{2}\ mu^2 = \frac{1}{2} mv^2 + \frac{1}{2} (2m)V^2$$

or $\quad u^2 - v^2 = 2V^2$

or $\quad (u - v)\ (u + v) = 2\ V^2 \qquad (2)$

Using Eq (1) in Eq (2), we have

$$2V(u + v) = 2V^2 \text{ or } u + v = V$$

or $\quad 2(u + v) = 2V \qquad (3)$

12. From Eqs. (1) and (3), we get $v = -u/3$. Hence the correct choice is (b).

13. Using $v = -u/3$ in Eq. (1), we get $V = 2u/3$. Hence the correct choice is (c).

14. Initial skinetic energy of the colliding mass is

$$K_i = \frac{1}{2}\ mu^2$$

Final kinetic energy, $K_f = \frac{1}{2} mv^2$

Loss in kinetic energy is

$$\Delta K = K_i - K_f = \frac{1}{2} mu^2 - \frac{1}{2} mv^2$$

$\therefore$ Fractional loss $= \frac{\Delta K}{K_i}$

$$= \frac{\frac{1}{2} mu^2 - \frac{1}{2} mv^2}{\frac{1}{2} mu^2}$$

$$= \frac{u^2 - v^2}{u^2} = 1 - \left(\frac{v}{u}\right)^2$$

$$= 1 - \left(-\frac{1}{3}\right)^2 \qquad (\because\ v = -u/3)$$

$$= \frac{8}{9}, \text{ which is choice (c).}$$

Questions 15 to 17 are based on the following passage

Passage VI

A body A of mass m_1 moving with a velocity u makes a head-on elastic collision with a body B of mass m_2 initially at rest.

15. After the collision, body B will move with the greatest speed if
(a) $m_1 >> m_2$ (b) $m_1 << m_2$
(c) $m_1 = m_2$ (d) $m_1 = 2\ m_2$

16. After the collision, body B will move with the greatest momentum if

(a) $m_1 >> m_2$ (b) $m_1 << m_2$

(c) $m_1 = m_2$ (d) $m_1 = 2\ m_1$

17. After the collision, body B will move with the greatest kinetic energy if

(a) $m_1 >> m_2$ (b) $m_1 << m_2$

(c) $m_1 = m_2$ (d) $m_2 = 2\ m_1$

SOLUTION

Since the collision is elastic, both linear momentum and kinetic energy are conserved. Thus

$$m_1\ u_1 = m_1\ v_1 + m_2\ v_2 \quad (1)$$

and $$\frac{1}{2} m_1\ u_1^2 = \frac{1}{2}\ m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad (2)$$

Solving for v_1 and v_2, we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\ u_1 \quad (3)$$

and $$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right) u_1 \quad (4)$$

15. Equation (4) gives the recoil speed of body B which can rewritten as

$$v_2 = \frac{2u_1}{\left\{1 + \left(\frac{m_2}{m_1}\right)\right\}}$$

Now v_2 will be maximum if the denominator is the minimum, i.e. if $\frac{m_2}{m_1} << 1$ or $m_2 << m_1$. Thus body B will move with the greatest speed if its mass is very small compared to the mass of body A. Hence the correct choice is (a).

16. The momentum of body B after the collision is

$$p_2 = m_2\ v_2 = \frac{2m_1 m_2 u}{(m_1 + m_2)}$$

The denominator $(m_1 + m_2)$ can rewritten as

$$m_1 + m_2 = \left(\sqrt{m_1}\right)^2 + \left(\sqrt{m_2}\right)^2 - 2\sqrt{m_1 m_2} + 2\sqrt{m_1 m_2}$$

$$= \left(\sqrt{m_1} - \sqrt{m_2}\right)^2 + 2\sqrt{m_1 m_2}$$

$$\therefore \quad p_2 = \frac{2m_1 m_2 u}{\left(\sqrt{m_1} - \sqrt{m_2}\right)^2 + 2\sqrt{m_1 m_2}}$$

Momentum p_2 will be maximum if the denominator is the minimum, i.e. if $\left(\sqrt{m_1} - \sqrt{m_2}\right)^2 = 0$ since a perfect square can never be negative. Thus for p_2 to be maximum $\sqrt{m_1} = \sqrt{m_2}$ or $m_1 = m_2$. Hence body B will move with the greatest momentum if its mass is equal to the mass of body A. Hence the correct choice is (c).

17. The kinetic energy of body B after the collision is

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 \left(\frac{2m_1 u}{m_1 + m_2}\right)^2 = \frac{2m_1^2 m_2 u^2}{(m_1 + m_2)^2}$$

The denominator $(m_1 + m_2)^2$ can be rewritten as

$$(m_1 + m_2)^2 = (m_1 + m_2)^2 - 4m_1\ m_2 + 4m_1\ m_2$$

$$= (m_1 - m_2)^2 + 4\ m_1\ m_2$$

$$\therefore \quad K_2 = \frac{2m_1^2 m_2 u^2}{(m_1 - m_2)^2 + 4m_1 m_2}$$

Energy K_2 will be maximum if the denominator is the minimum, i.e. if $(m_1 - m_2)^2 = 0$ or $m_1 = m_2$. Hence body B will move with maximum energy if ita mass is equal to the mass of body A. Thus the correct choice is (c).

Questions 18 to 20 are based on the following passage

Passage VII

A body P of mass m moving along the positive x-direction with velocity u collides with a body Q of mass M initially at rest. After the collision body P moves along the positive y-direction and body Q moves along a direction making an angle θ below the x-axis. The collision is assumed to be elastic.

18. The angle θ is give by

(a) $\theta = \tan^{-1}\left(\frac{M}{m}\right)$ (b) $\theta = \tan^{-1}\left(\frac{m}{M}\right)$

(c) $\theta = \tan^{-1}\left(\frac{M-m}{M+m}\right)$ (d) $\theta = \tan^{-1}\left(\frac{M+m}{M-m}\right)$

19. If $M = 2m$, the speed of body P after the collision will be

(a) $\frac{u}{2}$ (b) $\frac{u}{\sqrt{2}}$

(c) $\frac{u}{3}$ (d) $\frac{u}{\sqrt{3}}$

20. If $M = 2m$, the kinetic energy gained by body Q due to collision is

(a) $\frac{2}{3}\ mu^2$ (b) $\frac{1}{3}\ mu^2$

(c) $\frac{1}{2}\ mu^2$ (d) mu^2

SOLUTION

Refer to Fig. 7.32.

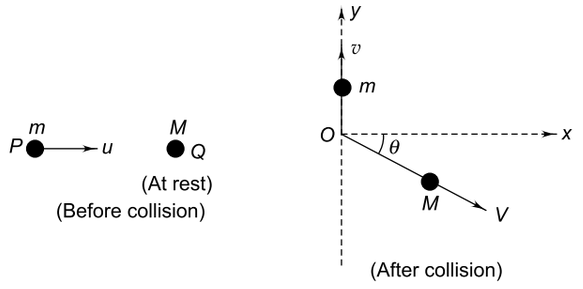


Fig. 7.32

Conservation of x and y components of momentum gives

$$mu = MV\cos\theta \quad (1)$$

$$0 = mv - MV\sin\theta$$

$$\Rightarrow \quad mv = MV\sin\theta \quad (2)$$

From conservation of kinetic energy we have

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \quad (3)$$

18. Using Eqs. (1) and (2) in Eq. (3), we get

$$\sin\theta = \left(\frac{M-m}{2M}\right)^{1/2} \text{ and } \cos\theta = \left(\frac{M+m}{2M}\right)^{1/2}$$

Hence the correct choice is (c).

19. If $M = 2m$, then $\sin\theta = \left(\frac{2m-m}{2\times 2m}\right)^{1/2} = \frac{1}{2}$

$$\Rightarrow \quad \theta = 30°$$

Dividing Eq. (2) by Eq. (1), we get

$$v = u\tan\theta = u\tan(30°) = \frac{u}{\sqrt{3}}$$

Hence the correct choice is (d).

20. Similarly we get $V = \frac{u}{\sqrt{3}}$. Therefore, the increase in K.E. of Q is $\frac{1}{2}MV^2 - 0 = \frac{1}{2}MV^2$

$$= \frac{1}{2}\times 2m\times\left(\frac{u}{\sqrt{3}}\right)^2 = \frac{1}{3}mu^2.$$

Hence the correct choice is (b).

Questions 21 to 24 are based on the following passage

Passage VIII

A ball of mass m is dropped from a height h on a smooth horizontal floor. The coefficient of restitution is e.

21. The total distance covered by the ball before it comes to rest is

(a) $h\left(\frac{1+e}{1-e}\right)$ (b) $h\left(\frac{1+e}{1-e}\right)^2$

(c) $h\left(\frac{1+e^2}{1-e^2}\right)$ (d) $h\left(\frac{1+e^2}{1-e^2}\right)^{1/2}$

22. The total time taken by the ball to come to rest is

(a) $\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$ (b) $\sqrt{\frac{2h}{g}}\left(\frac{1+e^2}{1-e^2}\right)$

(c) $\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)^2$ (d) $\sqrt{\frac{2h}{g}}\left(\frac{1+e^2}{1-e^2}\right)^{1/2}$

23. The total momentum imparted by the ball to the floor is

(a) $m\sqrt{2hg}\left(\frac{1+e^2}{1-e^2}\right)$ (b) $m\sqrt{2hg}\left(\frac{1+e}{1-e}\right)$

(c) $m\sqrt{2hg}$ (d) $me\sqrt{2hg}$

24. The average force exerted by the ball on the wall is

(a) mg (b) $mg\left(\frac{1+e}{1-e}\right)$

(c) $mg\left(\frac{1+e^2}{1-e^2}\right)$ (d) zero

SOLUTION

21. The ball will hit the floor with a speed $v = \sqrt{2gh}$. It will rebounce with a speed $v_1 = ev$ and will rise to a height h_1 given by

$$v_1^2 = 2g\,h_1$$

which gives $h_1 = \frac{v_1^2}{2g} = \frac{e^2v^2}{2g} = \frac{e^2\times 2gh}{2g} = e^2h.$

Between the first and the second impact with the floor, the distance covered by the ball is $2h_1 = 2e^2h$.

Between the second and the third impact, the distance covered $= 2e^4h$ and so on. Therefore, the total distance covered by the ball before it stops is

$$H = h + 2e^2h + 2e^4h + \ldots$$

$$= h + 2e^2h(1 + e^2 + \ldots.)$$

$$= h + \frac{2e^2h}{(1-e^2)} = h\left(\frac{1+e^2}{1-e^2}\right)$$

Hence the correct choice is (c).

22. Total time taken $\Delta t = \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} +$

$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h}{g}}\, e\,(1 + e + ...)$$

$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h}{g}}\left(\frac{e}{1-e}\right) = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$

Hence the correct choice is (a).

23. Total momentum transferred is

$$\Delta p = m[v + 2(v_1 + v_2 + ...)]$$
$$= m[v + 2(ev + e^2v + ...)]$$
$$= m[v + 2ev(1 + e + ...)]$$
$$= m\left[v + \frac{2ev}{1-e}\right] = mv\left(\frac{1+e}{1-e}\right)$$
$$= m\sqrt{2gh}\left(\frac{1+e}{1-e}\right)$$

Thus the correct choice is (b).

24. Average force $= \dfrac{\text{momentum transferred}}{\text{time taken}} = \dfrac{\Delta p}{\Delta t}$

$$= \frac{m\sqrt{2gh}\left(\frac{1+e}{1-e}\right)}{\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)} = mg$$

Hence the correct choice is (a).

Questions 25 to27 are based on the following passage

Passage IX

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in Fig. 7.33. The angle of the incline suddenly change from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic (g = 10 m/s^2)

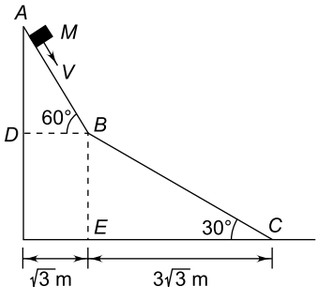


Fig. 7.33

IIT, 2008

25. The speed of the block at point B immediately after it strikes the second incline is

(a) $\sqrt{60}$ ms^{-1} (b) $\sqrt{45}$ ms^{-1}

(c) $\sqrt{30}$ ms^{-1} (d) $\sqrt{15}$ ms^{-1}

26. The speed of the block at point C, immediately before it leaves the second incline is

(a) $\sqrt{120}$ ms^{-1} (b) $\sqrt{105}$ ms^{-1}

(c) $\sqrt{90}$ ms^{-1} (d) $\sqrt{75}$ ms^{-1}

27. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

(a) $\sqrt{30}$ ms^{-1} (b) $\sqrt{15}$ ms^{-1}

(c) 0 (d) $-\sqrt{15}$ ms^{-1}

SOLUTION

25. Let v_B be the speed of the block just before it strikes the second incline.

$$\frac{1}{2}mv_B^2 = mg \times AD = mg \times BD \tan 60°$$

$$\Rightarrow \quad v_B = \left(2 \times 10 \times \sqrt{3} \tan 60°\right)^{1/2} = \sqrt{60}\ \text{ms}^{-1}$$

This velocity can be resolved into two components. $v_B \cos 30°$ along the second incline and $v_B \sin 30°$ perpendicular to it (see Fig.7.34)

In an inelastic collision, the perpendicular component becomes zero after the collision. Hence the speed of the block at point B immediately after the collision is $v_B \cos 30° = \sqrt{60} \times \dfrac{\sqrt{3}}{2} = \sqrt{45}$ ms^{-1}.

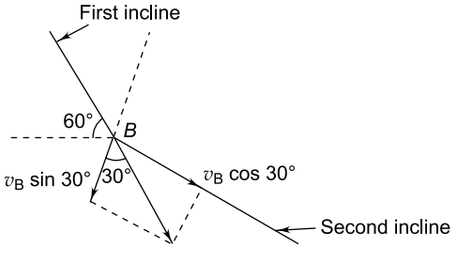


Fig. 7.34

26. Let v_C be the speed at C. From conservation of energy, gain in K.E. = loss in P.E., i.e.

$$\frac{1}{2}mv_C^2 - \frac{1}{2}mv_B^2 = mg \times BE = mg \times EC \tan 30°$$

$$\Rightarrow \quad v_C^2 = v_B^2 + 2g \times 3\sqrt{3} \tan 30°$$

$$= 45 + 2 \times 10 \times 3\sqrt{3} \times \frac{1}{\sqrt{3}} = 105$$

$$\Rightarrow \quad v_C = \sqrt{105} \text{ ms}^{-1}$$

27. If the collision is perfectly elastic, the perpendicular component $v_B \sin 30° = \sqrt{60} \times \frac{1}{2} = \sqrt{15}$ ms^{-1} is reversed after the collision as shown in the Fig.7.35. The resultant vertical component is $\sqrt{15} \sin 60° - \sqrt{45} \sin 30° = 0$, which is choice (c).

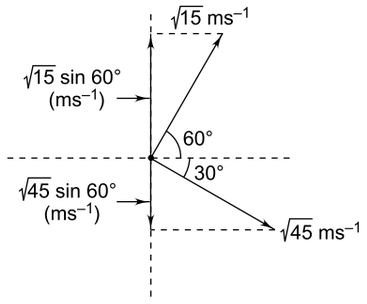


Fig. 7.35

SECTION IV

Matching

Match the Statements in Column I with the Processes in Column II

Column I	***Column II***
(a) Collision of two light nuclei to form a heavier nucleus	(p) Elastic collision
(b) A speeding bullet getting embedded in a wooden plank	(q) Inelastic collision
(c) Collision of neutron with a heavy unstable nucleus	(r) Nuclear fission
(d) Collision in which there is no loss of kinetic energy	(s) Nuclear fusion

ANSWER

(a) → (s) (b) → (q)
(c) → (r) (d) → (b)

SECTION V

Reason-Assertion Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true; Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true but Statement-2 is not the correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

1. Statement-1
Two identical balls B and C lie on a horizontal smooth straight groove so that they are touching. A third identical ball A moves at a speed v along the groove and collides with B (see Fig. 7.36). If the collisions are perfectly elastic, then after the collision, balls A and B will come to rest and ball C moves with velocity v to the right.

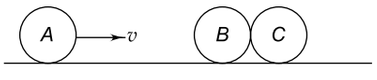


Fig. 7.36

Statement-2
In an elastic collision, linear momentum and kinetic energy are both conserved.

2. **Statement-1**
Two bodies A and B of masses m and 2 m respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with a velocity u and collides elastically with A as shown in Fig. 7.37. At a certain instant t_0 after the collision, it is found that the velocities of A and B are the same = $u/3$.

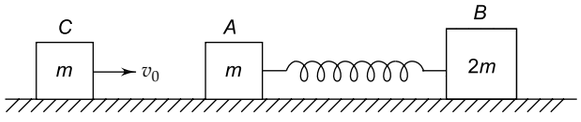


Fig. 7.37

Statement-2
In an elastic collision, the kinetic energy of the system is conserved.

3. **Statement-1**
In an inelastic collision between two bodies, the total energy does not change after the collision but the kinetic energy of the system decreases.
Statement-2
The loss of kinetic energy appears as heat in the system.

4. **Statement-1**
In a collision between two bodies, each body exerts an equal and opposite force on the other at each instant of time during the collision.
Statement-2
The total energy of the system is conserved.

5. **Statement-1**
The term 'collision' between two bodies does not necessarily mean that the two bodies actually strike against each other.
Statement-2
In physics, a collision is said to take place if the one body influences the motion of the other.

6. **Statement-1**
In an inelastic collision, the two colliding bodies stick to each other after the collision and move with a common velocity.
Statement-2
There is a lost of total kinetic energy in an inelastic collision.

7. **Statement-1**
In a collision between two bodies, the linear momentum each body remains constant.
Statement-2
If no external force acts, the total linear momentum of a system is conserved.

8. **Statement-1**
In an elastic collision between two bodies, the energy of each body is conserved.
Statement-2
The total energy of an isolated system is conserved.

9. **Statement-1**
The total energy of a system is always conserved irrespective of whether external forces act on the system.
Statement-2
The total energy of an isolated system is always conserved.

10. **Statement-1**
A body P of mass M moving with speed u collides head-on and elastically with a body Q of m initially at rest. If $m << M$, body Q will have a maximum speed equal to 2 u after the collision.
Statement-2
In an elastic collision, the momentum and kinetic energy are both conserved.

11. **Statement-1**
A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.
Statement-2
The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

IIT, 2007

12. **Statement-1**
In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.
Statement-2
In an elastic collision, the linear momentum of the system is conserved.

IIT, 2007

SOLUTIONS

1. The correct choice is (a). Linear momentum will be conserved if A comes to rest and B and C move to the right with a velocity $v/2$ each or A, B and C all move to the right with velocity $v/3$ each. It is easy to see that in these two cases, the kinetic energy is not conserved. Hence the only result of the collision is the one given in Statement-1.

2. The correct choice is (b). Let C collide with A at $t = 0$. Since the collision in elastic and A and C have equal masses, C will come to rest and A will move to the right with velocity u and at this instant the spring is uncompressed and B is at rest. Hence at $t = 0$, the momentum of the system = mu. When A moves to the right, it compresses the spring and as a result B beings to move to the right. Let v be the common velocity of A and B at time t_0. From the principle of conservation of linear momentum, we have
Momentum of C before collision = momentum of A after collision + momentum of B after collision

or $\quad mu = mv + (2m)\, v \Rightarrow v = \dfrac{u}{3}$.

3. The correct choice is (a). The total energy (which includes all forms of energy) is conserved in any process.

4. The correct choice is (b). Statement-1 follows from Newton's third law of motion.

5. The correct choice is (a).

6. The correct choice is (d). The two colliding body need not get stuck after an inelastic collision.

7. The correct choice is (d). Since the velocities of the two bodies change due to collision, the linear momentum of each body will change but the total linear momentum of the system of two bodies is conserved.

8. The correct choice is (d). Due to change in velocity, the energy of each body changes on collision but the total energy of the system of two bodies is conserved.

9. The correct choice is (d). If an external force acts on a system, it is accelerated which will increase the total energy.

10. The correct choice is (a). If v and V are the velocities of Q and P after the collision, then from conservation of momentum and kinetic energy, we have

$$Mu = mv + MV \Rightarrow M(u - V) = mv \qquad (1)$$

$$\frac{1}{2}Mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

$$\Rightarrow \qquad M(u - V)(u + V) = mv^2 \qquad (2)$$

From Eqs. (1), (2), we get

$$v = \frac{2Mu}{(M+m)} = \frac{2u}{\left(1+\dfrac{m}{M}\right)}$$

If $M >> m$, then v is maximum equal to $2u$ (since $\dfrac{m}{M} \to$ zero).

11. Statement-1 is true. The decrease in mechanical energy is smaller when the block is made to go up on the inclined surface because some part of the kinetic energy is converted into gravitational potential energy. Statement-2 is false. The coefficient of friction does not depend on the angle of inclination of the plane.

12. For elastic collisions both the statements are true but statement-2 is not the correct explanation of statement-1. Linear momentum is conserved in elastic as well as inelastic collision. But in an elastic collision the total kinetic energy of the system is also conserved. Statement-1 is obtained if we use both the conservation of linear momentum as well as the conservation of kinetic energy in the case of an elastic collision.

SECTION VI

Integer Answer Type

1. A shell of mass 4 kg, initially at rest explodes into three fragments. Two of the fragments, each of mass 1 kg are found to move with a speed of 2 ms $^{-1}$ each in mutually perpendicular directions. Find the total energy (in joule) released in the explosion.

IIT, 1987

2. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position and released (see Fig. 7.38). The bob hits the wall, the restitution coefficient being $2/\sqrt{5}$. What is the minimum number of impacts with wall after which the amplitude becomes less than 60°?

IIT, 1988

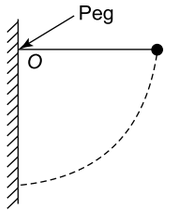


Fig. 7.38

3. A cart is moving along the + x direction with a velocity of 4 ms^{-1}. A person on the cart throws a stone with a velocity of 6 ms^{-1} relative to himself. In the reference frame of the cart, the stone is thrown in the y-z plane making an angle of 30° with vertical z-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from the branch of a tree. A completely inelastic collision occurs in which the stone gets embedded in the object. Find the speed in ms^{-1} of the combined mass immediately after the collision with respect to an observer on the ground.

IIT, 1997

4. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (**in m/s**) of the object C. [see Fig. 7.39]

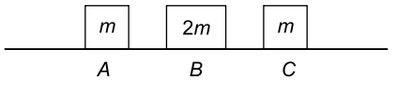


Fig. 7.39

IIT, 2009

SOLUTIONS

1. Refer to Fig. 7.40.

$$p_1 = 1 \text{ kg} \times 2 \text{ ms}^{-1} = 2 \text{ kg ms}^{-1}$$

$$p_2 = p_1 = 2 \text{ kg ms}^{-1}$$

$$p = \sqrt{p_1^2 + p_2^2} = 2\sqrt{2} \text{ kg ms}^{-1}$$

From the conservation of momentum, we have

$$\vec{p}_3 + \vec{p} = 0$$

$$\therefore \quad p_3 = -p = -2\sqrt{2} \text{ kg ms}^{-1}$$

$$\Rightarrow \quad 2 \times v = -2\sqrt{2} \Rightarrow v = -\sqrt{2} \text{ ms}^{-1}$$

Total energy released

$$= \frac{1}{2} \times 1 \times (2)^2 + \frac{1}{2} \times 1 (2)^2 + \frac{1}{2} \times 2 \times \left(-\sqrt{2}\right)^2$$

$$= 2 + 2 + 2 = 6 \text{ J}$$

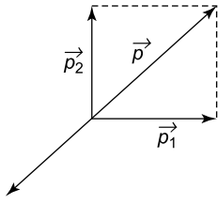


Fig. 7.40

2. Refer to Fig. 7.41. When the bob is at the horizontal position A, its height above B = length of the string = l. Therefore, potential energy at $A = mgl$. When the bob is released, it hits the wall at B and the entire potential energy is converted into kinetic energy. If v is the velocity with which the bob hits the wall, then

$$\frac{1}{2} mv^2 = mgl \text{ or } v = \sqrt{2gl} \qquad \text{(i)}$$

The speed of the bob after the first rebound is

$$v_1 = ev$$

where e is the coefficient of restitution. The speed of the bob after the second rebound will be $v_2 = ev_1 = e^2v$. Thus, the speed of the bob after n rebounds will be

$$v_n = e^n v$$

If the ball rises to a position C at a height $h = BD$ after n rebounds, then from the principle of conservation of energy, we have

$$\frac{1}{2} mv_n^2 = mgh$$

$$h = \frac{v_n^2}{2g} = \frac{\left(e^n v\right)^2}{2g} = \frac{e^{2n} v^2}{2g} \qquad (2)$$

Using Eq. (1) in (2), we get

$$h = \frac{2gle^{2n}}{2g} = le^{2n} \qquad (3)$$

For θ_n is the angle the string subtends with vertical after n rebounds, it follows from the figure that

$$h = OB - OD = l - l \cos \theta_n = l (1 - \cos \theta_n) \qquad (4)$$

From Eqs. (3) and (4), we have

$$le^{2n} = l (1 - \cos \theta_n) \text{ or } e^{2n} = 1 - \cos \theta_n)$$

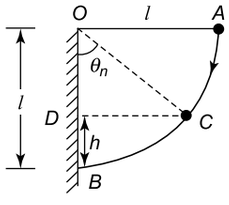


Fig. 7.41

For θ_n to be less than 60°, i.e. cos θ_n is greater than $\frac{1}{2}$, (1 – cos θ_n) must be less than $\frac{1}{2}$. Thus

$$e^{2n} < \frac{1}{2}$$

Given, $e = \frac{2}{\sqrt{5}}$. Therefore,

$$\left(\frac{2}{\sqrt{5}}\right)^{2n} < \frac{1}{2} \text{ or } \left(\frac{4}{5}\right)^{n} < \frac{1}{2}$$

or $(0.8)^n < \frac{1}{2}$ or $n \log (0.8) < \log (0.5)$

or $-(0.0969)n < -0.03010$ or $n > \frac{0.3010}{0.0969}$ or $n > 3.1$

The least integer greater than 3.1 is 4. Hence $n = 4$.

3. The x, y and z components of the initial velocity of the stone are (Fig. 7.42)

$$u_x = 4 \text{ ms}^{-1}$$
$$u_y = (6 \text{ ms}^{-1}) \sin 30° = 3 \text{ ms}^{-1}$$
$$u_z = (6 \text{ ms}^{-1}) \cos 30° = 3\sqrt{3} \text{ ms}^{-1}$$

When the stone reaches the highest point A of its parabolic path, the vertical i.e. z component of its velocity becomes zero but the x and y components remain unchanged, i.e.

$$v_x = u_x = 4 \text{ ms}^{-1}$$
$$v_y = u_y = 3 \text{ ms}^{-1}$$
$$v_z = 0$$

The speed of the stone at the highest point on its trajectory with respect to an observer on the ground is

$$v = (v_x^2 + v_y^2 + v_z^2)^{1/2}$$
$$= (4^2 + 3^2 + 0^2)^{1/2} = 5 \text{ ms}^{-1}$$

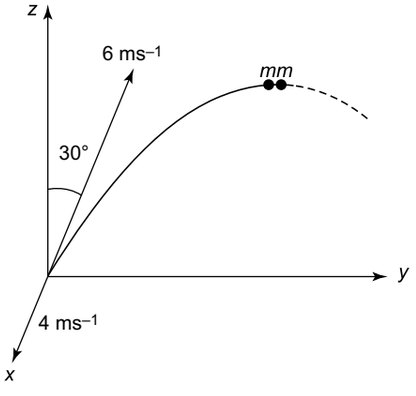


Fig. 7.42

4. Elastic collision of A with B. (Fig. 7.43)

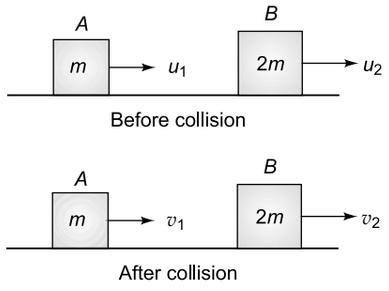


Fig. 7.43

Momentum conservation gives

$$mu_1 + 2mu_2 = mv_1 + 2mv_2$$
$$\Rightarrow 9m + 0 = mv_1 + 2mv_2$$
$$\Rightarrow \quad 9 = v_1 + 2v_2 \qquad (1)$$

Conservation of kinetic energy gives

$$\frac{1}{2} m \times (9)^2 = \frac{1}{2} mv_1^2 + \frac{1}{2}(2m)v_2^2$$
$$\Rightarrow \quad 81 = v_1^2 + 2v_2^2 \qquad (2)$$

Solving (1) and (2), we get we get $v_1 = -3 \text{ ms}^{-1}$ and $v^2 = 6 \text{ ms}^{-1}$

Inelastic collision between B and C (Fig. 7.44)

In a perfectly inelastic collision the two bodies stick together.

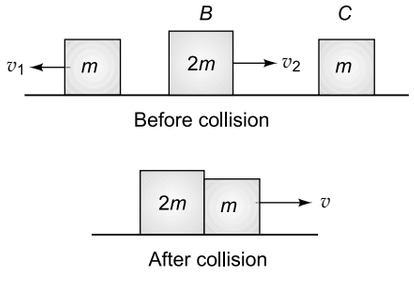


Fig. 7.44

Momentum conservation gives

$$2mv_2 + 0 = (2m + m)v$$
$$\Rightarrow \quad v = \frac{2}{3} v_2 = \frac{2}{3} \times 6 = 4 \text{ ms}^{-1}$$

8 Chapter

Rigid Body Rotation

REVIEW OF BASIC CONCEPTS

8.1 CENTRE OF MASS OF DISCRETE PARTICLES

For a system of particles, the centre of mass is defined as that point where the entire mass of the system is imagined to be concentrated, for considerations of its translational motion.

If $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, ... $\mathbf{r}_n$ are the position vectors of masses m_1, m_2, m_3, ...m_n respectively, the centre of mass of the system is

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 + ... + m_n\mathbf{r}_n}{m_1 + m_2 + m_3 + ... + m_n}$$

$$= \frac{\sum_{N=1}^{n} m_n\mathbf{r}_n}{\sum_{N=1}^{n} m_n} = \frac{\sum_{N=1}^{n} m_n\mathbf{r}_n}{M}$$

where M is the total mass of the system of particles.

$\mathbf{r}_{CM}$ is the weighted average of all the position vectors of the particles of the system, the contribution of each particle being proportional to its mass.

For a system consisting of two particles, the centre of mass is

$$\mathbf{r}_{cm} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

If the masses are equal, i.e. $m_1 = m_2$, then

$$\mathbf{r}_{CM} = \frac{1}{2}\ (\mathbf{r}_1 + \mathbf{r}_2)$$

which means that the centre of mass lies exactly in the middle of the line joining the two masses.

EXAMPLE 8.1

Three particles of masses $m_1 = m$, and $m_2 = 2m$ and $m_3 = 3m$ are placed at the corners of an equilateral triangle of side a as shown in Fig. 8.1. Locate the centre of mass of the system.

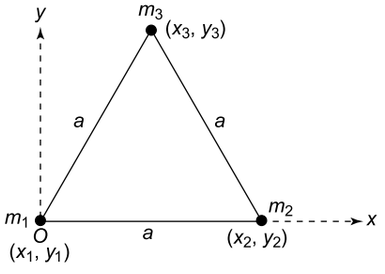


Fig. 8.1

SOLUTION

Take any one particle to be located at the origin O. The x and y coordinates of m_1, m_2, and m_3 respectively are $x_1 = 0$ and $y_1 = 0$, $x_2 = a$ and $y_2 = 0$ and $x_3 = \frac{a}{2}$ and $y_3 = \frac{\sqrt{3}a}{2}$

The x and y coordinates of the centre of mass are

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + 2m \times a + 3m \times \frac{a}{2}}{m + 2m + 3m} = \frac{7a}{12}$$

and $$y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + 2m \times 0 + 3m \times \frac{\sqrt{3}a}{2}}{m + 2m + 3m}$$

$$= \frac{\sqrt{3}a}{2}$$

EXAMPLE 8.2

Four particles of masses 1 kg, 1 kg, 2 kg, and 2 kg, are placed at the corners of a square of side 12 cm as shown in Fig. 8.2. Find the position vector of the centre of mass of the system.

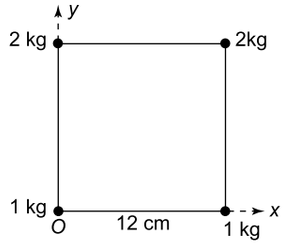


Fig. 8.2

SOLUTION

$$x_{CM} = \frac{1 \times 0 + 1 \times 12 + 2 \times 12 + 2 \times 0}{1 + 1 + 2 + 2} = 6 \text{ cm}$$

$$y_{CM} = \frac{1 \times 0 + 1 \times 0 + 2 \times 12 + 2 \times 12}{1 + 1 + 2 + 2} = 8 \text{ cm}$$

The position vector of the centre of mass is

$$\mathbf{r}_{CM} = \left(6\hat{\mathbf{i}} + 8\hat{\mathbf{j}}\right) \text{cm}$$

8.2 CENTRE OF MASS OF A BODY HAVING CONTINUOUS DISTRIBUTION OF MASS

If a body has continuous distribution of mass, the position of its centre of mass is determined by dividing the body into a very large number of extremely small elements. If dm is the mass of the element and it is at a distance x and y from the origin of chosen coordinate system, then x and y coordinates of the centre of mass are given by

$$x_{CM} = \frac{\int x\, dm}{\int dm}$$

and $$y_{CM} = \frac{\int y\, dm}{\int dm}$$

EXAMPLE 8.3

Locate the centre of mass of a uniform rod of mass M and length L.

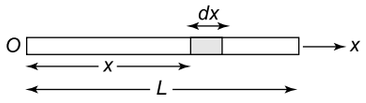


Fig. 8.3

SOLUTION

We assume that the rod lies along the x-axis with one end at origin O.

Mass per unit length of the rod = $\frac{M}{L}$

Mass of element of length dx is [Fig 8.3]

$$dm = \frac{M}{L} dx$$

The x-coordinate of the centre of mass is

$$x_{CM} = \frac{\int_0^L x\, dm}{\int_0^L dm} = \frac{1}{M} \int_0^L \frac{M}{L} x\, dx = \frac{L}{2}$$

The centre of mass is at the centre of the rod.

EXAMPLE 8.4

A non-uniform rod of length L is lying along the x-axis with one end at origin O as shown in Fig. 8.4. The linear mass density (i.e. mass per unit length) λ varies with x as $\lambda = a + bx$, where a and b are constants. Find the distance of the centre of mass from origin O.

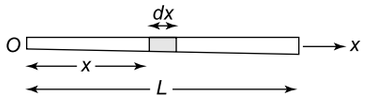


Fig. 8.4

SOLUTION

Mass of element is $dm = \lambda\, dx = (a + bx)\, dx$

$$x_{CM} = \frac{\int_0^L x\, dm}{\int_0^L dm} \quad \text{(i)}$$

$$\int_0^L x\,dm = \int_0^L (a+bx)\,x\,dx$$

$$= a\left|\frac{x^2}{2}\right|_0^L + b\left|\frac{x^3}{3}\right|_0^L = \frac{aL^2}{2} + \frac{bL^3}{3} \qquad \text{(ii)}$$

$$\Rightarrow \int_0^L x\,dm = \frac{L^2}{6}(3a+2bL)$$

And $$\int_0^L dm = \int_0^L (a+bx)\,dx = aL + \frac{bL^2}{2} = \frac{L}{2}(2a+bL) \qquad \text{(iii)}$$

Using (ii) and (iii) in (i), we get

$$x_{CM} = \frac{L(3a+2bL)}{3(2a+bL)}$$

EXAMPLE 8.5

Locate the centre of mass of a uniform semicircular ring of radius R and linear mass density λ.

SOLUTION

Let us take the centre of the ring at origin O. consider a small element of arc length dl of the ring. Let θ be the angle which the radius vector of the element makes with the x-axis as shown in Fig. 8.5.

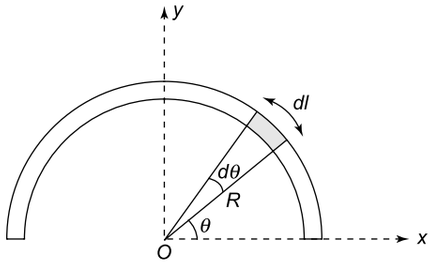


Fig. 8.5

Let $d\theta$ be the angle subtended by the element at the centre. Then $dl = R\,d\theta$. Mass of the element is

$$dm = \lambda dl = \lambda R d\theta$$

The x and y components of radius vector R are $x = R\cos\theta$ and $y = R\sin\theta$. Then

$$x_{CM} = \frac{\int x\,dm}{\int dm} = \frac{\lambda R^2 \int_0^{\pi} (\cos\theta)\,d\theta}{\lambda R \int_0^{\pi} d\theta} = \frac{\lambda R^2}{\lambda R\pi} |\sin\theta|_0^{\pi}$$

$$= \frac{R}{\pi}(\sin\pi - \sin 0) = 0$$

$$y_{CM} = \frac{\int y dm}{\int dm} = \frac{\lambda R^2 \int_0^{\pi} (\sin\theta)\,d\theta}{\lambda R \int_0^{\pi} d\theta} = \frac{\lambda R^2}{\lambda R\pi} |-\cos\theta|_0^{\pi}$$

$$= -\frac{R}{\pi}(\cos\pi - \cos 0) = \frac{2R}{\pi}$$

Thus, the centre of mass is at a distance of $\frac{2R}{\pi}$ from origin O on the y-axis.

8.3 FINDING CENTRE OF MASS OF A SYSTEM WHEN A PART OF ITS MASS IS REMOVED

Consider of system of mass M. If a mass m is removed, the remaining mass = $M - m$ which may written as $M + (-m)$. Then the x and y coordinates of the centre of mass of the remaining portion are given by

$$x_{CM} = \frac{Mx - mx'}{M - m}$$

and $$y_{CM} = \frac{My - my'}{M - m}$$

where x and y are the coordinates of the centre of mass of the complete part and x' and y' are the coordinates of the centre of mass of the removed part.

EXAMPLE 8.6

Four particles, each of mass 1 kg, are placed at the corners of a square of side 12 cm. If mass m_3 is removed, find the shift in the centre of mass of the system (see Fig. 8.6).

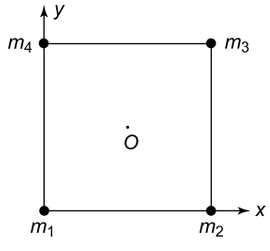


Fig. 8.6

SOLUTION

As shown in Example 2, the x and y coordinates of the original system are $x_{CM} = 6$ cm and $y_{CM} = 6$ cm

If mass m_3 is removed, the x and y coordinates of the centre of mass of the remaining system are

$$x'_{CM} = \frac{1\times 0 + 1\times 12 + 1\times 0}{1+1+1} = 4 \text{ cm}$$

$$y'_{CM} = \frac{1\times 0+1\times 0+1\times 12}{1+1+1} = 4 \text{ cm}$$

Thus $\mathbf{r} = 6\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

and $\mathbf{r}' = 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$\therefore$ shift $\Delta\mathbf{r} = \mathbf{r} - \mathbf{r}' = \left(6\hat{\mathbf{i}} + 6\hat{\mathbf{j}}\right) - \left(4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right)$

$= 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

EXAMPLE 8.7

From a uniform thin disc of radius R and mass M, a circular portion of radius $r = R/2$ is removed as shown in Fig. 8.7. Find the centre of mass of the remaining part of the disc.

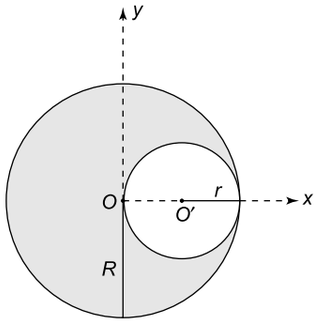


Fig. 8.7

SOLUTION

Let the centre of the disc be at origin O. The centre of mass of the complete disc will be at O (by symmetry) and its x and y coordinates are (0, 0). Mass per unit area of the disc $= \frac{M}{\pi R^2}$. Therefore, mass of removed portion is

$$m = \frac{M}{\pi R^2} \times \pi r^2.$$

$$= \frac{M}{\pi R^2} \times \pi \left(\frac{R}{2}\right)^2 = \frac{M}{4}.$$

The x and y coordinates of the centre of mass of the removed portion are

$$x' = r = \frac{R}{2} \text{ and } y' = 0$$

The x and y coordinates of the remaining portion (shown shaded) are

$$x_{CM} = \frac{Mx - mx'}{M - m}$$

$$= \frac{M\times 0 - \frac{M}{4}\times\frac{R}{2}}{M - \frac{M}{4}} = -\frac{R}{6}$$

$$y_{CM} = \frac{My - my'}{M - m} = \frac{M\times 0 - \frac{M}{4}\times 0}{M - \frac{M}{4}} = 0$$

The negative sign of x_{CM} indicates that the centre of mass of the remaining portion is located at a distance $R/6$ towards the left of the center O of the complete disc.

Alternative Method

It is clear that centre of mass of the remaining portion of the disc will shift to the left of O. Let G be the centre of mass of the remaining portion of the disc (Fig. 8.8). Let $OG = x$. Equating the moments of Mg and mg about G, we have

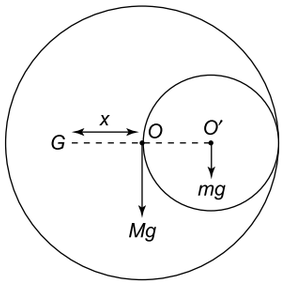


Fig. 8.8

$$Mg \times OG = mg \times O'G$$

$$\Rightarrow \quad M \times x = \frac{M}{4} \times \left(\frac{R}{2} + x\right)$$

$$\Rightarrow \quad x = \frac{R}{6}$$

Thus the centre of mass shifts by $R/6$ to the left of O.

EXAMPLE 8.8

From a thin uniform square lamina of side a a square of side $a/2$ is removed from its corner as shown in Fig. 8.9. Find the centre of mass of the remaining portion (shown shaded) of the lamina.

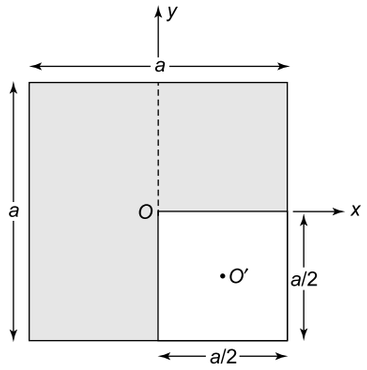


Fig. 8.9

SOLUTION

Let M be the mass of the complete lamina. Mass of the removed portion is $m = \frac{M}{4}$. The centre of mass of the complete lamina is taken to be at origin O ($x = 0, y = 0$). The coordinates of the centre of mass O' of the removed portion are $x' = \frac{a}{4}$ and $y' = -\frac{a}{4}$.

The x and y coordinates of the centre of mass of the remaining portion are

$$x_{\text{CM}} = \frac{Mx - mx'}{M - m}$$

$$= \frac{M \times 0 - \frac{M}{4} \times -\frac{a}{4}}{M - \frac{M}{4}} = -\frac{a}{12}$$

and

$$y_{\text{CM}} = \frac{My - my'}{M - m}$$

$$= \frac{M \times 0 - \frac{M}{4} \times -\frac{a}{4}}{M - \frac{M}{4}} = +\frac{a}{12}$$

8.4 VELOCITY AND ACCELERATION OF CENTRE OF MASS OF A SYSTEM OF PARTICLES

If $\mathbf{r}_1$, $\mathbf{r}_2$, ... $\mathbf{r}_n$ are the position vectors of masses m_1, m_2, ... m_n, respectively, th\e position vector of the centre of mass of the system of particles is given by

$$\mathbf{r}_{\text{CM}} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \cdots + m_n\mathbf{r}_n}{m_1 + m_2 + \cdots + m_n}$$

The velocity of the centre of mass is given by

$$\mathbf{v}_{\text{CM}} = \frac{d\mathbf{r}_{CM}}{dt}$$

$$= \frac{m_1\frac{d\mathbf{r}_1}{dt} + m_2\frac{d\mathbf{r}_2}{dt} + \ldots + m_n\frac{d\mathbf{r}_n}{dt}}{m_1 + m_2 + \ldots + m_n}$$

$$\Rightarrow \quad \mathbf{v}_{\text{CM}} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \ldots + m_n\mathbf{v}_n}{m_1 + m_2 + \ldots + m_n}$$

The acceleration of the centre of mass is given by

$$a_{\text{CM}} = \frac{d\mathbf{v}_{\text{CM}}}{dt}$$

$$= \frac{m_1\frac{d\mathbf{v}_1}{dt} + m_2\frac{d\mathbf{v}_2}{dt} + \cdots + m_n\frac{d\mathbf{v}_n}{dt}}{m_1 + m_2 + \cdots + m_n}$$

$$\Rightarrow \quad \boldsymbol{a}_{\text{CM}} = \frac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + \cdots + m_n\mathbf{a}_n}{m_1 + m_2 + \cdots + m_n}$$

or

$$\mathbf{a}_{\text{CM}} = \frac{\mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum \mathbf{F}_{\text{ext}}}{M}$$

If $\sum \mathbf{F}_{\text{ext}} = 0$, then $\mathbf{a}_{\text{CM}} = 0$, i.e $\text{v}_{\text{CM}} = \text{constant}$. Hence if *no net external force acts on a system, its centre of mass will remain at rest or will move with a constant velocity*.

EXAMPLE 8.9

A boy of mass m = 50 kg stands at the end A of a flat plank AB of wood of mass M = 100 kg and length l = 10 m floating in the still water in a lake. The end B of the plank is at a distance of 30 m from the shore of the lake as shown in Fig. 8.10. The boy walks a distance of 6 m on the plank towards the shore. How far is the boy from the shore now? Neglect viscosity of water.

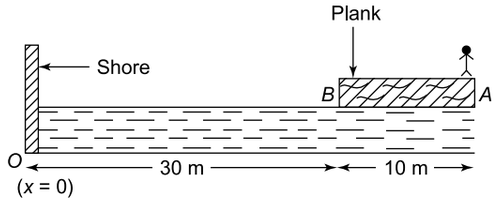


Fig. 8.10

SOLUTION

Initially the centre of mass of the system (plank + boy) is at rest. To walk, the boy exerts a force in the backward direction. The plank in turns exerts a reaction force on the boy in the forward direction.

These forces are internal to the system. Since no external force acts, the centre of mass of the system remains at rest even when the boy walks on the plank.

Let the shore be at the origin O ($x = 0$). Initially let x be the distance of the centre of mass of the plank from O. Then the distance of the centre of mass of the system (plank + boy) from O will be

$$x_{CM} = \frac{M \times x + m \times (30+10)}{M+m}$$

$$= \frac{100x + 50 \times 40}{100+50} = \frac{100x + 2000}{150}$$

Since the boy moves towards the shore and the centre of mass of the system has to remain at rest, the plank will move away from the shore. If x' is the distance moved by the plank, the distance of the centre of mass from the shore when the boy walks 6 m on the plank is given by

$$x'_{CM} = \frac{100(x+x') + 50(40-6+x')}{100+50}$$

$$= \frac{100x + 150x' + 1700}{150}$$

Since $x_{CM} = x'_{CM}$,

$$\frac{100x+2000}{150} = \frac{100x+150x'+1700}{150}$$

$$\Rightarrow \quad x' = \frac{300}{150} = 2 \text{ m}$$

$\therefore$ Distance of the boy from the shore = 40 – 6 + 2 = 36 m.

EXAMPLE 8.10

A car of mass 1000 kg is moving with a velocity of 10 ms^{-1} towards another car of mass 1500 kg moving with a velocity of 15 ms^{-1} in the same direction. Find the velocity of the centre of mass of the two cars.

SOLUTION

$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{1000 \times 10 + 1500 \times 15}{1000+1500} = 13 \text{ ms}^{-1}$$

8.5 MOMENTUM CONSERVATION AND CENTRE OF MASS MOTION

We have seen that

$$\sum \mathbf{F}_{ext} = M\mathbf{a}_{CM} = M\frac{d\mathbf{v}_{CM}}{dt} = \frac{d}{dt}(M\mathbf{v}_{CM})$$

$$\Rightarrow \quad \sum \mathbf{F}_{ext} = \frac{d\mathbf{P}}{dt}$$

where $\mathbf{P} = M\mathbf{v}_{CM}$ is the *total linear momentum of the system of particles which is equal to the product of the total mass of the system and the velocity of the centre of mass.*

If $\sum \mathbf{F}_{ext} = 0$, $\frac{d\mathbf{P}}{dt} = 0 \quad \Rightarrow \quad \mathbf{P}$ = constant.

Thus, if no net external force acts on a system, the total linear momentum of the system remains constant; the total linear momentum being the vector sum of linear momentum of individual particles, i.e.

$$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2 + \cdots + \mathbf{P}_n$$

EXAMPLE 8.11

A boy of mass $m = 40$ kg is standing on a stationary long plank of mass $M = 260$ kg floating on still water in a lake. He starts running with a velocity $v = 6$ ms^{-1} relative to the plank. Find the velocity of the boy relative to a stationary observer on the bank of the lake.

SOLUTION

Since the system (boy + plank) is initially at rest, its momentum is zero. Since no external force acts on the system, the momentum of the system will remain zero. Let us assume that the boy runs in the positive x-direction.

Velocity of boy relative to the plank is

$$\mathbf{v}_{bp} = v\hat{\mathbf{i}}$$

Now $\mathbf{v}_{bp} = \mathbf{v}_b - \mathbf{v}_p$

where $\mathbf{v}_b$ = velocity of boy relative to ground

and $\mathbf{v}_p$ = velocity of plank relative to ground

Hence $\mathbf{v}_b = \mathbf{v}_{bp} + \mathbf{v}_p$

$$= v\hat{\mathbf{i}} + v_p\hat{\mathbf{i}}$$

$$= (v + v_p)\hat{\mathbf{i}}$$

Total momentum of plank + boy = 0

$$\Rightarrow \quad M\mathbf{v}_p + m\mathbf{v}_b = 0$$

$$\Rightarrow \quad Mv_p\hat{\mathbf{i}} + mv_b\hat{\mathbf{i}} = 0$$

$$\Rightarrow \quad Mv_p\hat{\mathbf{i}} + m(v + v_p)\hat{\mathbf{i}} = 0$$

which gives $v_p = -\frac{mv}{M+m}$

$\therefore$ Velocity of boy is

$$\mathbf{v}_b = \left(v - \frac{mv}{M+m}\right)\hat{\mathbf{i}} = \frac{Mv}{M+m}\hat{\mathbf{i}}$$

$$= \left(\frac{260 \times 6}{260 + 40}\right)\hat{\mathbf{i}}$$

$$= 5.2\,\hat{\mathbf{i}}\ \text{ms}^{-1}$$

Hence the velocity of boy relative to a stationary observer is 5.2 ms^{-1} in the direction along which the boy is running.

8.6 TORQUE

If a force **F** acts on a particle *P* whose position vector with respect to the origin of an inertial reference frame is **r**, the torque $\boldsymbol{\tau}$ acting on the particle with respect to the origin is defined as [Fig. 8.11]

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

In terms of magnitudes,

$$\tau = r\,F\,\sin\,\theta = F\,(r\,\sin\,\theta) = Fr_\perp$$

where θ is the angle between vectors **r** and **F**.

Torque $\boldsymbol{\tau}$ is a vector quantity. Its magnitude is given by $\tau = rF \sin\theta$; its direction is normal to the plane containing vectors **r** and **F** and can be determined by the *right–hand screw rule.*

Unit of Torque Torque has the same dimensions as those of work (both being force times distance) viz. ML^2T^{-2}. The two are, however, very different quantities. Work is a scalar, torque is a vector. To distinguish between the two we express work in joule and torque in newton–metre (N m).

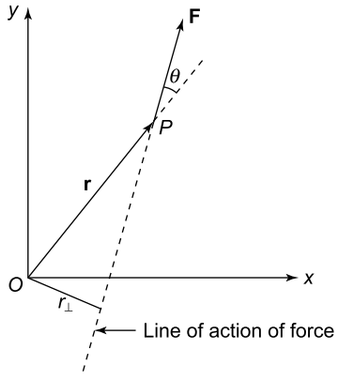


Fig. 8.11

Couple

Two equal antiparallel forces having different lines of action constitute a couple.

The moment of couple or torque = $Fr_\perp$ (Fig. 8.12)

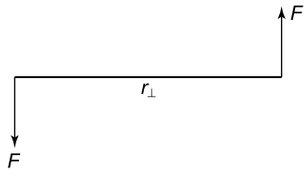


Fig. 8.12

= magnitude of either force × perpendicular distance between the two antiparallel forces

Work done by torque

If a force *F* acts on a rigid body at perpendicular $r_\perp$ from the axis of rotation, the work done by the force in rotating the body through an angle $\Delta\theta$ is given by

$$\Delta W = Fr_\perp\ \Delta\theta = \tau\,\Delta\theta$$

= magnitude of torque × angular displacement

$$\text{Power} = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$$

where ω is the angular velocity.

EXAMPLE 8.12

A force $\mathbf{F} = \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)$ newton acts on a particle whose position vector with respect to origin *O* is $\mathbf{r} = \left(4\hat{\mathbf{i}} - 5\hat{\mathbf{j}}\right)$ metre. Find the magnitude and direction of the torque.

SOLUTION

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$= \left(4\hat{\mathbf{i}} - 5\hat{\mathbf{j}}\right) \times \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)$$

$$= 4\hat{\mathbf{i}} \times 2\hat{\mathbf{i}} + 12\hat{\mathbf{i}} \times \hat{\mathbf{j}} - 10\hat{\mathbf{j}} \times \hat{\mathbf{i}} - 15\hat{\mathbf{j}} \times \hat{\mathbf{j}}$$

$$= 0 + 12\hat{\mathbf{k}} + 10\hat{\mathbf{k}} - 0$$

$$= 22\ \hat{\mathbf{k}}\ \text{newton metre}$$

The magnitude of torque is 22 Nm and its direction is along the positive *z*-axis.

EXAMPLE 8.13

A rectangular plate *OPQR* of dimensions 2 m × 3 m lies in the *x-y* plane as shown in Fig. 8.13. A force $\mathbf{F} = \left(3\hat{\mathbf{i}} + 5\hat{\mathbf{j}}\right)$ newton is applied at point *Q*. Find the

torque of **F** (a) about origin *O*, (b) about point *P* and (c) about *x*-axis, *y*-axis and *z*-axis.

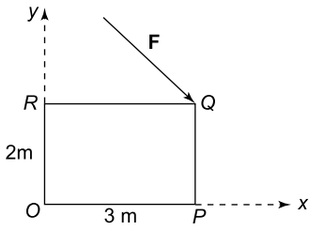


Fig. 8.13

SOLUTION

(a) $\mathbf{r} = \overrightarrow{OQ} = \left(3\hat{\mathbf{i}} + 2\mathbf{j}\right)$ metre

Torque about *O* is

$$\tau_O = \mathbf{r} \times \mathbf{F}$$

$$= \left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\right) \times \left(3\hat{\mathbf{i}} + 5\hat{\mathbf{j}}\right)$$

$$= 9\hat{\mathbf{i}} \times \hat{\mathbf{i}} + 15\hat{\mathbf{i}} \times \hat{\mathbf{j}} + 6\hat{\mathbf{j}} \times \hat{\mathbf{i}} + 10\hat{\mathbf{j}} \times \hat{\mathbf{j}}$$

$$= 0 + 15\hat{\mathbf{k}} - 6\hat{\mathbf{k}} + 0 = 9\hat{\mathbf{k}} \text{ Nm}$$

(b)Torque about *P* is

$$\boldsymbol{\tau}_P = \overrightarrow{PQ} \times \mathbf{F}$$

$$= 2\hat{\mathbf{j}} \times \mathbf{F}$$

$$= 2\hat{\mathbf{j}} \times \left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\right) = -6\hat{\mathbf{k}} \text{ Nm}$$

(c)Torque about *x*-axis is

$$\boldsymbol{\tau}_x = \boldsymbol{\tau}_o \cdot \hat{\mathbf{i}} = 9\hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$$

Torque about *y*-axis is

$$\boldsymbol{\tau}_y = \boldsymbol{\tau}_o \cdot \hat{\mathbf{j}} = 9\hat{\mathbf{k}} \cdot \hat{\mathbf{j}} = 0$$

Torque about *z*-axis is

$$\boldsymbol{\tau}_z = \mathbf{z}_O \cdot \hat{\mathbf{k}} = 9\hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 9 \text{ Nm}$$

8.7 ANGULAR MOMENTUM

The angular momentum **L** of a particle *P* with respect to the origin of an inertial reference frame is defined as [Fig. 8.14]

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where **r** is the position vector of the particle and **p** its linear momentum. In terms of magnitudes,

$$L = rp \sin \theta = r_\perp \times p$$

where θ is the angle between vectors **r** and **p**. The dimensions of angular momentum are ($ML^2 T^{-1}$) and its SI unit is kg m^2s^{-1}.

The direction of **L** is perpendicular to the plane containing the vectors **r** and **p** and its sense is given by the *right–hand rule*.

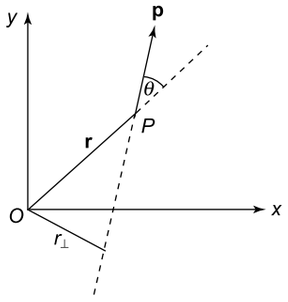


Fig. 8.14

EXAMPLE 8.14

A body of mass $m = 200$ g is moving parallel to the *x*-axis with a velocity $\mathbf{v} = 30$ cms^{-1} in the *x*-*y* plane as shown in Fig. 8.15. Calculate the magnitude of its angular momentum about origin *O* at any time *t*.

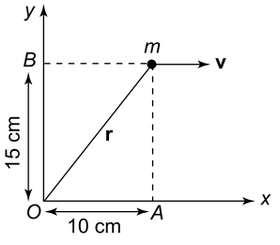


Fig. 8.15

SOLUTION

Magnitude of angular momentum = $r_\perp p$

$= r_\perp mv$

$= OB \times m \times v$

$= 0.15 \times 0.2 \times 0.3$

$= 9 \times 10^{-3}$ kg m^2s^{-1}

EXAMPLE 8.15

A body of mass $m = 200$ g is projected with a velocity $u = 5$ ms^{-1}at an angle $\theta = 30°$ with the horizontal. Calculate the magnitude of the angular momentum of the body about the point of projection when it is at the highest point of its trajectory. Take $g = 10$ ms^{-2}.

SOLUTION

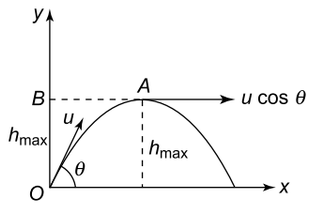


Fig. 8.16

Refer to Fig. 8.16. At the highest point A, the body has only horizontal velocity

$$v = u \cos \theta$$

$$h_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

Magnitude of angular momentum of the body about O when it is at point A is

$$L = mv \times OB$$
$$= mv \times h_{max}$$
$$= m u \cos\theta \times \frac{u^2 \sin^2 \theta}{2g}$$
$$= \frac{mu^3 \sin^2 \theta \cos\theta}{2g}$$
$$= \frac{0.2 \times (5)^3 \times \sin^2 (30°) \cos(30°)}{2 \times 10}$$
$$= \frac{0.2 \times 125 \times \frac{1}{4} \times \frac{\sqrt{3}}{2}}{20}$$
$$= 0.27 \text{ kg m}^2\text{s}^{-1}$$

8.8 RELATION BETWEEN TORQUE AND ANGULAR MOMENTUM

In linear motion, the relation between force **F** and linear momentum **p** is

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

In rotational motion, the relation between torque $\boldsymbol{\tau}$ and angular momentum **L** is

$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$$

which states that *the torque acting on a particle is equal to the rate of change of angular momentum.*

8.9 ANGULAR IMPULSE

In linear motion, impulse **I** is defined as

$$\mathbf{I} = \int \mathbf{F}\, dt$$
$$= \int \frac{d\mathbf{p}}{dt}\, dt$$
$$= d\mathbf{p} = \text{change in linear momentum}$$

In rotational motion, angular impulse **J** is defined as

$$\mathbf{J} = \int \boldsymbol{\tau}\, dt$$
$$= \int \frac{d\mathbf{L}}{dt}\, dt$$
$$= d\mathbf{L} = \text{change in angular momentum}$$

8.10 LAW OF CONSERVATION OF ANGULAR MOMENTUM

If no external torque acts, the total angular momentum of a body or a system of particles is conserved.

We have seen that the rate of change of angular momentum of a particle is equal to the torque produced by the total force. If, in a certain situation, the torque itself vanishes, then it follows that the angular momentum of the particle will remain constant. This is the law of conservation of the angular momentum of a particle. One trivial situation is when the force vanishes. Then the torque vanishes too. The particle then moves freely in a straight line in accordance with Newton's first law in which case both linear and angular momenta are conserved.

A general situation is when the torque vanishes without the force itself vanishing. The torque $\boldsymbol{\tau}$ will vanish if the component $\mathbf{F}_\perp$ (the angular component) of **F** vanishes but the radial component $\mathbf{F}_{II}$ does not. The radial component $\mathbf{F}_{II}$ is the component of **F** along the radius (or position) vector **r**. Hence, *if the force acting on the particle is purely radial (i.e. if it is directed along or against its position vector) then the torque acting on the particle vanishes and its angular momentum is conserved and so is its areal velocity.*

8.11 MOMENT OF INERTIA

The moment of inertia of a rigid body about a particular axis may be defined as the sum of the products of the masses of all the particles constituting the body and the squares of their respective distances from the axis of rotation, i.e.

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2$$
$$= \sum_{N=1}^{n} m_n r_n^2$$

Its value depends upon the particular axis about which the body rotates and the way the mass is distributed in the body with respect to the axis of rotation.

In the case of a body which does not consist of separate, discrete particles but has a continuous and homogeneous distribution of matter in it, the summation is replaced by integration, so that

$$I = \int r^2 dm$$

where dm is the mass of an infinitesimally small element of the body at a distance r from the axis of rotation.

Moment of inertia is a scalar quantity. Its SI unit is kg m^2 and its dimensions are (ML^2).

8.12 RADIUS OF GYRATION

The radius of gyration of a body about its axis of rotation may be defined as the distance from the axis of rotation at which, if the entire mass of the body were concentrated, its moment of inertia about the given axis would be the same as with its actual distribution of mass. It is usually denoted by the letter K. If M is the mass of the body, its moment of inertia I in terms of its radius of gyration K can be written $I = MK^2$

8.13 MOMENT OF INERTIA AND ROTATIONAL KINETIC ENERGY

Kinetic energy of a rotating body is related to moment of inertia as

$$KE = \frac{1}{2} I\omega^2$$

where ω is the angular velocity (or frequency) of the body.

8.14 MOMENT OF INERTIA AND TORQUE

The magnitude of torque is given by $\tau = I\alpha$
where α is the angular acceleration of the body.

8.15 MOMENT OF INERTIA AND ANGULAR MOMENTUM

The magnitude of angular momentum of a rotating body is given by $L = I\omega$

8.16 PARALLEL AXES THEOREM

If M is the total mass of a body and h the distance between two parallel axes, then according to parallel axes theorem $I = I_{CM} + Mh^2$

8.17 PERPENDICULAR AXES THEOREM

The theorem of perpendicular axes for a body of plane lamina states that the moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its own plane and intersecting each other at the point where the perpendicular axis passes through it.

If I_x and I_y are the moments of inertia of a plane lamina about the perpendicular axes x and y respectively which lie in the plane of the lamina and intersect each other at O, then the moment of inertia I of the lamina about an axis passing through O and perpendicular to its plane is given by

$$I = I_x + I_y$$

8.18 EXPRESSIONS FOR MOMENT OF INERTIA OF BODIES OF REGULAR SHAPES ABOUT PARTICULAR AXES OF ROTATION

	Shape of body		*Axis of Rotation*	*Expression for Moment of Inertia*
1.	Circular ring of mass M and radius R	(i)	through centre, perpendicular to plane of ring	MR^2
		(ii)	any diameter	$(1/2)\,MR^2$
		(iii)	any tangent in the plane of ring	$(3/2)\,MR^2$
		(iv)	any tangent perpendicular to plane of ring	$2\,MR^2$
2.	Circular disc of mass M and radius R	(i)	through centre, perpendicular to plane of disc	$(1/2)\,MR^2$
		(ii)	any diameter	$(1/4)\,MR^2$
		(iii)	tangent in the plane of the disc	$(5/4)\,MR^2$
		(iv)	tangent perpendicular to plane of disc	$(3/2)\,MR^2$
3.	Sphere of mass M and radius R	(i)	any diameter	$(2/5)\,MR^2$
		(ii)	any tangent plane	$(7/5)\,MR^2$
4.	Cylinder of mass M, radius R and length L	(i)	own axis	$(1/2)\,MR^2$
		(ii)	through centre perpendicular to length	$M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$
		(iii)	through end faces and $\perp$ to length	$M\left(\frac{R^2}{4}+\frac{L^2}{3}\right)$

Contd.

	Shape of body		*Axis of Rotation*	*Expression for Moment of Inertia*
5.	One dimensional rod of mass M and length L	(i)	centre of rod and $\perp$ to length	$ML^2/12$
		(ii)	one end and $\perp$ to length	$ML^2/3$
6.	Rectangular lamina of mass M, length L and breadth B	(i)	length of lamina and in its plane	$MB^2/3$
		(ii)	breadth of lamina and in its plane	$ML^2/3$
		(iii)	centre of lamina and parallel to length or breadth in its plane	$\frac{MB^2}{12}$ or $\frac{ML^2}{12}$
		(iv)	centre of lamina and $\perp$ to its plane	$M\left(\frac{L^2+B^2}{12}\right)$
		(v)	centre of length and $\perp$ to its plane	$M\left(\frac{L^2}{12}+\frac{B^2}{3}\right)$
		(vi)	centre of breadth and $\perp$ to its plane	$M\left(\frac{L^2}{3}+\frac{B^2}{12}\right)$
7.	Rectangular block of mass M, length L, breadth B and height H	(i)	through centre or block and parallel to length or breadth or height of the block	$M\left(\frac{B^2+H^2}{12}\right)$ or $M\left(\frac{H^2+L^2}{12}\right)$ or $M\left(\frac{L^2+B^2}{12}\right)$
		(ii)	through end face and parallel to length or breadth or height of the block	$M\left(\frac{H^2}{3}+\frac{B^2}{12}\right)$ or $M\left(\frac{L^2}{3}+\frac{H^2}{12}\right)$ or $M\left(\frac{B^2}{3}+\frac{L^2}{12}\right)$

8.19 KINEMATICS OF ROTATIONAL MOTION WITH CONSTANT ANGULAR ACCELERATION

Consider a body rotating with an initial angular velocity ω_0. It is given a constant angular acceleration α (by applying a constant torque) for a time t. As a result it acquires a final angular velocity ω and suffers an angular displacement θ in time t. The equations of rotational motion are

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

and $$2\alpha\theta = \omega^2 - \omega_0^2$$

EXAMPLE 8.16

The moment of inertia of a uniform circular disc of mass M and radius R about an axis passing through its centre and perpendicular to its plane is $\frac{1}{2}MR^2$. Find the moment of inertia of the disc

(a) about any diameter
(b) about an axis passing through a point on the edge of the disc and perpendicular to the disc
(c) about a tangent in the plane of the disc.

SOLUTION

The plane of the disc is the x-y plane.

(a) Using perpendicular axes theorem [Fig. 8.17(a)]

$$I_x + I_y = I_z$$

Now $I_z = I_C = \frac{1}{2}MR^2$ (given). From symmetry $I_y = I_x$.

$$\therefore \quad 2I_x = \frac{1}{2}MR^2 \Rightarrow I_x = I_y = \frac{1}{4}MR^2$$

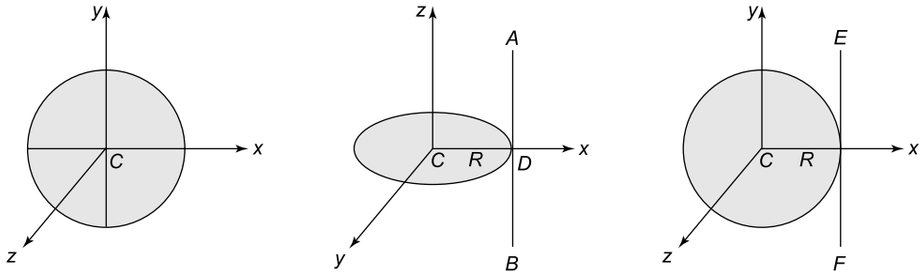
(b) using parallel axes theorem [Fig. 8.17(b)]

$$I_{AB} = I_z + M(CD)^2$$
$$= I_C + MR^2$$
$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

(c) using parallel axes theorem [Fig. 8.17(c)]

$$I_{EF} = I_y + MR2$$

$$= \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$



(a) (b) (c)

Fig. 8.17

EXAMPLE 8.17

A thin uniform disc of mass M and radius R has concentric hole of radius r. Find the moment of inertia of the disc about an axis passing through its centre and perpendicular to its plane.

SOLUTION

Mass per unit area of the disc is [Fig. 8.18]

$$m = \frac{M}{\pi\left(R^2 - r^2\right)}$$

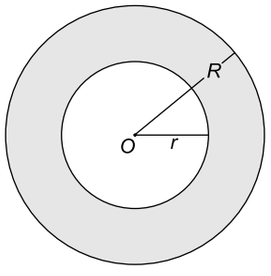


Fig. 8.18

Mass of the disc if it was complete (i.e. without hole) is

$$M_1 = m \times \pi R^2$$

$$= \frac{M}{\pi\left(R^2 - r^2\right)} \times \pi R^2 = \frac{M\,R^2}{R^2 - r^2}$$

Mass of the removed portion is

$$M_2 = m \times \pi r^2 = \frac{Mr^2}{R^2 - r^2}$$

Since the two portions are concentric, the moment of inertia of the given disc about the given axis is

$$I = \frac{1}{2}M_1R^2 - \frac{1}{2}M_2\,r^2$$

$$= \frac{1}{2}\left[\frac{MR^4}{R^2 - r^2} - \frac{Mr^4}{R^2 - r^2}\right]$$

$$= \frac{M}{2}\left[\frac{R^4 - r^4}{R^2 - r^2}\right] = \frac{1}{2}M\left(R^2 + r^2\right)$$

EXAMPLE 8.18

Find the moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its centre and making an angle θ with the rod.

SOLUTION

Divide the rod into a very large number of extremely small elements each of length dx. Consider one such element at a distance x from the centre O of the rod (Fig. 8.19).

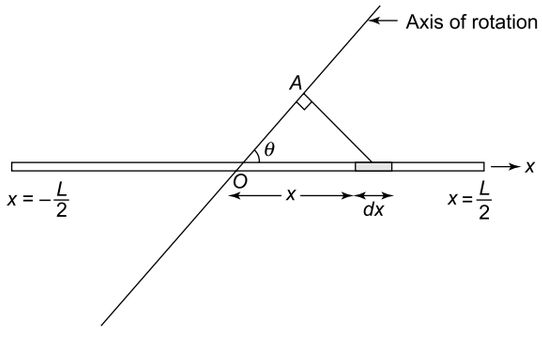


Fig. 8.19

Mass of element is $dm = \frac{M}{L}dx$

Perpendicular distance of the element from the axis of rotation is

$$r = OA = x\sin\theta$$

Moment of inertia of the rod about the given axis is

$$I = \int dm\,r^2 = \int \frac{M}{L}dx \times (x\sin\theta)^2$$

$$= \frac{M \sin^2\theta}{L} \int_{x=-\frac{L}{2}}^{x=+\frac{L}{2}} x^2 dx$$

$$= \frac{M \sin^2\theta}{L} \left| \frac{x^3}{3} \right|_{-L/2}^{+L/2}$$

$$= \frac{ML^2}{12} \sin^2\theta$$

If $\theta = 90°$, $I = \frac{ML^2}{12}$

EXAMPLE 8.19

The radius of gyration K of a hollow sphere of mass M and radius R about a certain axis is equal to R. Find the distance of that axis from the centre of the sphere.

SOLUTION

Let x be the distance of the axis from the centre of the sphere [Fig. 8.20]. From parallel axes theorem

$$I_{XY} = I_{AB} + Mx^2$$

$$\Rightarrow \quad MK^2 = \frac{2}{3} MR^2 + Mx^2$$

Given $K = R$. Hence

$$R^2 = \frac{2}{3} R^2 + x^2$$

$$\Rightarrow \qquad x = \frac{R}{\sqrt{3}}$$

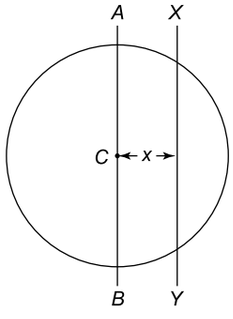


Fig. 8.20

EXAMPLE 8.20

Figure 8.21 shows a section (a part) of circular disc of radius R. The mass of the section is M. Find the moment of inertia of the section of the disc about an axis passing through its centre O and perpendicular to its plane.

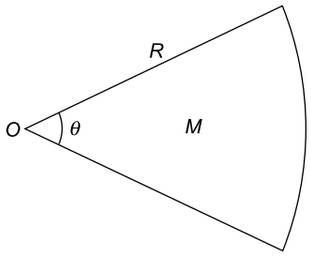


Fig. 8.21

SOLUTION

Area of the section $A = \pi R^2 \times \frac{\theta}{2\pi} = \frac{R^2\theta}{2}$

where θ is in radian [Fig 8.22].

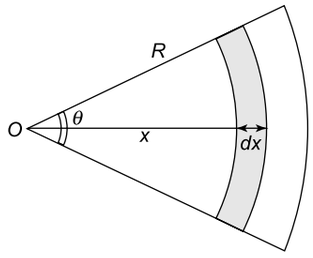


Fig. 8.22

$$\text{Mass per unit area} = \frac{M}{A}$$

$$= \frac{2M}{R^2\theta}$$

$$\text{Area of strip is } dA = \left(\pi(x+dx)^2 - \pi x^2\right) \times \frac{\theta}{2\pi}$$

$$\simeq 2\pi x dx \times \frac{\theta}{2\pi} = (xdx)\theta$$

$$\text{Mass of strip is } dm = \frac{2M}{R^2\theta} \times dA = \frac{2M}{R^2\theta} \times (xdx)\theta$$

$$= \frac{2M}{R^2} xdx$$

$\therefore$ Moment of inertia of the section about the given axis is

$$I = \int dm x^2 = \frac{2M}{R^2} \int_0^R x^3 dx$$

$$= \frac{1}{2} MR^2$$

EXAMPLE 8.21

Two particles of masses m_1= 1 kg and m_2 = 2 kg are connected by a rigid bar of length L = 1.2 m of negligible mass. The system rotates about an axis perpendicular to the rod and at a distance x from mass m_1. Find the value of x for which the moment of inertia about the given axis is minimum. what is the minimum moment of inertia?

SOLUTION

From Fig 8.23, it follows that

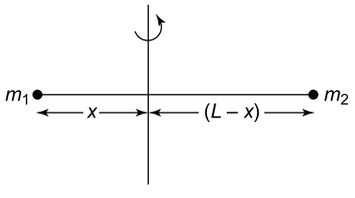


Fig. 8.23

$$I = m_1x^2 + m_2\,(L - x)^2 \qquad \text{(i)}$$

I will be minimum if $\frac{dI}{dx} = 0$ and $\frac{d^2I}{dx^2} > 0$

Differentiating (i)

$$\frac{dI}{dx} = 2m_1x + 2m_2\,(L - x)\,(-1) = 0 \qquad \text{(ii)}$$

$$\Rightarrow \quad x = \frac{m_2L}{m_1 + m_2} = \frac{2 \times 1.2}{(1+2)} = 0.8 \text{ m}$$

Differentiating (ii)

$\frac{d^2I}{dx^2} = 2m_1 + 2m_2$, which is positive for any value of x.

Substituting x in (i), we have

$$I_{\min} = m_1\left(\frac{m_2L}{m_1 + m_2}\right)^2 + m_2\left(L - \frac{m_2L}{m_1 + m_2}\right)^2$$

$$= \frac{m_1m_2L^2}{m_1 + m_2} = \frac{1 \times 2 \times (1.2)^2}{1+2} = 0.96 \text{ kg m}^2$$

EXAMPLE 8.22

A giant wheel of radius 2.0 m and mass 100 kg is initially at rest. (a) What torque should be applied to it so that it acquires an angular frequency of 300 r.p.m. in 10 s? (b) Find the kinetic energy when it is rotating at 300 r.p.m.

SOLUTION

Given $\omega_0 = 0$, $\nu = 300$ r.p.m. $= \frac{300}{60} = 5$ Hz. Therefore

$$\omega = 2\pi\nu = 10\pi \text{ rad s}^{-1} \text{ and } t = 10 \text{ s}$$

Using $\omega = \omega_0 + \alpha t$, we have $10\pi = 0 + 10\alpha$

$\Rightarrow \alpha = \pi$ rad s^{-2}

(a) Torque required is $\tau = I\alpha = \left(\frac{1}{2}MR^2\right)\alpha$

$$= \left(\frac{1}{2} \times 100 \times 2^2\right) \times \pi$$

$$= 200\pi = 628 \text{ Nm}$$

(b) K.E. $= \frac{1}{2}I\omega^2 = \frac{1}{2} \times \left(\frac{1}{2} \times 100 \times 2^2\right) \times (10\pi)^2$

$$= 10^4\pi^2 = 9.9 \times 10^4 \text{ J}$$

EXAMPLE 8.23

A stationary horizontal uniform disc of mass M and radius R is free to rotate about an axis passing through its centre and perpendicular to its plane. A torque $\tau = a\theta + b$ is applied to it, where θ is the angular displacement and a and b are positive constants. Obtain the expression for the angular velocity of the disc as a function of θ.

SOLUTION

$$\tau = I\alpha$$

or $$\alpha = \frac{\tau}{I}$$

$$\Rightarrow \quad \frac{d\omega}{dt} = \frac{a\theta + b}{I}$$

$$\Rightarrow \quad \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{a\theta + b}{I}$$

$$\Rightarrow \quad \omega\, d\omega = \left(\frac{a\theta + b}{I}\right)d\theta = \frac{a}{I}\theta\, d\theta + \frac{b}{I}d\theta$$

Integrating

$$\int_0^\omega \omega d\omega = \frac{a}{I}\int_0^\theta \theta d\theta + \frac{b}{I}\int_0^\theta d\theta$$

$$\Rightarrow \quad \frac{\omega^2}{2} = \frac{a}{I}\frac{\theta^2}{2} + \frac{b\theta}{I}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{1}{I}\left(a\theta^2 + 2b\theta\right)}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2}{MR^2}\left(a\theta^2 + 2b\theta\right)}$$

$$\left(\because I = \frac{1}{2}\text{MR}^2\right)$$

EXAMPLE 8.24

A thin uniform rod AB of mass M and length L is hinged at one end A to the horizontal floor. Initially it stands vertically. It is allowed to fall freely in a vertical plane.

(a) What is the angular acceleration of the rod when it is at an angle θ with the vertical?

(b) With what linear speed will the end B hit the floor?

SOLUTION

(a) The entire mass of the rod acts at its centre of mass C. $AC = L/2$. The magnitude of the torque due to weight Mg is [Fig. 8.24]

$$\tau = Mg \times r_\perp$$

$$= Mg \times AD = Mg \times \frac{L}{2}\sin\theta$$

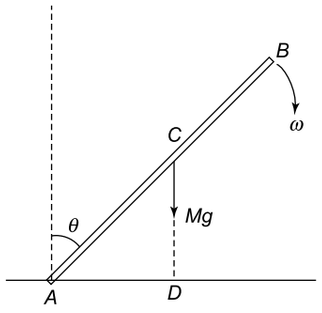


Fig. 8.24

Moment of inertia of the rod about A is $I = \frac{ML^2}{3}$

$\therefore$ Angular acceleration $\alpha = \frac{\tau}{I} = \frac{MgL\sin\theta \times 3}{2 \times ML^2}$

$$= \frac{3g\sin\theta}{L}$$

(b) When the end B hits the floor, the vertical distance through which C falls is $L/2$. From the law of conservation of energy,

Loss in P.E. = gain in K.E

$$Mg \times \frac{L}{2} = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

$\Rightarrow$ $$\omega = \sqrt{\frac{3g}{L}}$$

Linear speed of end $B = L\omega = \sqrt{3gL}$

EXAMPLE 8.25

A particle of mass m is released from rest at point P located at a distance x_0 from origin O on the x-axis as shown in Fig. 8.25. It falls vertically along the negative y-axis.

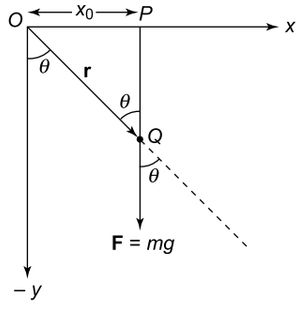


Fig. 8.25

(a) Find the magnitude and direction of the torque acting on the particle at time t when it reaches point $\boldsymbol{Q}$ whose position vector with respect to O is $\mathbf{r}$.

(b) Find the magnitude of the angular momentum of the particle about O at this time t.

(c) Show that, in this example, $\tau = \frac{dL}{dt}$

SOLUTION

(a) The torque is due to force of gravity $\mathbf{F} = m\mathbf{g}$. The magnitude of the torque of F about O is

$$\tau = rF\sin\theta = r\,mg\sin\theta$$

$$= rmg\,\frac{x_0}{r} \qquad \left(\because \sin\theta = \frac{x_0}{r}\right)$$

$\Rightarrow$ $$\tau = mg\,x_0$$

From right hand rule, the direction of the torque is into the page .

(b) The magnitude of angular momentum about O is

$$L = rp\sin\theta = rmv\sin\theta$$

From $v = u + at$ we have $v = 0 + gt = gt$. Therefore

$$L = rm \times gt \times \frac{x_0}{r} = mg\,x_0 t$$

(c) $$\frac{dL}{dt} = \frac{d}{dt}(mgx_0 t) = mgx_0 = \tau$$

8.20 ROLLING MOTION WITHOUT SLIPPING

(1) Total Kinetic Energy

The total kinetic energy of a body which is moving as well as rotating is equal to the sum of translational K.E. and rotational K.E., i.e.

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

where m = mass of the body, v = linear velocity of its centre of mass, I = moment of inertia of the body about an axis passing through its centre of mass and ω = angular velocity of rotation.

(2) Instantaneous Velocity of a point on a Rolling Body

Consider a wheel of radius $R(= AC = BC)$ rolling without slipping on a horizontal rough surface (Fig. 8.26).

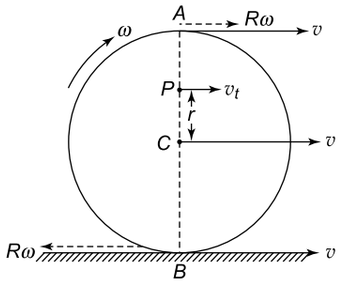


Fig. 8.26

Every point on the wheel has instantaneous velocity. For a point P at a distance r from the centre of mass C, the instantaneous velocity is the vector sum of velocity $\mathbf{v}$ of the centre of mass and tangential velocity $\mathbf{v}_t$ of point P relative to the centre of mass, i.e.

$$\mathbf{v}_P = \mathbf{v} + \mathbf{v}_t$$

where $v_t = r\omega$. Vector $\mathbf{v}_t$ is directed along the tangent to the circle of radius r about C.

For point A, $v_t = R\omega$ and $v = r\omega$. Since these velocities are in the same direction, the instantaneous velocity of A is

$$v_A = v + v_t = R\omega + R\omega = 2\ R\omega$$

For point B in contact with the horizontal surface,

$$v_B = v + v_t = R\omega - R\omega = 0$$

If a body rolls on a surface without slipping, the instantaneous velocity of the point of contact with the surface is zero.

(3) A body Rolling without slipping on a Rough Horizontal surface

Horizontal force F is applied at the centre of mass of a body (disc, ring, cylinder or sphere) of mass M and radius R (Fig. 8.27) on a rough horizontal surface. If f is the frictional force, and the body rolls without slipping $a_{CM} = \alpha R$.

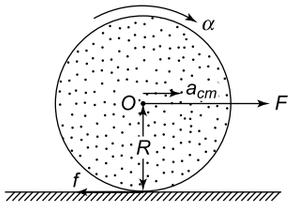


Fig. 8.27

$$F - f = Ma_{CM} \qquad \text{(i)}$$

$$\tau = fR = I\alpha$$

$$\Rightarrow f = \frac{I\alpha}{R} = \frac{Ia_{CM}}{R^2} \qquad \text{(ii)}$$

From (i) and (ii), acceleration of centre of mass is

$$a_{CM} = \frac{F}{M\left(1 + \frac{I}{MR^2}\right)}$$

and

$$f = \frac{F}{\left(1 + \frac{MR^2}{I}\right)}$$

For a disc $I = \frac{1}{2}MR^2 \Rightarrow a_{CM} = \frac{2F}{3M}$

For a ring $I = MR^2 \Rightarrow a_{CM} = \frac{F}{2M}$

For a solid cylinder $I = \frac{1}{2}MR^2 \Rightarrow a_{CM} = \frac{2F}{3M}$

For a hollow cylinder $I = MR^2 \Rightarrow a_{CM} = \frac{F}{2M}$

For a solid sphere $I = \frac{2}{5}MR^2 \Rightarrow a_{CM} = \frac{5F}{7M}$

For a hollow sphere $I = \frac{2}{3}MR^2 \Rightarrow a_{CM} = \frac{3F}{5M}$

If the force is applied tangentially to the body as shown in Fig. 8.28, then

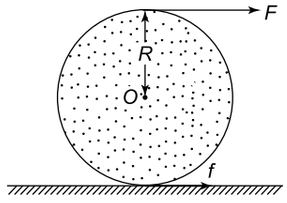


Fig. 8.28

$$a_{CM} = \frac{2F}{M\left(1 + \frac{I}{MR^2}\right)}$$

and

$$f = \frac{F\left(1 - \frac{I}{MR^2}\right)}{1 + \frac{I}{MR^2}}$$

(4) A body Rolling without slipping on a Rough Inclined Plane

A body (ring, disc, cylinder or sphere) of mass M and radius R is rolling (without slipping) down a rough inclined plane of inclination θ (Fig. 8.29).

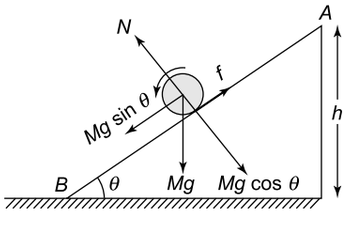


Fig. 8.29

For linear motion parallel to the plane

$$Mg \sin\theta - f = Ma \qquad \text{(i)}$$

where a = linear acceleration of the centre of mass.

For rotational motion about the axis through the centre of mass

$$\tau = I\ \alpha$$

$$\Rightarrow \qquad Rf = I\alpha \Rightarrow f = \frac{I\alpha}{R} = \frac{Ia}{R^2} = (\because\ a = R\alpha) \qquad \text{(ii)}$$

I is the moment of inertia about the centre of mass.

Using (i) and (ii), we get

$$a = \frac{g\sin\theta}{\left(1+\frac{I}{MR^2}\right)}$$

For a ring $I = MR^2 \Rightarrow a = \frac{g\sin\theta}{2}$

For a disc $I = \frac{1}{2}MR^2 \Rightarrow a = \frac{2g\sin\theta}{3}$

For a solid cylinder $I = \frac{1}{2}MR^2 \Rightarrow a = \frac{2g\sin\theta}{3}$

For a hollow cylinder $I = MR^2 \Rightarrow a = \frac{g\sin\theta}{2}$

For a hollow sphere $I = \frac{2}{3}MR^2 \Rightarrow a = \frac{3g\sin\theta}{5}$

For a solid sphere $I = \frac{2}{5}MR^2 \Rightarrow a = \frac{5g\sin\theta}{7}$

Frictional force is $f = \frac{Ia}{R^2} = \frac{Mg\sin\theta}{\left(1+\frac{MR^2}{I}\right)}$

Condition for rolling without slipping

To prevent slipping, $f \le \mu N$, where μ is the coefficient of static friction between the body and the plane and $N = Mg\cos\theta$ is the normal reaction.

Hence to avoid slipping,

$$\frac{Mg\sin\theta}{1+\frac{MR^2}{I}} \le \mu\, Mg\cos\theta$$

$$\Rightarrow \quad \mu \ge \frac{\tan\theta}{\left(1+\frac{MR^2}{I}\right)}$$

EXAMPLE 8.26

A solid cylinder of mass M and radius R is released from rest from top A of an inclined plane of height h and inclination θ as shown in Fig. 8.30. The cylinder rolls without slipping. Find (a) the speed at which it reaches bottom B of the plane and (b) the time it takes to reach B.

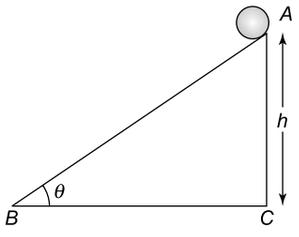


Fig. 8.30

SOLUTION

The acceleration of the cylinder is

$$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}} = \frac{2g\sin\theta}{3} \qquad \left(\because I = \frac{1}{2}MR^2\right)$$

Distance travelled is $s = AB = \frac{h}{\sin\theta}$

(a) Using $v^2 - u^2 = 2as$, we have

$$v^2 - 0 = 2 \times \frac{2g\sin\theta}{3} \times \frac{h}{\sin\theta}$$

$$\Rightarrow \quad v = \sqrt{\frac{4gh}{3}}$$

Speed v can also be found from the law of conservation of energy. As the cylinder moves from A to B, it loses P.E. and gains K.E.

Loss in P.E. = gain in K.E.

or $$Mgh = \frac{1}{2} = Mv^2 + \frac{1}{2}I\omega^2$$

Now $I = \frac{1}{2}MR^2$ and $\omega = \frac{v}{R}$. Therefore

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2} \times \frac{1}{2}\, MR^2 \times \left(\frac{v}{R}\right)^2$$

$$= \frac{3}{4}Mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{4gh}{3}}$$

(b) From $v = u + at$, we have

$$\sqrt{\frac{4gh}{3}} = 0 + \frac{2g\sin\theta}{3}t$$

$$\Rightarrow \quad t = \sqrt{\frac{3h}{g}} \cdot \frac{1}{\sin\theta}$$

EXAMPLE 8.27

A billiard ball has mass $M = 250$ g and radius $R = 2.5$ cm and is initially at rest. A rod held horizontal at a height h above centre C hits the ball. The ball begins to roll without slipping. Find the value of h [see Fig. 8.31].

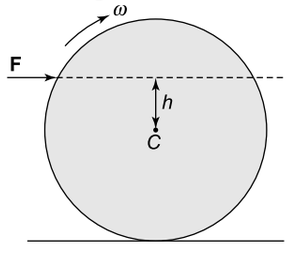


Fig. 8.31

SOLUTION

The horizontal force **F** imparts a linear impulse

$$I = \int F dt = \text{change in linear momentum}$$

$$\Rightarrow \quad I = Mv - 0 = Mv = MR\omega \qquad \text{(i)}$$

where v is the velocity of the centre of mass of the ball. Since it rolls without slipping, $v = R\omega$, where ω is the angular velocity.

The torque due to **F** imparts an angular impulse

$$J = I\,h = \text{change in angular momentum}$$

$$= I\omega - 0 = I\omega$$

$$\text{or} \quad Ih = \left(\frac{2}{5}MR^2\right)\omega \quad \left(\because I = \frac{2}{5}MR^2\right) \qquad \text{(ii)}$$

Dividing (ii) by (i)

$$h = \frac{\frac{2}{5}MR^2\omega}{MR\omega} = \frac{2R}{5} = \frac{2 \times 2.5\text{ cm}}{5}$$

$$= 1.0\text{ cm}$$

EXAMPLE 8.28

A turntable of radius R = 10 m is rotating making 98 revolutions in 10 s with a boy of mass m = 60 kg standing at its centre. He starts running along a radius. Find the frequency of the turntable when the boy is 4 m from the centre. The moment of inertia of the turntable about its axis is 1000 kg m^2.

SOLUTION

Initial moment of inertia of the system is

M_1 = M.I. of turntable + M.I. of boy at the centre

$$= 1000 + 0 = 1000\text{ kg m}^2$$

Initial frequency ν_1 = 9.8 rev/sec

Final moment of the system is

M_2 = M.I. of turntable + M.I. of boy at a distance 4 m from the centre of turn table

$$= 1000 + 60 \times (4)^2 = 1960\text{ kg m}^2$$

Since no external torque acts, the angular momentum of the system is conserved, i.e.

$$I_2\omega_2 = I_1\omega_1 \Rightarrow I_2\nu_2 = I_1\nu_1$$

$$\Rightarrow \quad \nu_2 = \frac{I_1\nu_1}{I_2} = \frac{1000 \times 9.8}{1960} = 5\text{ rev/s}$$

$$= 5\text{ Hz}$$

EXAMPLE 8.29

A uniform rod AB of length L = 1 m is sliding along two mutually perpendicular surfaces OP and OQ as shown in Fig. 8.32. When the rod subtends an angle $\theta = 30°$ with OQ, the end B has a velocity $\sqrt{3}$ ms^{-1}. Find the velocity of end A at that time.

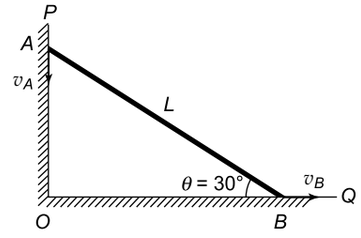


Fig. 8.32

SOLUTION

$OB = x$, $OA = y$ and $x^2 + y^2 = L^2$ and $x = L\cos\theta$

Differentiating $x^2 + y^2 = L^2$ with respect to t we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\Rightarrow \quad 2xv_B + 2yv_A = 0$$

$$\Rightarrow \quad v_A = -\frac{x}{y}v_B$$

$$\therefore \quad |v_A| = \frac{x}{y}v_B = v_B\cot\theta$$

$$= \sqrt{3} \times \cot 30°$$

$$= \sqrt{3} \times \frac{1}{\sqrt{3}} = 1\text{ ms}^{-1}$$

EXAMPLE 8.30

A rope is wound around a hollow cylinder of mass M = 3 kg and radius R = 40 cm. If the rope is pulled with a force F = 30 N, find (a) the angular acceleration of the cylinder and (b) the linear acceleration of the rope.

SOLUTION

For a hollow cylinder $I = MR^2$

(a) Torque on cylinder is $\tau = FR \Rightarrow I\alpha = FR$. Therefore

$$a = \frac{FR}{I} = \frac{FR}{MR^2} = \frac{F}{MR} = \frac{30}{3 \times 0.4}$$

$$= 25\text{ rad s}^{-2}$$

(b) Linear acceleration of rope is

$$a = R\alpha = 0.4 \times 25 = 10 \text{ ms}^{-2}$$

EXAMPLE 8.31

A uniform rod AB of mass $M = 0.4$ kg and length $L = 1$ m lies on a horizontal frictionless table with its end A pivoted to the table. A ball of mass $m = 0.2$ kg moving along the surface of the table with velocity $u = 4 \text{ ms}^{-1}$ perpendicular to the rod collides with the free end B of the rod. If the collision is elastic, find (a) the velocity of the ball immediately after the collision and (b) the angular velocity of the rod after collision.

SOLUTION

Refer to Fig. 8.33.

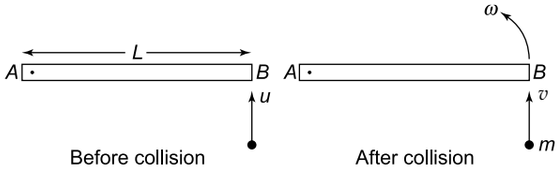


Fig. 8.33

(a) Let v be the velocity of the ball just after collision. Since the collision is perfectly elasitc, $e = 1$, i.e.

Velocity of approach = velocity of separation

or $\quad u = \omega L - v$

$$\Rightarrow \quad \omega = \frac{u + v}{L} \qquad \text{(i)}$$

Since there is no external torque, the angular mementum about A is conserved, i.e.

$$mu\ L = mvL + I\omega$$

$$= MvL + \frac{ML^2}{3}\ \omega$$

$$\Rightarrow \quad \omega = \frac{3\,(u - v)m}{ML} \qquad \text{(ii)}$$

From (i) and (ii), we get

$$v = \left(\frac{3m - M}{3m + M}\right) u \qquad \text{(iii)}$$

$$= \left(\frac{3 \times 0.2 - 0.4}{3 \times 0.2 + 0.4}\right) \times 4 = 0.8 \text{ ms}^{-1}$$

(b) Using (iii) in (i), we get

$$\omega = \frac{6\,mu}{(3\,m + M)\,L}$$

$$= \frac{6 \times 0.2 \times 4}{(3 \times 0.2 + 0.4) \times 1} = 4.8 \text{ rad s}^{-1}$$

EXAMPLE 8.32

A uniform rod AB of mass M and length L is hinged at one end A. It is released from rest at a horizontal position. Find the angular acceleration of the rod and the linear acceleration of its centre of mass as it falls.

SOLUTION

Refer to Fig. 8.34.

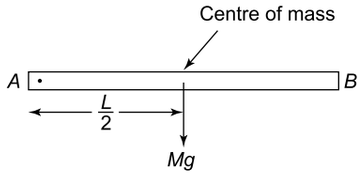


Fig. 8.34

$$\text{Torque about } A = Mg \times \frac{L}{2} \Rightarrow I\alpha = \frac{MgL}{2}$$

or $$\alpha = \frac{MgL}{2I} = \frac{3g}{2L} \qquad \left(\because\ I = \frac{ML^2}{3}\right)$$

Linear acceleration of centre of mass is

$$a_{CM} = \frac{L}{2} \times \alpha = \frac{L}{2} \times \frac{3g}{2L} = \frac{3g}{4}$$

8.21 TRANSLATIONAL AND ROTATIONAL EQUILIBRIUM

A body is said to be in equilibrium if its state of motion does not change with time.

Translational Equilibrium

A body is in translational equilibrium if the total force acting on it is zero; the total force is equal to the vector sum (resultant) of the individual forces acting on the body, i.e.

$$\mathbf{F}_{\text{total}} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_n$$

If a body is in translational equilibrium, the linear momentum does not change with time, i.e. $\mathbf{p}$ = constant

Rotational Equilibrium

A body is in rotational equilibrium if the total torque acting on the body is zero; the total torque is equal to the vector sum of the individual torques acting on the body, i.e.

$$\boldsymbol{\tau}_{\text{total}} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + \ldots\ldots + \boldsymbol{\tau}_n$$

If the body is in rotational equilibrium, the angular momentum about the axis of rotation does not change with time, i.e. $\mathbf{L}$ = constant

Consider a uniform rigid rod AB of negligible mass. Two parallel forces of equal magnitude F are applied perpendicular to the rod at ends A and B as shown in Fig. 8.35(a). Let C be the mid-point of rod.

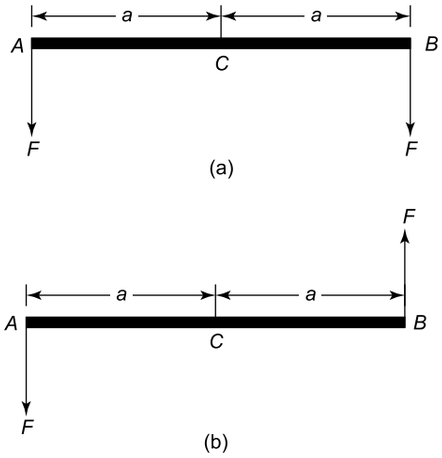


Fig. 8.35

Taking the anticlockwise torque to be positive and clockwise torque to be negative, anticlockwise torque about $C = Fa$ and clockwise torque about $C = -Fa$. Total torque about $C = Fa - Fa = 0$. Total Force $= F + F = 2F$. Hence the rod is in rotational equilibrium but not in translational equilibrium.

If the forces act in opposite directions as shown in Fig. 8.34(b), total torque $= Fa + Fa = 2Fa$ and total force $= F - F = 0$. Hence, in this case, the rod is in translational equilibrium but not in rotational equilibrium.

A pair of equal and opposite forces having different lines of action is known as a couple. A couple produces a torque which produces rotation without translation. For example, opening the cap of a bottle or opening a tap. Torque is also called moment of force.

Principle of Moments

Consider a uniform rod AB of negligible mass pivoted at a point along its length as shown in Fig. 8.36. Forces F_1 and F_2 act at ends A and B are such that the rod is in translational as well as rotational equilibrium. Let R be the normal reaction of the pivot.

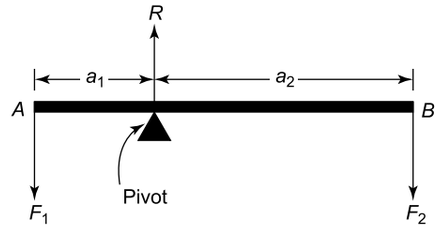


Fig. 8.36

For translational equilibrium $F_{total} = 0 \Rightarrow R - F_1 - F_2 = 0$
For rotational equilibrium $\tau_{total} = 0 \Rightarrow F_1 a_1 - F_2 a_2 = 0$
or $\quad F_2 a_2 = F_1 a_1$

i.e. anticlockwise torque = clockwise torque.
This is the principle of moments.

EXAMPLE 8.33

A uniform metal bar AB of length 100 cm and mass $M = 2$kg is supported on two knife-edges placed 20 cm from each end. A mass of $m = 3$kg is suspended at a distance of 40 cm from end A. Find the normal reactions at the knife-edges. Take $g = 10$ ms^{-2}.

SOLUTION

Refer to Fig. 8.37

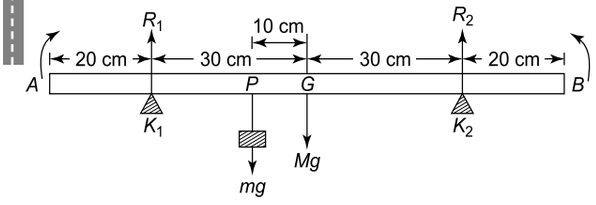


Fig. 8.37

Let R_1 and R_2 be the normal reaction at K_1 and K_2. The rod is in translational as well as rotational equilibrium.

For translational equilibrium, $F_{total} = 0$, Hence

$mg + mg - R_1 - R_2 = 0$

$\Rightarrow R_1 + R_2 = (m + m)g = (3 + 2) \times 10 = 50$ N (i)

For rotational equilibrium, $\tau_{total} = 0$, Hence

Clockwise moment about G = anticlockwise moment about G

$\Rightarrow R_1 \times (K_1G) = R_2 (K_2G) + mg \times (PG)$

$\Rightarrow \quad R_1 \times 30 = R_2 \times 30 + 3 \times 10 \times 10$

$\Rightarrow \quad R_1 = R_2 + 10$

$\Rightarrow \quad R_1 - R_2 = 10$ (ii)

From Eqs. (i) and (ii) we get $R_1 = 30$ N and $R_2 = 20$ N

EXAMPLE 8.34

A uniform rod AB of length 1.0 m and mass 5.0 kg leans on a frictionless vertical wall and a rough horizontal floor with end B touching the wall and end A at a distance of 40 cm from the wall as shown in Fig. 8.38(a) Find

(a) Normal reaction of the wall and normal reaction of the floor,

(b) the frictional force at end A,

(c) the coefficient of friction between the floor and end A, and

(d) the reaction force at A.

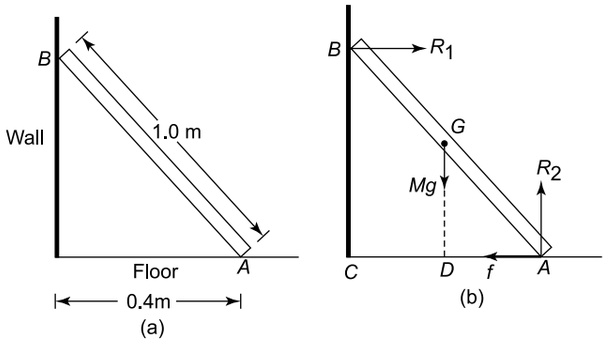


Fig. 8.38

SOLUTION

Refer to Fig. 8.38(b). $AB = 1.0$ m, $AG = BG = 0.5$ m, $AC = 0.4$m. G is the centre of mass of the rod.

$$BC = \sqrt{(AB)^2 - (AC)^2} = \sqrt{(1.0)^2 - (0.4)^2} = 0.92 \text{ m}$$

The forces acting on the rod are

(i) normal reactions R_1 and R_2 at B and A

(ii) weight Mg of rod acting at G and

(iii) frictional force f between the floor and end A of the rod.

(a) Since the frictional force f prevents the rod from sliding away from the wall, force f must be directed towards the wall.

For translational equilibrium, $F_{\text{total}} = 0$. Hence

$$R_1 = f \text{ (horizontal direction)}$$

and $$R_2 = Mg \text{ (vertical direction)}$$

For rotational equilibrium, $\tau_{\text{total}} = 0$. Hence clockwise moment about A = anticlockwise moment about A

$$\Rightarrow R_1 \times BC = R_2 \times 0 + Mg \times AD + f \times 0$$

$$\Rightarrow R_1 \times 0.92 = Mg \times 0.2 = 5 \times 10 \times 0.2 = 10$$

$$\Rightarrow R_1 = 10.9 \text{ N}$$

Also $$R_2 = Mg = 5 \times 10 = 50 \text{ N}$$

[Note that the perpendicular distance between R_2 and the axis of rotation at A is zero and perpendicular distance of f from the axis of rotation at A is also zero]

(b) Frictional force $f = R_1 = 10.9$ N

(c) Coefficient of friction between the floor and end A is

$$\mu = \frac{f}{R_2} = \frac{10.9}{50} = 0.22$$

(d) The reaction force F at A is the resultant of f and R_2

$$F = \sqrt{f^2 + R_2^2}$$

$$= \sqrt{(10.9)^2 + (50)^2} = 51.2 \text{ N}$$

NOTE

At end B, reaction force = normal reaction R_1 because the wall is frictionless.

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. A gun of mass M is initially at rest on a horizontal frictionless surface. It fires a bullet of mass m with a velocity v at an angle θ with the horizontal. After firing, the centre of mass of the gun-bullet system

(a) moves with a velocity mv/M opposite to the direction of motion of the bullet.

(b) moves with a velocity $mv \cos\theta/M$ in the horizontal direction

(c) moves with a velocity $mv \sin\theta/M$ in the horizontal direction

(d) remains at rest

2. A thin uniform circular disc has a radius R. A square portion of diagonal equal to R is cut out from it. The distance between the centre of mass of the remaining portion of the disc from the centre of the complete disc is

(a) $\frac{R}{2\pi}$ (b) $\frac{R}{(2\pi-1)}$

(c) $\frac{R}{(2\pi+1)}$ (d) $\frac{R}{2(2\pi-1)}$

3. A carpet of mass M is rolled along its length in the form of a cylinder of radius R and kept on a rough floor. If the carpet is unrolled, without sliding, to a radius $R/2$, the decrease in potential energy is

(a) $\frac{1}{2}\ MgR$ (b) $\frac{3}{4}MgR$

(c) $\frac{5}{8}\ MgR$ (d) $\frac{7}{8}\ MgR$

4. A ring of radius r has its mass non-uniformly distributed over its circumference with centre at the origin. If x is the distance of the centre of mass of the ring from its centre, then

(a) $x = r$ (b) $x < r$

(c) $x > r$ (d) $0 \le x \le r$

5. Two particles of equal mass have velocities $\boldsymbol{v}_1 = a\,\hat{\mathbf{i}}$ and $\boldsymbol{v}_2 = a\,\hat{\mathbf{j}}$. The acceleration of first particle is $\mathbf{a}_1 = b\,(\hat{\mathbf{i}}+\hat{\mathbf{j}})$ where a and b are constants. If the acceleration of the second particle is zero, the centre of mass of the two particles moves along a

(a) straight line (b) circle

(c) ellipse (d) parabola

6. A solid sphere, released from rest from the top of an inclined plane of inclination θ_1, rolls without sliding and reaches the bottom with speed v_1 and its time of descent is t_1. The same sphere is then released from rest from the top of another inclined plane of inclination θ_2 but of the same height, rolls without sliding and reaches the bottom with speed v_2 and its time of descent is t_2. If $\theta_2 > \theta_1$, then

(a) $v_2 > v_1$; $t_2 < t_1$

(b) $v_2 = v_1$; $t_2 < t_1$

(c) $v_2 < v_1$; $t_2 > t_1$

(d) $v_2 = v_1$; $t_2 = t_1$

7. The moment of inertia of a uniform rod of mass M and length L about an axis passing through its centre and inclined to it at an angle $\theta = 60°$ is

(a) $\frac{ML^2}{3}$ (b) $\frac{ML^2}{4}$

(c) $\frac{ML^2}{12}$ (d) $\frac{ML^2}{16}$

8. A solid cylinder of mass M and radius R is rolling without slipping on a horizontal plane with a speed v. It then rolls up an inclined plane of inclination θ to a maximum height given by

(a) $\frac{v^2}{2g}$ (b) $\frac{v^2}{2g}\ \sin\theta$

(c) $\frac{3v^2}{4g}$ (d) $\frac{3v^2}{4g}\sin\theta$

9. A uniform rod AB of mass m and length L is suspended by two strings C and D of negligible mass as shown in Fig. 8.39. When string D is cut, the tension in string C will be

(a) $\frac{mg}{4}$ (b) mg

(c) $2\ mg$ (d) $4\ mg$

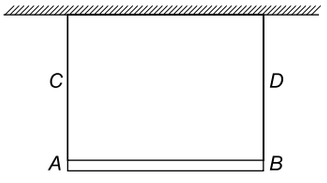


Fig. 8.39

10. A body of mass m is projected with a velocity u at an angle of 60° with the horizontal. The magnitude of the angular momentum about the point of projection when the body is at the highest point of its trajectory is

(a) $\frac{mu^3}{2g}$ (b) $\frac{2mu^3}{5g}$

(c) $\frac{3mu^3}{16g}$ (d) $\frac{mu^3}{4g}$

11. A cubical block of side L and mass m rests on a rough horizontal surface. A horizontal force F is applied normal to one of its faces at a point that is directly above the centre of the face at a height $2L/3$ above the base. The minimum force F required to topple the block before sliding is

(a) $\frac{2mg}{3}$ (b) $\frac{3mg}{2}$

(c) $\frac{3mg}{4}$ (d) $\frac{4mg}{3}$

12. If the earth were to suddenly contract to half its present size, without any change in its mass, the duration of the new day will be

(a) 6 hours (b) 12 hours

(c) 18 hours (d) 30 hours

13. A circular ring of mass M and radius R is rotating about its axis at an angular frequency ω. Two blocks, each of mass m, are gently placed on the opposite ends of a diameter of the ring. The angular frequency of the ring becomes ω'. The ratio ω'/ω is

(a) $\dfrac{M}{(M+2m)}$ (b) $\dfrac{2M}{(M+2m)}$

(c) $\dfrac{2m}{M}$ (d) $\dfrac{M}{2m}$

14. A solid sphere rolls down from the top of an inclined plane. Its velocity on reaching the bottom of the plane is v. When the same sphere slides down from the top of the plane, its velocity on reaching the bottom is v'. The ratio v'/v is

(a) $\sqrt{\dfrac{3}{5}}$ (b) 1

(c) $\sqrt{\dfrac{7}{5}}$ (d) $\dfrac{3}{\sqrt{5}}$

15. A circular disc is rolling down an inclined plane without slipping. If the angle of inclination is 30°, the acceleration of the disc down the inclined plane is

(a) g (b) $\dfrac{g}{2}$

(c) $\dfrac{g}{3}$ (d) $\dfrac{\sqrt{2}}{3}\ g$

16. A block of mass M is released from the top of an inclined plane. Its velocity on reaching the bottom of the plane is v. A circular disc of the same mass M rolls down the incline plane from the top. Its velocity on reaching the bottom is v'. The ratio v'/v will be

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\sqrt{\dfrac{2}{3}}$

(c) 1 (d) $\dfrac{2\sqrt{2}}{3}$

17. Two circular loops A and B of radii R and $2R$ respectively are made of the same wire. Their moments of inertia about the axis passing through the centre and perpendicular to their plane are I_A and I_B respectively. The ratio I_A/I_B is

(a) 1 (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{4}$ (d) $\dfrac{1}{8}$

18. A small coin is placed at a distance r from the centre of a gramophone record. The rotational speed of the record is gradually increased. If the coefficient of friction between the coin and the record is μ, the minimum angular frequency of the record for which the coin will fly off is given by

(a) $\sqrt{\dfrac{2\mu g}{r}}$ (b) $\sqrt{\dfrac{\mu g}{2r}}$

(c) $\sqrt{\dfrac{\mu g}{r}}$ (d) $2\sqrt{\dfrac{\mu g}{r}}$

19. In Q.18, what would be the minimum angular frequency at which two identical coins, placed one on top of the other, at the same location on the record, will fly off?

(a) $\sqrt{\dfrac{\mu g}{r}}$ (b) $\sqrt{\dfrac{2\mu g}{r}}$

(c) $2\sqrt{\dfrac{\mu g}{r}}$ (d) $2\sqrt{\dfrac{2\mu g}{r}}$

20. Three particles, each of mass m, are placed at the corners of a right angled triangle as shown in Fig. 8.40. If $OA = a$ and $OB = b$, the position vector of the centre of mass is (here **i** and **j** are unit vectors along x and y axes respectively).

(a) $\dfrac{1}{3}\ (a\mathbf{i} + b\mathbf{j})$ (b) $\dfrac{1}{3}\ (a\mathbf{i} - b\mathbf{j})$

(c) $\dfrac{2}{3}\ (a\mathbf{i} + b\mathbf{j})$ (d) $\dfrac{2}{3}\ (a\mathbf{i} - b\mathbf{j})$

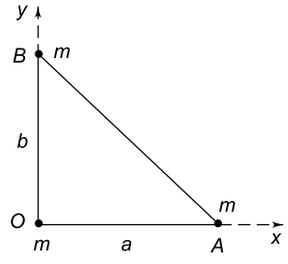


Fig. 8.40

21. A sphere of mass M and Radius R is released from the top of an inclined plane of inclination θ. The minimum coefficient of friction between the plane and the sphere so that it rolls down the plane without sliding is given by

(a) $\mu = \tan\ \theta$ (b) $\mu = \dfrac{2}{3}\ \tan\ \theta$

(c) $\mu = \dfrac{2}{5}\ \tan\ \theta$ (d) $\mu = \dfrac{2}{7}\ \tan\ \theta$

22. Three thin metal rods, each of mass M and length L, are welded to form an equilateral triangle. The moment of inertia of the composite structure about an axis passing through the centre of mass of the structure and perpendicular to its plane is

(a) $\frac{ML^2}{2}$ (b) $\frac{ML^2}{4}$

(c) $\frac{ML^2}{8}$ (d) $\frac{ML^2}{12}$

23. Four thin metal rods, each of mass M and length L, are welded to form a square $ABCD$ as shown in Fig. 8.41. What is the moment of inertia of the composite structure about a line which bisects rods AB and CD and perpendicular to the plane of the structure?

(a) $\frac{ML^2}{6}$ (b) $\frac{ML^2}{3}$

(c) $\frac{ML^2}{2}$ (d) $\frac{2ML^2}{3}$

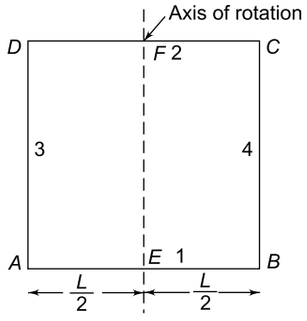


Fig. 8.41

24 . A thin uniform metallic triangular sheet of mass M has sides $AB = BC = L$. What is its moment of inertia about axis AC lying in the plane of the sheet? (Fig. 8.42)

(a) $\frac{ML^2}{12}$

(b) $\frac{ML^2}{6}$

(c) $\frac{ML^2}{3}$

(d) $\frac{2ML^2}{3}$

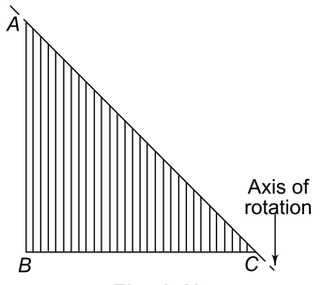


Fig. 8.42

25. A solid, homogeneous sphere of mass M and radius R is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of the sphere

(a) total kinetic energy is conserved

(b) the angular momentum of the sphere about the point of contact with the plane is conserved

(c) only the rotational kinetic energy about the centre of mass is conserved

(d) the angular momentum about the centre of mass is conserved.

26. I_1, I_2, I_3 and I_4 are respectively the moments of inertia of a thin square plate $ABCD$ of uniform thickness about axes 1, 2, 3 and 4 which are in the plane of the plate (Fig. 8.43). The moment of inertia of the plate about an axis passing through thecentre O and perpendicular to the plane of the plate is

(a) $2(I_1 + I_2)$ (b) $2(I_3 + I_4)$

(c) $I_1 + I_3$ (d) $I_1 + I_2 + I_3 + I_4$

IIT, 1992

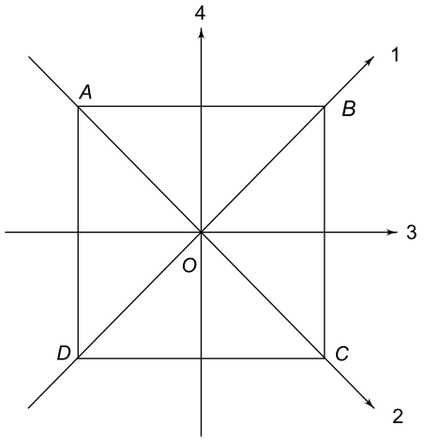


Fig. 8.43

27. The speed of a homogeneous solid sphere after rolling down an inclined plane of vertical height h, from rest without sliding is

(a) $\sqrt{\frac{10}{7}gh}$ (b) $\sqrt{gh}$

(c) $\sqrt{\frac{6}{5}gh}$ (d) $\sqrt{\frac{4}{3}gh}$

28. If a sphere is rolling, the ratio of its rotational energy to the total kinetic energy is given by

(a) 7 : 10 (b) 2 : 5

(c) 10 : 7 (d) 2 : 7

29. A cart of mass M is tied at one end of a massless rope of length 10 m. The other end of the rope is in the hands of a man of mass M. The entire system is on a smooth horizontal surface. The man is at $x = 0$ and the cart at $x = 10$ m. If the man pulls the cart by the rope, the man and the cart will meet at a point

(a) $x = 0$

(b) $x = 5$ m

(c) $x = 10$ m

(d) they will never meet.

30. A mass m is moving with a constant velocity along a line parallel to the x-axis, away from the origin. Its angular momentum with respect to the origin

(a) is zero (b) remains constant
(c) goes on increasing (d) goes on decreasing.

IIT, 1997

31. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(a) I (b) $I \sin^2 \theta$

(c) $I \cos^2 \theta$ (d) $I \cos^2 \left(\frac{\theta}{2}\right)$

IIT, 1998

32. A smooth sphere A is moving on a frictionless horizontal surface with angular speed ω and centre of mass velocity v. It collides elastically and head-on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B respectively. Then

(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$
(c) $\omega_A = \omega$ (d) $\omega_B = \omega$

IIT, 1999

33. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in Fig. 8.44. The magnitude of angular momentum of the disc about the origin O is

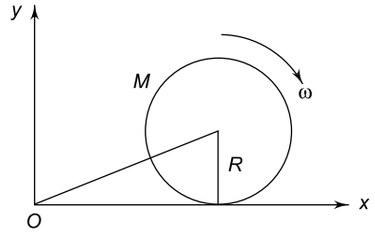


Fig. 8.44

(a) $\frac{1}{2} MR^2\omega$ (b) $MR^2\omega$

(c) $\frac{3}{2} MR^2\omega$ (d) $2\ MR^2\omega$

IIT, 1999

34. A cubical block of side a is moving with a velocity v on a horizontal smooth plane as shown in Fig. 8.45. It hits a ridge at point O. The angular speed of the block after it hits the ridge at O is

(a) $\frac{3v}{4a}$ (b) $\frac{3v}{2a}$

(c) $\frac{\sqrt{3}\,v}{\sqrt{2}\,a}$ (d) zero

IIT, 1999

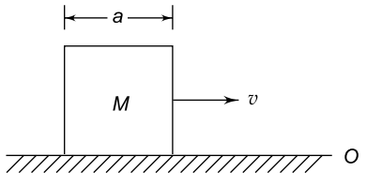


Fig. 8.45

35. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown in Fig. 8.46. The moment of inertia of the loop about the axis XX' is

(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$

(c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

IIT, 2000

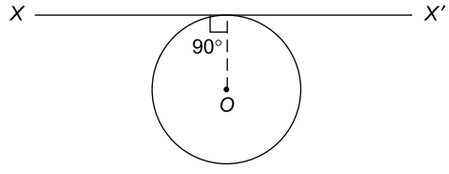


Fig. 8.46

36. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $\frac{M\omega^2 L}{2}$ (b) $M\ \omega^2\ L$

(c) $\frac{M\omega^2 L}{4}$ (d) $\frac{M\omega^2 L^2}{2}$

IIT, 1992

37. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown in Fig. 8.47. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are

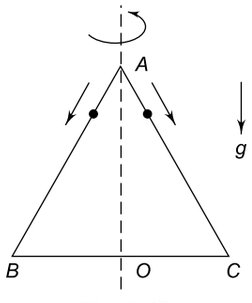


Fig. 8.47

(a) angular velocity and total energy (kinetic and potential).
(b) total angular momentum and total energy.
(c) angular velocity and moment of inertia about the axis of rotation.
(d) total angular momentum and moment of inertia about the axis of rotation.

IIT, 2000

38. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown in Fig. 8.48. If the coefficeint of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is

(a) infinitesimal (b) $mg/4$
(c) $mg/2$ (d) $mg(1 - \mu)$

IIT, 2000

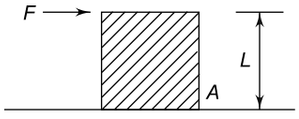


Fig. 8.48

39. The angular velocity of a body changes from ω_1 to ω_2 without applying a torque but by changing the moment of inertia about its axis of rotation. The ratio of the corresponding radii of gyration is

(a) $\omega_1 : \omega_2$ (b) $\sqrt{\omega_1} : \sqrt{\omega_2}$
(c) $\sqrt{\omega_2} : \sqrt{\omega_1}$ (d) $\omega_2 : \omega_1$

40. A thin uniform rod AB of mass m and length L is hinged at one end A to the level floor. Initially it stands vertically and is allowed to fall freely to the floor in the vertical plane. The angular velocity of the rod when its end B strikes the floor is:

(a) $\frac{mg}{L}$ (b) $\left(\frac{mg}{3L}\right)^{1/2}$
(c) $\frac{g}{L}$ (d) $\left(\frac{3g}{L}\right)^{1/2}$

41. Moment of inertia of uniform horizontal solid cylinder of mass M about an axis passing through its edge and perpendicular to the axis of the cylinder when its length is 6 times its radius R is:

(a) $\frac{39MR^2}{4}$ (b) $\frac{39MR}{4}$
(c) $\frac{49MR}{4}$ (d) $\frac{49MR^2}{4}$

42. If A is the areal velocity of a planet of mass M, its angular momentum is

(a) M (b) $2MA$
(c) A^2M (d) AM^2

43. One end of a thin uniform rod of length L and mass M_1 is rivetted to the centre of a uniform circular disc of radius 'r' and mass M_2 so that both are coplanar. The centre of mass of the combination from the centre of the disc is:
(Assume that the point of attachment is at the origin)

(a) $\frac{L(M_1 + M_2)}{2M_1}$ (b) $\frac{LM_1}{2(M_1 + M_2)}$
(c) $\frac{2(M_1 + M_2)}{LM_1}$ (d) $\frac{2LM_1}{(M_1 + M_2)}$

44. Two circular loops A and B of radii r_A and r_B respectively are made from a uniform wire. The ratio of their moments of inertia about axes passing through their centres and perpendicular to their planes is $\frac{I_B}{I_A} = 8$, then $\left(\frac{r_B}{r_A}\right)$ is equal to

(a) 2 (b) 4
(c) 6 (d) 8

45. A body of mass 'm' is tied to one end of a spring and whirled round in a horizontal plane with a constant angular velocity. The elongation in the spring is one centimeter. If the angular velocity is doubled, the elongation in the spring is 5 cm. The original length of the spring is:

(a) 16 cm (b) 15 cm
(c) 14 cm (d) 13 cm

46. A particle performs uniform circular motion with an angular momentum L. If the angular frequency of the particle is doubled, and kinetic energy is halved, its angular momentum becomes:

(a) $4L$ (b) $2L$
(c) $\frac{L}{2}$ (d) $\frac{L}{4}$

47. A uniform rod of length 1 metre is bent at its mid-point to make 90° angle. The distance of the centre of mass from the centre of the rod is

(a) 36.1 cm (b) 25.2 cm
(c) 17.7 cm (d) zero

48. A mass is whirled in a circular path with constant angular velocity and its angular momentum is L. If the string is now halved keeping the angular velocity the same, the angular momentum is

(a) $\frac{L}{4}$ (b) $\frac{L}{2}$
(c) L (d) $2L$

49. A solid sphere is rotating in free space. If the radius of the sphere is increased, keeping its mass the same, which one of the following will not be affected?
(a) Moment of inertia
(b) Angular momentum
(c) Angular velocity
(d) Rotational kinetic energy

50. A solid sphere is rotating about its diameter. Due to increase in room temperature, its volume increases by 0.5%. If no external torque acts, the angular speed of the sphere will
(a) increase by nearly $\frac{1}{3}\%$
(b) decrease by nearly $\frac{1}{3}\%$
(c) increase by nearly $\frac{1}{2}\%$
(d) decrease by nearly $\frac{2}{3}\%$

51. The height of a solid cylinder is four times its radius. It is kept vertically at time $t = 0$ on a belt which is moving in the horizontal direction with a velocity $v = 2.45\ t^2$ where v is in ms^{-1} and t is in second. If the cylinder does not slip, it will topple over at time t equal to
(a) 1 s (b) 2 s
(c) 3 s (d) 4 s

52. A circular portion of diameter R is cut out from a uniform circular disc of mass M and radius R as shown in Fig. 8.49. The moment of inertia of the remaining (shaded) portion of the disc about an axis passing through the centre O of the disc and perpendicular to its plane is
(a) $\frac{15}{32}\ MR^2$ (b) $\frac{7}{16}\ MR^2$
(c) $\frac{13}{32}\ MR^2$ (d) $\frac{3}{8}\ MR^2$

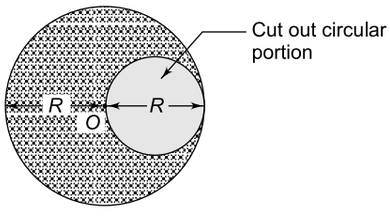


Fig. 8.49

53. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with angular velocity ω_0 about its axis perpendicular to the rod and passing through its mid point (see Fig. 8.50). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is
(a) $\frac{M\omega_0}{M+3m}$ (b) $\frac{M\omega_0}{M+6m}$
(c) $\frac{(M+6m)\omega_0}{M}$ (d) ω_0

IIT, 1988

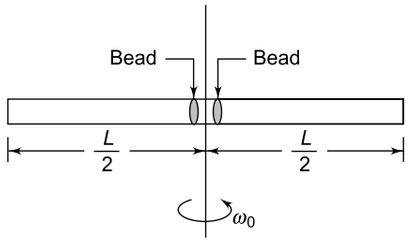


Fig. 8.50

54. Two particles A and B, initially at rest, move towards each other under a mutual force of attraction. At the instant when the speed of A is V and that *of* B is $2V$, the speed of the centre of mass of the system is
(a) 0 (b) V
(c) $1.5V$ (d) $3V$

IIT, 1982

55. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is (Fig. 8.51)
(a) $\frac{1}{2}\ MR^2$
(b) $\frac{1}{4}\ MR^2$
(c) $\frac{1}{8}\ MR^2$
(d) $\sqrt{2}\ MR^2$

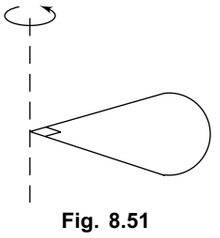


Fig. 8.51

IIT, 2001

56. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the center of mass is
(a) 30 m/s (b) 20 m/s
(c) 10 m/s (d) 5 m/s

IIT, 2002

57. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are:
(a) **up** the incline while ascending and **down** the incline while descending
(b) **up** the incline while ascending as well as descending
(c) **down** the incline while ascending and **up** the incline while descending
(d) **down** the incline while ascending as well as descending.

IIT, 2002

58. The angular momentum of a particle moving in a circular orbit with a constant speed remains conserved about
(a) any point on the circumference of the circle
(b) any point inside the circle
(c) any point outside the circle
(d) the centre of the circle

IIT, 2003

59. A particle moves in a circular orbit with uniform angular speed. However, the plane of the circular orbit is itself rotating at a constant angular speed. We may then say
(a) the angular velocity as well as the angular acceleration of the particle are both constant
(b) neither the angular velocity nor the angular acceleration of the particle are constant
(c) the angular velocity of the particle varies but its angular acceleration is a constant
(d) the angular velocity of the particle remains constant but its angular acceleration varies

60. A cylinder of mass m and radius r is rotating about its axis with a constant speed v. Its kinetic energy is
(a) $2\ mv^2$ (b) mv^2
(c) $\frac{1}{2}\ mv^2$ (d) $\frac{1}{4}\ mv^2$

61. A circular disc of mass m and radius r is rolling on a smooth horizontal surface with a constant speed v. Its kinetic energy is
(a) $\frac{1}{4}\ mv^2$ (b) $\frac{1}{2}\ mv^2$
(c) $\frac{3}{4}\ mv^2$ (d) mv^2

62. Two solid spheres A and B, each of radius R, are made of materials of densities ρ_A and ρ_B respectively. Their moments of inertia about a diameter are I_A and I_B respectively. The ratio I_A/I_B is
(a) $\sqrt{\frac{\rho_A}{\rho_B}}$ (b) $\sqrt{\frac{\rho_B}{\rho_A}}$
(c) $\frac{\rho_A}{\rho_B}$ (d) $\frac{\rho_B}{\rho_A}$

63. A cylinder, released from the top of an inclined plane, rolls without sliding and reaches the bottom with speed v_r. Another identical cylinder, released from the top of the same inclined plane, slides without rolling and reaches the bottom with speed v_s. Then
(a) $v_r > v_s$ (b) $v_r < v_s$
(c) $v_r = v_s$ (d) $v_r = v_s = 0$

64. In the rectangular lamina $ABCD$ shown in Fig. 8.52, $a = AB = BC/2$. The moment of inertia of the lamina is the minimum along the axis passing through
(a) BC (b) AB
(c) HF (d) EG

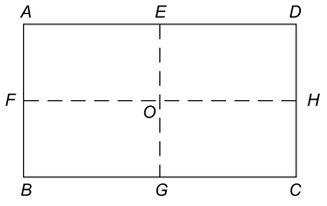


Fig. 8.52

65. A uniform rod of length L is suspended from one end such that it is free to rotate about an axis passing through that end and perpendicular to the length. What minimum speed must be imparted to the lower end so that the rod completes one full revolution?
(a) $\sqrt{2gL}$ (b) $2\sqrt{gL}$
(c) $\sqrt{6gL}$ (d) $2\sqrt{2gL}$

66. A circular disc of radius R is free to oscillate about an axis passing through a point on its rim and perpendicular to its plane. The disc is turned through an angle of 60° and released. Its angular velocity when it reaches the equilibrium position will be
(a) $\sqrt{\frac{g}{3R}}$ (b) $\sqrt{\frac{2g}{3R}}$
(c) $\sqrt{\frac{2g}{R}}$ (d) $2\sqrt{\frac{2g}{R}}$

67. A massless and inextensible cord is wound round the circumference of a circular ring of mass M and radius R. The ring is free to rotate about an axis passing through its centre and perpendicular to its

plane. A mass m is attached at the free end of the cord and is at rest. The angular speed of the ring when mass m has fallen through at height h is

(a) $\sqrt{\frac{2gh}{R^2}}$ (b) $\sqrt{\frac{2mgh}{MR^2}}$

(c) $\sqrt{\frac{2mgh}{(M+m)R^2}}$ (d) $\sqrt{\frac{2mgh}{(M+2m)R^2}}$

68. The moment of inertia of a hollow sphere of mass M and internal and external radii R and $2R$ about an axis passing through its centre and perpendicular to its plane is

(a) $\frac{3}{2}MR^2$ (b) $\frac{13}{32}MR^2$

(c) $\frac{31}{35}MR^2$ (d) $\frac{62}{35}MR^2$

69. A man, standing on a turn-table, is rotating at a certain angular frequency with his arms outstretched. He suddenly folds his arms. If his moment of inertia with folded arms is 75% of that with outstretched arms, his rotational kinetic energy will

(a) increase by 33.3%
(b) decrease by 33.3%
(c) increase by 25%
(d) decrease by 25%

IIT, 2004

70. A uniform disc of radius R is rolling (without slipping) on a horizontal surface with an angular speed ω as shown in Fig. 8.53. O is the centre of the disc, points A and C are located on its rim and point B is at a distance $\frac{R}{2}$ from O. During rolling, the points A, B and C lie on the vertical diameter at a certain instant of time. If v_A, v_B and v_C are the linear speeds of points A, B and C respectively at that instant, then

(a) $v_A = v_B = v_C$ (b) $v_A > v_B > v_C$

(c) $v_A = 0$, $v_C = \frac{4}{3}v_B$ (d) $v_A = 0$, $v_C = 2v_B$

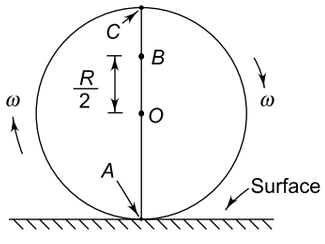


Fig. 8.53

IIT, 2004

71. A particle is moving in the $x - y$ plane with a constant velocity along a line parallel to the x-axis, away from the origin. The magnitude of its angular momentum about the origin.

(a) is zero
(b) remains constant
(c) goes on increasing
(d) goes on decreasing

IIT, 2005

72. A thin uniform disc has mass M and radius R. A circular hole of radius $R/3$ is made in the disc as shown in Fig. 8.54. The moment of inertia of the remaining portion of disc about an axis passing through O and perpendicular to the plane of the disc is

(a) $\frac{1}{9}MR^2$ (b) $\frac{2}{9}MR^2$

(c) $\frac{1}{3}MR^2$ (d) $\frac{4}{9}MR^2$

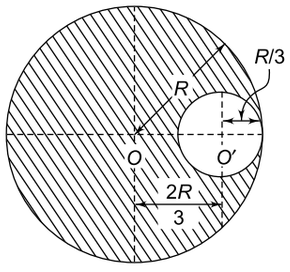


Fig. 8.54

IIT, 2005

73. A solid metallic sphere of radius R having moment of inertia equal to I about its diameter is melted and recast into a solid disc of radius r of a uniform thickness. The moment of inertia of the disc about an axis passing through its edge and perpendicular to its plane is also equal to I. The ratio r/R is

(a) $\frac{2}{\sqrt{15}}$ (b) $\frac{2}{\sqrt{10}}$

(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{2}}$

IIT, 2006

74. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum length $h = \frac{3v^2}{4g}$, with respect to the initial position. The object is (see Fig. 8.55)

(a) ring (b) solid sphere
(c) hollow sphere (d) disc

IIT, 2007

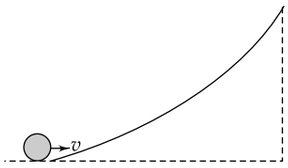


Fig. 8.55

75. The mass per unit length of a non-uniform rod OP of length L varies as

$$m = k\frac{x}{L}$$

where k is a constant and x is the distance of any point on the rod from end O. The distance of the centre of mass of the rod from end O is

(a) $\frac{L}{3}$ (b) $\frac{2L}{3}$

(c) $\frac{L}{2}$ (d) $\frac{2L}{\sqrt{3}}$

76. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is.

(a) $2\ M\omega^2 L$ (b) $M\omega^2 L$

(c) $\frac{1}{2}M\omega^2 L$ (d) $\frac{3}{2}\ M\omega^2 L$

77. A uniform thin rod of mass M and length L is hinged by a frictionless pivot at its end O, as shown Fig. 8.56. A bullet of mass m moving horizontally with a velocity v strikes the free end of the rod and gets embedded in it. The angular velocity of the system about O just after the collision is

(a) $\frac{mv}{L(M+m)}$ (b) $\frac{2mv}{L(M+2m)}$

(c) $\frac{3mv}{L(M+3m)}$ (d) $\frac{mv}{LM}$

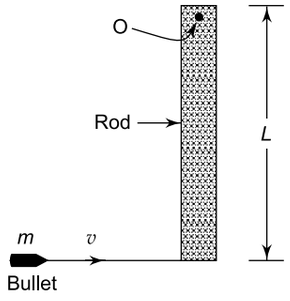


Fig. 8.56

78. A gramophone record of mass M and radius R is rotating at an angular velocity ω. A coin of mass m is gently placed on the record at a distance $r = R/2$ from its centre. The new angular velocity of the system is

(a) $\frac{2\omega M}{(2M+m)}$ (b) $\frac{2\omega M}{(M+2m)}$

(c) ω (d) $\frac{\omega M}{M}$

79. A bolck of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

(a) at $\theta = 30°$, the block will start sliding down the plane

(b) the bock will remain at rest on the plane up to certain θ and then it will topple

(c) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles

(d) at $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ.

IIT, 2009

ANSWERS

1. (d)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (b)
7. (d)	**8.** (c)	**9.** (a)	**10.** (c)	**11.** (c)	**12.** (a)
13. (a)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (d)	**18.** (c)
19. (a)	**20.** (a)	**21.** (d)	**22.** (a)	**23.** (d)	**24.** (a)
25. (b)	**26.** (c)	**27.** (a)	**28.** (d)	**29.** (b)	**30.** (b)
31. (a)	**32.** (c)	**33.** (c)	**34.** (a)	**35.** (d)	**36.** (a)
37. (b)	**38.** (c)	**39.** (c)	**40.** (d)	**41.** (d)	**42.** (b)
43. (b)	**44.** (a)	**45.** (b)	**46.** (d)	**47.** (c)	**48.** (a)
49. (b)	**50.** (b)	**51.** (a)	**52.** (c)	**53.** (b)	**54.** (a)
55. (a)	**56.** (c)	**57.** (b)	**58.** (d)	**59.** (c)	**60.** (d)

61. (c)	**62.** (c)	**63.** (b)	**64.** (c)	**65.** (c)	**66.** (b)
67. (c)	**68.** (d)	**69.** (a)	**70.** (c)	**71.** (b)	**72.** (d)
73. (a)	**74.** (d)	**75.** (b)	**76.** (c)	**77.** (c)	**78.** (a)
79. (b)					

SOLUTIONS

1. Since there is no external force acting on the gun-bullet system, the centre of mass of the system remains at rest.

2. Let σ = mass per unit area of the disc. Mass of the cut-out portion $m_1 = R^2\sigma/2$ and mass of the remaining portion is (see Fig. 8.57)

$$m_2 = \left(\pi R^2 - \frac{R^2}{2}\right)\sigma$$

Let O_1 be the centre of mass of the remaining portion. The centre of mass of the square is at O_2. Taking moments of m_1 g and m_2 g about O,

$$m_1\ g\ x_1 = m_2\ g\ x_2$$

$$\Rightarrow \quad \frac{R^2\sigma}{2} \times x_1 \times g = \left(\pi R^2 - \frac{R^2}{2}\right)\sigma \times x_2 \times g$$

$$\Rightarrow \quad x_2 = \frac{R}{2(2\pi - 1)} \qquad (\because\ x_1 = R/2)$$

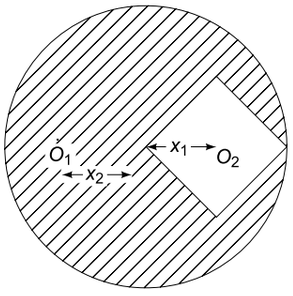


Fig. 8.57

3. The entire mass M of the carpet can be assumed to be concentrated at its centre of mass which is originally at a height R above the floor. So its original P.E. = MgR. When the carpet is unrolled to a radius $R/2$, its centre of mass will be at a height $R/2$ above the floor, but the mass left over unrolled is

$$m = \frac{M(R/2)^2}{R^2} = \frac{M}{4}$$

$$\text{and its P.E.} = mg\ (R/2) = \frac{M}{4} \times g \times \frac{R}{2} = \frac{MgR}{8}$$

$$\therefore \quad \text{Decrease in P.E.} = MgR - \frac{MgR}{8} = \frac{7}{8}\,MgR$$

4. Since the mass is not distributed uniformly over the circumference of the ring, its centre of mass may lie anywhere between its centre and the circumference. Hence the correct choice is (d).

5. $v_{CM} = \dfrac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$

$$= \frac{1}{2}\ (\mathbf{v}_1 + \mathbf{v}_2) \qquad (\because\ m_1 = m_2)$$

$$= \frac{a}{2}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$$

$$\boldsymbol{a}_{CM} = \frac{m_1\boldsymbol{a}_1 + m_2\,\boldsymbol{a}_2}{m_1 + m_2} = \frac{\boldsymbol{a}_1}{2} \qquad (\because\ \boldsymbol{a}_2 = 0)$$

$$= \frac{b}{2}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$$

Since vectors $\mathbf{v}_{CM}$ and $\boldsymbol{a}_{CM}$ are parallel to each other, the centre of mass will move along a straight line.

6. In rolling without slipping, the mechanical energy is conserved. Since both the inclined planes are of the same height, $v_2 = v_1$. The acceleration of the sphere rolling down the plane is

$$a = \frac{g\sin\theta}{1 + \dfrac{I}{MR^2}}$$

Since $\theta_2 > \theta_1$; $\quad a_2 > a_1$. Hence

$$\frac{t_1}{t_2} = \frac{\sin\theta_2}{\sin\theta_1}$$

Thus $t_1 > t_2$. Hence the correct choice is (b).

7. Mass of element of length dx is $dm = \dfrac{M}{L}\,dx$.

Perpendicular distance of the element from the axis of rotation = $x \sin \theta$. Therefore moment of inertia about the axis of rotation AB is (Fig. 8.58)

$$I = \int_{-L/2}^{+L/2} dm(x\sin\theta)^2$$

$$= \frac{M}{L}\ \sin^2\theta \int_{-L/2}^{L/2} x^2 dx$$

$$= \frac{ML^2}{12} \sin^2\theta = \frac{ML^2}{12} \sin^2(60°) = \frac{ML^2}{16}$$

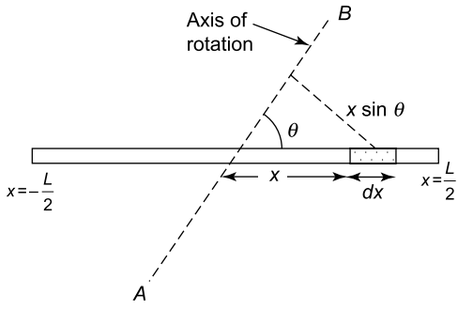


Fig. 8.58

8. K.E. $= \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$

$$= \frac{1}{2} Mv^2 + \frac{1}{2} \times \left(\frac{1}{2} MR^2\right) \times \left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2} Mv^2 + \frac{1}{4} Mv^2 = \frac{3}{4} Mv^2$$

From the principle of conservation of energy, Loss in K.E. = gain in P.E.

or $\frac{3}{4} Mv^2 = Mgh$

$\Rightarrow$ $h = \frac{3v^2}{4g}$, which is choice (c).

9. When string D is cut, the rod will rotate about point A. Let a be the linear acceleration of the centre of mass and T the tension in string C, then (see Fig. 8.59)

$$mg - T = ma \qquad (1)$$

Torque of force mg about A is $\tau = Mg \times \frac{L}{2}$.

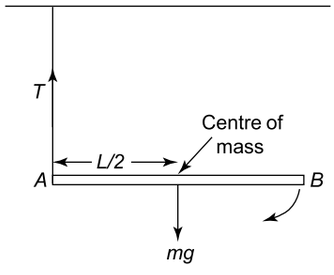


Fig. 8.59

Moment of inertia of the rod about A is $I = ML^2/3$. Therefore, the angular acceleration of the rod about A is

$$\alpha = \frac{\tau}{I} = \frac{3g}{2L} \qquad (2)$$

Also $a = \frac{L}{2}\alpha$ (3)

Using this in (2), we get $a = \frac{3g}{4}$. Then from Eq. (1)

$$mg - T = \frac{3mg}{4} \Rightarrow T = \frac{mg}{4}$$

So the correct choice is (a).

10. At the highest point, the velocity of the body is $v = u \cos 60° = u/2$ along the horizontal direction. Hence the perpendicular distance of its linear momentum from the point of projection = maximum height attained.

$$r_1 = h_{max} = \frac{u^2 \sin^2 60°}{2g} = \frac{3u^2}{8g}$$

$\therefore$ Magnitude of angular momentum $= m\, r_1\, v$

$$= m \times \frac{3u^2}{8g} \times \frac{u}{2}$$

$$= \frac{3mu^3}{16g}$$

11. Torque of F about A is [see Fig. 8.60]

$$\tau_1 = F \times \frac{2L}{3}$$

Since the weight mg acts at the centre of mass, the torque of the weight about A is

$$\tau_2 = mg \times \frac{L}{2}$$

The minimum force required to topple the block is given by

$$(\tau_1)_{min} = \tau_2 \text{ or } F_{min} \times \frac{2L}{3} = mg \times \frac{L}{2}$$

$\Rightarrow$ $F_{min} = \frac{3mg}{4}$

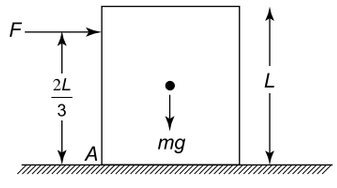


Fig. 8.60

12. Let M be the mass and R the initial radius of the earth. If ω is the angular velocity of the rotation of the earth, the duration T of the day is

$$T = \frac{2\pi}{\omega}$$

Let R' be the radius of the earth after contraction and ω' its angular velocity. From the conservation of angular momentum, we have

$$I\omega = I'\omega'$$

where $I\left(=\frac{2}{5}MR^2\right)$ and $I'\left(=\frac{2}{5}MR'^2\right)$ are the moments of inertia of the earth before and after contraction, respectively.

$$\therefore \quad \frac{2}{5}MR^2\omega = \frac{2}{5}MR'^2\omega' \text{ or } \omega' = \frac{R^2\omega}{R'^2} = 4\omega$$

$$(\because R' = R/2)$$

The duration T' of the new day will be

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{4\omega} = \frac{T}{4},$$

$$T' = \frac{24 \text{ hours}}{4} = 6 \text{ hours}$$

13. From the law of conservation of angular momentum, we have

$$I\omega = I'\omega'$$

Here $I = MR^2$ and $I' = (M + 2m)\,R^2$. Therefore

$$\frac{\omega'}{\omega} = \frac{I}{I'} = \frac{M}{(M+2m)}$$

Hence the correct choice is (a).

14. For rolling : $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}Mv^2 + \frac{1}{2} \times \left(\frac{2}{5}MR^2\right) \times \frac{v^2}{R^2}$$

$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$$

$$\left(\because I = \frac{2}{5}MR^2 \text{ and } \omega = \frac{v}{R}\right)$$

For sliding : $Mgh = \frac{1}{2}Mv'^2$. Therefore

$$\frac{1}{2}Mv'^2 = \frac{7}{10}Mv^2$$

or $\quad \frac{v'}{v} = \sqrt{\frac{7}{5}}$, which is choice (c).

15. The acceleration down the plane is given by

$$a = \frac{g\sin\theta}{\left(1 + \frac{I}{MR^2}\right)}$$

For disc, $I = \frac{1}{2}MR^2$. Using this and $\theta = 30°$, we get $a = g/3$, which is choice (c).

16. The acceleration of the block sliding down the plane is

$$a = g \sin\theta$$

where θ is the angle of inclination. If l is the length of the inclined plane, the velocity of the block on reaching the bottom is given by

$$v^2 = 2\,al = 2g \sin\theta \times l$$

or $\quad v = \sqrt{2gl\sin\theta}$

The acceleration of the disc rolling down the plane is (as shown above)

$$a' = \frac{2}{3}\,g\sin\theta$$

Therefore, the velocity of the disc on reaching the bottom is given by

$$v'^2 = 2a'l = \frac{4}{3}\,gl\sin\theta$$

or $\quad v' = 2\sqrt{\frac{gl\sin\theta}{3}}$, $\therefore \quad \frac{v'}{v} = \sqrt{\frac{2}{3}}$

Hence the correct choice is (b).

17. Let μ be the mass per unit length of the wire. The mass of loop A is $M_A = 2\pi R\mu$ and mass of loop B is $M_B = 4\pi R\mu$. Their moments of inertia respectively are $I_A = M_A R_A^2 = 2\,\pi R\mu \times R^2 = 2\pi\,\mu\,R^3$

and $\quad I_B = M_B R_B^2 = 4\,\pi R\mu \times (2R)^2$

$$= 16\,\pi\mu R^3$$

$$\therefore \quad \frac{I_A}{I_B} = \frac{1}{8}$$

Hence the correct choice is (d).

18. Let m be the mass of the coin. It will fly off when the centripetal force $mr\omega^2$ just exceeds the force of friction μmg. The minimum ω is given by

$$mr\omega^2 = \mu mg \text{ or } \omega = \sqrt{\frac{\mu g}{r}}$$

Hence the correct choice is (c).

19. The minimum angular frequency is independent of the mass. Hence the correct answer is still $\sqrt{\mu g / r}$ which is choice (a).

20. The (x, y) co-ordinates of the masses at O, A and B respectively are (refer to Fig. 8.40 on page 8.23) $(x_1 = 0, y_1 = 0)$, $(x_2 = a, y_2 = 0)$ and $(x_3 = 0, y_3 = b)$ The (x, y) co-ordinates of the centre of mass are

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + m \times a + m \times 0}{m + m + m} = \frac{a}{3}$$

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + m \times 0 + m \times b}{m + m + m} = \frac{b}{3}$$

The position vector of the centre of mass is $x_{CM}\,\mathbf{i} + y_{CM}\,\mathbf{j}$

$= \frac{a}{3}\,\mathbf{i} + \frac{b}{3}\,\mathbf{j} = \frac{1}{3}(a\mathbf{i} + b\mathbf{j})$, which is choice (a).

21. When the sphere rolls down the plane, its acceleration is given by (see also solution to Q. 16).

$$a = \frac{g \sin\theta}{1 + \frac{I}{MR^2}}$$

where K is the radius of gyration of the sphere about its diameter. Now, the moment of inertia of the sphere about its diameter is

$$I = \frac{2}{5}\,MR^2,$$

$$\text{Therefore,} \quad a = \frac{g \sin\theta}{1 + \frac{2}{5}} = \frac{5}{7}\,g \sin\theta \qquad \text{(i)}$$

For rolling without sliding, the frictional force f provides the necessary torque τ which is given by

$$\tau = \text{force} \times \text{moment arm} = fR$$

But $\tau = I\alpha$, where α is the angular acceleration of the sphere. Thus, $I\alpha = fR$. Also, linear acceleration $a = \alpha R$. Therefore,

$$f = \frac{I\alpha}{R} = \frac{Ia}{R^2} = \frac{2}{5}\,Ma \qquad \left(\because I = \frac{2}{5}MR^2\right)$$

Now, force of friction = $\mu \times$ normal reaction $= \mu Mg \cos\theta$. Thus $\mu Mg \cos\theta = \frac{2}{5}\,Ma$

$$\text{or} \qquad a = \frac{5}{2}\,\mu g \cos\theta \qquad \text{(ii)}$$

Equating (i) and (ii) we have

$$\frac{5}{7}\,g \sin\theta = \frac{5}{2}\,\mu g \cos\theta \text{ or } \mu = \frac{2}{7}\tan\theta$$

Hence the correct choice is (d).

22. Given $PQ = QR = RP = L$. The centre of mass is located at centroid C which cuts lines PS, QT and UR in the ratio 2 : 1. Let $h = CS = CT = UC$. In ΔPQS, we have (see Fig. 8.61)

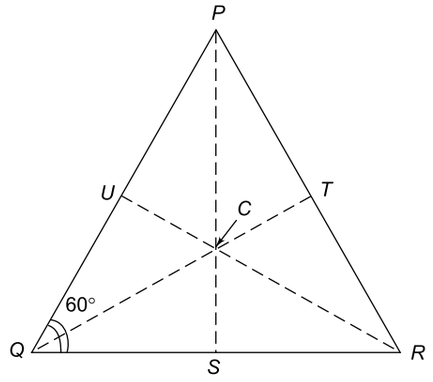


Fig. 8.61

$$PS = PQ \sin 60° = L \sin 60° = \frac{\sqrt{3}}{2}\,L.$$

$$\therefore \quad h = \frac{PS}{3} = \frac{1}{3} \times \frac{\sqrt{3}}{2}\,L = \frac{L}{2\sqrt{3}}$$

Since the structure consists of three identical rods, its moment of inertia about an axis passing through its centre of mass C and perpendicular to its plane is, from parallel axes theorem,

$$I_c = 3\,(I + Mh^2)$$

where I is the moment of inertia of each rod about the axis passing through its centre and perpendicular to its length, which is given by

$$I = \frac{ML^2}{12} \quad \text{Also} \quad Mh^2 = M\left(\frac{L}{2\sqrt{3}}\right)^2 = \frac{ML^2}{12}$$

$$\therefore \quad I_c = 3\left(\frac{ML^2}{12} + \frac{ML^2}{12}\right) = 3 \times \frac{ML^2}{6} = \frac{ML^2}{2}$$

Hence the correct choice is (a).

23. Refer to Fig. 8.41 on page 8.24. Moment of inertia is a scalar quantity. So the moment of inertia of the structure is the sum of the moments of inertia of the four rods about the specified axis of rotation, i.e.,

$$I = I_1 + I_2 + I_3 + I_4$$

where I_1= moment of inertia of rod 1 about an axis passing through its centre E and perpendicular to its plane $= \frac{ML^2}{12}$,

I_2= moment of inertia of rod 2 about an axis passing through its centre F and perpendicular to its plane $= \frac{ML^2}{12}$,

I_3 = moment of inertia of rod 3 about a parallel axis at a distance $\frac{L}{2}$ from it $= M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{4}$, and

I_4 = moment of inertia of rod 4 about a parallel axis at a distance $\frac{L}{2}$ from it = $\frac{ML^2}{4}$.

$$\therefore \quad I = \frac{ML^2}{12} + \frac{ML^2}{12} + \frac{ML^2}{4} + \frac{ML^2}{4}$$

$$= \frac{2}{3} ML^2,$$ which is choice (d).

24. Refer to Fig. 8.62. It is clear from the figure that the moment of inertia of triangular sheet $ABC = \frac{1}{2}$ × moment of inertia of a square sheet $ABCD$ about its diagonal AC or $I_t = \frac{1}{2} I_s$. Now, mass of square sheet = $M + M = 2M$. Therefore,

$$I_s = (2M) \frac{L^2}{12} = \frac{ML^2}{6}$$

$$\therefore \quad I_t = \frac{I_s}{2} = \frac{ML^2}{12}$$

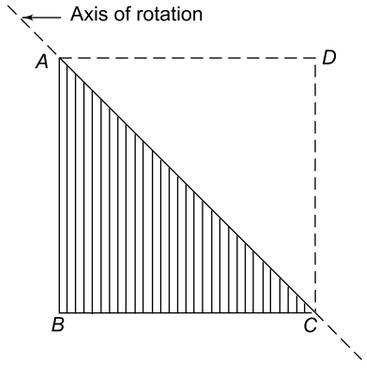


Fig. 8.62

Hence the correct choice is (a).

25. The correct choice is (b) because the angular momentum of the sphere about the point of contact with plane surface also includes the angular momentum about the centre of mass.

26. Because of symmetry about axes 1 and 2, $I_1 = I_2$. Similarly, $I_3 = I_4$. From perpendicular axes theorem, it follows that the moment of inertia of the plate about an axis passing through the centre and perpendicular to the plane of the plate is

$$I = I_1 + I_2 = I_3 + I_4 = 2I_1 = 2I_3 \qquad (\because I_1 = I_2,\ I_3 = I_4)$$

or $I_1 = I_3$.

Thus $I = I_1 + I_2 = I_3 + I_4 = I_1 + I_3$.

Hence the correct choice is (c).

27. Let the mass of the sphere be M and R its radius. When it is at rest at the top of the inclined plane, its energy is entirely potential given by PE = Mgh

If the sphere rolls down the plane, it acquires a linear velocity v and an angular velocity ω. Its total kinetic energy at any time t = translational KE + rotational KE

or $\text{KE} = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

Here $I = \frac{2}{5} MR^2$ and $\omega = \frac{v}{R}$. Therefore,

$$\text{KE} = \frac{1}{2} Mv^2 + \frac{1}{2} \times \frac{2}{5} MR^2 \times \frac{v^2}{R^2}$$

$$= \frac{1}{2} Mv^2 + \frac{1}{5} Mv^2 = \frac{7}{10} Mv^2$$

From the law of conservation of energy, PE = KE

or $Mgh = \frac{7}{10} Mv^2$ which gives $v = \sqrt{\frac{10gh}{7}}$

Hence the correct choice is (a).

28. For a rolling sphere, the rotation kinetic energy and the total (translational + rotational) kinetic energy respectively are

$$K_r = \frac{1}{2} I\omega^2 \text{ and } K_t = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

The moment of inertia of a sphere of mass m and radius r about its centre is $I = \frac{2}{5} mr^2$. Also $\omega = v/r$. Therefore $I\omega^2 = \frac{2}{5} mr^2 \times \frac{v^2}{r^2} = \frac{2}{5} mv^2$. Thus $K_r = \frac{1}{5} mv^2$ and $K_t = \frac{1}{2} mv^2 + \frac{1}{5} mv^2 = \frac{7}{10} mv^2$. Hence

$$\frac{K_r}{K_t} = \frac{\frac{1}{5} mv^2}{\frac{7}{10} mv^2} = \frac{2}{7}$$

Hence the correct choice is (d).

29. Since there is no external force, the cart and the man will meet at their centre of mass. Since their masses are equal, the centre of mass is located at the mid-point between them, i.e. at x = 5 m. Hence the correct choice is (b).

30. Refer to Fig. 8.63. The angular momentum of the mass at point $P(x, y)$ about origin O is defined as

$$\mathbf{L} = m\mathbf{r} \times \mathbf{v} = m(x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) \times (v\hat{\mathbf{i}})$$

$$= myv\ (-\hat{\mathbf{k}})$$

$$(\because \hat{\mathbf{i}} \times \hat{\mathbf{i}} = 0 \text{ and } \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}})$$

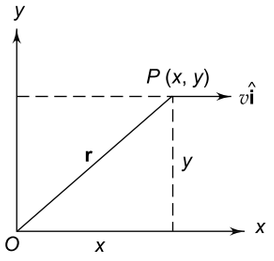


Fig. 8.63

Now m and v are constants. Also y remains constant as the mass moves parallel to the x-axis. Hence L remains constant. Thus the correct choice is (b).

31. Refer to Fig. 8.64. According to the perpendicular axes theorem, the moment of inertia of the plane about the z-axis is

$$I_z = I_x + I_y$$

with $I_x = I_y$. The square plate lies in the x-y plane. Since the directions of the x and y axes is arbitrary, the only restriction being that the angle between them is 90°, it follows that the moment of inertia will not change if the axes are rotated through any angle θ in the plane of the plate. This can be proved as follows.

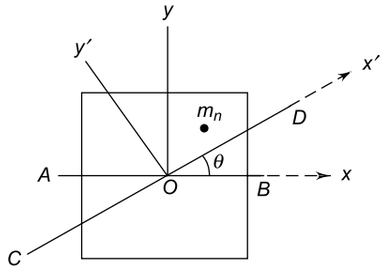


Fig. 8.64

Consider a particle of the plate of mass m_n located at a point $P(x_n, y_n)$ in the x-y plane. Then $I_{AB} = \Sigma\, m_n y_n^2$
The moment of inertia about line CD will be $I_{CD} = \Sigma\, m_n y'^2{}_n$
The coordinates (x_n, y_n) and (x_n', y_n') are related as

$$x_n' = x_n \cos\theta - y_n \sin\theta$$

$$y_n' = y_n \sin\theta - y_n \cos\theta$$

$$\text{Now } I_{CD} = \Sigma m_n y_n'^2 = \Sigma m_n (x_n \sin\theta - y_n \cos\theta)^2$$

$$= \Sigma(m_n x_n^2) \sin^2\theta + \Sigma(m_n y_n^2) \cos^2\theta - 2\,\Sigma(m_n x_n y_n) \sin\theta \cos\theta$$

From symmetry, it follows that $I_{AB} = \Sigma m_n x_n^2 = \Sigma m_n y_n^2$ and $\Sigma m_n x_n y_n = 0$. Hence

$$I_{CD} = I_{AB}(\sin^2\theta + \cos^2\theta) + 0 = I_{AB} = I$$

Thus the correct choice is (a).

32. Since there is no friction between the sphere and the horizontal surface and also between the spheres themselves, there will be no transfer of angular momentum from sphere A to sphere B due to the collision. Since the collision is elastic and the spheres have the same mass, the sphere A only transfers its linear velocity v to sphere B. Sphere A will continue to rotate with the same angular speed ω at a fixed location. Hence the correct choice is (c).

33. Refer to Fig. 8.65. Let $OC = R_C$ and let $\mathbf{v}_c$ be the velocity of the centre of mass of the disc. The linear momentum of the centre of mass is $\mathbf{p}_c = M\mathbf{v}_c$
If $\mathbf{L}_c$ is the angular momentum of the disc about C, then the angular momentum about origin O is

$$\mathbf{L}_0 = \mathbf{L}_c + \mathbf{R}_c \times \mathbf{p}_c$$

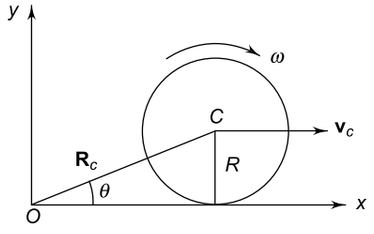


Fig. 8.65

$$\therefore \text{ Magnitude of } L_0 = I_c\omega + R_c \times Mv_c \sin\theta$$

$$= \frac{1}{2} MR^2\omega + M R_c v_c \sin\theta \qquad \left(\because I_c = \frac{1}{2} M R^2\right)$$

$$= \frac{1}{2} MR^2\omega + MR \times R\omega$$

$$(\because R_c \sin\theta = R \text{ and } v_c = R\omega)$$

$$= \frac{3}{2} MR^2\omega$$

Hence the correct choice is (c).

34. When the block hits the ridge at point O, it will start rotating about an axis passing through O and perpendicular to the plane of the paper (Fig. 8.66).

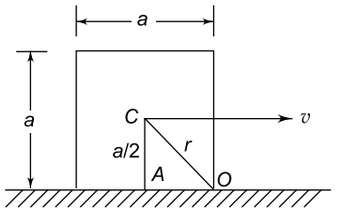


Fig. 8.66

Since no external torque acts on the block, its an-gular momentum is conserved. Angular momentum of the block before it hits the ridge is

$$L_i = Mv \times AC = Mv \times \frac{a}{2} = \frac{1}{2}\ Mva \quad (1)$$

Angular momentum of the block after it hits the ridge is

$$L_f = I_O\ \omega \quad (2)$$

where I_O is the moment of inertia of the block about an axis passing through O and perpendicular to the plane of the block and ω is the angular speed of rotation of the block. From the parallel axes theorem, we have

$$I_O = I_c + Mr^2$$

Now, the moment of inertia about C is $I_c = \frac{1}{6}\ Ma^2$ and $r^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = \frac{a^2}{2}$. Hence

$$I_O = \frac{1}{6}\ Ma^2 + \frac{1}{2}\ Ma^2 = \frac{2}{3}\ Ma^2 \quad (3)$$

Using Eqs. (3) in (2), we have $L_f = \frac{2}{3}\ Ma^2\omega$

Since the angular momentum is conserved, $L_i = L_f$

From Eqs (1) and (4), we get (4)

$$\frac{2}{3}\ Mva = \frac{2}{3}\ Ma^2\omega \quad \text{or} \quad \omega = \frac{3v}{4a}$$

Hence the correct choice is (a).

35. Let m be the mass of the loop and r its radius. The moment of inertia of the loop about an axis passing through the centre O is (Fig. 8.67)

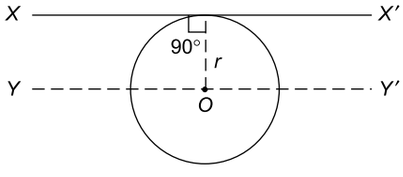


Fig. 8.67

$$I_O = \frac{1}{2}\ mr^2$$

From the parallel axes theorem, the moment of inertia bout XX' is

$$I = I_O + mr^2 = \frac{1}{2}\ mr^2 + mr^2 = \frac{3}{2}\ mr^2$$

The mass of the loop, $m = \rho L$ and radius $r = L/2\pi$. Hence

$$I = \frac{3}{2} \times \rho L \times \left(\frac{L}{2\pi}\right)^2 = \frac{3\rho L^3}{8\pi^2}$$

Thus the correct choice is (d).

36. The entire mass of the liquid can be regarded as being concentrated at the centre of mass of the tube which is at a distance of $r = \frac{L}{2}$ from the axis of revolution. The force exerted by the liquid at the other end of the tube is the centripetal force of a mass M revolving in a circle of radius $r = \frac{L}{2}$. Thus

$$F_c = \frac{Mv^2}{r} = \frac{M(r\omega)^2}{r} = Mr\omega^2 = \frac{ML\omega^2}{2}$$

Hence the correct choice is (a).

37. As no external torque acts on the system, the angular momentum L is conserved. As the beads slide down, the moment of inertia of the system will change. From $L = I\omega$, angular velocity ω will also change. Since the total energy cannot change, the correct choice is (b).

38. Torque due to F about A is $\tau_1 = FL$

Since the weight mg acts through the centre of mass of the block (which is at a distance of $L/2$ from the slide of the block) the torque due to weight mg about A is

$$\tau_2 = mg\ \left(\frac{L}{2}\right)$$

The minimum force required to topple the block is obtained when τ_1 is slightly greater than τ_2, i.e. in limit

$$(\tau_1)_{\min} = \tau_2 \ \text{ or } \ F_{\min}\ L = mg\ \left(\frac{L}{2}\right)$$

or $$F_{\min} = \frac{mg}{2}$$

Hence the correct choice is (c).

39. The magnitude of angular momentum of a rotating body is given by $L = I\omega$. If no torque acts, the angular momentum is conserved, i.e. $I\omega$ = constant. Hence $I_1\omega_1 = I_2\omega_2$. If K_1 and K_2 are the corresponding radii of gyration, then $I_1 = MK_1^2$ and $I_2 = MK_2^2$. Hence

$$MK_1^2\ \omega_1 = MK_2^2\ \omega_2$$

or $$\frac{K_1}{K_2} = \frac{\sqrt{\omega_2}}{\sqrt{\omega_1}}$$, which is choice (c).

40. Since the centre of gravity of the rod is at its centre, the loss in PE when its end B strikes the floor = $mg\ \left(\frac{L}{2}\right) = \frac{1}{2}\ mg\ L$. Gain in KE = $\frac{1}{2}\ I\omega^2$ where I is the moment of inertia of the rod about an axis passing through its end and perpendicular to its length which is given by $I = \frac{1}{3}\ mL^2$. Now gain in

KE = loss in PE, i.e.

$$\frac{1}{2}\times\frac{1}{3}\ mL^2\times\omega^2=\frac{1}{2}\ mgL$$

which gives $\omega=\sqrt{\frac{3g}{L}}$, which is choice (d).

41. Given: $l = 6R$. From parallel axes theorem, the moment of inertia about the given axis is given by

$$I = M\left(\frac{R^2}{4}+\frac{l^2}{3}\right)$$

$$= M\left[\frac{R^2}{4}+\frac{(6R)^2}{3}\right]$$

$$= M\left(\frac{R^2}{4}+\frac{36R^2}{3}\right)=\frac{49MR^2}{4}$$

Hence the correct choice is (d).

42. Areal velocity $A=\dfrac{\text{area swept by radius vector}}{\text{time taken}}$

Assuming that the orbit of the planet is a circle of radius R, then

$$A=\frac{\pi R^2}{T}$$

Now, time period $T=\dfrac{2\pi}{\omega}$. Hence

$$A=\frac{\pi R^2}{2\pi/\omega}=\frac{R^2\omega}{2}$$

or $$\omega=\frac{2A}{R^2}$$

Angular momentum $L = I\omega = (MR^2)\times\dfrac{2A}{R^2}$

$$= 2\ MA$$

Hence the correct choice is (b).

43. The mass of the rod can be considered to be concentrated at its centre ($x = L/2$) where $x = 0$ is the origin. Hence

$$R_{CM}=\frac{M_1\times L/2+0}{M_1+M_2}=\frac{LM_1}{2(M_1+M_2)}$$

Hence the correct choice is (b).

44. $I_A = m_A r_A^2$ and $I_B = m_B r_B^2$. Hence

$$\frac{I_B}{I_A}=\left(\frac{m_B}{m_A}\right)\times\left(\frac{r_B}{r_A}\right)^2 \qquad \text{(i)}$$

Let k be the mass per unit length of the wire. Then the masses of loops A and B are

$$m_A = (2\pi r_A)k \text{ and } m_B = (2\pi r_B)k$$

$$\therefore\quad \frac{m_B}{m_A}=\frac{r_B}{r_A} \qquad \text{(ii)}$$

Using (ii) in (i) and putting $\dfrac{r_B}{r_A}=8$ (given), we have

$$8=\left(\frac{r_B}{r_A}\right)^3 \text{ or } \frac{r_B}{r_A}=2, \text{ which is choice (a).}$$

45. Let L cm be the original length of the spring and k be the spring constant. Then

$$m(L + x_1)\ \omega_1^2 = kx_1$$

and $$m(L + x_2)\ \omega_2^2 = kx_2$$

Dividing, we get

$$\left(\frac{L+x_1}{L+x_2}\right)\times\left(\frac{\omega_1}{\omega_2}\right)^2=\frac{x_1}{x_2} \qquad \text{(i)}$$

Given $x_1 = 1$ cm, $x_2 = 5$ cm and $\omega_2 = 2\omega_1$. Using these in (i) solving, we get $L = 15$ cm, which is choice (b).

46. $L = I\omega = \dfrac{\frac{1}{2}I\omega^2}{2\omega}=\dfrac{K}{2\omega}$, where $K=\frac{1}{2}\ I\omega^2$ is the kinetic energy. If ω is doubled and K is halved, the value of L becomes one-fourth. Hence the correct choice is (d).

47. Rod POQ of length $l = 100$ cm is bent at its mid-point O so that $\angle POQ = 90°$ (see Fig. 8.68).

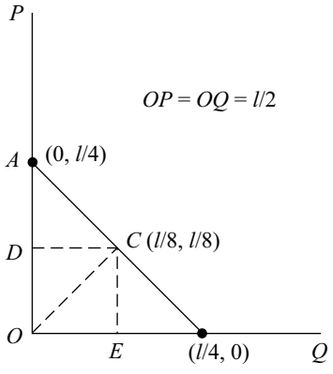


Fig. 8.68

The mass of part PO of length $l/2$ can be taken to be concentrated at its mid-point A whose coordinates are $(0, l/4)$ and of part OQ of length $l/2$ at its mid-point B whose coordinates are $(l/4, 0)$. The centre of mass of these two equal masses is at mid-point C between A and B. The coordinates of C are $(l/8, l/8)$.

$$\therefore\quad OC=\sqrt{(OE)^2+(CE)^2}=\sqrt{\left(\frac{l}{8}\right)^2+\left(\frac{l}{8}\right)^2}$$

$= \dfrac{l}{\sqrt{32}} = \dfrac{100\text{ cm}}{\sqrt{32}} = 17.7$ cm, which is choice (c).

48. $L = mr^2\,\omega$. For given m and ω, $L \propto r^2$. If r is halved, the angular momentum L becomes one-fourth. Hence the correct choice is (a).

49. Since no torque acts on the sphere, its angular momentum $L = (I\omega)$ remains unchanged, where I is the moment of inertia and ω is the angular velocity of the sphere. If the radius of the sphere is changed, I and hence ω will both change. Also, rotational kinetic energy $\left(=\dfrac{1}{2}I\omega^2\right)$ will also change. Hence the correct choice is (b).

50. $V = \dfrac{4}{3}\,\pi r^3$ or $\log V = \log\left(\dfrac{4\pi}{3}\right) + 3\log r$.

Differentiating, We have

$$\frac{\delta V}{V} = 3\frac{\delta r}{r} \text{ or } \frac{\delta r}{r} = \frac{1}{3}\frac{\delta V}{V} = \frac{1}{3}\times 0.5\% = \frac{1}{6}\%$$

Since no external torque acts, $I\omega$ = constant or $\dfrac{2}{5}$ $mr^2\omega$ = constant or $r^2\omega$ = constant (c)

or $2\log r + \log\omega = \log c$. Differentiating, we have

$$\frac{2\delta r}{r} + \frac{\delta\omega}{\omega} = 0$$

or $$\frac{\delta\omega}{\omega} = -2\frac{\delta r}{r} = -2\times\frac{1}{6}\% = -\frac{1}{3}\%$$

The negative sign indicates that ω decreases. Hence the correct choice is (b).

51. The cylinder will topple when the torque mgr equals the torque $ma\,\dfrac{h}{2}$ (see Fig. 8.69)

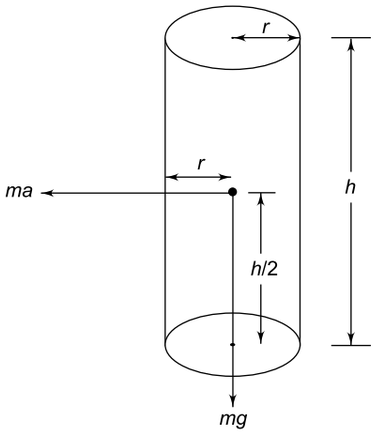


Fig. 8.69

or $a = \dfrac{2gr}{h} = \dfrac{g}{2}$ $\quad (\because h = 4r)$ $\quad$ (i)

Now $v = 2.45\,t^2$

$$\therefore\ a = \frac{dv}{dt} = \frac{d}{dt}\left(2.45t^2\right) = 4.9t \qquad \text{(ii)}$$

Equating (i) and (ii), we get $t = \dfrac{g}{2\times 4.9} = \dfrac{9.8}{9.8} = 1$ s.

Hence the correct choice is (a).

52. Moment of inertia of complete disc about O is $I = \dfrac{1}{2}\,MR^2$. Mass of the cut-out part is $m = \left(\dfrac{M}{4}\right)$. The moment of inertia of the cut-out portion about its own centre $I_0 = \dfrac{1}{2}\,mr^2 = \dfrac{1}{2}\left(\dfrac{M}{4}\right)\left(\dfrac{R}{2}\right)^2 = \dfrac{1}{32}\,MR^2$ because $r = R/2$. From the parallel axes theorem, the moment of inertia of the cut out portion about O is

$$I_c = I_0 + mr^2 = \frac{1}{32}\,MR^2 + \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$$

$$= \frac{3}{32}\,MR^2$$

$\therefore$ Moment of inertia of the shaded portion about O is $I_s = I - I_c = \dfrac{1}{2}\,MR^2 - \dfrac{3}{32}\,MR^2 = \dfrac{13}{32}\,MR^2$, which is choice (c).

53. From the principle of conservation of angular momentum, $I_0\omega_0 = I\omega$, where I_0 and ω_0 are the moment of inertia and angular velocity when the beads are at the centre of the rod and I and ω those when the beads are at the ends of the rod.

$$I_0 = \frac{ML^2}{12}$$

and $$I = \frac{ML^2}{12} + \frac{mL^2}{4} + \frac{mL^2}{4} = \frac{L^2}{12}\,(M + 6m)$$

$$\therefore\quad \frac{ML^2}{12}\,\omega_0 = \frac{(M+6m)\,\omega L^2}{12}$$

or $$\omega = \frac{M\omega_0}{(M+6m)}$$

Hence the correct choice is (b).

54. Since no external force acts on the system, the centre of mass will remain at rest. Hence the correct choice is (a).

55. Area of complete disc $= \pi R^2$. Area of one quarter sector $OAB = \dfrac{1}{4}\,\pi R^2$. Mass of this sector $= M$. Mass per unit area of the sector is

$$m_o = \frac{M}{\pi R^2/4} = \frac{4M}{\pi R^2} \quad (1)$$

Divide the sector into a large number of cylindrical shells. Consider an element of mass dm at a distance r from the centre O and having thickness dr (see Fig. 8.70). Then

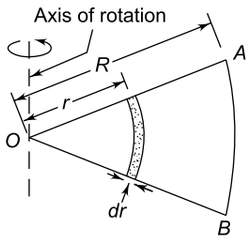


Fig. 8.70

$$dm = \left(\frac{2\pi r dr}{4}\right) m_o$$

The moment of inertia of the sector about the axis of rotation is

$$I = \int_0^R r^2 dm = \frac{2\pi m_0}{4}\int_0^R r^3 dr = \frac{\pi m_0 R^4}{8} \quad (2)$$

Using (1) in (2), we get $I = \frac{1}{2} MR^2$, which is choice (a).

56. The velocity v_{CM} of the centre of mass can be obtained by using the principle of conservation of linear momentum,

$$MV = (M + m)\, v_{CM}$$

$$\text{or } v_{CM} = \frac{MV}{(M+m)} = \frac{10\text{ kg} \times 14\text{ ms}^{-1}}{(10+4)\text{ kg}} = 10\text{ ms}^{-1}$$

Hence the correct choice is (c).

57. When a cylinder rolls up or down an inclined plane, its angular acceleration is always directed down the plane. Hence the frictional force acts up the inclined plate when the cylinder rolls up or down the plane. Thus, the correct choice is (b).

58. The correct choice is (d).

59. The angular velocity vector, being normal to the orbit, is constantly changing its direction. However, the rate of change of this vector is constant, therefore, the angular acceleration remains constant. Hence the correct choice is (c).

60. The kinetic energy (which is rotational) is $\frac{1}{2} I\omega^2$.

Now, The moment of inertia $I = \frac{1}{2} mr^2$ and $\omega = \frac{v}{r}$. Therefore, KE $= \frac{1}{2} \times \frac{1}{2}\, mr^2 \times \left(\frac{v}{r}\right)^2$ $= \frac{1}{4}\, mv^2$, which is choice (d).

61. The kinetic energy of a rolling disc consists of two parts: translational energy $= \frac{1}{2}\, mv^2$ and rotational energy $= \frac{1}{2}\, I\omega^2$.

$$\therefore \text{ KE} = \frac{1}{2}\, mv^2 + \frac{1}{2}\, I\omega^2$$

$$= \frac{1}{2}\, mv^2 + \frac{1}{2} \times \left(\frac{1}{2} mr^2\right) \times \left(\frac{v}{r}\right)^2 \quad \left(\because I = \frac{1}{2}\, mr^2\right)$$

$$= \frac{1}{2}\, mv^2 + \frac{1}{4}\, mv^2 = \frac{3}{4}\, mv^2$$

Hence the correct choice is (c).

62. Mass of sphere A, $M_A = \frac{4}{3}\, \pi R^3 \rho_A$, mass of sphere B, $M_B = \frac{4}{3}\, \pi R^3 \rho_B$. Now, $I_A = \frac{2}{5}\, M_A R^2$ and $I_B = \frac{2}{5} M_B R^2$. Therefore,

$$\frac{I_A}{I_B} = \frac{M_A}{M_B} = \frac{\rho_A}{\rho_B}$$

Hence the correct choice is (c).

63. When the cylinder rolls without sliding, the acceleration down the plane is (see solutions to Q. 16)

$$a_r = \frac{2}{3}\, g \sin\theta$$

When the cylinder slides without rolling, the acceleration is

$$a_s = g \sin\theta$$

where θ is the inclination of the plane.

If h is the height of the inclined plane, their speeds on reaching the bottom are given by

$$v_r = \sqrt{2a_r h} \text{ and } v_s = \sqrt{2a_s h}$$

Since $a_s > a_r$, it follows that $v_s > v_r$, which is choice (b).

64. $$I_{BC} = \frac{m(AB)^2}{3} = \frac{ma^2}{3}$$

$$I_{AB} = \frac{m(BC)^2}{3} = \frac{4}{3} ma^2$$

$$I_{HF} = \frac{m(AB)^2}{12} = \frac{ma^2}{12}$$

$$I_{EG} = \frac{m(BC)^2}{12} = \frac{ma^2}{3}$$

Thus, the moment of inertia about HF is the minimum, which is choice (c).

65. In one full revolution the increase in PE $= MgL$, where M is the mass of the rod. Therefore,

$$MgL = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

or $\omega = \sqrt{\frac{6g}{L}}$. Now $v = L\omega = L\sqrt{\frac{6g}{L}} = \sqrt{6gL}$

Hence the correct choice is (c).

66. PE at $\theta = 60°$ is $Mgh\ (1 - \cos\theta)$ where h is the distance between the axis of rotation and the centre of mass of the disc. Thus $h = R$. Gain in KE when the disc reaches the equilibrium position $= \frac{1}{2} I\omega^2$

where $I = I_{C.M.} + Mx^2 = \frac{1}{2} MR^2 + MR^2 = \frac{3}{2} MR^2$.

Here x is the distance between the centre of mass and the axis of rotation, i.e. $x = R$.

Now PE = KE gives

$$MgR\ (1 - \cos 60°) = \frac{3}{4} MR^2\ \omega^2$$

which gives $\omega = \sqrt{\frac{2g}{3R}}$, which is choice (b).

67. Loss in PE = gain in rotational KE. Thus

$$mgh = \frac{1}{2}\ (I + mR^2)\ \omega^2 = \frac{1}{2}\ (MR^2 + mR^2)\ \omega^2$$

$$= \frac{1}{2}\ R^2\ (M + m)\ \omega^2$$

or $\omega = \sqrt{\frac{2mgh}{(M+m)\,R^2}}$. Hence the correct choice is (c).

68. We obtain the given hollow sphere as if a solid sphere of radius R has been removed from a solid sphere of radius $2R$. The mass of the given hollow sphere is (here ρ is the density of the material of the sphere)

$$M = M_1 - M_2$$

where $M_1 = \frac{4}{3}\ \pi (2R)^3\ \rho$ and $M_2 = \frac{4}{3}\ \pi R^3 \rho$ are the masses of spheres of radii $2R$ and R respectively.

$$\therefore \qquad M = \frac{28}{3}\ \pi R^3 \rho \qquad \text{(i)}$$

The moment of inertia of the given hollow sphere is

$$I = \frac{2}{5}\ M_1\ (2R)^2 - \frac{2}{5}\ M_2 R^2$$

$$= \frac{2}{5} \times \frac{4}{3}\pi\ (2R)^3\ \rho (2R)^2 - \frac{2}{5} \times \frac{4}{3}\left(\pi R^3 \rho\right) R^2$$

$$= \frac{2}{5}\ (32 - 1)\ \frac{4}{3}\ \pi R^5 \rho \qquad \text{(ii)}$$

Using (i) in (ii), we get $I = \frac{62}{35}\ MR^2$, which is choice (d).

69. Let I_1 and ω_1 be the moment of inertia and angular frequency when his arms are outstretched and I_2 and ω_2 those when his arms are folded. Then

$$I_1\omega_1 = I_2\omega_2$$

Given $I_2 = \frac{3}{4}\ I_1$. Hence $I_1\ \omega_1 = \frac{3}{4}\ I_1\ \omega_2$

or $\omega_2 = \frac{4}{3}\ \omega_1$.

Initial KE is $K_1 = \frac{1}{2}\ I_1\ \omega_1^2$ and final KE is

$$K_2 = \frac{1}{2} I_2\ \omega^2{}_2 = \frac{1}{2} \times \frac{3I_1}{4} \times \left(\frac{4\omega_1}{3}\right)^2$$

$$= \frac{4}{3}\left(\frac{1}{2} I_1 \omega_1^2\right) = \frac{4}{3} K_1$$

$\therefore$ Percentage increase in KE $= \frac{K_2 - K_1}{K_1} \times 100$

$$= \frac{\frac{4}{3}K_1 - K_1}{K_1} \times 100 = \frac{100}{3} = 33.3\%$$

Hence the correct choice is (a).

70. The disc is rolling about the point O. Thus the axis of rotation passes through the point A and is perpendicular to the plane of the disc. From the relation $v = r\omega$ where r is the distance of the point on the rim about the axis of rotation, we have

$$v_A = 0,\ v_B = (AB)\ \omega = \frac{3R\omega}{2}$$

and $\qquad v_C = (AC)\ \omega = 2R\omega$

Hence $\frac{v_B}{v_c} = \frac{3R\omega}{2} \times \frac{1}{2R\omega} = \frac{3}{4}$. Thus the correct choice is (c).

71. Refer to Fig. 8.71. The angular momentum of the particle at point $P(x, y)$ about origin O is given by

$$\mathbf{L} = m\,\mathbf{r} \times \mathbf{v}$$

$$= m \times \left(x\,\hat{i} + y\,\hat{j}\right)$$

$$= -myv\ \hat{k}$$

$$(\because\ \hat{i} \times \hat{i} = 0 \text{ and } \hat{j} \times \hat{i} = -\hat{k})$$

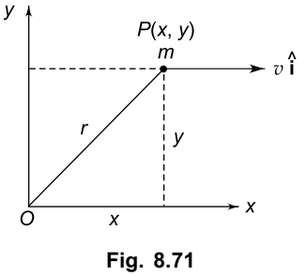


Fig. 8.71

Now, mass m and velocity v are constant. Also y remain constant as the particle moves parallel to the x-axis. Hence L remains constant. Thus the correct choice is (b).

72. Mass per unit area of the disc = $\frac{M}{\pi R^2}$. Therefore, mass of the removed portion (hole of radius $R/3$) is

$$m = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = \frac{M}{9}$$

The moment of inertia of the complete disc about an axis passing through its centre O and perpendicular to its plane is

$$I = \frac{1}{2} MR^2$$

Using the parallel axes theorem, the moment of inertia of the removed portion of the disc about the axis passing through O and perpendicular to the plane of the disc is

I' = MI of mass m about $O' + m \times OO'$

$$= \frac{1}{2} m\left(\frac{R}{3}\right)^2 + m \times \left(\frac{2R}{3}\right)^2$$

$$= \frac{1}{2} \times \frac{M}{9} \times \frac{R^2}{9} + \frac{M}{9} \times \frac{4R^2}{9} = \frac{1}{18} MR^2$$

Therefore, the moment of inertia of the remaining portion of the disc about $O = I - I' = \frac{1}{2} MR^2 - \frac{1}{18} MR^2 = \frac{4}{9} MR^2$. Hence the correct choice is (d).

73. Let M be the mass of the sphere. The mass of the disc will also be M. The moment of inertia of the sphere about its diameter is

$$I_s = \frac{2}{5} MR^2$$

The moment of inertia of the disc about its edge and perpendicular to its plane is (using parallel axes theorem)

$$I_d = I_{cm} + Mh^2 = \frac{1}{2} Mr^2 + Mr^2 = \frac{3}{2} Mr^2$$

Given $I_s = I_d$. Hence, we have

$$\frac{2}{5} MR^2 = \frac{3}{2} Mr^2$$

which gives $\frac{r}{R} = \frac{2}{\sqrt{15}}$, which is choice (a).

74. From the principle of conservation of mechanical energy, we have

$$\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = mgh$$

$$\frac{1}{2} mv^2 + \frac{1}{2} I\left(\frac{v}{R}\right)^2 = mg \times \frac{3v^2}{4g}$$

On solving, we get $I = \frac{mR^2}{2}$. Hence the object is a disc, which is choice (d).

75. Consider a rod OP of length L lying along the x-axis with O as the origin (Fig. 8.72). Consider a small element AB of length dx at a distance x from O.

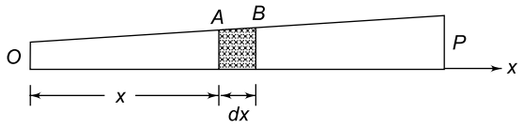


Fig. 8.72

Mass of element AB $(= dm) = mdx = \frac{k}{L}(xdx)$

The distance of the centre of mass from O is given by

$$x_{CM} = \frac{\int (dm)\, x}{\int (dm)} = \frac{\frac{k}{L}\int_0^L x^2\, dx}{\frac{k}{L}\int_0^L x\, dx}$$

$$= \frac{\left|\frac{x^3}{3}\right|_0^L}{\left|\frac{x^2}{2}\right|_0^L} = \frac{L^3/3}{L^2/2} = \frac{2L}{3}$$

The correct choice is (b).

76. When the tube AB is rotated about its end A in a horizontal plane with a uniform angular velocity, all points of the tube rotate with same angular velocity. Consider a small element of the liquid of length dr at a distance r from the axis of rotation. The mass of this element is (see Fig. 8.73).

$$m = \left(\frac{M}{L}\right) dr$$

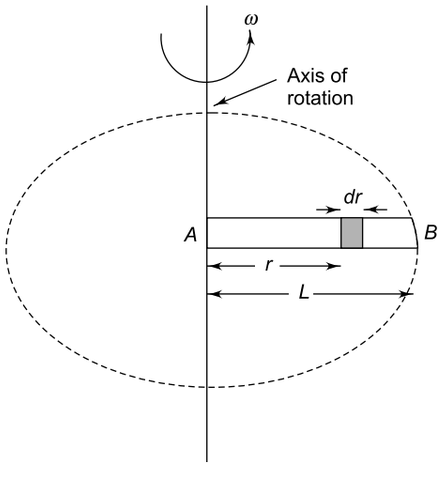


Fig. 8.73

Force exerted by the element is

$$dF = m\ r\ \omega^2 = \left(\frac{M}{L}\right)\omega^2 r\ dr$$

Total force exerted by the liquid at end B is

$$F = \int_0^L \left(\frac{M}{L}\right)\omega^2 r\ dr = \left(\frac{M}{L}\right)\omega^2 \int_0^L r\ dr$$

$$= \left(\frac{M}{L}\right)\omega^2 \left|\frac{r^2}{2}\right|_0^L = \frac{1}{2}\ M\omega^2 L$$,which is choice (c).

77. Let ω be the angular velocity acquired by the system (rod + bullet) immediately after the collision. Since no external torque acts, the angular momentum of the system is conserved. Thus

$$mvL = I\omega \qquad (1)$$

where I is the moment of inertia of the system about an axis passing through O and perpendicular to the rod. Thus

I = M.I. of rod about O + M.I. of bullet stuck at its lower end about O

$$= \frac{1}{3}\ ML^2 + mL^2 = \frac{1}{3}\ (M + 3m)L^2 \qquad (2)$$

Using Eq. (1) in Eq. (2), we have

$$mvL = \frac{1}{3}\ (M + 3m)L^2\omega$$

or $$\omega = \frac{3mv}{L\,(M + 3m)}$$

Hence the correct choice is (c).

78. The initial angular momentum of the rotating record is

$$L = I\omega$$

where $I = \frac{1}{2}\ MR^2$.

Let ω' be the angular velocity of the record when the coin of mass m is placed on it at a distance r from its centre. The angular momentum of the system becomes

$$L' = (I + mr^2)\omega'$$

Since no external torque acts on the system, the angular momentum is conserved, i.e.

$$L' = L \text{ or } (I + mr^2)\omega' = I\omega$$

or $$\omega' = \frac{I\omega}{I + mr^2} = \frac{\frac{1}{2}MR^2\omega}{\frac{1}{2}MR^2 + mr^2}$$

or $$\omega' = \frac{\omega}{\left(1 + \frac{2mr^2}{MR^2}\right)}$$

Putting $r = R/2$, we find that the correct choice is (a)

79. The block will just begin to slide if the downward force $mg\ sin\ \theta$ just overcomes the frictional force, i.e. if $mg \sin\theta = \mu N = \mu\ mg \cos\theta \Rightarrow \tan\theta = \mu = \sqrt{3} \Rightarrow \theta = 60°$ [see Fig. 8.74]

The block will topple if the torque due to normal reaction N about O just exceeds the torque due to $mg \sin\theta$ about 0, i.e.

$$N \times OA = mg\ \sin\theta \times OB$$

$$\Rightarrow \quad mg \cos\theta \times 5 \text{ cm} = mg \sin\theta \times \frac{15}{2} \text{ cm}$$

$$\Rightarrow \quad \tan\theta = \frac{2}{3} \Rightarrow \theta = 34°$$

Since θ for toppling is less than θ for sliding, the correct choice is (b).

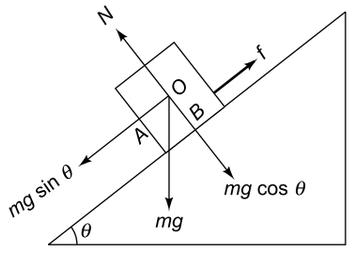


Fig. 8.74

SECTION II

Multiple Choice Questions with One or More Choice Correct

1. In the HCl molecule, the separation between the nuclei of hydrogen and chlorine atoms is 1.27 Å. If the mass of a chlorine atom is 35.5 times that of a hydrogen atom, the centre of mass of the HCl molecule is at a distance of

(a) $\frac{35.5 \times 1.27}{36.5}$ Å from the hydrogen atom

(b) $\frac{35.5 \times 1.27}{36.5}$ Å from the chlorine atom

(c) $\frac{1.27}{36.5}$ Å from the hydrogen atom

(d) $\frac{1.27}{36.5}$ Å from the chlorine atom

2. Choose the correct statements from the following:

(a) The position of the centre of mass of a system of particles does not depend upon the internal forces between particles.

(b) The centre of mass of a solid may lie outside the body of the solid.

(c) A body tied to a string is whirled in a circle with a uniform speed. If the string is suddenly cut, the angular momentum of the body will not change from its initial value.

(d) The angular momentum of a comet revolving around a massive star, remains constant over the entire orbit.

3. Which of the following statements are correct?

(a) When a body rolls on a surface, the force of friction acts in the same direction as the direction of motion of the centre of mass of the body.

(b) During rolling, the instantaneous speed of the point of contact is zero.

(c) During rolling, the instantaneous acceleration of the point of contact is zero.

(d) A wheel moving down a perfectly frictionless inclined plane will slip and not roll on the plane.

4. In which of the following is the angular momentum conserved?

(a) The planet Neptune moves in an elliptical orbit round the sun with the sun at one of the foci of the ellipse

(b) An electron describes a Sommerfieldian elliptical orbit round the nucleus

(c) An α-particle, approaching a nucleus, is scattered by the force of electrostatic repulsion between the two

(d) A boy whirls a stone, tied to a string, in a horizontal circle

5. Four tiny masses are connected by a rod of negligible mass as shown in Fig. 8.75.

(a) The moment of inertia of the system about axis AB is 50 ma^2

(b) The radius of gyration od the system about axis AB is $\sqrt{5}\,a$.

(c) The moment of inertia of the system about axis CD is 10 ma^2

(d) The radius of gyration of the system about axis CD is a.

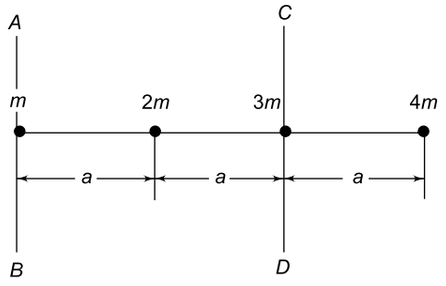


Fig. 8.75

6. The moment of inertia of a uniform circular disc of mass M and radius R about its centre and normal to its plane is $\frac{1}{2}\,MR^2$. Then

(a) its radius of gyration about the centre is R.

(b) the moment of inertia of the disc about its diameter is $\frac{1}{4}\,MR^2$.

(c) the moment of inertia of the disc about an axis passing through a point on its edge and normal to the disc is $\frac{3}{2}\,MR^2$.

(d) the moment of inertia of the disc about a tangent in the plane of the disc is $\frac{5}{4}\,MR^2$.

7. A molecule consists of two atoms, each of mass m, separated by a distance a. The rotational kinetic energy of the molecule is K and its angular frequency is ω. I is the moment of inertia of the molecule about its centre of mass. Then

(a) $I = ma^2$ (b) $I = \frac{1}{2}\,ma^2$

(c) $\omega = \frac{1}{a}\sqrt{\frac{K}{m}}$ (d) $\omega = \frac{2}{a}\sqrt{\frac{K}{m}}$

8. A circular ring of mass m and radius r rolls down an inclined plane of height h. When it reaches the bottom of the plane its angular velocity is ω and its rotational kinetic energy is K. Then

(a) $\omega = \frac{1}{r}\sqrt{gh}$ (b) $\omega = \frac{1}{2r}\sqrt{gh}$

(c) $K = mgh$ (d) $K = \frac{1}{2}mgh$

9. A rope is wound round a solid cylinder of mass M and radius R. If the rope is pulled with a force F, the cylinder acquires an anglular acceleration α and the rope acquires a linear acceleration a. Then

(a) $\alpha = \frac{F}{MR}$ (b) $\alpha = \frac{2F}{MR}$

(c) $a = \frac{2F}{M}$ (d) $a = \frac{F}{M}$

10. A solid sphere rotating about its diameter at an angular frequency ω has rotational kinetic energy K. When it is cooled so that its radius reduces to $\frac{1}{n}$ of its original value, the new values of ω and K become ω' and K' respectively. Then

(a) $\frac{\omega'}{\omega} = n$ (b) $\frac{\omega'}{\omega} = n^2$

(c) $\frac{K'}{K} = n$ (d) $\frac{K'}{K} = n^2$

11. Three forces act on a wheel of radius 20 cm as shown in Fig. 8.76.

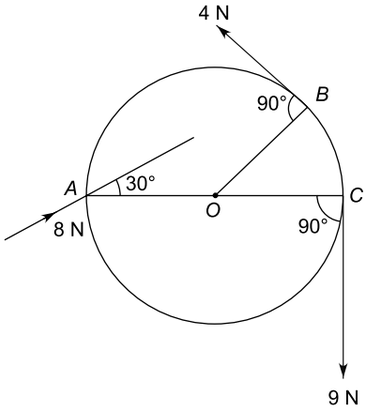


Fig. 8.76

(a) The torque produced by the force of 8N is 0.8 Nm clockwise.

(b) The torque produced by the force of 4 N is 0.8 Nm anticlockwise.

(c) The torque produced by the force of 9 N is zero.

(d) The net torque produced by the forces is 1.8 Nm clockwise.

12. A solid cylinder rolls down a rough inclined plane without slipping as shown in Fig. 8.77. Choose the correct statement (s) from the following.

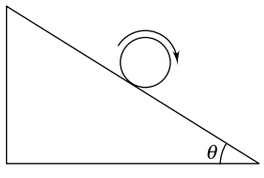


Fig. 8.77

(a) If θ is decreased, the force of friction will decrease.

(b) Frictional force is dissipative.

(c) Frictional force = $\mu mg \cos\theta$, where m is the mass of the cylinder and μ the coefficient of friction between the cylinder and the plane.

(d) Frictional force helps rotational motion of the cylinder but opposes its translational motion.

13. A spherical ball is released from rest from point A on a hemispherical surface and it rises up to a point C as shown in Fig. 8.78. Part AB of the surface is rough and the ball rolls from A to B without slipping. Part BC of the surface is frictionless. K_A, K_B and K_C are kinetic energies of the ball at points A, B and C respectively. Which of the following is/are correct?

(a) $K_A = K_C$, $h_A = h_C$ (b) $K_B > K_A$, $h_A < h_C$

(c) $K_B > K_C$, $h_A > h_C$ (d) $K_C > K_A$, $h_A > h_C$

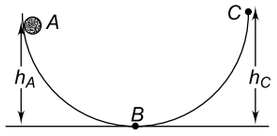


Fig. 8.78

14. The moment of inertia of a uniform disc about its diameter is I. Then the moment of inertia about an axis

(a) passing through its centre and perpendicular to its plane is $2I$.

(b) tangential to its plane is $5I$.

(c) tangential and perpendicular to its plane is $6I$.

(d) all the above choices are correct.

15. A wire of mass M, length L and density ρ is bent to form a circular ring of radius R. Then, the moment of inertia of the ring about its diameter is

(a) $\frac{1}{2}MR^2$ (b) $\pi R^4 L\rho/2$

(c) $\frac{ML^2}{8\pi^2}$ (d) $\frac{\rho R^2 L^3}{8\pi}$

16. A uniform solid sphere and a solid cylinder of the same mass and the same diameter are released from rest on the top of an inclined plane of inclination θ. If they roll down the plane without slipping, then

(a) the acceleration of each down the plane is $g \sin\theta$.

(b) the ratio of the accelerations of the sphere and the cylinder is 15:14.

(c) the ratio of the times taken by the sphere and the cylinder to reach the bottom of the plane is $\sqrt{14} : \sqrt{15}$.

(d) the sphere and the cylinder will reach the bottom with the same speed.

17. A rope is wound around a hollow cylinder of mass 3 kg and radius 20 cm. The rope is pulled with a constant force of 30 N. If α is the angular acceleration of the cylinder and a the linear acceleration of the rope, then

(a) $\alpha = 50$ rad s^{-2} (b) $\alpha = 40$ rad s^{-2}
(c) $a = 30$ ms^{-2} (d) $a = 10$ ms^{-2}

18. A rope of negligible mass is wound around the circumference of a bicycle wheel (without tyre) of diameter 1 m. A mass of 2 kg is attached to the end of the rope and is allowed to fall from rest. The mass falls 2 m is 4 s. The axle of the wheel is horizontal and the wheel rotates in the vertical plane. Take g = 10 ms^{-2} and neglect the friction due to air.

(a) the linear acceleration of the wheel is 0.25 m s^{-2}.
(b) the angular acceleration of the wheel is 0.5 rad s^{-2}.
(c) the magnitude of the torque acting on the wheel is 10 Nm.
(d) the moment of inertia of the wheel about the horizontal axle is 20 kg m^2.

19. A smooth sphere A is moving on a horizontal frictionless surface with angular speed ω and centre of mass velocity v. It collides head-on with an identical sphere B at rest. After the collision their angular speeds are ω_A and ω_B respectively. If the collision is elastic and the friction is neglected, then

(a) $\omega_A = \omega$ (b) $\omega_B = 0$
(c) $\omega_A < \omega_B$ (d) $\omega_A = \omega_B$

IIT, 1999

20. The torque acting on a body about a given point is given by $\vec{\tau} = \vec{A} \times \vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. It follows that

(a) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time.
(b) the component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time.
(c) the magnitude of $\vec{L}$ does not change with time.
(d) All the above choices are correct.

IIT, 1998

21. A uniform bar of length $6a$ and mass 8 m lies on a horizontal frictionless table. Two point masses m and $2m$ moving in opposite directions but in the same horizontal plane with speeds $2v$ and v respectively strike the bar at distance a and $2a$ from one end and stick to the bar after the collision. Then after the collision

(a) the velocity of centre of mass is zero.
(b) the angular speed of the bar with the masses stuck to it is $\frac{v}{5a}$.
(c) the moment of inertia of the bar with masses stuck to it about the axis passing through the end of the bar and perpendicular to its plane is 30 ma^2.
(d) the total energy of the bar $\frac{3}{5}mv^2$.

IIT, 1991

22. A disc of mass M and radius R is rolling with angular speed ω on a horizontal surface as shown in Fig. 8.79. The magnitude of angular momentum of the disc about the origin O is (here v is the linear velocity of the disc)

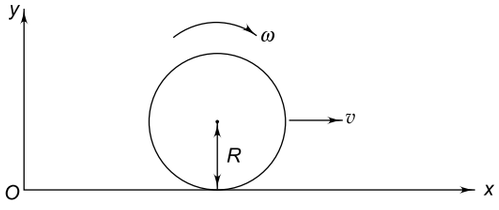


Fig. 8.79

(a) $\frac{3}{2}MR^2\omega$ (b) $MR^2\omega$
(c) MRv (d) $\frac{3}{2}MRv$

IIT, 1999

23. Which of the following statements is/are correct about a particle moving in a circle with a constant speed?

(a) The linear velocity and acceleration vectors are perpendicular to each other.
(b) The linear velocity vector is always perpendicular to the angular velocity vector.
(c) The force acting on the particle is radial.
(d) The force does no work on the particle.

24. The position vector of a particle with respect toorigin O is $\vec{r}$. If the torque acting on the particle is zero, then

(a) the linear momentum of the particle remains constant.
(b) the angular momentum of the particle about O remains constant.
(c) the force applied to the particle is perpendicular to $\vec{r}$.
(d) the force applied to the particle is parallel to $\vec{r}$.

25. A block of mass m is connected to a spring of spring constant k through a fixed pulley of radius r as shown in Fig. 8.80. The mass of the pulley is $2m$. The block is pulled down by x_0 from the equilibrium position and released. The spring has negligible mass. Then

(a) the total energy of the system is $\frac{1}{2}kx_0^2$.

(b) the velocity of the block when it is at a distance x from the equilibrium position is $\left[\frac{k}{2m}\left(x_0^2 - x^2\right)\right]^{1/2}$.

(c) the velocity of the block is maximum when $x = x_0$.

(d) the velocity of the block is zero when $x = x_0$.

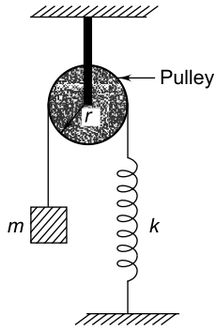


Fig. 8.80

26. A rod of mass $M = 0.9$ kg and length $L = 1$ m is suspended at O as shown in Fig. 8.81. A bullet of mass $m = 100$ g moving with velocity $v = 80$ ms^{-1} in the horizontal direction strikes the end P of the rod and gets embedded in it. If I is the moment of inertia of the system and ω is the angular velocity immediately after the collision, then

(a) $I = 0.4$ kgm^2

(b) $I = 0.3$ kgm^2

(c) $\omega = 20$ rad s^{-1}

(d) $\omega = 26.7$ rad s^{-1}

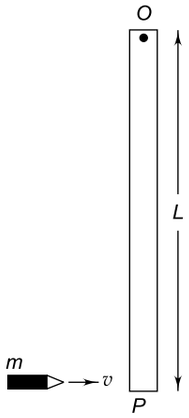


Fig. 8.81

27. A wheel is initially at rest. A constant torque τ acts on it for a time t. As a result, the wheel acquires an angular acceleration α and angular velocity ω. If the angular displacement produced is θ, then

(a) $\theta \propto t^2$ (b) α = constant

(c) $\omega \propto t$ (d) power $\propto \omega$

28. A string of length L and of negligible mass hangs from a support O. The other end of the string carries a mass m which is moved in a horizontal circle of radius R to form a conical pendulum. If the string makes an angle $\theta = 60°$ with the vertical, then

(a) the speed of the body along the circle is $v = \sqrt{Rg}$.

(b) the tension in the string is $2mg$.

(c) the horizontal component of the angular momentum of the body point O is $\frac{\sqrt{3}}{2}\, mL\sqrt{\frac{Lg}{2}}$.

(d) the magnitude of the torque acting on the body about point O is $\frac{\sqrt{3}}{2}mgL$.

29. A solid cylindrical roller of mass M and radius R is rolled on a rough horizontal surface by applying a horizontal force F. If a_{CM} is the linear acceleration of the centre of mass and f is the frictional force between the roller and the surface, then

(a) $a_{CM} = \frac{F}{M}$ (b) $a_{CM} = \frac{2F}{3M}$

(c) $f = \frac{F}{3}$ (d) f = zero

30. If the resultant of all the external forces acting on a system of particles is zero, then for an inertial frame, one can surely say that

(a) linear momentum of the system does not change in time

(b) kinetic energy of the system does not change in time

(c) angular momentum of the system does not change in time

(d) potential energy of the system does not change in time

IIT, 2009

31. A sphere is rolling without slipping on a fixed horizontal plane surface. In Fig. 8.82, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,

(a) $\vec{V}_C - \vec{V}_A = 2\left(\vec{V}_B - \vec{V}_C\right)$

(b) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(c) $\left|\vec{V}_C - \vec{V}_A\right| = 2\left|\vec{V}_B - \vec{V}_C\right|$

(d) $\left|\vec{V}_C - \vec{V}_A\right| = \left|\vec{V}_B\right|$

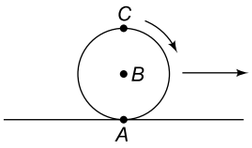


Fig. 8.82

IIT, 2009

32. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision [See Fig. 8.83]

(a) the ring has pure rotation about its stationary CM.

(b) the ring comes to a complete stop.

(c) friction between the ring and the ground is to the left.

(d) there is no friction between the ring and the ground.

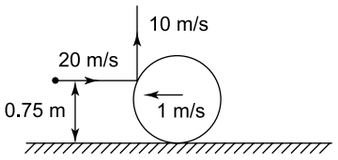


Fig. 8.83

IIT, 2011

ANSWERS AND SOLUTIONS

1. Since, most of the mass of an atom is concentrated in its nucleus and the size of a nucleus (which is of the order of 10^{-15} m) is very small compared to the separation between the hydrogen and chlorine atom (which is 1.27×10^{-10} m), the atoms can be treated as point masses.

The centre of mass of HCl molecule will be on the line joining the two atoms. Let us say that the H atom is located at, say, $x = 0$ and the *Cl* atom at $x = x$. Let x_{CM} be the position of the centre of mass between $x = 0$ and $x = x$. If m_1 and m_2 are the masses of *H* and *Cl* atoms, then from the definition of centre of mass, we have

$$x_{CM} = \frac{m_1 \times 0 + m_2 x}{m_1 + m_2} = \frac{m_2 x}{m_1 + m_2}$$

Now x = separation between *H* and *Cl* atoms = 1.27 Å and $m_2 = 35.5\ m_1$. Hence

$$x_{CM} = \frac{35.5\ m_1 \times 1.27\,\text{Å}}{m_1 + 35.5\ m_1} = \frac{35.5 \times 1.27\,\text{Å}}{36.5}$$

This gives the distance of the centre of mass from the hydrogen atom. The distance of the centre of mass from the chlorine atom is

$$1.27\ \text{Å} - \frac{35.5 \times 1.27}{36.5}\ \text{Å} = \frac{1.27\,\text{Å}}{36.5}$$

Hence the correct choices are (a) and (d).

2. The only incorrect statement is (c). Since no external torque acts on the body even after the string is cut, the angular momentum will remain unchanged.

3. Statement (a) is correct. The direction of motion of the centre of mass is the direction along which the body rolls. Since the force of friction is opposite to the direction of the velocity of the point of contact, the force of friction acts in the direction of motion of the centre of mass. Statement (b) is also correct. At each instant of time, the point of contact is momentarily at rest. Statement (c) is incorrect. Since the body is rotating while it is rolling, the direction of the velocity is changing with time. Hence the acceleration of the point of contact is not zero. Statement (d) is correct. Rolling cannot take place in the absence of friction because it is the frictional force that provides the necessary torque which makes the body roll on a surface. Hence the correct choices are (a), (b) and (d).

4. The angular momentum is conserved in the four cases. The object, in each case, is moving under the action of a central (radial) force. The torque due to a radial force is zero. Since $\tau = \frac{dL}{dt}$; the angular momentum L does not change with time.

5. All four choices are correct.

The moment of inertia about *AB* is

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$
$$= m \times 0 + 2m \times (a)^2 + 3m \times (2a)^2 + 4m \times (3a)^2$$
$$= 0 + 2\ ma^2 + 12ma^2 + 36ma^2 = 50ma^2$$

The radius of gyration K is given by

$$I = MK^2 = (m_1 + m_2 + m_3 + m_4)K^2$$
$$= (m + 2m + 3m + 4m)K^2 = 10mK^2$$

or $$K = \sqrt{\frac{I}{10m}} = \sqrt{\frac{50ma^2}{10m}} = \sqrt{5}\ a$$

The moment of inertia about *CD* is

$$I = m \times (2a)^2 + 2\ m \times (a)^2 + 3m \times 0 + 4\ m \times (a)^2$$
$$= 4\ ma^2 + 2ma^2 + 0 + 4ma^2 = 10ma^2$$

The radius of gyration is

$$= \sqrt{\frac{I}{10ma}} = \sqrt{\frac{10ma^2}{10m}} = a$$

6. The correct choices are (b), (c) and (d).

Let us consider two perpendicular diameters, one along the x-axis and the other along the y-axis. Then

$$I_x = I_y = \frac{1}{4}\ MR^2$$

According to the perpendicular axes theorem, the moment of inertia of the disc about an axis passing through the centre is

$$I_c = I_x + I_y = \frac{1}{4}\ MR^2 + \frac{1}{4}\ MR^2$$
$$= \frac{1}{2}\ MR^2$$

which is choice (b).

Since the disc is uniform, its centre of mass coincides with its centre. Therefore, the moment of inertia of the disc about an axis passing through its centre of mass and normal to its plane is

$$I_{\text{CM}} = I_C = \frac{1}{2}\ MR^2$$

According to the theorem of parallel axes, the moment of inertia of the disc about an axis passing through a point on its edge and normal to its plane is given by

$$I_e = I_{\text{CM}} + Mh^2 = \frac{1}{2}\ MR^2 + MR^2 \qquad (\because\ h = R)$$
$$= \frac{3}{2}\ MR^2.$$

From the parallel axes theorem, the moment of inertia of the disc about a tangent is $\frac{1}{4} MR^2 + MR^2 = \frac{5}{4} MR^2$.

7. The correct choices are (b) and (d).

Since the two atoms have the same mass, the centre of mass is at a distance of $a/2$ from each atom. Therefore, the moment of inertia of the molecule about its centre of mass is

$$I = m\left(\frac{a}{2}\right)^2 = m\left(\frac{a}{2}\right)^2 = \frac{ma^2}{2}$$

Kinetic energy is $k = \frac{1}{2} I\omega^2$, which gives

$$\omega = \sqrt{\frac{2k}{I}} = \sqrt{\frac{2k}{ma^2} \times \frac{2}{1}} = \frac{2}{a}\sqrt{\frac{k}{m}}$$

8. $$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} mv^2 + \frac{1}{2} (mr^2)\omega^2 \qquad (\because\ I = mr^2)$$
$$= \frac{1}{2} mr^2\omega^2 + \frac{1}{2} mr^2\omega^2 \qquad (\because\ v = r\omega)$$
$$= mr^2\omega^2$$
$$\Rightarrow \quad \omega = \frac{1}{r}\sqrt{gh}\ .$$ Also

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2} (mr^2) \times \frac{gh}{r^2} = \frac{1}{2} mgh$$

Hence the correct choices are (a) and (d).

9. $I = \frac{1}{2} MR^2$ and torque $\tau = FR = I\alpha$. Hence

$$\alpha = \frac{FR}{I} = \frac{FR}{\frac{1}{2} MR^2} = \frac{2F}{MR}$$

Also $$a = \alpha R = \frac{2F}{M}$$

Thus the correct choices are (b) and (c).

10. The correct choices are (b) and (d).

$$I\omega = I'\omega',\ I = \frac{2}{5}\ mr^2,\ I' = \frac{2}{5} m\left(\frac{r}{n}\right)^2 = \frac{I}{n^2}\ .$$

Hence $I\omega = \frac{I'}{n^2}\omega'$

or $$\omega' = n^2\omega$$

$$K = \frac{1}{2} I\omega^2,\ K' = \frac{1}{2} I'\omega'^2 = \frac{1}{2} \times \frac{I}{n^2} (n^2\omega)^2$$
$$= \left(\frac{1}{2} I\omega^2\right) n^2 = n^2 K.$$

11. The torques produced by forces 8 N, 4 N and 9 N respectively are

$\tau_1 = 8\ \text{N} \times 0.2\ \text{m} \times \sin 30° = 0.8\ \text{Nm}$ (clockwise)

$\tau_2 = 4\ \text{N} \times 0.2\ \text{m} \times \sin 90° = 0.8\ \text{Nm}$ (anticlockwise)

$\tau_3 = 9\ \text{N} \times 0.2\ \text{m} \times \sin 90° = 1.8\ \text{Nm}$ (clockwise)

Net torque = 0.8 – 0.8 + 1.8 = 1.8 Nm clockwise

Hence the correct choices are (a), (b) and (d).

12. Refer to the Fig. 8.84. Here f is the frictional force. The linear acceleration of the centre of mass of the rolling cylinder is given by

$$a_{\text{cm}} = \frac{g\sin\theta}{1 + \frac{I_{\text{cm}}}{mR^2}} \qquad (1)$$

where R is the radius of the cylinder and I_{cm} is the moment of inertia of the cylinder about the centre of mass which is given by

$$I_{\text{cm}} = \frac{1}{2} mR^2$$

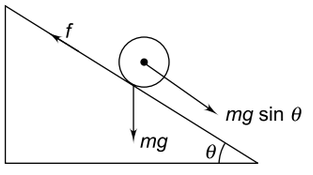


Fig. 8.84

Using this in Eq. (1), we have

$$a_{\text{cm}} = \frac{g\sin\theta}{1 + \frac{mR^2}{2mR^2}} = \frac{2g\sin\theta}{3} \qquad (2)$$

Now, for linear motion, we have

$$mg\sin\theta - f = ma_{\text{cm}} \qquad (3)$$

Using Eq. (2) in Eq. (3), we get

$$mg\sin\theta - f = \frac{2mg\sin\theta}{3}$$

$$\text{which gives } f = \frac{mg\sin\theta}{3} \qquad (4)$$

It follows from Eq. (4) that if θ is decreased, f will also decrease. Hence choice (a) is correct. As the cylinder is rolling down, the point of application of the frictional force is at rest at any given instant. Hence no work is done by the frictional force, i.e. the frictional force is not dissipative. Therefore, statement (b) is wrong. Statement (c) is also wrong as f is given by Eq. (4). Statement (d) is correct because the frictional force provides torque $= fR$ to help rotational motion but it will oppose translational motion. Hence the correct choice are (a) and (d).

13. The ball is at rest at point A. Hence its kinetic energy K_A (rotational + translational) is zero, it has only gravitational potential energy mgh_A. As it is released at A, it begins to roll (due to friction in part AB) thus acquiring kinetic energy at the expense of gravitational potential energy. When it reaches point B, its kinetic energy consists of both rotational and translational energy. At point B its potential energy is zero. At point B, the rotational kinetic energy is $\frac{1}{2} I\omega^2$, where ω is the angular velocity at point B and I is the moment of inertia of the ball about its centre. Since part BC is frictionless, the torque on the ball is zero. Hence its angular momentum $L = I\omega$ remains constant in part BC. Hence, the angular velocity ω of the ball remains constant in part BC. The translational kinetic energy is converted into gravitational potential energy. At point C, the translational kinetic energy of the ball is zero; it has rotational kinetic energy $\frac{1}{2} I\omega^2$ and gravitational potential energy $mg\,h_C$. Thus

$$K_A = 0 \qquad (1)$$

$$K_B = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \qquad (2)$$

where v is the linear velocity of the centre of mass of the ball and

$$K_C = \frac{1}{2} I\omega^2 \qquad (3)$$

It follows from (1), (2) and (3) that $K_B > K_C > K_A$. Now total energy at A is $E_A = 0 + mgh_A = mgh_A$ and total energy at C is $E_C = \frac{1}{2} I\omega^2 + mg\,h_C$. From the law of conservation of energy $E_A = E_C$, i.e. $mg\,h_A = \frac{1}{2} I\omega^2 + mg\,h_C$ or $mg\,(h_A - h_C) = \frac{1}{2} I\omega^2$. Since the right hand side of this equation is positive, $h_A > h_C$. Hence the correct choices are (c) and (d).

14. From perpendicular axes theorem, $I_x + I_y = I_c$. Hence $I_c = I + I = 2\,I$. [see Fig. 8.85(a)]

we knows that $I_c = \frac{1}{2} MR^2 \quad \Rightarrow \quad MR^2 = 2I_c = 4I$.

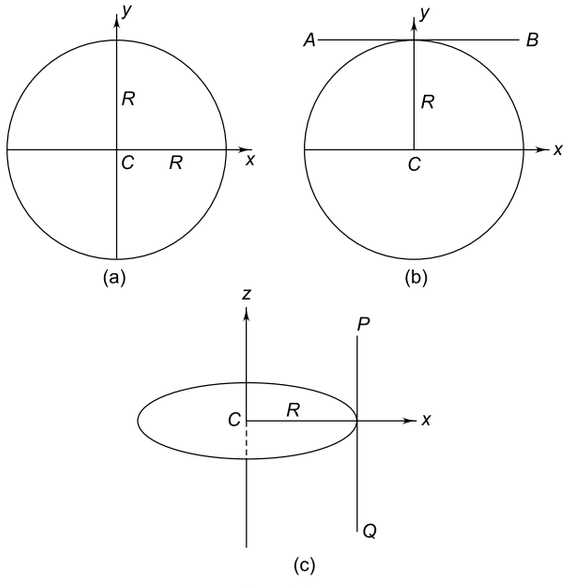


Fig. 8.85

From parallel axes theorem [see Fig. 8.84 (b)], we have

$$I_{AB} = I_x + MR^2 = I + 4\,I = 5\,I.$$

Using parallel axes theorem [see Fig. 8.84 (c)], we have

$$I_{PQ} = I_c + MR^2 = 2I + 4I = 6\,I.$$

Hence the correct choice is (d).

15. $$\rho = \frac{M}{\pi R^2 L} \quad \Rightarrow \quad M = \pi R^2 L\rho \qquad (1)$$

$$L = 2\pi R \quad \Rightarrow \quad R = \frac{L}{2\pi} \qquad (2)$$

The moment of inertia of a ring about an axis passing through its centre and perpendicular to its plane $= MR^2$. From the parallel axes theorem, the moment of inertia of the ring about its diameter $= \frac{1}{2} MR^2$. Using Eqs. (1) and (2) we find that all the four choices are correct.

16. Acceleration $a = \dfrac{g\sin\theta}{\left(1 + \dfrac{I}{MR^2}\right)}$

If s is the distance along the inclined plane, the velocity when the body reaches the bottom of the plane is given by $v^2 - 0 = 2as \Rightarrow v = \sqrt{2as}$ and the time taken to reach the bottom is

$$t = \sqrt{\frac{2s}{a}} = \left[\frac{2s\left(1+\frac{I}{MR^2}\right)}{g\sin\theta}\right]^{1/2}$$

For cylinder: $I_c = \frac{1}{2}MR^2$

For sphere: $I_s = \frac{2}{5}MR^2$

Hence $a_c = \frac{2g\sin\theta}{3}$ and $a_s = \frac{5g\sin\theta}{7}$

giving $\frac{a_s}{a_c} = \frac{15}{14}$

Also $\frac{v_s}{v_c} = \sqrt{\frac{a_s}{a_c}}$ and $\frac{t_s}{t_c} = \sqrt{\frac{a_c}{a_s}}$

Thus the correct choices are (b) and (c).

17. $I = MR^2$ and $\tau = FR$.

$$\alpha = \frac{\tau}{I} = \frac{FR}{MR^2} = \frac{F}{MR} = \frac{30}{3\times 0.2} = 50 \text{ rad s}^{-2}$$

$a = R\alpha = 0.2 \times 50 = 10 \text{ ms}^{-2}$

The correct choices are (a) and (d).

18. $s = ut + \frac{1}{2}at^2 \Rightarrow 2 = 0 + \frac{1}{2} \times a \times (4)^2$

$\Rightarrow a = 0.25 \text{ ms}^{-2}$.

$$\alpha = \frac{a}{R} = \frac{0.25}{0.5} = 0.5 \text{ rad s}^{-2}$$

$\tau = mg \times$ moment arm $= 2 \times 10 \times 0.5 = 10$ Nm

$$I = \frac{\tau}{\alpha} = \frac{10}{0.5} = 20 \text{ kg m}^2$$

Hence all the four choices are correct.

19. Since the collision is elastic and head-on and the spheres have the same mass, they will exchange their velocity after the collision, i.e. A comes to rest and B moves with velocity v. Since there is no friction, the torque on each sphere is zero. Hence their angular speeds remain unchanged on collision. Thus the correct choices are (a) and (b).

20. Given $\vec{\tau} = \vec{A} \times \vec{L}$ (1)

We know that $\vec{\tau} = \frac{d\vec{L}}{dt}$. Hence $\frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$. This means that $\frac{dL}{dt}$ is perpendicular to both $\vec{L}$ and $\vec{A}$.

Hence choice (a) is correct.

Now $\vec{L} \cdot \vec{L} = L^2$, where L is the magnitude of $\vec{L}$.

Differentiating, we have

$$\frac{d}{dt}(\vec{L} \cdot \vec{L}) = \frac{d}{dt}(L^2)$$

$$\Rightarrow \quad \vec{L} \cdot \frac{d\vec{L}}{dt} + \frac{d\vec{L}}{dt} \cdot \vec{L} = 2L\frac{dL}{dt}$$

$$\Rightarrow \quad 2\vec{L} \cdot \frac{d\vec{L}}{dt} = 2L\frac{dL}{dt}$$

Since $\vec{L} \perp \frac{d\vec{L}}{dt}$; $\vec{L} \cdot \frac{d\vec{L}}{dt} = 0$. Hence

$L\frac{dL}{dt} = 0 \Rightarrow \frac{dL}{dt} = 0 \Rightarrow L =$ constant. Hence choice (c) is correct. Since $\frac{d\vec{L}}{dt} \perp \vec{A}$, choice (b) is also correct. Thus the correct choice is (d).

21. Since no external force is applied, the linear momentum is conserved. Hence

$(8m + m + 2m)v_{CM} = 2m(-v) + m(2v) + 8m \times 0$

where v_{CM} is the velocity of the centre of mass. This gives $v_{CM} = 0$.

The moment of inertia of the system is

$$I = 2ma^2 + m(2a)^2 + \frac{1}{12} \times 8m \times (6a)^2$$

$$= 2ma^2 + 4ma^2 + 24ma^2 = 30\,ma^2$$

Since no external torque is applied, the angular momentum of the system is conserved. If ω is the angular speed, then

$$2mv \times a + m \times 2v \times 2a = I\omega^2$$

$$\Rightarrow \quad 6mva = 30ma^2\omega^2 \quad \Rightarrow \quad \omega = \frac{v}{5a}$$

The system has no translational K.E. Hence

Total K.E. = K.E. of rotation

$$= \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}30\,ma^2 \times \left(\frac{v}{5a}\right)^2 = \frac{3}{5}mv^2$$

Hence all the four choices are correct.

22. The angular momentum about O is

$$\vec{L}_O = \vec{L}_{CM} + M\left(\vec{R} \times \vec{v}\right)$$

Its magnitude is $\left(\because \vec{R} \perp \vec{v}\right)$ and $L_{CM} = I\omega$

$$L_O = I\omega + MRv$$

$$= \left(\frac{1}{2}MR^2\right)\omega + MR \times R\omega \qquad (\because v = R\omega)$$

$$= \frac{3}{2}MR^2\omega$$

$$= \frac{3}{2} MR^2 \times \left(\frac{v}{R}\right) = \frac{3}{2} MRv$$

Hence the correct choices are (a) and (d).

23. All the four choices are correct. The linear velocity $\vec{v}$, angular velocity $\vec{\omega}$ and radius vector $\vec{r}$ are related as

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Hence $\vec{v}$ is perpendicular to both $\vec{\omega}$ and $\vec{r}$. Hence choice (b) is correct. The acceleration (and hence force) is centripetal (towards the centre) and $\vec{v}$ is $\perp \vec{r}$. Hence choice (a) and (c) are correct. Since $\vec{F}$ is $\perp \vec{r}$, the work done is zero. Hence choice (d) is also correct.

24. $\vec{\tau} = \vec{r} \times \vec{F}$ and $\tau = \frac{d\vec{L}}{dt}$. Hence the correct choices are (b) and (d).

25. Total energy before the block is released = $\frac{1}{2} kx_0^2$.

After the block is released, the total energy when it is at a distance x from the equilibrium position = K.E. of block + rotational K.E. of pulley + P.E. stored in the spring = $\frac{1}{2} mv^2 + \frac{1}{2} I_p \omega^2 + \frac{1}{2} kx^2$.

where $I_p = \frac{1}{2} m_p r^2 = \frac{1}{2} \times (2m) r^2 = mr^2$.

From the principle of conservation of energy, we have ($\because v = r\omega$)

$$\frac{1}{2} kx_0^2 = \frac{1}{2} mv^2 + \frac{1}{2} \times (mr^2) \times \left(\frac{v}{r}\right)^2 + \frac{1}{2} kx^2$$

which gives $v = \left[\frac{k}{2m}\left(x_0^2 - x^2\right)\right]^{1/2}$.

Thus v is maximum when $x = 0$. Hence the correct choices are (a), (b) and (d).

26. $I = mL^2 + \frac{1}{3} ML^2 = 0.1 \times (1)^2 + \frac{1}{3} \times 0.9 \times (1)^2$

$= 0.4 \text{ kgm}^2$

From conservation of angular momentum, we have

$$mvL = I\omega$$

$$\Rightarrow \quad \omega = \frac{mvL}{I} = \frac{0.1 \times 80 \times 1}{0.4} = 20 \text{ rad s}^{-1}.$$

The correct choices are (a) and (c).

27. $\omega = \omega_0 + \alpha t \Rightarrow \omega = 0 + \alpha t = \alpha t$. Since τ is constant and $\tau = I\alpha$, it follows that α is constant. Hence $\omega \propto t$. Also $\theta = \frac{1}{2} \alpha t^2$. Hence $\theta \propto t^2$. Also power $P = \tau\omega$. Hence $P \propto \omega$. Thus all the four choices are correct.

28. Refer to Fig. 8.86. $OA = OB = OC = L$

$R = l \sin \theta$, $OD = L \cos \theta$

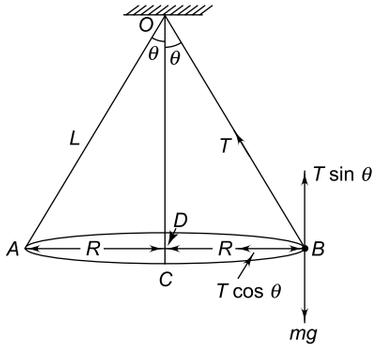


Fig. 8.86

$$T \sin \theta = mg \qquad (1)$$

$$T \cos \theta = \frac{mv^2}{R} \qquad (2)$$

Dividing (1) by (2), we get

$v = \sqrt{Rg \tan \theta} = \sqrt{Rg \tan 60°} = \sqrt{1.73Rg}$

Hence choice (a) is incorrect. From (1) and (2) we get

$$T^2 = (mg)^2 + \frac{m^2}{R^2} (v^2)^2$$

$$= (mg)^2 + \frac{m^2}{R^2} \times (Rg \tan 60°)^2$$

$$= (mg)^2 + 3(mg)^2 = 4(mg)^2$$

$$\therefore \quad T = 2\ mg$$

Horizontal component of angular momentum about O

$$= mv \times OD = mv\ L \cos \theta$$

$$= m \times \sqrt{Rg \tan \theta} \times L \cos \theta$$

$$= mL \cos \theta \sqrt{Lg \tan \theta \sin \theta}$$

$$= mL \sin \theta \sqrt{Lg \cos \theta}$$

$$= mL \sin 60° \sqrt{Lg \cos 60°}$$

$$= \frac{\sqrt{3}}{2} mL \sqrt{\frac{Lg}{2}}, \text{ which is choice (c).}$$

Torque about $O = mgR = mgL \sin \theta$

$$= mgL \sin 60° = \frac{\sqrt{3}}{2} mgL$$

Hence the correct choices are (b), (c) and (d).

29. Since the frictional force acts opposite to the direction of motion, the equation of translational motion is

$$F - f = Ma_{CM} \qquad (1)$$

For rotational motion, we have

$$\tau = I\alpha$$

Since the torque is due to frictional force, $\tau = fR$.

Hence $$fR = \left(\frac{1}{2}MR^2\right)\frac{a_{CM}}{R}$$

$$\left(\because I = \frac{1}{2}MR^2, \alpha = \frac{a_{CM}}{R}\right)$$

or $$f = \frac{Ma_{CM}}{2} \qquad (2)$$

Equations (1) and (2) give $a_{CM} = \frac{2F}{3M}$ and $f = \frac{F}{3}$.

So the correct choices are (b) and (c).

30. The only correct choice is (a)

31. Refer to Fig. 8.87.

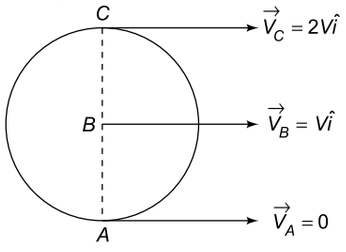


Fig. 8.87

$\vec{V}_A = 0$, Let $\vec{V}_B = V\hat{i}$, then $\vec{V}_C = 2V\hat{i}$

$\therefore$ $\vec{V}_C - \vec{V}_A = 2V\hat{i} - 0 = 2V\hat{i}$

$\vec{V}_C - \vec{V}_B = 2V\hat{i} - V\hat{i} = V\hat{i}$

$\vec{V}_B - \vec{V}_C = V\hat{i}$

$\vec{V}_B - \vec{V}_A = V\hat{i}$

Hence the correct choice are (b) and (c)

32. Let M be the mass of the ring and m that of the ball and let V and v be their velocity before collision. The initial momentum of the system (ring and ball) in the horizontal direction is

$$\vec{p}_i = M\vec{V} + m\vec{v}$$

$$= 2 \times 1 + 0.1 \times (-20)$$

$$= 2 - 2 = 0$$

From conservation of mementum, the final momentum of the system $\vec{p}_f = 0$ in the horizontal direction. Hence $V_{cm} = 0$ for the ring, i.e. the ring has pure rotation about its centre of mass. So choice (a) is correct.

The total initial angular momentum of the system about the point of collision is

$$L_i = mvr - I\omega$$

$$= mvr - MR^2\frac{V}{R}$$

$$= mvr - MRV$$

$$= 0.1 \times 20 \times 0.75 - 2 \times 0.5 \times 1$$

$$= 1.5 - 1 = 0.5 \text{ kg m}^2 \text{ s}^{-1}$$

From the conservation of angular momentum, the final angular velocity must be anticlockwise. Hence the friction between the ring and the ground is to the left. So the correct choices are (a) and (c).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 2 are based on the following passage

Passage I

Quantization

In physics some measurable quantities are quantized. A physical quantity is said to be quantized if it can have only discrete (not continuous) values. Some examples of quantities which are quantized are mass, charge and energy. For sub-atomic particles (called fundamental particles) the angular momentum is quantized. Fundamental particles such as electrons and protons have a certain intrinsic angular momentum of their own. This angular momentum is called spin angular momentum. The spin angular momentum of a fundamental particle is quantized and its value is given by

$$S = n\frac{h}{2\pi}$$

where h is the Planck's constant = 6.63×10^{-34} Js and n is a number called the spin quantum number. The value of n for electrons, protons, positrons and antiprotons can be $+\frac{1}{2}$ and $-\frac{1}{2}$. Pions have $n = 0$.

1. Which of the following is not quantized ?

(a) Mass (b) Energy

(c) Linear momentum (d) Charge

2. Choose the correct statements from the following.
(a) Electric charge on a charged body can only be an integral multiple of the smallest possible charge.
(b) Energy can have only discrete values.
(c) The spin angular momentum of an electron can be $+\frac{h}{4\pi}$ or $-\frac{h}{4\pi}$.
(d) The spin angular momentum of a pion is $+\frac{h}{2\pi}$ or $-\frac{h}{2\pi}$.

ANSWERS

1. The correct choice is (c).
2. The correct choices are (a), (b) and (c). The spin angular momentum of a pion is zero.

Questions 3 to 5 are based on the following passage

Passage II

A hollow sphere of mass M and radius R is initially at rest on a horizontal rough surface. It moves under the action of a constant horizontal force F as shown in Fig. 8.88.

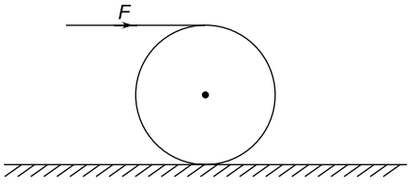


Fig. 8.88

3. The frictional force between the sphere and the surface
(a) retards the motion of the sphere
(b) makes the sphere move faster
(c) has no effect on the motion of the sphere
(d) is independent of the velocity of the sphere.

4. The linear acceleration of the sphere is
(a) $a = \frac{10F}{7M}$ (b) $a = \frac{7F}{5M}$
(c) $a = \frac{6F}{5M}$ (d) $a = \frac{F}{M}$

5. The frictional force between the sphere and the surface is
(a) $\frac{F}{2}$ (b) $\frac{F}{3}$
(c) $\frac{F}{4}$ (d) $\frac{F}{5}$

SOLUTIONS

3. If the horizontal force F is applied at the centre of mass of the sphere, then the frictional force opposes the translational motion of the sphere. If force F is applied above the centre of mass, the torque due to frictional force tends to rotate the sphere faster. Hence, in this case, frictional force f acts in the direction of motion, as shown in Fig. 8.89. Thus the correct choice is (b).

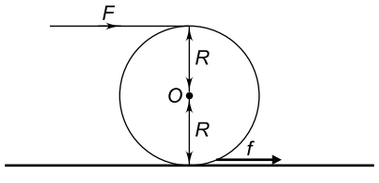


Fig. 8.89

4. Let a and α be the linear and angular accelerations of the sphere respectively. For translational motion,

$$F + f = Ma \qquad (1)$$

The magnitude of the net torque acting on the sphere = $FR - fR$. Hence, for rotational motion the equation is

$$FR - fR = I\alpha = \frac{Ia}{R} \qquad (\because a = \alpha R)$$

For a hollow sphere, $I = \frac{2}{3}MR^2$. Hence

$$FR - fR = \frac{2}{3}MR^2 \times \frac{a}{R} = \frac{2}{3}MRa$$

$$\Rightarrow \qquad F - f = \frac{2}{3}Ma \qquad (2)$$

Equations (1) and (2) give $a = \frac{6F}{5M}$, which is choice (c)

5. From Eqs. (1) and (2) we get $f = \frac{Ma}{6} = \frac{F}{5}$. Hence the correct choice is (d).

Questions 6 to 11 are based on the following passage

Passage III

Four solid spheres each of mass m and radius r are located with their centres on four corners of a square $ABCD$ of side a as shown in Fig. 8.90.

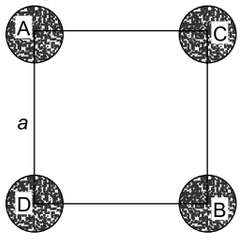


Fig. 8.90

6. The moment of inertia of sphere A about diagonal AB is

(a) $\frac{2}{3}mr^2$ (b) $\frac{2}{5}mr^2$

(c) $m\left(r^2+\frac{a^2}{4}\right)$ (d) $m\left(\frac{a^2}{4}-r^2\right)$

7. The moment of inertia of sphere C about diagonal AB is

(a) $\frac{2}{5}mr^2$ (b) $\frac{2}{5}(2r^2+3a^2)$

(c) $\frac{m}{5}(5r^2+3a^2)$ (d) $\frac{m}{10}(4r^2+5a^2)$

8. The moment of inertia of sphere B about side AD is

(a) $\frac{2}{5}mr^2$ (b) $\frac{m}{5}(5a^2+2r^2)$

(c) $\frac{m}{5}(2r^2+a^2)$ (d) $\frac{m}{5}(3a^2+5r^2)$

9. The moment of inertia of sphere D about side AD is

(a) $\frac{2}{5}mr^2$ (b) $\frac{2}{3}m(r^2+a^2)$

(c) $\frac{m}{5}(3a^2+4r^2)$ (d) $\frac{m}{5}(5a^2+2r^2)$

10. The moment of inertia of the system of four spheres about diagonal AB is

(a) $\frac{m}{5}(8r^2+5a^2)$ (b) $\frac{m}{5}(7r^2+4a^2)$

(c) $\frac{m}{5}(5r^2+8a^2)$ (d) $\frac{m}{5}(3r^2+5a^2)$

11. The moment of inertia of the system of four spheres about side AD is

(a) $\frac{2m}{5}(2r^2+5a^2)$ (b) $\frac{m}{5}(7r^2+5a^2)$

(c) $\frac{2m}{5}(4r^2+5a^2)$ (d) $\frac{m}{5}(3r^2+5a^2)$

SOLUTION

Refer to Fig. 8.91.

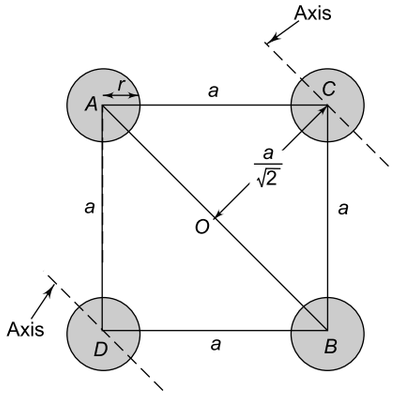


Fig. 8.91

The moment of inertia of spheres A and B about their common diameter $AB = \frac{2}{5}mr^2$ each. Also the moment of inertia of spheres C and D about an axis passing through their centre and parallel to $AB = \frac{2}{5}mr^2$ each. The distance of this axis (shown by broken lines) from the diagonal $AB = a/\sqrt{2}$. From the parallel axes theorem, the moment of inertia of spheres C and D about diagonal AB is

$$\frac{2}{5}mr^2 + m(CO)^2 = \frac{2}{5}mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{2}{5}mr^2 + \frac{ma^2}{2}$$

6. The correct choice is (b).

7. The correct choice is (d).

8. The correct choice is (b).

9. The correct choice is (a).

10. The moment of inertia of the system of four spheres about diagonal AB is

I_{AB} = MI of A about AB + MI of B about AB + MI of C about AB + MI of D about AB

$$= \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + \frac{1}{2}ma^2 + \frac{2}{5}mr^2 + \frac{1}{2}ma^2$$

$$= \frac{8}{5}\, mr^2 + ma^2 = m\left(\frac{8r^2}{5} + a^2\right)$$

The correct choice is (a).

11. Moment of inertia of sphere A about side AD = moment of inertia of sphere D about side AD = $\frac{2}{5}\, mr^2$. Using the parallel axes theorem, moment of inertia of sphere C about AD = moment of inertia of sphere B about $AD = \frac{2}{5}\, mr^2 + ma^2$. Hence the moment of inertia of the system of four spheres about side AD is

I_{AD} = MI of A about AD + MI of D about AD + MI of B about AD + MI of C about AD

$$= \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + ma^2 + \frac{2}{5}mr^2 + ma^2$$

$$= \frac{8}{5}\, mr^2 + 2\, ma^2 = m\left(\frac{8r^2}{5} + 2a^2\right)$$

The correct choice is (c).

Questions 12 to 15 are based on the following passage

Passage IV

A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has a elevated section and a horizontal part. The horizontal part is 1.0 m above the ground and the top of the track is 2.4 m above the ground. (See Fig. 8.92)

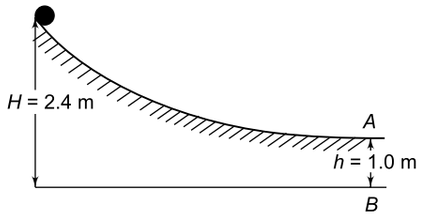


Fig. 8.92

12. If g = 10 ms^{-2}, the horizontal velocity when the sphere reaches point A is

(a) $\sqrt{5}$ ms^{-1} (b) $2\sqrt{5}$ ms^{-1}

(c) $\sqrt{7}$ ms^{-1} (d) $2\sqrt{7}$ ms^{-1}

13. If g = 10 ms^{-2}, the time taken by the sphere to fall through h = 1.0 m is

(a) $\frac{1}{\sqrt{5}}$ s (b) $\frac{2}{\sqrt{5}}$ s

(c) 0.1 s (d) 0.2 s

14. If g = 10 ms^{-2}, the distance on the ground with respect to point B (which is vertically below the end A of the track) is

(a) 1.0 m (b) 1.4 m

(c) 2.0 m (d) 2.8 m

15. Choose the correct statement/statements from the following.

(a) During its motion as a projectile after the sphere leaves the track at A, it will stop rotating.

(b) During its motion as a projectile after point A, the sphere will continue to rotate about its centre of mass.

(c) Due to rotation, the horizontal range of the sphere will be less than that found in Q.14 above.

(d) The rotation of the sphere has no effect on the horizontal range found in Q.14.

SOLUTION

12. The loss in potential energy when the sphere moves from the top of the track to point A = gain in total kinetic energy (translational and rotation), i.e.

$$Mg(H - h) = \frac{1}{2}\, Mv^2 + \frac{1}{2}\, I\omega^2$$

where $I = \frac{2}{5}\, MR^2$ and $\omega = \frac{v}{R}$. Thus

$$Mg(H - h) = \frac{1}{2}\, Mv^2 + \frac{1}{2} \times \frac{2}{5}\, MR^2 \times \left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}\, Mv^2 + \frac{1}{5}\, Mv^2 = \frac{7}{10}\, Mv^2$$

or $$v = \left[\frac{10(H-h)g}{7}\right]^{1/2}$$

$$= \left[\frac{10 \times (2.4 - 1.0) \times 10}{7}\right]^{1/2} = 2\sqrt{5} \text{ ms}^{-1}$$

The correct choice is (b).

13. Since the vertical component of velocity is zero, the time of flight is

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 1.0}{10}} = \frac{1}{\sqrt{5}}$ s, which is choice (a).

14. Horizontal range $= vt = 2\sqrt{5} \times \frac{1}{\sqrt{5}} = 2.0$ m, which is choice (c).

15. During its flight as a projectile from point A to the point where it hits the ground, the sphere will continue to rotate about the centre of mass because during the flight, the angular momentum of the sphere remains conserved. Due to this clockwise spinning of the sphere, the horizontal range decreases. This follows from Bernoulli's principle. Hence the correct choices are (b) and (c).

Questions 16 to 18 are based on the following passage

Passage V

A solid sphere of mass M and radius R is released from rest at the top of a frictionless inclined plane of length s and inclination θ. In case (a) it rolls down the plane without slipping and in case (b) it slides down the plane.

16. The ratio of the acceleration of the sphere in case (a) to that in case (b) is

(a) 1 (b) $\frac{2}{3}$

(c) $\frac{5}{7}$ (d) $\frac{7}{9}$

17. The ratio of the velocity of the sphere when it reaches the bottom of the plane in case (a) to that in case (b) is

(a) $\sqrt{2}$ (b) $\sqrt{\frac{2}{3}}$

(c) $\sqrt{\frac{5}{7}}$ (d) $\frac{\sqrt{7}}{3}$

18. The ratio of the time taken by the sphere to reach the bottom in case (a) to that in case (b) as

(a) 1 (b) $\sqrt{\frac{3}{2}}$

(c) $\sqrt{2}$ (d) $\sqrt{\frac{2}{3}}$

SOLUTION

16. In case (a), the acceleration down the plane is

$$a_1 = \frac{g\sin\theta}{\left(1+\frac{I}{MR^2}\right)} = \frac{5}{7}g\sin\theta \quad \left(\because I = \frac{2}{5}MR^2\right)$$

In case (b), the acceleration is

$$a_2 = g\sin\theta$$

Hence the correct choice is (c).

17. Using $v^2 = 2as$, we find that

$\frac{v_1}{v_2} = \sqrt{\frac{5}{7}}$, which is choice (c).

18. From $s = \frac{1}{2}at^2$, we find that the correct choice is (d).

Questions 19 to 22 are based on the following passage

Passage VI

Two blocks of masses $m_1 = 3\,m$ and $m_2 = m$ are attached to the ends of a string which passes over a frictionless fixed pulley (which is a uniform disc of mass $M = 2\,m$ and radius R) as shown in Fig. 8.93. The masses are then released.

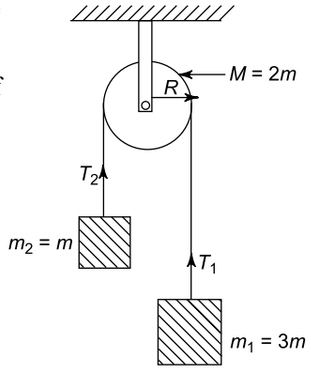


Fig. 8.93

19. The acceleration of the system is

(a) $2\,g$

(b) $\frac{2g}{3}$

(c) $\frac{2g}{5}$

(d) $\frac{3g}{7}$

20. Tension T_1 is

(a) $\frac{mg}{5}$ (b) $\frac{3mg}{5}$

(c) $\frac{7mg}{5}$ (d) $\frac{9mg}{5}$

21. Tension T_2 is

(a) $\frac{9mg}{5}$ (b) $\frac{7mg}{5}$

(c) mg (d) $\frac{3mg}{5}$

22. The magnitude of torque on the pulley is

(a) $2\,mgR$ (b) $\frac{2mgR}{3}$

(c) $\frac{2mgR}{5}$ (d) $3\,mgR$

SOLUTION

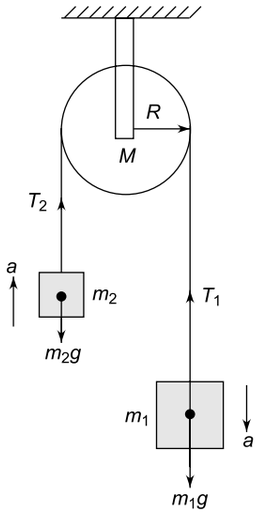


Fig. 8.94

Refer to Fig. 8.94. Since the pulley has a finite mass, two tensions T_1 and T_2 will not be equal. If a is the acceleration of the system, the equations of motion of masses

$$m_1g - T_1 = m_1a \quad \Rightarrow \quad 3mg - T_1 = 3ma \qquad (1)$$

and $$T_2 - m_2g = m_2a \quad \Rightarrow \quad T_2 - mg = ma \qquad (2)$$

The resultant tension $(T_1 - T_2)$ exerts a torque on the pulley which is given by

$$\tau = (T_1 - T_2)R \qquad (3)$$

Also, $\tau = I\alpha$ where $I = \frac{1}{2}MR^2$ and α is the angular acceleration which is given by

$$\alpha = \frac{a}{R}$$

$$\therefore \qquad \tau = \frac{1}{2}MR^2 \times \frac{a}{R} = \frac{1}{2}MRa = mRa \qquad (4)$$
$$(\because M = 2m)$$

From Eqs. (3) and (4) we get

$$T_1 - T_2 = ma \qquad (5)$$

19. Using Eqs. (1) and (2) and (5) and solving for a we get $a = \frac{2g}{5}$. Hence the correct choices is (c).

20. From Eqs. (1) and (2) we get $T_1 = 3m(g - a) = 3m\left(g - \frac{2g}{5}\right) = \frac{9mg}{5}$; which is choice (d).

21. Eqs. (1) and (2) give $T_2 = m(g + a) = m\left(g + \frac{2g}{5}\right) = \frac{7mg}{5}$ so choice (b) is correct.

22. From Eq. (4) $\tau = mRa = mR \times \frac{2g}{5} = \frac{2mgR}{5}$, which is choice (c).

Questions 23 to 26 are based on the following passage

Passage VII

A small solid sphere of mass m rolls without slipping on the track shown in Fig. 8.95. The radius of the circular part of the track is R. The sphere starts from rest from point P at a height $H = 4.5\,R$ above the bottom.

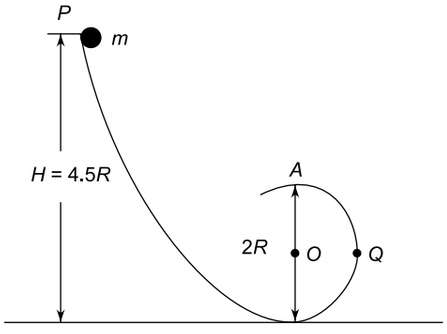


Fig. 8.95

23. The speed of the sphere when it reaches point Q on the track is

(a) $\sqrt{2gR}$ (b) $\sqrt{3gR}$

(c) $\sqrt{5gR}$ (d) $\sqrt{7gR}$

24. The horizontal force acting on the sphere when it is at point Q is

(a) mg (b) $2\ mg$

(c) $3\ mg$ (d) $5\ mg$

25. The magnitude of the force acting on the sphere when it is at point Q is

(a) $\sqrt{4.5}\ mg$ (b) $\sqrt{5}\ mg$

(c) $\sqrt{26}\ mg$ (d) $3\sqrt{3}\ mg$

26. What is the minimum value of height H so that the sphere can reach the top A of the circle?

(a) 2.4 R (b) 2.5 R

(c) 2.6 R (d) 2.7 R

SOLUTION

23. Loss of P.E. at Q = Gain in K.E.

$$mg(H-R)=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$$

$$\Rightarrow mg(4.5R-R)=\frac{1}{2}mv^2+\frac{1}{2}\times\left(\frac{2}{5}mR^2\right)\times\left(\frac{v}{R}\right)^2$$

$$=\frac{1}{2}mv^2+\frac{1}{5}mv^2=\frac{7}{10}mv^2$$

which gives $v=\sqrt{5gR}$. Hence the correct choice is (c).

24. At point Q the velocity is directed tangentially. Thus, the horizontal force acting on the sphere at point Q is the centripetal force directed towards the centre O of the circular part of the track and is given by

$$F_h=\frac{mv^2}{R}=\frac{m}{R}\times(5gR)=5mg,$$ which is choice (d).

25. The vertical force acting on the sphere is

F_v = weight of the sphere = mg

$\therefore$ Net force $F=\sqrt{F_h^2+F_v^2}=\sqrt{26}\,mg$

Thus the correct choice is (c).

26. The sphere will rise to point A if it has a minimum speed at A which satisfies

$$\frac{mv_A^2}{R}=mg \quad\Rightarrow\quad v_A=\sqrt{Rg}$$

The minimum value of H is given by

$$mg(H_{\min}-2R)=\frac{7}{10}\,mv_A^2=\frac{7}{10}mRg$$

$\Rightarrow$ $H=2R+0.7R=2.7R$, which is choice (d).

Questions 27 to 30 are based on the following passage

Passage VIII

A uniform disc of mass M and radius R rolls without slipping down a plane inclined at an angle θ with the horizontal.

27. The acceleration of the centre of mass of the disc is

(a) $g\sin\theta$ (b) $\frac{2g\sin\theta}{3}$

(c) $\frac{g\sin\theta}{3}$ (d) $\frac{2g\cos\theta}{3}$

28. The frictional force on the disc is

(a) $\frac{Mg\sin\theta}{3}$ (b) $\frac{2Mg\sin\theta}{3}$

(c) $Mg\sin\theta$ (d) none of these

29. The magnitude of torque acting on the disc is

(a) MgR (b) $MgR\sin\theta$

(c) $\frac{2MgR\sin\theta}{3}$ (d) $\frac{MgR\sin\theta}{3}$

30. If the disc is replaced by a ring of the same mass M and the same radius R, the ratio of the frictional force on the ring to that on the disc will be

(a) $\frac{3}{2}$ (b) 2

(c) $\sqrt{2}$ (d) 1

SOLUTION

The various forces acting on the disc are shown in the figure. As the disc rolls down the plane, the frictional force f acts upwards along the plane. This force f produces a torque which rotates the disc. If a is the linear acceleration of the centre of mass of the disc, the equation of motion is (see Fig. 8.96)

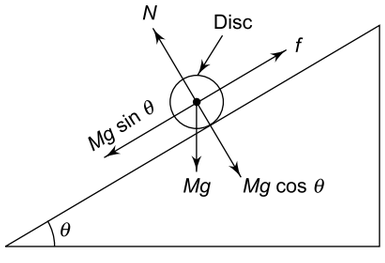


Fig. 8.96

$$Mg\sin\theta-f=Ma \qquad (1)$$

If α is the angular acceleration and I is the moment of inertia of the disc about the axis of rotation, the torque acting on it is

$$\tau=I\alpha$$

Now $I=\frac{1}{2}MR^2$, $\alpha=\frac{a}{R}$ and $\tau=fR$. Hence

$$fR=\frac{1}{2}MR^2\times\frac{a}{R}=\frac{1}{2}MRa$$

or $$f=\frac{1}{2}Ma \qquad (2)$$

27. From Eqs. (1) and (2), we get $a=\frac{2g\sin\theta}{3}$, which is choice (b).

28. $f = \frac{1}{2}Ma = \frac{Mg\sin\theta}{3}$, which is choice (a).

29. $\tau = fR = \frac{MgR\sin\theta}{3}$, which is choice (d).

30. For a ring, $I = MR^2$. The correct choice is (a).

Questions 31 to 33 are based on the following passage

Passage IX

A solid cylinder of mass M and radius R is mounted on a frictionless horizontal axle so that it can freely rotate about this axis. A string of negligible mass is wrapped round the cylinder and a body of mass m is hung from the string as shown in Fig. 8.97. The mass is released from rest.

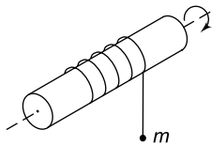


Fig. 8.97

31. The acceleration with which the mass falls is

(a) g (b) $\frac{mg}{M}$

(c) $\frac{mg}{(M+m)}$ (d) $\frac{2mg}{(M+2m)}$

32. The tension in the string is

(a) mg (b) $\frac{Mmg}{(M+m)}$

(c) $\frac{2Mmg}{(M+2m)}$ (d) $\frac{Mmg}{(M+2m)}$

33. The angular speed of the cylinder is proportional to h^n, where h is the height through which the mass falls. The value of n is

(a) zero (b) 1

(c) $\frac{1}{2}$ (d) 2

SOLUTION

Referring to Fig. 8.98, the acceleration a of the falling body is governed by the equation

$$ma = mg - T \qquad (1)$$

where T is the tension in the string.

Torque on cylinder is

$$\tau = TR = I\alpha$$

$$\Rightarrow \quad T = \frac{I\alpha}{R}$$

$$= \left(\frac{1}{2}MR^2\right) \times \frac{a}{R^2}$$

$$= \frac{1}{2}Ma \qquad (2)$$

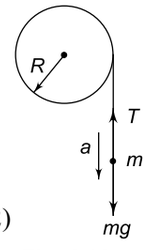


Fig. 8.98

31. From Eqs. (1) and (2) we get

$$a = \frac{2mg}{(M+2m)}$$, which is choice (d).

32. Equations (1) and (2) give $T = \frac{mMg}{(M+2m)}$, which is choice (d).

33. From conservation of energy, we have

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mR^2\omega^2 + \frac{1}{2} \times \left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \frac{1}{4}(2m + M)R^2\omega^2$$

$$\omega = \left[\frac{4mgh}{(M+2m)R^2}\right]^{1/2}$$

Thus $\omega \propto h^{1/2}$. Hence the correct choice is (c).

Questions 34 to 36 are based on the following passage

Passage X

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

IIT, 2007

34. The ratio x_1/x_2 is

(a) 2 (b) $\frac{1}{2}$

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

35. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(a) $\frac{2I\omega}{3t}$ (b) $\frac{9I\omega}{2t}$

(c) $\frac{9I\omega}{4t}$ (d) $\frac{3I\omega}{2t}$

36. The loss of kinetic energy during the above process is

(a) $\frac{I\omega^2}{2}$ (b) $\frac{I\omega^2}{3}$

(c) $\frac{I\omega^2}{4}$ (d) $\frac{I\omega^2}{6}$

SOLUTION

34. Rotational kinetic energy of disc A is

$$\frac{1}{2}\ kx_1^2 = \frac{1}{2}\ I\ (2\omega)^2 \qquad (1)$$

For disc B, the energy is

$$\frac{1}{2}\ kx_2^2 = \frac{1}{2}(2I)^2\omega \qquad (2)$$

From (1) and (2) we get $\frac{x_1^2}{x_2^2} = 2 \Rightarrow = \frac{x_1}{x_2} = \sqrt{2}$.

35. When disc B is brought in contact with disc A, let ω' be their common angular velocity and I' the moment of intertia of the sysem about the axis of rotation. From the law of conservation of angular momentum, we have

$$I'\omega' = I\ (2\omega) + (2I)\omega = 4I\omega$$

Since the discs are mounted coaxially, $I' = I + 2I = 3I$

$$\therefore \quad 3\ I\omega' = 4I\omega \Rightarrow \omega' = \frac{4\omega}{3}$$

Now, change in angular momentum of disc A

$$= I\ (2\omega) - I\left(\frac{4\omega}{3}\right) = \frac{2I\omega}{3}$$

Frictional torque on A is

τ = rate of change of angular mmentum

$$= \frac{\text{change in angular momentum}}{\text{Time}} = \frac{2I\omega}{3t}$$

36. Initial kinetic energy of the system is

$$E_1 = \frac{1}{2}\ I(2\omega)^2 + \frac{1}{2}\ (2I)\omega^2 = 3I\omega^2$$

Final kinetic energy of the system is

$$E_2 = \frac{1}{2}I'(\omega')^2 = \frac{1}{2}(3I)\left(\frac{4\omega}{3}\right)^2 = \frac{8}{3}\ I\omega^2$$

$\therefore$ Loss of kinetic energy $= E_1 - E_2$

$$= 3I\omega^2 - \frac{8}{3}\ I\omega^2 = \frac{1}{3}\ I\omega^2$$

Questions 37 to 39 are based on the following passage

Passage XI

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in Fig. 8.99. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. the unstretched length of the each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ.

37. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is.

(a) $-kx$ (b) $-2kx$

(c) $-\frac{2kx}{3}$ (d) $-\frac{4kx}{3}$

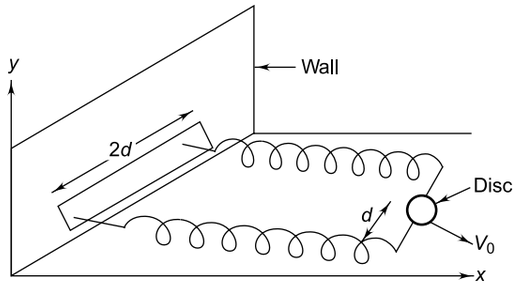


Fig. 8.99

IIT, 2008

38. The centre of mass of the disk undregoes simple harmonic motion with angular frequency ω equal to

(a) $\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{2k}{M}}$

(c) $\sqrt{\frac{2k}{3M}}$ (d) $\sqrt{\frac{4k}{3M}}$

39. The maximum value of V_0 for which the disk will roll without slipping is

(a) $\mu g\sqrt{\frac{M}{k}}$ (b) $\mu g\sqrt{\frac{M}{2k}}$ (c) $\mu g\sqrt{\frac{3M}{k}}$ (d) $\mu g\sqrt{\frac{5M}{2k}}$

SOLUTION

37. Let α be the linear acceleration of the disk. For translational motion (see Fig. 8.100)

$$Ma = f - 2kx \qquad (1)$$

Let α be the angular acceleration of the disc, then for rotational motion

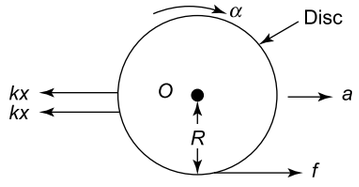


Fig. 8.100

Torque $\tau = I\alpha$

or $\quad -fR = \left(\frac{1}{2}MR^2\right)\times\frac{a}{R}\ (\because\ a = \alpha R) \qquad (2)$

Eliminating f from Eqs. (1) and (2), we get

$$a = -\frac{4kx}{3M}$$

Net external force $F = Ma = -\frac{4kx}{3}$.

38. Acceleration $a = -\frac{4kx}{3M} = -\omega^2 x$. Hence the motion is simple harmonic whose angular frequency is

$$\omega = \sqrt{\frac{4k}{3M}}.$$

39. When V_0 is maximum, x is maximum ($= x_{max}$) and the frictional force is maximum and is given by $f_{max} = \mu Mg$.

From Eqs. (1) and (2) above, we have

$$Ma = \mu Mg - 2kx_{max} \qquad (3)$$

and $\quad -\mu MgR = \frac{1}{2}MRa \qquad (4)$

Eliminating a from Eqs. (3) and (4), we get

$$kx_{max} = \frac{3}{2}\mu Mg \qquad (5)$$

Maximum K.E. of disc $= \frac{3}{4}MV^2_{max} = kx^2_{max}$ gives

$$V_{max} = \mu g\sqrt{\frac{3M}{k}} \quad \text{[(use eq.5)]}$$

SECTION IV

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. Statement-1
A solid cylinder of mass M and radius R rolls down an inclined plane of height h. The rotational kinetic energy of the cylinder when it reaches the bottom of the plane is $Mgh/3$.

Statement-2
The total energy of the cylinder remains constant throughout its motion.

2. Statement-1
Two bodies A and B initially at rest, of masses $2m$ and m respectively move towards each other under mutual gravitational force of attraction. At the instant when the speed of A is v and that of B is $2v$, the speed of the centre of mass of system is $4v/3$.

Statement-2
The speed of the centre of mass of a system changes if an external force acts on the system.

3. **Statement-1**
The angular momentum of a particle moving in a circular orbit with a constant speed remains conserved about any point on the circumference of the circle.
Statement-2
If no net torque acts, the angular momentum of a system is conserved.

4. **Statement-1**
Two blocks of masses M and m (with $M > m$) are connected by a spring of negligible mass and placed on a horizontal frictionless surface. An impulse gives a velocity V to the heavier block in the direction of the lighter block. The velocity of the centre of mass is
$$v_{CM} = \frac{MV}{(M+m)}$$
Statement-2
The principle of conservation of linear momentum is applicable to the centre of mass motion.

5. **Statement-1**
A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is $ML\,\omega^2$.
Statement-2
The entire mass of the liquid can be regarded as being concentrated at the centre of mass of the tube.

6. **Statement-1**
A thin wire of length L and mass m is bent into a circular loop of radius r as shown in Fig. 8.101. The moment of inertia of the loop about the XX' is $3mL^2/8\pi^2$.

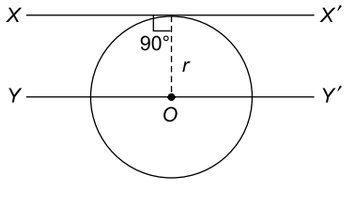


Fig. 8.101

Statement-2
According to the parallel axes theorem, the moment of inertia of the loop about XX' = moment of inertia about $YY' + mr^2$.

7. **Statement-1**
A thin uniform rod AB of mass M and length L is hinged at one end A to the horizontal floor. Initially it stands vertically. It is allowed to fall freely on the floor in the vertical plane. The angular velocity of the rod when its ends B strikes the floor is $\sqrt{3g/L}$.
Statement-2
The angular momentum of the rod about the hinge remain constant throughout its fall to the floor.

8. **Statement-1**
For a particle moving in a circle with a constant speed, the direction of the centripetal force depends on whether the particle is moving clockwise or anticlockwise along the circle.
Statement-2
The centripetal force is directed radially towards the centre of the circle.

9. **Statement-1**
A body tied to a string is moved in a circle with a uniform speed. If the string suddenly breaks, the angular momentum of the body becomes zero.
Statement-2
The torque on the body equals the rate of change of angular momentum.

10. **Statement-1**
A solid cylinder and a solid sphere have the same mass M and the same radius R. If torques of equal magnitude are applied to them for the same time, the sphere will acquire greater angular speed.
Statement-2
For a given torque, the angular acceleration is inversely proportional to the moment of inertia.

11. **Statement-1**
If a disc rotating about its axis with a certain angular speed is gently placed on a horizontal frictionless surface, it will roll along the surface with the same angular speed.
Statement-2
No torque acts on the disc if the friction is absent.

12. **Statement-1**
Two solid spheres of the same radius, one made of steel and the other of aluminium are released from rest from the top of an inclined plane at the same time. The two spheres will reach the bottom at the same time.
Statement-2
The linear acceleration down the plane is independent of the mass of the sphere.

13. **Statement-1**
A sphere and a cylinder slide without rolling from rest from the top of an inclined plane. They will reach the bottom with the same speed.

Statement-2
Bodies of all shapes, masses and sizes slide down a plane with the same acceleration.

14. Statement-1
If a cylinder rolling with angular speed ω suddenly breaks up into two equal halves of the same radius, the angular speed of each piece becomes 2ω.

Statement-2
If no external torque acts, the angular momentum of a system is conserved.

15. Statement-1
Friction is necessary for a body to roll on a surface.

Statement-2
Friction provides the necessary tangential force and torque.

16. Statement-1
A sphere is rolling on a rough surface in the direction of the arrow as shown in Fig. 8.102. The force of friction at the point of contact will be in the direction of the arrow.

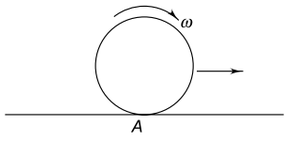


Fig. 8.102

Statement-2
Friction opposes motion.

17. Statement-1
During perfect rolling, the force of friction becomes zero.

Statement-2
The speed at the point of contact becomes zero.

18. Statement-1
During rolling, the acceleration of the point of contact is non zero.

Statement-2
The direction of the velocity changes with time.

SOLUTIONS

1. The correct choice is (a).
From the law of conservation of energy, we have
Potential energy = Translational kinetic energy + Rotational kinetic energy

or $$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$$

or $$Mgh = \frac{1}{2} MR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \frac{3}{4} MR^2\omega^2$$

or $$\omega^2 = \frac{4gh}{3R^2}$$

Now the rotational kinetic energy = $\frac{1}{2} I\omega^2$.

Substituting for ω^2 and I, we have

Rotational kinetic energy = $\frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{4gh}{3R^2}$

$$= \frac{Mgh}{3}$$

2. The correct choice is (d). Mutual gravitational force is internal to the system. Since no external force acts on the system, the centre of mass (which is initially at rest) will remain at rest.

3. The correct choice is (d). Since the centripetal force is radial (directed towards the centre of the circle), the torque due to this force is zero about the centre. Hence, angular momentum remains conserved only about the centre of the circle.

4. The correct choice is (a). From the principle of conservation of linear momentum, we have

$$MV = (M+m)v_{CM}$$

which gives $$v_{CM} = \frac{MV}{(M+m)}$$

5. The correct choice is (d). The entire mass of the liquid can be regarded as being concentrated at the centre of mass of the tube which is at a distance of $r = \frac{L}{2}$ from the axis of revolution. The force exerted by the liquid at the other end of the tube is the centripetal force of a mass M revolving in a circle or radius $r = \frac{L}{2}$.

Thus

$$F_c = \frac{Mv^2}{r} = \frac{M(r\omega)^2}{r} = Mr\omega^2 = \frac{ML\omega^2}{2}$$

6. The correct choice is (a). The moment of inertia of the loop about an axis passing through the centre O is

$$I_O = \frac{1}{2} mr^2$$

From the parallel axes theorem, the moment of inertia about XX' is

$$I = I_O + mr^2 = \frac{1}{2} mr^2 + mr^2 = \frac{3}{2} mr^2$$

Now $L = 2\pi r$ or $r = L/2\pi$. Therefore,

$$I = \frac{3m}{2} \times \left(\frac{L}{2\pi}\right)^2 = \frac{3mL^2}{8\pi^2}$$

7. The correct choice is (c). Loss in P.E. = gain in rotational K.E. As the centre of mass of the rod falls through a distance $L/2$, the loss in P.E. = $\frac{MgL}{2}$.
Gain in K.E. = $\frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$
Equating the two, we have

$$\left(\frac{MgL}{2}\right) = \left(\frac{ML^2\omega^2}{6}\right) \quad \Rightarrow \quad \omega = \sqrt{\frac{3g}{L}}$$

8. The correct choice is (d).

9. The correct choice is (d). Since no external torque acts on the body even after the string breaks, the angular momentum will remain unchanged.

10. The correct choice is (a). For cylinder, $I_c = \frac{1}{2} MR^2$ and for sphere $I_s = \frac{2}{5} MR^2$. Also $\tau = I\alpha$ or $\alpha = \frac{\tau}{I}$.
For a given τ, α is inversely proportional to I. Hence

$$\frac{\alpha_s}{\alpha_c} = \frac{I_c}{I_s}$$

Since $I_c > I_s$; $\alpha_s > \alpha_c$. Since radius R is the same for both, $a_s > a_c$. Since time is the same, $\omega_s > \omega_c$.

11. The correct choice is (d). Since no torque acts, the disc will not roll on the surface; it will simply keep of rotating at the point where it is placed.

12. The correct choice is (a). The linear acceleration is

$$a = \frac{g\sin\theta}{\left(1 + \frac{I}{MR^2}\right)} = \frac{5g\sin\theta}{7} \qquad \left(\because I = \frac{2}{5}MR^2\right)$$

Thus the acceleration is independent of the mass of the sphere. Hence the two spheres of the same radius will reach at the same time.

13. The correct choice is (a). If a body slides down an inclined plane, its acceleration is

$$a = g(\sin\theta - \mu\cos\theta)$$

which depends only on g, θ and μ.

14. The correct choice is (a). If I is the moment of inertia of the complete cylinder, the moment of inertia of each piece becomes $I/2$. Since $L = I\omega$ is constant, the angular speed of each piece becomes 2ω.

15. The correct choice is (a).

16. The correct choice is (a). The direction of the linear velocity at A, the point of contact is to the left (opposite to the direction of the arrow). Since friction opposes motion, the direction of the frictional force at A will be in the direction of the arrow, i.e. in the direction along which the sphere is rolling.

17. The correct choice is (a). The effect of frictional force is to decrease the speed of the body at the point of contact. When speed is zero, perfect rolling begins and the force of friction becomes zero. Hence no work is done.

18. The correct choice is (a). Since the body is rotating while it is rolling, the direction of the velocity is changing with time. Hence the instantaneous acceleration of the point of contact is not zero.

SECTION V

Integer Answer Type

1. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in Fig. 8.103. Find the distance (in cm) of the centre of mass of the remaing part of the plate from the original centre of mass of the complete plate.

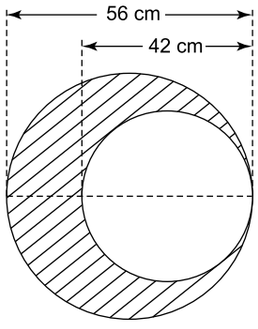


Fig. 8.103

IIT, 1980

2. A cubical block of side L and mass 300 g rests on a rough horizontal surface. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre of the face at a height $3L/4$ above the base. Find the minimum value of force F (in newton) for which the block does not slide before toppling. Take $g = 10\ \text{ms}^{-2}$

IIT, 1984

3. A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part. The horizontal part is 1.0 m above the ground and the top of the track is 2.4 m above the ground. (see Fig. 8.104)

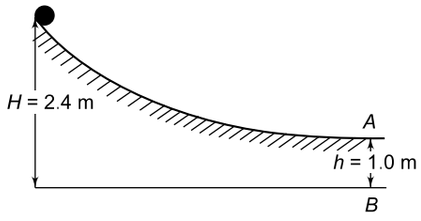


Fig. 8.104

If $g = 10\ \text{ms}^{-2}$, find the distance (in metre) of the point at which the sphere falls on the ground with respect to point B (which is vertically below the end A of the track).

IIT, 1987

4. A stone of mass m, tied to the end of a string, is whirled in a horizontal circle. The length of the string is gradually decreased, keeping the angular momentum of the stone about the centre of the circle constant. At an instant when the radius of the circle is r, the tension in the string is $T = Ar^{-n}$, where A is a constant. Find the value of n.

IIT, 1993

5. A block X of mass $m = 0.5$ kg is held by a string on a frictionless inclined plane of inclination $\theta = 30°$. The string is wound on a uniform solid cylindrical drum Y of mass $M = 2$ kg and radius $R = 0.2$ m as shown in Fig. 8.105. The drum is given an initial angular velocity such that the block X starts moving up the plane. If $g = 10\text{ms}^{-2}$, find the tension (in newton) in the string during the motion.

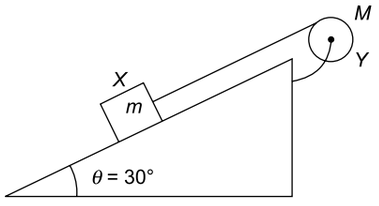


Fig. 8.105

IIT, 1994

6. A uniform disc of mass M and radius R is projected horizontally with velocity $u = 6\ \text{ms}^{-1}$ on a rough horizontal floor so that it starts off with a purely sliding motion at $t = 0$. After t_0 second, it acquires a purely roiling motion as shown in Fig. 8.106.
Calculate velocity v of the centre of mass (in ms^{-1}) of the disc at t_0.

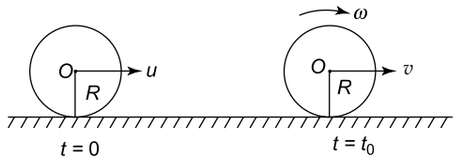


Fig. 8.106

IIT, 1997

7. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in Fig. 8.107. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of $0.3\ \text{m/s}^2$.
The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is $(P/10)$. Find the value of P. Take $g = 10\ \text{ms}^{-2}$.

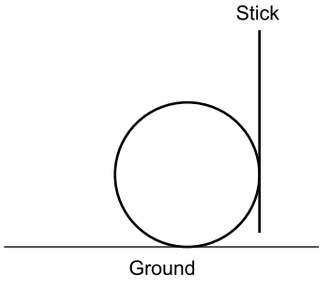


Fig. 8.107

IIT, 2011

8. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centres at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}\ \text{kg m}^2$, then N is (see Fig. 8.108)

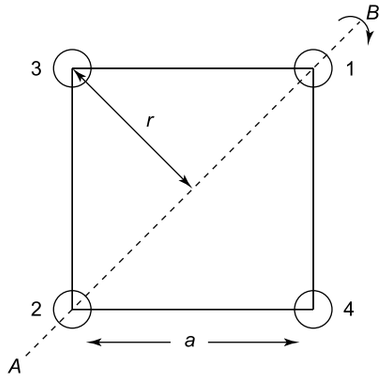


Fig. 8.108

IIT, 2011

SOLUTION

1. O is the original position of the centre of mass. O_1 is the position of the centre of mass of the cut out portion. Let O_2 be the position of the centre of mass of the remaining portion of the plate. Let W_1

be the weight of the cut out portion and W_2 that of the remaining portion. (see Fig. 8.109)

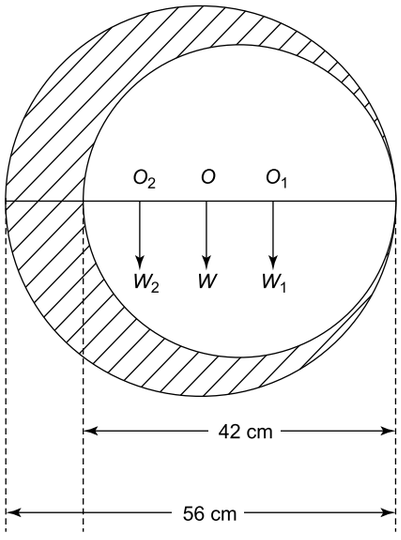


Fig. 8.109

Area of cut out portion $= p \times (21)^2$ cm^2

$= 441\pi$ cm^2

Area of the complete plate $= \pi \times (28)^2$ cm^2

$\therefore$ Area of the remaining portion $= \pi[(28)^2 - (21)^2]$

$=343\pi$ cm^2

Since the plate is uniform, mass is proportional to area. Hence

$$\frac{W_1}{W_2} = \frac{m_1 g}{m_2 g} = \frac{m_1}{m_2} = \frac{441\pi}{343\pi} = \frac{9}{7}$$

Taking moments about O, we have

$$W_1 \times OO_1 = W_2 \times OO_2$$

$$\Rightarrow \quad OO_2 = \frac{W_1}{W_2} \times OO_1$$

$$= \frac{9}{7} \times 7 \text{ cm} = 9 \text{ cm}$$

2. Torque of F about A is (Fig. 8.110)

$$\tau_1 = F \times \frac{3L}{4}$$

Since the weight mg of the block acts at the centre of the mass of the block, the torque of weight about A is

$$\tau_2 = mg \times \frac{L}{2}$$

The minimum force required to topple the block before sliding is given by

$$(\tau_1)_{\min} = \tau_2$$

$$\Rightarrow \quad F_{\min} \times \frac{3L}{4} = mg \times \frac{L}{2}$$

$$\Rightarrow \quad F_{\min} = \frac{2mg}{3} = \frac{2 \times 0.3 \times 10}{3} = 2\text{N}.$$

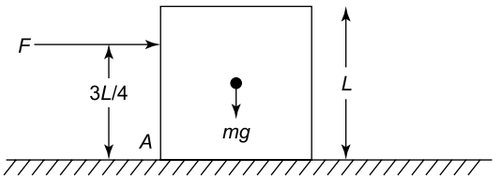


Fig. 8.110

3. The loss in potential energy when the sphere moves from the top of the track to point A = gain in total kinetic energy (translational and rotation), i.e.

$$Mg(H - h) = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$$

Where $I = \frac{2}{5} MR^2$ and $\omega = \frac{v}{R}$. Thus

$$Mg(H - h) = \frac{1}{2} Mv^2 + \frac{1}{2} \times \frac{2}{5} MR^2 \times \left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2} Mv^2 + \frac{1}{5} Mv^2 = \frac{7}{10} Mv^2$$

or $$v = \left[\frac{10(H-h)g}{7}\right]^{1/2}$$

$$= \left[\frac{10 \times (2.4 - 1.0) \times 10}{7}\right]^{1/2}$$

$$= 2\sqrt{5} \text{ ms}^{-1}$$

Since the vertical component of velocity is zero, the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1.0}{10}} = \frac{1}{\sqrt{5}} \text{ s}$$

Horizontal range $= vt = 2\sqrt{5} = 2.0$ m.

4. Angular momentum is mvr = constant (say k).

Therefore $v = \frac{k}{mr}$.

Also $$T = \frac{mv^2}{r} = \frac{mk^2}{m^2 r^3} = m^{-1} k^2 r^{-3} = Ar^{-3}$$

where $A = m^{-1}k^2$ is a constant. Comparing with $T = Ar^{-n}$, we get $n = 3$

5. The free body diagram of X is as shown in Fig. 8.111. If a is the linear acceleration of X, then

$$ma = T - mg \sin \theta \qquad (1)$$

If α is the angular acceleration of the drum, then

$$a = R\alpha$$

also torque $\tau = RT = I\alpha$

$$\Rightarrow \quad T = \frac{I\alpha}{R} = \frac{\frac{1}{2}MR^2\alpha}{R} = \frac{1}{2}MR\left(\frac{a}{R}\right)$$

$$\Rightarrow \quad T = \frac{1}{2}Ma \qquad (2)$$

Using (2) in (1)

$$ma = \frac{1}{2}Ma - mg \sin 30°$$

$$\Rightarrow \quad a = \frac{mg}{(M-2m)} \qquad (3)$$

Using (3) in (2) we have $T = \dfrac{Mmg}{2(M-2m)}$

$$= \frac{2\times 0.5\times 10}{2(2-2\times 0.5)} = 5 \text{ N}$$

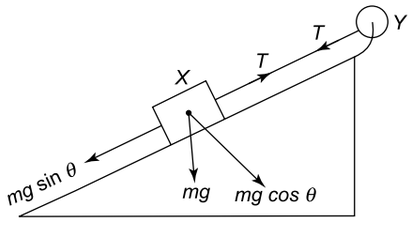


Fig. 8.111

6. Applying conservation of angular momentum about the point of contact of the disc with the horizontal surface, we have

$$MuR = MvR + \frac{1}{2}I\omega^2$$

$$= MvR + \frac{1}{2}\times (MR^2)\left(\frac{v}{R}\right)$$

$$\Rightarrow \quad u = v + \frac{v}{2}$$

$$\Rightarrow \quad v = \frac{2u}{3} = \frac{2\times 6}{3} = 4 \text{ ms}^{-1}$$

7. Refer to Fig. 8.112.

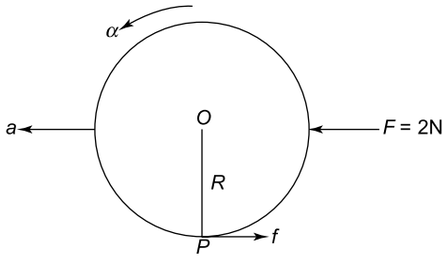


Fig. 8.112

$$f = \mu mg$$

The net torque about point P is

$$F \times R - fR = I_p\alpha$$

Where $\quad I_p = mR^2 + mR^2 = 2mR^2$

(parallel axes theorem)

and $\quad a = R\alpha$

Also $\quad f = \mu mg$

$$\therefore \quad F \times R - \mu mgR = (2\ mR^2) \times \left(\frac{a}{R}\right) = 2maR$$

$$\Rightarrow \quad F - \mu mg = 2\ ma$$

$$\Rightarrow 2 - \mu \times 2 \times 10 = 2 \times 2 \times 0.3$$

which gives $\mu = \dfrac{0.8}{2\times 10} = \dfrac{0.4}{10}$. Hence $P = 4$

8. Let M be the mass of each sphere and R its radius. The moment of inertia of the system about the diagonal AB of the square is

$$I = I_1 + I_2 + I_3 + I_4$$

where I_1, I_2, I_3 and I_4 are the moments of inertia of spheres 1, 2, 3 and 4 respectively about axis AB.

$$I_1 = I_2 = \frac{2}{5}MR^2$$

Using parallel-axes theorem

$$I_3 = I_4 = \frac{2}{5}\ MR^2 + Mr^2$$

$$= \frac{2}{5}MR^2 + M\left(\frac{a}{\sqrt{2}}\right)^2$$

$$\therefore \quad I = 2 \times \left(\frac{2}{5}MR^2\right) + 2\left[\frac{2}{5}MR^2 + \frac{Ma^2}{2}\right]$$

$$= \frac{8}{5}MR^2 + Ma^2$$

$$= \frac{8}{5} \times 0.5 \times \left(\frac{\sqrt{5}}{2}\times 10^{-2}\right)^2 + 0.5 \times (4 \times 10^{-2})^2$$

$$= 9 \times 10^{-4} \text{ kgm}^2. \text{ Hence } N = 9.$$

9 Chapter Gravitation

REVIEW OF BASIC CONCEPTS

9.1 NEWTON'S LAW OF GRAVITATION

Newton's law of universal gravitation states as follows:

'Any two particles of matter anywhere in the universe attract each other with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, the direction of the force being along the line joining the particles, i.e. (Fig. 9.1)

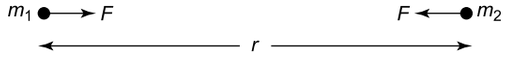


Fig. 9.1

$$F \propto \frac{m_1 m_2}{r^2}$$

where F is the magnitude of the force of attraction between two particles of masses m_1 and m_2 separated by a distance r.

In the form of an equation the law is written as

$$F = \frac{G m_1 m_2}{r^2}$$

where G is a constant called the universal gravitation constant. The value of this constant is to be determined experimentally and is found to be

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}.$$

9.2 GRAVITATIONAL FORCE DUE TO MULTIPLE MASSES

If a system consists of more than two masses, the gravitational force experienced by a given mass due to all other masses is obtained from the principle of superposition which states that '*the gravitational force experienced by one mass is equal to the vector sum of the gravitational forces exerted on it by all other masses taken one at a time.*'

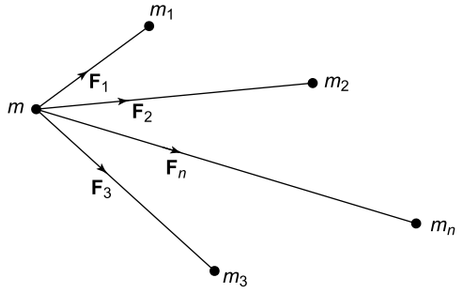


Fig. 9.2

The gravitational force on mass m due to masses m_1, m_2, m_3, ... m_n is given by (Fig. 9.2)

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{F}_n$$

NOTE

(1) Gravitational force is always attractive.
(2) Gravitational force between two masses does not depend the medium between them.
(3) Gravitational force acts along the straight line joining the centres of the two bodies.

EXAMPLE 9.1

Two bodies A and B of masses $m_1 = 1$ kg and $m_2 = 16$ kg respectively are placed 1.0 m apart. A third body C of mass $m = 3$ kg is placed on the line joining A and B. At what distance from A should C be placed so that it experiences no gravitational force?

SOLUTION

Let x metre be the distance between A and C (Fig. 9.3)

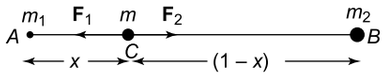


Fig. 9.3

Force exerted by A on C is

$$F_1 = \frac{G m_1 m}{x^2} \text{ directed towards } A$$

Force exerted by B on C is

$$F_2 = \frac{G m_2 m}{(1-x)^2} \text{ directed towards } B$$

C will experience no force if $F_1 = F_2$, i.e.

$$\frac{G m_1 m}{x^2} = \frac{G m_2 m}{(1-x)^2}$$

$$\Rightarrow \quad \frac{m_2}{m_1} = \frac{(1-x)^2}{x^2}$$

$$\Rightarrow \quad 16 = \frac{(1-x)^2}{x^2}$$

$$\Rightarrow \quad 4 = \frac{1-x}{x}$$

which gives $x = 0.2$ m. Note that if body C is placed to the left of body A or to the right of body B, it will experience a finite gravitational force.

EXAMPLE 9.2

Three bodies, each of mass m, are placed at the vertices of an equilateral triangle of side a as shown in Fig. 9.4. Find the magnitude and direction of the force experienced by the body at vertex A.

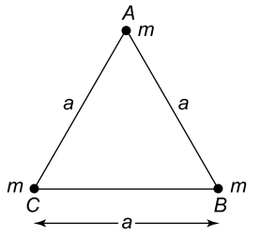


Fig. 9.4

SOLUTION

The forces exerted on the body at A by bodies at B and C are shown in Fig. 9.5.

$$F_1 = F_2$$

$$= \frac{Gm^2}{a^2} = F \text{ (say)}$$

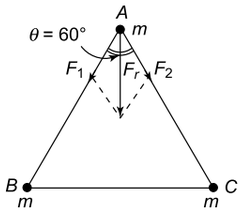


Fig. 9.5

The magnitude of the resultant force is

$$F_r = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$

$$= \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ}$$

$$= \sqrt{3}\, F = \sqrt{3}\,\frac{Gm^2}{a^2}, \text{ directed vertically downwards.}$$

9.3 ACCELERATION DUE TO GRAVITY

Considering the earth as an isolated mass, a force is experienced by a body near it. This force is directed towards the centre of the earth and has a magnitude mg, where g is the acceleration due to gravity.

$$F = mg = \frac{GmM}{R^2}$$

where M is the mass of the earth and R its radius (nearly constant for a body in the vicinity of the earth)

$$\therefore \quad g = \frac{GM}{R^2}$$

All bodies near the surface of the earth fall with the same acceleration which is directed towards the centre of the earth.

9.4 VARIATION OF G

1. Variation with altitude The acceleration due to gravity of a body at a height h above the surface of the earth is given by

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

where g is the acceleration due to gravity on the surface of the earth. If h is very small compared to R, we can use binomial expansion and retain terms of order h/R. We then get

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

Thus, the acceleration due to gravity decreases as the altitude (h) is increased.

2. Variation with depth The acceleration due to gravity at a depth d below the surface of the earth is given by

$$g_d = g\left(1 - \frac{d}{R}\right)$$

This equation shows that the acceleration due to gravity decreases with depth. At the centre of the earth where $d = R$, $g_d = 0$. Thus the acceleration due to gravity is maximum at the surface of the earth, decreases with increase in depth and becomes zero at the centre of the earth.

3. Variation with Latitude Due to the rotation of earth about its axis, the value of g varies with latitude, i.e. from one place to another on the earth's surface. At poles, the effect of rotation on g is negligible. At the equator, the effects of rotation on g is the maximum. In general, the value of acceleration due to gravity at a place decreases with the decrease in the latitude of the place.

The acceleration due to gravity at a place on earth where the latitude is ϕ is given by

$$g_\phi = g - \omega^2 R \cos^2 \phi \ ;$$

ω = angular velocity of rotation of earth.

At equator, $\phi = 0 \Rightarrow g_e = g - R\omega^2$ (minimum)

At poles, $\phi = 90^\circ \Rightarrow g_p = g$ (maximum)

$$g_\phi = g - 0.0337 \cos^2 \phi$$

Thus the value of g varies slightly from place to place on earth. Variation of g with altitude and depth is shown in Fig. 9.6.

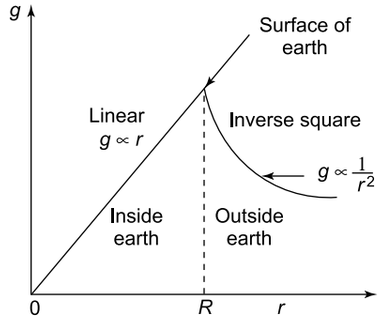


Fig. 9.6 Variation of g (gravitational acceleration)

EXAMPLE 9.3

The acceleration due to gravity on the surface of the moon is 1.67 ms^{-2}. The mass of the earth is about 80 times that of the moon. Estimate the ratio of the radius of the earth to that of the moon.

SOLUTION

$$g_e = \frac{G M_e}{R_e^2} \text{ and } g_m = \frac{G M_m}{R_m^2}$$

$$\therefore \quad \frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_m} \right)^{1/2}$$

$$= \left(80 \times \frac{1.67}{9.8} \right)^{1/2} \simeq 3.7$$

EXAMPLE 9.4

Assuming that the earth is a sphere of radius R, at what altitude will the value of acceleration due to gravity be half its value at the surface of the earth?

SOLUTION

$$\frac{g'}{g} = \left(\frac{R}{R+h} \right)^2$$

$$\Rightarrow \quad \frac{1}{2} = \left(\frac{R}{R+h} \right)^2$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{R}{R+h}$$

which gives $h = \left(\sqrt{2} - 1\right) R.$

EXAMPLE 9.5

A body weighs 63 N on the surface of the earth. How much will it weigh at a height equal to half the radius of the earth?

SOLUTION

$$W = mg = 63 \text{ N}$$

$$W' = mg' = mg \left(\frac{R}{R+h} \right)^2$$

$$\therefore \quad W' = W \left(\frac{R}{R+h} \right)^2$$

$$= 63 \times \left(\frac{R}{R + R/2} \right)^2 = 28 \text{ N}$$

EXAMPLE 9.6

Assuming the earth to be a sphere of uniform mass density, find the percentage decrease in the weight of a body when it is taken to the end of a tunnel 32 km below the surface of the earth. Radius of earth = 6400 km.

SOLUTION

$$g' = g \left(1 - \frac{d}{R} \right) = g \left(1 - \frac{32}{6400} \right) = \frac{199\, g}{200}$$

Decrease in weight = $mg - mg'$

$$= mg \left(1 - \frac{199}{200} \right) = \frac{mg}{200}$$

$$\text{Percentage decrease} = \frac{mg/200}{mg} \times 100 = 0.5\%$$

EXAMPLE 9.7

At what depth below the surface of the earth will the value of acceleration due to gravity become 90% of its value at the surface? $R = 6.4 \times 10^6$ m.

SOLUTION

$$g' = 0.9\ g. \qquad g' = g\left(1 - \frac{d}{R}\right)$$

$$\therefore \qquad 0.9 = 1 - \frac{d}{R}$$

$$\Rightarrow \qquad d = 0.1\ R = 6.4 \times 10^5 \text{ m}$$

9.5 GRAVITATIONAL FIELD INTENSITY

Just as the region around a magnet has magnetic field and the region around a charge has electric field, the region around a mass has gravitational field.

The gravitational field intensity (or simply gravitational field) at a point is defined as the gravitational force experienced by a unit mass placed at that point.

Consider the gravitational field of a body of mass M. To find the strength of the field at a point P at a distance r from M, we place a small mass m at P. The gravitational force exerted on m by M is (Fig. 9.7)

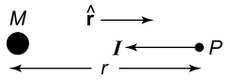


Fig. 9.7

$$F = \frac{GMm}{r^2}$$

By definition, the gravitational field (intensity) of M at P is given by

$$I = \frac{F}{m} = \frac{GM}{r^2}$$

I is a vector quantity. In vector form

$$\mathbf{I} = -\frac{GM}{r^2}\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from M to P, i.e radially away from M. The negative sign indicates that $\mathbf{I}$ directed radially inwards towards M.

The SI unit of I is N kg^{-1}.

In three dimensions, if mass M is located at the origin, the magnetic field at a $P(x, y, z)$ is given by

$$\mathbf{I} = -\frac{GM}{r^2}\hat{\mathbf{r}}$$

where $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ represents the position of point P with respect to mass M at the origin.

For a many body system, the principle of superposition holds for gravitational field (intensities) just as it holds for gravitational forces, i.e.

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \cdots + \mathbf{I}_n$$

where $\mathbf{I}_1, \mathbf{I}_2, \ldots \mathbf{I}_n$ are the gravitational field intensities at a point due to bodies of masses $M_1, M_2, \ldots M_n$.

For continuous mass distributions (i.e rigid bodies), we divide the body into an infinitely large number of infinitesimally small elements. Then the gravitational field intensity is given by

$$I = \int dI$$

Gravitational Field due to some continuous Mass Distributions

(1) Gravitational field due to a circular ring of mass M and radius R at a point at a distance r from the centre and on the axis of the ring is given by

$$I = \frac{GMr}{(R^2 + r^2)^{3/2}}$$

(2) Gravitational field due to a thin spherical shell of mass M and radius R at a point P at a distance $r > R$ from the centre of shell,

$$I = \frac{GM}{r^2} \text{ (outside the shell)}$$

Inside the shell, $I = 0$

(3) Gravitational field of a solid sphere of mass M and radius R is

$$I = \frac{GM}{r^2} \text{ for } r > R$$

$$I = \frac{GM}{R^2} \text{ for } r = R$$

$$I = \frac{GMr}{R^3} \text{ for } r < R$$

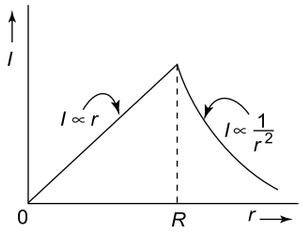


Fig. 9.8

Figure 9.8 shows the variation of the gravitational field of a solid sphere.

9.6 GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy of a system of two masses M and m held a distance r apart is defined as the amount of work done to bring the masses from infinity to their respective locations along any path and without any acceleration. (see Fig. 9.9)

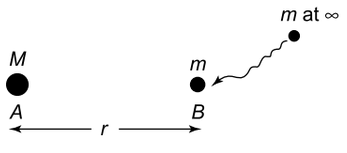


Fig. 9.9

Work done to bring mass M from infinity to A is $W_1 = 0$.
Work done to bring mass m from $r = \infty$ to $r = r$ is

$$W_2 = \int_{\infty}^{r} \mathbf{F} \cdot d\mathbf{r}$$

$$= \int_{\infty}^{r} F\, dr \cos\theta$$

Since mass M will attract mass m, angle θ between $\mathbf{F}$ and $d\mathbf{r}$ is zero. Hence

$$W_2 = \int_{\infty}^{r} \frac{GMm}{r^2} dr$$

$$= GMm \int_{\infty}^{r} r^{-2} dr = -\frac{GMm}{r}$$

Total work $\quad W = W_1 + W_2 = -\dfrac{GMm}{r}$

$\therefore$ Gravitational potential energy of the system is

$$U = W = -\frac{GMm}{r}$$

The zero of potential energy is assumed to be at $r = \infty$. The negative sign indicates the potential energy is negative for any finite separation between the masses and increases to zero at infinite separation.

Expression for Increase in Gravitational Potential Energy

If the body m is moved away from M, the potential energy of the system increases. [see Fig. 9.10]

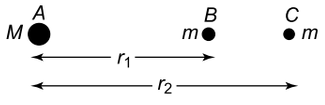


Fig. 9.10

$$\text{P.E. at } B = -\frac{GMm}{r_1}$$

$$\text{P.E. at } C = -\frac{GMm}{r_2}$$

$$\therefore \quad \text{Increase in P.E.} = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$$

$$= GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

If the body of mass M is the earth, then the increase in gravitational P.E. when a body of mass m is taken from the surface of the earth to a height h above the surface is given by (see Fig. 9.11), R = radius of the earth.

$$\Delta U = \text{P.E. at } Q - \text{P.E at } P$$

$$= -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GmMh}{R(R+h)}$$

If $h << R$; $\quad \Delta U = \dfrac{GmMh}{R^2} = mgh$

$$\left(\because \frac{GM}{R^2} = g\right)$$

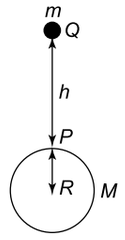


Fig. 9.11

9.7 GRAVITATIONAL POTENTIAL

Gravitational potential at a point P in the gravitational field of a body of M is defined as the amount of work done to bring a unit mass from infinity to that point along any path and without any acceleration, i.e., (Fig. 9.12)

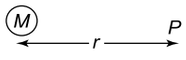


Fig. 9.12

$$V = \frac{W}{m} = -\frac{GMm}{r \times m}$$

$$\Rightarrow \qquad V = -\frac{GM}{r}$$

Potential V is a scalar quantity. Hence the gravitational potential at a point P due to a number of masses $m_1, m_2, \ldots m_n$ at distances $r_1, r_2, \ldots r_n$ respectively from P is given by

$$V = V_1 + V_2 + \cdots + V_n$$

$$= -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2} + \cdots + \frac{m_n}{r_n}\right)$$

The SI unit of V is J kg^{-1}.

Relation between Gravitational Field Intensity (I) and Gravitational Potential (V)

Gravitational field intensity and gravitational potential at a point are related as

$$I = -\frac{dV}{dr}$$

Gravitational Potential due to a Spherical Shell

(i) At a point outside the shell, $V = -\dfrac{GM}{r}$ (for $r > R$) where M is the mass and R is the radius of the shell.

(ii) At a point on the surface of the shell, $V = -\dfrac{GM}{R}$

(iii) At a point inside the shell, $V = -\dfrac{GM}{R}$ (for $r < R$)

Figure 9.13 shows the variation V with r for a spherical shell.

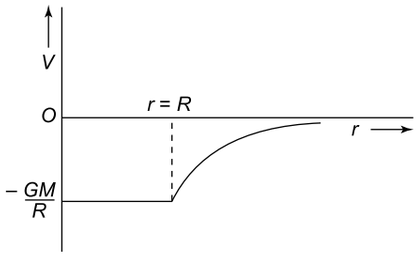


Fig. 9.13

Gravitational Potential due to a Solid sphere of mass M and radius R

(i) For points outside the sphere ($r > R$),

$$V = -\frac{GM}{r}$$

(ii) For points inside the sphere ($r < R$),

$$V = -\frac{3GM}{R^3}\left(\frac{R^2}{2} - \frac{r^2}{6}\right)$$

(ii) At the centre of the sphere ($r = 0$)

$$V = -\frac{3GM}{2R}$$

(iv) On the surface of the sphere ($r = R$)

$$V = -\frac{GM}{R}$$

EXAMPLE 9.8

Two masses $m_1 = 100$ kg and $m_2 = 8100$ kg are held 1 m apart.

(a) At what point on the line joining them is the gravitational field equal to zero? Find the gravitational potential at that point.

(b) Find the gravitational potential energy of the system. Given $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

SOLUTION

Gravitational field at P due to m_1 is [Fig. 9.14]

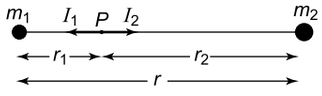


Fig. 9.14

$$I_1 = \frac{Gm_1}{r_1^2} \text{ directed towards } m_1$$

Gravitational field at P due to m_2 is

$$I_2 = \frac{Gm_2}{r_2^2} \text{ directed towards } m_2$$

Gravitational field at P will be zero if

$$I_1 = I_2 \quad \Rightarrow \quad \frac{Gm_1}{r_1^2} = \frac{Gm_2}{r_2^2}$$

or $$\frac{m_2}{m_1} = \left(\frac{r_1}{r_2}\right)^2 \quad \Rightarrow \quad 81 = \left(\frac{r_1}{1 - r_1}\right)^2$$

($\because r_2 = r - r_1$ and $r = 1$ m)

$$\therefore \qquad 9 = \frac{r_1}{1 - r_1} \quad \Rightarrow \quad r_1 = 0.1 \text{ m}$$

Gravitational potential at P is

$$V = V_1 + V_2 = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$$

$$= -6.67 \times 10^{-11} \times \left(\frac{100}{0.1} \times \frac{8100}{0.9}\right)$$

$$= -6.67 \times 10^{-7} \text{ J kg}^{-1}$$

(b) Gravitational potential energy of the system is

$$\text{G.P.E.} = -\frac{G m_1 m_2}{r}$$

$$= -\frac{6.67 \times 10^{-11} \times 100 \times 8100}{1}$$

$$= -5.4 \times 10^{-5} \text{ J}$$

EXAMPLE 9.9

Three equal masses, each equal to m, are kept at the vertices of an equilateral triangle of side a. Find the gravitational field and gravitational potential at the centroid of the triangle.

SOLUTION

Refer to Fig. 9.15.

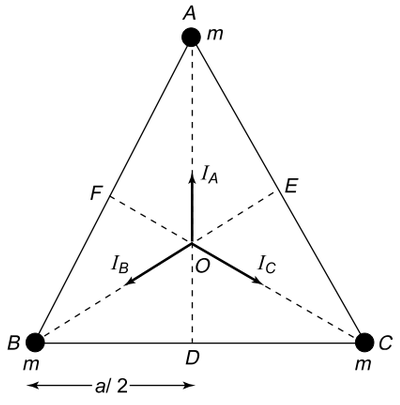


Fig. 9.15

The centroid O divides the lines AD, BE and CF in the ratio 2:1. Also $AO = BO = CO = r$ (say). Now $AO = \frac{2}{3} AD$ and

$$AD = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}\, a}{2}$$

$$\therefore \quad AO = \frac{2}{3} \times \frac{\sqrt{3}\, a}{2} = \frac{a}{\sqrt{3}}, \text{ i.e. } r = \frac{a}{\sqrt{3}}$$

The gravitational fields at O due to masses m at A, B and C are $I_A = I_B = I_C = \frac{Gm}{r^2} = I$. Their directions are shown in Fig. 9.15. The angle between any two of them is $\theta = 120°$.

The resultant of I_B and I_C is

$$I' = \sqrt{I^2 + I^2 + 2I^2 \cos 120°}$$

$= I$ directed vertically down.

I' will cancel with I_A. Hence the net gravitational field at O is zero.

The gravitational potential at O is

$$V = V_1 + V_2 + V_3$$

$$= -\frac{Gm}{r} - \frac{Gm}{r} - \frac{Gm}{r}$$

$$= -\frac{3Gm}{r} = -3\sqrt{3}\,\frac{Gm}{a} \qquad \left(\because r = \frac{a}{\sqrt{3}}\right)$$

EXAMPLE 9.10

The gravitational potential at a height h = 1600 km above the surface of the earth is -4.0×10^7 Jkg^{-1}. Assuming the earth to be a sphere of radius R = 6400 km, find the value of the acceleration due to gravity at that height.

SOLUTION

Let $r = R + h$. Then

$$V = -\frac{GM}{r}$$

and $$g' = \frac{GM}{r^2}$$

$$\therefore \quad \frac{g'}{|V|} = \frac{GM/r^2}{GM/r} = \frac{1}{r} = \frac{1}{R+h}$$

$$\Rightarrow \quad g' = \frac{|V|}{(R+h)} = \frac{4.0 \times 10^7 \text{ J kg}^{-1}}{(6.4 + 1.6) \times 10^6 \text{ m}}$$

$$= 5.0 \text{ J kg}^{-1} \text{ m}^{-1} = 5.0 \text{ ms}^{-2}$$

EXAMPLE 9.11

Two particles of masses m_1 and m_2 are initially at rest at an infinite distance apart. They begin to move towards each other due to gravitational attraction. Find the ratio of their accelerations and speeds when the separation between them becomes r.

SOLUTION

Since no external force acts on the system, the acceleration of their centre of mass is zero, i.e.

$$a_{CM} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$

$$\Rightarrow \quad 0 = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$

$$\Rightarrow \quad m_1 a_1 = -m_2 a_2$$

$$\Rightarrow \quad \frac{a_1}{a_2} = -\frac{m_2}{m_1}$$

The negative sign indicates that they move in opposite directions.

Let v_1 and v_2 be the speeds of two masses when they are at a distance r. Due to gravitational attraction, they gain speed as they approach each other. Hence their kinetic energy increases and gravitational potential energy (G.P.E) decreases. From the conservation of energy,

Loss in G.P.E. = gain in K.E.

$(\text{G.P.E.})_i - (\text{G.P.E.})_f = (\text{K.E.})_f - (\text{K.E.})_i$

$$\Rightarrow \quad 0 - \left(-\frac{Gm_1 m_2}{r}\right) = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - 0$$

$$\Rightarrow \quad \frac{G\, m_1\, m_2}{r} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \qquad \text{(i)}$$

Since no external force acts, the total momentum of the system is conserved, i.e., $p_i = p_f$ or

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$v_1 = \left(\frac{2Gm_2^2}{r(m_1 + m_2)}\right)^{1/2}$$

and $$v_2 = \left(\frac{2G\, m_1^2}{r(m_1 + m_2)}\right)^{1/2}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{m_2}{m_1}$$

9.8 ESCAPE VELOCITY

The escape velocity is the minimum velocity with which a body must be projected in order that it may escape the earth's gravitational pull. The magnitude of the escape velocity is given by

$$v_e = \sqrt{\frac{2MG}{R}}$$

where M is the mass of the earth and R its radius. Substituting the known values of G, M and R, we get $v_e = 11.2$ kms^{-1}. The expression for the escape velocity can be written in terms of g as

$$v_e = \sqrt{2gR}$$

The escape velocity is independent of the mass of the body and the direction of projection.

9.9 SATELLITES

A body moving in an orbit around a much larger and massive body is called a satellite. The moon is the natural satellite of the earth.

Orbital Velocity Let us assume that a satellite of mass m goes around the earth in a circular orbit of radius r with a uniform speed v. If the height of the satellite above the earth's surface is h, then $r = (R + h)$, where R is the mean radius of the earth. The centripetal force $\frac{mv^2}{r}$ necessary to keep the satellite in its circular orbit is provided by the gravitational force $\frac{GmM}{r^2}$ between the earth and the satellite. This means that

$$\frac{mv^2}{r} = G\frac{mM}{r^2}$$

where M is the mass of the earth. Thus

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$$

Now the acceleration due to gravity on earth's surface is given by

$$g = \frac{GM}{R^2}$$

or $$GM = gR^2$$

Substituting for GM we get

$$v = R\sqrt{\frac{g}{(R+h)}}$$

If the satellite is a few hundred kilometres above the earth's surface (say 100 to 300 km), we can replace $(R + h)$ by R. The error involved in this approximation is negligible since the radius of the earth $(R) = 6.4 \times 10^6$ m = 6400 km.

Thus, we may write

$$v = \sqrt{gR} = \sqrt{9.8 \times 6.4 \times 10^6}$$
$$= 7.9 \times 10^3 \text{ ms}^{-1} \simeq 8 \text{ km s}^{-1}$$

Periodic Time The periodic time T of a satellite is the time it takes to complete one revolution and it is given by (since $r = R + h$)

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

If $h << R$, we have $v = \sqrt{gR}$. Hence

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \simeq 2\pi\sqrt{\frac{R}{g}}$$

Total Energy of a Satellite

Total energy E = K.E. + P.E.

$$= \frac{1}{2}mv^2 - \frac{GmM}{r}$$
$$= \frac{GmM}{2r} - \frac{GmM}{r} \qquad \left(\because v = \sqrt{\frac{GM}{r}}\right)$$

$$\Rightarrow \qquad E = -\frac{GmM}{2r} = -\frac{GmM}{2(R+h)}$$

NOTE

(1) $E = \frac{\text{P.E.}}{2}$

(2) $E = -$ (K.E.)

(3) The total energy is negative which implies that the satellite is bound by the gravitational field of the earth. The binding energy = $\frac{GmM}{2r}$. This energy must be given to the orbiting satellite to escape to infinity.

Angular Momentum

The magnitude of angular momentum of a satellite is given by

$$L = mvr$$
$$= m\sqrt{\frac{GM}{r}}\, r$$

$$\Rightarrow \qquad L = m\sqrt{GMr}$$

Geostationary Satellites A Geostationary satellite is a particular type used in communication. A number of communication satellites are launched which remain in

fixed positions at a specified height above the equator. They are called *geostationary* or *synchronous satellites* used in international communication.

For a satellite to appear fixed at a position above a certain place on the earth, it must corotate with the earth so that its orbital period around the earth is exactly equal to the rotational period of the earth about its axis of rotation.

This condition is satisfied if the satellite is put in orbit at a height of about 36,000 km above the earth's surface.

Trajectory of a Satellite for Different Speeds

Orbital speed of a satellite is

$$v_0 = \sqrt{\frac{GM}{r}}$$

Escape velocity is

$$v_e = \sqrt{\frac{2GM}{r}}$$

Let v be the speed given to a satellite.

(i) If $v < v_0$, the satellite will follow an elliptical path with the centre of the earth as the farthest focus. The satellite will not complete the orbit and will fall to the earth.

(ii) If $v = v_0$, the satellite will follow a circular path with the centre of the earth as the centre of the circular orbit.

(iii) If $v_0 < v < v_e$, the satellite will follow an elliptical orbit with the centre of the earth as the nearer focus.

(iv) If $v = v_e$, the satellite will escape the gravitational pull of the earth following a parabolic path.

(v) If $v > v_e$, the satellite will escape the gravitational pull of the earth following a hyperbolic path.

9.10 KEPLER'S LAWS OF PLANETARY MOTION

Johannes Kepler formulated three laws which describe planetary motion. They are as follows:

1. *Law of orbits* Each planet revolves about the sun in an elliptical orbit with the sun at one of the focii of the ellipse. The orbit of a planet is shown in Fig. 9.16(a) in which the two focii F_1 and F_2, are far apart. For the planet earth, F_1 and F_2 are very close together. In fact, the orbit of the earth is practically circular.
2. *Law of areas* A line drawn from the sun to the planet (termed the radius) sweeps out equal areas in equal intervals of time. In Fig. 9.16(b) P_1, P_2, P_3 and P_4 represent positions of a planet at different times in its orbit and S, the position of the sun.

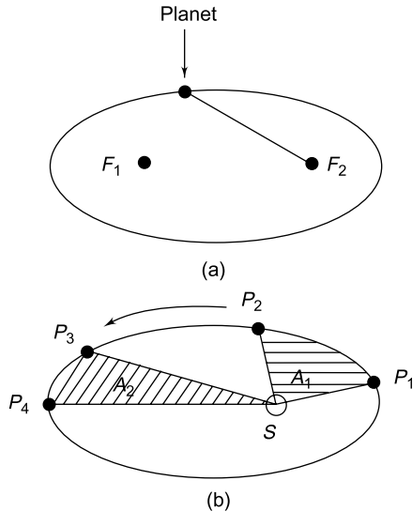


Fig. 9.16

According to Kepler's second law, if the time interval between P_1 and P_2 equals the time interval between P_3 and P_4, then area A_1 must be equal to area A_2. Also the planet has the greater speed in its path from P_1 to P_2 than in its path from P_3 to P_4.

3. *Law of periods* The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun. If T_1 represents the period of a planet about the sun, and r_1 its mean distance, then

$$T_1^2 \propto r_1^3$$

If T_2 represents the period of a second planet about the sun, and r_2 its mean distance, then for this planet

$$T_2^2 \propto r_2^3$$

These two relations can be combined since the factor of proportionality is the same for both. Thus

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

EXAMPLE 9.12

The mass of Jupiter is 318 times that of the earth and its radius is 11.2 times that of the earth. Calculate the escape velocity from Jupiter's surface. Given the escape velocity from earth'surface = 11.2 km s^{-1}.

SOLUTION

For Jupiter : $v_J = \sqrt{\frac{2M_J G}{R_J}}$

For Earth : $v_J = \sqrt{\frac{2M_E G}{R_E}}$

$$\therefore \quad \frac{v_J}{v_E} = \sqrt{\frac{M_J}{M_E} \times \frac{R_E}{R_J}}$$

$$= \sqrt{318 \times \frac{1}{11.2}} = 5.33$$

$\therefore \quad v_J = 5.33\ v_E = 5.33 \times 11.2 = 59.7$ km s^{-1}

EXAMPLE 9.13

Calculate the escape velocity of a body at a height 1600 km above the surface of the earth. Radius of earth = 6400 km.

SOLUTION

Work required to move a body of mass m from $r = R + h$ to $r = \infty$ is

$$W = \int_{R+h}^{\infty} F dr$$

$$= GmM \int_{R+h}^{\infty} \frac{dr}{r^2} = \frac{GmM}{R+h}$$

If v_e is the escape velocity, then

$$\frac{1}{2} m v_e^2 = \frac{GmM}{R+h}$$

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2gR^2}{R+h}} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

Given $R = 6.4 \times 10^6$ m, and $h = 1.6 \times 10^6$ m and $g = 9.8$ ms^{-2}

$$\therefore \quad v_e = \sqrt{\frac{2 \times 9.8 \times (6.4 \times 10^6)^2}{(6.4 + 1.6) \times 10^6}}$$

$$= 10 \times 10 \text{ ms}^{-1}$$

$$= 10 \text{ km s}^{-1}$$

EXAMPLE 9.14

A rocket is launched vertically from the surface of the earth with an initial velocity equal to half the escape velocity. Find the maximum height attained by it in terms of R where R is the radius of the earth. Ignore atmospheric resistance.

SOLUTION

On the surface of the earth, the total energy of the rocket is

$$E_i = \text{K.E.} + \text{P.E.} = \frac{1}{2} m v^2 - \frac{GmM}{R}$$

At the highest point, $v = 0$. If h is the maximum height attained, the energy of the rocket at height h is

$$E_f = -\frac{GmM}{(R+h)}$$

From conservation of energy, $E_i = E_f$, i.e.,

$$\frac{1}{2} m v^2 - \frac{GmM}{R} = -\frac{GmM}{(R+h)}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R} \times \frac{h}{R+h}} = v_e \sqrt{\frac{h}{R+h}}$$

Given $v = \frac{v_e}{2}$. Hence $\frac{v_e}{2} = v_e \sqrt{\frac{R}{R+h}} \quad \Rightarrow \quad h = \frac{R}{3}$

EXAMPLE 9.15

A body is dropped from a height h equal to $\frac{R}{2}$, where R is the radius of the earth. Show that it will hit the surface of the earth with a speed $v = v_e/\sqrt{3}$, where v_e is the escape velocity from the surface of the earth.

SOLUTION

Total energy of the body at height h is

$$E_i = -\frac{GmM}{(R+h)} \qquad \text{(i)}$$

Total energy when it hits the surface of the earth is

$$E_f = \frac{1}{2} m v^2 - \frac{GmM}{R} \qquad \text{(ii)}$$

From conservation of energy, $E_i = E_f$, i.e.

$$-\frac{GmM}{(R+h)} = \frac{1}{2} m v^2 - \frac{GmM}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R}} \times \sqrt{\frac{h}{(R+h)}}$$

$$= v_e \sqrt{\frac{R}{(R+h)}}$$

For $h = \frac{R}{2}$, $v = \frac{v_e}{\sqrt{3}}$.

EXAMPLE 9.16

Show that the minimum energy required to launch a satellite of mass m from the surface of the earth in a circular orbit at an altitude $h = R$, where R is the radius of the earth is $\frac{3mgR}{4}$ where M is the mass of the earth.

SOLUTION

Total energy of the satellite orbiting the earth is

$$E_1 = -\frac{GmM}{2r}$$

$$= -\frac{GmM}{2(R+h)} = -\frac{GmM}{4R} \qquad (\because\ h = R)$$

Total energy when the satellite was at rest on the surface of the earth is

$$E_2 = \text{K.E.} + \text{P.E.}$$

$$= 0 - \frac{GmM}{R} = -\frac{GmM}{R}$$

$\therefore$ Minimum energy required is

$$E_{\min} = E_1 - E_2$$

$$= -\frac{GmM}{4R} - \left(-\frac{GmR}{R}\right)$$

$$= \frac{3GmM}{4R} = \frac{3}{4}mgR$$

EXAMPLE 9.17

A satellite of mass $m = 100$ kg is in a circular orbit at a height $h = R$ above the surface of the earth where R is the radius of the earth. Find

(a) the acceleration due to gravity at any point on the path of the satellite,

(b) the gravitational force on the satellite and

(c) the centripetal force on the satellite.

SOLUTION

(a) $g' = g\left(\frac{R}{R+h}\right)^2 = 9.8 \times \left(\frac{R}{R+R}\right)^2 = \frac{9.8}{4}$

$= 2.45\ \text{ms}^{-2}$

(b) Gravitational force on satellite is

$$F_g = mg' = 100 \times 2.45 = 245\ \text{N}$$

(c) Centripetal force $F_c = \frac{mv^2}{r} = \frac{GmM}{r^2}$

$$\left(\because g = \frac{GM}{R^2}\right)$$

or $F_c = \frac{GmM}{R^2} \times \frac{R^2}{r^2}$

$$= mg \times \left(\frac{R}{R+h}\right)^2 = mg' = 245\ \text{N}.$$

NOTE

Since $F_g = F_c$, the satellite is a freely falling body and is, therefore, weightless.

EXAMPLE 9.18

A body projected vertically upwards from the surface of the earth with a certain velocity rises to a height of 10 m. How high will it rise if it is projected with the same velocity vertically upwards from a planet whose density is one-third that of the earth and radius half that of earth? Ignore atmospheric resistance.

SOLUTION

Since the kinetic energy of the body is the same in both the cases, loss in K.E. = gain in P.E. will be equal, i.e.,

$$mg_{\text{p}}h_{\text{p}} = mg_{\text{e}}h_e$$

$$\Rightarrow \qquad h_{\text{p}} = h_{\text{e}} \times \frac{g_{\text{e}}}{g_{\text{p}}}$$

Now $g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4\pi}{3} R^3\rho = \frac{4\pi}{3} GR\rho$

$$\therefore \qquad \frac{g_{\text{e}}}{g_{\text{p}}} = \frac{R_{\text{e}}}{R_{\text{p}}} \times \frac{\rho_{\text{e}}}{\rho_{\text{p}}} = 2 \times 3 = 6$$

$$\therefore \qquad h_{\text{p}} = 6h_{\text{e}} = 6 \times 10 = 60\ \text{m}$$

EXAMPLE 9.19

A satellite of mass 2000 kg is orbiting the earth at an altitude $R/2$, where R is the radius of the earth. What extra energy must be given to the satellite to increase its altitude to R? Given $R = 6.4 \times 10^6$ m.

SOLUTION

In the first case, $r_1 = R + \frac{R}{2} = \frac{3R}{2}$ and in the second case, $r_2 = R + R = 2R$.

Required energy $= E_2 - E_1$

$$= -\frac{GmM}{2r_2} - \left(-\frac{GmM}{2r_1}\right)$$

$$= -\frac{GmM}{4R} + \frac{GmM}{3R}$$

$$= \frac{GmM}{12R}$$

$$= \frac{mgR}{12} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

$$= \frac{2000 \times 9.8 \times \left(6.4 \times 10^6\right)}{12}$$
$$= 1.04 \times 10^8 \text{ J}$$

EXAMPLE 9.20

Two bodies of masses m_1 and m_2 are held at a distance r apart. Show that at the point where the gravitational field due to them is zero, the gravitational potential is given by

$$V = -\frac{G}{r}\left(m_1 + m_2 + 2\sqrt{m_1 m_2}\right)$$

SOLUTION

Let the gravitational field be zero at point P (Fig. 9.17). Then

$$\frac{G m_1}{r_1^2} = \frac{G m_2}{r_2^2}$$

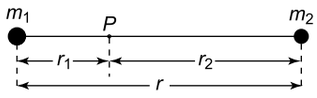


Fig. 9.17

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{\sqrt{m_1}}{\sqrt{m_2}}$$

$$\Rightarrow \quad \frac{r_1}{r - r_1} = \frac{\sqrt{m_1}}{\sqrt{m_2}}$$

$$\Rightarrow \quad r_1 = \frac{r\sqrt{m_1}}{\left(\sqrt{m_1} + \sqrt{m_2}\right)} \qquad \text{(i)}$$

Also $r_2 = r - r_1$

$$= r - \frac{r\sqrt{m_1}}{\left(\sqrt{m_1} + \sqrt{m_2}\right)}$$

$$= \frac{r\sqrt{m_2}}{\left(\sqrt{m_1} + \sqrt{m_2}\right)} \qquad \text{(ii)}$$

Gravitational potential at P is

$$V = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2} \qquad \text{(iii)}$$

Using (i) and (ii) in (iii) and simplifying, we get

$$V = -\frac{G}{r}\left(m_1 + m_2 + 2\sqrt{m_1 m_2}\right)$$

EXAMPLE 9.21

The distance of a planet from the sun is 10 times that of the earth. Find the period of revolution of the planet around the sun.

SOLUTION

From Kepler's law of periods, $T^2 \propto r^3$. Therefore

$$\frac{T_p^2}{T_e^2} = \frac{r_p^3}{r_e^3} = (10)^3 = 1000$$

$$\therefore \quad T_p = T_e \times \sqrt{1000} = 1 \text{ year} \times 31.6 = 31.6 \text{ years}$$

EXAMPLE 9.22

A satellite is revolving in a circular orbit close to the surface of the earth with a speed v. What minimum additional speed must be imparted to it so that it escapes the gravitational pull of the earth? Radius of earth = 6.4×10^6 m.

SOLUTION

$$v = \sqrt{gR} \text{ and } v_e = \sqrt{2gR}$$

$\therefore$ Additional speed required is

$$v_e - v = \left(\sqrt{2} - 1\right)\sqrt{gR}$$
$$= 0.414 \times \sqrt{9.8 \times 6.4 \times 10^6}$$
$$= 3.28 \times 10^3 \text{ ms}^{-1}$$
$$= 3.28 \text{ km s}^{-1}$$

EXAMPLE 9.23

A body of mass m is placed at the centre of a spherical shell of radius R and mass M. Find the gravitational potential on the surface of the shell.

SOLUTION

Gravitational potential on the surface of the shell due to body of mass m is

$$V_b = -\frac{Gm}{R}$$

Gravitational potential on the surface of the shell due to shell itself is

$$V_s = -\frac{GM}{R}$$

$$\therefore \quad V = V_b + V_s = -\frac{G}{R}(m + M)$$

EXAMPLE 9.24

A tunnel is drilled from the surface of the earth to its centre. A body of mass m is dropped into the tunnel. Find the speed with which the body hits the bottom of the tunnel. The mass of earth is M and its radius is R.

SOLUTION

Let v be the required speed. Gain in K.E. = loss in P.E. = P.E. at the surface – P.E. at the centre. The potential energy at the centre of the sphere = $-\frac{3}{2}\frac{GmM}{R}$.

Hence

$$\frac{1}{2}mv^2 = -\frac{GmM}{R} - \left(-\frac{3}{2}\frac{GmM}{R}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR} \quad \left(\because g = \frac{GM}{R^2}\right)$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. A body of mass m is placed at the centre of a spherical shell of radius R and mass M. The gravitational potential on the surface of the shell is

(a) $-\frac{G}{R}(M+m)$ (b) $-\frac{G}{R}(M-m)$

(c) $-\frac{G}{R}\left(\frac{mM}{M+m}\right)$ (d) $-\frac{G}{R}\left(\frac{mM}{M-m}\right)$

2. The magnitude of angular momentum of the earth revolving round the sun is proportional to R^n where R is the distance between the earth and the sun. The value of n is (assume the orbit to be circular).

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) 2

3. A tunnel is drilled from the surface of the earth to its centre. The radius of the earth is R and g is the acceleration due to gravity on its surface. A body of mass m is dropped into the tunnel. If M is the mass of the earth, the speed with which the body hits the bottom of the tunnel is

(a) $\frac{m}{M}\sqrt{gR}$ (b) $\frac{M}{m}\sqrt{gR}$

(c) $\sqrt{2gR}$ (d) $\sqrt{gR}$

4. A satellite is orbiting at a height R above the surface of the earth where R is the radius of the earth. By what percentage must the energy of the satellite be increased so that it orbits at a height 2 R above the surface of the earth?

(a) 25% (b) 33.3%

(c) 50% (d) 66.7%

5. If E_1 is the energy required to raise a satellite to a height $h = R$ (radius of the earth) above the surface of the earth and E_2 is the energy required to put it into a circular orbit at that height, then the ratio E_1/E_2 is

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

6. If g is the acceleration due to gravity on the surface of the earth, the gain in potential energy of a satellite of mass m raised from the earth's surface to a height equal to the radius R of the earth is

(a) $mgR/4$ (b) $mgR/2$

(c) mgR (d) $2\,mgR$

7. The escape velocity of a body projected vertically upwards from the surface of the earth is v. If the body is projected at an angle of 30° with the horizontal, the escape velocity would be

(a) $v/2$ (b) $\sqrt{3}\,v/2$

(c) $2v$ (d) v

8. The escape velocity of a body at a height h above the surface of the earth is (g = acceleration due to gravity on the surface of the earth and R = radius of the earth) is

(a) $\sqrt{2gR}$ (b) $\sqrt{2g(R+h)}$

(c) $\sqrt{\frac{2gR^2}{(R+h)}}$ (d) $\sqrt{\frac{gR^2}{2(R+h)}}$

9. A small planet is revolving around a very massive star in a circular orbit of radius r with a period of revolution T. If the gravitational force between the planet and the star were proportional to $r^{-5/2}$, then T would be proportional to

(a) $r^{3/2}$ (b) $r^{5/3}$

(c) $r^{7/4}$ (d) r^3

10. If both the mass and the radius of the earth decrease by 1%, the acceleration due to gravity on the surface of the earth will

(a) decrease by 1% (b) increase by 1%
(c) increase by 2% (d) remain unchanged.

11. Two masses M_1 and M_2 are separated by a distance r. A particle of mass m is placed exactly mid-way between them. The minimum speed with which the particle should be projected so as to escape to infinity is

(a) $v = 2\left[\frac{G(M_1+M_2)}{r}\right]^{1/2}$

(b) $v = \left[\frac{2G(M_1+M_2)}{r}\right]^{1/2}$

(c) $v = 2\left[\frac{G(M_1+M_2)^2}{mr}\right]^{1/2}$

(d) $v = \left[\frac{2G(M_1+M_2)^2}{mr}\right]^{1/2}$

12. The areal velocity of a planet of mass m moving along an elliptical orbit around the sun is

(a) proportional to $\sqrt{m}$ (b) proportional to m
(c) proportional to $\frac{1}{m}$ (d) independent of m

13. A planet of mass m revolves around the sun in an elliptical orbit of semimajor axis a. If M is the mass of the sun, the speed of the planet when it is at a distance x from the sun is

(a) $\sqrt{GM\left(\frac{1}{x}-\frac{1}{2a}\right)}$ (b) $\sqrt{GM\left(\frac{1}{2x}-\frac{1}{a}\right)}$
(c) $\sqrt{GM\left(\frac{2}{x}-\frac{1}{a}\right)}$ (d) $\sqrt{\frac{GMa}{2x^2}}$

14. A satellite revolves around a planet with a speed v in a circular orbit of radius r. If R is the radius of the planet, the acceleration due to gravity on its surface is

(a) $g = \frac{v^2 r}{R^2}$ (b) $g = \frac{v^2 R}{r^2}$
(c) $g = \frac{v^2}{r}$ (d) $g = \frac{v^2}{R}$

15. Three spheres, each of mass M and radius R, are kept such that each touches the other two. The magnitude of the gravitational force on any one sphere due to the other two is

(a) $\frac{GM^2}{2R^2}$ (b) $\frac{3GM^2}{2R^2}$
(c) $\frac{\sqrt{3}GM^2}{2R^2}$ (d) $\frac{\sqrt{3}GM^2}{4R^2}$

16. An extremely small and dense neutron star of mass M and radius R is rotating at an angular frequency ω. If an object is placed at its equator, it will remain stuck to it due to gravity if

(a) $M > \frac{R\omega}{G}$ (b) $M > \frac{R^2\omega^2}{G}$
(c) $M > \frac{R^3\omega^2}{G}$ (d) $M > \frac{R^2\omega^3}{G}$

17. What is the minimum energy required to launch a satellite of mass m from the surface of the earth of radius R in a circular orbit at an altitude of $2R$?

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$
(c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{3R}$

18. Two stars, each of mass m and radius R are approaching each other for a head-on collision. They start approaching each other when their separation is $r >> R$. If their speeds at this separation are negligible, the speed with which they collide would be

(a) $v = \sqrt{Gm\left(\frac{1}{R}-\frac{1}{r}\right)}$

(b) $v = \sqrt{Gm\left(\frac{1}{2R}-\frac{1}{r}\right)}$

(c) $v = \sqrt{Gm\left(\frac{1}{R}+\frac{1}{r}\right)}$

(d) $v = \sqrt{Gm\left(\frac{1}{2R}+\frac{1}{r}\right)}$

19. Consider a particle of mass m suspended vertically by a string at the equator. Let R and M denote the radius and the mass of the earth respectively. If ω is the angular velocity of earth's rotation about its own axis, the tension in the string is equal to

(a) $G\frac{mM}{2R^2}$ (b) $G\frac{mM}{R^2}$
(c) $G\frac{mM}{R^2} - m\omega^2 R$ (d) $G\frac{mM}{R^2} + m\omega^2 R$

20. Infinite number of masses, each of mass m, are placed along a straight line at distances of r, $2r$, $4r$, $8r$, etc. from a reference point O. The gravitational field intensity at point O will be

(a) $\frac{5Gm}{4r^2}$ (b) $\frac{4Gm}{3r^2}$
(c) $\frac{3Gm}{2r^2}$ (d) $\frac{2Gm}{r^2}$

21. In Q. 20, the magnitude of the gravitational potential at point O will be

(a) $\frac{Gm}{2r}$ (b) $\frac{Gm}{r}$

(c) $\frac{3Gm}{2r}$ (d) $\frac{2Gm}{r}$

22. A satellite in force-free space sweeps stationary interplanetary dust at a rate $dM/dt = \alpha v$, where M is the mass and v is the velocity of the satellite and α is a constant. The acceleration of the satellite is

(a) $-\frac{2\alpha v}{M}$ (b) $-\frac{\alpha v^2}{M}$

(c) $+\frac{\alpha v^2}{M}$ (d) $-\alpha v^2$

23. The time of revolution of a satellite is T. Its kinetic energy is proportional to

(a) $\frac{1}{T}$ (b) $\frac{1}{T^2}$

(c) $\frac{1}{T^3}$ (d) $T^{-2/3}$

24. A solid sphere of uniform density and radius 4 units is located with its centre at origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (– 2, 0, 0) and B(2, 0, 0) respectively are taken out of the solid leaving behind spherical cavities as shown in Fig. 9.18. Choose the incorrect statement from the following.

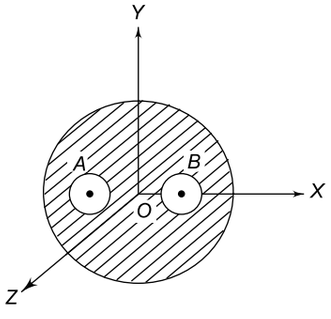


Fig. 9.18

(a) the gravitational force due to this object at the origin is zero.
(b) the gravitational force at point B (2, 0, 0) is zero.
(c) the gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$.
(d) the gravitational potential is the same at all points of the circle $y^2 + z^2 = 4$.

IIT, 1993

25. Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move toward each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is

(a) $\left[\frac{2G(m_1 + m_2)}{r}\right]^{1/2}$

(b) $\left[\sqrt{\frac{2G}{r}}\,\frac{(m_1 + m_2)}{2}\right]^{1/2}$

(c) $\left[\frac{r}{2G(m_1 m_2)}\right]^{1/2}$

(d) $\left(\frac{2G}{r} m_1 m_2\right)^{1/2}$

26. If the distance between the earth and the sun were half its present value, the number of days in a year would have been

(a) 64.5 (b) 129
(c) 182.5 (d) 730

IIT, 1996

27. An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0. Its potential energy is

(a) $-E_0$ (b) $1.5\ E_0$
(c) $2\ E_0$ (d) E_0

IIT, 1997

28. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Which of the following statements is correct?

(a) The acceleration of S is always directed towards the centre of the earth.
(b) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.
(c) The total mechanical energy of S remains constant.
(d) The linear momentum of S remains constant in magnitude.

IIT, 1998

29. The distance between the sun and the earth is r and the earth takes time T to make one complete revolution around the sun. Assuming the orbit of the earth around the sun to be circular, the mass of the sun will be proportional to

(a) $\frac{r^2}{T}$ (b) $\frac{r^2}{T^2}$

(c) $\frac{r^3}{T^2}$ (d) $\frac{r^3}{T^3}$

30. A meteor of mass M breaks up into two parts. The mass of one part is m. For a given separation r the

mutual gravitational force between the two parts will be the maximum if

(a) $m = \frac{M}{2}$ (b) $m = \frac{M}{3}$

(c) $m = \frac{M}{\sqrt{2}}$ (d) $m = \frac{M}{2\sqrt{2}}$

31. A body of mass m is raised to a height h above the surface of the earth of mass M and radius R until its gravitational potential energy increases by $\frac{1}{3}\ mgR$. The value of h is

(a) $\frac{R}{3}$ (b) $\frac{R}{2}$

(c) $\frac{mR}{(M+m)}$ (d) $\frac{mR}{M}$

32. Two balls A and B are thrown vertically upwards from the same location on the surface of the earth with velocities $2\sqrt{\frac{gR}{3}}$ and $\sqrt{\frac{2gR}{3}}$ respectively, where R is the radius of the earth and g is the acceleration due to gravity on the surface of the earth. The ratio of the maximum height attained by A to that attained by B is

(a) 2 (b) 4

(c) 8 (d) $4\sqrt{2}$

33. A uniform sphere of mass M and radius R exerts a force F on a small mass m situated at a distance of $2R$ from the centre O of the sphere. A spherical portion of diameter R is cut from the sphere as shown in Fig. 9.19. The force of attraction between the remaining part of the sphere and the mass m will be

(a) $\frac{7F}{9}$ (b) $\frac{2F}{3}$

(c) $\frac{4F}{9}$ (d) $\frac{F}{3}$

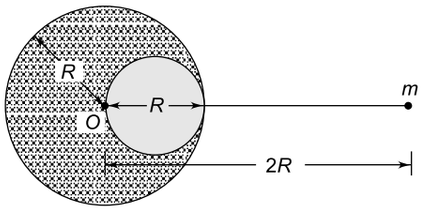


Fig. 9.19

34. The centres of a ring of mass m and a sphere of mass M of equal radius R, are at a distance $\sqrt{8}\ R$ apart as shown in Fig. 9.20. The force of attraction between the ring and the sphere is

(a) $\frac{2\sqrt{2}}{27}\frac{GmM}{R^2}$ (b) $\frac{GmM}{8R^2}$

(c) $\frac{GmM}{9R^2}$ (d) $\frac{\sqrt{2}}{9}\frac{GmM}{9R^2}$

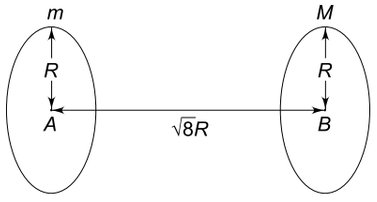


Fig. 9.20

35. Two objects of masses m and $4m$ are at rest at infinite separation. They move towards each other under mutual gravitational attraction. Then, at a separation r, which of the following is true?

(a) The total energy of the system is not zero.
(b) The force between them is not zero.
(c) The centre of mass of the system is at rest.
(d) All the above are true.

IIT, 1994

36. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius 1.01 R. The period of the second satellite is longer than that of the first by approximately

(a) 0.5% (b) 1.0%

(c) 1.5% (d) 3.0%

IIT, 1995

37. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is

(a) 1 (b) $\sqrt{2}$

(c) 4 (d) 2

IIT, 2001

38. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

(a) $4\ Mg/k$ (b) $2\ Mg/k$

(c) Mg/k (d) $Mg/2k$

IIT,2002

39. A geo-stationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred kilometers above the earth's surface (R_{Earth} = 6400 km) will approximately be

(a) (1/2) h (b) 1 h
(c) 2 h (d) 4 h

IIT, 2002

40. A satellite of mass m is moving in a circular orbit of radius R above the surface of a planet of mass M and radius R. The amount of work done to shift the satellite to a higher orbit of radius $2R$ is (here g is the acceleration due to gravity on planet's surface)

(a) mgR (b) $\frac{mgR}{6}$

(c) $\frac{mMgR}{(M+m)}$ (d) $\frac{mMgR}{6(M+m)}$

41. The change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the earth is (here R is the radius of the earth)

(a) $\left(\frac{n}{n+1}\right) mgR$ (b) $\left(\frac{n}{n-1}\right) mgR$

(c) $nmgR$ (d) $\frac{mgR}{n}$

42. A body of mass m is dropped from a height nR above the surface of the earth (here R is the radius of the earth). The speed at which the body hits the surface of the earth is

(a) $\sqrt{\frac{2gR}{(n+1)}}$ (b) $\sqrt{\frac{2gR}{(n-1)}}$

(c) $\sqrt{\frac{2gRn}{(n-1)}}$ (d) $\sqrt{\frac{2gRn}{(n+1)}}$

43. Two solid spheres of radii r and $2r$, made of the same material, are kept in contact. The mutual gravitational force of attraction between them is proportional to

(a) $\frac{1}{r^4}$ (b) $\frac{1}{r^2}$

(c) r^2 (d) r^4

44. A comet is moving in a highly elliptical orbit round the sun. When it is closest to the sun, its distance from the sun is r and its speed is v. When it is farthest from the sun, its distance from the sun is R and its speed will be

(a) $v\left(\frac{r}{R}\right)^{1/2}$ (b) $v\left(\frac{r}{R}\right)$

(c) $v\left(\frac{r}{R}\right)^{3/2}$ (d) $v\left(\frac{r}{R}\right)^{2}$

45. The value of the acceleration due to gravity at the surface of the earth of radius R is g. It decreases by 10% at a height h above the surface of the earth. The gravitational potential at this height is

(a) $-\frac{gR}{\sqrt{10}}$ (b) $-\frac{2gR}{\sqrt{10}}$

(c) $-\frac{3gR}{\sqrt{10}}$ (d) $-\frac{4gR}{\sqrt{10}}$

46. The radius of the earth is R and g is the acceleration due to gravity on its surface. What should be the angular speed of the earth so that bodies lying on the equator may appear weightless?

(a) $\sqrt{\frac{g}{R}}$ (b) $\sqrt{\frac{2g}{R}}$

(c) $\sqrt{\frac{g}{2R}}$ (d) $2\sqrt{\frac{g}{R}}$

47. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 (as shown in Fig. 9.21) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.

(a) $W_1 > W_3 > W_2$ (b) $W_1 = W_2 = W_3$

(c) $W_1 < W_3 < W_2$ (d) $W_1 < W_2 < W_3$

IIT, 2003

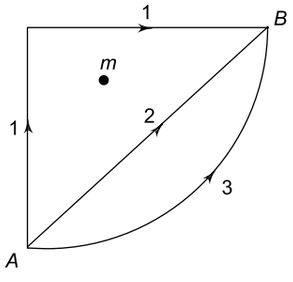


Fig. 9.21

48. A binary star system consists of two stars of masses M_1 and M_2 revolving in circular orbits of radii R_1 and R_2 respectively. If their respective time periods are T_1 and T_2, then

(a) $T_1 > T_2$ if $R_1 > R_2$

(b) $T_1 > T_2$ if $M_1 > M_2$

(c) $T_1 = T_2$

(d) $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2}$

IIT, 2006

49. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$ where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance

$r(0 < r < \infty)$ from the centre of the system is represented by (see Fig. 9.22)

IIT, 2008

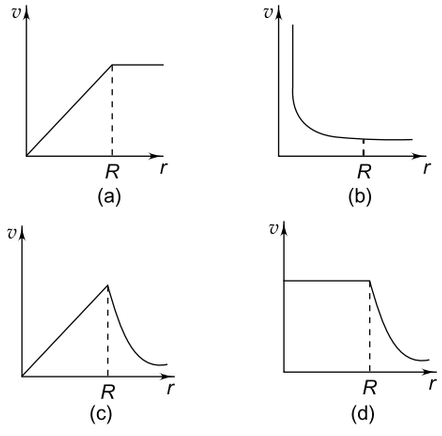


Fig. 9.22

50. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

(a) $\frac{1}{2}mV^2$ (b) mV^2

(c) $\frac{3}{2}mV^2$ (d) $2mV^2$

IIT, 2011

ANSWERS

1. (a)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (b)
7. (d)	**8.** (c)	**9.** (c)	**10.** (b)	**11.** (a)	**12.** (d)
13. (c)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (b)
19. (c)	**20.** (b)	**21.** (d)	**22.** (b)	**23.** (d)	**24.** (b)
25. (a)	**26.** (b)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (a)
31. (b)	**32.** (c)	**33.** (a)	**34.** (a)	**35.** (d)	**36.** (c)
37. (d)	**38.** (b)	**39.** (c)	**40.** (b)	**41.** (a)	**42.** (d)
43. (d)	**44.** (b)	**45.** (c)	**46.** (a)	**47.** (b)	**48.** (c)
49. (c)	**50.** (b)				

SOLUTIONS

1. Gravitational potential on the surface of the shell due to the body of mass m is

$$V_b = -\frac{Gm}{R}$$

Gravitational potential on the surface of the shell due to shell itself is

$$V_s = -\frac{GM}{R}$$

$\therefore\ V = V_b + V_s = -\frac{G}{R}(M + m)$, which is choice (a).

2. $\frac{mv^2}{R} = \frac{GmM}{R^2} \Rightarrow v = \sqrt{\frac{GM}{R}}$

Angular momentum $L = mvR = m \times \sqrt{\frac{GM}{R}} \times R$

$= m(GMR)^{1/2}$.

i.e. $L \propto R^{1/2}$. So the correct choice is (a).

3. Let v be the required speed.

Gain in K.E. = loss in P.E.

= P.E. at the surface – P.E. at the centre of the earth

$$\Rightarrow \quad \frac{1}{2}mv^2 = -\frac{GmM}{R} - \left(-\frac{3}{2}\frac{GmM}{R}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR} \quad \left(\therefore g = \frac{GM}{R^2}\right)$$

4. The total energy of a satellite in orbital radius r is

$$E = \text{K.E.} + \text{P.E.} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$= \frac{GMm}{2r} - \frac{GMm}{r} \quad \left(\therefore v = \sqrt{\frac{GM}{r}}\right)$$

$$= -\frac{GMm}{2r}$$

In the first case, $r = R + R = 2R$. Hence

$$E = -\frac{GMm}{4R}$$

In the second case, $r = R + 2R = 3R$. Hence

$$E' = -\frac{GMm}{6R}$$

Increase in energy $\Delta E = E' - E$

$$= -\frac{GMm}{6R} - \left(-\frac{GMm}{4R}\right)$$

$$= \frac{GMm}{12R}$$

Percentage increase $= \dfrac{\Delta E}{|E|} \times 100$

$$= \frac{GMm/12R}{GMm/4R} \times 100 = 33.3\%$$

5. E_1 = P.E. at $h = R$ – P.E. at $h = 0$

= P.E. at $r = 2R$ – P.E. at $r = R$

$$= -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{2R}$$

$$E_2 = -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3GMm}{4R}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{2}{3}$$

6. P.E. on the surface of the earth $= -\dfrac{GMm}{R}$

P.E. at a height $h\ (= R) = -\dfrac{GMm}{2R}$

$\therefore$ Gain in P.E. $= -\dfrac{GMm}{2R} - \left(-\dfrac{GMm}{R}\right)$

$$= \frac{GMm}{2R} = \frac{1}{2}\, mgR \quad \left(\therefore\ g = \frac{GM}{R^2}\right)$$

7. The correct choice is (d) because the escape velocity is independent of the direction along which the body is projected.

8. Escape velocity at height $h = \sqrt{2g'(R+h)}$, where $g' = g\left(\dfrac{R}{R+h}\right)^2$. Hence the correct choice is (c).

9. Since the gravitational force provides the necessary centripetal force,

$$\frac{mv^2}{r} \propto r^{-5/2}$$

$$\Rightarrow \quad \frac{mv^2}{r} = kr^{-5/2}, \qquad k = \text{constant}$$

$$\therefore \quad v = \sqrt{\frac{k\, r^{-3/2}}{m}}$$

$$T = \frac{2\pi r}{v} = 2\pi r \times \frac{\sqrt{m}}{\sqrt{kr^{-3/2}}}$$

i.e. $T \propto r^{7/4}$, which is choice (c)

10. $g = \dfrac{GM}{R^2}$

$$\therefore \quad \frac{\Delta g}{g} = \frac{\Delta m}{M} - \frac{2\Delta R}{R}$$

$$= -1\% - 2 \times (-1\%) = +1\%$$

Hence g will increase by 1%.

11. Distance of m from M_1 or $M_2 = r/2$. Therefore,

$$\text{Total P.E.} = -\frac{GmM_1}{r/2} - \frac{GmM_2}{r/2}$$

$$= \frac{2Gm}{r}\ (M_1 + M_2)$$

If v is the required velocity of projection, the total initial energy is

$$E_i = \frac{1}{2}\, mv^2 - \frac{2Gm}{r}\ (M_1 + M_2)$$

At infinity, $E_f = 0$. Putting $E_i = E_f$, we get

$$v = 2\left[\frac{G(M_1 + M_2)}{r}\right]^{1/2}$$

12. A real velocity $\dfrac{dA}{dt} = \dfrac{L}{2m}$, where L is the magnitude of angular momentum of the planet about the sun. $L = mvr \sin\theta$. Hence,

$$\frac{dA}{dt} = \frac{mvr\sin\theta}{2m} = \frac{vr\sin\theta}{2}$$

So the correct choice is (d).

13. Total energy of the planet in an elliptical orbit of semimajor axis a is

$$E_1 = -\frac{GmM}{2a}$$

Total energy of the planet when it is at a distance x from the sun (here v = speed of the plane at that instant) is

$$E_2 = \text{K.E.} + \text{P.E.}$$

$$= \frac{1}{2}\, mv^2 - \frac{GmM}{x}$$

From conservation of energy, $E_1 = E_2$, i.e.

$$-\frac{GmM}{2a} = \frac{1}{2}\, mv^2 - \frac{GmM}{x}$$

which gives $v = \sqrt{GM\left(\frac{2}{x} - \frac{1}{a}\right)}$, which is choice (c).

14. $\frac{mv^2}{r} = \frac{GmM}{r^2}$

$\Rightarrow GM = v^2 r$

$$g = \frac{GM}{R^2} = \frac{v^2 r}{R^2}$$

So the correct choice is (a).

15. Force between any two spheres is

$$F = \frac{GM^2}{(2R)^2} = \frac{GM^2}{4R^2}$$

This is the force exerted on any sphere (say A) by the other two spheres B and C (Fig. 9.23). Thus the resultant force on sphere A is

$$F_r = \sqrt{F^2 + F^2 + 2F^2 \cos 60°}$$

$$= \sqrt{3}\, F = \frac{\sqrt{3}\, GM^2}{4R^2}, \text{ which is choice (d).}$$

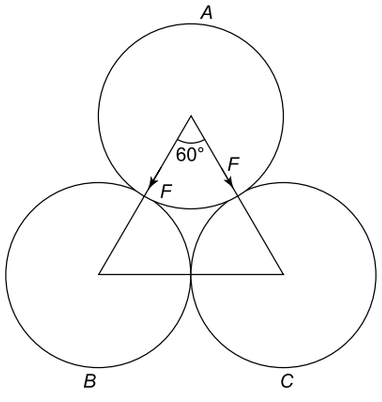


Fig. 9.23

16. An object of mass m, placed at the equator of the star, will experience two forces: (i) an attractive force due to gravity towards the centre of the star and (ii) an outward centrifugal force due to the rotation of the star. The centrifugal force arises because the object is in a rotating (non-inertial) frame; this force is equal to the inward centripetal force but opposite in direction. Force on object due to gravity is

$$F_g = \frac{GmM}{R^2}$$

Centrifugal force on the object is

$$F_c = mR\omega^2$$

The object will remain stuck to the star and not fly off if

$$F_g > F_c$$

or $$\frac{GmM}{R^2} > mR\omega^2 \text{ or } M > \frac{R^3\omega^2}{G}$$

Hence the correct choice is (c).

17. Consider a satellite of mass m moving with a speed v at an altitude r (measured from the centre of the earth). Then

$$\text{Kinetic energy (KE)} = \frac{1}{2}\, mv^2$$

$$\text{Gravitational potential energy (PE)} = -\frac{GmM}{r}$$

where M is the mass of the earth.

For a satellite in circular orbit, we have

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \quad \text{or} \quad v^2 = \frac{GM}{r}$$

or $$\frac{1}{2}\, mv^2 = \frac{GmM}{2r}$$

i.e. $$\text{KE} = \frac{GmM}{2r}$$

Thus the KE of a satellite in a circular orbit is numerically half its PE but opposite in sign. The total energy of the satellite in orbit is

$$E = \text{KE} + \text{PE} = \frac{GmM}{2r} - \frac{GmM}{r}$$

$$= -\frac{GmM}{2r}$$

It is given that $r = 2R + R = 3R$, where R is the radius of the earth.

$$\therefore \quad E = -\frac{GmM}{6R}$$

Now PE on the surface of the earth $= -\frac{GmM}{R}$

$\therefore$ Minimum energy required ($E_{\min}$)

$$= -\frac{GmM}{6R} - \left(-\frac{GmM}{R}\right)$$

$$= \frac{5GmM}{6R}$$

Hence the correct choice is (a).

18. The speeds of stars at separation r are negligible. Therefore, their energy is entirely potential at this separation (since KE = 0)

$$E_1 = (\text{PE at } r) = -\frac{Gm_1 m_2}{r}$$

$$= -\frac{Gm^2}{r}$$

As the stars approach each other under gravitational attraction, they begin to acquire speed and hence

kinetic energy at the expense of potential energy. When they eventually collide, the separation between their centres is

$$r = R + R = 2R$$

At $r = 2R$, the total energy is

$$E_2 = \text{PE at } (r = 2R) + \text{KE at } (r = 2R)$$

$$= -\frac{Gm^2}{2R} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

or $$E_2 = -\frac{Gm^2}{2R} + mv^2$$

From the principle of conservation of energy, $E_1 = E_2$, i.e.

$$-\frac{Gm^2}{r} = -\frac{Gm^2}{2R} + mv^2$$

which gives $$v = \sqrt{Gm\left(\frac{1}{2R} - \frac{1}{r}\right)}$$

Hence the correct choice is (b).

19. The acceleration due to gravity at a place on the surface of the earth is given by

$$g' = g - \omega^2 R \cos\phi$$

where ϕ is the latitude of the place. At the equator, $\phi = 0$. Therefore

$$g' = g - \omega^2 R$$

Therefore, the tension in the string will be

$$mg' = mg - m\omega^2 R$$

$$= \frac{GmM}{R^2} - m\omega^2 R$$

Hence the correct choice is (c).

20. The gravitational field intensity at point O will be

$$I = Gm\left[\frac{1}{r^2} + \frac{1}{(2r)^2} + \frac{1}{(4r)^2} + \frac{1}{(8r)^2} + \ldots\right]$$

$$= \frac{Gm}{r^2}\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \cdots\right]$$

$$= \frac{Gm}{r^2}\left[\frac{1}{2^0} + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots\right]$$

$$= \frac{Gm}{r^2}\left(\frac{1}{1 - \frac{1}{2^2}}\right) = \frac{Gm}{r^2}\left(\frac{1}{1 - \frac{1}{4}}\right) = \frac{4Gm}{3r^2}$$

Hence the correct choice is (b).

21. The gravitational potential, in magnitude, at point O is

$$V = Gm\left[\frac{1}{r} + \frac{1}{2r} + \frac{1}{4r} + \frac{1}{8r} + \cdots\right]$$

$$= \frac{Gm}{r}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right]$$

$$= \frac{Gm}{r}\left[\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\right]$$

$$= \frac{Gm}{r}\left(\frac{1}{1 - \frac{1}{2}}\right) = \frac{2Gm}{r}$$

Hence the correct choice is (d).

22. From Newton's second law of motion, force is the rate of change of momentum, i.e.

$$F = \frac{d}{dt}(Mv) = \frac{dM}{dt} \cdot v = \alpha v^2 \qquad \left(\because \frac{dM}{dt} = \alpha v\right)$$

$\therefore$ Retardation $= \frac{F}{M} = \frac{\alpha v^2}{M}$ or acceleration $= -\frac{\alpha v^2}{M}$. Hence the correct choice is (b).

23. The orbital speed of a satellite at a height r from the centre of the earth is given by $v = \sqrt{\frac{GM}{r}}$

where M is the mass of the earth. If m is the mass of the satellite, its kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM}{r}$$

Thus, K is proportional to $\frac{1}{r}$. Now, the time period of the satellite is given by $T = 2\pi\sqrt{\frac{r^3}{gR^2}}$

where R is the radius of the earth. Thus $T \propto r^{3/2}$. But $K \propto \frac{1}{r}$. Hence $K \propto T^{-2/3}$, which is choice (d).

24. The distance of each cavity from the centre O is the same. Since the two cavities are symmetrical with respect to the centre O and the mass of the sphere can be regarded as being concentrated at the centre O, the gravitational force due to the sphere is zero at the centre. Hence choice (a) is correct. For the same reason, the gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$ whose radius is 6 units and at all points of the circle $y^2 + z^2 = 4$. Hence choices (c) and (d) are also correct. But the gravitational force at point B cannot be zero.

25. Initially when the two masses are at an infinite distance from each other, their gravitational potential energy is zero. When they are at a distance r from each other the gravitational P.E. is

$$\text{PE} = -\frac{G m_1 m_2}{r}$$

The minus sign indicates that there is a decrease in P.E. This gives rise to an increase in kinetic energy. If v_1 and v_2 are their respective velocities when they are a distance r apart, then, from the law of conservation of energy, we have

$$\frac{1}{2} m_1 v_1^2 = \frac{G m_1 m_2}{r}$$

or $$v_1 = \sqrt{\frac{2 G m_2}{r}}$$

and $$\frac{1}{2} m_2 v_2^2 = \frac{G m_1 m_2}{r}$$

or $$v_2 = \sqrt{\frac{2 G m_1}{r}}$$

Therefore, their relative velocity of approach is

$$v_1 + v_2 = \sqrt{\frac{2 G m_2}{r}} + \sqrt{\frac{2 G m_1}{r}}$$

$$= \sqrt{\frac{2G}{r}(m_2 + m_1)}$$

Hence the correct choice is (a).

26. According to Kepler's law of periods,

$$\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R_1}{R_1/2}\right)^{3/2}$$

$$= (2)^{3/2} = 2\sqrt{2}$$

$$\therefore \quad T_2 = \frac{T_1}{2\sqrt{2}} = \frac{365 \text{ days}}{2\sqrt{2}} = 129 \text{ days}.$$

27. For a satellite, we have

$$\text{Kinetic energy} = \frac{GmM}{2r}$$

$$\text{Potential energy} = -\frac{GmM}{r}$$

Total energy $E_0 = \text{KE} + \text{PE}$

$$= \frac{Gm M}{2r} - \frac{Gm M}{r} = -\frac{Gm M}{2r} = \frac{\text{PE}}{2}$$

or PE = $2E_0$. Hence the correct choice is (c).

28. For elliptical orbit, the earth is at one focus of the ellipse. For spherical bodies, the gravitational force is central (or radial). Hence statement (a) is correct. The gravitational force exerts no torque on the satellite. Hence the angular momentum of S remains constant in magnitude as well as direction. Hence choice (b) is incorrect. For elliptical orbit, the distance of the satellite from the earth varies periodically. Hence potential energy, kinetic energy and linear momentum vary periodically. Hence choices (c) and (d) are also incorrect.

29. Let m and M be the masses of the earth and the sun respectively and v the speed of the earth in circular orbit. To keep the earth in circular orbit, the gravitational force $\frac{GmM}{r^2}$ must balance the centripetal force $\frac{mv^2}{r}$, i.e.

$$\frac{GmM}{r^2} = \frac{mv^2}{r}$$

or $$M = \frac{v^2 r}{G}$$

Also $v = \frac{2\pi r}{T}$. Using this, we get $M = \frac{4\pi^2 r^3}{T^2 G}$ or $M \propto \frac{r^3}{T^2}$.

Hence the correct choice is (c).

30. Mass of the second part = $M - m$. Gravitational force between the two parts is

$$F = \frac{G(M-m)m}{r^2} = \frac{G}{r^2}(Mm - m^2)$$

F will be maximum if $\frac{dF}{dm} = 0$ and $\frac{d^2F}{dm^2}$ is negative.

Now, $\frac{dF}{dm} = \frac{G}{r^2}(M - 2m)$. Setting $\frac{dF}{dm} = 0$, we get $M - 2m = 0$ or $m = \frac{M}{2}$. Now

$$\frac{d^2F}{dm^2} = -\frac{2G}{r^2}, \text{ which is negative.}$$

Hence the correct choice is (a).

31. PE on the surface of earth $= -\frac{GMm}{R}$

PE at a height h above the surface of earth $= -\frac{GMm}{(R+h)}$

$$\therefore \quad \text{Increase in PE} = -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$$

$$= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right) = GMm\left[\frac{h}{R(R+h)}\right]$$

$$= \frac{gRmh}{(R+h)} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

$\therefore$ PE will increase by $\frac{1}{3}mgR$ at a value of h given by

$$\frac{gRmh}{(R+h)} = \frac{1}{3}mgR$$

or $\frac{h}{R+h} = \frac{1}{3}$ or $h = \frac{R}{2}$, which is choice (b).

32. If h is the maximum height attained, then we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$$

which gives $v^2 = \frac{2ghR}{(R+h)} \quad \left(\because g = \frac{GM}{R^2}\right)$

For ball A, we have $\frac{4gR}{3} = \frac{2gh_AR}{(R+h_A)} \Rightarrow h_A = 4R$

For ball B, we have $\frac{2gR}{3} = \frac{2gh_BR}{(R+h_B)} \Rightarrow h_B = \frac{R}{2}$

$\therefore \frac{h_A}{h_B} = 8$, which is choice (c).

33. The force of attraction between the complete sphere and mass m is

$$F = \frac{GmM}{(2R)^2} = \frac{GmM}{4R^2} \qquad \text{(i)}$$

Mass of complete sphere is $M = \frac{4\pi}{3}R^3\rho$. Mass of the cut out portion is $m_0 = \frac{4\pi}{3}\left(\frac{R}{2}\right)^3\rho$. Thus, $m_0 = \frac{M}{8}$. The distance between the centre of the cut out portion and mass $m = 2R - \frac{R}{2} = \frac{3R}{2}$.

Hence the force of attraction between the cut out portion and mass m is

$$f = \frac{Gm_0m}{(3R/2)^2} = \frac{G(M/8)m}{9R^2/4} = \frac{GmM}{4R^2} \times \frac{2}{9}$$

Using (i), we get $f = \frac{2F}{9}$. Therefore, the force of attraction between the remaining part of the sphere and mass $m = F - f = F - \frac{2F}{9} = \frac{7F}{9}$ which is choice (a).

34. Refer to Fig. 9.24. Let μ be the mass per unit length of the ring. $L = 2\pi R$ is the length of the ring. Consider a small element of length dx of the ring located at C. Then

Force along BC is $f = \frac{GM\mu\,dx}{(3R)^2}$. Therefore, force along BA is $dF = f\cos\theta = \frac{GM\mu\,dx}{9R^2} \frac{\sqrt{8}R}{3R} = \frac{\sqrt{8}}{27}\frac{GM\,\mu dx}{R^2}$

$\therefore$ Total force $= \frac{\sqrt{8}}{27}\frac{GM}{R^2}\int \mu\,dx = \frac{\sqrt{8}}{27}\frac{GMm}{R^2}$

because $\int \mu\,dx = \mu \times L = m$, the mass of the ring. Hence the correct choice is (a).

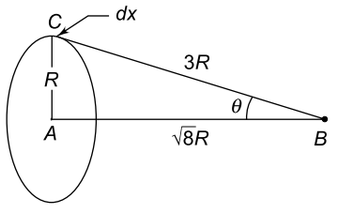


Fig. 9.24

35. At a finite separation, the total kinetic energy of the system of two masses and the force between them are both finite. Since the two masses are at rest initially and there is no external force, the centre of mass cannot move. Hence the correct choice is (d).

36. According to Kepler's law of period, $T^2 = kR^3$ where k is a constant. Taking logarithm of both sides, we have

$$2\log T = \log k + 3\log R$$

Differentiating, we get

$$2\frac{\delta T}{T} = 0 + 3\frac{\delta R}{R}$$

or $\frac{\delta T}{T} = \frac{3}{2}\frac{\delta R}{R} = \frac{3}{2} \times \left(\frac{1.01R - R}{R}\right) \times 100$

$= 1.5\%$

Hence the correct choice is (c).

37. The acceleration due to gravity at a height h above the surface of the earth is given by

$$g_2 = g_1\left(\frac{R}{R+h}\right)^2$$

where g_1 is the value at the surface of the earth. Now

$$T_2 = 2\pi\sqrt{\frac{l}{g_2}} \text{ and } T_1 = 2\pi\sqrt{\frac{l}{g_1}}$$

$$\therefore \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \frac{R+h}{R} = \frac{R+R}{R} = 2 \quad (\because h = R)$$

Hence the correct choice is (d).

38. Let x be the extension in the spring when it is loaded with mass M. The change in gravitational potential energy = Mgx. This must be the energy stored in the spring which is given by $\frac{1}{2}kx^2$. Thus

$$\frac{1}{2}kx^2 = Mgx \text{ or } x = \frac{2Mg}{k}, \text{ which is choice (b).}$$

39. For a satellite of mass m moving with a velocity v in a circular orbit of radius r around the earth of mass M, we have

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \text{ or } v = \sqrt{\frac{GM}{r}}$$

Now $v = \frac{2\pi r}{T}$. Thus $\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$ or $T \propto r^{3/2}$.

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \qquad (1)$$

Given r_2 = 6400 km and r_1 = 36000 km. For a geostationary satellite T_1 = 24 h. Using these values in (1), we have get $T_2 = 24 \times \left(\frac{64}{360}\right)^{3/2}$ = 1.8 h. Hence the closest choice is (c).

40. PE at a distance r from the centre of the planet $= -\frac{GMm}{r}$

$$\text{Initial PE} = -\frac{GMm}{R+R} = -\frac{GMm}{2R}$$

$$\text{Final PE} = -\frac{GMm}{R+2R} = -\frac{GMm}{3R}$$

Now, work done = increase in PE

$$= \frac{GMm}{R}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{GMm}{6R} = \frac{1}{6}mgR \left(\because g = \frac{GM}{R^2}\right)$$

Hence the correct choice is (b).

41. $\text{Change in PE} = \frac{GMm}{R} - \frac{GMm}{(n+1)R} = \left(\frac{n}{n+1}\right)mgR$

Hence the correct choice is (a).

42. From the principle of conservation of energy, we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+nR)}$$

which gives $v^2 = \frac{2nR}{(n+1)}\frac{GM}{R^2} = \frac{2nRg}{(n+1)}$

$$\left(\because g = \frac{GM}{R^2}\right)$$

Hence the correct choice is (d).

43. If ρ is the density of the material of each sphere, then the mass of the sphere of radius r is $M_1 = \frac{4\pi}{3}r^3\rho$ and the mass of the sphere of radius $2r$ is $M_2 = \frac{4\pi}{3}(2r)^3\rho$.

Distance between their centres is $d = r + 2r = 3r$.

$$\text{Now } F = \frac{GM_1M_2}{d^2} = \frac{G \times \left(\frac{4\pi}{3}\right)r^3\rho \times \frac{4\pi}{3}(2r)^3\rho}{9r^2}$$

which gives $F \propto r^4$, which is choice (d).

44. The angular momentum of the comet is constant over the entire orbit. Hence $vr = VR$ or $V = v\left(\frac{r}{R}\right)$, which is choice (b).

45. $g_h = \frac{GM}{(R+h)^2}$

Also $g = \frac{GM}{R^2}$. Thus $\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$.

Given $g_h = \frac{90g}{100}$

$$\therefore \frac{R^2}{(R+h)^2} = \frac{9}{10} \text{ or } (R+h) = \frac{\sqrt{10}R}{3}$$

$$\text{Potential} = -\frac{GM}{(R+h)} = -\frac{GMR^2}{R^2(R+h)} = -\frac{gR^2}{(R+h)}$$

$$= -\frac{3gR}{\sqrt{10}}$$

Hence the correct choice is (c).

46. At the equator, the value of g is

$$g' = g - R\omega^2$$

where ω is the angular speed of the earth. For bodies to appear weightless at the equator, $g' = 0$, i.e.

$$g - R\omega^2 = 0$$

which gives $\omega = \sqrt{\frac{g}{R}}$. Hence the correct choice is (a).

47. Gravitational force is conservative. The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle. Hence the correct choice is (b).

48. In a binary star system, the two stars move under their mutual gravitational force. Therefore, their angular velocities and hence their time periods are equal. Thus the correct choice is (c).

49. If M is the total mass of the system of particles, the orbital speed of the test mass is

$$v = \sqrt{\frac{GM}{r}}$$

For $r \le R$, $v = \sqrt{\frac{G \times \frac{4\pi}{3}r^3\rho_0}{r}}$ which gives $v \propto r$,

i.e. v increases linearly with r up to $r = R$. Hence choices (b) and (d) are wrong.

For $r > R$, the whole mass of the system is $M = \frac{4\pi}{3} R^3 \rho_0$, which is constant. Hence for $r > R$,

$$v = \sqrt{\frac{GM}{r}}$$

i.e, $v \propto \frac{1}{\sqrt{r}}$. Hence the correct choice is (c)

50. Let M be the mass of the earth and r be the orbital radius of the satellite. The energy needed so that the object of mass m escapes from $x = r$ to $x = \infty$ is

$$E = GMm\int_r^\infty \frac{dx}{x^2} = \frac{GMm}{r} \qquad \text{(i)}$$

The orbital speed V is given by

$$\frac{mV^2}{r} = \frac{GMm}{r^2} \quad \Rightarrow \quad mV^2 = \frac{GMm}{r} \qquad \text{(ii)}$$

Using (ii) in (i), $E = mV^2$.

SECTION II

Multiple Choice Questions with one or More Choices Correct

1. A satellite is moving around the earth in a stable circular orbit. Choose the correct statements from the following.
- (a) It is moving at a constant speed.
- (b) Its angular momentum remains constant.
- (c) It is acted upon by a force directed away from the centre of the earth which counter balances the attraction by the earth.
- (d) It behaves as if it were as freely falling body.

2. The escape velocity from a planet depends upon
- (a) the mass of the body
- (b) the mass of the planet
- (c) the average radius of the planet
- (d) the average density of the planet

3. The orbital velocity of a body in a stable orbit around a planet depends upon
- (a) the average radius of the planet
- (b) the height of the body above the planet
- (c) the acceleration due to gravity
- (d) the mass of the orbiting body

4. Choose the correct statements from the following:
- (a) The equivalence of inertial and gravitational mass has provided a clue to the deeper understanding of gravitation .
- (b) At poles, the effect of rotation of earth on the value of g is the minimum.
- (c) Very massive rockets and extremely tiny particles, such as the molecules of a gas, require the same initial velocity to escape from the earth.
- (d) A geostationary satellite, if imparted the necessary velocity, can be put in orbit at any height above the earth.

5. Choose the correct statements from the following:
- (a) The magnitude of the gravitational force between two bodies of mass 1 kg each and separated by a distance of 1 m is 9.8 N.
- (b) Higher the value of the escape velocity for a planet, the higher is the abundance of lighter gases in its atmosphere.
- (c) The gravitational force of attraction between two bodies of ordinary mass is not noticeable because the value of the gravitation constant is extremely small.
- (d) Force of friction arises due to gravitational attraction.

6. Choose the wrong statements from the following.
- (a) It is possible to shield a body from the gravitational field of another body by using a thick shielding material between them.
- (b) The escape velocity of a body is independent of the mass of the body and the angle of projection.
- (c) The acceleration due to gravity increases due to the rotation of the earth.
- (d) The gravitational force exerted by the earth on a body is greater than that exerted by the body on the earth.

7. A comet is revolving around the sun in a highly elliptical orbit. Which of the following will remain constant throughout its orbit?
- (a) Kinetic energy
- (b) Potential energy
- (c) Total energy
- (d) Angular momentum

8. A satellite is orbiting the earth. If its distance from the earth is increased, its
(a) angular velocity would increase
(d) linear velocity would increase
(c) angular velocity would decrease
(d) time period would increase.

9. For two satellites at distance R and $7R$ above the earth's surface, the ratio of their
(a) total energies is 4 and potential and kinetic energies is 2
(b) potential energies is 4
(c) kinetic energies is 4
(d) total energies is 4.

10. A satellite is orbiting the earth in a circular orbit of radius r. Its
(a) kinetic energy varies as $1/r$
(b) angular momentum varies as $1/\sqrt{r}$
(c) linear momentum varies as $1/\sqrt{r}$
(d) frequency of revolution varies as $1/r^{3/2}$.

11. If both the mass and radius of the earth decrease by 1%, the value of
(a) acceleration due to gravity would decrease by nearly 1%
(b) acceleration due to gravity would increase by 1%
(c) escape velocity from the earth's surface would decrease by 1%
(d) the gravitational potential energy of a body on earth's surface remains unchanged.

12. An object is taken from a point P to another point Q in a gravitational field,
(a) assuming the earth to be spherical, if both P and Q lie on earth's surface the work done is zero.
(b) If P is on earth's surface and Q above it, the work done is minimum when it is taken along the straight line PQ.
(c) The work done depends only on the positions of P and Q and is independent of the path along which the particle is taken.
(d) there is no net work done if the object is taken from P to Q and then brought back to P, along any path.

13. A satellite is moving in a circular orbit around the earth with a speed equal to half the escape velocity from the earth. The radius of the earth is R and h is the height of the satellite above the surface of the earth. If the satellite is suddenly stopped in its orbit and allowed to fall freely, it will hit the surface of the earth with a speed v. Then

(a) $h = \frac{R}{2}$ (b) $h = R$

(c) $v = \sqrt{2gR}$ (d) $v = \sqrt{gR}$

14. Two stars of masses m and $2\,m$ are co-rotating about their centre of mass. Their centres are at a distance r apart. If r is much larger than the size of the stars, then their
(a) common period of revolution is proportional to $r^{3/2}$.
(b) orbital velocities are in the ratio 2 : 1.
(c) kinetic energies are in the ratio 1 : 2 .
(d) angular momenta are in the ratio 1 : 4.

15. A space-ship is orbiting close to the surface of the earth at a speed v. The radius of the earth is R and g is the acceleration due to gravity close to the surface of the earth. An additional speed of v_0 is to be imparted to the space-ship so that it overcomes the gravitational pull of the earth. Then

(a) $v = \sqrt{Rg}$ (b) $v = \sqrt{2Rg}$

(c) $v_0 = \sqrt{Rg}(\sqrt{2}+1)$ (d) $v_0 = \sqrt{Rg}(\sqrt{2}-1)$

16. A satellite of mass m is moving in a circular orbit of radius r around a planet of mass M.
(a) The magnitude of angular momentum with respect to the centre of the orbit is $m\sqrt{GMr}$, where G is the gravitation constant.
(b) The magnitude of the angular momentum is $mR\sqrt{2gr}$ where g is the acceleration due to gravity on the surface of the planet.
(c) The direction of angular momentum is parallel to the plane of the orbit.
(d) The direction of angular momentum is perpendicular to the plane of the orbit.

IIT, 1997

17. Two bodies of masses $m_1 = m$ and $m_2 = 4\,m$ are placed at a distance r apart. The gravitational field is zero at a point P at a distance x from mass m_1. The gravitational potential at point P is V. Then (U is the gravitational P.E. of the system)

(a) $x = \frac{r}{3}$ (b) $x = \frac{2r}{3}$

(c) $U = -\frac{7G}{r}$ (d) $U = -\frac{9G}{r}$

18. A small planet is revolving with speed v around a very massive star in a circular orbit of radius r with a period of revolution T. If the gravitational force between the planet and the star were inversely proportional to r, then
(a) v is independent of r.
(b) v decreases if r is increased.
(c) T increases if r is increased.
(d) T is independent of r.

19. The total energy of a satellite of mass m moving with speed v around the earth of mass M in a

circular orbit of radius r is directly proportional to
(a) m (b) M
(c) v (d) r

20. Assuming the earth to be a sphere of radius R and uniform density, if the acceleration due to gravity at a point at a distance r from the centre of the earth is g, then
(a) $g \propto r$ for $r < R$ (b) $g \propto \frac{1}{r}$ for $r < R$
(c) $g \propto r^2$ for $r > R$ (d) $g \propto \frac{1}{r^2}$ for $r > R$

21. A satellite is revolving around a planet in a circular orbit. During its motion, the satellite begins to experience a resistive force (possibly due to cosmic dust). As a result
(a) its kinetic energy will decrease
(b) its angular speed will decrease
(c) its time period of revolution will increase
(d) its angular momentum will decrease.

22. If a satellite revolving around the earth is moved from one stable orbit to another stable orbit of a higher orbital radius, then its
(a) time period will increase
(b) centripetal force will decrease.
(c) gravitational potential energy will increase
(d) angular momentum will remain unchanged

23. A comet is moving around the sun in a highly elliptical orbit. Its
(a) kinetic energy and gravitational potential energy both change over the orbit
(b) total energy remains constant throughout the orbit
(c) linear momentum changes in magnitude as well as direction over the orbit
(d) angular momentum remains constant over the orbit.

IIT, 1998

24. A body of mass m is released from rest from above at a distance r from the centre of the earth of mass M and radius R. If the air resistance is neglected, it will strike the surface of the earth with a velocity v. If g is the acceleration due to gravity on earth's surface,
(a) $v = \sqrt{2Rg}$ if $r >> R$
(b) $v = \sqrt{Rg}$ if $r = 2R$
(c) $v = \sqrt{\frac{Rg}{2}}$ if $r = 4R$
(d) $v = \frac{1}{2}\sqrt{Rg}$ if $r = 8R$

25. A satellite of mass m is launched from the surface of the earth in a circular orbit at a height $h = R$ above the surface of the earth; R being the radius of the earth. If g is the acceleration due to gravity on the surface of the earth and the air resistance is neglected,
(a) the increase in gravitational potential energy is mgR
(b) the velocity with which it was projected from the earth is $\sqrt{gR}$
(c) the total energy spent in launching the satellite in the circular orbit is $\frac{3}{4}\ mgR$
(d) the acceleration due to gravity at the site of the satellite is $g/4$.

26. Two satellites of the same mass are put in the same orbit round the earth but they revolve in opposite directions. They undergo an inelastic collision and stick together.
(a) The ratio of the potential energy of the system before and just after the collision is 2 : 1
(b) The ratio of the total energy of the system before and just after the collision is 1 : 2
(c) After the collision, the combined mass will continue moving in the same orbit
(d) After the collision, the combined mass will fall freely under the gravity of the earth.

27. A solid sphere of uniform density and radius 4 units is located with its centre at origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (– 2, 0, 0) and B(2, 0, 0) respectively are taken out of the solid leaving behind spherical cavities as shown in Fig. 9.25.

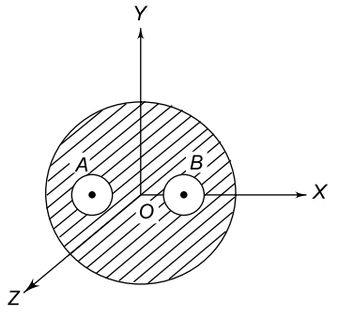


Fig. 9.25

(a) the gravitational force due to this object at the origin is zero.
(b) the gravitational force at point B (2,0,0) is zero.
(c) the gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$.
(d) the gravitational potential is the same at all points of the circle $y^2 + z^2 = 4$.

IIT, 1993

28. The magnitudes of the gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then

(a) $\frac{F_1}{F_2}=\frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(b) $\frac{F_1}{F_2}=\frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(c) $\frac{F_1}{F_2}=\frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

(d) $\frac{F_1}{F_2}=\frac{r_2}{r_1}$ if $r_1 > R$ and $r_2 > R$

IIT, 1994

ANSWERS AND SOLUTIONS

1. The correct choices are (a), (b) and (d).

2. The correct choices are (b), (c) and (d).

3. The correct choices are (a), (b) and (c).

4. The only incorrect statement is (d). A geostationary satellite can be put in orbit only at a height of 35,870 km above the earth.

5. Statement (a) is incorrect; the magnitude of the force is 6.67×10^{-11} N. Statement (b) is correct. If the escape velocity for a planet is high, the thermal speeds of lighter gases are less than the escape velocity. Therefore, lighter gases are not able to escape. Statement (c) is also correct. Statement (d) is incorrect; force of friction arises due to electrical forces. Hence correct choices are (b) and (c).

6. Statement (a) is incorrect; there is no method by which a body can be shielded from the gravitational field of another body. Statement (b) is correct. Statement (c) is wrong. As the earth rotates about its axis, a body on the surface of the earth also rotates with it. Since the body is in a rotating (non–inertial) frame, it experiences an outward centrifugal force against the inward force of gravity. As a result, the acceleration due to gravity decreases due to rotation. Statement (d) is incorrect, the forces are equal in magnitude but opposite in direction. Hence choices (a), (c) and (d) are wrong.

7. The correct choices are (c) and (d). The kinetic energy of the comet changes because its speed in the orbit keeps changing. The potential energy changes because the distance of the comet from the sun keeps changing for an elliptical orbit.

8. Linear velocity or orbital velocity is $v=\sqrt{\frac{GM}{r}}$, where r is the distance of the satellite from the centre of the earth. Therefore v decreases as r is increased. Also $v = r\omega$, where ω is the angular velocity. Therefore

$$\omega = \frac{v}{r} = \frac{1}{r}\sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{r^3}}$$

Thus ω decreases with increase in r. The time period of the satellite is given by

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Thus T increases as r is increased. Hence the correct choices are (c) and (d).

9. Distances of the two satellites from the centre of the earth are $r_1 = 2R$ and $r_2 = 8R$ respectively. R = earth's radius. Their potential energies are

$$V_1 = -\frac{GmM}{r_1}$$

and

$$V_2 = -\frac{GmM}{r_2}$$

Their ratio is $\frac{V_1}{V_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4.$

The kinetic energy of a satellite can be obtained from relation

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

or $K = \frac{1}{2}mv^2 = \frac{GmM}{2r}$

Thus $K_1 = \frac{GmM}{2r_1}$ and $K_2 = \frac{GmM}{2r_2}$

The ratio of their kinetic energies is

$$\frac{K_1}{K_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4.$$

Their total energies are

$$E_1 = -\frac{GmM}{r_1} + \frac{GmM}{2r_1} = -\frac{GmM}{2r_1}$$

and $E_2 = -\frac{GmM}{r_2} + \frac{GmM}{2r_2} = -\frac{GmM}{2r_2}$

Their ratio is $\frac{E_1}{E_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4.$

Hence the correct choices are (b), (c) and (d).

10. Kinetic energy $\frac{1}{2}\ mv^2 = \frac{GmM}{2r}$ or KE $\propto \frac{1}{r}$. Thus choice (a) is correct. Angular momentum = mvr = $m \times \sqrt{\frac{GM}{r}} \times r = m\ \sqrt{GMr}$, which is proportional to $\sqrt{r}$. Hence choice (b) is wrong. Linear momentum $mv = m \times \sqrt{\frac{GM}{r}}$, which is proportional to $\frac{1}{\sqrt{r}}$. Hence choice (c) is correct. The frequency of revolution is

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{GM}{r^3}} \quad \text{i.e.} \quad \nu \propto \frac{1}{r^{3/2}}$$

Hence the correct choices are (a), (c) and (d).

11. The acceleration due to gravity is

$$g = \frac{GM}{R^2}$$

The new value of g would be

$$g' = \frac{G(0.99M)}{(0.99R)^2} \simeq 1.01\ \frac{GM}{R^2} \simeq 1.01\ g$$

Thus g would increase by about 1%. The new escape velocity would be

$$v'_e = \sqrt{\frac{2 \times 0.99M \times G}{0.99R}} = \sqrt{\frac{2MG}{R}} = v_e$$

Thus the escape velocity will remain unchanged. The potential energy of a body of mass m on earth's surface would be

$$-\frac{GM(0.99M)}{(0.99R)} = -\frac{GmM}{R}$$

Thus the potential energy will also remain unchanged. Hence the correct choices are (b) and(d).

12. Work done is independent of the path chosen and depends only on the initial and final positions of the object. Also the work done on any closed path in a gravitational field will be zero. Since every point on the surface of the earth is at the same potential, no work is done for points on the surface of the earth. Hence the correct choices are (a), (c) and (d).

13. If M is the mass of the earth, the escape velocity is

$$v_e = \sqrt{\frac{2GM}{R}}$$

For a satellite of mass m and orbital radius r (= its distance from the centre of the earth), the orbital speed v is given by

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \text{ or } v = \sqrt{\frac{GM}{r}}$$

But $$v = \frac{1}{2}\ v_e = \frac{1}{2}\sqrt{\frac{2GM}{R}}$$

$$= \sqrt{\frac{GM}{2R}}$$

$$\therefore \quad \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{2R}}$$

or $r = 2R$. Height above earth = $2R - R = R$.

Potential energy of the satellite in its orbit is

$$E_1 = -\frac{GmM}{r} = -\frac{GmM}{2R} \qquad (\because r = 2R)$$

The kinetic energy is zero because the satellite is stopped. Potential energy of the satellite on the surface of the earth is

$$E_2 = -\frac{GmM}{R}$$

$$\therefore \quad \text{Loss of PE} = E_1 - E_2$$

$$= -\frac{GmM}{2R} - \left(-\frac{GmM}{R}\right)$$

$$= \frac{GmM}{2R}$$

This is converted into kinetic energy. If v is the speed with which the satellite hits the surface of the earth, then from the law of conservation of energy, we have

$$\frac{1}{2}\ mv^2 = \frac{GmM}{2R}$$

or $$v^2 = \frac{GM}{R} = gR \quad \left(\because g = \frac{GM}{R^2}\right)$$

The correct choices are (b) and (d).

14. The distance x of the star of mass m from the centre of mass is given by

$$\frac{m}{x} = \frac{2m}{(r-x)}$$

which gives $x = \frac{r}{3}$. The orbital speed v_1 of the star of mass $m_1 = m$ is given by (here $m_2 = 2\ m$)

$$\frac{Gm_1 m_2}{r^2} = \frac{m_1 v_1^2}{x} = \frac{m_1 v_1^2}{r/3}$$

which gives $v_1 = \sqrt{\frac{Gm_2}{3r}} = \sqrt{\frac{2Gm}{3r}}$ (i)

$$\therefore \text{ Time period } (T) \text{ of } m = \frac{2\pi x}{v_1} = \frac{2\pi r}{3} \times \sqrt{\frac{3r}{2GM}}$$

$$= \pi \sqrt{\frac{2}{3GM}}\ (r)^{3/2}$$

or $T \propto r^{3/2}$.

The orbital speed v_2 of the star of mass $m_2 = 2m$ is given by

$$\frac{Gm_1m_2}{r^2} = \frac{m_2v_2^2}{(r-x)} \text{ or } \frac{Gm(2m)}{r^2} = \frac{2mv_2^2}{2r/3}$$

or $$v_2 = \sqrt{\frac{2Gm}{3r}} = v_1 \qquad \text{[see Eq. (i)]}$$

Now $K_1 = \frac{1}{2} m_1 v_1^2$ and $K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2}(2m) v_2^2$

$$\therefore \quad \frac{K_1}{K_2} = \frac{v_1^2}{2v_2^2} = \frac{1}{2}, \text{ since } v_1 = v_2.$$

Angular momentum of m_1 is $L_1 = m_1v_1 x = \frac{mv_1r}{3}$

Angular momentum of m_2 is $L_2 = m_2v_2(r-x)$

$$= \frac{2mv_2 \times 2r}{3}$$

$$\therefore \quad \frac{L_1}{L_2} = \frac{1}{4} \qquad (\text{since } v_1 = v_2)$$

The correct choices are (a), (c) and (d).

15. The orbital velocity of the space-ship of mass M and orbital radius r is given by

$$v = \sqrt{\frac{GM}{r}}$$

where $r = R + h$; R being the radius of the earth and h the height of the space-ship above the surface of the earth. For a space-ship close to the earth′s surface $r = R$. Therefore

$$v = \sqrt{\frac{GM}{R}} = \sqrt{Rg} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

The space-ship will overcome the gravitational pull of the earth if it is given an additional v_0 such that $v + v_0 = v_e$ where v_e is the escape velocity which is given by

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2Rg}$$

The required addition velocity is

$$v_0 = v_e - v = \sqrt{2Rg} - \sqrt{Rg}$$

$$= \sqrt{Rg}\,(\sqrt{2} - 1)$$

The correct choices are (a) and (d).

16. At a certain instant of time let r be the radius vector of the satellite from the centre of its circular orbit. If the velocity of the satellite is v as shown in Fig. 9.26, its angular momentum is given by

$$\mathbf{L} = \mathbf{r} \times (m\mathbf{v})$$

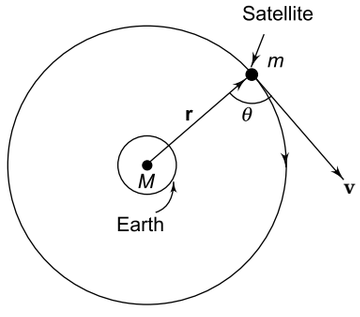


Fig. 9.26

The magnitude of angular momentum is

$$L = mrv \sin\theta$$

where θ is the angle between vectors **r** and **v**. For a circular orbit, $\theta = 90°$. Therefore

$$L = mrv \qquad (1)$$

The gravitational force of attraction on the satellite is GMm/r^2 which provides the necessary centripetal force mv^2/r, i.e.

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

or $$(mrv) = m\sqrt{GMr} \qquad (2)$$

From Eqs. (1) and (2), we have

$$L = m\sqrt{GMr}$$

This gives the magnitude of angular momentum. The direction of angular momentum is perpendicular to the plane of the orbit.

The correct choices are (a) and (d).

17. Consider a point P at a distance x from mass m_1; its distance from mass m_2 is $(r - x)$. The net gravitational field at point P will be zero if

$$\frac{Gm_1}{x^2} = \frac{Gm_2}{(r-x)^2}$$

or

$$\frac{\sqrt{m_1}}{x} = \frac{\sqrt{m_2}}{(r-x)}$$

which gives $$x = \frac{r\sqrt{m_1}}{(\sqrt{m_1} + \sqrt{m_2})} \qquad (1)$$

$$\therefore \quad (r-x) = \frac{r\sqrt{m_2}}{(\sqrt{m_1} + \sqrt{m_2})} \qquad (2)$$

The gravitational potential at point P is given by

$$U = -\frac{Gm_1}{x} + \left(-\frac{Gm_2}{r-x}\right) \qquad (3)$$

Using (1) and (2) in (3), we have

$$U = -G\left[m_1 \times \frac{(\sqrt{m_1} + \sqrt{m_2})}{r\sqrt{m_1}} + m_2 \frac{(\sqrt{m_1} + \sqrt{m_2})}{r\sqrt{m_2}}\right]$$

$$= -\frac{G}{r}\left[\sqrt{m_1}\left(\sqrt{m_1}+\sqrt{m_2}\right)+\sqrt{m_2}\left(\sqrt{m_1}+\sqrt{m_2}\right)\right]$$

$$= -\frac{G}{r}\left[m_1+m_2+2\sqrt{m_1 m_2}\right] \qquad (4)$$

Putting $m_1 = m$ and $m_2 = 4m$ in Eqs. (1) and (4) we find that the correct choices are (a) and (d).

18. $F = \dfrac{k}{r}$, where k is a constant.

$$\frac{mv^2}{r} = \frac{k}{r} \Rightarrow v = \sqrt{\frac{k}{m}}, \text{ which is independent of } r.$$

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{m}{k}}\, r \Rightarrow T \propto r.$$

Hence the correct choices are (a) and (c).

19. Total energy $E = -\dfrac{GmM}{2r}$. Hence the correct choices are (a) and (b).

20. The correct choices are (a) and (d).

21. Due to friction, the satellite will slow down. Hence its kinetic energy will decrease. Now $v = R\omega$. Therefore, a decrease in v results in a decrease in ω. Since $T = 2\pi/\omega$, time period T will increase. The angular momentum will remain unchanged because the torque due to a radial (central) force is zero. Hence the correct choices are (a), (b) and (c).

22. Orbital speed $v = \sqrt{\dfrac{GM}{r}}$. If r increases, v will decrease. Time period $T = \dfrac{2\pi r}{v} = 2\pi\sqrt{\dfrac{r^3}{GM}}$. Hence T will increase. Centripetal force = gravitational force = $\dfrac{GmM}{r^2}$. Hence if r increases, centripetal force will decrease. Gravitational P.E. = $-\dfrac{GM}{r}$. If r increase, P.E. becomes less negative, i.e. it increases. Since no external torque acts, the angular momentum is conserved. Hence all the four choices are correct.

23. The speed of the comet is greater when it is closer to the sun than when it is farther from it. Hence both v and r change for an elliptical orbit. Thus choice (a) is correct. The total energy is conserved. The linear momentum changes because the velocity changes in magnitude as well as direction. Since the force is central, the angular momentum is conserved. Hence all the four choices are correct.

24. Since the initial velocity of the body is zero, its total energy is

$$E_i = -G\frac{Mm}{r}$$

When the body reaches the earth's surface, its velocity is v and its distance from the centre of the earth is the earth's radius R. Therefore

$$E_f = \frac{1}{2}mv^2 - G\frac{Mm}{R}$$

From energy conservation, $E_i = E_f$, i.e.

$$\frac{1}{2}mv^2 - G\frac{Mm}{R} = -G\frac{Mm}{r}$$

which gives

$$v = R\left[2g\left(\frac{1}{R}-\frac{1}{r}\right)\right]^{1/2}$$

Using this expression, we find that the correct choices are (a) and (b).

25. (a) Increase in P.E. $= -\dfrac{GmM}{(R+h)} - \left(-\dfrac{GmM}{R}\right)$

$$= \frac{GMm}{2R} \qquad (\because h = R)$$

$$= \frac{1}{2}mgR \qquad \left(\because g = \frac{GM}{R^2}\right)$$

(b) If v is the velocity of projection, then from energy conservation, we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$$

Using $h = R$, this equation gives $v = \sqrt{gR}$.

(c) Total energy of the satellite orbiting at $h = R$ is

$$E_1 = \text{K.E.} + \text{P.E.} = \frac{GMm}{4R} - \left(\frac{GMm}{2R}\right) = -\frac{GMm}{4R}$$

Total energy when the satellite is at rest on the surface of the earth is

$$E_2 = -\frac{GMm}{R}$$

Energy required is $E = E_1 - E_2$

$$= -\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right) = \frac{3}{4}\frac{GMm}{R} = \frac{3}{4}mgR$$

(d) Value of g at h $(= R) = g\left(\dfrac{R}{R+h}\right)^2 = \dfrac{g}{4}$.

Hence the correct choices are (b), (c) and (d).

26. (a) P.E. of a satellite in orbit $= -\dfrac{GMm}{r}$. Therefore, P.E. of two satellites before collision is

$$(\text{P.E.})_i = 2 \times \left(-\frac{GMm}{r}\right) = -\frac{2GMm}{r}$$

After the collision, they stick together. So the combined mass is $2m$. Therefore, P.E. just after collision is

$$(\text{P.E.})_f = -\frac{GM(2m)}{r} = -\frac{2GMm}{r}$$

Hence the ratio of P.E. before and after collision is 1 : 1.

(b) Total energy of a satellite in orbit is

$$\text{K.E.} + \text{P.E.} = \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right) = -\frac{GMm}{2r}$$

Therefore, the total energy of the two satellites before collision is

$$E_i = 2 \times \left(-\frac{GMm}{2r}\right) = -\frac{GMm}{r}$$

Let V be the velocity of the combined mass. Final momentum = $(2m)V$. Since the satellites have opposite velocities, initial momentum = $mv - mv = 0$. Now final momentum = initial momentum, i.e. $2mV = 0$ or $V = 0$. Hence the combined mass has no kinetic energy; it has only potential energy. Thus, total energy of the system just after the collision is

$$E_f = -\frac{GM(2m)}{r} = -\frac{2GMm}{r}$$

$$\therefore \quad \frac{E_i}{E_f} = \frac{1}{2}$$

Since the combined mass comes to rest, the centripetal force disappears ($\because$ velocity $V = 0$). Therefore, the combined mass will not move in an orbit; it will fall freely under the gravity of the earth.

Hence the correct choices are (b) and (c).

27. The distance of each cavity from the centre O is the same. Since the two cavities are symmetrical with respect to the centre O and the mass of the sphere can be regarded as being concentrated at the centre O, the gravitational force due to the sphere is zero at the centre. Hence choice (a) is correct. For the same reason, the gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$ whose radius is 6 units and at all points of the circle $y^2 + z^2 = 4$. Hence choices (c) and (d) are also correct. But the gravitational force at point B cannot be zero.

28. (i) For $r > R$, $F = \frac{GM}{r^2}$. Therefore,

$$F_1 = \frac{GM}{r_1^2} \text{ and } F_2 = \frac{GM}{r_2^2}. \text{ Hence } \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$$

So choice (b) is correct and choice (d) is wrong.

(ii) For $r < R$, $F = \frac{GMr}{R^3}$. Therefore,

$$F_1 = \frac{GMr_1}{R^3},\ F_2 = \frac{GMr_2}{R^3}. \text{ Hence } \frac{F_1}{F_2} = \frac{r_1}{r_2}$$

So choice (a) is correct and choice (c) is wrong.

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage

Passage I

A satellite of mass m is revolving in a circular orbit of radius r around the earth of mass M. The speed of the satellite in its orbit is one-fourth the escape velocity from the surface of the earth.

1. The height of the satellite above the surface of the earth is (R = radius of earth)

(a) $2R$ (b) $3R$
(c) $5R$ (d) $7R$

2. The magnitude of angular momentum of the satellite is

(a) $m\sqrt{GMR}$ (b) $\frac{m}{2}\sqrt{GMR}$
(c) $\frac{m}{2\sqrt{2}}\sqrt{GMR}$ (d) $2m\sqrt{GMR}$

3. If the total energy of the satellite is E, its potential energy is

(a) $-E$ (b) E
(c) $2E$ (d) $-2E$

SOLUTION

1. $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$ (1)

$$v = \frac{v_e}{4} = \frac{1}{4}\sqrt{\frac{2GM}{R}} \quad (2)$$

Equations (1) and (2) give $h = 7R$, which is choice (d).

2. $L = mvR = m\sqrt{\dfrac{GM}{8R}} \times R = \dfrac{m}{2\sqrt{2}}\sqrt{GMR}$

$(\because\ h = 7\ R)$

The correct choice is (c).

3. The correct choice is (c).

Questions 4 to 8 are based on the following passage

Passage II

Considering the earth as an isolated mass, a force is experienced by a body at any distance from it. This force is directed towards the centre of the earth and has a magnitude mg, where m is the mass of the body and g is the acceleration due to gravity. The value of the acceleration due to gravity decreases with increase in the height above the surface of the earth and with increase in the depth below the surface of the earth. Even on the surface of the earth, the value of g varies from place to place and decreases with decrease in the latitude of the place.

4. Assuming the earth to be a sphere of uniform mass density, which of the graphs shown in Fig. 9.27 represents the variation g with distance r from the centre of the earth (R = radius of earth)

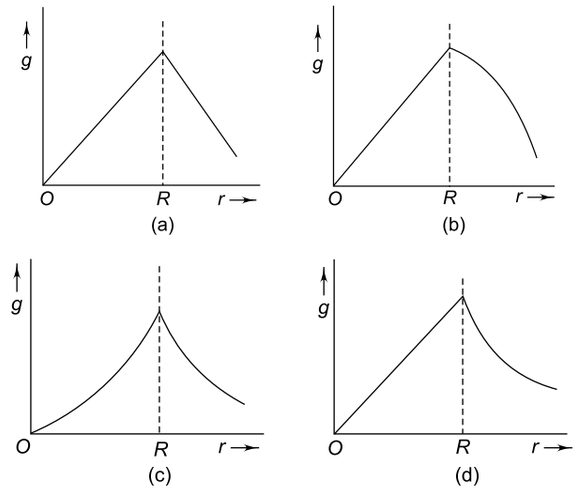


Fig. 9.27

5. A body weighs 63 N on the surface of the earth. How much will it weigh at a height equal to half the radius of the earth?

(a) 63 N (b) 32.5 N
(c) 28 N (d) none of these

6. Assuming the earth to be a sphere of uniform mass density, the weight of a body when it is taken to the end of a tunnel 32 km below the surface will (radius of earth = 6400 km)

(a) decrease by 0.5%
(b) decrease by 1%
(c) increase by 0.5%
(d) increase by 1%

7. If g_p is the acceleration due to gravity at the poles and g_e that at the equator, then

(a) $g_p < g_e$ (b) $g_p > g_e$
(c) $g_p = q_e$ (d) $g_e = 0$

8. If a tunnel is dug along a diameter of the earth and a body is dropped from one end of the tunnel,

(a) it will fall and come to rest at the centre of the earth where its weight becomes zero.
(b) it will emerge from the other end of the tunnel.
(c) it will execute simple harmonic motion about the centre of the earth.
(d) it will accelerate till it reaches the centre and decelerate after that eventually coming to rest at the other end of the tunnel.

SOLUTION

4. At a height h above the surface of the earth,

$$g_h = \frac{g_0 R^2}{(R+h)^2}$$

At a depth d below the surface of the earth

$$g_d = g_0\left(1 - \frac{d}{R}\right)$$

where g_0 = acceleration due to gravity at the surface of the earth. Hence the correct choices is (d).

5. $W = W_0\left(\dfrac{R}{R+h}\right)^2 = 63 \times \left(\dfrac{R}{R+\dfrac{R}{2}}\right)^2 = 28$ N

6. $g = g_0\left(1 - \dfrac{d}{R}\right) = g_0\left(1 - \dfrac{32}{6400}\right) = \dfrac{199 g_0}{200}$

$\therefore$ Decrease in weight = $mg_0 - mg$

$$= mg_0\left(1 - \frac{199}{200}\right) = \frac{mg_0}{200}$$

Hence the correct choice is (a).

7. Due to the rotation of the earth about its axis, $g_p > g_e$.

8. The correct choice is (c).

Questions 9 to 12 are based on the following passage

Passage III

The escape velocity on a planet or a satellite is the minimum velocity with which a body must be projected from that planet so that it escapes the gravitational pull of the planet goes into outer space. We obtain the expression for the escape velocity by equating the work required to move the body from the surface of the planet to infinity with the initial kinetic energy given to the body. The escape velocity from a planet of mass M and radius R is given by

$$v_e = \sqrt{\frac{2MG}{R}} = \sqrt{2gR}$$

where g is the acceleration due to gravity on the surface of the planet and G is the gravitation constant.

9. Choose the only incorrect statement from the following. The escape velocity from a planet

(a) is independent of the mass of the body which is projected.

(b) is independent of the direction in which the body is projected.

(c) depends on the mass and radius of the planet.

(d) will be less than the value given by the expression $v_e = \sqrt{\frac{2MG}{R}}$ if the planet has an atmosphere.

10. The mass of Jupiter is about 319 times that of the earth and its radius is about 11 times that of the earth. The ratio of the escape velocity on Jupiter to that on earth is

(a) $\sqrt{29}$ (b) 29

(c) $\frac{1}{\sqrt{29}}$ (d) $\frac{1}{\sqrt{29}}$

11. If R is the radius of the earth and g the acceleration due to gravity on its surface, the escape velocity of a body projected from a satellite orbiting the earth at a height $h = R$ from the surface of the earth will be

(a) $\sqrt{gR}$ (b) $\sqrt{2gR}$

(c) $\sqrt{3gR}$ (d) $2\sqrt{gR}$

12. A body is dropped from a height equal to half the radius of the earth. If v_e is the escape velocity on earth and air resistance is neglected, it will strike the surface of the earth with a speed

(a) $\frac{v_e}{\sqrt{2}}$ (b) $\frac{v_e}{2}$

(c) $\frac{v_e}{\sqrt{3}}$ (d) $\frac{v_e}{3}$

SOLUTION

9. The only incorrect statement is (d). Due to atmospheric friction, a part of the initial kinetic energy is lost as heat. Hence the actual value of the escape velocity is greater than the value obtained from the given expression.

10. $\frac{v_J}{v_E} = \sqrt{\frac{M_J}{M_E} \times \frac{R_E}{R_J}} = \sqrt{319 \times \frac{1}{11}} = \sqrt{29}$

Hence the correct choice is (a).

11. The escape velocity at a height h is given by

$$v'_e = \sqrt{2g'(R+h)}$$

where g' is the acceleration due to gravity at height h,

$$g' = g\left(\frac{R}{R+h}\right)^2$$

For $h = R$, we get $v'_e = \sqrt{gR}$, which is choice (a).

12. The correct choice is (c). Use conservation of energy, i.e.

Total energy at h ($= R/2$)

= Total energy when the body strikes the earth

$$\Rightarrow -\frac{GmM}{(R+h)} = \frac{1}{2}mv^2 - \frac{GmM}{R}$$

SECTION IV

Assertion Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1

A body is projected up with a velocity equal to half the escape velocity from the surface of the earth. If R is the radius of the earth and atmospheric resistance is neglected, it will attain a height $h = R/3$.

Statement-2

The gravitational potential is $-GM/R$ on the surface of the earth and it increases with height; M being the mass of the earth.

2. Statement-1

The total energy (kinetic + potential) of a satellite moving in a circular orbit around the earth is half its potential energy.

Statement-2

The gravitational force obeys the inverse square law of distance.

3. Statement-1

Two bodies of masses $m_1 = m$ and $m_2 = 3m$ are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach when they are at a separation

$$r \text{ is } v = \sqrt{\frac{2Gm}{r}}$$

Statement-2

The gain in the kinetic energy of each body equals the loss in its gravitational potential energy.

4. Statement-1

An astronaut inside a massive space-ship orbiting around the earth will experience a finite but small gravitational force.

Statement-2

The centripetal force necessary to keep the space-ship in orbit around the earth is provided by the gravitational force between the earth and the space-ship.

5. Statement-1

The escape velocity varies with altitude and latitude of the place from where it is projected.

Statement-2

The escape velocity depends on the gravitational potential at the point of projection.

6. Statement-1

A comet orbits the sun in a highly elliptical orbit. Its potential energy and kinetic energy both change over the orbit but the total energy remains constant throughout the orbit.

Statement-2

For a comet, only the angular momentum remains constant throughout the orbit.

7. Statement-1

The acceleration due to gravity decreases due to rotation of the earth.

Statement-2

A body on the surface of the earth also rotates with it in a circular path. A body in a rotating (non-inertial) frame experiences an outward centrifugal force against the inward force of gravity.

SOLUTIONS

1. The correct choice is (b). Use $v_e = \sqrt{\frac{2Gm}{R}}$ and total energy at r $(= R + h)$ = total initial energy, i.e.

$$-\frac{GmM}{r} = \frac{1}{2}mv^2 - \frac{GmM}{R}$$

2. The correct choice is (a). The centripetal force needed for circular motion is provided by the gravitational force. Since the gravitational force obeys the inverse square law of distance, the orbital velocity of the satellite is given by

$$v = \sqrt{\frac{GM}{r}}$$

where M = mass of earth and r = orbital radius.

Therefore

$$\text{Kinetic energy} = \frac{1}{2}\ mv^2 = \frac{GmM}{2r}$$

where m = mass of the satellite. From the inverse square law of distance, we find that the potential of the satellite is given by

$$\text{Potential energy} = -\frac{GmM}{r}$$

$\therefore$ Total energy E = K.E. + P.E.

$$= \frac{GmM}{2r} + \left(-\frac{GmM}{r}\right)$$

$$= -\frac{GmM}{2r} = \frac{\text{P.E.}}{2}$$

3. The correct choice is (d). Initially when the two masses are at an distance from each other, their gravitational potential energy is zero. When they are at a distance r from each other the gravitational P.E. is

$$\text{P.E} = -\frac{Gm_1m_2}{r}$$

The minus sign indicates that there is a decrease in P.E. This gives rise to an increase in kinetic energy. If v_1 and v_2 are their respective velocities when they are at a distance r apart, then, from the law of conservation of energy, we have

$$\frac{1}{2}m_1v_1^2 = \frac{Gm_1m_2}{r} \text{ or } v_1 = \sqrt{\frac{2Gm_2}{r}}$$

and $$\frac{1}{2}m_1v_2^2 = \frac{Gm_1m_2}{r} \text{ or } v_2 = \sqrt{\frac{2Gm_1}{r}}$$

Therefore, their relative velocity of approach is

$$v = v_1 + v_2 = \sqrt{\frac{2Gm_2}{r}} + \sqrt{\frac{2Gm_1}{r}}$$

$$= \sqrt{\frac{2G}{r}(m_2 + m_1)}$$

Putting $m_1 = m$ and $m_2 = 3m$, we get $v = 2\sqrt{\frac{2Gm}{r}}$

4. The correct choice is (b). Because the centripetal force equals the gravitational force exerted by the earth on the space-ship, the astronaut does not experience any gravitational force of the earth. The only force of gravity that an astronaut in an orbiting space-ship experiences is that which is due to the gravitational force exerted by the space-ship. Since the space-ship is very massive, this force is finite but very small.

5. The correct choice is (a). The gravitational potential at a point varies with the altitude and latitude of the place.

6. The correct choice is (c).

7. The correct choice is (a).

10 Chapter Elasticity

REVIEW OF BASIC CONCEPTS

10.1 ELASTICITY

The ability of a body to regain its original shape and size when the deforming force is withdrawn, is known as elasticity.

10.2 STRESS

When a deforming force is applied to a body, it reacts to the applied force by developing a reaction (or restoring) force which, from Newton's third law, is equal in magnitude and opposite in direction to the applied force. *The reaction force per unit area of the body which is called into play due to the action of the applied force is called stress.* Stress is measured in units of force per unit area, i.e. Nm^{-2}. Thus

$$\text{Stress} = \frac{F}{A}$$

where F is the applied force and A is the area over which it acts.

10.3 STRAIN

When a deforming force is applied to a body, it may suffer a change is size or shape. Strain is defined as the ratio of the change in size or shape to the original size or shape of the body. Strain is a number; it has no units or dimensions.

The ratio of the change in length to the original length is called *longitudinal strain*. The ratio of the change in volume to the original volume is called *volume strain*. The strain resulting from a change in shape is called *shearing strain*.

10.4 HOOKES' LAW

Hookes' law states that, within the elastic limit, the stress developed in a body is proportional to the strain produced in it. Thus the ratio of stress to strain is a constant. This constant is called the modulus of elasticity. Thus

$$\text{Modulus of elasticity} = \frac{\text{stress}}{\text{strain}}$$

Since strain has no unit, the unit of the modulus of elasticity is the same as that of stress, namely, Nm^{-2}.

10.5 YOUNG'S MODULUS

Suppose that a rod of length L and a uniform cross-sectional area A is subjected to a longitudinal pull. In other words, two equal and opposite forces are applied at its ends.

$$\text{Stress} = \frac{F}{A}$$

The stress in the present case is called linear stress, tensile stress, or extensional stress. If the direction of the force is reversed so that ΔL is negative, we speak of *compressional strain* and *compressional stress*. If the elastic limit is not exceeded, then from Hooke's law

$$\text{Stress} \propto \text{strain}$$

or

$$\text{Stress} = Y \times \text{strain}$$

or

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F}{A} \cdot \frac{L}{\Delta L} \qquad (1)$$

where Y, the constant of proportionality, is called the *Young's modulus* of the material of the rod and may be defined as the *ratio of the linear stress to linear strain, provided the elastic limit is not exceeded.* Since strain has no unit, the unit of Y is Nm^{-2}.

10.6 BULK MODULUS

Solids, liquids and gases can be deformed by subjecting them to a uniform normal pressure P in all directions.

Stress and pressure have the same dimension (force per unit area), but pressure is not the same thing as

stress. Pressure is the force per unit area acting on the surface of a system, *the force being everywhere perpendicular to the surface* so that, for a uniform pressure, the force per unit area is the same.

Pressure is a particular kind of stress which changes only the volume of the substance and not its shape. The substance may be a solid, liquid or gas. A small increase in pressure ΔP applied to a substance decreases its volume from, say, V to $V - \Delta V$ so that ΔV is the small decrease in volume. The volume strain is given by

$$\text{Volume strain} = -\frac{\Delta V}{V}$$

The bulk modulus is defined as the ratio of the excess pressure and the corresponding volume strain, i.e.

$$B = \frac{\Delta P}{\left(-\frac{\Delta V}{V}\right)} = -\frac{\Delta P V}{\Delta V} \qquad (2)$$

If ΔP is positive, ΔV will be negative and vice versa. The negative sign in our definition of bulk modulus B ensures that B is always positive. The SI unit of B is Nm^{-2}. The reciprocal of B is known as *compressibility*.

The bulk modulus of a gas depends on the pressure. Under isothermal conditions (i.e. when the temperature is kept constant), the bulk modulus of a gas is equal to its pressure P. Under adiabatic conditions (i.e. when heat is not allowed to leave or enter the system), the bulk modulus is equal to γP, where $\gamma (= C_p/C_v)$ is the ratio of the molar heat capacities of the gas at constant pressure and constant volume. Thus

$$\text{Isothermal bulk modulus} = P$$
$$\text{Adiabatic bulk modulus} = \gamma P.$$

10.7 SHEAR MODULUS OR MODULUS OF RIGIDITY

Shear is a particular kind of stress which only solids can withstand. The solid is deformed by changing its shape without changing its size. The body does not move or rotate as a whole: there is a relative displacement of its contiguous layers.

Consider a solid in the form of a rectangular cube as in Fig. 10.1.

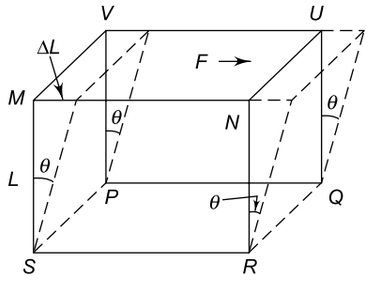


Fig. 10.1

Suppose the lower face $PQRS$ is held fixed and a force F is applied parallel to the upper face $MNUV$. As a result of this, the lines joining the two faces turn through an angle θ. We say that the face $MNRS$ is sheared through an angle θ (measured in radians). The angle θ is called the *shear strain or the angle of shear* and is a measure of the degree of deformation.

If A is the area of the face $MNUV$, the ratio F/A is the shearing stress. It is found that for small deformation, the shearing stress is proportional to the shear strain, i.e.

$$\frac{F}{A} \propto \theta, \quad \frac{F}{A} = \eta\theta, \quad n = \frac{F}{A\theta}$$

The quantity n is called the *shear modulus* or the modulus of rigidity. Referring to Fig. 10.1, if θ is small,

$$\theta \simeq \tan\theta = \frac{\Delta L}{L}$$

so that

$$\eta = \left(\frac{F}{A}\right)\cdot\frac{L}{\Delta L}$$

This equation looks similar to Eq. (1) for Young's modulus with the difference that F/A here is the *tangential stress* and not longitudinal stress.

10.8 POISSON'S RATIO

When a wire is stretched with a force, apart from an increase in its length, there is a slight decrease in its diameter, i.e. both shape and volume change under longitudinal stress.

The ratio of the decrease ΔD in diameter to the original diameter D is called *lateral strain*, i.e. strain at right angles to the deforming force. Thus

$$\text{Lateral strain} = \frac{\text{change in diameter}}{\text{original diameter}} = \frac{\Delta D}{D}$$

$$\text{Also} \quad \text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

The ratio of the two is called Poisson's ratio and is denoted by σ. Hence,

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{\Delta D/D}{\Delta L/L}$$

$$= \frac{\Delta D}{D}\cdot\frac{L}{\Delta L}$$

Since it is a ratio between two types of strain, σ is dimensionless. Theoretically, one can show that it must be less than 0.5. For most solids it lies between 1/4 and 1/3, and for rubber it is very nearly 0.5.

10.9 ENERGY STORED IN A STRAINED WIRE: STRAIN ENERGY

If a wire is stretched, the energy stored per unit volume is given by

$$U = \frac{1}{2} \text{ stress} \times \text{strain}$$

$$= \frac{1}{2} S \times \varepsilon$$

where S = stress, ε = strain

Since $$Y = \frac{S}{\varepsilon}$$

$$U = \frac{1}{2} S \times \varepsilon = \frac{1}{2} Y\varepsilon^2 = \frac{1}{2} \frac{S^2}{Y}$$

10.10 THERMAL STRESSES

If a metal rod fixed rigidly at its ends is heated or cooled, then due to expansion or contraction, tensile or compressive stress is set up in the rod. These stresses are called thermal stresses. If a rod of length L is free to expand or contract and its temperature is changed by ΔT, the change in its length is given by

$$\Delta L = \alpha L \, \Delta T$$

where α is the coefficient of linear expansion of the rod. Now from Eq. (1), we have

$$\Delta L = \frac{FL}{AY}$$

$$\therefore \quad F = \alpha AY\Delta T$$

Thus, the thermal stress in the rod is

$$\frac{F}{A} = \alpha Y \Delta T$$

Similarly, if a fluid is contained in a vessel such that its volume cannot change, then a change in temperature results in a change in pressure. The thermal stress is then given by

$$\Delta P = \gamma B \Delta T$$

where B is the bulk modulus of the fluid and γ is its coefficient of volume expansion or contraction.

10.11 TORQUE

If a rod or wire of length l and radius r is fixed at one end and the other end is twisted through an angle θ (in radian) by applying a torque, the restoring torque is given by

$$\tau = \frac{\pi \eta r^4 \theta}{2l}$$

Where η is the shear modulus of the material of the rod or wire.

EXAMPLE 10.1

A steel wire (original length = 2 m, diameter = 1mm) and a copper wire (original length = 1m, diameter = 2 mm) are loaded as shown in Fig. 10.2. Find the ratio of extension of steel wire to that of copper wire. Young's modulus of steel = 2×10^{11} Nm^{-2} and that of copper = 1×10^{11} Nm^{-2}.

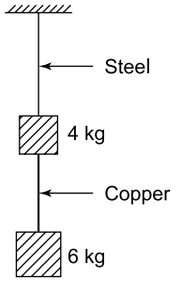


Fig. 10.2

SOLUTION

$$Y = \frac{FL}{A\Delta L}$$

$$\Rightarrow \quad \Delta L = \frac{FL}{YA} = \frac{4FL}{Y \times \left(\pi d^2\right)}$$

For steel wire $F_s = (6 + 4) \times 9.8 = 98$ N

For copper wire $F_e = 6 \times 9.8 = 58.8$ N

$$(\Delta L)_s = \frac{4F_s \, L_s}{Y_s \times \left(\pi d_s^2\right)} \text{ and } (\Delta L)_c = \frac{4F_c \, L_c}{Y_c \times \left(\pi d_c^2\right)}$$

$$\therefore \quad \frac{(\Delta L)_s}{(\Delta L)_c} = \frac{F_s}{F_c} \times \frac{L_s}{L_c} \times \frac{Y_c}{Y_s} \times \left(\frac{d_c}{d_s}\right)^2$$

$$= \frac{98}{58.8} \times \frac{2}{1} \times \frac{1}{2} \times (2)^2$$

$$= \frac{20}{3}$$

EXAMPLE 10.2

When the pressure on all sides of a metal cube is increased by 10^7 Pa, its volume decreases by 0.015% . Find the bulk modulus of the metal.

SOLUTION

$\Delta P = 10^7$ Pa, $\dfrac{\Delta V}{V} = -\dfrac{0.015}{100}$

$$\therefore \quad B = -\frac{\Delta P}{\Delta V / V} = -\frac{10^7}{-0.015/100}$$

$$= 6.67 \times 10^{10} \text{ Pa or Nm}^{-2}$$

EXAMPLE 10.3

A rubber cube is subjected to a pressure of 10^6 Pa on all sides. As a result, each side of the cube decreases by 1%. Calculate the bulk modulus of rubber.

SOLUTION

Volume of cube is $V = L^3$. Therefore

$$\frac{\Delta V}{V} = \frac{3\Delta L}{L} = 3 \times -1\% = -3\% = -\frac{3}{100}$$

$$\therefore\ B = -\frac{\Delta \mathrm{P}}{\Delta \mathrm{V}/\mathrm{V}} = -\frac{10^6}{-3/100} = 3.33 \times 10^7\ \mathrm{Nm}^{-2}$$

EXAMPLE 10.4

The base of a rubber cube of side 3.0 cm is clamped. A horizontal force F of 2.7 N is applied on the top face. Calculate (a) the angle of shear and (b) the horizontal displacement of the top face of the eraser. Shear modulus of rubber = $1.5 \times 10^5\ \mathrm{Nm}^{-2}$.

SOLUTION

Refer to Fig. 10.1 and also to Fig. 10.3.

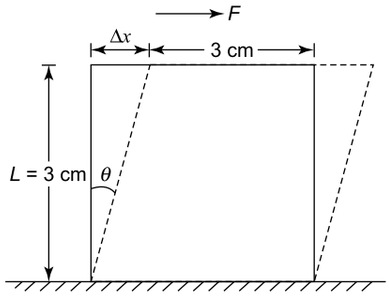


Fig. 10.3

Area of top face (A) = 3 cm × 3 cm

$= 9\ \mathrm{cm}^2 = 9 \times 10^{-4}\ \mathrm{m}^2$

$$\text{Shearing Stress} = \frac{F}{A} = \frac{2.7}{9 \times 10^{-4}} = 3 \times 10^3\ \mathrm{Nm}^{-2}$$

$$\text{(a) Angle of Shear } \theta = \frac{\text{shearing stress}}{\text{shear modulus}}$$

$$= \frac{3 \times 10^3}{1.5 \times 10^5} = 2.0 \times 10^{-2}\ \text{rad}$$

$$\text{(b) } \tan\theta = \frac{\Delta x}{L} \Rightarrow \Delta x = L\tan\theta = L\theta \quad (\because\ \theta \text{ is small})$$

$= 3\ \mathrm{cm} \times 2.0 \times 10^{-2}$

$= 6 \times 10^{-2}\ \mathrm{cm}$

$= 0.6\ \mathrm{mm}$

EXAMPLE 10.5

In this example, Statement-I is followed by Statement-II. Choose

(a) if statement-II is true and statement-II is true and statement-II is the correct explanation for statement-I.

(b) if statement-I is true and statement-II is true but statement-II is not the correct explanation for statement-I.

(c) if statement-I is true and statement-II is false.

(d) if statement-I is false and statement-II is true.

(i) *Statement-I*: A wire of length L extends by an amount ΔL when a block of weight W is hung from it as shown in Fig. 10.4(a). If the same wire goes over a frictionless pulley and two blocks each of weight W are hung at its ends as shown in Fig 10.4(b), the wire will extend by $2\Delta L$, if the elastic limit is not exceeded.

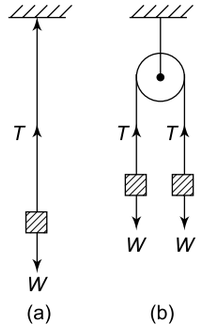


Fig. 10.4

Statement-II: For a given load, the extension is proportional to the length of the wire.

(ii) *Statement-I*: Steel is more elastic than rubber.

Statement-II: For a given deforming force, a steel wire is extended by an amount less than rubber band of the same radius and the same length.

(iii) *Statement-I*: Figure 10.5 shows the stress-strain curves for two materials 1 and 2. It follows from the graphs that material 1 has higher Young's modulus then material 2.

Statement-II: Within elastic limit, the slope of the stress-strain graph is greater for material 1 than for material 2.

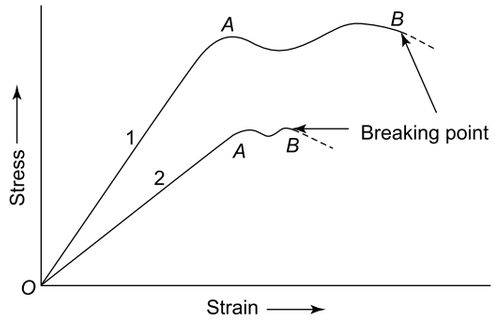


Fig. 10.5

(iv) *Statement-I*: It follows from Fig. 10.5 than material 1 is more brittle than material 2.

Statement-II: The plastic range is more for material 1 than for material 2.

(v) *Statement-I*: A hollow shaft is stronger than a solid shaft both made of the same material and the same external dimensions.

Statement-II: To produce a given twist, a greater torque is needed for the hollow shaft than for the solid shaft.

SOLUTIONS

(i) Statement-II is true. Since the length of the wire and tension in wire in cases (a) and (b) shown in Fig. 10.4 are the same, the extension in the wire will be the same equal to ΔL in both case. So Statement-I is false. The correct choice is (d)

(ii) $Y_s > Y_r$, i.e. $\dfrac{FL}{AY_s} < \dfrac{FL}{AY_r}$ or $(\Delta L)_s < (\Delta L)_r$

So Statement-II is true. This means than steel develops less strain that rubber and hence steel, can withstand greater stress than rubber. So Statement-I is also true. The correct choice is (a).

(iii) Young's modulus = slope of the straight portion of the stress – strain graph. Hence Young's modulus of material 1 is greater than that of material 2. So the correct choice is (a).

(iv) The plastic range AB of material 2 is less than that of material 1. So the breaking strain of material 2 is less than that of material 1. Hence material 2 is more brittle than material 1. So, Statement-I is false and Statement-I is true.

(v) The torque per unit angular twist for a solid shaft is

$$\frac{\tau_s}{\theta} = \frac{\pi\eta r^4}{2L} \qquad (1)$$

If r_1 and r_2 are the external and internal radii of the hollow shaft, the torque per unit angular twist is

$$\frac{\tau_h}{\theta} = \frac{\pi\eta(r_1^4 - r_2^4)}{2L}$$

$$= \frac{\pi\eta}{2L}(r_1^2 - r_2^2)(r_1^2 + r_2^2) \qquad (2)$$

Since the volumes are the same, $\pi(r_1^2 - r_2^2)L = \pi r^2 L$.

Using this eq. (2), we get

$$\frac{\tau_h}{\theta} = \frac{\pi\eta r^2}{2L}(r_1^2 + r_2^2)$$

$$= \frac{\tau_s}{\theta}\left(1 + \frac{r_2^2}{r^2}\right) \qquad (\because\ r_1 = r)$$

i.e. $\tau_h > \tau_s$. So the correct choice is (a).

EXAMPLE 10.6

A heavy and thick rubber rope is hung frame support. It is stretched by its own weight. The strain in the rope is

(a) the maximum near the support.

(b) the maximum near the free end of the rope

(c) the maximum at the centre of the rope

(d) the same everywhere.

SOLUTION

If M is mass of the rope and L its length, then the tension at a point P at a distance y from the free end is (see Fig. 10.6)

$$T_p = \frac{Mg}{L} \times y$$

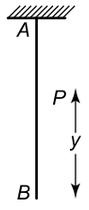


Fig. 10.6

Thus the tension is maximum at A and zero at B. Now

$$\text{Strain} = \frac{\Delta L}{L} = \frac{T}{AY}$$

Hence the strain is maximum near A and minimum near B. So the correct choice is (a).

EXAMPLE 10.7

A metal rod of length 1.0 m and cross-sectional area 1.0 mm^2 is fixed horizontally between two rigid supports. The tension in the rod is zero when the temperature is 40°C. Find the tension in the rod when the temperature falls to 20°C. Young's modulus of metal = 2.0×10^{11} Nm^{-2} and coefficient of linear expansion = 1.0×10^{-5} K^{-1}.

SOLUTION

Thermal stress = $\alpha Y \Delta T$

$$\Rightarrow \quad \frac{T}{A} = \alpha Y \Delta T$$

$$\Rightarrow \quad T = A\alpha Y \Delta T$$

$$= (1.0 \times 10^{-5}) \times (1.0 \times 10^{-4}) \times (2.0 \times 10^{11}) \times (40 - 20)$$

$$= 40 \text{ N}$$

EXAMPLE 10.8

A bob of mass 1 kg is suspended by a rubber cord 2.0 m long and of cross-sectional area 1.0 cm^2 . It is revolved in a horizontal circle of radius 1.0 m at a speed of 4 ms^{-1}. If Young's modulus of rubber is 5×10^8 Nm^{-2}, calculate the extension of the cord. Take $g = 10$ ms^{-2}.

SOLUTION

Refer to Fig. 10.7. Let T be the tension in the cord.

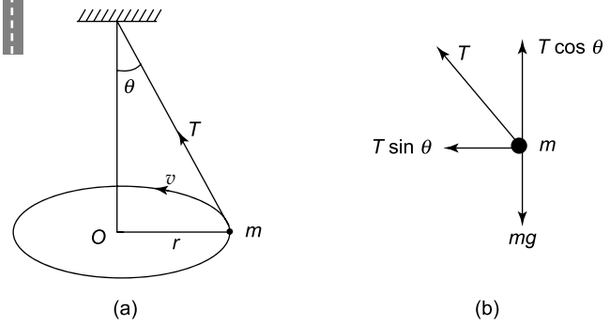


Fig. 10.7

Figure 10.7(b) shows the free body diagram of the bob. From the diagram, it follows that

$$T \sin\theta = \frac{mv^2}{r} \text{ (centripetal force)}$$

and $T \cos\theta = mg$

Squaring and adding these equations, we get

$$T = \sqrt{\left(\frac{mv^2}{r}\right)^2 + (mg)^2}$$

$$= \sqrt{\left(\frac{1.0 \times 4^2}{1.0}\right)^2 + (1.0 \times 10)^2}$$

$$= 18.87 \text{ N}$$

Now $$\Delta L = \frac{TL}{AY} = \frac{18.87 \times 2.0}{(1.0 \times 10^{-4}) \times (5 \times 10^8)}$$

$$= 7.5 \times 10^{-4} \text{ m} = 0.75 \text{ mm}$$

EXAMPLE 10.9

A wire of length L is suspended from a support. Its length becomes L_1 when its is loaded with a block of mass m_1 and L_2 when it is loaded with a block of mass m_2.

Then

(a) $L = \sqrt{L_1 L_2}$ (b) $L = \frac{1}{2}(L_1 + L_2)$

(c) $L = \frac{L_1 m_2 + L_2 m_1}{(m_1 + m_2)}$ (d) $L = \frac{L_1 m_2 - L_2 m_1}{(m_2 - m_1)}$

SOLUTION

$$\Delta L = \frac{mgL}{AY}$$

$$\Delta L_1 = \frac{m_1 gL}{AY}$$

$$\Rightarrow \quad L_1 - L = \frac{m_1 gL}{AY} \qquad \text{(i)}$$

and $$\Delta L_2 = \frac{m_2 gL}{AY}$$

$$\Rightarrow \quad L_2 - L = \frac{m_2 gL}{AY} \qquad \text{(ii)}$$

Dividing (i) by (ii)

$$\frac{L_1 - L}{L_2 - L} = \frac{m_1}{m_2} \Rightarrow L = \frac{L_1 m_2 - L_2 m_1}{m_2 - m_1}$$

So the correct choice is (d).

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?
(a) Length = 50 cm, diameter = 0.5 mm
(b) Length = 100 cm, diameter = 1 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 300 cm, diameter = 3 mm

IIT, 1981

2. When a wire of length L is stretched with a tension F, it extends by l. If the elastic limit is not exceeded, the amount of energy stored in the wire is

(a) Fl (b) $\frac{1}{2} Fl$

(c) $\frac{Fl^2}{L}$ (d) $\frac{Fl^2}{2L}$

IIT, 1990

3. A wire, suspended from one end, is stretched by attaching a mass of 2 kg to the lower end. The wire stretches by 1 mm. The ratio of the energy gained by the wire to the gravitational energy lost by the mass in dropping a distance of 1 mm is (Take g = 10 ms^{-2})

(a) 1 (b) 1/2

(c) 1/3 (d) 1/4

4. When the pressure on a metal cube is increased by 10^7 Pa, its volume decreases by 0.015%. The bulk modulus of the metal (in Nm^{-2}) is

(a) 1.5×10^{10} (b) 3.33×10^{10}

(c) 6.67×10^{10} (d) 7.5×10^{10}

5. A rubber cube is subjected to a pressure of 10^6 Pa on all sides. As a result, each side of the cube decreases by 1%. The bulk modulus of rubber is (in Nm^{-2})

(a) 1×10^8 (b) 3×10^7

(c) 3.33×10^7 (d) 6.67×10^7

6. Two wires A and B made of the same material have their lengths in the ratio 1 : 2 and their diameters in the ratio 2 : 1. If they are stretched with the same force, the ratio of the increase in the length of A to that of B is

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

7. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional areas are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the Young's modulii of copper and steel are 1.0×10^{11} Nm^{-2} and 2.0×10^{11} Nm^{-2}, the total extension of the composite wire is

(a) 1.25 mm (b) 1.50 mm

(c) 1.75 mm (d) 2.0 mm

8. Two springs of equal lengths and equal cross-sectional areas are made of materials whose Young's modulii are in the ratio of 3 : 2. They are suspended and loaded with the same mass. When stretched and released, they will oscillate with time periods in the ratio of

(a) $\sqrt{3} : \sqrt{2}$ (b) 3 : 2

(c) $3\sqrt{3} : 2\sqrt{2}$ (d) 9 : 4

9. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. The work one in stretching the wire by an amount x is given by

(a) $\frac{YA\,x^2}{L}$ (b) $\frac{YA\,x^2}{2L}$

(c) $\frac{YA\,L^2}{x}$ (d) $\frac{YA\,L^2}{2x}$

IIT, 1987

10. A solid sphere of radius R and made of a material of bulk modulus K is completely immersed in a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, the fractional change in the radius of the sphere, $\delta R/R$ is given by

(a) $\frac{Mg}{KA}$ (b) $\frac{Mg}{2KA}$

(c) $\frac{Mg}{3KA}$ (d) $\frac{Mg}{4KA}$

11. A steel wire of cross-sectional area 3×10^{-6} m^2 can withstand a maximum strain of 10^{-3}. Young's modulus of steel is 2×10^{11} Nm^{-2}. The maximum mass the wire can hold is (Take $g = 10$ ms^{-2})

(a) 40 kg (b) 60 kg

(c) 80 kg (d) 100 kg

12. A rubber eraser 3 cm × 1 cm × 8 cm is clamped at one end with the 8 cm edge vertical. A horizontal force of 2.4 N is applied at the free end (the top face). If the shear modulus of rubber is 1.6×10^5 Nm^{-2}, the horizontal displacement of the top face will be

(a) 1 mm (b) 2 mm

(c) 3 mm (d) 4 mm

13. The density of water at the surface of the ocean is ρ. If the bulk modulus of water is B, what is the density of ocean water at a depth where the pressure is nP_0, where P_0 is the atmospheric pressure?

(a) $\frac{\rho B}{B-(n-1)P_0}$ (b) $\frac{\rho B}{B+(n-1)P_0}$

(c) $\frac{\rho B}{B-nP_0}$ (d) $\frac{\rho B}{B+nP_0}$

14. One end of a uniform rod of mass M and cross-sectional area A is suspended from rigid support and an equal mass M is suspended from the other end. The stress at the mid-point of the rod will be

(a) $\frac{2Mg}{A}$ (b) $\frac{3Mg}{2A}$

(c) $\frac{Mg}{A}$ (d) zero

15. Two rods of different materials, having coefficients of linear expansion α_1 and α_2 and Young's modulii Y_1 and Y_2, are fixed between two rigid walls. The rods are heated to the same temperature. There is no bending of rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods will be equal provided $Y_1 : Y_2$ is equal to

(a) 2 : 3 (b) 1 : 1

(c) 3 : 2 (d) 4 : 9

16. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the ceiling and a weight W_1 is suspended from its lower end. If S is the area of cross-section of the wire, the stress in the wire at a height $3L/4$ from its lower end is

(a) W_1/S (b) $\left(W_1 + \frac{W}{4}\right)\Big/S$

(c) $\left(W_1 + \frac{3W}{4}\right)\Big/S$ (d) $(W_1 + W)/S$

17. A uniform wire (Young's modulus 2×10^{11} Nm^{-2}) is subjected to a longitudinal tensile stress of 5×10^7 Nm^{-2}. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is

(a) 1.5×10^{-4} (b) 1.0×10^{-4}

(c) 0.5×10^{-4} (d) 0.25×10^{-4}

IIT, 1993

18. An elastic spring of unstretched length L and force constant k is stretched by a small length x. It is further stretched by another small length y. The work done in the second stretching is

(a) $\frac{1}{2}\ ky^2$ (b) $\frac{1}{2}\ k\ (x^2 + y^2)$

(c) $\frac{1}{2}\ k(x + y)^2$ (d) $\frac{1}{2}\ ky(2x + y)$

IIT, 1994

19. The length of an elastic string is a metre when the longitudinal tension is 4 N and b metre when the tension is 5 N. The length of the string (in metre) when the longitudinal tension is 9 N is

(a) $a - b$ (b) $5b - 4a$

(c) $2b - \frac{a}{2}$ (d) $4a - 3b$

IIT, 1987

20. The Poisson's ratio of a material is 0.4. If a force is applied to a wire of this material, there is a decrease of cross-sectional area by 2%. The percentage increase in its length is :

(a) 3% (b) 2.5%

(c) 1% (d) 0.5%

21. Fig. 10.8 shows a graph of the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, the Young's modulus of the material of the wire is

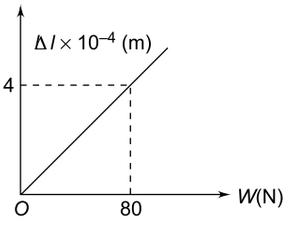


Fig. 10.8

(a) 2×10^{11} N/m^2 (b) 2×10^{-11} N/m^2

(c) 3×10^{12} N/m^2 (d) 3×10^{-12} N/m^2

IIT, 2003

22. A light rod of length L is suspended from a support horizontally by means of two vertical wires A and B of equal length as shown in Fig. 10.9. The cross-sectional area of A is half that of B and the Young's modulus of A is twice that of B. A weight W is hung as shown. The value of x so that W produces equal stress in wires A and B is

(a) $\frac{L}{3}$ (b) $\frac{L}{2}$

(c) $\frac{2L}{3}$ (d) $\frac{3L}{4}$

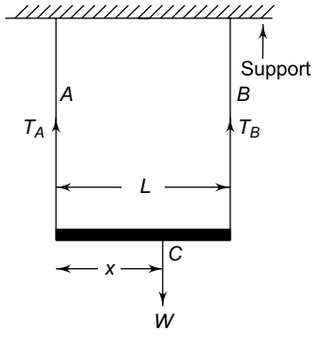


Fig. 10.9

23. In Q. 22 above, the value of x at which W produces equal strain in wires A and B is

(a) $\frac{L}{4}$ (b) $\frac{L}{2}$

(c) $\frac{2L}{3}$ (d) $\frac{4L}{5}$

24. A wire breaks when subjected to a stress S. If ρ is the density of the material of the wire and g, the acceleration due to gravity, then the length of the wire so that it breaks by its own weight is

(a) ρgS (b) $\frac{\rho g}{S}$

(c) $\frac{gS}{\rho}$ (d) $\frac{S}{\rho g}$

25. A stone of mass m is attached to one end of a wire of cross-sectional area A and Young's modulus Y. The stone is revolved in a horizontal circle at a speed such that the wire makes an angle θ with the vertical. The strain produced in the wire will be.

(a) $\frac{mg\cos\theta}{AY}$ (b) $\frac{mg}{AY\cos\theta}$

(c) $\frac{mg\sin\theta}{AY}$ (d) $\frac{mg}{AY\sin\theta}$

26. The density of a metel at a normal pressure is ρ. Its density when it is subjected to an excess pressure p is ρ'. If B is the bulk modulus of the metal, the ratio ρ'/ρ is

(a) $\frac{1}{1-p/B}$ (b) $1+\frac{p}{B}$

(c) $\frac{1}{1-p/B}$ (d) $1 + B/p$

27. A thin uniform metallic rod of mass M and length L is rotated with a angular velocity ω in a horizontal plane about a vertical axis passing through one of its ends. The tension in the middle of the rod is

(a) $\frac{1}{2}ML\omega^2$ (b) $\frac{1}{4}ML\omega^2$

(c) $\frac{1}{8}ML\omega^2$ (d) $\frac{3}{8}ML\omega^2$

28. A rubber cord of mass M, length L and cross-sectional area A is hung vertically from a ceiling. The Young's modulus of rubber is Y. If the change in the diameter of the cord due to its own weight is neglected, the increase in its length due to its own weight is

(a) $\frac{MgL}{AY}$ (b) $\frac{MgL}{2AY}$

(c) $\frac{2MgL}{AY}$ (d) $\frac{MgL}{4AY}$

29. The volume of a wire remains unchanged when the wire is subjected to a certain tension. The Poisson's ratio of the material of the wire is

(a) 0.25 (b) 0.3

(c) 0.4 (d) 0.5

ANSWERS

1. (a)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (d)
7. (a)	**8.** (a)	**9.** (b)	**10.** (c)	**11.** (b)	**12.** (d)
13. (a)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (d)
19. (b)	**20.** (b)	**21.** (a)	**22.** (c)	**23.** (b)	**24.** (d)
25. (b)	**26.** (a)	**27.** (d)	**28.** (b)	**29.** (d)	

SOLUTIONS

1. Extension $\Delta L = \frac{FL}{AY} = \frac{4FL}{\pi d^2 Y}$

Since the wires are made of the same material, Y is the same for all wires. For a given tension F,

$$\Delta L \propto \frac{L}{d^2}$$

The value of $\frac{L}{d^2}$ is the largest for L = 50 cm and d = 0.5 mm.

2. Energy stored per unit volume = $\frac{1}{2}$ (stress × strain).

If A is the cross-sectional area of the wire, the total energy store in the wire is

$$U = \frac{1}{2}\text{ (stress} \times \text{strain)} \times \text{volume of wire}$$

$$= \frac{1}{2}\times\frac{F}{A}\times\frac{l}{L}\times AL$$

$$= \frac{1}{2}Fl$$

3. Energy gained by the wire is

$$U_1 = \frac{1}{2}Fl = \frac{1}{2}mgl$$

$$= \frac{1}{2}\times 2 \times 10 \times (1 \times 10^{-3}) = 0.01 \text{ J}$$

Gravitational P.E. lost by the mass is

$U_2 = mgh = 2 \times 10 \times (1 \times 10^{-3}) = 0.02$ J

Hence the correct choice is (b).

4. $\Delta P = 10^7$ Pa, $\dfrac{\Delta V}{V} = -\dfrac{0.015}{100} = -1.5 \times 10^{-4}$

Bulk modulus $B = -\dfrac{\Delta P}{\Delta V / V}$

$$= -\frac{10^7}{-1.5 \times 10^{-4}} = 6.67 \times 10^{10} \text{ Nm}^{-2}$$

5. $V = L^3$. Therefore

$$\frac{\Delta V}{V} = \frac{3\Delta L}{L} = 3 \times (-1\%) = -3\%$$

$$B = -\frac{\Delta P}{\Delta V / V} = -\frac{10^6}{-3\%} = 3.33 \times 10^7 \text{ Nm}^{-2}$$

6. $\Delta L = \dfrac{FL}{AY} = \dfrac{4FL}{\pi d^2 Y}$

Since F and Y are the same for wires A and B,

$$\frac{(\Delta L)_A}{(\Delta L)_B} = \left(\frac{L_A}{L_B}\right) \times \left(\frac{d_B}{d_A}\right)^2 = \frac{1}{2} \times \left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

7. When a wire of length L, cross-sectional area A and Young's modulus Y is stretched with a force F, the extension l in the wire is given by

$$l = \frac{FL}{AY}$$

Since F and A are the same for the two wires, we have

For copper wire $\quad l_c = \dfrac{FL_c}{AY_c}$

For steel wire, $\quad l_s = \dfrac{FL_s}{AY_s}$

$\therefore \quad l_s = l_c \dfrac{L_s}{L_c} \cdot \dfrac{Y_c}{Y_s}$

$$= 1 \text{ mm} \times \left(\frac{0.5\,\text{m}}{1.0\,\text{m}}\right) \times \left(\frac{1.0 \times 10^{11} \text{Nm}^{-2}}{2.0 \times 10^{11} \text{Nm}^{-2}}\right)$$

$= 0.25$ mm

$\therefore$ Total extension $= 1 + 0.25 = 1.25$ mm.

Hence the correct choice is (a).

8. Young's modulus $Y = \dfrac{F}{A} \cdot \dfrac{L}{l}$

Force constant $k = \dfrac{F}{l} = \dfrac{YA}{L}$

where l is the extension in the spring of original length L and cross-sectional area A when a force $F = Mg$ is applied. Now, the time period of vertical oscillations is given by

$$T = 2\pi \sqrt{\frac{M}{k}} = 2\pi \sqrt{\frac{ML}{YA}}$$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{Y_2}{Y_1}} = \sqrt{\frac{3}{2}}$$

Hence the correct choice is (a).

9. Work done per unit volume of the wire

$$= \frac{1}{2} \text{ (stress} \times \text{strain)}.$$

$$Y = \frac{\text{stress}}{\text{strain}}$$

$\therefore \quad \text{stress} = Y \times \text{strain}$

$\therefore$ Work done per unit volume

$$= \frac{1}{2} \times Y \times (\text{strain})^2 = \frac{1}{2} \times Y \times \left(\frac{x}{L}\right)^2$$

But, volume of the wire $= A \times L$

$$\therefore \text{ Work done } = \frac{1}{2} \times Y \times \left(\frac{x}{L}\right)^2 \times A \times L$$

$$= \frac{YAx^2}{2L}$$

Hence the correct choice is (b).

10. Pressure exerted by the piston on the liquid when a mass M is placed on the piston, $P = Mg/A$. This pressure is exerted by the liquid equally in all directions. Therefore, the surface of the sphere experiences a force P per unit area. The stress on the sphere is $P = Mg/A$. Now, the volume of the sphere is

$$V = \frac{4\pi R^3}{3}$$

Due to stress, the change in the volume of the sphere is

$$\delta V = \delta\left(\frac{4\pi R^3}{3}\right) = \frac{4\pi}{3} \cdot 3R^2 \delta R$$

$$= 4\pi R^2 \delta R$$

$$\therefore \quad \text{Volume strain } \frac{\delta V}{V} = \frac{3\delta R}{R}$$

By definition, bulk modulus

$$K = \frac{\text{stress}}{\text{strain}} = \frac{Mg/A}{3\delta R / R}$$

or $\quad \dfrac{\delta R}{R} = \dfrac{Mg}{3KA}$

Hence the correct choice is (c).

11. Maximum stress = Young's modulus × maximum strain

$$= 2 \times 10^{11} \times 10^{-3} = 2 \times 10^8 \text{ Nm}^{-2}$$

$\therefore$ Maximum force (F) = maximum stress × area

$= 2 \times 10^8 \times 3 \times 10^{-6} = 600$ N

Maximum mass = $\frac{F}{g} = \frac{600}{10} = 60$ kg, which is choice (b).

12. Figure 10.10 shows a rubber eraser $ABCD$ clamped at end AB and a horizontal F applied at the free face DC resulting in a displacement $DD' = CC'$. The shear angle is given by

$$\tan\theta = \frac{DD'}{AD}$$

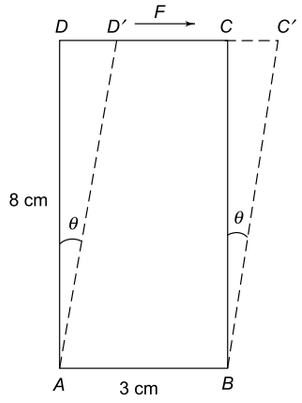


Fig. 10.10

Since θ is small, $\tan\theta = \theta = \frac{DD'}{AD}$

Area of top face = 3 cm × 1 cm

$= 3$ cm^2 $= 3 \times 10^{-4}$ m^2

$\therefore$ Shearing stress $= \frac{F}{A} = \frac{2.4}{3\times10^{-4}}$

$= 8 \times 10^3$ Nm^{-2}

$\therefore$ Shear angle $\theta = \frac{\text{shearing stress}}{\text{shear modulus}}$

$= \frac{8\times10^3}{1.6\times10^5} = 0.05$ rad

or $\frac{DD'}{AD} = 0.05$ or $DD' = 0.05 \times AD$

$= 0.05 \times 8$ cm $= 0.4$ cm $= 4$ mm

Hence the correct choice is (d).

13. Pressure at the surface of the ocean = P_0, the atmospheric pressure. Pressure at a depth = nP_0 (given).

$\therefore$ Increase in pressure (ΔP) $= nP_0 - P_0$

$= (n-1)P_0$

Let V be the volume of a certain mass M of water at the surface, so that $M = \rho V$. The decrease in volume under pressure ΔP is

$$\Delta V = \frac{V\Delta P}{B}$$

$\therefore$ Volume of the same mass M of water at the given depth is

$$V' = V - \Delta V = V - \frac{V\Delta P}{B}$$

$$= V\left(1-\frac{\Delta P}{B}\right) = \frac{V}{B}(B-\Delta P)$$

$\therefore$ Density of water at that depth is

$$\rho' = \frac{M}{V'} = \frac{\rho V}{V'} = \frac{\rho V}{\frac{V}{B}(B-\Delta P)}$$

$$= \frac{\rho B}{B-\Delta P} = \frac{\rho B}{B-(n-1)P_0}$$

Hence the correct choice is (a).

14. Since the rod is uniform, half its weight can be taken to act at its mid-point. Therefore, stress at mid-point is

$$\frac{\text{weight of suspended mass + weight of half the rod}}{\text{cross-sectional area}}$$

$$= \frac{Mg + \frac{1}{2}Mg}{A} = \frac{3Mg}{2A},$$ which is choice (b).

15. Young's modulus $Y = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon}$, where $\varepsilon = \frac{l}{L}$

Now, if the temperature of a rod is increased by θ, the increase in its length due to thermal expansion is

$$l = \alpha L\theta$$

$\therefore$ Strain $\varepsilon = \frac{l}{L} = \alpha\theta$. Now stress is

$$\sigma = Y\varepsilon = Y\alpha\theta$$

For the two rods, the stress is

$$\sigma_1 = Y_1\alpha_1\theta \text{ and } \sigma_2 = Y_2\alpha_2\theta$$

But $\sigma_1 = \sigma_2$ (given). Hence $Y_1\alpha_1\theta = Y_2\alpha_2\theta$ or $\frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$. Hence the correct choice is (c).

16. Since the wire is uniform, i.e. its mass per unit length is constant over its entire length L, the total donward weight = the weight of the suspended mass + weight of length $\frac{3L}{4}$ of the wire or $F = W_1 + \frac{3W}{4}$.

$\therefore$ Stress $= \frac{\text{force}}{\text{area}} = \frac{F}{S} = \frac{W_1 + \frac{3}{4}W}{S}$

Hence the correct choice is (c)

17. $\frac{\Delta L}{L} = \frac{\text{stress}}{Y} = \frac{5\times10^7}{2\times10^{11}} = 2.5 \times 10^{-4}$. Given $\frac{\Delta V}{V} = \frac{0.02}{100} = 2\times10^{-4}$

Now, $V = \pi r^2 L$. Therefore,

$$\frac{\Delta V}{V} = \frac{\Delta(\pi r^2 L)}{\pi r^2 L} = \frac{2\pi r\,\Delta r\,L + \pi r^2 L\,\Delta L}{\pi r^2 L}$$

$$= 2\frac{\Delta r}{r} + \frac{\Delta L}{L}$$

or $\frac{2\Delta r}{r} = \frac{\Delta V}{V} - \frac{\Delta L}{L} = 2\times10^{-4} - 2.5\times10^{-4}$

$$= -0.5\times10^{-4}$$

or $\frac{\Delta r}{r} = -0.25\times10^{-4}$

$\therefore$ Fractional decrease in $r = 0.25\times10^{-4}$ which is choice (d).

18. Potential energy stored in the spring when it is extended by x is

$$U_1 = \frac{1}{2}kx^2$$

Potential energy stored in the spring when it is further extended by y is

$$U_2 = \frac{1}{2}k(x+y)^2$$

$\therefore$ Work done $= U_2 - U_1 = \frac{1}{2}k(x+y)^2 - \frac{1}{2}kx^2$

$$= \frac{1}{2}ky(2x+y)$$

19. If L is the initial length, then the increase in length by a tension F is given by

$$l = \frac{FL}{\pi r^2 Y}$$

Hence $a = L + l = L + \frac{4L}{\pi r^2 Y} = L + 4c$ (1)

and $b = L + \frac{9L}{\pi r^2 Y} = L + 5c$ (2)

where $c = \frac{L}{\pi r^2 Y}$. Solving (1) and (2) for L and c, we get $L = 5a - 4b$ and $c = b - a$. For $F =$ 9N, we have

$$x = L + \frac{9L}{\pi r^2 Y} = L + 9c$$

$$= (5a - 4b) + 9(b - a) = 5b - 4a$$

Hence the correct choice is (b).

20. Poisson's ratio is defined as

$$\sigma = \frac{\Delta d/d}{\Delta l/l} \text{ or } \frac{\Delta l}{l} = \frac{\Delta d/d}{\sigma} \quad \text{(i)}$$

where d is the diameter and l is the length of the wire. The area of cross-section is

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

or $\log A = \log\left(\frac{\pi}{4}\right) + 2\log d$

Differentiating, we have

$$\frac{\Delta A}{A} = 2\frac{\Delta d}{d}$$

or $\frac{\Delta d}{d} = \frac{1}{2}\frac{\Delta A}{A} = \frac{1}{2} \times 2\% = 1\%$

$$\left(\because \frac{\Delta A}{A} = 2\%, \text{ given}\right)$$

Using this in (i) we have

$$\frac{\Delta l}{l} = \frac{1\%}{0.4} = 2.5\% \qquad (\because \sigma = 0.4)$$

Hence the correct choice is (b).

21. Stress $= \frac{\text{force}}{\text{area}} = \frac{W}{A}$. Strain $= \frac{\Delta l}{l}$. Young's modulus is given by

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{W/A}{\Delta l/l} = \frac{W}{\Delta l} \times \frac{l}{A} \quad \text{(i)}$$

Now, slope of graph is $\frac{\Delta l}{W} = \frac{4\times10^{-4}}{80}$ m/N. Using this in (i), we get (given $l = 1$ m and $A = 10^{-6}$ m^2)

$$Y = \frac{80}{4\times10^{-4}} \times \frac{1}{10^{-6}} = 2\times10^{11} \text{ N/m}^2$$

22. Let T_A and T_B be the tensions in wires A and B respectively. If a_A and a_B are the respective cross-sectional areas, then

$$\text{Stress in wire } A = \frac{T_A}{a_A}$$

$$\text{Stress in wire } B = \frac{T_B}{a_B}$$

The stress in wires A and B will be equal if

$$\frac{T_A}{a_A} = \frac{T_B}{a_B} \text{ or } \frac{T_A}{T_B} = \frac{a_A}{a_B} = \frac{1}{2} \text{ (given).}$$

Since the system is in equilibruim, the moments of forces T_A and T_B about C will be equal (see Fig. 10.3), i.e.

$$T_A \times x = T_B \times (L - x)$$

or $\dfrac{T_A}{T_B} = \dfrac{L-x}{x}$ or $\dfrac{1}{2} = \dfrac{L-x}{x}$

which gives $x = \dfrac{2L}{3}$. Hence the correct choice is (c).

23. Strain $= \dfrac{\text{Stress}}{\text{Young's modulus}} = \dfrac{T}{aY}$

Strain in wires A and B will be equal if

$$\frac{T_A}{a_A Y_A} = \frac{T_B}{a_B Y_B}$$

or $$\frac{T_A}{T_B} = \frac{Y_A}{Y_B} \times \frac{a_A}{a_B} = 2 \times \frac{1}{2} = 1$$

which gives $T_A = T_B$. Equating moments about C, we have

$$T_A \times x = T_B \times (L - x)$$

Which gives $x = L - x$ or $x = \dfrac{L}{2}$ ($\because T_A = T_B$). Hence the correct choice is (b).

24. Let L be the required length of wire. If A is its area of cross-section and ρ is the density of its material, the weight of the wire is

$$W = mg = Al\,\rho g$$

$\therefore$ Stress $= \dfrac{\text{weight}}{\text{area}} = \dfrac{AL\rho g}{A} = L\rho g$

or $S = L\rho g \Rightarrow L = S/\rho g$

25. Refer to Fig. 10.11. For vertical equilibruim

$$T\cos\theta = mg$$

or $$T = \frac{mg}{\cos\theta}$$

If L is the original length of the wire, the increase in length is

$$l = \frac{TL}{AY}$$

$\therefore$ Strain $= \dfrac{l}{L} = \dfrac{T}{AY} = \dfrac{mg}{AY\cos\theta}$

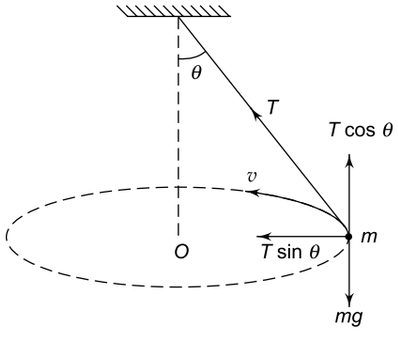


Fig. 10.11

26. Let V be the volume of the metal at normal pressure.

From $B = -\dfrac{p}{\Delta V/V}$, the decrease in the volume when the metal is subjected to an excess pressure p is given by

$$|\Delta V| = \frac{pV}{B}$$

$\therefore$ New volume of metal is $V' = V - \dfrac{pV}{B} = V\left(1 - \dfrac{p}{B}\right)$

Now mass of the metal is $m = \rho V$. Therefore, its new density is

$$\rho' = \frac{m}{V'} = \frac{\rho V}{V\left(1 - \frac{p}{B}\right)} = \frac{\rho}{1 - \frac{p}{B}}$$

or $\dfrac{\rho'}{\rho} = \dfrac{1}{1 - p/B}$, which is choice (a).

27. Let m be the mass per unit length of the rod. Then $M = mL$.

Consider a small element of length dx located at C at a distance x from A. (Fig. 10.12)

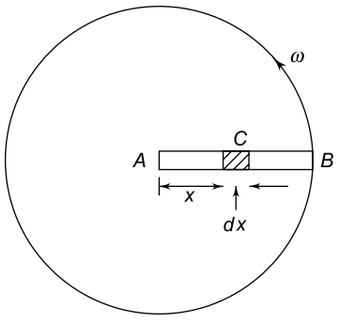


Fig. 10.12

The mass of element of length $dx = mdx$. The centripetal force at C is

$$dF = (mdx)\, x\omega^2 \qquad (1)$$

The force (or tension) at point C is due to the outer portion CB of the rod, i.e. portion from $x = x$ to $x = L$. Therefore tension at point C is

$$F = \int_{x=x}^{x=L} (mdx)\, x\omega^2$$

$$= m\omega^2 \int_{x=x}^{x=L} x\,dx = \frac{1}{2} m\omega^2 (L^2 - x^2)$$

Now, $m = \dfrac{M}{L}$. Therefore,

$$F = \frac{1}{2}\frac{M}{L}\, \omega^2\, (L^2 - x^2)$$

$$= \frac{1}{2}\, ML\omega^2 \left(1 - \frac{x^2}{L^2}\right)$$

If r is the radius of the rod and ρ its density, the mass of the rod $M = \pi r^2 L\rho$. Therefore,

$$F = \frac{1}{2}\ \pi r^2 \rho L^2 \omega^2 \left(1 - \frac{x^2}{L^2}\right) \qquad (2)$$

Thus the tension in the rod varies with x, it is zero at $x = L$, i.e. at point B and maximum at $x = 0$, i.e. at A where the rod is pivoted.

Tension in the middle $\left(x = \frac{L}{2}\right)$ of the rod is

$$F' = \frac{1}{2}\ \pi r^2 \rho L^2 \omega^2 \left(1 - \frac{1}{4}\right)$$

$$= \frac{3}{8}\ \pi r^2 \rho L^2 \omega^2$$

Thus the correct choice is (d).

28. Let M be the mass of the rubber cord, L its length, A its cross-sectional area (assumed constant). Let us first find the elongation dl of an element AB of length dy at a distance y from the fixed end (Fig. 10.13). The force due to the weight of the cord is maximum at $P(y = 0)$ and zero at $Q(y = L)$. Therefore, the force acting at the lower end B of the element is

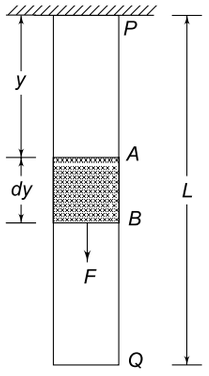


Fig. 10.13

$$F = \frac{Mg}{L}\ (L - y) \qquad (1)$$

This force is responsible for the elongation of element AB. Now

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{dl/dy} = \frac{Fdy}{Adl}$$

or $$dl = \frac{Fdy}{AY} \qquad (2)$$

Using Eq. (1) in Eq. (2), we get

$$dl = \frac{Mg}{LAY}\ (L - y)dy$$

To obtain the total elongation l of the cord, we integrate from $y = 0$ to $y = L$. Thus

$$l = \int dl = \frac{Mg}{LAY}\int_0^L (L - y)\ dy$$

$$= \frac{Mg}{LAY}\left| Ly - \frac{y^2}{2}\right|_0^L$$

$$= \frac{Mg}{LAY}\left(L^2 - \frac{L^2}{2}\right) = \frac{MgL}{2AY}$$

The correct choice is (b).

29. Poisson's ratio σ is defined as

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta r/r}{\Delta l/l}$$

where r is the radius of the wire and l its length and Δr is the change in r and Δl the change in l when the wire is subjected to tension.

Volume of wire before elongation is

$$V_1 = \pi r^2 l$$

Volume of wire after elongation is

$$V_2 = \pi(r - \Delta r)^2\ (l + \Delta l)$$

Given $V_1 = V_2$. Thus

$$\pi r^2 l = \pi(r - \Delta r)^2\ (l + \Delta l)$$

$$= \pi[r^2 - 2r(\Delta r) + (\Delta r)^2]\ (l + \Delta l)$$

$$= \pi r^2(l + \Delta l) - 2\ \pi r\ \Delta r\ (l + \Delta l)$$

$$+ \pi(\Delta r)^2\ (l + \Delta l)$$

Since Δr and Δl are very small, terms of order $\Delta r\ \Delta l$ and $(\Delta r)^2$ and higher can be ignored. Then, we have

$$\pi r^2 l = \pi r^2 l + \pi r^2 \Delta l - 2\ \pi r l\ \Delta r$$

or $$r\Delta l = 2\ l\Delta r \quad \text{or} \quad \frac{\Delta l}{l} = 2\ \frac{\Delta r}{r}$$

$$\therefore \qquad \sigma = \frac{\Delta r/r}{\Delta l/l} = \frac{1}{2} = 0.5,$$

which is choice (d).

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Figure 10.14 shows the stress–strain graphs for materials *A* and *B*.

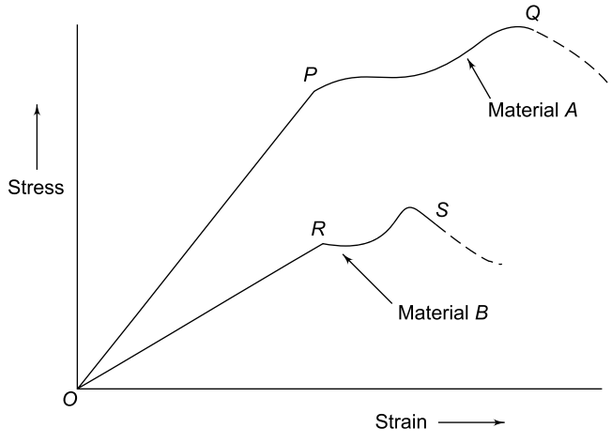


Fig. 10.14

From the graph it follows that
(a) material *A* has a higher Young's modulus
(b) material *B* is more ductile
(c) material *A* is more brittle
(d) material *A* can withstand a greater stress.

2. Two wires *A* and *B* have the same cross-section and are made of the same material, but the length of wire *A* is twice that of *B*. Then, for a given load
(a) the extension of *A* will be twice that of *B*
(b) the extensions of *A* and *B* will be equal
(c) the strain in *A* will be half that in *B*
(d) the strains in *A* and *B* will be equal.

3. Two wires *A* and *B* have equal lengths and are made of the same material, but the diameter of wire *A* is twice that of wire *B*. Then, for a given load
(a) the extension of *B* will be four times that of *A*
(b) the extensions of *A* and *B* will be equal
(c) the strain in *B* is four times that in *A*
(d) the strains in *A* and *B* will be equal.

4. In Q. 3, above
(a) Young's modulus of *A* is twice that of *B*
(b) Young's modulus is the same for both *A* and *B*
(c) *A* can withstand a greater load before breaking than *B*
(d) *A* and *B* will withstand the same load before breaking

5. Choose the correct statements from the following:
(a) Steel is more elastic than rubber.
(b) The stretching of a coil spring is determined by the Young's modulus of the wire of the spring.
(c) The frequency of a tuning fork is determined by the shear modulus of the material of the fork.
(d) When a material is subjected to a tensile (stretching) stress, the restoring forces are caused by interatomic attraction.

6. A uniform wire of Young's modulus Y is stretched by a force within the elastic limit. If S is the stress produced in the wire and ε is the strain in it, the potential energy stored per unit volume is given by

(a) $\frac{1}{2}\varepsilon S$ (b) $\frac{1}{2}Y\varepsilon^2$

(c) $\frac{S^2}{2Y}$ (d) $\frac{1}{2}Y\varepsilon S$

7. A wire AB of length $2l$ and cross-sectional area a is stretched without tension between fixed points A and B (Fig. 10.15). The wire is pulled at the centre into shape ACB such that $d \ll l$. The tension in the string is T and the strain produced in it is ε. If the Young's modulus of the material of wire is Y. Then

(a) $\varepsilon = \frac{d}{2l}$ (b) $\varepsilon = \frac{d^2}{2l^2}$

(c) $T = aY\left(\frac{d}{2l}\right)$ (d) $T = aY\left(\frac{d^2}{2l^2}\right)$

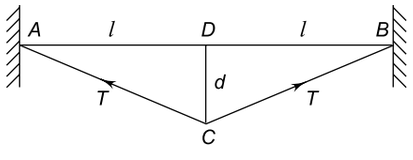


Fig. 10.15

ANSWERS AND SOLUTIONS

1. The slope of the linear portion of the curve gives the Young's modulus of the material. The slope of the linear portion OP for material A is greater than that of the linear portion OR for material B. Hence statement (a) is correct. The plastic region for material A (from P to Q) is greater than that (from

R to S) for material B, which indicates that material A is more ductile. Hence statement (b) is incorrect. The breaking stress for material B (i.e. stress corresponding to point S) is less than that for ma-terial A (i.e. stress corresponding to point Q), which implies that material B can break move easily than material A. Thus material B more brittle. Hence choice (c) is also incorrect. Material A is stronger than material B because it can withstand a greater stress before it breaks. The breaking stress is the stress corresponding to point Q for material A and to point S for material B. Hence the correct choice are (a) and (d).

2. We know that

$$Y = \frac{FL}{Al}$$

$$\therefore \quad l = \frac{FL}{AY}$$

Since the two wires are made of the same material, the Young's modulus Y is the same for both. Since load F and the cross-sectional A are the same,

$$l \propto L$$

i.e. extension is proportional to the length of the wire. Hence choice (a) is correct. The strain in a wire is given by

$$\frac{l}{L} = \frac{F}{AY}$$

Since F, A and Y are the same, the strain in wires A and B will be equal. Thus the correct choices are (a) and (d).

3. Area of cross-section $A = \pi d^2/4$; where d is the diameter of the wire. Therefore

$$l = \frac{4FL}{\pi d^2 Y}$$

Since F, L and Y are the same for wires A and B, $l \propto 1/d^2$, i.e. the extension is inversely proportional to the square of the diameter. Hence choice (a) is correct. The strain is

$$\frac{l}{L} = \frac{4F}{\pi d^2 Y}$$

Thus, strain $\propto 1/d^2$. Hence the correct choices are (a) and (c).

4. Since wires A and B are made of the same material they have the same Young's modulus. Now breaking load = breaking stress × cross-sectional area, the wire having a greater area of cross-section can withstand a greater load. Hence the correct choices are (b) and (c).

5. Statement (a) is correct because the Young's modulus of steel is greater than that of rubber. Statement (b) is incorrect. If a spring is stretched, the total length of the wire in the coil and the volume of the wire, both do not change. Only the shape of the coils of the wire undergoes a change. Hence it is the shear modulus that determines the stretching of a coil. Statement (c) is also incorrect. The bending moment of the prongs of a tuning fork is determined by the Young's modulus of its material. Hence the restoring force on the prongs depends on Young's modulus, which determines the frequency of the fork. Statement (d) is correct. When the material is not subjected to any stress, its atoms are in their normal (equilibrium) positions. When a tensile stress is applied, the separation R between the atoms becomes greater than the equilibrium separation R_0. For $R > R_0$, the interatomic forces are attractive.

6. The correct choices are (a), (b) and (c).

7. $AC + CB = 2\,AC = 2(l^2 + d^2)^{1/2}$. Increase in length is

$$\Delta L = AC + CB - AB = 2(l^2 + d^2)^{1/2} - 2l$$

$$\text{Stress} = \frac{T}{a}.$$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{2(l^2 + d^2)^{1/2} - 2l}{2l}$$

If $d \ll l$, $(l^2 + d^2)^{1/2} = l\left(1 + \frac{d^2}{l^2}\right)^{1/2} = l\left(1 + \frac{d^2}{2l^2}\right)$.
Therefore

$2(l^2 + d^2)^{1/2} - 2l = 2l\left(1 + \frac{d^2}{2l^2}\right) - 2l = \frac{d^2}{l}$. Thus

$$\therefore \quad \text{Strain} = \frac{\Delta L}{L} = \frac{d^2}{2l^2}$$

Now $$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/a}{d^2/2l^2}$$

or $$T = a\,Y\left(\frac{d^2}{2l^2}\right)$$

Hence the correct choices are (b) and (d).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage

Passage I

Elasticity: The atoms in solids are held together by inter-atomic forces. The average locations of the atoms in a lattice do not change with time. Since the atoms are almost lacking in mobility, their kinetic energy is negligibly small. It is this lack of mobility which makes a solid rigid. This rigidity is the cause of elasticity in solids. In some solids such as steel, the atoms are bound together by larger inter-atomic forces than in solids such as aluminium. Thus, the elastic behaviour varies from solid to solid. Even fluids exhibit elasticity. All material bodies get deformed when subjected to a suitable force. The ability of a body to regain its original shape and size is called elasticity. The deforming force per unit area is called stress. The change in the dimension (length, shape or volume) divided by the original dimension is called strain. The three kinds of stresses are tensile stress, shearing stress and volumetric stress. The corresponding strains are called tensile strain, shearing strain and volume strain. According to Hookes' law, within the elastic limit stress is proportional to strain. The ratio stress/strain is called the modulus of elasticity.

1. Figure 10.16 is the load-extension curve for a metallic wire. Over which region is Hookes' law obeyed?

(a) OA (b) AB
(c) BC (d) CD

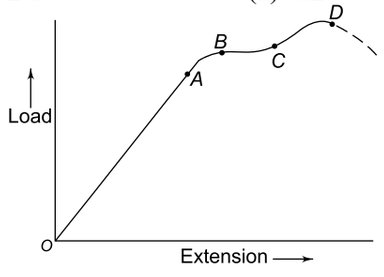


Fig. 10.16

2. In Q.1 above, the metal shows plastic behaviour beyond point

(a) A (b) B
(c) C (d) D

3. Figure 10.17 shows the strain-stress graphs for materials A and B. From the graph it follows that

(a) material A has a higher Young's modulus than B.
(b) material B has a higher Young's modulus than A.
(c) for a given strain, material B can withstand greater stress than A.
(d) for a given stress, the strain produced in material B is more than that in material A.

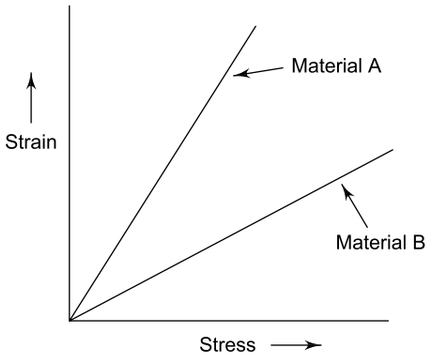


Fig. 10.17

4. Choose the correct statements from the following.

(a) Steel is more elastic than rubber.
(b) Fluids have Young's modulus as well as shear modulus.
(c) Solids have Young's modulus, bulk modulus as well as shear modulus.
(d) Bulk modulus of water is greater than that of copper.

ANSWERS

1. Hooke's law is obeyed in the linear portion of the graph. Hence the correct choice is (a).

2. The correct choice is (b).

3. Young's modulus $= \dfrac{\text{Stress}}{\text{Strain}}$, which is, therefore, given by the reciprocal of the strain versus stress graph. The correct choices are (b) and (c).

4. The correct choices are (a) and (c).

Questions 5 to 7 are based on the following passage

Passage II

Two rods P and Q of different metals having the same area A and the same length L are placed between two rigid walls as shown in Fig. 10.18. The coefficients of linear expansion of P and Q are α_1 and α_2 respectively and their Young's modulii are Y_1 and Y_2. The temperature of both rods is now raised by T degrees.

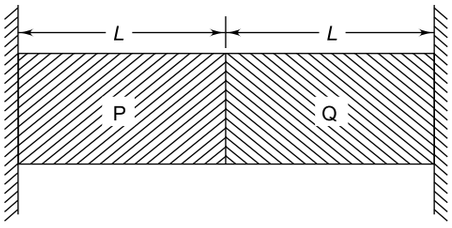


Fig. 10.18

5. The force exerted by one rod on the other is

(a) $F = \dfrac{TA(\alpha_1 + \alpha_2)}{\left(\dfrac{1}{Y_1} + \dfrac{1}{Y_2}\right)}$

(b) $F = TA\ Y_1Y_2(\alpha_1 + \alpha_2)$

(c) $F = TA(Y_1 + Y_2)\alpha_1\alpha_2$

(d) None of these

6. The new length of the rod P is

(a) $L_1 = L\left[1 + \alpha_1 T + \dfrac{F}{AY_1}\right]$

(b) $L_1 = L\left[1 - \alpha_1 T + \dfrac{F}{AY_1}\right]$

(c) $L_1 = L\left[1 + \alpha_1 T - \dfrac{F}{AY_1}\right]$

(d) $L_1 = L\left[1 - \alpha_1 T - \dfrac{F}{AY_1}\right]$

7. The new length of rod Q is

(a) $L_2 = L\left[1 + \alpha_2 T + \dfrac{F}{AY_2}\right]$

(b) $L_2 = L\left[1 - \alpha_2 T + \dfrac{F}{AY_2}\right]$

(c) $L_2 = L\left[1 + \alpha_2 T - \dfrac{F}{AY_2}\right]$

(d) $L_2 = L\left[1 - \alpha_2 T - \dfrac{F}{AY_2}\right]$

SOLUTION

$$\Delta L_1 = L\alpha_1 T \quad \text{and} \quad \Delta L_2 = L\alpha_2 T$$

$$\therefore \quad \Delta L_1 + \Delta L_2 = LT(\alpha_1 + \alpha_2) \qquad (1)$$

Since the walls are rigid, this increase in length cannot occur. Hence a force F appears between them. The decrease in length of the rods due to this force is

$$\Delta L_1' = \frac{FL}{Y_1 A} \quad \text{and} \quad \Delta L_2' = \frac{FL}{Y_2 A}$$

$$\therefore \quad \Delta L_1' + \Delta L_2' = \frac{FL}{A}\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right) \qquad (2)$$

5. Since the total length cannot change, the increase in total length due to heating = decrease in total length due to the compressive force F. Equating (1) and (2), we find that the correct choice is (a).

6. New length of rod P = original length + increase in length due to heating – decrease in length due to F. Hence.

$$L_1 = L + L\alpha_1 T - \frac{FL}{AY_1}$$

So the correct choice is (c).

7. The correct choice is (c).

Questions 8 to 11 are based on the following passage

Passage III

One end of a string of length L and cross-sectional area A is fixed to a support and the other end is fixed to a bob of mass m. The bob is revolved in a horizontal circle of radius r with an angular velocity ω such that the string makes an angle θ with the vertical.

8. The angular velocity ω is equal to

(a) $\sqrt{\dfrac{g\sin\theta}{r}}$ (b) $\sqrt{\dfrac{g\cos\theta}{r}}$

(c) $\sqrt{\dfrac{g\tan\theta}{r}}$ (d) $\sqrt{\dfrac{g\cot\theta}{r}}$

9. The tension T in the string is

(a) $\dfrac{mg}{\cos\theta}$ (b) $\dfrac{mg}{\sin\theta}$

(c) $\dfrac{mg}{\tan\theta}$ (d) $m(g^2 + r^2\omega^4)^{1/2}$

10. The increase ΔL in length of the string is

(a) $\dfrac{TL}{AY}$ (b) $\dfrac{MgL}{AY\cos\theta}$

(c) $\dfrac{MgL}{AY\sin\theta}$ (d) $\dfrac{MgL}{AY}$

11. The stress in the string is

(a) $\frac{mg}{A}$ (b) $\frac{mg}{A}\left(1-\frac{r}{L}\right)$ (c) $\frac{mg}{A}\left(1+\frac{r}{L}\right)$ (d) $\frac{mg}{A}\left(\frac{r}{L}\right)$

SOLUTION

Refer to Fig. 10.19. The vertical component $T\cos\theta$ of tension T balances the weight mg and the horizontal component $T\sin\theta$ provides the necessary centripetal force. Thus

$T\cos\theta = mg$ (1)

$T\sin\theta = mr\omega^2$ (2)

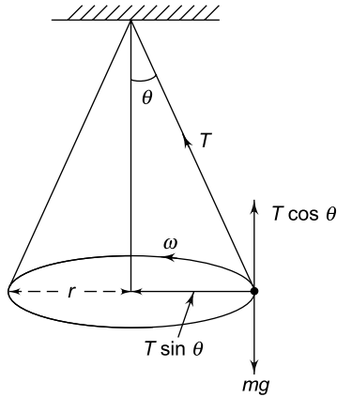


Fig. 10.19

8. Dividing (1) and (2)

$$\tan\theta = \frac{r\omega^2}{g}$$

or $$\omega = \sqrt{\frac{g\tan\theta}{r}}$$

Hence the correct choice is (c).

9. From Eqs. (1) and (2) we find that the correct choices are (a) and (d).

10. Stress $= \frac{T}{A}$. Also strain $\frac{\Delta L}{L} = \frac{\text{stress}}{Y}$

$$\therefore \quad \Delta L = \frac{TL}{AY} = \frac{mg}{\cos\theta}\times\frac{L}{AY}$$

Hence the correct choices are (a) and (b)

11. The correct choice is (a).

Questions 12 to 14 are based on the following passage

Passage IV

A thin rod of negligible mass and cross-sectional area $4\times10^{-6}\text{m}^2$, suspended vertically from one end, has a length of 0.5 m at 100°C. The rod is cooled to 0°C. Young's modulus = 10^{11} Nm^{-2}, coefficient of linear expansion = 10^{-5} K^{-1} and $g = 10$ ms^{-2}.

12. The decrease in the length of the rod on cooling is

(a) 2×10^{-4}m (b) 3×10^{-4}m (c) 4×10^{-4}m (d) 5×10^{-4}m

13. What mass must be attached at the lower end of the rod so that the rod is prevented from contracting on cooling?

(a) 40 kg (b) 30 kg (c) 20 kg (d) 10 kg

14. The total energy stored in the rod is

(a) 0.1 J (b) 0.2 J (c) 0.3 J (d) 0.4 J

SOLUTION

12. The decrease in the length of the rod on cooling is

$$\Delta L = L\alpha\,\Delta t = 0.5\times10^{-5}\times100$$
$$= 5\times10^{-4}\text{ m}$$

So the correct choice is (d).

13. If the rod is to be prevented from contracting, the mass m attached at the lower end must increase its length by an amount $\Delta L = 5\times10^{-4}$ m. Now

$$Y = \frac{mgL}{A\Delta L}$$

or $$m = \frac{YA\Delta L}{gL} = \frac{10^{11}\times4\times10^{-6}\times5\times10^{-4}}{10\times0.5}$$

$= 40$ kg, which is choice (a).

14. Energy stored in the rod is

$$U = \frac{1}{2}F\times\Delta L = \frac{1}{2}mg\Delta L$$
$$= \frac{1}{2}\times40\times10\times5\times10^{-4}$$
$$= 0.1\text{ J}.$$

Thus the correct choice is (a).

Questions 15 to 17 are based on the following passage

Passage V

Two blocks of masses m and $M = 2\,m$ are connected by means of a metal wire of cross-sectional area A passing over a frictionless fixed pulley as shown in Fig. 10.20. The system is then released.

15. The common acceleration of the blocks is

(a) g (b) $\frac{g}{3}$ (c) $\frac{2g}{3}$ (d) $\frac{3g}{2}$

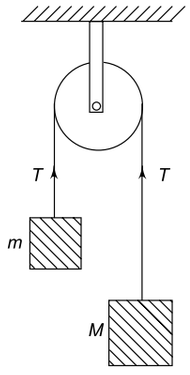


Fig. 10.20

16. The stress produced in the wire is

(a) $\frac{mg}{A}$ (b) $\frac{2mg}{3A}$

(c) $\frac{3mg}{4A}$ (d) $\frac{4mg}{3A}$

17. If $m = 1$ kg, $A = 8 \times 10^{-9}$ m^2, the breaking stress = 2×10^9 Nm^{-2} and $g = 10$ ms^{-2}, the maximum value of M for which the wire will not break is

(a) 4 kg (b) 6 kg

(c) 8 kg (d) 10 kg

SOLUTION

If a is the common acceleration of the blocks and T the tension in the wire. Then the equations of motion of the blocks are

$$Mg - T = Ma \quad (1)$$

and $$T - mg = ma \quad (2)$$

15. Adding Eq. (1) and Eq. (2), we get

$$a = \frac{(M-m)g}{(M+m)}$$

For $M = 2m$, we get $a = g/3$, which is choice (b).

16. From Eqs. (1) and (2), $T = m(g + a) = m(g + g/3) = 4mg/3$.

$\therefore$ Stress $= \frac{T}{A} = \frac{4mg}{3A}$, which is choice (d).

17. Breaking stress is the maximum stress the wire can withstand. From Eqs. (1) and (2)

$$T = \frac{2mMg}{(M+m)}$$

$\therefore$ Breaking stress $= \frac{T}{A} = \frac{2mg}{A\left(1+\frac{m}{M_{\max}}\right)}$.

Using the given values, we get $M_{\max} = 4$ kg, which is choice (a).

SECTION IV

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. Statement-1
The length of an elastic string of initial length L is a metre when the tension is 4 N and b metre when the tension is 5 N. The length of the string (in metre) when the tension is 9 N is $(a + b - L)$
Statement-2
The extension of an elastic string is proportional to the initial length of the string.

2. Statement-1
Steel is more elastic than rubber.
Statement-2
The Young's modulus of steel is greater than that of rubber.

3. Statement-1
The stretching of an elastic spring is determined by the shear modulus of the material of the spring.
Statement-2
For a given stretching force, the amount of stretching depends on the force constant of the spring.

4. **Statement-1**
Figure 10.21 shows that stress-strain curves for two different types of rubber. Rubber *A* rather than rubber *B* should be used as a car tyre.

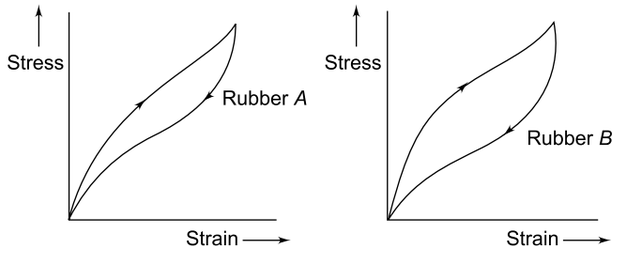


Fig. 10.21

Statement-2
Rubber *A* dissipates larger amount heat energy than rubber *B*.

5. **Statement-1**
Two wires *A* and *B* have the same cross-sectional area and are made of the same material but the length of wire *A* is twice that of *B*. For a given load, the extension of *A* will be twice that of *B*.
Statement-2
For a given load the extension of a wire is proportional to its length.

6. **Statement-1**
Two wires *A* and *B* have the same cross-sectional area and are made of the same material but the length of wire *A* is the twice that *B*. For a given load, the strain in wire *A* is twice that in *B*.
Statement-2
For a given load, the extension in a wire is proportional to its length.

7. **Statement-1**
Two wires *A* and *B* have equal lengths and are made of the same material but the diameter of wire *A* is twice that of *B*. For a given load, the extension of *B* will be four times that of *A*.
Statement-2
For a given load, the extension of a wire is inversely proportional to its area of cross-section.

8. **Statement-1**
Two wires *A* and *B* are made of the same material. The length of wire *A* is twice that of *B* but the diameter of *A* is half that of *B*. For a given load, the strain produced in *B* will be twice that in *A*.
Statement-2
For a given load, the extension produced in a wire is directly proportional to its length and inversely proportional to the area of cross-section.

9. **Statement-1**
When a material is subjected to a tensile (stretching) stress, the restoring forces are caused by interatomic attraction.
Statement-2
Restoring force is called into play in an elastic material by an inherent property of the material and not due to interatomic attraction.

10. **Statement-1**
When a material is subjected to a compressional stress, the restoring forces are caused by interatomic repulsion.
Statement-2
The atoms of a material never repel.

ANSWERS

1. The correct choice is (d). If L is the initial length, then the increase in length by a tension F is given by

$$l = \frac{FL}{\pi r^2 Y}$$

Hence $$a = L + l = L + \frac{4L}{\pi r^2 Y} = L + 4c \quad (1)$$

and $$b = L + \frac{5L}{\pi r^2 Y} = L + 5c \quad (2)$$

where $c = \frac{L}{\pi r^2 Y}$. Solving (1) and (2) for L and c, we get $L = 5a - 4b$ and $c = b - a$. For $F = 9$ N, we have

$$x = L + \frac{9L}{\pi r^2 Y} = L + 9c$$

$$= (5a - 4b) + 9(b - a) = 5b - 4a$$

2. The correct choice is (a).
3. The correct choice is (b). If a spring is stretched, the total length of the wire of the coil and the volume of the wire, both do not change. Only the shape (or configuration) of the coils of the wire undergoes a change. Hence the stretching of a spring is not determined by Young's modulus or bulk modulus. It is determined by the shear modulus.
4. The correct choice is (c). The area of the hysteresis loop for rubber *A* much smaller than that for rubber *B*. This implies that rubber *A* dissipates a smaller

amount of heat energy than rubber *B*. Consequently, tyres made of rubber *A* will not get heated to a high temperature. This prevents wear and tear of tyres.

5. $\Delta L = \dfrac{FL}{AY}$. Since the two wires are made of the same material, the Young's modulus Y is the same. Since F and A also the same, $\Delta L \propto L$. Hence the correct choice is (a).

6. The correct choice is (d).

$$\text{Strain} = \frac{\Delta L}{L} = \frac{F}{AY}$$

Since F, A and Y are the same for the two wires, the strains in them are equal.

7. Area of cross-section $A = \pi d^2/4$, where d is the diameter of the wire. Thus

$$\Delta L = \frac{4F}{\pi d^2} \cdot \frac{L}{Y} \quad \text{or} \quad \Delta L \propto \frac{1}{d^2}$$

Hence the correct choice is (a).

8. Strain $\dfrac{\Delta L}{L} = \dfrac{4F}{\pi d^2 Y}$. Thus strain $\propto \dfrac{1}{d^2}$.

Hence strain in *B* will be four times that in *A*. Thus the correct choice is (d).

9. The correct choice is (c). When the material is not subjected to any stress, its atoms are in their normal (equilibrium) positions. When a tensile stress is applied, the distance R between atoms becomes greater than their equilibrium separation R_0. For $R > R_0$, the interatomic force is attractive and this force provides the restoring force under which the material regains its original shape and size when the stress is removed.

10. The correct choice is (c). When the material is subjected to a compressional stress, R becomes less than R_0 and in this case the interatomic force is repulsive which causes the restoring force.

SECTION V

Integer Answer Type

1. A light rod *AB* of length 2m is suspended from the ceiling horizontally by means of two vertical wires as shown in Fig. 10.22. One of the wires is made of steel of cross-section 0.1 cm^2 and the other of brass of cross-section 0.2 cm^2. The Young's modulus of brass is 1.0×10^{11} Nm^{-2} and of steel is 2.0×10^{11} Nm^{-2}. A weight W is hung at point C at a distance x from end A. It is found that the stress in the two wires is the same when $x = \dfrac{n}{3}$ metre. Find the value of n.

IIT, 1980

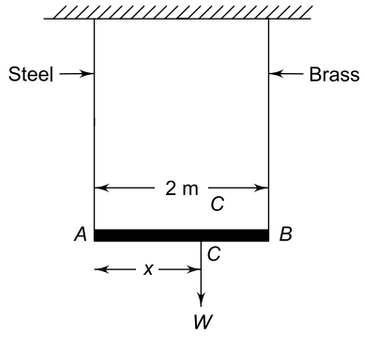


Fig. 10.22

2. In Q.2 above, find the value of x (in metre) so that the strains in the two wires are the same.

IIT, 1980

3. A metal wire of negligible mass, length 1 m and cross-sectional area 10^{-6} m^2 is kept on a smooth horizontal table with one end fixed on the table. A ball of mass 2 kg is attached to the other end of the wire. When the wire and the ball are rotated with angular velocity of 20 rad s^{-1}, it is found that the wire is elongated by 10^{-3} m. If the Young's modulus of the metal is $n \times 10^{11}$ Nm^{-2}, find the value of n.

IIT, 1992

4. In Q.10 above, if the angular velocity is gradually increased to 100 rad s^{-1}, the wire breaks. If the breaking stress is $x \times 10^{10}$ Nm^{-2}, find the value of x.

IIT, 1992

5. A body of mass 3.14 kg is suspended from one end of a wire of length 10.0 m. The radius of the wire is changing uniformly from 9.8×10^{-4} m at one end to 5.0×10^{-4} m at the other end. Find the change in the length of the wire in mm. Young's modulus of the material of wire is 2×10^{11} Nm^{-2}.

IIT, 1994

6. Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free

end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that $L >>$ diameter of the wire. Then the value of 'm' in kg is nearly.

IIT, 2011

SOLUTIONS

1. If T_s and T_b are the tensions in steel and brass wires, then the stress in them will be the same if

$$\frac{T_s}{A_s} = \frac{T_b}{A_b} \Rightarrow \frac{T_s}{T_b} = \frac{A_s}{A_b} = \frac{0.1\text{ cm}^2}{0.2\text{ cm}^2} = \frac{1}{2}$$

Since the system is in equilibrium, the moments of forces T_s and T_b about C will be equal, i.e. (see Fig. 10.23)

$$T_s \times x = T_b \times (2 - x)$$

$$\Rightarrow \quad x = 2(2 - x) \Rightarrow x = \frac{4}{3} \text{ metre} \quad (\because T_b = 2T_s)$$

Thus the value of $n = 4$

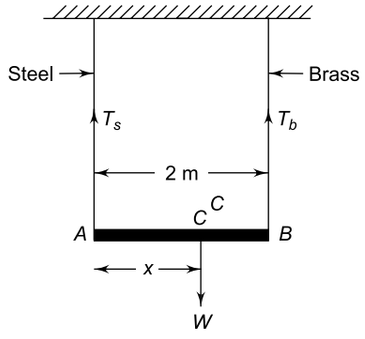


Fig. 10.23

2. Strain $= \dfrac{\text{stress}}{Y} = \dfrac{T}{AY}$

The strain in the two wires will be equal if

$$\frac{T_s}{A_s Y_s} = \frac{T_b}{A_b Y_b}$$

$$\Rightarrow \quad \frac{T_s}{T_b} = \frac{A_s}{A_b} \times \frac{Y_s}{Y_b} = \frac{0.1}{0.2} \times \frac{2.0 \times 10^{11}}{1.0 \times 10^{11}} = 1$$

Equating moments about C, we have

$$T_s \times x = T_b\ (2 - x)$$

which gives $x = 1$ metre ($\because T_s = T_b$)

3. Let $R = L =$ length of the wire = radius of the circle. The centrifugal force $F = mR\omega^2$ produces the stress as at result of which the wire elongates.

Thus $$Y = \frac{FL}{A\Delta L} = \frac{mR\omega^2 L}{A\Delta L} = \frac{mR^2\omega^2}{A\Delta L} \qquad (1)$$

Given $m = 2$ kg, $R = 1$ m, $\omega = 20$ rad s^{-1}, A $= 10^{-6}$m^2 and $\Delta L = 10^{-3}$ m. Substituting these values in (1) and solving, we get

$$Y = 8 \times 10^{11}\ \text{Nm}^{-2}$$

Thus $n = 8$.

4. If the wire breaks at $\omega_{\max} = 100$ rad s^{-1}, then the breaking stress is

$$\frac{F_{\max}}{A} = \frac{mR\omega^2{}_{\max}}{A} = \frac{2 \times 1 \times (100)^2}{10^{-6}} = 2 \times 10^{10}\ \text{Nm}^{-2}.$$

Thus $x = 2$.

5. Figure 10.24 shows a wire PQ of length L having radius r_1 at end P increasing uniformly to a radius r_2 at end Q. Consider an element AB of the wire of length dy at a distance y from end P. Let r be the radius of the wire at y and $(r + dr)$ at $(y + dy)$. As the radius increases uniformly from P to Q.

$$\frac{dr}{dy} = \text{constant, say } C$$

where $$C = \frac{r_2 - r_1}{L}. \text{ Thus}$$

$$dy = \frac{dr}{C}$$

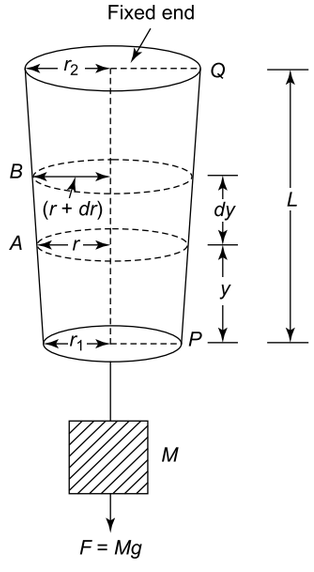


Fig. 10.24

Let us first find the extension dl produced in the element of length dy due to a force $F = Mg$. Stress in the element $= F/\pi r^2$ and strain in it $= dl/dy$. Hence

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/\pi r^2}{dl/dy} = \frac{F dy}{\pi r^2 dl}$$

But $dy = \frac{dr}{C}$. Therefore,

$$Y = \frac{F dr}{\pi r^2 C dl} \quad \text{or} \quad dl = \frac{F dr}{\pi C Y r^2}$$

Integrating from $r = r_1$ to $r = r_2$, we obtain the total extension l produced in the wire by a force F which is given by

$$l = \int dl = \frac{F}{\pi C Y}\int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= -\frac{F}{\pi C Y}\left|\frac{1}{r}\right|_{r_1}^{r_2} = \frac{F}{\pi C Y}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{F(r_2 - r_1)}{\pi C Y r_1 r_2}$$

Now $F = Mg$ and $C = \frac{r_2 - r_1}{L}$. Hence

$$l = \frac{MgL}{\pi Y r_1 r_2}$$

Substituting the given values of M, L, Y, r_1 and r_2 and $g = 9.8\ \text{ms}^{-2}$, we have

$$l = \frac{3.14 \times 9.8 \times 10.0}{3.14 \times 2 \times 10^{11} \times 9.8 \times 10^{-4} \times 5.0 \times 10^{-4}}$$

$$= 10^{-3}\ \text{m} = 1\ \text{mm}.$$

6. Change in length $\Delta L = L\,\alpha\,\Delta T$ (i)

Also $Y = \frac{mgL}{A\Delta L} \Rightarrow \Delta L = \frac{mgL}{YA}$ (ii)

Equation (i) and (ii) we get ($\because A = \pi r^2$)

$$m = \frac{\alpha \Delta T Y \times \pi r^2}{g}$$

$$= \frac{(10^{-5}) \times (10) \times (10^{11}) \times 3.14 \times (1 \times 10^{-3})^2}{9.8}$$

$$= 3.2\ \text{kg} \simeq 3\ \text{kg}$$

11 Chapter

Hydrostatics (Fluid Pressure and Buoyancy)

REVIEW OF BASIC CONCEPTS

11.1 FLUID PRESSURE

Pressure is defined as the force exerted normally on a unit area of the surface of a fluid and is given by

$$P = \frac{F}{A}$$

In the **SI** system, the unit of pressure is newton per square metre (Nm^{-2}) which is also called pascal (Pa). Thus

$$1 \text{ Pa} = 1 \text{ Nm}^{-2}$$

11.2 PASCAL'S LAW

Blaise Pascal (1623–1662), a French scientist, discovered a principle which tells us how force (or pressure) can be transmitted in a fluid. Pascal's law states that *pressure in a fluid in equilibrium is the same everywhere* (if the effect of gravity can be neglected).

11.3 DENSITY AND RELATIVE DENSITY

The density of a substance is defined as the mass per unit volume of the substance. The SI unit of density is kilogram per cubic metre (kg m^{-3}).

$$1 \text{ g cm}^{-3} = 1000 \text{ kg m}^{-3}$$

The relative density of a substance is the ratio of its density to that of water, i.e.

$$\text{Relative density} = \frac{\text{density of substance}}{\text{density of water}}$$

Being a ratio of two similar quantities, relative density is just a number; it has no units.

11.4 ATMOSPHERIC PRESSURE

Like all gases, air also has weight and hence exerts pressure. Just as water pressure is caused by the weight of water, the weight of all the air above the earth causes an atmospheric pressure. The atmosphere exerts this pressure not only on the earth's surface, but also on the surface of all objects on the earth including living being.

The atmospheric pressure at sea level is

$$P_0 = 1.01 \times 10^5 \text{ Pa}$$

11.5 GAUGE PRESSURE

The pressure due to a column of a liquid is called gauge pressure. The gauge pressure due to a column of height h below the free surface of a liquid at rest is given by

$$P = h\rho g$$

where ρ is the density of the fluid and g, the acceleration due to gravity. The pressure is the same at all points at the same horizontal level. The pressure at any point in a fluid contained in a vessel is independent of the shape or size of the vessel.

11.6 HYDROSTATIC PRESSURE

The pressure at any point in a fluid is equal to the sum of the atmospheric pressure P_0 acting on its surface and the gauge pressure $h\rho g$ due to the weight of the fluid above that point which is at a depth h below the surface of the fluid. This pressure is given by

$$P = P_0 + h\rho g$$

EXAMPLE 11.1

A tank of square cross-section (1.0 m × 1.0 m) is filled with a liquid of relative density 1.2 upto a height of 1.5 m. Find the thrust exerted by the liquid column at the bottom of the tank and on a vertical side of the tank. Take $g = 10 \text{ ms}^{-2}$.

SOLUTION

Density of liquid $= 1.2 \times 10^3 \text{ kg m}^{-3}$

Pressure due to liquid column = $h\rho g$

$$= 1.5 \times (1.2 \times 10^3) \times 10$$
$$= 1.8 \times 10^4 \text{ Pa}$$

This pressure is uniformly distributed over the bottom of the tank.

Thrust at the bottom of tank = pressure × base area

$$= (1.8 \times 10^4) \times (1.0 \times 1.0)$$
$$= 1.8 \times 10^4 \text{ N}$$

On the vertical side, the pressure due to liquid column is zero at the top and 1.8×10^4 Pa at the bottom. Therefore, the average pressure exerted on the vertical side $= \frac{1}{2}(0 + 1.8 \times 10^4) = 0.9 \times 10^4$ Pa. Area of vertical side in contact with liquid $= 1.0 \times 1.5 = 1.5$ m^2

$\therefore$ Average thrust on the vertical side

$$= (0.9 \times 10^4) \times 1.5$$
$$= 1.35 \times 10^4 \text{ N}$$

EXAMPLE 11.2

A cylindrical jar of cross-sectional area A is filled with a liquid of density ρ to a height h. It carries a tight-fitting piston of negligible mass. Find the pressure at the bottom of the jar when a block of mass m is placed on the piston . The atmospheric pressures is P_0.

SOLUTION

Total force acting on the base of the jar is

$$F = (P_0 + h\rho g)\, A + mg$$

$\therefore$ Pressure at the bottom of the jar is

$$P = \frac{F}{A} = P_0 + h\rho g + \frac{mg}{A}$$

EXAMPLE 11.3

A large tank with a square base of side 1.0 m is divided into two compartments by a vertical partition in the middle as shown in Fig. 11.1. There is a small hinged door of size 2.0 × 2.5 cm at the bottom of partition. Water is filled in one compartment and oil of relative density 1.5 in the other both to the same height h = 2.0 m. Find the force necessary to keep the door closed.

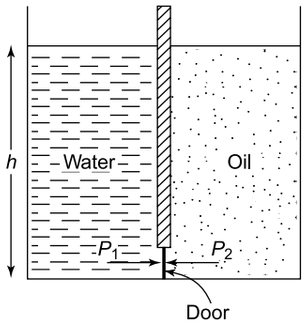


Fig. 11.1

SOLUTION

Density of water (ρ_1) = 1000 kg m^{-3}, density of oil (ρ_2) = 1500 kgm^{-3} and area of the door (A) = 2.0 × 2.5 = 5cm^2 = 5×10^{-4} m^2.

Lateral pressure by water on the face of the door is

$$p_1 = h\rho_1 g$$

Total pressure $p_1 = P_0 + h\rho_1 g$ (P_0 = atmospheric pressure)

Total pressure on the door due to oil is

$$P_2 = P_0 + h\rho_2 g$$

$\therefore$ Net pressure difference = $P_2 - P_1 = (\rho_2 - \rho_1)\, hg$

Net force = $(P_2 - P_1)\, A = (\rho_2 - \rho_1)\, hgA$

$$= (1500 - 1000) \times 2.0 \times 9.8 \times 5 \times 10^{-4}$$
$$= 4.9 \text{ N}$$

EXAMPLE 11.4

A horizontal tube OP of length L and of uniform cross-sectional area A is open at end O and has a small hole at the other end P. The tube is filled with a liquid of density ρ and then rotated about the axis passing through O with an angular velocity ω. Find the pressure exerted by the liquid at end P at the instant when length $L/2$ of the liquid is left in the tube.

SOLUTION

Consider a small element of the liquid of length dr at a distance r from O (Fig. 11.2). Mass of element is $m = A\,\rho\,d\,r$.

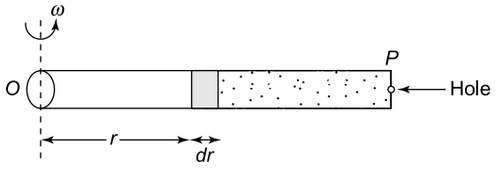


Fig. 11.2

Outwards force (centrifugal force) acting on the element is

$$dF = mr\omega^2 = A\rho\omega^2\, rdr$$

Integrating from $r = L/2$ to $r = L$, we have

$$F = A\rho\omega^2 \int_{L/2}^{L} rdr = A\rho\omega^2 \left|\frac{r^2}{2}\right|_{L/2}^{L} = \frac{3}{8} A\rho\omega^2 L^2$$

$\therefore$ Pressure at $P = \frac{F}{A} = \frac{3}{8}\rho\omega^2 L^2$

EXAMPLE 11.5

A U-tube contains mercury in both sides of its arms. A glycerine column of length 10 cm in introduced in one of the arms. Oil is poured in the other arm until the upper surfaces of oil and glycerine are at the same horizontal level. Find the length of the oil column. Given density of glycerine = 1300 kgm^{-3}, density of oil = 800 kg m^{-3} density of mercury = 13600 kg m^{-3}.

SOLUTION

Since glycerine is denser then oil, the level of mercury in the left arm will be higher than that in the right arm (Fig. 11.3)

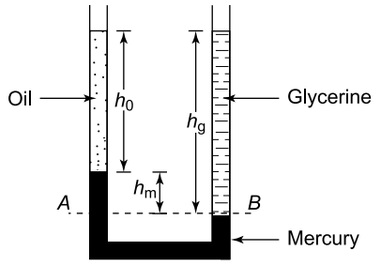


Fig. 11.3

Pressure at $A = P_0 + h_0\rho_0 g + h_m\rho_m g$ (i)

Pressure at B = $P_0 + h_g\rho_g g$ (ii)

Since pressure at the same horizontal level is the same (Pascal's law), equating (i) and (ii) we get

$h_0\,\rho_0 + h_m\,\rho_m = h_g\,\rho_g$

$\Rightarrow\ h_0\,\rho_0 + (h_g - h_0)\rho_m = h_g\,\rho_g$

$\Rightarrow\ h_0 \times 800 + (0.1 - h_0) \times 13600 = 0.1 \times 1300$

$(\because\ h_g = 10\text{ cm} = 0.1\text{m})$

$\Rightarrow\ h_0 = 0.096\text{ m} = 9.6\text{ cm}$

11.7 PRESSURE DIFFERENCE IN AN ACCELERATED LIQUID

Consider a liquid of density ρ in a container. If the container is given an acceleration a, say, to the right, the liquid miniscus will no longer remain horizontal, it will be inclined at an angle θ as shown in Fig. 11.4.

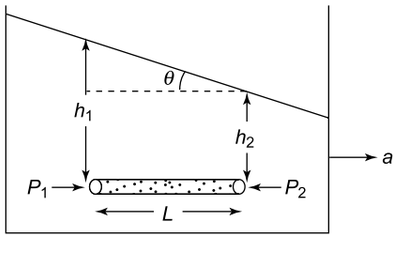


Fig. 11.4

Consider a small element (shown shaded) of length L of the liquid. Let ΔA be the cross-sectional area of the element. The mass of the element is $m = \rho L \Delta A$. When an acceleration a is given as shown, the equation motion of the element is

$$P_1\Delta A - P_2\,\Delta A = ma = (\rho L \Delta \text{A})\,a$$

where P_1 and P_2 are the pressures at the ends of the element.

Thus $P_1 - P_2 = \rho La$ (i)

$\Rightarrow\ (h_1 - h_2)\,\rho g = \rho La$

$\Rightarrow\ h_1 - h_2 = \dfrac{La}{g}$ (ii)

It follows from (i) that $P_1 > P_2$. From (ii) we have

$$\tan\,\theta = \frac{h_1 - h_2}{L} = \frac{a}{g}$$

EXAMPLE 11.6

A liquid is contained in a rectangular vessel fastened on a cart. A constant force is applied to the cart. As a result, the level becomes inclined at an angle of 30° with the horizontal. What acceleration is produced by force?

SOLUTION

$$\tan\,\theta = \frac{a}{g}$$

$\Rightarrow \tan 30° = \dfrac{a}{g}$

$\Rightarrow\ a = 9.8 \times \tan 30° = \dfrac{9.8}{\sqrt{3}} = 5.66\text{ ms}^{-2}$

11.8 ARCHIMEDES' PRINCIPLE

Archimedes' principle states as follows:

'*When a solid body is wholly or partly immersed in a fluid, it experiences an upward thrust or buoyant force equal to the weight of the fluid displaced by it.*' The word 'fluid' includes both liquids and gases. The principle is a general one and holds for solids of any shape and for all fluids.

Law of Floatation The necessary condition for a body to float in a fluid is that the weight of the fluid displaced by it must be equal to the weight of the body. This is the law of floatation.

11.9 APPLICATIONS

1. Buoyant force = weight of the liquid displaced by the immersed portion of the body = $V\rho g$ where V is the volume immersed, ρ is the density of the liquid.

Case (a). If the density (σ) of the body is less than that of the liquid ($\sigma < \rho$), then the body will float with volume V_0 outside liquid and V_i inside liquid such that (V = volume of the body)

$$V_i \rho g = V\sigma g$$

$$\Rightarrow \quad \frac{V_i}{V} = \frac{\sigma}{\rho} \text{ and } \frac{V_0}{V} = \frac{\rho - \sigma}{\sigma} \; (\because V = V_0 + V_i)$$

Case (b) if $\sigma = \rho$, the body will float completely immersed, i.e. $V_0 = 0$.

Case (c). If $\sigma > \rho$, the body will sink to the bottom.

2. Tension T in the string tied to a body immersed in a liquid ($\sigma > \rho$).

Case (a) The system is at rest (Fig. 11.5)

$$T = V\sigma g - V\rho g$$
$$= V(\sigma - \rho) g$$

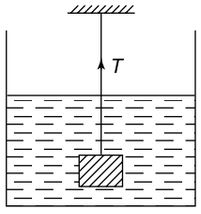


Fig. 11.5

Case (b) When the system is accelerating, then

$$T = V(\sigma - \rho) g'$$

(i) If the system moves up with acceleration a,

$$g' = g + a \text{ and } T' = V(\sigma - \rho)(g + a)$$

(ii) If the system moves down with acceleration a ($< g$)

$$g' = g - a \text{ and } T' = V(\sigma - \rho)(g - a)$$

(iii) If the system falls freely, $a = g$ and $T' = 0$

3. $\sigma < \rho$ and the body is kept immersed in a liquid by a string fixed at the bottom of the beaker (Fig. 11.6)

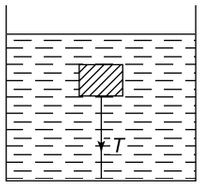


Fig. 11.6

(i) If the system is at rest,

$$T = V(\rho - \sigma) g$$

(ii) If the system moves up with acceleration a,

$$T' = V(\rho - \sigma) g' \text{ where } g' = (g + a)$$

(iii) If the system moves down with acceleration a ($< g$)

$$T' = V(\rho - \sigma) \text{ where } g' = (g - a)$$

(iv) If the system falls freely, $T' = 0$

4. A toy boat carrying an object is floating in water in a beaker. The object is droped into water.
   (1) If the object is made of wood (of density less than that of water), it will float and the level of water in the beaker remains unchanged.
   (2) If object is denser than water, it will sink and the water level will fall.
5. A piece of ice is floating in a liquid. If the ice melts, the level of water
   (a) remains unchanged if the liquid is water,
   (b) rises if the relative density of the liquid is greater than 1,
   (c) falls if the relative density of the liquid is less than 1.
6. A piece of ice with an object embedded in it is floating in water. If all the ice melts, the water level
   (a) falls if the object sinks in water
   (b) remains unchanged if the object floats in water
7. An object of density ρ and volume V floats at the interface of two liquids 1 and 2 of densities ρ_1 and ρ_2 with volume V_1 in liquid 1 and V_2 in liquid 2 (Fig. 11.7) Then, for equilibrium ($\rho_1 < \rho < \rho_2$)

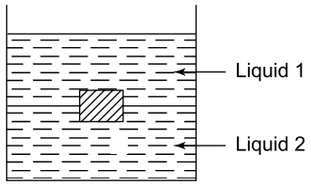


Fig. 11.7

Weight of body = buoyant force

i.e. $$V\rho g = V_1 \rho_1 g + V_2 \rho_2 g \qquad \text{(i)}$$

Also $$V_1 + V_2 = V \qquad \text{(ii)}$$

Eqs (i) and (ii) give

$$V_1 = \frac{V(\rho_2 - \rho)}{(\rho_2 - \rho_1)}, \; V_2 = \frac{V(\rho - \rho_1)}{(\rho_2 - \rho_1)}$$

and $$\frac{V_2}{V_1} = \frac{(\rho - \rho_1)}{(\rho_2 - \rho)}$$

EXAMPLE 11.7

A cubical metal block of edge 5 cm is suspended from a support by a massless spring and immersed in water in a beaker as shown in Fig. 11.8. The relative density of metal is 7. Find the tension in the string when the whole system

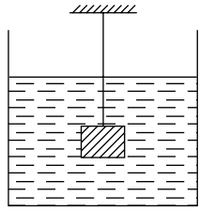


Fig. 11.8

(a) is at rest

(b) moves up with an acceleration of 2 ms^{-2}

(c) moves down with an acceleration of 2 ms^{-2}

Take $g = 10$ ms^{-2}

SOLUTION

Density of water (ρ) = 1000 kg m^{-3}

Density of block (σ) = 7000 kg m^{-3}

Volume of block (V) = 5 cm × 5 cm × 5 cm = 125 cm^3 = 1.25×10^{-4} m^3

Weight of block $W = mg = \sigma Vg$

Upthrust U = weight of water displaced = ρVg

Figure 11.8 shows the free body diagrams

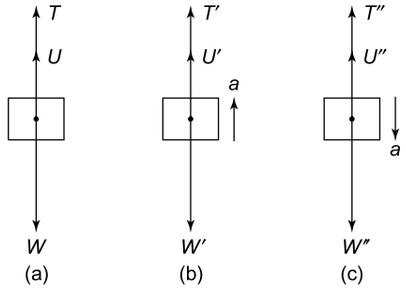


Fig. 11.9

(a) From Fig. 11.9 (a)

$T + U = W$

$\Rightarrow \quad T = W - U = \sigma Vg - \rho Vg$

$= (\sigma - \rho)\, Vg$

$= (7000 - 1000) \times (1.25 \times 10^{-4}) \times 10$

$= 7.5$ N

(b) In this case $g_{eff} = g + a$[Fig.11.9 (b)]

$\therefore \quad T' = W' - U' = \sigma\, Vg_{eff} - \rho Vg_{eff}$

$= (\sigma - \rho)\, V\,(g + a)$

$= (7000 - 100) \times (1.25 \times 10^{-4}) \times (10 + 2)$

$= 9.0$ N

(c) In this case $g_{eff} = g - a$ [Fig. 11.9(c)]

$\therefore \quad T'' = (\sigma - \rho)\, V\,(g - a)$

$= (7000 - 1000) \times (1.25 \times 10^{-4}) \times (10 - 2)$

$= 6.0$ N

EXAMPLE 11.8

A cubical block of wood (density = 800 kg m^{-3}) of side 5 cm floats on the surface of water with its lower face just touching a vertical spring fixed at the bottom of the container. When a body of mass m = 75 g is placed on top of the block, it floats in water with its top face in level with water. Find the value of spring constant k.

Take g = 10 ms^{-2}

SOLUTION

Let x cm be the height of the block above the surface of water (Fig. 11.10). From the law of floatation, upthrust = weight of the block (W)

or weight of water displaced = weight of the block, i.e.

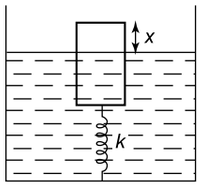


Fig. 11.10

$(5 - x) \times 10^{-2} \times (5 \times 10^{-2})^2 \times 1000\, g = (5 \times 10^{-2})^3 \times 800\, g$

which gives x = 1cm

When a body of mass m = 75 g = 0.075 kg is placed on the block, it depresses by x = 1cm = 1×10^{-2} m. If k is the spring constant, the force in the spring is $f = kx = k \times 10^{-2}$. The upthrust now is

$U = (5 \times 10^{-2})^3 \times 1000 \times g = 1.25$ N

$(\because\ g = 10\ \text{ms}^{-2})$

Figure 11.11 shows the free body diagram. For equilibrium

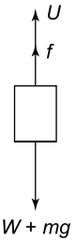


Fig.11.11

$W + mg = U + f$ (i)

Now $W = (5 \times 10^{-2})^3 \times 800 \times 10 = 1$ N, $f = k \times 10^{-2}$, m = 0.075 kg and U = 1.25 N.

Using these values in (i),

$1 + 0.075 \times 10 = 1.25 + 10^{-2}\, k$

$\Rightarrow \quad k = 50$ Nm^{-1}

EXAMPLE 11.9

A cylinder of radius R and height h and mass M is suspended by a string in a liquid of density ρ where it stays vertical with its upper surface at a depth h_1 below the surface of the liquid. Find the force at the bottom of the cylinder.

SOLUTION

Pressure at top of cylinder is (Fig. 11.12)

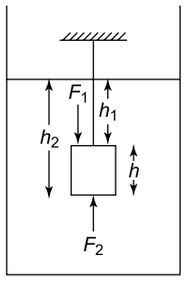


Fig. 11.12

$$P_1 = h_1\rho g$$

Pressure at the bottom of cylinder is

$$P_2 = h_2\rho g$$

If A is the cross-sectional area of the cylinder, the net force at the bottom is

$$F = (p_2 - p_1)\,A = (h_2 - h_1)\,\rho Ag$$

$$= h\rho Ag = V\rho g$$

where V = volume of cylinder.

EXAMPLE 11.10

In example 11.9 above, what will be the force at the bottom if a hemispherical portion of radius R is removed from the bottom of the cylinder. The volume of the remaining part of the cylinder is V' and the top of the cylinder is now at a depth h' below the liquid surface as shown in Fig. 11.13.

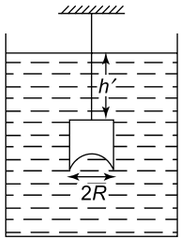


Fig. 11.13

SOLUTION

From Archimedes' principle,

Force at bottom = force at top + $V'\rho g$,

$$= (h' \times \pi R^2) \times \rho g + V'\rho g$$

$$= \rho g\,[V' + \pi R^2 h']$$

EXAMPLE 11.11

A cubical block of steel 10 cm on each side is floating on mercury in a vessel. The density of steel is 7.8×10^3 kg m^{-3} and that of mercury is 13.6×10^3 kg m^{-2}.

(a) Find the height of the block above the mercury level.

(b) If water is poured into the vessel until it just covers the steel block, find the height of the water column.

SOLUTION

Volume of block $= 10 \times 10 \times 10 = 10^3\ \text{cm}^3 = 10^{-3}\ \text{m}^3$

(a) Let h_1 be the height of the block above the mercury surface. From the law of floatation,

Weight of mercury displaced = weight of block

i.e. $(0.1 - h_1) \times 0.1 \times 0.1 \times 13.6 \times 10^3 g = (7.8 \times 10^3) \times 10^3\,g$

$\Rightarrow \quad h_1 = 0.043\ \text{m} = 4.3\ \text{cm}$

(b) Let h_2 be the height of water column required. Then

Weight of block = weight of water displaced + weight of mercury displaced

i.e. $(7.8 \times 10^3) \times 10^3 g = h_2 \times 0.1 \times 0.1 \times 10^3 \times g$

$$+ (0.1 - h_2) \times 0.1 \times 0.1 \times 13.6 \times 10^3 \times g$$

$\Rightarrow \quad h_2 = 0.046\ \text{m} = 4.6\ \text{cm}$

EXAMPLE 11.12

A spring balance reads 10 kg when a bucket of water is suspended from it. What will be the reading of the spring balance when

(a) an ice cube of mass 1.5 kg is put into the bucket?

(b) an iron piece of 7.2 kg suspended from another spring is immersed with half its volume inside water in the bucket?

Relative density of iron is 7.2

SOLUTION

(a) When an ice cube of mass 1.5 kg is put into the bucket, the total mass suspended from the spring balance = 10 kg + 1.5 kg = 11.5 kg. Hence the balance will read 11.5 kg.

(b) Density of iron = 7.2×10^3 kg m^{-3}. Therefore volume of iron piece is

$$V = \frac{\text{mass}}{\text{density}} = \frac{7.2}{7.2 \times 10^3} = 10^{-3}\ \text{m}^3$$

Volume of iron immersed in water = $\frac{V}{2}$

$\therefore$ Weight of water displaced = $\frac{V}{2} \times 1000 \times g$ newton

$$= \frac{10^{-3}}{2} \times 1000 \times g = 0.5\ g$$

= weight of 0.5 kg

This is the buoyant force on the iron piece. Hence, according to Newton's third law, the iron piece will exert an equal force on water in the downward direction. Hence the balance will now read = 10 kg + 0.5 kg = 10.5 kg

11.10 SURFACE TENSION AND SURFACE ENERGY

Surface tension is the force acting per unit length of an imaginary line on a liquid surface; the direction of the force being perpendicular to the line and tangential to the liquid surface.

The SI unit of surface tension is N m^{-1} and its dimensional formula is [$ML°T^{-2}$]. Consider a frame *ABCD* having a wire *PQ* of length *L* which can slide along sides *AB* and *CD*. The frame is dipped in a liquid (e.g. soap solution) and taken out. We get a film of liquid within *PBCQ* (Fig. 11.14). Since the film has two surface each of length *L*, the force due to surface tension acting on wire *PQ* is

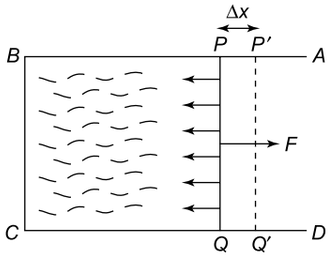


Fig. 11.14

$$F = \sigma \times 2\ L = 2\sigma\ L$$

This force is directed inwards to the left and has to be applied to the right to hold the wire *PQ* in place. Hence, if the area of the film has to be increased, work has to be done against the force of surface tension. This work is stored as potential energy called surface energy. Work done to move the wire from a position *PQ* to a position *P′Q′* is

$$\Delta W = F\Delta x = \sigma\ \ (2L\Delta x) = \sigma\ \Delta \text{A}$$

where ΔA = increase in the surface area of the film. Thus work done = surface tension × increase in surface area of the film. Another SI unit of surface tension is Jm^{-2}.

EXAMPLE 11.13

A rectangular film of a liquid is extended from 2 cm × 3 cm to 2 cm × 3.5 cm. If the work done is 3×10^{-5} J, find the surface tension of the liquid.

SOLUTION

Increase in surface area of the film is

$$= (2\text{ cm} \times 3.5\text{ cm}) - (2\text{ cm} \times 3\text{ cm})$$

$$= 1\text{ cm}^2 = 1 \times 10^{-4}\text{ m}^2$$

Since the film has two surfaces, $\Delta A = 2 \times 10^{-4}$ m^2

$$\sigma = \frac{\Delta W}{\Delta A} = \frac{3\times10^{-5}}{2\times10^{-4}} = 0.15\text{ J m}^{-2}\text{ or N m}^{-1}$$

EXAMPLE 11.14

A U- shaped wire frame is dipped in a soap solution and removed.A thin film is formed in the frame. A light slider supports a weight of 1.5×10^{-2} N, which includes the weight of the slider (Fig. 11.15). The length of the slider is 30 cm. Find the surface tension of soap solution.

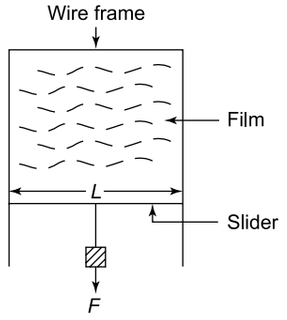


Fig. 11.15

SOLUTION

$$F = 2\sigma\ L \Rightarrow \sigma = \frac{F}{2L} = \frac{1.5\times10^{-2}}{2\times0.3}$$

$$= 2.5 \times 10^{-2}\text{ N m}^{-1}$$

11.11 EXCESS PRESSURE

When the free surface of a liquid is curved, there is a difference of pressure between the liquid side and the vapour side of the surface.

(i) Excess pressure inside a liquid drop of radius *r* is given by

$$p = \frac{2\sigma}{r}$$

where σ is the surface tension of the liquid.

(ii) Excess pressure inside a liquid bubble of radius *r* is given by

$$p = \frac{4\sigma}{r}$$

(iii) Excess pressure inside an air bubble of radius *r* in a liquid of surface tension σ is given by

$$p = \frac{2\sigma}{r}$$

If the pressure outside is *P*, the total pressure inside bubble $= P + \frac{2\sigma}{r}$

$$= P_0 + h\rho g + \frac{2\sigma}{r}$$

where *h* = depth of bubble below the free surface of the liquid of density ρ.

11.12 WORK DONE IN BLOWING A SOAP BUBBLE

Suppose the radius of a soap bubble is increased from r_1 to r_2 by blowing. Then, since there are two surfaces of the film, initial energy = $2 \times (4\pi\ r^2{}_1\sigma)$ and final energy = $2 \times (4\pi\ r^2{}_2\sigma)$

∴ Work done = final energy – initial energy

or $W = 8\pi\sigma\,(r_2^2 - r_1^2)$

(i) Work done in forming a bubble of radius r is (since $r_2 = r$ and $r_1 = 0$)

$$W = 8\pi\sigma\, r^2$$

(ii) Work done in doubling the radius of a bubble from $r_1 = r$ to $r_2 = 2r$ is

$$W = 8\pi\sigma\,[(2r)^2 - r^2] = 24\pi\sigma\, r^2$$

(iii) Work done in splitting a drop of radius R into n identical drops, each of radius r, is obtained as follows:

Initial surface area $= 4\pi R^2$

Final surface area $= n \times 4\pi r^2$

$\therefore$ Work done is $W = 4\pi(nr^2 - R^2)\sigma$

Since the volume remains unchanged,

$$\frac{4}{3}\pi r^3 n = \frac{4}{3}\pi R^3$$

or $n^{1/3}\, r = R$. Hence

$$W = 4\pi\, R^2\,(n^{1/3} - 1)\,\sigma$$

EXAMPLE 11.15

Calculate the work done (or energy needed) to split a spherical drop of mercury of diameter 1 cm into 8 identical drops. The surface tenstion of mercury $= 0.035\ \text{N m}^{-1}$.

SOLUTION

As shown in Example 11.14, $r = \dfrac{R}{2}$

Surface area of the big drop $= 4\pi R^2$

Surface area of 8 small drops $= 8 \times 4\pi r^2$

$$= 8 \times 4\pi \times \left(\frac{R}{2}\right)^2 = 8\pi R^2$$

$\therefore$ Increase in surface area $\Delta A = 8\pi R^2 - 4\pi R^2 = 4\pi R^2$

Work done $= \sigma \times \Delta A$

$$= 0.035 \times 4 \times 3.14 \times (0.5 \times 10^{-2})^2$$

$$= 1.1 \times 10^{-5}\text{J}$$

EXAMPLE 11.16

Eight spherical droplets, each of radius r of a liquid of density ρ and surface tension σ coalesce to form one big drop. If s is the specific heat capacity of the liquid, find the rise in the temperature of the liquid in this process.

SOLUTION

In this process, energy is evolved as heat because there is a decrease in surface area. Radius of big drop is $R = 2\,r$. Decrease in surface area is

$$\Delta\text{A} = 8 \times 4\pi r^2 - 4\pi R^2$$

$$= 32\ \pi r^2 = 4\pi(2r)^2$$

$$= 16\ \pi r^2$$

$\therefore$ Energy evolved is $Q = \sigma\,\Delta A = 16\ \pi\,\sigma\, r^2$

Mass of big drop is $m = \dfrac{4\pi}{3}\ R^3\rho = \dfrac{32}{3}\ r^3\rho$

If ΔT is the rise in temperature, then

$$Q = ms\Delta T$$

$$\Rightarrow \quad 16\ \pi\sigma\, r^2 = \frac{32}{3}\ r^3\rho s\Delta\text{T}$$

$$\Rightarrow \quad \Delta T = \frac{3\sigma}{2r\rho s}$$

EXAMPLE 11.17

An air bubble of radius 1 mm is formed at a depth of 50 cm inside a large container of soap solution. Calculate the pressure inside the bubble. Surface tension soap solution $= 0.05\ \text{Nm}^{-1}$, density of soap solution $= 1200\ \text{kg m}^{-3}$, atmospheric pressure $= 1.013 \times 10^5$ Pa.

SOLUTION

$$P = P_0 + \frac{2\sigma}{R} + h\rho g$$

$$= 1.013 \times 10^5 + \frac{2 \times 0.05}{\left(1 \times 10^{-3}\right)} + 0.5 \times 1200 \times 9.8$$

$$= 1.073 \times 10^5\ \text{Pa}$$

11.13 ANGLE OF CONTACT

The shape of meniscus of water in a narrow glass tube is concave upwards [Fig. 11.16 (a)] while the shape of meniscus of mercury in a narrow glass tube is convex upwards [Fig. 11.16 (b)].

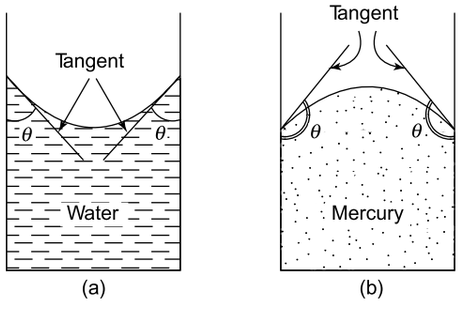


Fig. 11.16

The angle of contact (θ) between a liquid and a solid surface is defined as the angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid.

The value of angle of contact depends upon

(i) the nature of the liquid and solid in contact,
(ii) the nature of the medium above the free surface of the liquid and
(iii) the temperature of the liquid.

NOTE

Generally the angle of contact for liquids which wet glass is acute and obtuse for liquids which do not wet glass.

11.14 CAPILLARITY

The rise or fall of a liquid in a capillary tube is known as capillarity. The height to which a liquid of surface tension σ and density ρ rises in a capillary tube of radius r is given by

$$h = \frac{2\sigma\cos\theta}{\rho r g}$$

where θ is the angle of contact. For pure water and clean glass, $\theta \simeq 0°$ in which case $\cos\theta \simeq \cos 0° = 1$ and we have

$$h = \frac{2\sigma}{\rho r g}$$

For mercury and glass, $\theta \simeq 140°$ so that $\cos\theta$ is negative. Hence mercury falls in a capillary tube, i.e.the level of mercury in the capillary tube is lower than the level outside.

NOTE

Surface tension of a liquid decreases with increase in temperature.

EXAMPLE 11.18

A narrow glass tube of diameter 1.0 mm is dipped vertically in a container of water. The surface tension of water is 0.07 N m^{-1} and the angle of contact with glass is zero. (a) Find the height to which water rises in the tube. (b) To what height will water rise if the tube is held slanting making an angle of 60° with the vertical? Also find the length which the water occupies in the tube. Take g = 10 m s^{-2}.

SOLUTION

(a) $h = \dfrac{2\sigma\cos\theta}{r\rho g} = \dfrac{2\times 0.07\times\cos 0°}{\left(0.5\times10^{-3}\right)\times10^{3}\times10}$

$= 2.8 \times 10^{-2}$ m = 2.8 cm

(b) The vertical height h of water column will remain the same = 2.8 cm (Fig. 11.17).

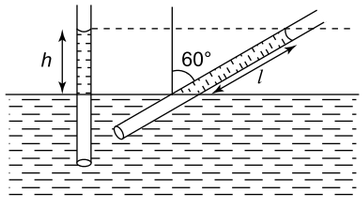


Fig. 11.17

Length of water column is $l = \dfrac{h}{\cos 60°} = 2h = 5.6$ cm

EXAMPLE 11.19

Water rises to a height of 10.0 cm in a capillary tube dipped in water. When the same tube is dipped in mercury, it is depressed by 2.6 cm. Compare the surface tensions of mercury and water. Relative density of mercury = 13.6. Angle of contact for water is zero and for mercury is 135°.

SOLUTION

$$\sigma_1 = \frac{h_1\rho_1 g r}{\cos\theta_1} \text{ and } \sigma_2 = \frac{h_2\rho_1 g r}{2\cos\theta_2}$$

$$\therefore \quad \frac{\sigma_2}{\sigma_1} = \frac{h_2}{h_1}\times\frac{\rho_2}{\rho_1}\times\frac{\cos\theta_1}{\cos\theta_2}$$

$$= \left(\frac{-2.6}{10.0}\right)\times 13.6\times\frac{\cos 0°}{\cos 135°}$$

$$= \left(\frac{-2.6}{10}\right)\times 13.6\times\frac{1}{(-0.707)} = 5.0$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. A piece of metal weighs x newton in air, y newton when completely immersed in water and z newton when completely immersed in a liquid. The relative density of the liquid is

(a) $\frac{y-z}{x-y}$ (b) $\frac{y-z}{x-z}$

(c) $\frac{x-y}{x-z}$ (d) $\frac{x-z}{x-y}$

2. A wooden ball of relative density 0.75 falls into a pond from a height of 1 m. If viscous forces due to air and water are neglected, the ball will sink in water to a depth of

(a) $\frac{2}{3}$ m (b) 3 m

(c) 6 m (d) 9 m

3. A piece of cork of density 250 kg m^{-3} is immersed in water to a depth of 1 m and released. If viscous forces due to water and air are neglected, the piece of cork will jump to what height above the surface of water?

(a) 1 m (b) 2 m

(c) 3 m (d) 4 m

4. A cylinder of mass m, cross-sectional area a and relative density σ (> 1) hanging from a string is lowered into water in a vessel until it is completely immersed. If A is the base area of the vessel, the increase in pressure at the bottom of the vessel due to the immersion of the cylinder is

(a) $\frac{mg}{\sigma A}$ (b) $\frac{mg}{\sigma a}$

(c) $\frac{mg\sigma}{A}$ (d) $\frac{mg\sigma}{a}$

5. A space-ship is revolving around the earth at an altitude where the acceleration due to gravity is $g/2$. The air pressure inside the cabin is maintained at 76 cm of mercury. A barometer of tube length 100 cm is hanging on the wall of the cabin. The mercury in the barometer will rise to a height equal to

(a) 38 cm (b) 76 cm

(c) 100 cm (d) zero

6. A block floats in a liquid contained in a beaker (Fig. 11.18). The beaker is placed on the floor of an elevator. If the elevator descends with acceleration a (< g), the upthrust on the block due to the liquid

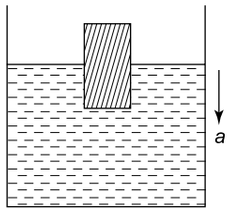


Fig. 11.18

(a) is equal to the weight of the liquid displaced

(b) is greater than the weight of the liquid displaced

(c) is less than the weight of the liquid displaced

(d) becomes equal to zero

7. A liquid stands at the same level in arms A and B of a U-tube. If the U-tube is given a constant acceleration a (< g) towards the right as shown in Fig. 11.19, the level of liquid in limb A rises to a height h above the level in limb B.

If the length of the horizontal part of the U-tube is L, the value of h is given by

(a) $\frac{La}{g}$ (b) $\frac{Lg}{a}$

(c) $L\left(1+\frac{a}{g}\right)$ (d) $L\left(1-\frac{a}{g}\right)$

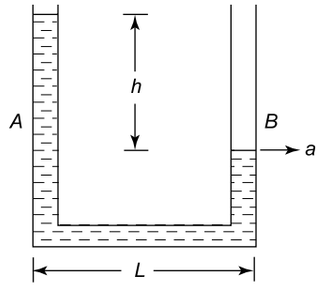


Fig. 11.19

8. A cube of wood supporting a mass of 200 g just floats in water. When the mass is removed, the cube rises by 2 cm. What is the size of the cube?

(a) 6 cm (b) 8 cm

(c) 10 cm (d) 12 cm

9. A stone of relative density k is released from rest on the surface of a lake. If viscous effects are ignored, the stone sinks in water with an acceleration of

(a) $g\ (1 - k)$ (b) $g\ (1 + k)$

(c) $g\left(1-\frac{1}{k}\right)$ (d) $g\left(1+\frac{1}{k}\right)$

10. Two identical cylindrical vessels, each of base area A, have their bases at the same horizontal level. They contain a liquid of density ρ. In one vessel the height of the liquid is h_1 and in the other $h_2 > h_1$. When the two vessels are connected, the work done by gravity in equalizing the levels is

(a) $2\rho\ Ag\ (h_2 - h_1)^2$ (b) $\rho Ag\ (h_2 - h_1)^2$

(c) $\frac{1}{2}\ \rho Ag\ (h_2 - h_1)^2$ (d) $\frac{1}{4}\ \rho Ag\ (h_2 - h_1)^2$

11. A cylindrical jar has radius r. To what height h should it be filled with a liquid so that the force exerted by the liquid on the sides of the jar equals the force exerted on the bottom?

(a) $h = \frac{r}{2}$ (b) $h = r$

(c) $h = 2r$ (d) $h = 4r$

12. A glass tube of radius r is dipped vertically into a container of mercury with its lower end at a depth h below the mercury surface. If σ is the surface tension of mercury, what must be the gauge pressure of air in the tube to blow a hemispherical bubble at the lower end?

(a) $\frac{2\sigma}{r} + hg\rho$ (b) $\frac{2\sigma}{r} - hg\rho$

(c) $\frac{4\sigma}{r} + hg\rho$ (d) $\frac{4\sigma}{r} - hg\rho$

13. A small drop of water of surface tension σ is squeezed between two clean glass plates so that a thin layer of thickness d and area A is formed between them. If the angle of contact is zero, the force required to pull the plates apart is

(a) $\frac{\sigma A}{d}$ (b) $\frac{2\sigma A}{d}$

(c) $\frac{4\sigma A}{d}$ (d) $\frac{8\sigma A}{d}$

14. A spherical small ball of density ρ is gently released in a liquid of density $\sigma(\rho > \sigma)$. The initial acceleration of the free fall of the ball will be

(a) $\left(\frac{\rho+\sigma}{\rho}\right) g$ (b) $\left(\frac{\rho-\sigma}{\sigma}\right) g$

(c) $\left(\frac{\rho-\sigma}{\rho}\right) g$ (d) g

15. The time period of a simple pendulum is T. The pendulum is oscillated with its bob immersed in a liquid of density σ. If the density of the bob is ρ and viscous effect is neglected, the time period of the pendulum will be

(a) $\left(\frac{\rho}{\rho-\sigma}\right)^{1/2} T$ (b) $\left(\frac{\sigma}{\rho-\sigma}\right)^{1/2} T$

(c) $\left(\frac{\rho}{\sigma}\right)^{1/2} T$ (d) $\left(\frac{\sigma}{\rho}\right)^{1/2} T$

16. A wooden block of mass m and density ρ is tied to a string; the other end of the string is fixed to the bottom of a tank. The tank is filled with a liquid of density σ with $\sigma > \rho$. What is the tension in the string.

(a) $\left(\frac{\sigma-\rho}{\sigma}\right) mg$ (b) $\left(\frac{\sigma-\rho}{\rho}\right) mg$

(c) $\frac{\rho mg}{\sigma}$ (d) $\frac{\sigma mg}{\rho}$

17. A wooden ball of density σ is released from the bottom of a tank which is filled with a liquid of density ρ ; $(\rho > \sigma)$ up to a height h_1. The ball rises in the liquid, emerges from its surface and attains a height h_2 in air. If viscous effects are neglected, the ratio h_2/h_1 is

(a) $\left(\frac{\rho}{\sigma}+1\right)$ (b) $\left(\frac{\rho}{\sigma}-1\right)$

(c) $\frac{\rho}{\sigma}$ (d) $\frac{\sigma}{\rho}$

18. Two blocks A and B are made of different kinds of wood. Block A floats in water with $\frac{1}{4}$ th of its above the surface of water. Block B floats in water with $\frac{2}{3}$ rds of its volume below the surface of water. The ratio of the densities of A and B is

(a) 3 : 2 (b) 5 : 3

(c) 9 : 8 (d) 4 : 3

19. Equal masses of two substances of densities ρ_1 and ρ_2 are mixed together. The density of the mixture would be

(a) $\frac{1}{2}(\rho_1 + \rho_2)$ (b) $\sqrt{\rho_1\rho_2}$

(c) $\frac{\rho_1\rho_2}{(\rho_1+\rho_2)}$ (d) $\frac{2\rho_1\rho_2}{(\rho_1+\rho_2)}$

20. Equal volumes of two substances of densities ρ_1 and ρ_2 are mixed together. The density of the mixture would be

(a) $\frac{1}{2}(\rho_1 + \rho_2)$ (b) $(\rho_1 + \rho_2)$

(c) $\sqrt{\rho_1\rho_2}$ (d) $\frac{\rho_1\rho_2}{(\rho_1+\rho_2)}$

21. A capillary tube of radius r is immersed in water and water rises in it to a height h. The mass of water in the capillary tube is 5g. Another capillary tube of radius $2r$ is immersed in water. The mass of water that will rise in this tube is

(a) 2.5 g (b) 5.0 g

(c) 10 g (d) 20 g

22. When a capillary tube of radius r is immersed in a liquid of density ρ, the liquid rises to a height h in it. If m is the mass of the liquid in the capillary tube, the potential energy of this mass of the liquid in the tube is

(a) $mgh/4$ (b) $mgh/2$

(c) mgh (d) $2mgh$

23. The volume of an air bubble is doubled as it rises from the bottom of a lake to its surface. The atmo-

spheric pressure is 75 cm of mercury and the ratio of the density of mercury to that of lake water is 40/3. What is the depth of the lake?

(a) 10 m (b) 15 m
(c) 20 m (d) 25 m

24. In a surface tension experiment with a capillary tube, water rises upto 0.1 m. If the same experiment is repeated in an artificial satellite, which is revolving around the earth, water will rise in the capillary tube upto a height of

(a) 0.1 m
(b) 0.2 m
(c) 0.98 m
(d) full length of the capillary tube.

25. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect the effect of gravity. Then the pressure in the compartment is

(a) the same everywhere
(b) lower in the front side
(c) lower in the rear side
(d) lower in the upper side

IIT, 1999

26. A vessel contains oil (density 0.8 g cm^{-3}) over mercury (density 13.6 g cm^{-3}). A homogeneous sphere floats with half volume immersed in mercury and the other half in oil. The density of the material of the sphere in g cm^{-3} is

(a) 3.3 (b) 6.4
(c) 7.2 (d) 12.8

IIT, 1988

27. The height to which a liquid rises or falls in a capillary tube is directly proportional to

(a) the radius of the capillary
(b) the surface tension of the liquid
(c) the density of the liquid
(d) the angle of contact.

28. The density of air in earth's atmosphere decreases with height as

$$\rho = \rho_0 \, e^{-kh}$$

where ρ_0 = density of air at sea level and k is a constant. The atmospheric pressure at sea level is

(a) $\frac{\rho_0 g}{2k}$ (b) $\frac{\rho_0 g}{k}$

(c) $\frac{2\rho_0 g}{k}$ (d) $\frac{\rho_0 g}{\sqrt{2}k}$

29. A cubical vessel of height 1 m is full of water. The work done in pumping water out of the vessel is

(a) 49 J (b) 98 J
(c) 4900 J (d) 9800 J

30. Two water droplets coalesce to form a large drop. In this process,

(a) energy is liberated
(b) energy is absorbed
(c) energy is neither liberated nor absorbed
(d) a small amount of mass is converted into energy in accordance with Einstein's mass–energy equivalence relation $E = mc^2$.

31. A solid iron ball and a solid aluminium ball of the same diameter are released together on a deep lake. Which ball will reach the bottom first?

(a) Aluminium ball
(b) Iron ball
(c) Both balls will reach the bottom at the same time
(d) The aluminium ball will never reach the bottom and will remain suspended in the lake

32. Two spherical soap bubbles formed in vacuum have diameters 3.0 mm and 4.0 mm. They coalesce to form a single spherical bubble. If the temperature remains unchanged, the diameter of the bubble so formed will be

(a) 5.0 mm (b) 5.8 mm
(c) 6.2 mm (d) 7.0 mm

33. A liquid of density ρ and surface tension σ rises to a height h in a capillary tube of diameter d. The weight of the liquid in the capillary tube is

(a) $2\,\pi\sigma h$ (b) $\frac{2\pi\sigma h^2}{d}$

(c) $\pi\sigma d$ (d) $\frac{\pi\sigma d^2\rho}{h}$

34. In Q. 33 above, the potential energy of the liquid in the capillary tube is

(a) $h\rho g$ (b) $\frac{2\pi\sigma^2}{\rho g}$

(c) $2\pi\sigma^2\rho g$ (d) $\frac{2\pi\sigma^2}{\rho g h}$

35. A needle of length l and density ρ will float on a liquid of surface tension σ if its radius r is less than or equal to

(a) $\sqrt{\frac{2\sigma}{\pi\rho lg}}$ (b) $\sqrt{\frac{2\sigma l}{\pi\rho g}}$

(c) $\sqrt{\frac{\sigma}{\pi\rho g}}$ (d) $\sqrt{\frac{2\sigma}{\pi\rho g}}$

36. A film of water is formed between two straight parallel wires, each 10 cm long and at a separation of 0.5 cm. The work that must be done to increase the separation between the wires by 1 mm is (surface tension of water = 7.0×10^{-2} Nm^{-1})

(a) 7.0×10^{-5} N (b) 1.4×10^{-5} N
(c) 7.0×10^{-7} N (d) 1.4×10^{-7} N

37. Two separate air bubbles of radii r_1 and r_2 ($r_2 > r_1$) formed of the same liquid come together to form a double bubble. The radius of the internal film surface common to both bubbles is

(a) $\dfrac{r_1 r_2}{r_2 - r_1}$ (b) $\dfrac{r_1 r_2}{r_2 + r_1}$

(c) $\dfrac{1}{2}(r_1 + r_2)$ (d) $(r_2 - r_1)$

38. Find the depth at which an air bubble of radius 0.7 mm will remain in equilibrium in water. Given, surface tension of water = 7.0×10^{-2} Nm^{-1}. Take g = 10 ms^{-2}.
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 5 cm

39. If a number of identical droplets of water, each of radius r, coalesce to form a single drop of radius R, the resulting rise in the temperature of water is given by (here ρ is the density of water, s its specific heat and σ its surface tension)

(a) $\dfrac{\sigma}{\rho s}\left(\dfrac{1}{r} - \dfrac{1}{R}\right)$ (b) $\dfrac{3\sigma}{\rho s}\left(\dfrac{1}{r} - \dfrac{1}{R}\right)$

(c) $\dfrac{\sigma}{\rho s}\left(\dfrac{1}{r} + \dfrac{1}{R}\right)$ (d) $\dfrac{3\sigma}{\rho s}\left(\dfrac{1}{r} + \dfrac{1}{R}\right)$

40. Work W is required to be done to form a spherical bubble of volume V from a given soap solution. How much work is needed to form a spherical bubble of volume $2V$?
(a) $2\ W$ (b) $\sqrt{2}\ W$
(c) $(2^{1/3})\ W$ (d) $(4^{1/3})\ W$

41. A circular wire frame of radius R is dipped in a soap solution of surface tension σ. When it is taken out a thin soap film is formed inside the frame. The force on the frame will be
(a) $\pi\sigma R$ (b) $2\ \pi\sigma R$
(c) $4\ \pi\sigma R$ (d) $8\ \pi\sigma R$

42. A thin loop of a thread floats on a soap film formed inside a wire frame which is kept horizontal. When the film is pierced, the loop arranges in the form of a circle of radius r. If σ is the surface tension of the soap solution, the tension in the string will be
(a) $\pi\sigma R$ (b) $2\ \pi\sigma R$
(c) $4\ \pi\sigma R$ (d) $8\ \pi\sigma R$

43. A ring of external and internal radii r_1 and r_2 just touches the horizontal surface of a liquid of surface tension σ. The force required to pull the ring away from the surface is
(a) $2\pi\ (r_1 + r_2)\sigma$ (b) $2\pi\ (r_1 - r_2)\sigma$
(c) $4\pi\ (r_1 + r_2)\sigma$ (d) $4\pi\ (r_1 - r_2)\sigma$

44. A long metal rod of length l and relative density σ is held vertically with its lower end just touching the surface of water. The speed of the rod when it just sinks in water is given by
(a) $\sqrt{2gl}$ (b) $\sqrt{2gl\sigma}$
(c) $\sqrt{2gl\left(1 - \dfrac{1}{2\sigma}\right)}$ (d) $\sqrt{2gl(2\sigma - 1)}$

45. A sphere of relative density σ and diameter D has concentric cavity of diameter d. It will just float on water in a tank if the ratio D/d is
(a) $\dfrac{\sigma}{(\sigma - 1)}$ (b) $\dfrac{(\sigma + 1)}{\sigma}$
(c) $\left(\dfrac{\sigma}{\sigma - 1}\right)^{1/3}$ (d) $\left(\dfrac{\sigma + 1}{\sigma}\right)^{1/3}$

46. A large block of ice 5 m thick has a vertical hole drilled through it and is floating in the middle of a lake. The minimum length of the rope required to scoop up bucket full of water through the hole is (the relative density of ice = 0.9)
(a) 1 m (b) 0.9 m
(c) 0.5 m (d) 0.45 m

IIT, 1982

47. A capillary tube is immersed vertically in water and the height of the water column is x. When this arrangement is taken into a mine of depth d, the height of the water column is y. If R is the radius of the earth, the ratio $\dfrac{x}{y}$ is
(a) $\left(1 - \dfrac{d}{R}\right)$ (b) $\left(1 + \dfrac{d}{R}\right)$
(c) $\left(\dfrac{R - d}{R + d}\right)$ (d) $\left(\dfrac{R + d}{R - d}\right)$

48. When an air bubble of radius r rises from the bottom to the surface of a lake, its radius becomes $5r/4$, the atmospheric pressure being equal to 10 m height of water column. If the temperature remains constant and the surface tension is neglected, the depth of the lake is
(a) 3.53 m (b) 6.53 m
(c) 9.53 m (d) 12.53 m

49. Water rises to a height h in a capillary tube of area of cross-section a. To what height will water rise in a capillary tube of area of cross-section $4a$?
(a) $\dfrac{h}{4}$ (b) $\dfrac{h}{2}$
(c) $2\ h$ (d) $4\ h$

50. If W is the amount of work done in blowing a bubble of volume, V, what will be the amount of work

done to blow a bubble of volume 8 V?
(a) 2 W (b) 4 W
(c) 8 W (d) 16 W

51. If a million tiny droplets of water of the same radius coalesce into one larger drop, the ratio of the surface energy of the large drop to the total surface energy of all the droplets will be
(a) $1:10$ (b) $1:10^2$
(c) $1:10^4$ (d) $1:10^6$

52. A container of a large uniform cross-sectional area A resting on a horizontal surface holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $\frac{H}{2}$ as shown in Fig. 11.20. The lower density liquid is open to atmosphere. A homogeneous solid cylinder of length $L\left(L<\frac{H}{2}\right)$, cross-sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical to the liquid-liquid interface with length $\frac{L}{4}$ in the denser liquid. The density of the solid is

(a) $\frac{3d}{2}$ (b) $\frac{4d}{3}$

(c) $\frac{5d}{4}$ (d) 3d

IIT, 1995

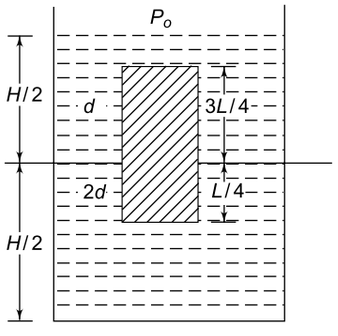
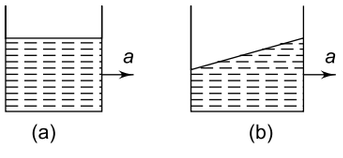


Fig. 11.20

53. A vessel containing water is given a constant acceleration towards the right, along a straight horizontal path. Which of the diagrams (Fig. 11.21) represents the surface of the liquid

IIT, 1981



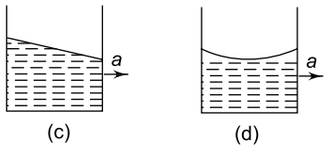


Fig. 11.21

54. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ where it stays vertical (see Fig. 11.22). The upper part of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder exerted by the liquid is

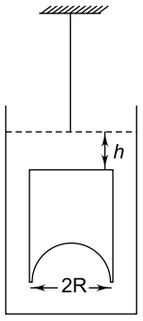


Fig. 11.22

(a) Mg
(b) $Mg - V\rho g$
(c) $Mg + \pi R^2 h\rho g$
(d) $\rho g\ (V + \pi R^2 h)$

IIT, 2001

55. A wooden block with a coin placed on its top, floats in water as shown in Fig. 11.23. The distances l and h are shown. After some time the coin falls into the water. Then

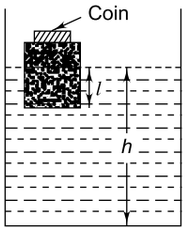


Fig. 11.23

(a) l decreases and h increases
(b) l increases and h decreases
(c) both l and h increase
(d) both l and h decrease

IIT, 2005

56. Water is filled up to a height h in a beaker of radius R as shown in Fig. 11.24. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section $ABCD$ of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude

(a) $|\ 2P_0Rh + \pi R^2\rho gh - 2\ RT|$
(b) $|\ 2P_0Rh + R\rho gh^2 - 2\ RT|$
(c) $|\ P_0\pi R^2 + R\rho gh^2 - 2\ RT|$
(d) $|\ P_0\pi R^2 + R\rho gh^2 + 2\ RT|$

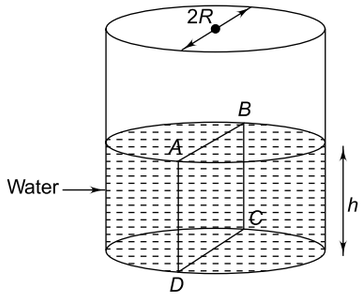


Fig. 11.24

IIT, 2007

57. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the value is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown Fig. 11.25. Just after opening the value.

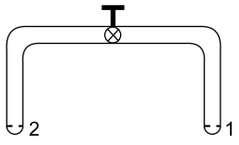


Fig. 11.25

(a) air form end 1 flows towards end 2. No change in the volume of the soap bubbles
(b) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases
(c) no change occurs
(d) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

IIT, 2008

ANSWERS

1. (d)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (c)
7. (a)	**8.** (c)	**9.** (c)	**10.** (d)	**11.** (b)	**12.** (a)
13. (b)	**14.** (c)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (c)
19. (d)	**20.** (a)	**21.** (c)	**22.** (b)	**23.** (a)	**24.** (d)
25. (b)	**26.** (c)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (a)
31. (b)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (d)	**36.** (d)
37. (a)	**38.** (c)	**39.** (b)	**40.** (d)	**41.** (c)	**42.** (b)
43. (a)	**44.** (c)	**45.** (c)	**46.** (c)	**47.** (a)	**48.** (c)
49. (b)	**50.** (b)	**51.** (b)	**52.** (c)	**53.** (c)	**54.** (c)
55. (d)	**56.** (b)	**57.** (b)			

SOLUTIONS

1. Upthrust = weight of liquid displaced. If V is the volume of the metal piece, then

$$x - y = V\,\rho_w\,g \qquad (1)$$

and $x - z = V\,\rho_l\,g$ (2)

Dividing (2) by (1) we get, relative density of liquid.

$$\frac{\rho_l}{\rho_w} = \frac{x-z}{x-y}$$

So the correct choice is (d).

2. The ball will hit the surface of water with a speed

$$v = \sqrt{2gh} = \sqrt{2\times g\times 1} = \sqrt{2g}\ \text{ms}^{-1}$$

When the ball enters the water, it experiences an upthrust U = weight of water displaced = $V\,\rho_w\,g$, where V = volume of the ball. Weight of the ball is $W = V\,\rho_b\,g$ where ρ_b = density of the ball. Therefore, the retardation of the ball moving inside water is

$$a = \frac{U-W}{m}$$

(m = mass of the ball = $\rho_b\ V$)

$$= \frac{V\,\rho_w\,g - V\,\rho_b\,g}{\rho_b V} = \frac{(1-\rho_b/\rho_w)g}{\rho_b/\rho_w}$$

$$\Rightarrow \qquad a = \left(\frac{1-0.75}{0.75}\right) g = \frac{g}{3}$$

Initial velocity with which the ball enters water is $v = \sqrt{2g}$. If it sinks to a depth H then (since the final velocity = 0, we have from ($v^2 - u^2 = 2aS$),

$$0 - 2g = 2 \times \left(-\frac{g}{3}\right) H$$

$\Rightarrow H = 3$ m, which is choice (b).

3. Let m be the mass of piece and V its volume. Then

$$V = \frac{m}{\rho}, \qquad \text{where } \rho = 250 \text{ kg m}^{-3}$$

When the piece is released, its upward acceleration is

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}}$$

$$= \frac{V\rho_w g - V\rho g}{\rho V} = \frac{1000g - 250g}{250} = 3g$$

$\therefore$ Velocity of the piece when it reaches the surface of water is $v = \sqrt{2ah} = \sqrt{2\times 3g\times 1} = \sqrt{6g}$.

It will jump to a height H above the surface of water, where H is given by

$$H = \frac{v^2}{2g} = \frac{6g}{2g} = 3 \text{ m}$$

4. Let ρ_c be the density of the cylinder and ρ_w that of water. Upthrust on cylinder $= \left(\frac{m}{\rho_c}\right)\rho_w g = \frac{mg}{\sigma}$ $\left(\because \sigma = \frac{\rho_c}{\rho_w}\right)$

From Newton's third law, the force exerted by the cylinder on water in the downward direction is

$$F = \frac{mg}{\sigma}$$

$\therefore$ Increase in pressure at the bottom of the vessel is

$$\frac{F}{A} = \frac{mg}{\sigma A}, \text{ which is choice (a).}$$

5. Inside the cabin $g_{\text{eff}} = 0$. Hence the pressure outside the barometer tube is 76 cm of mercury, but the pressure above the tube is zero. Hence mercury will rise to the full length of the tube of the barometer, i.e. 100 cm. So the correct choice is (c).

6. Due to downward acceleration of the elevator, the beaker exerts a force on the floor of the elevator. From Newton's third law, the floor exerts upward force on the beaker. Hence total upthrust on the block is greater than when the system was at rest. Hence the correct choice is (c).

7. Let p_1 and p_2 be the pressures at the bottom of limbs A and B, then

$p_1 - p_2 = h\rho g$ (ρ = density of liquid)

If A is the cross-sectional area of the U-tube, the force at the horizontal part of the tube is

$$F = (p_1 - p_2)A = h\rho g A \qquad (1)$$

Now $F = ma$, where $m = \rho AL$ is the mass of the liquid in length L. Hence

$$F = \rho A L a \qquad (2)$$

Equating (1) and (2), we get $h = \frac{La}{g}$. So the correct choice is (a).

8. Let the side of the cube be $l$cm. The volume of the cube above the surface of water = volume of water displaced due to mass of 200 g. Therefore, mass of displaced water is 200 g, its volume is 200 cm^3. Hence $2 \times l \times l = 200$ or $l = 10$ cm. Hence the correct choice is (c).

9. If m is the mass of the stone and V its volume, the weight of the water displaced by it $= \rho Vg = \rho' \times \frac{m}{\rho} \times g = \frac{mg}{k}$, where ρ' is the density of water. Therefore, $k = \rho/\rho'$. Thus, the buoyant force acting upwards is mg/k whereas the weight mg of the stone acts vertically downwards. Therefore, the net force in the downward direction $= mg - mg/k$ $= mg\left(1 - \frac{1}{k}\right)$. If a is the acceleration of the sinking stone, then

$$ma = mg\left(1 - \frac{1}{k}\right)$$

or $$a = g\left(1 - \frac{1}{k}\right)$$

Hence the correct choice is (c).

10. After the levels in the two vessels become equal, the increase in height of the liquid in one vessel is $\frac{1}{2}(h_2 - h_1)$ with the same decrease in height in the other. Thus, effectively a slab of liquid $\frac{1}{2}(h_2 - h_1)$ in thickness falls a vertical distance equal to its thickness under the action of gravity. Therefore, Work done by the gravity is

$$W = mg \times \frac{1}{2}(h_2 - h_1)$$

where mass of the slab m is given by

$$m = \rho \times V = \rho \times \frac{1}{2}(h_2 - h_1) \times A$$

Therefore $W = \frac{1}{2}\rho(h_2 - h_1) \times A \times g \times \frac{1}{2}(h_2 - h_1)$

$$= \frac{1}{4}\rho A g (h_2 - h_1)^2$$

11. Let a cylinder with radius of cross-section r be filled with a homogeneous liquid of density ρ up to a height h.

Pressure at the bottom of the cylinder, $p_1 = \rho gh$

Pressure at the top of the liquid surface, $p_2 = 0$

Average pressure on the sides of the cylinder,

$$p = \frac{p_1 + p_2}{2}$$

or $p = \rho\ g\ h/2$

Force on the sides of the vessel

$$= \text{average pressure} \times \text{area}$$

$$= \frac{\rho g h}{2} \times 2\ \pi\ r\ h$$

$$= \pi\ r\ \rho\ g h^2$$

Force on the bottom of the vessel $= p_1 \pi r^2 = \rho g h \pi r^2$

Two forces will be equal if

$\pi\ r\ \rho\ g\ h^2 = \rho\ g\ h\ \pi\ r^2$ or $h = r$

12. The correct choice is (a).

13. An extremely thin layer of a liquid can be regarded as a collection of a large number of semi-spherical drops. Hence the excess pressure across a thin layer of a liquid is $\frac{\sigma}{r}$ instead of $\frac{2\sigma}{r}$ as in the case of a spherical drop, where $r = d/2$. Therefore, excess pressure is

$$p = \frac{\sigma}{r} = \frac{\sigma}{d/2} = 2\ \frac{\sigma}{d}$$

$\therefore$ Force due to surface tension pushing the two plates together is

$$F = \text{excess pressure} \times \text{area of layer}$$

$$= \frac{2\sigma A}{d}$$

Hence the correct choice is (b).

14. Let m be the mass of the ball and V its volume. Its mass $m = \rho V$. The weight of the ball is

$$W = mg = \rho V g$$

The volume of the liquid displaced $= V$. If σ is the density of the liquid, the weight of the liquid displaced is the upthrust U it experiences.

$$U = V \sigma g$$

$\therefore$ The net downward force acting on the body is

$$F = W - U = (\rho - \sigma)\ Vg$$

The initial acceleration is

$$a = \frac{F}{m} = \frac{(\rho - \sigma) Vg}{\rho V}$$

$$= \left(\frac{\rho - \sigma}{\rho}\right) g$$

Hence the correct choice is (c).

15. Refer to solution of Q. 14. The net downward force acting on the bob is

$$F = (\rho - \sigma)\ Vg = \left(\frac{\rho - \sigma}{\rho}\right)\ mg$$

$(\because\ m = \rho V)$

or $mg' = \left(\frac{\rho - \sigma}{\rho}\right)\ mg$

or $g' = \left(\frac{\rho - \sigma}{\rho}\right)\ g;$

This is the effective acceleration due to gravity.

$$\therefore \quad T' = 2\pi\ \sqrt{\frac{l}{g'}} = 2\pi\ \sqrt{\frac{\rho l}{(\rho - \sigma)\ g}}$$

$$= \left(\frac{\rho}{\rho - \sigma}\right)^{1/2} T \qquad \left(\because T = 2\pi \sqrt{\frac{l}{g}}\right)$$

Hence the correct choice is (a).

16. Volume of the block $= \frac{m}{\rho}$. Now upthrust = weight of water displaced = weight of volume m/ρ of liquid = mass of volume m/ρ of liquid $\times\ g = \frac{m\sigma g}{\rho}$. This is the upward force on the block due to buoyancy. The downward force on the block = its weight $= mg$. The tension in the string is the net upward force on the block which is

$$T = \frac{m\sigma g}{\rho} - mg = \left(\frac{(\sigma - \rho)}{\rho}\right)\ mg$$

Hence the correct choice is (b).

17. Weight of the ball $W = mg = \sigma V g$. Upthrust $U = \rho V g$. Therefore, the net upward force acting on the ball is

$$F = U - W = (\rho - \sigma)\ Vg$$

Now, mass of the ball is $m = \sigma V$. Therefore, upward acceleration of the ball while it is rising in the liquid is

$$a = \frac{F}{m} = \frac{(\rho - \sigma) Vg}{\sigma V} = \left(\frac{\rho - \sigma}{\sigma}\right) g$$

Velocity of the ball on reaching the surface of water is

$$v = \sqrt{2a h_1} \qquad \text{(i)}$$

This is the initial upward velocity of the ball in air. If it rises to a height h_2 in air, we have

$$v = \sqrt{2g h_2} \qquad \text{(ii)}$$

Equating (i) and (ii), we have $a\ h_1 = g\ h_2$

or $\frac{h_2}{h_1} = \frac{a}{g} = \frac{\rho - \sigma}{\sigma} = \left(\frac{\rho}{\sigma} - 1\right)$

Hence the correct choice is (b).

18. Let V_1 be the volume of block A and ρ_1 is density. Then, from the principle of flotation, we have

$$\rho_1 V_1 g = \left(1-\frac{1}{4}\right) V_1 \times \rho \times g$$

or $$\rho_1 = \frac{3\rho}{4}$$

where ρ is the density of water. Similarly for block B we have

$$\rho_2 V_2 g = \frac{2V_2}{3} \rho g \text{ or } \rho_2 = \frac{2\rho}{3}$$

$$\therefore \quad \frac{\rho_1}{\rho_2} = \frac{3\rho}{4} \times \frac{3}{2\rho} = \frac{9}{8}$$

Hence the correct choice is (c).

19. The density of the mixture is

$$\rho = \frac{\text{total mass}}{\text{total volume}}$$

$$= \frac{m_1 + m_2 + m_3 + \cdots + m_n}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} + \frac{m_3}{\rho_3} + \cdots + \frac{m_n}{\rho_n}}$$

For two substances mixed together, we have

$$\rho = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}} = \frac{(m_1 + m_2)\rho_1\rho_2}{(m_1\rho_2 + m_2\rho_1)}$$

Given $m_1 = m_2 = m$. Therefore, $\rho = \frac{2\rho_1\rho_2}{(\rho_2 + \rho_1)}$.

Hence the correct choice is (d).

20. The density of the mixture is

$$\rho = \frac{m_1 + m_2}{V_1 + V_2} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

where V_1 and V_2 are the volumes of the two substances.

Given $V_1 = V_2 = V$. Therefore, $\rho = \frac{1}{2}(\rho_1 + \rho_2)$.

Hence the correct choice is (a).

21. Mass of water in first tube is $m = \pi r^2 h\rho$

Now, surface tension $\sigma = \frac{h\rho g r}{2} = \frac{h'\rho g r'}{2}$

where h' is the height to which water rises in the second tube and r' its radius. Since $r' = 2r$, $h' = h/2$, the mass of water in the second capillary tube is

$$m' = \pi r'^2 h' \rho = \pi (2r)^2 \frac{h}{2} \rho$$

$$= 2\pi r^2 h \rho = 2m = 2 \times 5 = 10 \text{ g}$$

Hence the correct choice is (c).

22. The mass of the liquid in a column between x and $x + dx$ is

$$dm = \pi r^2 \rho dx$$

Therefore, the potential energy of the liquid in a column of height h is

$$\int_0^h (\pi r^2) x \rho g dx = \pi r^2 \rho g \int_0^h x dx$$

$$= \pi r^2 \rho g \frac{h^2}{2}$$

$$= (\pi r^2 h \rho) gh/2 = mgh/2$$

Hence the correct choice is (b).

23. Let V_1 be the volume of the bubble and P_1 the pressure on it when it is at the bottom of the lake. If h is the depth of the lake, we have

$$P_1 = P_0 + h\rho g$$

(here ρ = density of water).

where P_0 is the atmospheric pressure. If V_2 is the volume of the bubble and P_2 the pressure on it when it reaches the surface of the lake, then

$$P_2 = P_0$$

$$V_2 = 2V_1 \text{ (given)}$$

From Boyle's law, $P_1V_1 = P_2V_2$ or $(P_0 + h\rho g) V_1 = P_0 \times 2V_1$, we have

$$h = \frac{P_0}{\rho g} = \frac{0.75 \times 40/3 \times 9.8}{1 \times 9.8} = 10 \text{ m}$$

Hence the correct choice is (a).

24. In a capillary tube water rises to a height h such that the hydrostatic pressure ($h\rho g$) becomes equal to the excess pressure $p = 2\sigma/R$, where R is the radius of curvature of the meniscus. A satellite in a stable orbit around the earth is in a state of weightlessness, i.e. $g = 0$. Thus the hydrostatic pressure becomes zero. Consequently, water will rise to the top to the tube. Hence the correct choice is (d).

25. Due to frictional force (which acts in a direction opposite to the direction of acceleration) on the rear face, the pressure in the rear side will be increased. Hence the pressure in the front side will be lowered. Thus the correct choice is (b).

26. Weight of sphere = weight of mercury displaced + weight of oil displaced

or $$V\rho g = \frac{V}{2} \times 13.6 \times g + \frac{V}{2} \times 0.8 \times g$$

or $$\rho = \frac{13.6 + 0.8}{2} = 7.2 \text{ g cm}^{-3}$$

Hence the correct choice is (c).

27. The height h is given by

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

Hence the only correct choice is (b).

28. The pressure due to a small layer of air of thickness dh at a height h above the earth is

$$dp = (dh)\ \rho g$$

According to the problem, density ρ varies with height h as

$$\rho = \rho_0\ e^{-kh}$$

$$\therefore \qquad dp = \rho_0 g\ e^{-kh}\ dh \qquad (1)$$

The atmospheric pressure P is due to the pressure exerted by all the air above the surface of the earth. Integrating (1) from $h = 0$ to $h = \infty$, we have

$$P = \rho_0 g \int_0^\infty e^{-kh} dh\ = \rho_0 g \left| \frac{e^{-kh}}{-k} \right|_0^\infty$$

$$= -\ \frac{\rho_0 g}{k}\ (e^{-\infty} - e^{-0}) = \frac{\rho_0 g}{k}$$

29. Let l be the dimension of each side of the cubical vessel. The mass of water contained in a height x is $l^2 x\ \rho$. Therefore, the work done is

$$W = \int_o^l \left(l^2 x \rho g\right)\ dx$$

$$= l^2 \rho g \int_o^l x dx\ = l^2 \rho g\ \frac{l^2}{2}$$

$$= \frac{l^4 \rho g}{2} = \frac{(1)^4 \times 1000 \times 9.8}{2} = 4900\ \text{J}$$

Hence the correct choice is (c).

30. When a big drop breaks up into smaller drops, the total surface area of the smaller drops is more that the surface area of the big drop. The increase in the surface area can be brought about by supplying energy. Thus a big drop has to absorb energy to break up into smaller drops. On the other hand, when smaller drops coalesce to form a big drop, there is a decrease in the surface area. Hence energy is liberated in this process, which is choice (a).

31. Since both balls have the same volume, they experience the same upthrust. Since the density of iron is greater than that of aluminium, the iron ball has a greater mass and therefore a greater weight; it therefore accelerates more and will reach the bottom before the aluminium ball. Hence the correct choice is (b).

32. Since the bubbles are in vacuum, the pressure of air inside them are $P_1 = \frac{4\sigma}{r_1}$ and $P_2 = \frac{4\sigma}{r_2}$, where $r_1 = 3.0$ mm and $r_2 = 4.0$ mm. Since the temperature remains unchanged, we have from Boyle's law

$$P_1\ V_1 + P_2\ V_2 = PV$$

$$\text{or} \quad \frac{4\sigma}{r_1} \cdot \frac{4}{3}\pi r_1^3 + \frac{4\sigma}{r_2} \cdot \frac{4}{3}\pi r_2^3 = \frac{4\sigma}{r} \cdot \frac{4}{3}\pi r^3 \qquad (1)$$

where r is the radius of the single bubble formed.

From (1), we get $r^2 = r_1^2 + r_2^2$ or $r = \sqrt{r_1^2 + r_2^2}$

$= \sqrt{(3.0)^2 + (4.0)^2}\ = 5.0$ mm, which is choice (a).

33. Weight of the liquid in the capillary tube (W) = mg = $\pi r^2\ h\rho g$.

Now $\sigma = \frac{h\rho r g}{2}$ or $h\rho g r = 2\sigma$. Hence

$$W = 2\ \pi\sigma r = \pi\sigma d \qquad (\because\ d = 2r)$$

Hence the correct choice is (c).

34. Since the mass of the liquid in the capillary tube can be considered to be concentrated at the centre of mass which is at a height $h/2$, the potential energy is

$$\text{PE} = mg\ \frac{h}{2} = \pi r^2\ h\rho g\ \frac{h}{2} = \frac{\pi (r h g)^2 \rho}{2g} \qquad (1)$$

Now $\sigma = \frac{h\rho r g}{2}$ or $rhg = \frac{2\sigma}{\rho}$ (2)

Using (2) in (1), we have

$$\text{PE} = \frac{\pi}{2g}\left(\frac{2\sigma}{\rho}\right)^2 \rho\ = \frac{2\pi\sigma^2}{\rho g}$$

Hence the correct choice is (b).

35. Force per unit area exerted on the needle due to liquid is

$$P_1 = \frac{\sigma}{r}$$

Pressure exerted by needle on the liquid is

$$P_2 = \frac{mg}{l(2r)} = \frac{\pi r^2 l \rho g}{2lr} = \frac{\pi r \rho g}{2}$$

For equilibrium, $P_1 = P_2$, i.e.

$$\frac{\sigma}{r} = \frac{\pi r \rho g}{2}$$

$$\text{or} \qquad r = \sqrt{\frac{2\sigma}{\pi \rho g}}$$

Hence the correct choice is (d).

36. $F = 2\ \sigma l$. Work done is $W = FS = 2\ \sigma l S$ $= 2 \times 7.0 \times 10^{-2} \times 0.1 \times 0.001 = 1.4 \times 10^{-7}$ N, which is choice (d).

37. Let r be the radius of the common surface and σ the surface tension of the liquid. When the bubbles come together, the difference of the excess pressures on either side of the common surface must be equal, i.e.

$$\frac{4\sigma}{r_1} - \frac{4\sigma}{r_2} = \frac{4\sigma}{r}$$

which gives $r = \dfrac{r_1 r_2}{r_2 - r_1}$, which is choice (a).

38. The bubble will be in equilibrium at a depth h if excess pressure $\dfrac{2\sigma}{r} = h\rho g$ or $h = \dfrac{2\sigma}{r\rho g}$. Given $\sigma = 7.0 \times 10^{-2}$ Nm^{-1}, $r = 0.7 \times 10^{-3}$ m, $\rho = 10^3$ kg m^{-3} and $g = 10$ ms^{-2}. Using these values we get $h = 4 \times 10^{-2}$ m = 4 cm, which is choice (c).

39. Work done is

$$W = 4\ \pi R^2\ (n^{1/3} - 1)\sigma = 4\ \pi R^2\ \left(\frac{R}{r} - 1\right) \quad (1)$$

$$(\because\ R = n^{1/3}\ r)$$

Heat produced $\quad Q = ms\Delta T = \dfrac{4}{3}\ \pi R^3\ \rho s \Delta T \quad (2)$

Equating (1) and (2) we find that choice (b) gives the correct expression for ΔT.

40. $W = 8\ \pi r^2 \sigma$. Now $V = \dfrac{4}{3}\ \pi r^3$. Hence

$$W = 8\ \pi\sigma\ \left(\frac{3V}{4\pi}\right)^{2/3}, \text{ i.e. } W \propto V^{2/3}$$

Hence the correct choice is (d).

41. Total length of wire is $L = 2\ \pi R$. Since the film is in air, force on frame $= 2\sigma \times L = 2\sigma \times 2\pi R = 4\pi\sigma R$. Hence the correct choice is (c).

42. Tension $= \sigma \times 2\pi r = 2\pi\sigma r$, which is choice (b).

43. Force $= 2\pi r_1\ \sigma + 2\pi r_2 \sigma = 2\pi(r_1 + r_2)\sigma$ which is choice (a).

44. Let the densities of metal and water be ρ and ρ_0 respectively and let x be the length of the rod immersed in water at an instant of time t. Then, acceleration at that instant = apparent weight divided by mass of the rod, i.e.

$$\frac{dv}{dt} = \frac{\pi r^2 l \rho g - \pi r^2 x \rho_0 g}{\pi r^2 l \rho}$$

$$= g - \frac{g x \rho_0}{l \rho}$$

$$= g\left(1 - \frac{x}{\sigma l}\right)$$

$$\text{or } \frac{dv}{dx} \cdot \frac{dx}{dt} = g\left(1 - \frac{x}{\sigma l}\right)$$

$$\text{or } v\ \frac{dv}{dx} = g\left(1 - \frac{x}{\sigma l}\right)$$

Integrating, we have

$$\frac{v^2}{2} = g\left| x - \frac{x^2}{2\sigma l}\right|_0^l = g\left(l - \frac{l}{2\sigma}\right)$$

$$\text{or } \quad v = \sqrt{2gl\left(1 - \frac{1}{2\sigma}\right)}$$ which is choice (c).

45. Let ρ be the density of the sphere and ρ_0 that of water. Volume of metal $= \dfrac{4}{3}\ \pi(R^2 - r^3)$ and volume of water displaced $= \dfrac{4}{3}\ \pi R^3$. From the principle of floatation, we have

$$\frac{4}{3}\ \pi (R^3 - r^3)\ \rho g = \frac{4}{3}\ \pi R^3\ \rho_0\ g$$

$$\text{or } \quad (R^3 - r^3)\ \frac{\rho}{\rho_0} = R^3$$

$$\text{or } \quad (R^3 - r^3)\sigma = R^3$$

$$\text{which gives } \frac{R}{r} = \left(\frac{\sigma}{\sigma - 1}\right)^{1/3}$$

Hence the correct choice is (c).

46. Let h be the height of the block and A its area of cross-section. Weight of ice block $= Ahg \times 0.9$. If x is the height of the block immersed in water, then the weight of water displaced $= Axg$. Equating the two we get $x = 0.9\ h$. Height above the surface of water $= h - 0.9\ h = 0.1\ h = 0.1 \times 5$ m $= 0.5$ m. Hence the correct choice is (c).

47. On earth's surface, $\sigma = \dfrac{x\rho r g}{2}$

In the mine, $\quad \sigma = \dfrac{y\rho r g_d}{2}$

Dividing, we get $\dfrac{x}{y} = \dfrac{g_d}{g} = \dfrac{g\left(1 - \dfrac{d}{R}\right)}{g}$

$$= 1 - \frac{d}{R}$$

Hence the correct choice is (a).

48. Let the depth of the lake be h metre. Since the temperature remains constant, we have, from Boyle's law.

$$P_1\ V_1 = P_2\ V_2$$

$$\text{or } (h + 10) \times \frac{4\pi}{3}\ r^3 = (10) \times \frac{4\pi}{3}\left(\frac{5r}{4}\right)^3$$

$$\text{or } \quad (h + 10) = 10 \times \frac{125}{64}$$

which gives $h = 9.53$ m. Hence the correct choice is (c).

49. Area of cross-section $a = \pi r^2$.

Therefore $r = \sqrt{a/\pi}$. In terms of a, the height to which a liquid rises in a capillary tube, is given by

$$h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2\sqrt{\pi}\sigma\cos\theta}{\sqrt{a}\rho g}$$

Thus, h is inversely proportional to $\sqrt{a}$. If a is increased 4 times, h will decrease by a factor of 2. Hence the correct choice is (b).

50. Work done to blow a bubble of radius r is

$$W = 4\pi\sigma r^2$$

In terms of volume $V = \frac{4\pi}{3} r^3$, we have

$$W = 4\pi\sigma \left(\frac{3V}{4\pi}\right)^{2/3}$$

Now, work done to blow a bubble of volume 8 V is

$$W = 4\pi\sigma \left(\frac{3\times 8V}{4\pi}\right)^{2/3}$$

$$= 4\pi\sigma \left(\frac{3V}{4\pi}\right)^{2/3} \times (8)^{2/3} = 4\ W$$

Hence the correct choice is (b).

51. Let r be the radius each droplet and R be the radius of the big drop which is made up of a million (10^6) tiny droplets. Since the total volume is the same, we have

$$V = 10^6 \times \frac{4\pi r^3}{3} = \frac{4\pi R^3}{3}$$

which gives $R^3 = 10^6\, r^3$ or $R = 100\, r$.

Now, the surface energy of a drop of radius r is given by

$$S = 4\pi r^2\sigma$$

where σ is the surface tension of water. The surface energy of one million drops will be

$$E_1 = 4\pi r^2\sigma \times 10^6$$

The surface energy of one big drop is

$$E_2 = 4\pi R^2\sigma$$

$$\therefore \frac{E_2}{E_1} = \left(\frac{R}{r}\right)^2 \times \frac{1}{10^6} = \left(\frac{100r}{r}\right)^2 \times \frac{1}{10^6} = \frac{1}{10^2}$$

Hence the correct choice is (b).

52. Mass of solid cylinder = cross-sectional area × length × density

$$= \frac{A}{5} \times L \times D = \frac{1}{5}\ ALD$$

$\therefore$ Weight of solid cylinder $(W) = \frac{1}{5}\ ALDg$

Buoyant force acting on the cylinder is

F_B = weight of denser liquid displaced + weight of lighter liquid displaced

$$= \frac{A}{5}\times\frac{L}{4} \times 2d \times g + \frac{A}{5} \times \frac{3L}{4} \times d \times g$$

$$= \left(\frac{1}{10} + \frac{3}{20}\right)\ ALdg = \frac{1}{4}\ ALdg$$

From the principle of floatation,

$$W = F_B$$

or $\frac{1}{5}\ ALDG = \frac{1}{4}\ ALdg$ or $D = \frac{5d}{4}$, Which is choice (c).

53. Any volume element V of the liquid (Fig. 11.26) experiences two forces, weight mg acting vertically downwards and the inertial force ma acting opposite to the direction of a. The resultant of mg and ma, namely F, is shown in Fig. 11.26. The hydrostatic thrust T on the volume element is equal and opposite to F. The level of the liquid will be perpendicular to the direction of the thrust. The correct choice is (c).

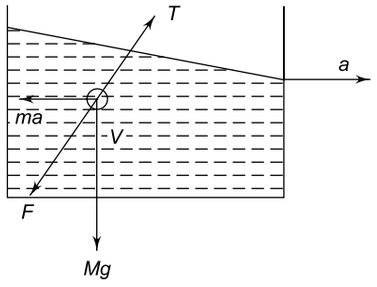


Fig. 11.26

54. Force on the bottom of the cylinder = weight of the remaining cylinder + force due to the liquid above the cylinder

$= Mg + Ah\rho g = Mg + \pi R^2\, h\rho g$, which is chioce (c).

55. When the coin falls into water, the downward force reduces causing the block to rise thus decreasing l. Since the density of the coin is greater then that of water, the volume of coin will be less than the volume of the liquid displaced, causing a decrease in h. Hence the correct choice is (d).

56. Let us find the force exerted on water in the right side of section $ABCD$. This force F = force f_1 due to pressure on the left side of section $ABCD$ + force f_2 due to surface tension . Now

f_1 = pressure at the center × area of $ABCD$

$= \left(P_0 + \frac{h}{2}\rho g\right) 2Rh$ directed towards right

f_2 = surface tension × length AB

$= T \times 2R = 2RT$ directed toward left

$\therefore \quad F = \left(P_0 + \frac{h}{2}\rho g\right) \times 2Rh - 2RT$

$= [2P_0\, Rh + R\rho g h^2 - 2RT]$

57. Excess pressure across a bubble is inversely proportional to the radius of the bubble, i.e. $\Delta p \propto \frac{1}{r}$. Since the value of r of the sub-hemispherical bubble is more than that for a hemispherical bubble, the excess pressure across the bubble at end 2 is less than that at end 1. Hence air from end 1 will flow towards end 2 resulting in a decrease in the volume of the bubble at end 1. Thus the correct choice is (b).

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Which physical quantity has the dimensional formula $ML^{-1}T^{-2}$?
(a) Pressure (b) Surface tension
(c) Stress (d) Young's modulus

2. Which of the following are dimensionless?
(a) Strain (b) Relative density
(c) Poisson's ratio (d) Shear modulus

3. The dimensions of pressure as the same as those of
(a) Stress (b) Young's modulus
(c) Surface tension (d) Bulk modulus.

4. Choose the correct statements from the following:
(a) A body will sink in a liquid if its weight is equal to or greater than the weight of the liquid displaced by it.
(b) A body will float in a liquid if its weight is equal to or less than the weight of the liquid displaced by it.
(c) When a body floats in a liquid, the portion of the body above the surface of the liquid is independent of the density of the body relative to that of the liquid.
(d) In still air, a hydrogen–filled balloon rises up to a certain height and then stops rising.

5. A spring balance A reads 2 kg with a block suspended from it. Another balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. When the block is immersed in water
(a) the balance A will read more than 2 kg
(b) the balance B will read more than 5 kg
(c) the balance A will read less than 2 kg
(d) the balance A will read 2 kg and the balance B will read 5 kg.

IIT, 1985

6. Choose the wrong statements from the following. A block of wood is floating in a lake. The apparent weight of the floating block is
(a) equal to its true weight
(b) less than its true weight
(c) more than its true weight
(d) equal to zero

7. Choose the wrong statements from the following. A boat carrying a large number of stones is floating in a water tank. If the stones are unloaded into the water in the tank, the level of water
(a) remains unchanged
(b) rises
(c) falls
(d) falls at first and then rises to the same level as before.

IIT, 1979

8. A cube of ice is floating in a liquid of relative density 1.2 contained in a beaker. When the ice melts, the level of water in the beaker
(a) rises
(b) falls
(c) remains unchanged if the liquid is water.
(d) rises if the liquid is water.

9. A piece of ice, with a stone frozen inside it, is floating in water contained in a beaker. When the ice melts, the level of water in the beaker
(a) rises
(b) falls
(c) remains unchanged
(d) falls at first and then rises to the same height as before.

10. Choose the wrong statements from the following. A block of wood floats in a liquid in a beaker with

3/4ths of its volume submerged under the liquid. If the beaker is placed in an enclosure that is falling freely under gravity, the block will
(a) float with 3/4ths of its volume submerged
(b) float completely submerged
(c) float with any fraction of its volume submerged
(d) sink to the bottom.

11. Choose the wrong statements from the following. A body floats in a liquid contained in a beaker. The whole system falls freely under gravity. The upthrust on the body due to the liquid is
(a) zero
(b) equal to the weight of the liquid displaced
(c) equal to the weight of the body in air
(d) equal to the weight of the immersed portion of the body.

12. A cubical block of steel 10 cm on each side is floating on mercury in a vessel. The height of the block above the mercury level is h_1. Water column of height h_2 is now poured into the vessel so that it just covers the steel block. Density of steel = 7.8×10^3 kg m^{-3} and that of mercury = 13.6×10^3 kg m^{-3}. The values of h_1 and h_2 are
(a) h_1 = 2.3 cm (b) h_1 = 4.3 cm
(b) h_2 = 4.6 cm (d) h_2 = 7.6 cm

13. A spring balance reads 10 kg when a bucket of water is suspended from it. It reads W_1 when an ice cube of mass 1.5 kg is put into the bucket and it reads W_2 when an iron piece (relative density 7.2) of mass 7.2 kg suspended from another string is immersed with half its volume inside the water in bucket. Then
(a) W_1 = 10 kg (b) W_1 = 11.5 kg
(c) W_2 = 10.5 kg (d) W_2 = 11 kg

14. A U-tube is filled with two immiscible liquids of densities ρ_1 and ρ_2 as shown in Fig. 11.27. If P_A, P_B, P_C and P_D are the pressures at points A, B, C and D respectively, then

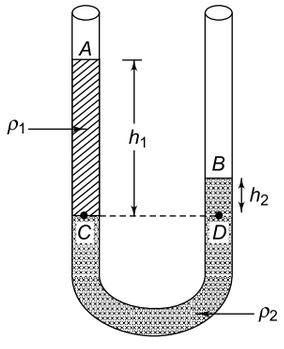


Fig. 11.27

(a) $P_A = P_B$ (b) $P_C = P_D$
(c) $\rho_1 > \rho_2$ (d) $\frac{h_1}{h_2} = \frac{\rho_2}{\rho_1}$

15. A wire is found to have a length L when it is loaded with a block of mass M and relative density n. When the block is immersed in water, the length of the wire reduces by x, then
(a) weight of water displaced when the block is immersed in water is $\frac{Mg}{n}$.
(b) the apparent loss of weight due to immersion is $Mg\left(1-\frac{1}{n}\right)$.
(c) the original length of the wire before it was loaded is $L_1 = L - nx$.
(d) $L_1 = L - x/n$.

16. A metal block weighs 2.4 N in air, 2.0 N in water and 1.9 N in a liquid. Then
(a) relative density of metal is 6.
(b) relative density of metal is 4.8.
(c) relative density of liquid is 0.8.
(d) relative density of liquid is 1.25.

17. A cylinder is partly filled with a liquid and then sealed. The pressure on the surface of the liquid is P and the force exerted at the bottom of the cylinder is F. If some air is removed from the cylinder with the help of a vacuum pump, then
(a) P will decrease and F will increase
(b) P and F both will decrease
(c) the liquid level in the cylinder rises
(d) the liquid level remains unchanged.

IIT, 1991

18. A U-tube of cross-sectional area a is filled with a liquid of density ρ. Air is blown into arm A until the liquid levels in the two arms are shown in Fig. 11.28. The blowing is then stopped. The initial restoring force is F. When the liquid comes to rest, the loss in potential energy is U. Then

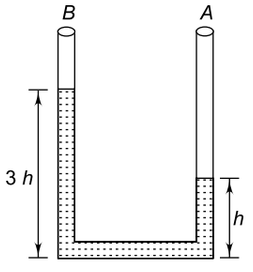


Fig. 11.28

(a) $F = 2a\rho gh$ (b) $F = a\rho gh$
(c) $U = 2a\rho gh^2$ (d) $U = a\rho gh^2$

19. A uniform cylinder of density ρ and cross-sectional aera A floats in equilibrium in two non-mixing liquids of densities ρ_1 and ρ_2 as shown in Fig.11.29. The length of the part of the cylinder in air is h and the lengths of the part of cylinder immersed in the liquid are h_1 and h_2 as shown in the figure.

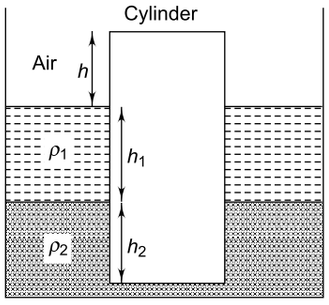


Fig. 11.29

(a) $h = h_1\left(\frac{\rho_1}{\rho}+1\right) - h_2\left(\frac{\rho_2}{\rho}+1\right)$

(b) $h = h_1\left(\frac{\rho_1}{\rho}-1\right) + h_2\left(\frac{\rho_2}{\rho}-1\right)$

(c) The cylinder is depressed in such a way that its top surface is just covered by the liquid of density ρ_1 and then released. The restoring force acting on the cylinder is $F = hA\rho_2 g$.

(d) In choice (c) above, the acceleration of the cylinder is $a = \frac{h(\rho_1+\rho_2)g}{(h+h_1+h_2)\rho}$

IIT, 2002

20. A container of width $2a$ is filled with a liquid. A thin wire of mass per unit length μ is gently placed on the middle of the surface as shown in Fig.11.30. As a result the liquid surface is depressed by a distance y. The surface tension of liquid is given by

(a) $\sigma = \frac{\mu g}{2\cos\theta}$

(b) $\sigma = \frac{\mu g}{2\sin\theta}$

(c) If $y << a$, then $\sigma = \frac{\mu g a}{2y}$

(d) If $y << a$, then $\sigma = \frac{\mu g a}{y}$

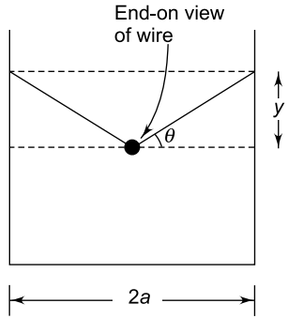


Fig. 11.30

IIT, 2004

21. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in Fig. 11.31 with a tension in the string. The arrangement is possible only if

(a) $d_A < d_F$ (b) $d_B > d_F$
(c) $d_A > d_F$ (d) $d_A + d_B = 2d_F$

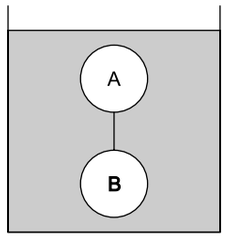


Fig. 11.31

IIT, 2011

ANSWERS AND SOLUTIONS

1. The correct choices are (a), (c) and (d).
2. The correct choices are (a), (b) and (c).
3. The correct choices are (a), (b) and (d).
4. Choice (a) is incorrect. A body will sink if its weight is greater than the upthrust. Choice (b) is correct. Choice (c) is incorrect. Choice (d) is correct. The density of hydrogen is less than that of air. Therefore, initially, the balloon rises because the upthrust due to air is greater than the weight of the hydrogen–filled balloon. The density of air decreases as we go up. Therefore, the upthrust on the balloon also decreases. It will stop rising when it attains a height at which the upthrust becomes equal to its weight.
5. Due to the upward force of buoyancy on the block exerted by the liquid, the apparent weight of the block

will be less than 2 kg. Hence choice (c) is correct. The hanging block exerts a downward force on the liquid (and the beaker) equal in magnitude to the upward buoyant force. Therefore, balance *B* will read more than 5 kg. Hence choice (b) is also correct.

6. Statements (a), (b) and (c) are wrong.

7. Let M and m be the masses of the boat and the stones respectively. The volume of the water displaced by the boat and the stones in it is

$$V = \frac{M+m}{\rho_w} \qquad (1)$$

where ρ_w is the density of water.

When the stones are unloaded into water, the volume of water displaced by the boat will be

$$V_b = \frac{M}{\rho_w}$$

If ρ_s is the density of the stones, the volume of water displaced by the stones will be

$$V_s = \frac{m}{\rho_s}$$

$\therefore$ Total volume of water displaced, in this case, is

$$V' = V_b + V_s$$

$$= \frac{M}{\rho_w} + \frac{m}{\rho_s} = \frac{1}{\rho_w}\left(M + \frac{m\rho_w}{\rho_s}\right) \qquad (2)$$

Since $\rho_s > \rho_w$, it follows from Eqs (1) and (2) that $V' < V$. Thus the volume of water displaced when the stones are unloaded into water is less than the volume of water displaced when the stones were in the boat. Hence the level of water in the tank will fall when the stones are unloaded into water. Hence the incorrect choices are (a), (b) and (d).

8. Figure 11.32 (a) shows a liquid of density ρ_l contained in a beaker filled up to level *A*. When a block of ice of mass m floats in the liquid, let the level of liquid rise to *B* as shown in Fig. 11.32 (b). *AB* is the increase in the level of the liquid. If V is the volume of the liquid displaced by the block of ice, then from the principle of floatation,

weight of ice block = weight of the liquid displaced

i.e. $mg = V\rho_l g$ or $V = \frac{m}{\rho_l}$ (1)

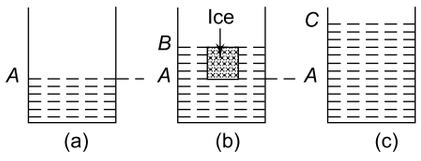


Fig. 11.32

When the ice melts completely, let the level of liquid + water formed be at *C* as shown in Fig. 11.32 (c). The volume contained in the height *AC* is due to the ice converted into water. Let this volume be V'. Then

$$V' = \frac{m}{\rho_w} \qquad (2)$$

where ρ_w is the density of water. From (1) and (2), we have

$$\frac{V'}{V} = \frac{\rho_l}{\rho_w}$$

Given $\frac{\rho_l}{\rho_w} = 1.2$. Hence $V' > V$. Hence the level of the liquid in the beaker will rise. If the liquid in the beaker has density less than that of water, then $V' < V$. Hence, in this case, the level of the liquid will fall. If the liquid in the beaker were water ($\rho_l = \rho_w$), $V' = V$. Hence, in this case, the level of the liquid would remain unchanged. Hence the correct choice are (b) and (c).

9. Let the mass of ice be m_1 and the mass of stone be m_2. The mass of the displaced water is equal to $(m_1 + m_2)$. If ρ is the density of water, the volume of water displaced is

$$V = (m_1 + m_2)/\rho$$

When the ice melts, additional volume of water obtained is m_1/ρ. The stone sinks in water, and displaces a volume of water equal to its own volume, which is m_2/ρ_s where ρ_s is the density of stone. Thus the total volume of extra water is

$$V' = \frac{m_1}{\rho} + \frac{m_2}{\rho_s}$$

Since $\rho_s > \rho$, $\frac{1}{\rho_s} < \frac{1}{\rho}$

$\therefore$ $V' < V$. Therefore, level of water in beaker decreases.

10. When the beaker, with the block floating in the liquid, is falling freely under gravity, both its weight and upthrust will be zero. Hence the block will float with any fraction of its volume submerged. Thus choice (a), (b) and (d) are wrong.

11. For a body falling freely under gravity, the effective value of g is zero. Hence the upthrust is zero. Thus choice (b), (c) and (d) are wrong.

12. Volume of steel block = $10 \times 10 \times 10 = 10^3$ cm^3
$= 10^{-3}$ m^3

(a) Let h_1 be the height of the block above the mercury surface.
Volume of mercury displaced

$= (0.1 - h_1) \times 0.1 \times 0.1 \text{ m}^3$

$\therefore$ Weight of mercury displaced

$= (0.1 - h_1) \times 0.1 \times 0.1 \times 13.6 \times 10^3\ g$ newton

This must be equal to the weight of block which is

$7.8 \times 10^3 \times 10^{-3} \times g$ newton $= 7.8 \times g$ newton

$\therefore\ (0.1 - h_1) \times 0.1 \times 0.1 \times 13.6 \times 10^3 \times g = 7.8 \times g$

which gives $h_1 = 0.0426$ m $= 4.26$ cm

(b) Let h_2 be the height of the water column required to just submerge the steel block. Thus

weight of the block = weight of water displaced + weight of mercury displaced.

i.e. $7.8 \times g = h_2 \times 0.1 \times 0.1 \times 1000 \times g + (0.1 - h_2) \times 0.1 \times 0.1 \times 13.6 \times 10^3 \times g$ which

gives $h_2 = 0.046$ m $= 4.60$ cm. Hence the correct choice are (b) and (c).

13. When an ice cube of mass 1.5 kg is dropped into the bucket, the total mass of the system suspended from the spring balance will be 10 kg + 1.5 kg = 11.5 kg. Hence the balance will rad 11.5 kg.

Since the relative density of iron is 7.2, its density is 7.2×10^3 kg m^{-3}. Therefore, the volume of the iron piece is

$$V = \frac{\text{mass}}{\text{density}} = \frac{7.2 \text{ kg}}{7.2 \times 10^3 \text{ kg m}^{-3}} = 10^{-3} \text{ m}^3$$

Now, volume of iron piece under water is $\frac{V}{2}$, which is the volume of water displaced by it. Hence

weight of water displaced

= volume displaced × density of water × g

$= \frac{V}{2} \times 1000 \times g$ newton

$= \frac{1}{2}\ g$ newton = weight of 0.5 kg

($\because\ V = 10^{-3}$ m^3)

This is the buoyant force exerted by water on the iron piece. Hence the iron piece will exert an equal force on water in the downward direction, which will increase the reading of the balance by 0.5 kg. Hence, the spring balance will read 10 kg + 0.5 kg = 10.5 kg. Hence correct choice are (b) and (c).

14. Pressure at A and B is atmopheric. Hence choice (a) is correct. For a liquid at rest the pressure at the same horizontal level is the same. So choice (b) is also correct. Now, if P_0 is the atmospheric pressure,

$$P_C = P_0 + \rho_1 h_1\ g \text{ and } P_D = P_0 + \rho_2\ h_2\ g$$

Since $P_C = P_D$, $\rho_1 h_1 = \rho_2 h_2$ or $\frac{h_1}{h_2} = \frac{\rho_2}{\rho_1}$.

Since $h_1 > h_2$; $\rho_1 < \rho_2$. Hence the correct choices are (a), (b) and (d).

15. Volume of block $V = \frac{M}{\rho}$ where ρ = density of block. Therefore, volume of water displaced = V.

Weight of water displaced is $W' = V\rho_w g = \frac{M\rho_w}{\rho} g$

$= \frac{Mg}{n}$ where ρ_w is the density of water and $n = \rho / \rho_w$. Hence choice (a) is correct.

Apparent loss of weight $= Mg - \frac{Mg}{n} = Mg\left(1 - \frac{1}{n}\right)$

which is choice (b).

If L_1 is the original length of the unloaded wire, then Young's modulus is $Y = \frac{F}{A}\frac{L}{\Delta L}$

Before the block immersed in water, $\Delta L = (L - L_1)$ and $F = Mg$. Hence

$$Y = \frac{MgL_1}{A(L - L_1)} \quad (1)$$

After the block is immersed in water, $\Delta L = (L - L_1 - x)$ and $F = Mg\left(1 - \frac{1}{n}\right)$. Hence

$$Y = \frac{Mg\left(1 - \frac{1}{n}\right)L_1}{A(L - L_1 - x)} \quad (2)$$

Equating (1) and (2) and simplyfing, we get $L_1 = L - nx$.

Hence the correct choices are (a), (b), and (c).

16. Relative density of metal

$$= \frac{\text{weight in air}}{\text{loss of weight in water}} = \frac{2.4}{2.4 - 2.0} = 6$$

Relative density of liquid

$$= \frac{\text{loss of weight in liquid}}{\text{loss of weight in water}}$$

$$= \frac{2.4 - 1.9}{2.4 - 2.0} = \frac{0.5}{0.4} = 1.25$$

Hence the correct choices are (a) and (d).

17. The weight of air above the liquid surface decreases. Hence P and F both will decrease. Since the volume of the liquid does not change, the level of the liquid will remain unchanged. Hence the correct choice are (b) and (d).

18. Since the liquid column in left arm is higher by $2h$ that in the right arm, the mass of this column of liquid

is $m = 2a\rho g$. The weight mg provides the restoring force of magnitude $F = mg = 2a\rho gh$. Hence choice (a) is correct. The liquid column will oscillate for some time and will eventually come to rest due to viscous effects. When this happens, the height of liquid column in each arm will be $2h$. Therefore, change in level in each arm = h. Hence loss of P.E. = mgh = $(a\rho h)gh = a\rho gh^2$. Hence the correct choice is (d).

19. From the principle of floatation,
weight of cylinder = net upthrust

$$\Rightarrow (h + h_1 + h_2)\, A\rho g = \rho_1 h_1 A + \rho_2 h_2 A$$

Solving for h, we find that choice (a) is wrong and choice (b) is correct.

When the cylinder is depressed such that its top surface is at the level of the liquid of density ρ_1, then the length of the cylinder immersed in ρ_1 remains equal to h_1 but the length of the cylinder immersed in ρ_2 increases by h. The extra upthrust due to this additional immersion in liquid of density ρ_2 is $F = hA\rho_2 g$ which provides the restoring force. So choice (c) is correct. Now total mass of cylinder is $M = (h + h_1 + h_2)\ \rho A$.

$$\therefore \text{Acceleration } a = \frac{F}{M} = \frac{h A \rho_2 g}{(h + h_1 + h_2)\rho A}$$

$$= \frac{h\rho_2 g}{\rho(h + h_1 + h_2)}$$

Hence choice (d) is wrong.

20. Let σ be the surface tension of the liquid and l be the length of the wire. Weight of wire is

$$W = \mu l g$$

and force due to surface tension is $f = \sigma l$. The forces acting on the wire are shown in Fig. 11.33.

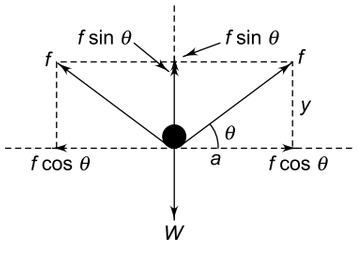


Fig. 11.33

The horizontal components $f \cos\theta$ cancel each other but the vertical components $f \sin\theta$ add up. Therefore, for equilibrium, we have

$$2f \sin\theta = W$$

or $$2\sigma l \sin\theta = \mu l g$$

or $$\sigma = \frac{\mu g}{2\sin\theta} \qquad (1)$$

It is clear from the diagram that

$$\tan\theta = \frac{y}{a}$$

If $y << a$, angle θ will be very small and $\tan\theta \simeq \sin\theta$ where θ is in radian. Thus

$$\sin\theta = \frac{y}{a} \qquad (2)$$

Using (2) in (1), we get

$$\sigma = \frac{\mu g a}{2y}$$

Hence the correct choices are (b) and (c).

21. Let V be the volume of each sphere and let T be the tension in the string.
Buoyant force on sphere A is $U_A = d_F Vg$
Buoyant force on sphere B is $U_B = d_F Vg$
Weight of A is $W_A = d_A Vg$
Weight of B is $W_B = d_B Vg$
The free body diagrams of A and B are as follows. (see Fig. 11.34)
For equilibrium,

$$U_A = T + W_A \quad \text{and} \quad U_B + T = W_B$$

i.e. $$d_F Vg = T + d_A Vg \qquad (i)$$

and $$d_F Vg + T = d_B Vg \qquad (ii)$$

From Eq. (i) $d_F = \frac{T}{Vg} + d_A$. Hence $d_F > d_A$. So choice (a) is correct.

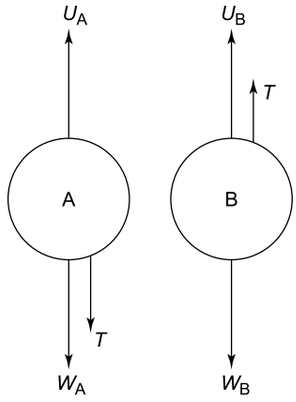


Fig. 11.34

From Eq. (ii) $d_B = \frac{T}{Vg} + d_F$. Hence $d_B > d_F$. So choice (b) is also correct.

Eliminating T from Eqs. (i) and (ii) we get
$2d_F = d_A + d_B$, which is choice (d).
So the correct choices are (a), (b) and (d).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage

Passage I

The earth's atmosphere is a mixture of gases such as nitrogen, oxygen, tiny amounts of carbon dioxide and traces of other gases. Because of the pull of gravity, these gases tend to accumulate near the surface of the earth. The weight of these gases causes an atmospheric pressure of about 10^5 Pa on the surface of the earth. Because the density of air decreases with height (h) above the surface of the earth, the atmospheric pressure (P) decreases with h. In fact P varies with h as

$$P = P_0\ e^{-h/h_0}$$

where P_0 = atmospheric pressure at sea-level ($h = 0$) and h_0 is a constant.

1. The dimensional formula for constant h_0 is
(a) $[M^0L^0T^0]$ (b) $[M^0L\ T^0]$
(c) $[M\ L^0T^0]$ (d) none of these

2. The pressure gradient dP/dh is proportional to ρ^n where ρ is the density of air. The value of n is
(a) zero (b) $\frac{1}{2}$
(c) 1 (d) 2

3. Which of the graphs shown in Fig 11.35. represents the variation of P with h?

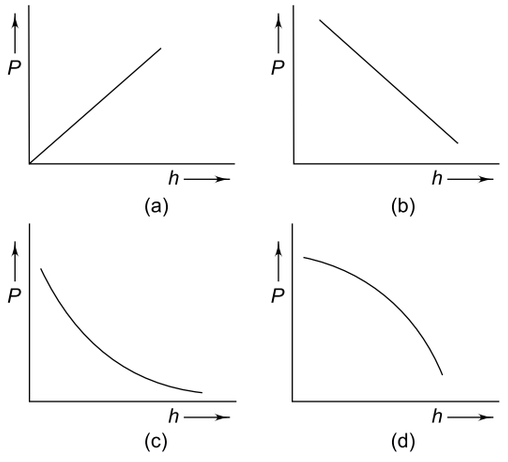


Fig. 11.35

4. The height at which the atmospheric pressure is half its value at sea level is
(a) $h = h_0$ (b) $h = 2h_0$
(c) $h = h_0 \log_e(2)$ (d) $h = h_0e^{-1/2}$

SOLUTION

1. Since the exponent is dimensionless the dimension of h_0 is the same as that of h. Hence the correct choice is (b).

2. $\frac{dP}{dh} = -\frac{P_0}{h_0}e^{-h/h_0} = -\frac{P}{h_0}$

For a given mass m of a gas at a constant temperature, $PV = k$ (constant) giving $P = \frac{k}{V} = \frac{k\rho}{m}$

$(\because\ \rho = m/V)$

Thus $\frac{dP}{dh} = -\frac{k}{mh_0}\ \rho \propto \rho$. Hence the correct choice is (c).

3. P decreases exponentially with h. So the correct choice is (c).

4. $P = P_0/2$ at a value of h given by

$$\frac{P_0}{2} = P_0 e^{-h/h_0}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-h/h_0}$$

$$\Rightarrow \quad 2 = e^{h/h_0}$$

$$\Rightarrow \quad \log_e(2) = \frac{h}{h_0}$$

$$\Rightarrow \quad h = h_0 \log_e(2), \text{ which is choice (c)}$$

Questions 5 to 10 are based on the following passage

Passage II

If a body is immersed in a fluid, it exeperinces an upward force called the buoyant force. The magnitude of buoyant force depends on (a) the size or volume of the body and (b) the density of the fluid. How large is this buoyant force? The answer to this question was discovered by the Greek Philosopher and mathematician Archimedes (287 – 212 B.C.) and is known as Archimedes Principle which states that '*When a body is immersed wholly or partly in*

a fluid, it experiences an upthrust equal to the weight of the fluid displaced by it.' This principle holds for bodies of any shape and for fluids of even non-uniform density. The necessary condition for a body to float in a liquid is that the weight of the liquid displaced by the immersed portion of the body must be equal to the weight of the body.

5. A block of wood floats in a liquid in a beaker with 3/4ths of its volume submerged under the liquid. If the beaker is placed in an enclosure that is falling freely under gravity, the block will
(a) float with 3/4ths of its volume submerged
(b) float completely submerged
(c) float with any fraction of its volume submerged
(d) sink to the bottom.

6. A common hydrometer has a uniform stem graduated downwards from 0, 1, 2, … up to 10. When floating in pure water it reads 0 and in a liquid of relative density 1.5, it reads 10. What is the relative density of a liquid in which it reads 5?
(a) 1.15 (b) 1.20
(c) 1.25 (d) 1.30

7. A piece of copper having an internal cavity weighs 264 g in air and 221 g in water. The density of copper is 8.8 g cm^{-3}. What is the volume of the cavity?
(a) 12 cm^3 (b) 13 cm^3
(c) 14 cm^3 (d) 15 cm^3

8. A concrete sphere of radius R has a cavity of radius r which is packed with sawdust. The relative densities of concrete and sawdust are 2.4 and 0.3 respectively. For this sphere to float with its entire volume submerged under water, the ratio of the mass of concrete to the mass of sawdust will be
(a) 8 (b) 4
(c) 3 (d) zero

9. A cube floating in mercury contained in a beaker has one-third of its volume submerged in mercury. Water is poured into the beaker till the cube is completely covered. What fraction of the volume of the cube is now submerged in mercury? The relative density of mercury = 13.6.
(a) 0.38 (b) 0.28
(c) 0.18 (d) 0.08

10. A cubical block of density ρ floats completely immersed in two non-mixing liquids of densities ρ_1 and ρ_2 as shown in Fig. 11.36. The relation between ρ, ρ_1 and ρ_2 is

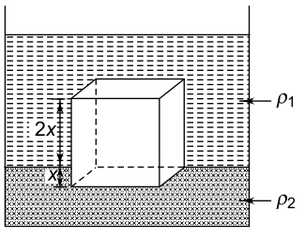


Fig. 11.36

(a) $\rho = \frac{1}{2}(\rho_1 + \rho_2)$

(b) $\rho = 2\rho_1 + \rho_2$

(c) $\rho = \frac{1}{3}(2\rho_1 + \rho_2)$

(d) $\rho = \frac{1}{3}(\rho_1 + 2\rho_2)$

SOLUTION

5. When the beaker, with the block floating in the liquid, is falling freely under gravity, both its weight and upthrust will be zero. Hence the block will float with any fraction of its volume submerged. Thus the correct choice is (c).

6. Let the reading of the hydrometer be x when it is dipped in water. When it is dipped in a liquid of relative density 1.5, its reading will be $(x - 10)$. Therefore,

$$x \times 1 \times g = (x - 10) \times 1.5 \times g$$

which gives $x = 1.5\,x - 15$ or $x = 30$. Let ρ be the relative density of the liquid in which the hydro meter reads $(x - 5)$, then

$$x \times 1 \times g = (x - 5) \times \rho \times g$$

or $$x = (x - 5)\,\rho$$

or $$\rho = \frac{x}{x-5} = \frac{30}{30-5} = 1.2$$

Hence the correct choice is (b).

7. Apparent loss of mass of copper = 264 – 221 = 43 g. This is the mass of water displaced by copper piece when it is immersed in water. Volume of copper piece with cavity = 43 cm^3. Volume of copper only is $\frac{m}{\rho} = \frac{264}{8.8} = 30\ cm^3$. Therefore, the volume of the cavity = 43 – 30 = 13 cm^3, which is choice (b).

8. Let m be the mass of concrete and ρ its density and let m' be the mass of sawdust and ρ' its density. Then (see Fig. 11.36)

$$m = \frac{4\pi}{3}(R^3 - r^3)\rho$$

and $m' = \frac{4\pi}{3} r^3 \rho'$

$$\therefore \quad \frac{m}{m'} = \frac{R^3 - r^3}{r^3} \cdot \frac{\rho}{\rho'} \qquad \text{(i)}$$

Since the entire volume $V = \frac{4\pi}{3} R^3$ of the sphere is submerged under water, we have, from the principle of flotation,

Weight of concrete + weight of sawdust = weight of volume V water displaced

or $mg + m'g = V\rho_0 g$ or $m + m' = V\rho_0$

where ρ_0 is the density of water. Thus

$$\frac{4\pi}{3}(R^3 - r^3)\rho + \frac{4\pi}{3} r^3 \rho' = \frac{4\pi}{3} R^3 \rho_0$$

or $(R^3 - r^3)d + r^3 d' = R^3 \qquad \text{(ii)}$

where $d = \rho/\rho_0$ and $d' = \rho'/\rho_0$ are the relative densities of concrete and sawdust respectively. Equation (ii), on simplification, gives

$$\frac{R^3}{r^3} = \frac{(d - d')}{(d - 1)}$$

or $$\frac{R^3}{r^3} - 1 = \frac{(d - d')}{(d - 1)} - 1$$

or $$\frac{R^3 - r^3}{r^3} = \frac{(1 - d')}{(d - 1)} \qquad \text{(iii)}$$

Using (iii) in (i) and noting that $\frac{\rho}{\rho'} = \frac{d}{d'}$, we have

$$\frac{m}{m'} = \frac{(1 - d')}{(d - 1)} \times \frac{d}{d'} = \frac{(1 - 0.3)}{(2.4 - 1)} \times \frac{2.4}{0.3} = 4$$

Hence the correct choice is (b).

9. Let the volume of the cube be V and ρ its density. Weight of the cube in air = $V\rho g$. Weight of mercury displaced when one-third of the volume is submerged = $\frac{1}{3} V\rho_m$, where ρ_m is the density of mercury. Then

$$V\rho g = \frac{1}{3} V\rho_m \text{ or } \rho = \frac{\rho_m}{3}$$

Let x be the fraction of the volume of the cube submerged in mercury when water is poured to cover the cube. Then, the fraction submerged in water = $(1 - x)$. Now

weight of cube in air = weight of mercury displaced + weight of water displaced

$$V\rho g = xV\rho_m g + (1 - x)\, V\rho_\omega g$$

or $$\rho = x\rho_m + (1 - x)\, \rho_\omega$$

where ρ_ω is the density of water. We have seen above that $\rho = \frac{\rho_m}{3}$. Therefore,

$$\frac{1}{3}\, \rho_m = x\rho_m + (1 - x)\, \rho_\omega$$

or $$\left(\frac{1}{3} - x\right) \frac{\rho_m}{\rho_\omega} = (1 - x)$$

Now $\frac{\rho_m}{\rho_\omega}$ = relative density of mercury = 13.6. Thus

$$\left(\frac{1}{3} - x\right) \times 13.6 = (1 - x)$$

which gives $x = 0.28$, Which is choice (b).

10. Each side of the block = $3x$. For floating, weight of the block = upthrust due to liquid of density ρ_1 + upthrust due to liquid of density ρ_2

$\therefore\ (3x)^3\, \rho g = (3x)^2 \times 2x \times \rho_1 g + (3x)^2 \times x \times \rho_2 g$

$$\Rightarrow \qquad \rho = \frac{1}{3}\, (2\rho_1 + \rho_2)$$

Thus the correct choice is (c).

Questions 11 to 13 are based on the following passage

Passage III

A tank has a cylindrical hole H of diameter $2r$ at its bottom as shown in Fig. 11.37. A cylindrical block B of diameter $4r$ and height h is placed on the hole H to prevent the flow of liquid through the hole. The liquid in the tank stands at a height h_1 above the top face of the block. The density of liquid is ρ and that of the block is $\rho/3$.

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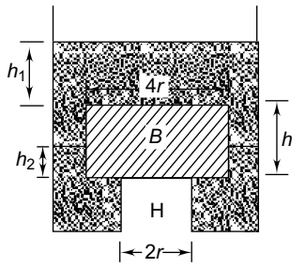

Fig. 11.37

11. If the liquid is gradually taken out of the tank, the height h_1 of the liquid surface above the top face of the block for which the block just begins to rise is

(a) $\frac{2h}{3}$ (b) $\frac{3h}{4}$

(c) $\frac{5h}{3}$ (d) $2h$

12. If the liquid level is further lowered so that it stands at a depth h_2 above the bottom face of the block as shown in the figure, then the maximum value of h_2 so that the block does not move is

(a) $\frac{4h}{9}$ (b) $\frac{h}{3}$

(c) $\frac{2h}{3}$ (d) $\frac{5h}{9}$

13. If the liquid level is lowered below h_2, then

(a) the block will never rise

(b) the block will start rising if $h_2 = \frac{h}{3}$

(c) the block will start rising if $h_2 = \frac{h}{4}$

(d) the block will start rising if $h_2 = \frac{h}{5}$

SOLUTION

11. Area of hole $= \pi (2r)^2 = 4\pi r^2 = A$. The area of the block $= \pi (4r)^2 = 16\ \pi r^2 = 4A$. The area of the lower face of block in contact with liquid $= 4A - A = 3A$. The block starts rising when upthrust = weight. (See Fig. 11.38)

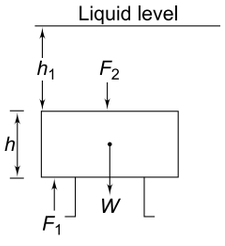

Fig. 11.38

The upthrust $U = F_1 - F_2$

$$= \rho g\ (h_1 + h)\ 3A - \rho g h_1 \times 4A$$

Weight of block $W = \frac{\rho}{3} gh \times 4A$

Now $U = W$, i.e.

$$\rho g(h_1 + h)3A - 4\rho g\ h_1 A = \frac{4}{3}\ \rho g\ hA$$

which gives $h_1 = \frac{5h}{3}$.

12. The block will not move if the upthrust U equals the weight W of the block (see Fig.11.39) i.e. if

$$\rho g h_2 \times 3A = \frac{\rho}{3} gh \times 4A$$

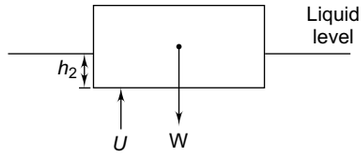

Fig. 11.39

which gives $h_2 = \frac{4h}{9}$. Hence the correct choice is (a).

13. If the liquid level is lowered below h_2, the upthrust will become less than the weight of the block. Therefore, the block will not rise. Hence the correct choice is (a).

SECTION IV

Assertion-Reason Type Questions

In the following questions, Statement-1(Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is true, statement-2 is true and Statement-2 is the correct explanation of Statement-1.

(b) Statement-1 is true, statement-2 is true but Statement-2 is *not* the correct explanation of Statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

1. Statement-1

A closed compartment containing gas is moving with some acceleration in the horizontal direction. If the effect of gravity is neglected, the pressure in the compartment will be higher in the rear side than in the front side.

Statement-2

Pascal's law holds only for a fluid at rest.

2. **Statement-2**
   A block of wood floats on water in a beaker with a fraction n of its volume submerged under water. If the system is taken to the moon (where the acceleration due to gravity is 1/6 of that on earth), the fraction of the volume of the block submerged under water will be $n/6$.
   Statement-2
   The buoyant force depends on the value of the acceleration due to gravity.
3. **Statement-3**
   A cylinder fitted with a movable piston contains a certain amount of a liquid in equilibrium with its vapour. The temperature of the system is kept constant with the help of a thermostat. When the volume of the vapour is decreased by moving the piston inwards, the vapour pressure does not increase.
   Statement-2
   Vapour in equilibrium with its liquid, at a constant temperature, does not obey Boyle's law.
4. **Statement-1**
   A body floats in a liquid with a fraction n of its volume above the surface of the liquid. If the system is taken to a planet where the acceleration due to gravity is greater than that on earth, the fraction n will decrease.
   Statement-2
   For floatation, the weight of the body is equal to the weight of the liquid displaced.
5. **Statement-1**
   In still air, a hydrogen-filled balloon rises up to a certain height and then stops rising.
   Statement-2
   The upthrust depends on the density of hydrogen relative to that of air.
6. **Statement-1**
   It is easier to move our arms and legs when our body is immersed in water than in air.
   Statement-2
   The average density of our body is greater than that of water.
7. **Statement-1**
   The excess pressure inside a liquid drop of radius R is $2\sigma/R$, where σ is the surface tension the liquid and excess pressure in side a bubble of radius R is $4\sigma/R$..
   Statement-2
   A bubble has two surfaces in contact with vapour whereas a drop has one surface in contact with vapour.
8. **Statement-1**
   The total pressure inside an air bubble of radius R at a depth h below the surface of a lake is $\left(P_0 + \frac{4\sigma}{R} + h\rho g\right)$ where ρ is the density of water and P_0 is the atmospheric pressure.
   Statement-2
   Total perssure = atmospheric pressure + excess pressure inside the bubble + hydrostatic pressure.

SOLUTION

1. The correct choice is (b), Due to acceleration of the compartment in the forward direction, the frequency of collisions of molecules of the gas will be highter at the rear face than at the front face.
2. The correct choice is (d). Although the buoyant force on the block depends on the value of g, the fraction of the volume of the block under a liquid is independent of g and depends on the density of the block relative to that of the liquid.
3. The correct choice is (a). If the valume of the vapour is decreased, at constant temperature, a part of the vapour will condense into liquid such that the vapour pressure remains unchanged.
4. The correct choice is (d). Although the upthrust on the body depends on the value of acceleration due to gravity, the fraction of its volume submerged under liquid is independent of g and depends on the density of the body relative to that of the liquid. Therefore, the fraction n remains the same.
5. The correct choice is (a). Initially, the balloon rises because the upthrust due to air is greater than the weight of the hydrogen-filled balloon. As the balloon rises up, the density of air decreases, therefore, the upthrust on the balloon also decreases. It will stop rising when it attains a height at which the upthrust becomes equal to its weight.
6. The correct choice is (c). Since the density of water is greater than that of the air, the upthrust on our limbs is greater in water than in air. Therefore, the apparent weight of the limbs is less in water than in air.
7. The correct choice is (a).
8. The correct choice is (d). Because a bubble inside water has only one surface in contact with vapour, total pressure
$$= P_0 + \frac{2\sigma}{R} + h\rho g$$

12 Chapter Hydrodynamics (Bernoulli's Theorem and Viscosity)

REVIEW OF BASIC CONCEPTS

12.1 VISCOSITY

When a fluid flows , there exists a relative motion between the layers of the fluid. Internal force acts which destroys this relative motion.This force is called viscous force. The viscous force between two layers of a fluid is given by

$$F = -\eta A \frac{dv}{dx}$$

where A is the area of the layer, $\frac{dv}{dx}$ is the velocity gradient and η is called the coefficient of viscosity of the fluid.

The SI unit of η is Nsm^{-2} which is called poiseuilli (Pl) or pascal second (Pa–s). The dimensional formula of η is $[ML^{-1} T^{-1}]$.

12.2 STOKES' LAW

The viscous force experienced by a small spherical body of radius r moving with a small velocity v through a homogeneous fluid of coefficient of viscosity η is given by

$$F = 6\pi\eta rv$$

This relation is called Stokes' law.

12.3 TERMINAL VELOCITY

If a body is released in a viscous fluid, it is accelerated due to gravity and its velocity begins to increase. Hence viscous force on it also increases. A stage is reached when the velocity is such that the viscous force F becomes equal to ($W - U$), where W is the weight of the body and U is the upthrust (Fig. 12.1). Then no net force acts on the body and it falls with a constant velocity called the terminal velocity (v_t). For a speherical body of radius r and density ρ falling in a fluid of density σ and coefficient of viscosity η, the terminal velocity is given by

$$v_t = \frac{2(\rho - \sigma)r^2 g}{9\eta}$$

For a body moving in a given fluid, $v_t \propto r^2$

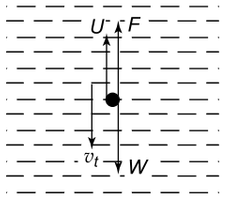

Fig. 12.1

NOTE

For a liquid the coefficient of viscosity decreases with increase in temperature whereas for a gas it increases with increase in temperature. Hence the terminal velocity of a body falling in a liquid increases with increase in temperature.

12.4 POISEUILLI'S FORMULA

The volume of a liquid flowing per second through a capillary tube of radius r when its ends are maintained at a pressure difference p is given by

$$Q = \frac{\pi p r^4}{8\eta l}$$

Where l is the length of the tube and η is the coefficient of viscosity of the liquid.

Capillaries Connected in Series

If two capillaries of lengths l_1 and l_2 and radii r_1 and r_2 are connected in series across constant pressure difference p, then the fluid resistance R is given by

$$R = R_1 + R_2 = \frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}$$

As the volume of liquid flowing per second is the same through both capillaries.

$$Q = Q_1 = Q_2 = \frac{p}{R_1 + R_2}$$

If p_1 and p_2 are the pressure differences across individual capillaries, then

$$P = p_1 + p_2$$

Capillaries Connected in Parallel

If two capillaries are connected in parallel across constant pressure difference p, then the fluid resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

where $$R_1 = \frac{8\eta l_1}{\pi r_1^4} \text{ and } R_2 = \frac{8\eta l_2}{\pi r_2^4}$$

The volume of liquid flowing per second through the capillary of radius r_1 is

$$Q_1 = \frac{p}{R_1}$$

For the capillary of radius r_2, we have

$$Q_2 = \frac{p}{R_2}$$

EXAMPLE 12.1

A thin rectangular metal plate 10 cm × 5 cm is lying on a layer of glycerine of thickness 1 mm and coefficient of viscosity 0.8 Pa–s. Calculate the horizontal force needed to move the plate with a speed of 15 cm s^{-1} on the liquid.

SOLUTION

Area of plate (A) = 10 cm × 5 cm = 50 cm^2 = 50 × 10^{-4} m^2.

The plate is in contact with free surface of the liquid and the liquid in contact with the fixed surface below the layer is at rest. Hence, Δv = 15 cm s^{-1} = 15 × 10^{-2} ms^{-1} and Δx = 1 mm = 10^{-3} m . Therefore

Velocity gradient, $\frac{\Delta v}{\Delta x} = \frac{15 \times 10^{-2}}{10^{-3}} = 150 \text{ s}^{-1}$

The required force is

$$F = \eta A \frac{\Delta v}{\Delta x} = 0.8 \times (50 \times 10^{-4}) \times 150 = 0.6 \text{ N}$$

EXAMPLE 12.2

In Millikan's oil drop experiment, an oil drop of radius 0.02 mm and density 1.2×10^3 kg^{-3} falls in air. The coefficient of viscosity of air at the temperature of the experiment is 1.8×10^{-5} Pa–s. (a) Find the terminal velocity of the drop. (b) What is the viscous force on the drop at this velocity? Neglect buoyancy due to air.

SOLUTION

(a) If buoyancy is neglected ($\sigma = 0$), the terminal speed is

$$v_t = \frac{2\rho r^2 g}{9\eta}$$

$$= \frac{2 \times (1.2 \times 10^3)\,(0.02 \times 10^{-3})^2 \times 9.8}{9 \times (1.8 \times 10^{-5})}$$

$$= 5.8 \times 10^{-2} \text{ ms}^{-1} = 5.8 \text{ cm s}^{-1}$$

(b) Viscous force $F = 6\pi\eta r v_t$

$$= 6 \times 3.14 \times (1.8 \times 10^{-5}) \times (0.02 \times 10^{-3}) \times (5.8 \times 10^{-2})$$

$$= 3.9 \times 10^{-10} \text{ N}$$

EXAMPLE 12.3

Eight spherical rain drops of the same mass and radius are falling vertically through air with a terminal speed of 6 cm s^{-1}. If they coalesce to form one spherical drop, what will be its terminal speed?

SOLUTION

Let r be the radius of each small drop and R that of the big drop. Since the volume remains the same,

$$8 \times \left(\frac{4\pi}{3} r^3\right) = \frac{4\pi}{3} R^3 \Rightarrow R = 2r$$

Since terminal speed $\propto$ (radius)^2, if the radius is doubled, the terminal speed becomes four times = 6 cm s^{-1} × 4 = 24 cm s^{-1}

EXAMPLE 12.4

A body of relative density 5.0 is released from rest on the surface water filled to a height of 1.0 m in a tall cylinder. If viscous force is neglected, find the time

taken by the body to reach the bottom of the cylinder. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

Density of body (ρ) = 5000 kg m^{-3}

Density of water (σ) = 1000 kg m^{-3}

If V is the volume of the body, its mass (m) = $V\rho$

Weight of the body (W) = $mg = V\rho g$

Upthrust on the body (U) = $V\sigma g$

$\therefore$ Net downward force $F = W - U = (\rho - \sigma)\, Vg$

$$\therefore \text{ Acceleration } a = \frac{F}{m} = \frac{F}{\rho V} = \frac{(\rho - \sigma) Vg}{\rho V}$$

$$= \left(\frac{\rho - \sigma}{\rho}\right) g$$

From the relation $s = ut + \frac{1}{2} at^2$, we have

$$h = 0 + \frac{1}{2} at^2$$

$$\Rightarrow \qquad t = \sqrt{\frac{2h}{a}} = \left[\frac{2h\rho}{(\rho - \sigma)}\right]^{1/2}$$

$$= \left(\frac{2 \times 1.0 \times 5000}{(5000 - 1000) \times 10}\right)^{1/2}$$

$$= 0.5 \text{ s}$$

EXAMPLE 12.5

A vertical U-tube is filled with a liquid of density ρ = 1200 kg m^{-3} a shown in Fig. 12.2. The tube is rotated about a vertical axis with angular velocity ω such that the difference in levels of the liquid in the two arms is 25 cm. If x_1 = 0.6 m and x_2 = 0.4 m, find the value of ω. Take $g = 10\ \text{ms}^{-2}$.

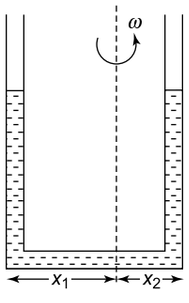

Fig. 12.2

SOLUTION

Consider a small element AB of the liquid of length dx at a distance x from the axis of rotation (Fig. 12.3).

Due to centripetal force, the liquid rises to height h_1 in the left arm and to a height h_2 in the right arm.

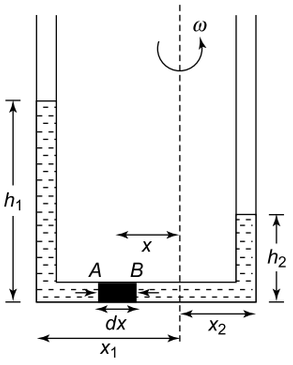

Fig. 12.3

Pressure at A is $p_1 = h_1 \rho g$

Pressure at B is $p_2 = h_2 \rho g$

Pressure difference $\Delta p = p_1 - p_2 = (h_1 - h_2)\, \rho g$

If a is the cross-sectional area of the tube, the net force on element AB is $F = a\, \Delta p = (h_1 - h_2)\, a\rho g$ (i)

Mass of element $dm = \rho a dx$

$\therefore$ Centripetal force is

$$F_e = \int_{x_2}^{x_1} dm\, x\omega^2 = \omega^2 \rho a \int_{x_2}^{x_1} x dx$$

$$= \frac{1}{2} \omega^2 \rho a\, (x_1^2 - x_2^2) \qquad \text{(ii)}$$

Equating (i) and (ii), we get

$$h_1 - h_2 = \frac{\omega^2}{2g} (x_1^2 - x_2^2)$$

$$\Rightarrow \qquad 0.25 = \frac{\omega^2}{2 \times 10} [(0.6)^2 - (0.4)^2]$$

$$\Rightarrow \qquad \omega = 5 \text{ rad s}^{-1}$$

EXAMPLE 12.6

Glycerine flows steadily through a horizontal pipe of length 1.2 m and diameter 2 cm. If the amount of glycerine collected at one end is 3.14×10^{-3} kg s^{-1}, find the pressure difference between the ends of the pipe. Density of glycerine = 1.2×10^3 kg m^{-3}. Viscosity of glycerine = 0.8 N sm^{-2}

SOLUTION

Given l = 1.2 m, r = 1 cm = 1×10^{-2} m, η = 0.8 Nsm^{-2}

Volume of glycerine flowing per second is

$$Q = \frac{3.14 \times 10^{-3}\ \text{kg s}^{-1}}{1.2 \times 10^{3}\ \text{kg m}^{-3}}$$

$$= \frac{3.14 \times 10^{-6}}{1.2}\ \text{m}^3\text{s}^{-1}$$

From Poiseuilli's formula, we have

$$p = \frac{8\eta l Q}{\pi r^4}$$

Substituting the above values, we get $p = 6.4 \times 10^2$ Pa.

12.5 STREAMLINE OR LAMINAR FLOW

Streamline or Laminar flow is the flow in which every particle of the liquid follows the same path and has exactly the same velocity in magnitude and direction as that of the preceding particle at a given point in the flow.

The actual path followed by the particles in a regular flow is called a streamline, which can be straight or curved. The tangent at a point on a streamline gives the direction of the liquid flow at that point.

12.6 CRITICAL VELOCITY AND REYNOLD'S NUMBER

The liquid flow remains steady or streamline if its velocity does not exceed a limiting value called the critical value, which is given by

$$v_c = \frac{k\eta}{\rho r}$$

where η = coefficient of viscosity of the liquid, ρ = density of the liquid, r = radius of the pipe in which the liquid flows and k is a dimensionless constant called Reynold's number.

If the velocity of the liquid exceeds the critical velocity, the flow becomes irregular causing the liquid to flow in a disorderly fashion. Such a flow is called turbulent flow.

The value of k is usually very high. If k is less than 2000, the flow is streamline. If the value of k exceeds 2000, the flow becomes turbulent.

12.7 EQUATION OF CONTINUITY OF FLOW

If a_1 and a_2 are the areas of cross-section at two sections of a tube of a variable cross-section and v_1 and v_2 are the velocities of flow crossing these sections, then

$$a_1 v_1 = a_2 v_2 \text{ or } av = \text{constant}$$

This means that smaller the area of cross-section, higher is of the liquid flow and *vice versa*. This is called equation of continuity of flow and it holds only if the flow is streamline.

12.8 BERNOULLI'S THEOREM

Bernoulli's theorem states that *the total energy of an incompressible and non-viscous liquid in a streamline flow remains constant throughout the flow; the total energy being the sum of pressure energy, potential energy and kinetic energy of the liquid.*

$$PV + mgh + \frac{1}{2}mv^2 = \text{constant}$$

or $$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constrant} \qquad \left(\because \rho = \frac{m}{V}\right)$$

12.9 VELOCITY OF EFFLUX

A liquid is filled up to a height H in a vessel which has a small hole at a depth h below the surface of the liquid (Fig. 12.4).

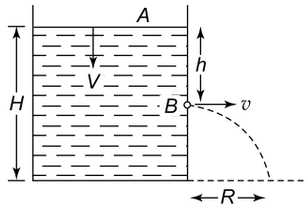

Fig. 12.4

When the hole is unplugged, the velocity v with which the liquid comes out of the hole is called the velocity of efflux. Let V be the velocity with which the free surface of the liquid falls in the vessel. Applying Bernoulli's theorem to points A and B

$$P_A + \frac{1}{2}\rho V^2 + \rho gH = P_B + \frac{1}{2}\rho v^2 + \rho g\,(H - h)$$

Since $P_A = P_B = P_0$ (atmospheric pressure) and $AV = av$ (equation of continuity) where A = cross-sectional area of the vessel and a = cross-sectional area of hole, we have

$$P_0 + \frac{1}{2}\rho\frac{a^2v^2}{A^2} + \rho gH = P_0 + \frac{1}{2}\rho v^2 + \rho g\,(H - h)$$

which gives $$v = \left[\frac{2gh}{\left(1 - \frac{a^2}{A^2}\right)}\right]^{1/2}$$

Since $A >> a$, $v = \sqrt{2gh}$

The time taken by the liquid emerging from the hole to hit the ground is

$$t = \sqrt{\frac{2(H-h)}{g}}$$

$\therefore$ Horizontal Range $R = v\,t = 2\sqrt{h(H-h)}$

NOTE

(1) Horizontal range is maximum when $h = \frac{H}{2}$ and $R_{max} = H$.
(2) The horizontal range is the same if the hole is at a depth h below the surface of water or at a height h above the bottom of the vessel.

EXAMPLE 12.7

Water flows through a horizontal non-uniform pipe at the rate of 31.4 litre/min. Find the velocity of flow of water at the section of the pipe where the diameter is 2 m.

SOLUTION

Volume of water flowing per second is

$$Q = 31.4 \text{ litre/min} = \frac{31.4 \times 10^{-3} \text{ m}^3}{60 \text{ s}}$$

$$= \frac{3.14 \times 10^{-4}}{0.6} \text{ m}^3 \text{ s}^{-1}$$

Area of cross-section $A = \pi r^2 = 3.14 \times (1 \times 10^{-2})^2 \text{ m}^2$

$$\therefore \text{ Velocity of flow} = \frac{Q}{A} = \frac{3.14 \times 10^{-4}}{0.6 \times 3.14 (1 \times 10^{-2})^2}$$

$$= 1.67 \text{ ms}^{-1}$$

EXAMPLE 12.8

Water flows through a horizontal pipe of diameter 2 cm at a speed of 3 cm s^{-1}. The pipe has a nozzle of diameter 0.5 cm at its end. Find the speed of water emerging from the nozzle.

SOLUTION

$a_1 = \pi r_1^2$ and $a_2 = \pi r_2^2$. From continuity of flow,

$a_1 v_1 = a_2 v_2$, we have

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{\pi r_1^2}{\pi r_2^2} \times v_1$$

$$= \left(\frac{r_1}{r_2}\right)^2 v_1$$

$$= \left(\frac{1 \text{ cm}}{0.25 \text{ cm}}\right)^2 \times 3 \text{ cm s}^{-1} = 48 \text{ cm s}^{-1}$$

EXAMPLE 12.9

Find the maximum speed at which water should flow in a pipe of diameter 2 cm so that the flow remains laminar at 20°C. Viscosity of water at 20°C = 0.001 N s m^{-2}.

SOLUTION

The maximum value of Reynold's number for flow to be laminar is 2000. Hence

$$v_{max} = \frac{k_{max} \eta}{\rho r}$$

$$= \frac{2000 \times 0.001}{1000 \times (1 \times 10^{-2})} = 0.2 \text{ m s}^{-1}$$

$$= 20 \text{ cm s}^{-1}$$

EXAMPLE 12.10

Find the velocity of efflux of water from an orifice near the bottom of a tank of height 1.25 m full of water. Take $g = 10 \text{ ms}^{-2}$

SOLUTION

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1.25}$$

$$= 5 \text{ ms}^{-1}$$

EXAMPLE 12.11

Figure 12.5 shows a stream of water emerging from the opening of a tap. As the water falls through a height $h = PQ$, the cross-sectional area of the stream decreases from A to a. Obtain the expression for the rate of flow of water through the opening of the tap.

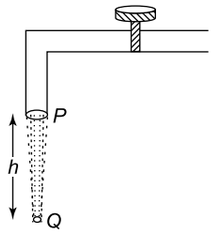

Fig. 12.5

SOLUTION

Let V be the velocity of water at P and v at Q. From the equation of continuity, $AV = av$. From Bernoulli's theorem,

$$P_P + \rho g h + \frac{1}{2} \rho V^2 = P_Q + \frac{1}{2} \rho g (0) + \frac{1}{2} \rho v^2$$

Now $P_P = P_Q = P_0$ (atmospheric pressure). Therefore,

$$P_0 + \rho g h + \frac{1}{2} \rho V^2 = P_0 + \frac{1}{2} \rho v^2$$

$$\Rightarrow \quad V^2 = v^2 - 2gh \qquad \text{(i)}$$

From $AV = av$, $v = \frac{AV}{a}$. Using this in (i) we get

$$V = \left[\frac{2gha^2}{A^2 - a^2}\right]^{1/2}$$

$\therefore$ Rate of flow $= AV = aA\left[\frac{2gh}{A^2 - a^2}\right]^{1/2}$

EXAMPLE 12.12

A liquid of density ρ is filled in a vessel of cross-sectional area A. The vessel has a hole of cross-sectional area a at a depth h below the free surface of the liquid. When the hole is unplugged, the liquid comes out of the hole with a velocity v. The volume of liquid coming out of the hole per second is Q. The reaction force exerted on the vessel by the emerging liquid is (assume $A >> a$)

(a) $av^2\rho$ (b) $2agh\rho$ (c) ρvQ (d) $\rho Q\sqrt{2gh}$

SOLUTION

Velocity of efflux $(v) = \sqrt{2gh}$ ($\because A >> a$)

Mass of liquid emerging from the hole per second $= \rho av$

$\therefore$ Momentum of water emerging per second $= (\rho av) \times v = \rho av^2$

From Newton's law, the force exerted by the vessel on the emerging water is

F = rate of changes of momentum $= \rho av^2$

From Newton's third law, the reaction force (back) force exerted by the emerging liquid on the vessel is

$$F = \rho av^2 = \rho a \times 2gh = 2a\rho gh \quad (\because v = \sqrt{2gh})$$

Volume of liquid emerging from the hole per second is

$$Q = \frac{\rho av}{\rho} = av = a\sqrt{2gh}$$

$\therefore$ $F = \rho Qv = \rho Q\sqrt{2gh}$

Thus all the four choices are correct.

EXAMPLE 12.13

Water flows out of a hole of cross-sectional area a = 2 cm^2 from a big tank. The water flows at a rate Q = 3.6 litre/min. Find the torque exerted by the emerging water on the tube of length L about print O.

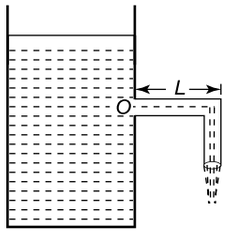

Fig. 12.6

SOLUTION

Velocity of efflux $v = \frac{Q}{a}$

Reaction force $F = \rho av^2 = \frac{\rho Q^2}{a}$

Rate of flow Q = 3.6 litre/min $= \frac{3.6 \times 10^{-3}}{60}$

$= 6 \times 10^{-5}$ m^3/s

$\therefore$ Torque about $O = F \times L$

$$= \frac{\rho Q^2 L}{a}$$

$$= \frac{10^3 \times (6 \times 10^{-5})^2 \times 0.14}{(2 \times 10^{-4})}$$

$= 2.52 \times 10^{-3}$ Nm

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. A tank of capacity 1000 litres is at a height of 30 m above the ground. A pump of power 1 kW situated on the ground is used to fill the tank with water using a pipe of diameter 2.4 cm. The time taken to fill the tank is (take g = 10 ms^{-2})

(a) 5 minutes (b) 10 minutes

(c) 20 minutes (d) 30 minutes

2. A rectangular tank is filled to the brim with water. It has a hole of diameter 0.5 cm at the bottom. When the hole is unplugged, the tank is emptied in time T. If the tank was half filled with water, it would be emptied in time

(a) $\frac{T}{\sqrt{2}}$ (b) $\frac{T}{2}$

(c) $\frac{T}{2\sqrt{2}}$ (d) $\frac{T}{4}$

3. If η is the coefficient of viscosity and σ is the surface tension, then the dimensions of $\frac{\sigma}{\eta}$ are the same as those of

(a) speed (b) momentum
(c) impulse (d) angular momentum

4. Eight identical spherical rain drops are falling down with terminal velocity v. If they combine to form one big spherical drop, its terminal velocity will be

(a) $\frac{v}{2}$ (b) $2v$
(c) $4v$ (d) $8v$

5. A large tank lying on a floor is filled with a liquid of density ρ. A small hole is made at a depth h below the surface of water in the tank. When the hole is unplugged, the horizontal range of the emerging liquid is R. The horizontal range is doubled if the pressure on the surface of water in the tank is increased by

(a) $h\rho g$ (b) $2\ h\rho g$
(c) $3h\rho g$ (d) $4\ h\rho g$

6. A rectangular tank full of water open at the top and having a base area A, is resting on a horizontal frictionless surface. It has two small holes, each of cross-sectional area a, on the opposite sides of the tank. The difference in the heights between the holes is h. The horizontal force which must be applied to the tank to keep at rest when the holes are unplugged is (ρ = density of water).

(a) $\rho\ g\ h\ A$ (b) $2\ \rho\ g\ h\ A$
(c) $\rho\ g\ h\ a$ (d) $2\ \rho\ g\ h\ a$

7. In a laminar flow of a liquid through a horizontal capillary tube of radius r, the rate of flow is Q when a pressure difference P is maintained between its ends. The rate of flow through another tube of the same length but radius $r/2$ when the pressure difference between its ends is $2P$ will be

(a) $8\ Q$ (b) $4\ Q$
(c) $\frac{Q}{4}$ (d) $\frac{Q}{8}$

8. Two capillary tubes A and B of the same radius r but of lengths $2l$ and l respectively are held horizontally. When a pressure difference P is maintained across A, the rate of flow of water through it is Q. If the tubes are connected in series and the same pressure difference P is maintained across the combination, the rate of flow of water through the combination will be

(a) Q (b) $\frac{2Q}{3}$
(c) $\frac{Q}{3}$ (d) $\frac{3Q}{4}$

9. An electric pump sends a liquid of density ρ through a horizontal pipe of cross-sectional area a with a speed v. The power of the pump is proportional to

(a) v (b) v^2
(c) v^3 (d) v^4

10. Water is flowing through a tube of radius r with a speed v. If this tube is joined to another tube of radius $r/2$, what is the speed of water in the second tube?

(a) $\frac{v}{4}$ (b) $\frac{v}{2}$
(c) $2\ v$ (d) $4\ v$

11. A small sphere of volume V falling in a viscous fluid acquires a terminal velocity v_t. The terminal velocity of a sphere of volume $8\ V$ of the same material and falling in the same fluid will be

(a) $\frac{v_t}{2}$ (b) v_t
(c) $2\ v_t$ (d) $4\ v_t$

12. A spherical steel ball released at the top of a long column of glycerine of length L, falls through a distance $L/2$ with accelerated motion and the remaining distance $L/2$ with a uniform velocity. If t_1 and t_2 denote the times taken to cover the first and second half and W_1 and W_2 the work done against gravity in the two halves, then

(a) $t_1 < t_2\ ;\ W_1 > W_2$ (b) $t_1 > t_2\ ;\ W_1 < W_2$
(c) $t_1 = t_2\ ;\ W_1 = W_2$ (d) $t_1 > t_2\ ;\ W_1 = W_2$

13. A cubical vessel of height 1 m is full of water. The work done in pumping water out of the vessel is

(a) 49 J (b) 98 J
(c) 4900 J (d) 9800 J

14. A solid iron ball and a solid aluminium ball of the same diameter are released together on a deep lake. Which ball will reach the bottom first?

(a) Aluminium ball
(b) Iron ball
(c) Both balls will reach the bottom at the same time
(d) The aluminium ball will never reach the bottom and will remain suspended in the lake

15. Two spheres of equal masses but radii R and $2R$ are allowed to fall in a liquid. The ratio of their terminal velocities is

(a) 1 : 4 (b) 1 : 2
(c) 1 : 32 (d) 2 : 1

16. A volume V of a viscous liquid flows per second due to a pressure head ΔP along a pipe of diameter d and length l. Instead of this pipe, a set of four pipes each of diameter $d/2$ and length $2l$ is connected to the same pressure head ΔP. The volume of the liquid flowing per second now is

(a) V (b) $\frac{V}{4}$

(c) $\frac{V}{8}$ (d) $\frac{V}{16}$

17. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of wa-ter and that the flow is steady. The cross-sectional area of the stream 0.15 m below the tap is (take $g = 10$ ms^{-2})

(a) 5.0×10^{-4} m^2 (b) 1.0×10^{-5} m^2

(c) 5.0×10^{-5} m^2 (d) 2.0×10^{-5} m^2

IIT, 1998

18. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both the holes are the same. Then R is equal to

(a) $\frac{L}{\sqrt{2\pi}}$ (b) $2\pi L$

(c) L (d) $\frac{L}{2\pi}$

IIT, 2000

19. Water stands at a depth H in a tank whose side walls are vertical. A hole is made in one of the walls at a height h below the water surface. The stream of water emerging from the hole strikes the floor at a distance R from the tank, where R is given by

(a) $R = \sqrt{h(H-h)}$ (b) $R = \sqrt{h(H+h)}$

(c) $R = 2\sqrt{h(H-h)}$ (d) $R = 2\sqrt{h(H+h)}$

20. In Q. 19, R is maximum if

(a) $h = \frac{H}{4}$ (b) $h = \frac{H}{3}$

(c) $h = \frac{H}{2}$ (d) $h = H$

21. A liquid is kept in a cylindrical vessel. When the vessel is rotated about its axis, the liquid rises at its sides. If the radius of the vessel is 0.05 m and the speed of rotation is 2 revolutions per second, the difference in the heights of the liquid at the centre and at the sides of the vessel will be (take $g = 10$ ms^{-2} and $\pi^2 = 10$)

(a) 2 cm (b) 4 cm

(c) 1 cm (d) 8 cm

22. A cylindrical tank of height H is completely filled with water. On its vertical side there are two tiny holes, one above the middle at a height h_1 and the other below the middle at a depth h_2. If the jets of water from the holes meet at the same point at the horizontal plane through the bottom of the tank then the ratio $\frac{h_1}{h_2}$ is

(a) 1 (b) 2

(c) 3 (d) 4

23. A water barrel having water upto a depth d is placed on a table of height h. A small hole is made on the wall of the barrel at its bottom. If the stream of water coming out of the hole falls on the ground at a horizontal distance 'R' from the barrel, then the value of d is

(a) $\frac{R^2}{h}$ (b) $\frac{4R^2}{h}$

(c) $\frac{R^2}{4h}$ (d) $\frac{h^2}{R}$

24. A liquid flows through two capillary tubes A and B connected in series. The length and radius of B are twice those of A. The ratio of the pressure difference across A to that across B is

(a) 8 (b) 4

(c) 2 (d) 1

25. A tiny sphere of mass m and density x is dropped in a tall jar of glycerine of density y. When the sphere acquires terminal velocity, the magnitude of the viscous force acting on it is

(a) $\frac{mgx}{y}$ (b) $\frac{mgy}{x}$

(c) $mg\left(1-\frac{y}{x}\right)$ (d) $mg\left(1+\frac{x}{y}\right)$

26. Two capillary tubes of the same length l and radii r and $2r$ are fitted to the bottom of a vessel with pressure head p in parallel with each other. What should be the radius of the single tube of the same length l that can replace the two so that the rate of flow is not affected?

(a) $(17)^{1/4}\, r$ (b) $17\, r$

(c) $8.5\, r$ (d) $\sqrt{17}\, r$

27. Tanks A and B open at the top contain two different liquids upto certain height in them. A hole is made to the wall of each tank at a depth h from the surface of the liquid. The area of the hole in A is twice that in B. If the liquid mass flux (i.e. rate of mass of liquid flowing per unit area) through each hole is equal, then the ratio of the densities of the liquids in A and B, is:

(a) $\frac{2}{1}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

28. A large container (with open top) of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area a in its side wall near the bottom. The container is keep on a smooth horizontal platform and contains a liquid of density ρ and mass m. If the liquid starts flowing out of the hole at time $t = 0$, the initial acceleration of the container is

(a) $\frac{ga}{A}$ (b) $\frac{gA}{a}$

(c) $\frac{2ga}{A}$ (d) $\frac{gA}{2a}$

29. In Q. 28 above, the velocity of the liquid when 75% of the liquid has drained out is

(a) $\sqrt{\frac{3mg}{4A\rho}}$ (b) $\sqrt{\frac{2mg}{A\rho}}$

(c) $2\sqrt{\frac{mg}{A\rho}}$ (d) $\sqrt{\frac{mg}{2A\rho}}$

30. A cylindrical tank having cross-sectional area A is filled with water to a height of 2.0 m. A circular hole of cross-sectional area a is opened at a height of 75 cm from the bottom. If $\frac{a}{A} = \sqrt{0.2}$, the velocity with which water emerges from the hole is ($g = 9.8$ ms^{-2}).

(a) 4.9 ms^{-1} (b) 4.95 ms^{-1}

(c) 5.0 ms^{-1} (d) 5.5 ms^{-1}

IIT, 2005

31. A syringe containing water is held horizontally with its nozzle at a height h above the ground as shown in Fig. 12.7. The cross-sectional areas of the piston and the nozzle are A and a respectively. The piston is pushed with a constant speed V. The horizontal range R of the stream of water on the ground is.

IIT, 2004

(a) $R = V\sqrt{\frac{2h}{g}}$ (b) $R = V\sqrt{\frac{g}{2h}}$

(a) $R = \frac{aV}{A}\sqrt{\frac{2h}{g}}$ (d) $R = \frac{AV}{a}\sqrt{\frac{2h}{g}}$

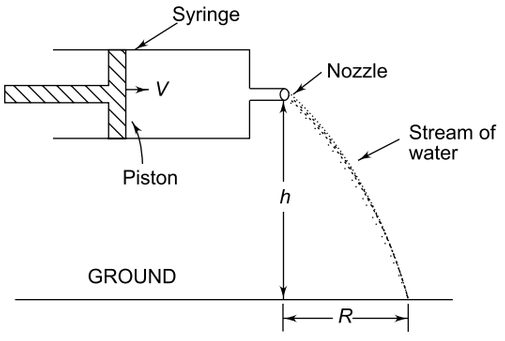

Fig. 12.7

32. A small metal sphere of radius r and density ρ falls from rest in a viscous liquid of density σ and coefficient of viscosity η. Due to friction heat is produced. The rate of production of heat when the sphere has acquired the terminal velocity is proportional to

(a) r^2 (b) r^3

(c) r^4 (d) r^5

IIT, 2004

33. A small spherical ball of radius r falls freely under gravity through a distance h before entering a tank of water. If, after entering the water, the velocity of the ball does not change, then h is proportional to

(a) r^2 (b) r^3

(c) r^4 (d) r^5

IIT, 1990

34. A wide vessel of uniform cross-section with a small hole in the bottom is filled with 40 cm thick layer of water and 30 cm thick layer of kerosene. The relative density of kerosene is 0.8. The inital velocity of flow of water streaming out of the hole is (take $g = 10$ ms^{-2})

(a) $\frac{2}{\sqrt{5}}$ ms^{-1} (b) $\frac{4}{\sqrt{5}}$ ms^{-1}

(c) $\frac{6}{\sqrt{5}}$ ms^{-1} (d) $\frac{8}{\sqrt{5}}$ ms^{-1}

ANSWERS

1. (b)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (d)
7. (d)	**8.** (b)	**9.** (c)	**10.** (d)	**11.** (d)	**12.** (d)
13. (c)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (a)
19. (c)	**20.** (c)	**21.** (a)	**22.** (a)	**23.** (c)	**24.** (a)
25. (c)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (d)	**30.** (c)
31. (d)	**32.** (d)	**33.** (c)	**34.** (d)		

SOLUTIONS

1. Work done against gravity is

$$W_1 = mgh = (\rho V) \times gh = \rho ghV$$

Work done against pressure difference is

$$W_2 = \Delta P \times V = h\rho gV$$

$\therefore$ Total work done $W = W_1 + W_2 = 2\,h\rho gV$

Power $P = \dfrac{W}{t}$. Therefore

$$t = \frac{W}{P} = \frac{2h\rho gV}{P}$$

$$= \frac{2\times(30)\times\left(10^3\right)\times(10)\times\left(10^3\times10^{-3}\right)}{10^3}$$

$$= 600 \text{ s} = 10 \text{ minutes}$$

2. If h is the height of the tank, the velocity of efflux is

$$v = \sqrt{2gh}$$

Initially the velocity of water at the top of the tank is zero. Therefore, average velocity of efflux is

$$v_a = \frac{v}{2} = \frac{1}{2}\sqrt{2gh} = \sqrt{\frac{gh}{2}}$$

If V is the volume of water in the tank when it is full,

$$T = \frac{V}{Av_a} = \frac{\sqrt{2}\,V}{A\sqrt{gh}}$$

where A = cross-sectional area of the hole.

When the tank is half full, $V = \dfrac{V}{2}$ and $h = \dfrac{h}{2}$.

Therefore, time taken to empty the tank will be

$$T' = \frac{\sqrt{2}\,V/2}{A\sqrt{gh/2}} = \frac{V}{A\sqrt{gh}} = \frac{T}{\sqrt{2}}$$

So the correct choice is (a).

3. $\left[\dfrac{\sigma}{\eta}\right] = \dfrac{\left[\mathrm{MT}^{-2}\right]}{\left[\mathrm{ML}^{-1}\mathrm{T}^{-1}\right]} = [\mathrm{LT}^{-1}]$

Hence the correct choice (a).

4. If r is the radius of each droplet, the radius of big drop = $2r$. Since terminal velocity $\propto r^2$, the terminal velocity of the big drop = $4v$, which is choice (c).

5. Velocity of efflux is $v = \sqrt{2gh}$. The horizontal range is doubled if v is doubled. From Bernoulli's theorem,

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\Rightarrow \quad \Delta P = \frac{1}{2}\rho[(2v)^2 - v^2]$$

$$= \frac{3}{2}\rho v^2$$

$$= \frac{3}{2}\rho \times 2gh = 3\rho hg$$

6. Let h_1 and h_2 be the depths of the holes below the surface of water.

The volume of water flowing out from the holes per second is av_1 and av_2, where $v_1 = \sqrt{2gh_1}$ and $v_2 = \sqrt{2gh_2}$. Hence mass of water flowing per unit time is $m_1 = \rho av_1$ and $m_2 = \rho av_2$. Therefore, momentum per second carried by water is

$$p_1 = m_1v_1 \text{ in one direction}$$

and $$p_2 = m_2v_2 \text{ in the opposite direction}$$

Hence force required to keep the tank at rest is

$$F = \text{rate of change of momentum}$$

$$m_1v_1 - m_2v_2$$

$$= \rho av_1^2 - \rho av_2^2$$

$$= \rho a \times 2gh_1 - \rho a \times 2gh_2$$

$$= 2\rho ag(h_1 - h_2)$$

$$= 2\rho agh$$

7. $Q = \dfrac{\pi Pr^4}{8\eta l}$; η = coefficient of viscosity of the liquid and l = length of the tube.

Since η and l are the same, $Q \propto Pr^4$. Hence if P is doubled and r becomes half, the rate of flow will become $Q/8$.

8. Since the pressure difference is proportional to the length of the tube, the pressure difference across A and B is $P_a = 2P/3$ and $P_b = P/3$. The rate of flow through the series combination is (here l' = equivalent length of combination)

$$Q' = Q_a + Q_b$$

or $$\frac{\pi Pr^4}{8\eta l'} = \frac{\pi(2P/3)r^4}{8\eta(2l)} + \frac{\pi(P/3)r^4}{8\eta l}$$

$$\Rightarrow \quad \frac{1}{l'} = \frac{1}{3l} + \frac{1}{3l}$$

$$\Rightarrow \qquad l' = \frac{3l}{2}$$

Hence $Q' = \dfrac{\pi Pr^4}{8\eta(3l/2)} = \dfrac{2}{3} \times \dfrac{\pi Pr^4}{8\eta l} = \dfrac{2Q}{3}$.

9. From work-energy principle, work done = increase in K.E. Therefore (V = volume of liquid in the pipe)

$$P = \frac{W}{t} = \frac{W}{V} \times \frac{V}{t}$$

$$= \frac{\text{K.E}}{\text{volume}} \times \frac{\text{volume}}{\text{time}}$$

$$= \frac{1}{2}\rho v^2 \times av$$

$$= \frac{1}{2}\rho a v^3$$

i.e. $P \propto v^3$, which is choice (c).

10. From the equation of continuity of flow, we have

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad \pi r_1^2 v_1 = \pi r_2^2 v_2$$

or $$\frac{v_2}{v_1} = \frac{r_1^2}{r_2^2} = \frac{r^2}{(r/2)^2} = 4$$

or $$v_2 = 4v_1 = 4v.$$

Hence the correct choice is (d).

11. We know that

$$v_t = \frac{2}{9}\frac{(\rho - \sigma) g r^2}{\eta} = kr^2,$$

where $$k = \frac{2}{9}\frac{(\rho - \sigma) g}{\eta}$$

Now $$V = \frac{4\pi}{3} r^3.$$

Therefore $$r^2 = \left(\frac{3V}{4\pi}\right)^{2/3}.$$ Thus

$$v_t = k \left(\frac{3}{4\pi}\right)^{2/3} \times V^{2/3}$$

$\therefore$ Terminal velocity of the sphere of volume 8 V will be

$$V_t' = k \left(\frac{3}{4\pi}\right)^{2/3} \times (8V)^{2/3} = 4v_t$$

$[\because (8)^{2/3} = 4]$

Hence the correct choice is (d).

12. The average velocity in the first half of the distance = $\dfrac{0+v}{2} = \dfrac{v}{2}$; while in the second half, the average velocity is v. Therefore, $t_1 > t_2$. The work done against gravity in both halves = $mgh = mgL/2$. Hence the correct choice is (d).

13. Let l be the dimension of each side of the cubical vessel. The mass of water contained in a height x is $l^2 x\,\rho$. Therefore, the work done is

$$W = \int_o^l \left(l^2 x \rho g\right) dx = l^2 \rho g \int_o^l x\,dx$$

$$= l^2\ \rho\ g\ \frac{l^2}{2}$$

$$= \frac{l^4 \rho g}{2} = \frac{(1)^4 \times 1000 \times 9.8}{2}$$

$$= 4900 \text{ J}$$

Hence the correct choice is (c).

14. Since both balls have the same volume, they experience the same upthrust. Since the density of iron is greater than that of aluminium, the iron ball has a greater mass and therefore a greater weight; it therefore accelerates more and will reach the bottom before the aluminium ball. Hence the correct choice is (b).

15. A sphere of mass m, radius r and density ρ falling in a viscous liquid of coefficient of viscosity η acquires a terminal velocity v_t if its net downward force = weight – upthrust = $W - U$ is balanced by the upward viscous force $F = 6\pi\eta r v_t$. Here $W = mg$ and $U = \dfrac{4\pi r^3}{3} \cdot \sigma g$, where $4\pi r^3/3$ is the volume of the sphere and σ the density of the liquid. Thus

$$6\ \pi\eta r v_t = mg - \frac{4\pi r^3}{3}\ \sigma g$$

$$= mg \left(1 - \frac{\sigma}{\rho}\right)$$

because $\dfrac{4\pi r^3}{3}$ = volume of sphere = $\dfrac{m}{\rho}$.

Since η, m, σ and ρ are constant for both spheres, rv_t = constant, i.e. $v_t \propto \dfrac{1}{r}$. Since the second sphere has twice the radius, its terminal velocity will be half that of the first sphere. Hence the correct choice is (d).

16. The volume of liquid flow per second is given by

$$V = \frac{\pi \Delta P r^4}{8\eta l} = \frac{\pi \Delta P d^4}{128\eta l} \quad (\therefore r = \frac{d}{2})$$

Thus $V \propto \dfrac{d^4}{l}$. If l is increased to $2l$ and d is reduced to $\dfrac{d}{2}$, V decreases by a factor of 1/32.

Since there are four such pipes in series, V will decrease by a factor of 1/8. Thus V becomes $V/8$. Hence the correct choice is (c).

17. The equation of continuity of flow is $v_1 a_1 = v_2 a_2$ where $v_1 = 1.0\ \text{ms}^{-1}$, $a_1 = 10^{-4}\ \text{m}^2$, v_2 = velocity of stream at $h = 0.15$ m below the tap. The value of v_2 is given by

$$v_2^2 = v_1^2 + 2gh$$
$$= 1.0 + 2 \times 10 \times 0.15 = 4.0$$

or $\quad v_2 = 2.0\ \text{ms}^{-1}$.

Now $\quad a_2 = \dfrac{v_1 a_1}{v_2} = \dfrac{1.0 \times 10^{-4}}{2.0}$

$$= 5.0 \times 10^{-5}\ \text{m}^2$$

Hence the correct choice is (c).

18. The ratio of volume of water flowing out per second is given by

$$\frac{V_1}{V_2} = \frac{v_1 a_1}{v_2 a_2} = \frac{v_1 (L)^2}{v_2 (\pi R^2)} \qquad (1)$$

The velocities of water flowing out are given by

$$v_1 = \sqrt{2gy} \quad \text{and} \quad v_2 = \sqrt{2g(4y)}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{\sqrt{2gy}}{\sqrt{8gy}} = \frac{1}{2} \qquad (2)$$

Using (2) in (1), we have

$$\frac{V_1}{V_2} = \frac{1}{2}\frac{L^2}{\pi R^2}$$

Given $\quad V_1 = V_2$. Therefore $1 = \dfrac{1}{2}\dfrac{L^2}{\pi R^2}$

or $\quad R = \dfrac{L}{\sqrt{2\pi}}$ which is choice (a).

19. Let h be the depth of the hole below the free surface of water. (see Fig. 12.8).

According to Torricelli's theorem, the velocity of efflux v of water through the hole is given by

$$v = \sqrt{2gh} \qquad (i)$$

The height through which water falls is

$$S = H - h$$

If t is the time taken by water to strike the floor, then

$$S = \frac{1}{2} gt^2$$

$$\text{or } H - h = \frac{1}{2} gt^2$$

$$\text{giving } t = \sqrt{\frac{2(H-h)}{g}} \qquad (ii)$$

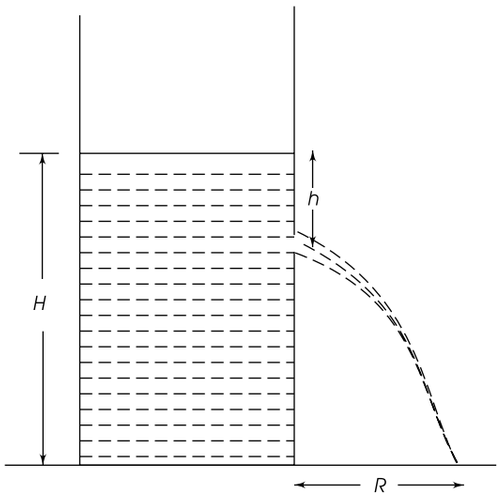

Fig. 12.8

The distance R where the emerging stream strikes the floor is given by

$$R = vt$$

Substituting for v and t from Eqs. (i) and (ii), we get

$$R = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}\ 2 \times \sqrt{h(H-h)}$$

Hence the correct choice is (c).

20. R will be maximum at value of h for which $\dfrac{dR}{dh} = 0$

and $\dfrac{d^2R}{dh^2} < 0$

Now $R = 2(hH - h^2)^{1/2}$

Differentiating with respect to h, we have

$$\frac{dR}{dh} = 2 \times \frac{1}{2}\ (hH - h^2)^{-1/2}\ (H - 2h)$$

$$= \frac{(H-2h)}{\left(hH - h^2\right)^{1/2}} \qquad (i)$$

It is clear that $\dfrac{dR}{dh}$ will be zero at a value of h given by

$$H - 2h = 0$$

i.e. $\quad h = \dfrac{H}{2}$

To find out whether $\dfrac{d^2R}{dh^2}$ is negative at this value of h, we differentiate Eq. (i) with respect to h to get

$$\frac{d^2R}{dh^2} = -\frac{2h}{(H-h)^{1/2}}\left\{1 + \frac{1}{4}\frac{(H-2h)}{h(H-h)}\right\}$$

Putting $h = H/2$, we have

$$\left(\frac{d^2R}{dh^2}\right)_{h=H/2} = -\sqrt{2H}$$

which is negative. Hence R will be maximum at $h = H/2$. Thus for range R to be maximum the hole must be exactly in the middle of the tank.

21. Using Bernoulli's theorem, we have

$$\frac{1}{2}\rho v^2 = p = \rho g h$$

or $$h = \frac{v^2}{2g} \qquad (1)$$

Now $v = r\omega = r(2\pi\nu)$. Using this in (1), we get

$$h = \frac{2\pi^2\nu^2 r^2}{g}$$

Given, $\nu = 2$ rev. per second, $r = 0.05$ m, $g = 10$ ms^{-2} and $\pi^2 = 10$. Using these values, we get $h = 0.02$ m $= 2$ cm, which is choice (a).

22. The horizontal range of a jet of water emerging from a hole at a height h below the surface of water is given by

$$R = 2\sqrt{h(H-h)}$$

The upper hole is at a height $\left(\frac{H}{2} + h_1\right)$ from the bottom and the lower hole is at a height $\left(\frac{H}{2} - h_2\right)$ from the bottom. Their depths from the surface are respectively $\left(\frac{H}{2} - h_1\right)$ and $\left(\frac{H}{2} + h_2\right)$. The horizontal ranges will be equal if

$$2\sqrt{\left(\frac{H}{2}+h_1\right)\left(\frac{H}{2}-h_1\right)} = 2\sqrt{\left(\frac{H}{2}+h_2\right)\left(\frac{H}{2}-h_2\right)}$$

which gives $h_1 = h_2$. Hence the correct choice is (a).

23. Velocity of efflux $(v) = \sqrt{2gd}$. Time taken $t = \sqrt{\frac{2h}{g}}$

$\therefore$ Range R = velocity × time $= vt$

$$= \sqrt{2gd} \times \sqrt{\frac{2h}{g}} = \sqrt{4dh}$$

or $R^2 = 4\,dh$ or $d = \frac{R^2}{4h}$, which is choice (c).

24. $Q_1 = \frac{\pi P_1 r_1^4}{8\eta l_1}$ and $Q_2 = \frac{\pi P_2 r_2^4}{8\eta l_2}$.

Since the tubes are connected in series, $Q_1 = Q_2$

or $$\frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2}$$

or $$\frac{P_1}{P_2} = \left(\frac{l_1}{l_2}\right) \times \left(\frac{r_2}{r_1}\right)^4 = \left(\frac{1}{2}\right) \times 2^4 = 8$$

Hence the correct choice is (a).

25. When the sphere acquires terminal velocity, the upward viscous force equals the apparent weight in glycerine. Therefore, the magnitude of the viscous force is (here r is the radius of the sphere)

$$F = \frac{4}{3}\pi r^3 (x-y)g = \frac{m}{x}(x-y)g = mg\left(1 - \frac{y}{x}\right)$$

Hence the correct choice is (c).

26. Radius R of the single tube is given by

$$\frac{\pi p R^4}{8\eta l} = \frac{\pi p r^4}{8\eta l} + \frac{\pi p (2r)^4}{8\eta l}$$

or $R^4 = r^4 + 16r^4$ or $R = (17)^{1/4} r$ which is choice (a).

27. Mass flux = rate of mass of liquid flowing per unit areas

$$= \frac{\text{mass}}{\text{area} \times \text{time}} = \frac{\text{mass}}{\text{volume}} \times \frac{\text{distance moved}}{\text{time taken}}$$

$$= \text{density} \times \text{velocity of flow}$$

$$= \rho v$$

Since mass flux is the same $\rho_1 v_1 = \rho_2 v_2$

According to continuity equation, $A_1 v_1 = A_2 v_2$

$\therefore$ $$\frac{\rho_1}{\rho_2} = \frac{v_2}{v_1} = \frac{A_1}{A_2} = \frac{A_1}{2A_1} = \frac{1}{2}$$

Hence the correct choice is (d).

28. Let h be the initial height of the liquid of density ρ in the container of cross-sectional area A. The mass of the liquid in the container initially is (Fig. 12.9)

$m = Ah\rho$

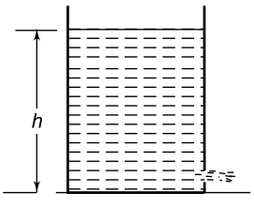

Fig. 12.9

From Torricelli's theorem, the velocity of the liquid flowing out of the hole is

$$v = \sqrt{2gh}$$

$\therefore$ Volume of liquid flowing out per unit time $= av$. Hence the mass of liquid flowing out per unit time $= \rho av$. Therefore, the momentum carried per unit time by the liquid flowing out is = (mass per unit time) × velocity = $(\rho av)v = \rho av^2$.

This is the rate of change of momentum of the liquid flowing out which is the force with which the liquid flows out at $t = 0$.

$$\therefore \text{ Initial acceleration} = \frac{\text{force}}{\text{mass}} = \frac{\rho a v^2}{Ah\rho} = \frac{av^2}{Ah}$$

$$= \frac{a \times 2gh}{Ah} \quad (\because v = \sqrt{2gh})$$

$$= \frac{2ga}{A}, \text{ which is choice (c).}$$

29. When 75% of the liquid has drained out, the height of the liquid in the container will be $h' = h/4$. For this height,

Velocity of liquid flowing out (v') $= \sqrt{2gh'}$

$$= \sqrt{2g \times \frac{h}{4}} = \sqrt{\frac{gh}{2}}$$

Now, $h = \frac{m}{A\rho}$. Hence

$$v' = \sqrt{\frac{gm}{2A\rho}}$$

30. Refer to Fig. 12.10. Let v be the velocity of efflux and V be the velocity with which the water level in the tank falls. From the equation of continuity of flow, we have $av = AV$ or

$$V = \frac{av}{A} \tag{1}$$

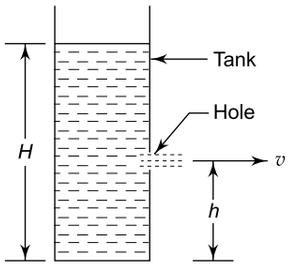

Fig. 12.10

From Bernoulli's theorem, we have

$$P_0 + \frac{1}{2}\rho V^2 + \rho g H = P_0 + \frac{1}{2}\rho v^2 + \rho g h$$

which gives $v^2 = V^2 + 2g(H - h)$ (2)

Using (1) in (2), we get

$$v^2 = \frac{2g(H-h)}{1 - \left(\frac{a}{A}\right)^2} = \frac{2 \times 9.8 \times (2.0 - 0.75)}{1 - 0.2} = 25$$

which gives $v = 5.0 \text{ ms}^{-1}$. Hence the correct choice is (c).

31. Let v be the horizontal speed of water when it emerges from the nozzle. From the equation of continuity, we have

$$AV = av$$

or $$v = \frac{AV}{a} \tag{1}$$

Let t be the time taken by the stream of water to strike the ground. The horizontal and vertical distances covered in time t are

$$R = vt \tag{2}$$

$$h = \frac{1}{2}gt^2 \tag{3}$$

From Eq. (3) we have $t = \sqrt{\frac{2h}{g}}$. Using this value in Eq. (2), we get

$$R = v\sqrt{\frac{2h}{g}} \tag{4}$$

Using Eqs. (1) and (4), we have

$$R = \frac{AV}{a}\sqrt{\frac{2h}{g}}, \text{ which is choice (d).}$$

32. The terminal velocity is

$$v_t = \frac{2}{9}\frac{(\rho - \sigma) r^2 g}{\eta}$$

The rate of production of heat (or power dissipated) is given by

$$P = fv_t, \text{ where } f = 6\pi\eta r v_t$$

$$\therefore \quad P = 6\pi\eta r\, v_t \times v_t$$

$$= 6\,\pi\eta r \left[\frac{2}{9}\frac{(\rho - \sigma) r^2 g}{\eta}\right]^2$$

$$= \left[\frac{8\pi g^2}{27\eta}(\rho - \sigma)^2\right] r^5$$

i.e. $P \propto r^5$. Hence the correct choice (d).

33. As indicated in the question as the ball enters the water it has already attained the terminal velocity. Terminal velocity is reached when the viscous force (acting upwards) balances the weight mg of the ball. Therefore,

$$6\,\pi\eta r v = mg$$

where $6\pi\eta r v$ is the viscous force on the ball. Here v its terminal velocity and η, the coefficient of viscosity of water. If ρ is the density of the material of the ball,

we have $m = \frac{4}{3}\pi r^3\rho$, $\therefore$ $6\,\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$

or $$v = \frac{2r^2\rho g}{9\eta} \quad (1)$$

Now $v^2 - 0 = 2gh \Rightarrow h = \frac{v^2}{2g}$ (2)

From (1) and (2) we find that $h \propto r^4$, which is choice (c).

34. Let ρ_w and ρ_k be the densities of water and kerosene. The initial weight of the liquid in the vessel = $h_w \rho_w ag + h_k \rho_k ag$ where h_w and h_k are the thicknesses of water and kerosene layers and a is the cross-sectional area of the vessel. Let this weight be equivalent to water layer of thickness h, then

$$h\rho_w ag = h_w\rho_w ag + h_k\rho_k ag$$

or $$h = \left\{h_w + h_k\left(\frac{\rho_k}{\rho_w}\right)\right\}$$

$$= 0.4 + 0.3 \times 0.8 = 0.64 \text{ m}$$

From Torricelli's theorem, the velocity of efflux is

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \times 10 \times 0.64} = \frac{8}{\sqrt{5}} \text{ ms}^{-1}$$

Hence the correct choice is (d).

SECTION II

Multiple Choice Questions with Two or More Choices Correct

1. Which of the following are dimensionless?
(a) Strain (b) Relative density
(c) Poisson's ratio (d) Reynold's number

2. The dimensions of pressure are the same as those of
(a) stress (b) modulus of elasticity
(c) surface tension (d) viscosity

3. Choose the correct statements from the following:
(a) The terminal velocity of a spherical body falling in a fluid depends on the diameter and density of the body as well on the density and viscosity of the fluid.
(b) To keep a piece of paper horizontal, we should blow under it and not over it.
(c) The velocity of fluid flow at any section of a pipe is directly proportional to the area of cross-section at that section.
(d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.

4. A liquid flows through a non-uniform pipe. The pressure in the pipe will be
(a) higher where the cross section is smaller
(b) lower where the cross section is smaller
(c) higher where velocity of the liquid is smaller
(d) the same throughout the pipe.

5. Choose the incorrect statements from the following. When a liquid flows through the narrow part of a nonuniform pipe,
(a) its velocity and pressure both increase
(b) its velocity and pressure both decrease
(c) its velocity decreases but its pressure increases
(d) its velocity increases but its pressure decreases

6. The rate of flow of a liquid through a capillary tube depends upon
(a) the pressure difference between the ends of the tube
(b) the radius of the tube
(c) the length of the tube
(d) the viscosity of the liquid.

7. The viscous force on a spherical body moving through a fluid depends upon
(a) the mass of the body
(b) the radius of the body
(c) the velocity of the body
(d) the viscosity of the fluid.

8. The terminal velocity of a spherical body falling through a fluid depends upon
(a) the density of the body
(b) the radius of the body
(c) the density of the fluid
(d) the viscosity of the fluid.

9. A horizontal tube OP of length L and of uniform cross-sectional area a is open at end O and has a small hole at the other end P. The tube is filled with a liquid of density ρ and then rotated about the axis passing through O at an angular velocity ω. At the instant when $L/2$ length of liquid column is left in the tube, the pressure exerted by the liquid at end P is p and the velocity of the liquid flow from the hole is v. If viscous effects are neglected

(a) $p = \frac{3}{4}\rho\omega^2 L^2$ (b) $p = \frac{3}{8}\rho\omega^2 L^2$

(c) $v = \sqrt{\frac{3}{2}}\omega L$ (d) $v = \frac{\sqrt{3}\omega L}{2}$

10. In a streamlined flow of a liquid through a tube of nonuniform cross-section,
(a) the speed of a particle at a given point is the same as that of the preceding particle passing that point.
(b) the velocity of a particle at a given point is the same as that of the preceding particle passing that point.
(c) the momenta of all particles passing a given point are the same.
(d) the kinetic energy of every particle passing a given point is the same.

11. A liquid is flowing steadily through a horizontal tube shown in Fig. 12.11. If P_A and P_B are the pressures at A and B and v_A and v_B, the velocity of flow at A and B, then
(a) $P_A = P_B$ because A and B are at the same horizontal level.
(b) P_A is greater than P_B.
(c) v_A is greater than v_B.
(d) v_A is less than v_B.

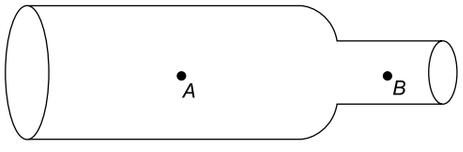

Fig. 12.11

12. The upper surface of the wing of an aircraft is more curved that the lower surface. It is flying horizontally. If v_1 and v_2 are the velocities of air along the upper and lower surfaces respectively and p_1 and p_2 the respective air pressures, then
(a) $v_1 > v_2$ (b) $v_1 < v_2$
(c) $p_1 > p_2$ (d) $p_1 < p_2$

13. In Q.12 above, if v_1 =100 ms^{-1}, v_2= 80 ms^{-1}, the density of air at the height of the aircraft = 1.25 kg m^{-3} and surface area of the wing is 20 m^2, then
(a) $p_2 - p_1 = 2.25 \times 10^3$ Pa
(b) $p_2 - p_1 = 4.5 \times 10^3$ Pa
(c) dynamic uplift on the wing is 4.5×10^4 N.
(d) dynamic uplift on the wing is 2.25×10^4 N

14. A syringe containing water is held horizontally with its nozzle at a hight h = 1.25 m above the ground as shown in Fig 12.12. The diameter of the piston is 5 times that the of nozzle. The piston is pushed with a constant speed of 20 cm s^{-1}. If g = 10 ms^{-2},
(a) The speed of water emerging from the nozzle is 5 ms^{-1}.
(b) The time taken by water to hit the ground is 0.5 s.
(c) The horizontal range R = 2.5 m.
(d) The magnitude of the velocity with which the water hits the ground is $5\sqrt{2}$ ms^{-1}.

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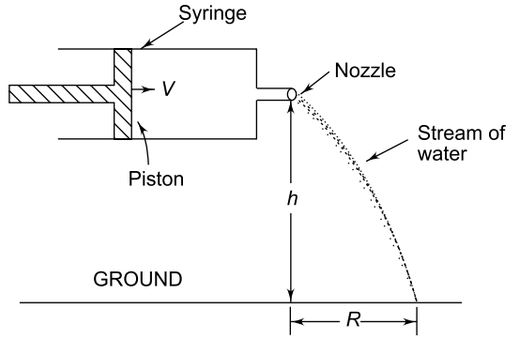


Fig. 12.12

15. A liquid of density ρ is contained in a cylindrical vessel of radius r. When the vesel is rotated about its axis at an angular velocity ω, the liquid rises by h at the sides as shown in Fig. 12.13. If p_c is the pressure of the liquid at the centre and p_s at the sides of the vessel, then

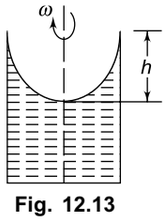


Fig. 12.13

(a) $p_c > p_s$ (b) $p_c < p_s$
(c) $h = \frac{r^2\omega^2}{2g}$ (d) $h = \frac{r^2\omega^2}{g}$

ANSWERS AND SOLUTIONS

1. All quantities are dimensionless.
2. The correct choices are (a) and (b).
3. Statement (a) is correct. Statement (b) is incorrect; we must blow over the paper and not under it. If we blow over a piece of paper, the velocity of air moving along the upper surface of the paper is higher than that along the lower surface. Therefore, from Bernoulli's principle, the air pressure on

the upper surface falls below the atmospheric pressure whereas that on the lower surface rises above the atmospheric pressure. Hence an upward thrust is exerted on the paper which keeps it horizontal. Statement (c) is wrong. Equation of continuity of flow (av = constant) tells us that v is inversely proportional to a. Statement (d) is correct. The fluid flowing out of a hole has velocity and therefore momentum in the outward direction. From the principle of conservation of momentum, the vessel must experience a recoil momentum in the backward direction and hence a backward thrust.

4. According to the Bernoulli's principle, the pressure of the liquid will be lower where the cross-section of the pipe is smaller. From the continuity condition, it follows that the velocity of the liquid flow will be higher where the cross-section of the pipe is smaller. Hence choices (b) and (c) are correct.

5. Choices (a), (b) and (c) are incorrect. Consider two points A and B in the pipe at the same horizontal level. At point A, let a_1 be the area of cross-section of the pipe, v_1 the velocity of fluid flow and p_1 the fluid pressure. Let a_2, v_2 and p_2 be the corresponding quantities at point B. Then from Bernoulli's theorem we have

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or $$(p_1 - p_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad \text{(i)}$$

If $a_2 < a_1$, it following from continuity equation $a_1v_1 = a_2v_2$, that $v_1 > v_2$. From Eq. (i) it follows that $p_2 < p_1$. Hence, at the narrow part of the pipe, the fluid velocity increases but its pressure decreases.

6. The correct choices are (a), (b), (c) and (d). The rate of flow is given by

$$Q = \frac{\pi p r^4}{8\eta l}$$

7. The correct choices are (b), (c) and (d). The viscous force is given by

$$F = 6\ \pi\ \eta\ r\ v$$

8. All the four choices are correct. The terminal velocity is given by

$$v_t = \frac{2(\rho - \sigma)r^2 g}{9\eta}$$

where ρ = density of the body and σ = density of the fluid

9. Consider a small element of the liquid of length dr at a distance r from O. The mass of the element $m = a\rho dr$. Therefore, the outward force (centrifugal force) acting on the element is (see Fig. 12.14)

$$dF = mr\omega^2 = a\rho\omega^2\ rdr$$

The total outward force F acting on the liquid column of length l at that instant is obtained by integrating this expression from $r = (L - l)$ to $r = L$. Thus

$$F = a\rho\omega^2 \int_{L-l}^{L} rdr = a\rho\omega^2 \left|\frac{r^2}{2}\right|_{(L-l)}^{L}$$

$$= \frac{1}{2}\ a\rho\omega^2\ [L^2 - (L - l)^2] = a\rho\omega^2 l\left(L - \frac{l}{2}\right)$$

Given $l = L/2$. Therefore, $F = \dfrac{3a\rho\omega^2 L^2}{8}$

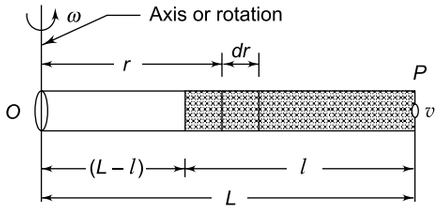


Fig. 12.14

Outward pressure at P is $p = \dfrac{F}{a} = \dfrac{3}{8}\rho\omega^2 L^2$. If v is the velocity of efflux due to this pressure, then

$$\frac{1}{2}\rho v^2 = p = \frac{3}{8}\rho\omega^2 L^2$$

which gives $$v = \frac{\sqrt{3}\omega L}{2}$$

Hence the correct choices are (b) and (d).

10. The correct choices are (b), (c) and (d).

11. Equation of continuity gives $v_A < v_B$. From Bernoulli's theorem $P_A + \dfrac{1}{2}\rho v_A^2 = P_B + \dfrac{1}{2}\rho v_B^2$. Since $v_B > v_A$, $p_B < p_A$. Thus the correct choices are (b) and (d).

12. As the aircraft moves, the streamlines of air flow curve around the wing and meet at the rear end at the same time. Hence velocity of air moving along the upper surface is higher than that moving along the lower surface. According to Bernoulli's principle, the air pressure on the upper surface is less than that on the lower surface of the wing. Hence the correct choices are (a) and (d).

13. From Bernoulli's principle,

$$\Delta p = p_2 - p_1 = \frac{1}{2}\rho\ (v_1^2 - v_2^2)$$

$$= \frac{1}{2} \times 1.25\ [(100)^2 - (80)^2\ ] = 2.25 \times 10^3 \text{Pa}$$

uplift = $\Delta p \times$ area of wing

$$= 2.25 \times 10^3 \times 20 = 4.5 \times 10^4 \text{ N}$$

Thus the corect choices are (a) and (c).

14. (a) Area of piston $A = \dfrac{\pi D^2}{4}$; D = diameter of piston

Area of nozzle $a = \dfrac{\pi d^2}{4}$; d = diameter of nozzle

From equation of continuity $AV = av$,

$$v = \frac{AV}{a} = \frac{D^2}{d^2} \times V = (5)^2 \times 0.2 = 5 \text{ ms}^{-1}$$

(b) $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 1.25}{10}} = 0.5$ s

(c) $R = vt = 5 \times 0.5 = 2.5$ m

(d) Horizontal velocity $v_x = v = 5 \text{ ms}^{-1}$

Vertical velocity at $t = 0.5$ is $v_y = g_t = 10 \times 0.5 = 5 \text{ ms}^{-1}$

$\therefore$ Magnitude of velocity $= (v_x^2 + v_y^2)^{1/2} = [(5)^2 + (5)^2] = 5\sqrt{2} \text{ ms}^{-1}$

Hence all choices are correct.

15. According to Bernoulli's theorem

$$p + \frac{1}{2} \rho v^2 = \text{constant}$$

When the cylindrical vessel is rotated at angular velocity ω about its axis, the velocity of the liquid at the sides is the maximum, given by

$$v_s = r\omega$$

where r is the radius of the vessel. Applying Bernoulli's theorem at the sides and at the centre of the vessel, we have (Fig. 12.8)

$$p_s + \frac{1}{2} \rho v_s^2 = p_c + \frac{1}{2} \rho v_c^2$$

where p_s = pressure at the sides, p_c = pressure at the centre and v_c = velocity of the liquid at the centre. Now, since $v_c = 0$, we have

$$p_c - p_s = \frac{1}{2} \rho v_s^2 = \frac{1}{2} \rho r^2 \omega^2 \quad (1)$$

Since p_c is greater than p_s, the liquid rises at the sides of the vessel. Let h be the difference in the levels of the liquid at the sides and at the centre (Fig. 12.8), then

$$p_c - p_s = \rho g h \quad (2)$$

From (1) and (2), we have

$$\rho g h = \frac{1}{2} \rho r^2 \omega^2 \quad \text{or} \quad h = \frac{r^2 \omega^2}{2g}$$

Hence the correct choices are (a) and (c).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage

Passage I

Stokes' Law: A body falling through a viscous medium experiences a retarding force resulting in absorption of energy by the medium in the form of heat. The motion of the body produces a relative motion between the different layers of the fluid. Consequently, it experiences a force which tends to retard its motion. When a small spherical body is dropped in a viscous liquid such as glycerine, it accelerates first, but soon begins to experience a retarding force. When the retarding force becomes equal to the effective weight of the body in the fluid, the body experiences no net force and falls with a constant velocity known as the *terminal velocity*.

George Stokes found that a small spherical body of radius r moving with a uniform velocity v in a fluid of coefficient of viscosity η experiences a retarding force F given by

$$F = 6 \pi \eta r v$$

1. From relation $F = 6 \pi \eta r v$, the dimensions of η are

(a) $ML^{-1}T^{-1}$ (b) $ML^{-1}T^{-2}$

(c) $ML^{-2}T^{-1}$ (d) $ML^{-2}T^{-2}$

2. Choose the correct statement. When a small body falls freely in a viscous fluid

(a) its speed increases indefinitely

(b) its speed first increases due to gravity and then decreases due to viscosity of the liquid, eventually the body comes to test in the liquid

(c) it accelarates first and then experiences deceleration until it acquires a constant velocity called terminal velocity

(d) the speed of the body remains constant throughout the motion.

3. If the upthrust on a body is negligible compared to its weight, the terminal velocity of a small spherical body falling through a viscous liquid depends upon
(a) the mass of the body
(b) the diameter of the body
(c) the viscosity of the liquid
(d) the acceleration due to gravity.

4. A body having a low density and relatively large surface area for its weight
(a) falls through a long distance before it acquires its terminal velocity
(b) falls through a short distance before it acquires its terminal velocity
(c) acquires a high terminal velocity
(d) can never acquire terminal velocity.

ANSWERS

1. The correct choice is (a)
2. The correct choice is (c)
3. All the four choice are correct. The terminal velocity (if upthrust is neglected) is given by

$$v_t = \frac{mg}{6\pi\eta r}$$

4. The correct choice is (b).

Questions 5 to 7 are based on the following passage

Passage II

When a liquid flows in a tube, there is a relative motion between adjacent layers of the liquid. This force is called the viscous force when tends to oppose the relative motion between the layers of the liquid. Newton was the first person to study the factors that govern the viscous force in a liquid. According to Newton's law of viscous flow, the magnitude of the viscous force on a certain layer of a liquid is given by

$$F = -\eta A \frac{dv}{dx}$$

where A is the area of the layer, $\frac{dv}{dx}$ is the velocity gradient at the layer and η is the coefficient of viscosity of the liquid.

5. If f is the frictional force between a solid sliding over another solid, and F is the viscous force when a liquid layer slides over another, then.
(a) f is independent of the area of the solid sliding over another solid.
(b) f depends on the relative velocity of one solid with respect to the other.
(c) F depends on the area of the layer of the liquid.
(d) F is independent of the relative velocity between adjacent layers.

6. The dimensional formula for the coefficient of viscocity is
(a) $[ML^{-1}T^{-1}]$ (b) $[MLT^{-1}]$
(c) $[ML^{-2}T^{-2}]$ (d) $[ML^{-1}T^{-2}]$

7. A river is 5 m deep. The velocity of water on its surface is 2 ms^{-1}. If the coefficient of viscosity of water is 10^{-3} Nsm^{-2}, the viscous force per unit area is
(a) 10^{-4} Nm^{-2}, (b) 2×10^{-4} Nm^{-2},
(c) 4×10^{-4} Nm^{-2}, (d) 5×10^{-4} Nm^{-2},

SOLUTION

5. In the case of solid sliding over another solid, the frictional force is independent of the area of contact and of the relative velocity between the solid surfaces. This is not true in the case of viscous force between layers of a liquid. Thus the correct choices are (a) and (c).

6. The numerical value of η is given by

$$\eta = \frac{F}{A(dv/dx)}$$

$$\therefore \quad [\eta] = \frac{MLT^{-2}}{L^2\left(\frac{LT^{-1}}{L}\right)} = [ML^{-1}T^{-1}]$$

7. The velocity of water in contact with river bed is zero.

$$\frac{|F|}{A} = \eta\frac{dv}{dx} = 10^{-3} \times \frac{2}{5} = 4 \times 10^{-4}\ Nm^{-2}.$$

Questions 8 to 11 are based on the following passage

Passage IV

A large container of negligible mass is open at the top and has a uniform cross-sectional area A. It has a small hole of cross-sectional area a ($<< A$) in its side wall near the bottom. The containar is kept on a horizontal frictionless floor and contains a liquid of density ρ and mass m filled up to a height h from the bottom.

8. The initial rate of change of momentum of the liquid flowing out from the hole is

(a) ρagh (b) ρAgh

(c) $2\rho agh$ (d) $2\rho Agh$

9. The initial force with which the liquid flows out from the hole is

(a) $\left(\frac{a}{A}\right)mg$ (b) $\left(\frac{A}{a}\right)mg$

(c) $\left(\frac{2a}{A}\right)mg$ (d) $\left(\frac{2A}{a}\right)mg$

10. The initial acceleration of the container is

(a) zero (b) g

(c) $\frac{Ag}{2a}$ (d) $\frac{2ag}{A}$

11. The velocity of efflux when 75% of the liquid has drained out is

(a) $\sqrt{\frac{gh}{2}}$ (b) $\sqrt{gh}$

(c) $\sqrt{\frac{3gh}{4}}$ (d) $\sqrt{\frac{3gh}{2}}$

SOLUTIONS

8. Pressure at the bottom (hole) is $P = h\rho g$.

$\therefore$ Force at the hole is

F = pressure $P \times$ area a of the hole = $h\rho ga$

From Torricelli's theorem, the velocity of the liquid flowing out of the hole is

$$v = \sqrt{2gh}$$

$\therefore$ Volume of liquid flowing out per unit time = av. Hence the mass of liquid flowing out per unit time = ρav. Therefore, the momentum carried per unit time by the liquid flowing out is = (mass per unit time) $\times$ velocity = $(\rho av)v = \rho av^2$.

Thus the rate of change of momentum of the liquid coming out of the hole = $\rho av^2 = \rho a \times 2gh = 2\rho agh$, which is choice (c).

9. Force = rate of change of momentum = $2\rho agh = \left(\frac{2a}{A}\right)mg$ ($\because$ $m = \rho Ah$). Hence the correct choice is (c).

10. Initial acceleration of container is due to the backward reaction force exerted on the container by the liquid emerging from the hole. The action force = rate of change of momentum of the emerging water = $\left(\frac{2a}{A}\right)mg$ which equal and opposite to the reaction force on the container. Hence the backward acceleration of the container = $\frac{\text{backward force}}{\text{mass}}$ = $\left(\frac{2a}{A}\right)g$, which is choice (d).

11. When 75% of the liquid has drained out, the height of the liquid in the container will be $h' = h/4$. For this height,

Velocity of liquid flowing out (v') = $\sqrt{2gh'}$

$= \sqrt{2g \times \frac{h}{4}} = \sqrt{\frac{gh}{2}}$, which is choice (a).

Questions 12 to 14 are based on the following passage

Passage V

A cylindrical tank of cross-sectional area A rests on a platform of height H from the ground as shown in Fig. 12.15. It has a hole of cross-sectional area a on the side wall at the bottom. The tank is filled with water to a height h. The plug is removed from the hole.

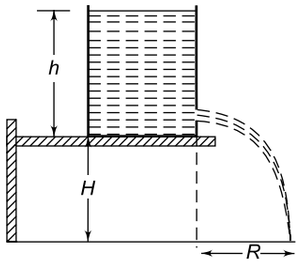


Fig. 12.15

12. The initial speed with which the water hits the ground is

(a) $\sqrt{2gh}$ (b) $[2g(H^2 + h^2)^{1/2}]^{1/2}$

(c) $\sqrt{2g(H+h)}$ (d) $\sqrt{2gH}$

13. The stream of water strikes the ground at a distance R given by

(a) $\sqrt{hH}$ (b) $\sqrt{2hH}$

(c) $2\sqrt{hH}$ (d) $\sqrt{(h^2 + H^2)}$

14. The time taken to empty the tank to one-fourth of its original volume of water is

(a) $\frac{A}{a}\sqrt{\frac{h}{2g}}$ (b) $\frac{A}{a}\sqrt{\frac{2h}{g}}$

(c) $\frac{A}{a}\sqrt{\frac{H}{2g}}$ (d) $\frac{a}{A}\sqrt{\frac{2H}{g}}$

SOLUTION

12. Velocity of efflux from the hole is $v_x = \sqrt{2gh}$. This is the horizontal velocity, which remains unchanged. Vertical velocity on striking the ground is $v_y = \sqrt{2gH}$ (use $u^2 = v^2 + 2as$).
The resultant speed with which the water strikes the ground is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2g(H+h)}$$

Hence the correct choice is (c).

13. Time taken by stream to strike the ground is $t = \sqrt{\frac{2H}{g}}$.

$\therefore R = v_x \times t = \sqrt{2gh} \times \sqrt{\frac{2H}{g}} = 2\sqrt{hH}$, which is choice (c).

14. As water flows through the orifice, the level of water in the tank keeps falling with time. If x is the height of the water level at an instant of time t, then the instantaneous speed of water flowing out is

$$v_x = \sqrt{2gx}$$

If a is the cross-sectional area of the orifice, the volume of water flowing per unit time at that instant $= av_x = a\sqrt{2gx}$. This must be equal to the rate at which the volume of water in the tank decreases at that instant which is given by $-A\,dx/dt$, where A is the cross-sectional area of the tank. The negative sign indicates that height x of water decreases with time. Thus

$$a\sqrt{2gx} = -A\frac{dx}{dt}$$

or
$$dt = -\frac{A\,dx}{a\sqrt{2gx}}$$

$$= -\frac{A}{a\sqrt{2g}}x^{-1/2}\,dx \qquad (1)$$

The time t taken to empty the tank to half its original volume is obtained by integrating (1) from $x = h$ to $x = h/4$. Thus

$$t = -\frac{A}{a\sqrt{2g}}\int_h^{h/4} x^{-1/2}\,dx$$

$$= -\frac{A}{a\sqrt{2g}}\left|\frac{x^{1/2}}{1/2}\right|_h^{h/4}$$

$$= -\frac{2A}{a\sqrt{2g}}\left[\left(\frac{h}{4}\right)^{1/2} - h^{1/2}\right]$$

$$= \frac{A}{a}\sqrt{\frac{h}{2g}}$$, which is choice (a).

Questions 15 to 17 are based on the following passage

Passage VI

A cylindrical tank is open at the top and has cross-sectional area a_1. Water is filled in it up to a height h. There is a hole of cross-sectional area a_2 at its bottom. Given $a_1 = 3a_2$.

15. The initial velocity with which the water falls in the tank is

(a) $\sqrt{2gh}$ (b) $\sqrt{gh}$

(c) $\sqrt{\frac{gh}{2}}$ (d) $\frac{1}{2}\sqrt{gh}$

16. The initial velocity with which the water emerges from the hole is

(a) $\frac{1}{2}\sqrt{gh}$ (b) $\sqrt{2gh}$

(c) $\frac{3}{2}\sqrt{gh}$ (d) $2\sqrt{2gh}$

17. The time taken to empty the tank is

(a) $\sqrt{\frac{2h}{g}}$ (b) $4\sqrt{\frac{h}{g}}$

(c) $6\sqrt{\frac{2h}{g}}$ (d) $8\sqrt{\frac{2h}{g}}$

SOLUTIONS

15. Let v_1 be the initial velocity of water in the tank and v_2 the initial velocity of flow of water from the hole, then from continuity of flow (Fig. 12.16),

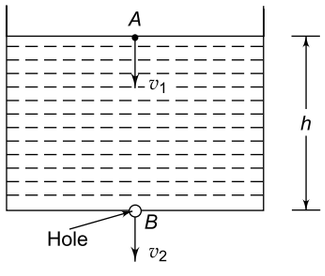


Fig. 12.16

$$a_1v_1 = a_2v_2$$

$$\Rightarrow \quad v_1 = \left(\frac{a_2}{a_1}\right)v_2 = \frac{v_2}{3} \qquad (1)$$

Applying Bernoulli's theorem at A and B,

$$P_0 + \rho gh + \frac{1}{2}\rho v_1^2 = P_0 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad v_1^2 = v_2^2 - 2gh \qquad (2)$$

Using (1) in (2), we get $v_1 = \frac{1}{2}\sqrt{gh}$, which is choice (d).

16. $v_2 = 3v_1 = \frac{3}{2}\sqrt{gh}$. Hence the correct choice is (c).

17. As h decreases with time, we have

$$v_1 = -\frac{dh}{dt}$$

$$\Rightarrow \qquad dt = -\frac{-dh}{v_1} = -\frac{2}{\sqrt{g}} \times h^{-1/2}\, dh$$

The time taken to empty the tank is obtained by integrating from $h = h$ to $h = 0$. Thus

$$t = -\frac{2}{\sqrt{g}} \int_h^0 h^{-1/2}\, dh$$

$$= -\frac{2}{\sqrt{g}} (0 - 2\sqrt{h}\,) = 4\sqrt{\frac{h}{g}}$$

Hence the correct choice is (b).

Questions 18 to 20 are based on the following passage

Passage VII

A container of large uniform cross-sectional area A resting on a horizontal surface, is filled with two non-mixing and non viscous liquids of densities d and $2d$, each to a height $H/2$ as shown in Fig.12.17. A tiny hole of cross-sectional area $a(<< A)$ is punched on the vertical side of the container at a height $h = H/4$.

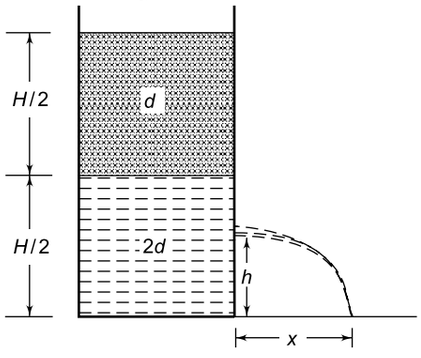


Fig. 12.17

18. The initial speed of efflux of the liquid from the hole is

(a) $2\sqrt{2\,gH}$ (b) $2\sqrt{gH}$

(c) $\sqrt{2\,gH}$ (d) $\sqrt{gH}$

19. The horizontal range x of the liquid initially is

(a) H (b) $\frac{H}{\sqrt{2}}$

(c) $\frac{H}{2}$ (d) $\frac{H}{2\sqrt{2}}$

20. The height h of the hole from the ground for which the horizontal range x is maximum is

(a) $\frac{2H}{3}$ (b) $\frac{3H}{2}$

(c) $\frac{3H}{4}$ (d) $\frac{H}{2}$

SOLUTION

18. Let v_1 be the initial velocity of the fall of liquid level in the container and v_2 the velocity of efflux from the hole. Then $Av_1 = av_2$ or $v_1 = \left(\frac{a}{A}\right) v_2$. since $a << A$, $v_2 \simeq 0$. Applying Bernoulli's theorem at a point on the top of the liquid and at the hole, we have

$$P_0 + \frac{1}{2}\, dv_1^2 + dg\left(\frac{H}{2}\right) + (2d)\, g\left(\frac{H}{2}\right)$$

$$= P_0 + \frac{1}{2}\,(2d)\, v_2^2 + (2d)\, gh$$

Putting $v_1 = 0$, we get

$$v_2 = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2}$$

Putting $h = H/4$, we get $v_2 = \sqrt{gH}$ which is choice (d).

19. Time of fall $t = \sqrt{\frac{2h}{g}}$. Horizontal range is

$$x = v_2\, t = \left[\left(\frac{3}{2}H - 2h\right)g\right]^{1/2} \times \sqrt{\frac{2h}{g}}$$

$$= [h(3H - 4h)]^{1/2} \qquad (1)$$

putting $h = H/4$, we get $x = \frac{H}{\sqrt{2}}$, which is choice (b).

20. To determine the height h at which the range x is maximum, we differentiate (1) with respect to h and set $\frac{dx}{dh} = 0$, i.e.

$$\frac{d}{dh}\ [3Hh - 4h^2]^{1/2} = 0$$

or $$\frac{1}{2}\,[3Hh - 4h^2]^{-1/2} \times (3H - 8h) = 0$$

which gives $3H - 8h = 0$ or $h = \frac{3}{8}\ H$. Notice that $(3Hh - 4h^2)^{-1/2}$ cannot be zero since this would give x = infinity which is not possible. Hence x is maximum ($= x_m$) at a value of h given by

$$h = \frac{3}{8}\ H$$

Using this value of h in (1), we get

$$x_m = \left[\frac{3}{8}\ H\left(3H - \frac{12}{8}\ H\right)\right]^{1/2} = \frac{3}{4}\ H$$

Hence the correct choice is (c).

SECTION IV

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1
To keep a piece of paper horizontal, we should blow over it and not under it.
Statement-2
In a steady flow of a fluid, the total energy of a given mass of a fluid is conserved.

2. Statement-1
If we plug a running water tap with our fingers, fast jets of water gush through the opening between in fingers.
Statement-2
The pressure of water at the opening increases due to the decrease in the area.

3. Statement-1
When a fluid flows out of a small hole in the sides of a vessel, a backward force is exerted on the vessel.
Statement-2
The total energy of a given mass of a fluid in motion is conserved.

4. Statement-1
The critical velocity of a liquid flowing through a tube is inversely proportional to the radius of the tube.
Statement-2
The velocity of a liquid flowing through a tube is inversely proportional to the cross-sectional area of the tube.

5. Statement-1
The viscous force experienced by a steel ball moving in a liquid is less than that experienced by an aluminuim ball of the same radius moving in the liquid with the same speed.
Statement-2
The density of steel is greater than that of aluminuim.

6. Statement-1
The terminal velocity of a small steel ball falling in a liquid is more than that of an identical aluminuim ball falling in the same liquid.
Statement-2
The density of steel is greater than that of aluminuim.

7. Statement-1
No net force acts on a body falling in a liquid with a velocity equal to the terminal velocity.
Statement-2
The weight of the body is balanced by the upward buoyant force.

8. Statement-1
A very light spinning ball can be held hanging in air by blowing air as shown in Fig 12.18.

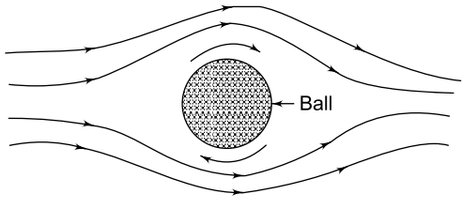


Fig. 12.18

Statement-2
The weight of the ball can be balanced by upthrust due to viscosity of air.

9. Statement-1
The uplift of the wing of an aircraft moving horizontally is caused by a pressure difference between the upper and lower faces of the wing.

Statement-2
The velocity of air moving along the upper surface is higher than that along the lower surface.

10. Statement-1
The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Statement-2
In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

IIT, 2005

SOLUTION

1. The correct choice is (a). If we blow over a piece of paper, the velocity of air moving along the upper surface of the paper is higher than that along the lower surface. From Bernoulli's theorem of conservation of energy of a fluid flow, the air pressure on the upper surface will be less that on the lower surface causing an uplift of the paper.

2. The correct choice is (c). The cross-sectional area of the opening between fingers is very small compared to that of the tap, From continuity of flow, the speed of water emerging from the opening is very high.

3. The correct choice is (c). The water emerging of from the hole has a momentum. From conservation of momentum, the vessel acquires a recoil momentum in the backward direction and hence a backward thrust.

4. The correct choice is (c).

5. The correct choice is (d). The viscous force is independent of the density of the body, it depends only on the radius and speed of the body and the viscosity of the fluid in which it moves ($F = 6\pi\eta r v$).

6. The correct choice is (b).

7. The correct choice is (c). The weight of the body is balanced by two upward forces, namely the buoyant force and viscous force.

8. The correct choice is (c). The ball drags some air with it while spinning. Therefore, the velocity of air at the lower surface increases. According to Bernoulli's theorem, the pressure of air at the upper surface becomes less than the atmospheric pressure (P_0) whereas the air pressure at the lower surface become more than P_0. This pressure difference causes a dynamic uplift of the ball.

9. The correct choice is (a)

10. The equation of continuity of streamline flow is Av = constant. When the hose pipe is held vertically up, the speed of the stream decreases with height. Hence the area of cross-section of the stream increases resulting it to spread like a fountain.

SECTION V

Integer Answer Type

1. A liquid is kept in a cylindrical vessel which is rotating about its axis. The liquid rises at its sides. If the radius of the vessel is 0.05 m and the speed of rotation is 2 revolutions per second, find the difference in the heights (in cm) of the liquid at the centre of the vessel and at its sides. Take $\pi^2 = 10$ and $g = 10\ \text{ms}^{-2}$.

IIT, 1987

SOLUTION

1. According to Bernoulli's theorem

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

When the cylindrical vessel is rotated at angular velocity ω about its axis, the velocity of the liquid at the sides is the maximum, given by

$$v_s = r\omega$$

where r is the radius of the vessel. Applying Bernoulli's theorem at the sides and at the centre of the vessel, we have (Fig. 12.19)

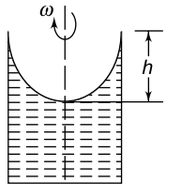


Fig. 12.19

$$p_s + \frac{1}{2}\rho v_s^2 = p_c + \frac{1}{2}\rho v_c^2$$

where p_s = pressure at the sides, p_c = pressure at the centre and v_c = velocity of the liquid at the centre.

Now, since $v_c = 0$, we have

$$p_c - p_s = \frac{1}{2}\rho v_s^2 = \frac{1}{2}\rho r^2\omega^2 \qquad (1)$$

Since p_c is greater than p_s, the liquid rises at the sides of the vessel. Let h be the difference in the levels of the liquid at the sides and at the centre, then

$$p_c - p_s = \rho g h \qquad (2)$$

From (1) and (2), we have

$$\rho g h = \frac{1}{2}\rho r^2\omega^2 \text{ or } h = \frac{r^2\omega^2}{2g}$$

Now $\omega = 2\pi n$, where n is the number of revolutions per second. Thus

$$h = \frac{4\pi^2 r^2 n^2}{2g} = \frac{4 \times 10 \times (0.05)^2 \times (2)^2}{2 \times 10} = 2 \text{ cm}$$

13 Chapter Simple Harmonic Motion

REVIEW OF BASIC CONCEPTS

13.1 SIMPLE HARMONIC MOTION

Simple harmonic motion (SHM) is the simplest kind of oscillatory motion in which a body, displaced from its stable equilibrium position, oscillates to and fro about the position when released. If the displacement (x) from the equilibrium position is small, the restoring force (F) acting on the body is given by

$$F = -kx$$

where k is a positive constant, known as the force constant. In the SI system k is expressed in *newton per metre* (N m^{-1}). The acceleration (a) of the body is given by

$$a = \frac{F}{m} = -\frac{k}{m}x = -\omega^2 x$$

where m is the mass of the body and $\omega = \sqrt{\frac{k}{m}}$

Thus acceleration (a) = – constant × displacement x.

The resulting motion is called simple harmonic motion. Thus, *a simple harmonic motion is a motion in which the acceleration (i) is proportional to the displacement from the equilibrium position and (ii) is directed towards the equilibrium position.*

13.2 CHARACTERISTICS OF SHM

The displacement x in SHM at time t is given by

$$x = A \sin(\omega t + \phi)$$

where the three constants A, ω and ϕ characterize the SHM, i.e., they distinguish one SHM from another. A SHM can also be described by a cosine function as

$$x = A \cos(\omega t + \delta)$$

We will use the sine function. A cosine function is equally valid.

(1) Amplitude The amplitude of SHM is the maximum (positive or negative) value of the displacement from the equilibrium position. Quantity A (which is the coefficient of the sine or cosine function) is the amplitude of SHM.

(2) Time period The smallest time interval during which the motion repeats itself is called time period or simply period of SHM. The angular frequency ω is related to time period T as

$$\omega = \frac{2\pi}{T}$$

Frequency ν of SHM is the number of complete oscillations completed in 1 second.

Thus, $$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

or $$\omega = \frac{2\pi}{T} = 2\pi\nu$$

Since $\omega = \sqrt{\frac{k}{m}}$, we have

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Angular frequency ω is the coefficient of time t in the sine or cosine function.

(3) Phase The quantity $(\omega t + \phi)$ is called the phase of SHM at time t; it describes the state of motion at that instant. The quantity ϕ is the phase at time $t = 0$ and is called the phase constant or initial phase or epoch of the SHM. The phase constant is the time-independent term in the cosine or sine function.

13.3 VELOCITY AND ACCELERATION IN SHM

The velocity V or the particle in SHM is given by

$$V = \frac{dx}{dt} = A\ \omega \cos\ (\omega\ t + \phi)$$

But $\sin\ (\omega\ t + \phi) = \frac{x}{A}$

$\therefore \quad \cos\ (\omega\ t + \phi) = \left(1 - \frac{x^2}{A^2}\right)^{1/2}$

Hence velocity $V = A\ \omega \left(1 - \frac{x^2}{A^2}\right)^{1/2}$

$$= \omega\ (A^2 - x^2)^{1/2}$$

The acceleration a of the particle in SHM is given by

$$a = \frac{d^2x}{dt^2}$$

$$= -\ A\ \omega^2 \sin\ (\omega\ t + \phi) = -\ \omega^2\ x$$

Notice that, when the displacement is maximum, i.e., when $x = |A|$, the velocity is zero but the acceleration is maximum $= |\omega^2\ A|$. But when the displacement is zero $(x = 0)$, the velocity is maximum $= |\omega A|$ and the acceleration is zero.

13.4 ENERGY IN SHM

At any instant of time t, the kinetic energy of the oscillator is given by

$$\text{K.E.} = \frac{1}{2}mV^2 = \frac{1}{2}mA^2\omega^2 \cos^2(\omega t + \phi)$$

At any displacement x from the equilibrium position,

$$\text{K.E.} = \frac{1}{2}m\omega^2(A^2 - x^2)$$

because $\cos^2(\omega t + \phi) = 1 - \sin^2(\omega t + \phi) = 1 - \frac{x^2}{A^2}$.

At that instant t, the potential energy of the oscillator is given by

$$\text{P.E.} = \frac{1}{2}kx^2 = \frac{1}{2}\ m\omega^2 A^2 \sin^2(\omega t + \phi)$$

because $k = m\omega^2$ and $x = A \sin\ (\omega t + \phi)$. In terms of displacement x,

$$\text{P.E.} = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$

Total energy of the oscillator is

$$E = \text{K.E.} + \text{P.E.}$$

$$= \frac{1}{2}mA^2\omega^2\left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right]$$

$$= \frac{1}{2}mA^2\omega^2$$

E is also obtained as follows:

$$E = \text{K.E.} + \text{P.E.}$$

$$= \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2x^2)$$

$$= \frac{1}{2}mA^2\omega^2$$

Thus, although the kinetic energy and potential energy of a simple harmonic oscillator both change with time t and displacement x, the total energy is independent of both x and t and hence remains constant. This is expected because friction has been neglected.

NOTE

(1) At the mean position, $x = 0$, K.E. is maximum $= \frac{1}{2}\ mA^2\omega^2$ and P.E. = 0

(2) At the extreme position, $x = \pm A$, K.E. = 0 and P.E. is maximum $= \frac{1}{2}mA^2\omega^2$.

(3) E is constant $= \frac{1}{2}mA^2\omega^2$ for all values of x.

Figure 13.1 shows the variation K.E., P.E. and E with t for the case $\phi = 0$.

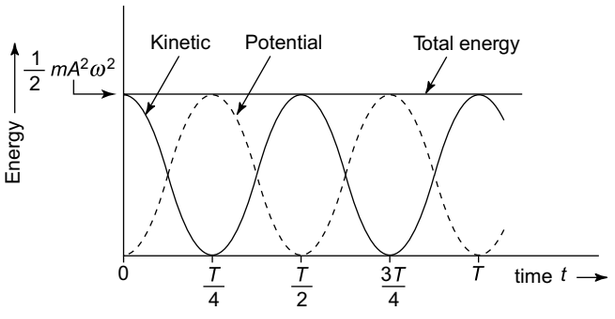


Fig. 13.1

The energy of an oscillator may decrease with time not only by friction (damping) but also due to radiation. The oscillating body imparts periodic motion to the particles of the medium in which it oscillates, thus producing waves. For example, a tuning fork or a string produces sound waves in the medium which results in a decrease in energy.

EXAMPLE 13.1

The displacement x of a body varies with time t as

$$x = a \sin\ (ct) + b \cos\ (ct)$$

where a, b and c are constants.

(a) Show that the motion of the body is simple harmonic.

(b) Find the amplitude A, time period T and phase constant ϕ of the motion in terms a, b and c.

SOLUTION

(a) Given $x = a \sin(ct) + b \cos(ct)$ (i)

Differentiating (i) w.r.t. time t, we get velocity

$$V = \frac{dx}{dt} = ac \cos(ct) - bc \sin(ct)$$

The acceleration is given by

$$a = \frac{dV}{dt} = \frac{d}{dt}[ac\cos(ct) - bc\sin(ct)]$$
$$= -c^2 [a \sin(ct) + b \cos(ct)]$$

$$\Rightarrow \quad a = -c^2 x \text{ [use Eq. (i)]}$$

Since $a \propto (-x)$, the motion is simple harmonic.

(b) Since the motion is simple harmonic;

$$x = A \sin(\omega t + \phi)$$
$$= A [\sin(\omega t) \cos\phi + \cos(\omega t) \sin\phi]$$
$$x = (A \cos\phi) \sin(\omega t) + (A \sin\phi) \cos(\omega t) \quad \text{(ii)}$$

Comparing (i) and (ii), we get

$$A \cos\phi = a \quad \text{(iii)}$$

$$A \sin\phi = b \quad \text{(iv)}$$

and $$\omega = c \Rightarrow \frac{2\pi}{T} = c \Rightarrow T = \frac{2\pi}{c}$$

Squaring and adding (iii) and (iv), we get

$$A = \sqrt{a^2 + b^2}$$

Dividing (iv) by (iii) we get

$$\tan\phi = \frac{b}{a} \Rightarrow \phi = \tan^{-1}\left(\frac{b}{a}\right)$$

EXAMPLE 13.2

Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) nonperiodic motion? Find the period for each periodic motion. Here k is a positive real constant.

(i) $\sin kt + \cos kt$, (ii) $\sin \pi t + 2\cos 2\pi t + 3 \sin 3\pi t$, (iii) $\cos\left(2kt + \frac{\pi}{3}\right)$, (iv) $\cos kt + 2 \sin^2 kt$, (v) e^{-kt} and (vi) $\log kt$.

SOLUTION

(i) $\sin kt + \cos kt$ represents simple harmonic motion since we can write it as $\sqrt{2} \sin\left(kt + \frac{\pi}{4}\right)$ or $\sqrt{2}\cos\left(kt - \frac{\pi}{4}\right)$. The coefficient of t in the argument of the sine or cosine function $= \frac{2\pi}{T}$, where T is the period. Hence $k = \frac{2\pi}{T}$ or $T = \frac{2\pi}{k}$, i.e. the period of the function is $\frac{2\pi}{k}$.

(ii) Each term in $(\sin\pi t + 2\cos 2\pi t + 3\sin 3\pi t)$ represents SHM.

The period T of the term $\sin\pi t$ is given by $\pi = \frac{2\pi}{T}$ or $T = 2$s. The period of the term $2\cos 2\pi t$ is 1 s, i.e. $T/2$ and the period of the term $3 \sin 3\pi t$ is 2/3 s, i.e. $T/3$. The sum of the three terms, however, does not represent SHM; it represents a periodic motion. By the time the first term completes one cycle, the second term completes two cycles and the third term completes three cycles. This shows that the sum represents a periodic motion with period $T = 2$s.

(iii) $\cos\left(2kt + \frac{\pi}{3}\right)$ represents an SHM whose period T is given by

$$2k = \frac{2\pi}{T}$$

or $$T = \frac{\pi}{k}$$

(iv) $\cos kt + 2 \sin^2 kt = \cos kt + (1 - \cos 2kt) = 1 + \cos kt - \cos 2kt$. The period of $\cos kt$ is $T = \frac{2\pi}{k}$ and that of $\cos 2kt$ is $\frac{\pi}{k} = \frac{T}{2}$. These two terms together have a period $T = \frac{2\pi}{k}$ as explained in (ii). The other term 1 is a constant independent of t and hence does not affect the period of the sum. Hence $(\cos kt + 2\sin^2 kt)$ represents a periodic motion with period $T = \frac{2\pi}{k}$.

(v) e^{-kt} decreases monotonically to zero as $t \to \infty$. It is an exponential function with a negative exponent of e, where $e \simeq 2.71828$. It is non-periodic.

(vi) Function log (kt) increases monotonically with time. Therefore, it never repeats itself and is a non-periodic function.

EXAMPLE 13.3

An oscillator has a time period $T = 4$s. If it is oscillating harmonically, find the time period of its kinetic energy.

SOLUTION

The kinetic energy of a simple harmonic oscillator varies with time as $\sin^2(\omega t)$, where $\omega = \dfrac{2\pi}{T}$.

Now $\sin^2(\omega t) = \dfrac{1}{2}(1 - \cos 2\omega t) = \dfrac{1}{2}(1 - \cos \omega' t)$,

where $\omega' = 2\omega \Rightarrow \dfrac{2\pi}{T'} = 2 \times \dfrac{2\pi}{T} \Rightarrow T' = \dfrac{T}{2}$.

Hence K.E. varies periodically with period $\dfrac{T}{2} = 2$ s.

NOTE

The P.E. also varies periodically with period $\dfrac{T}{2}$.

EXAMPLE 13.4

A body oscillates harmonically with amplitude 0.05 m. At a certain instant of time its displacement is +0.01 m and acceleration is –1.0 ms^{-2}. Find (a) the velocity of the oscillator at this instant and (b) the maximum velocity.

SOLUTION

$a = -\omega^2 x \Rightarrow -1.0 = -\omega^2 \times 0.01 \Rightarrow \omega = 10$ rad s^{-1}

Velocity when displacement $x = 0.01$ m is

$$V = \omega (A^2 - x^2)^{1/2}$$
$$= 10 \times [(0.05)^2 - (0.01)^2]^{1/2}$$
$$= 0.49 \text{ ms}^{-1}$$
$$V_{max} = A\omega = 0.05 \times 10 = 0.5 \text{ ms}^{-1}$$

EXAMPLE 13.5

The displacement x (in centimetres) of an oscillating particle varies with time t (in seconds) according to the equation

$$x = 2 \cos\left(0.5\pi t + \frac{\pi}{3}\right)$$

Find

(a) the amplitude of oscillation
(b) the time period of oscillation
(c) the maximum velocity of the particle
(d) the maximum acceleration of the particle.

SOLUTION

The displacement of the particle is given by

$$x = 2 \cos\left(0.5\pi t + \frac{\pi}{3}\right) \text{cm}$$

To find the amplitude and time period of the oscillation, we compare this equation with

$$x = A \cos(\omega t + \delta)$$

(a) Amplitude $A = 2$ cm

(b) Angular frequency $\omega = 0.5\pi$ rad s^{-1}

or $$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.5\pi} = 4\text{s}$$

(c) Maximum velocity $V_{max} = |A\omega|$
$$= 2 \times 0.5\pi = \pi \text{ cm s}^{-1}$$
$$= 3.14 \text{ cm s}^{-1}$$

(d) Maximum acceleration $a_{max} = |-\omega^2 A|$
$$= \omega^2 A$$
$$= (0.5\pi)^2 \times 2$$
$$= \frac{\pi^2}{2} \text{ cm s}^{-2}$$
$$= 4.935 \text{ cm s}^{-2}$$

EXAMPLE 13.6

A particle executes SHM of amplitude 25 cm and time period 3s. What is the minimum time required for the particle to move between two points located at 12.5 cm on either side of the mean position?

SOLUTION

Let A and B be the two extreme positions of the particle with O as the mean position. Displacements to the right of O are taken as positive while those to the left of O are taken as negative (Fig. 13.2)

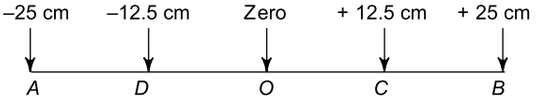


Fig. 13.2

Let the displacement of the particle in SHM be given by

$$x(t) = A \sin(\omega t + \phi) \qquad \text{(i)}$$

where $A = 25$ cm and $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{3}$ rad s^{-1}

Let us suppose that at time $t = 0$, the particle is at extreme position B. Setting $x = A$ at $t = 0$ in Eq. (i) we have

$$A = A \sin \phi$$

giving $\phi = \pi/2$.

Putting $\phi = \pi/2$ in Eq. (i), we get

$$x(t) = A \cos \omega t \qquad \text{(ii)}$$

Now let us say that the particle reaches point C at $t = t_1$ and point D at $t = t_2$. At C, the displacement $x(t_1) = +12.5$ cm and at D, it is $x(t_2) = -12.5$ cm (see Fig. 10.2). So from (ii) we have

$$+12.5 = 25 \cos \omega t_1$$

and $$-12.5 = 25 \cos \omega t_2$$

or $$\cos \omega t_1 = +0.5 \text{ or } \omega t_1 = \pi/3$$

and $$\cos \omega t_2 = -0.5 \text{ or } \omega t_2 = \frac{2\pi}{3}$$

Hence $$\omega(t_2 - t_1) = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$$

$$\therefore \quad t_2 - t_1 = \frac{\pi}{3\omega} = \frac{T}{6} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

or $$(t_2 - t_1)_{\min} = \frac{3}{6} = 0.5 \text{ s}$$

Notice that $\cos \omega t_2 = -0.5$ even for $t_2 = \frac{4\pi}{3}$. This value of t_2 does not correspond to the minimum time because this is the time at which the particle, moving to left, reaches A and then returns to D.

EXAMPLE 13.7

A particle is executing linear simple harmonic motion of amplitude A. At what displacement is the energy half kinetic and half potential?

SOLUTION

The energy will be half kinetic and half potential at a value of displacement x when K.E. = P.E., i.e.

$$\frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2 x^2$$

$$\Rightarrow \quad A^2 - x^2 = x^2$$

$$\Rightarrow \quad x = \frac{A}{\sqrt{2}}$$

EXAMPLE 13.8

A horizontal platform is executing SHM in the vertical direction with an amplitude of 1.0 m. A block of mass 5 kg is placed on the platform. What is the maximum frequency of platform's SHM so that the block is not detatched from the platform?

SOLUTION

The block will not be detatched from the platform, if the frequency of platform's SHM is such that the maximum acceleration of the platform equals the acceleration due to gravity, i.e.

$$A \omega^2_{\max} = g$$

$$\Rightarrow \quad \omega_{\max} = \sqrt{\frac{g}{A}} = \sqrt{\frac{9.8}{1.0}} = 3.13 \text{ rad s}^{-1}$$

$$\therefore \quad \nu_{\max} = \frac{\omega_{\max}}{2\pi} = \frac{3.13}{2 \times 3.14} \simeq 0.5 \text{ Hz}.$$

EXAMPLE 13.9

A body of mass m is attached by a string to a suspended spring of spring constant k. Both the string and the spring have negligible mass. The body is pulled down a distance A and released. Assuming that the string remains taut throughout the motion, find

(a) the maximum downward acceleration of the oscillating body and

(b) the maximum amplitude for which the string remains taut.

SOLUTION

(a) As long as the string remains taut, the restoring force will be proportional to displacement from the mean position. The motion of the body is simple harmonic whose amplitude is A and angular frequency is

$$\omega = \sqrt{\frac{k}{m}}$$

The maximum acceleration is $\omega^2 A = \frac{kA}{m}$

(b) If the tension in the string is T, the downward acceleration is

$$a = g - \frac{T}{m}$$

For the string to remain taut $T > 0$. Therefore, the maximum downward acceleration cannot exceed g, i.e.

$$a_{\max} = g$$

$$\Rightarrow \quad \omega^2 A_{\max} = g$$

$$\Rightarrow \quad A_{\max} = \frac{g}{\omega^2} = \frac{mg}{k} \qquad \left(\because \omega^2 = \frac{k}{m}\right)$$

EXAMPLE 13.10

At time $t = 0$, the displacement of a simple harmonic oscillator from the mean position is x_0 and the velocity is v_0. Obtain the expressions for the amplitude and phase constant of the oscillator in terms of x_0, v_0 and ω where ω is the angular frequency of the oscillator.

SOLUTION

For a simple harmonic oscillator

$$x = A \sin(\omega t + \phi) \quad \text{(i)}$$

and $$v = \frac{dx}{dt} = A\omega \cos(\omega t + \phi) \quad \text{(ii)}$$

Putting $t = 0$ and $x = x_0$ and $v = v_0$ in (i) and (ii), we have

$$x_0 = A \sin\phi \quad \text{(iii)}$$

and $$v_0 = A\omega \cos\phi \quad \text{(iv)}$$

From (iii) and (iv), we get

$$A = \left(x_0^2 + \frac{v_0^2}{\omega^2}\right)^{1/2}$$

and $$\tan\phi = \frac{\omega x_0}{v_0} \Rightarrow \phi = \tan^{-1}\left(\frac{\omega x_0}{v_0}\right)$$

NOTE

If at $t = 0$, $x = x_0$ and $v_0 = 0$, then $A = x_0$ and $\phi = \frac{\pi}{2}$. On the other hand, if at $t = 0$, $x = 0$ and $v = v_0$ then $A = \frac{v_0}{\omega}$ and $\phi = 0$

13.5 EXPRESSIONS FOR TIME PERIOD OF MASS-SPRING SYSTEM

(1) *Horizontal Oscillations of a Mass-Spring System*

Consider a block of mass m placed one horizontal frictionless surface and attached to a spring of negligible mass and spring constant k as shown in Fig. 13.3.

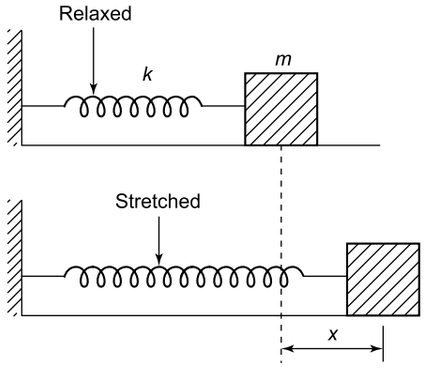


Fig. 13.3

The block is pulled to the right by a small distance x from the equilibrium position and released. The restoring force on the block is $F = -kx$. The acceleration of the block is

$$a = \frac{F}{m} = -\left(\frac{k}{m}\right)x \quad \text{(i)}$$

Since $a \propto (-x)$, the motion of the block is simple harmonic. Comparing Eq. (i) with $a = -\omega^2 x$, we get

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

$$\therefore \quad \text{Time period } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

(2) *Vertical Oscillations of a Mass-spring System*

Consider a massless spring suspended from a support. [Fig. 13.4 (a)]. A block of mass m is attached at the lower end, as a result, the string extends by an amount d given by [Fig. 13.4 (b)].

$$F = kd \Rightarrow mg = kd \quad \text{(i)}$$

This is the equilibrium state of the system.

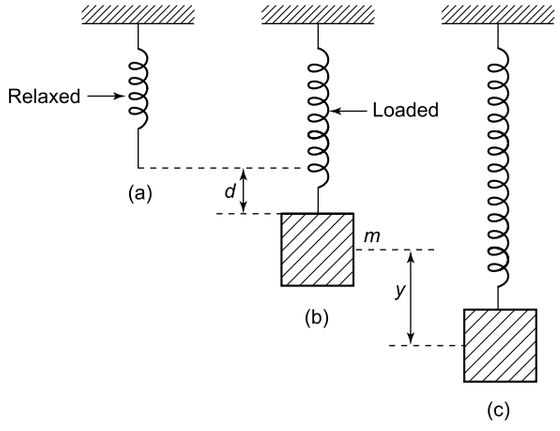


Fig. 13.4

When the body is pulled through a distance y from this position and released [Fig. 10.4 (c)], the restoring force is $F = -ky$ and the acceleration of the block is

$$a = \frac{F}{m} = -\left(\frac{k}{m}\right)y$$

Hence the motion is simple harmonic whose time period is

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(ii)}$$

Equation (i) determines k. Time period given by Eq. (ii) is the same as for horizontal oscillation. It depends only on m and k and is independent of gravity.

(3) Parallel Combination of Springs

Figure 13.5 shows the equilibrium state of a block connected to two springs which are joined in parallel. If the block is pulled down through a distance x, the extension produced in each spring will be x. The restoring forces in the springs are $F_1 = -k_1 x$ and $F_2 = -k_2 x$.

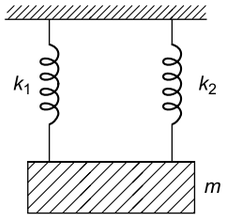


Fig. 13.5

Total restoring force $F = F_1 + F_2 = -(k_1 + k_2)x = -(k_p)\,x$ where $k_p = k_1 + k_2$ is the effective force constant of the parallel combination. The time period is given by

$$T = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$$

(4) Series Combination of Springs

Figure 13.6 shows the equilibrium state of a block connected to two springs which are joined in series. The block is pulled down by a distance x. Let x_1 and x_2 be the extensions produced in the springs. The restoring force in each spring will be the same equal to

$$F = -k_1 x_1 = -k_2 x_2$$

so that $$x_1 = -\frac{F}{k_1} \text{ and } x_2 = -\frac{F}{k_2}$$

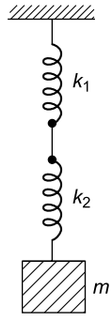


Fig. 13.6

Total extension $$x = x_1 + x_2$$

$$= -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= -F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

or $$F = -\left(\frac{k_1 k_2}{k_1 + k_2}\right) x = -k_s\, x$$

where $k_s = \dfrac{k_1 k_2}{k_1 + k_2}$ is the effective force constant of the series combination. The time period is

$$T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

(5) A Block Connected between Two Springs

Figure 13.7 shows the equilibrium state of a block connected between two springs

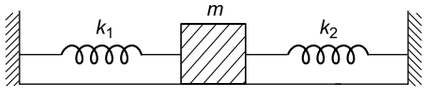


Fig. 13.7

If the block is displaced through a distance x, say to the right, the spring k_1 is extended by x and spring k_2 is compressed by x so that the restoring force exerted by each spring on the block is in the same direction (along the left). If F_1 and F_2 are the restoring forces, $F_1 = -k_1 x$ and $F_2 = -k_2 x$, the total restoring force is

$$F = -(k_1 + k_2)x = -kx$$

where $k = (k_1 + k_2)$ is the effective force constant of the system. The time period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$$

EXAMPLE 13.11

If a spring of force constant k is cut into two equal halves, what is the force constant of each half.

SOLUTION

If a force F produces an extension x in the spring, then

$$F = kx \qquad \text{(i)}$$

Since the extension produced by a force is proportional to the length of the spring, if a spring is cut into two equal halves, the same force F will produce an extension $x' = x/2$ in half the spring. If k' is the force constant of half the spring,

$$F = k'\,x' = k'\left(\frac{x}{2}\right) \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get $k' = 2k$.

EXAMPLE 13.12

Two blocks, each of mass m, are connected by a spring of force constant k and placed on a horizontal frictionless surface as shown in Fig. 13.8. Equal force F is applied to each block as shown. Find time period of the system when the force is removed.

SOLUTION

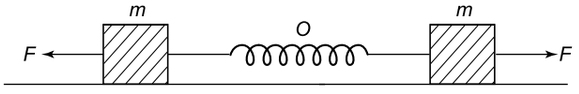


Fig. 13.8

When a force F is applied at each end of a spring, every coil of the spring is not elongated. The coil at point O in the middle of the spring is not elongated. This situation is the same as two springs each of length $l/2$ (where l is the length of the complete spring) joined to each other at point O. If k is the force constant of the complete spring, the force constant of each half = $2k$. Hence

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

EXAMPLE 13.13

A spring has a natural length of 50 cm and a force constant of 2.0×10^3 Nm^{-1}. A body of mass 10 kg is suspended from the spring. (i) What is the stretched length of the spring? (ii) If the body is pulled down further stretching the spring to a length of 58 cm, and then released, what is the frequency and amplitude of oscillation. Neglect the mass of the spring.

SOLUTION

Natural length of the spring = 50 cm = 0.50 m

$$k = 2.0 \times 10^3 \text{ N m}^{-1}, \quad m = 10 \text{ kg}$$

(i) The extension produced in the spring is

$$y = \frac{mg}{k} = \frac{10 \times 9.8}{2.0 \times 10^3}$$
$$= 0.049 \text{ m} = 4.9 \text{ cm}$$

$\therefore$ Stretched length of the spring = 0.50 + 0.049 = 0.549 m = 54.9 cm

(ii) The frequency of oscillation is given by

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi} \times \sqrt{\frac{2.0 \times 10^3}{10}}$$
$$= 2.25 \text{ Hz}$$

The length of the spring in the equilibrium position = 54.9 cm. If it is pulled to a length of 58 cm, the maximum displacement from the equilibrium position = 58 – 54.9 = 3.1 cm, which is the amplitude of oscillation.

EXAMPLE 13.14

A small trolley of mass 2.0 kg resting on a horizontal frictionless turntable is connected by a light spring to the centre of the table. The relaxed length of the spring is 35 cm. When the turntable is rotated at a speed of 300 rev/min, the length of the spring becomes 40 cm. Find the force constant of the spring.

SOLUTION

Mass of trolley (m) = 2.0 kg

Frequency of rotation (ν) = 300 rev. min^{-1} = $\frac{300}{60}$ = 5 rev. s^{-1}

$\therefore$ Angular frequency (ω) = $2\pi\nu = 2\pi \times 5 = 10\pi$ rad s^{-1}

The radius of the circle along which the trolley moves is

$$r = 40 \text{ cm} = 0.4 \text{ m}$$

When the table is rotated, the tension in the spring is equal to the centripetal force, i.e.

$$T = \frac{mv^2}{r} = mr\omega^2$$
$$= 2.0 \times 0.4 \times (10\pi)^2 = 790 \text{ N}$$

Now, extension produced in the spring by this force

$$= 40 - 35$$
$$= 5 \text{ cm} = 0.05 \text{ m}$$

$\therefore$ Force constant of spring = $\frac{\text{force}}{\text{extension}} = \frac{790\,\text{N}}{0.05\,\text{m}}$

$$= 1.58 \times 10^4 \text{ Nm}^{-1}$$
$$\simeq 1.6 \times 10^4 \text{ Nm}^{-1}$$

EXAMPLE 13.15

A tray of mass M = 12 kg is supported on two identical springs as shown in Fig. 13.9. When the tray is depressed a little and released, it executes an SHM of period 1.5s. (a) Find the force constant of each spring. (b) When a block of mass m is placed on the tray, the period of the SHM becomes 3.0s. Find the value of m.

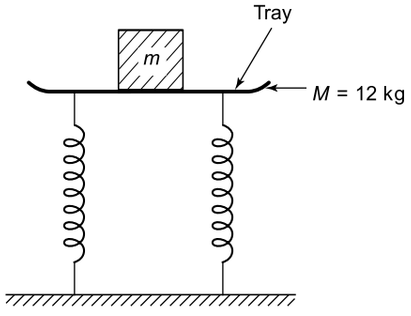


Fig. 13.9

SOLUTION

(a) The time period of oscillation of M is given by

$$T = 2\pi\sqrt{\frac{M}{2k}}$$

where k is the force constant of each spring. Therefore

$$k = \frac{2\pi^2 M}{T^2} = \frac{2 \times (3.142)^2 \times 12}{(1.5)^2}$$
$$= 105.3 \text{ N m}^{-1}$$

(b) The new period T_1 is now given by

$$T_1 = 2\pi\sqrt{\frac{M+m}{2k}}$$

so that $m + M = \dfrac{T_1^2 k}{2\pi^2}$

or $\quad m + 12 = \dfrac{(3)^2 \times 105.3}{2\times(3.142)^2} = 48$

or $\quad m = 36$ kg

13.6 EXPRESSIONS FOR TIME PERIOD OF SOME OTHER SYSTEMS

(1) A Ball Oscillating in a Concave Mirror

A small spherical steel ball is placed a little away from the centre of a concave mirror whose radius of curvature is R. When the ball is released, it begins to oscillate about the centre.

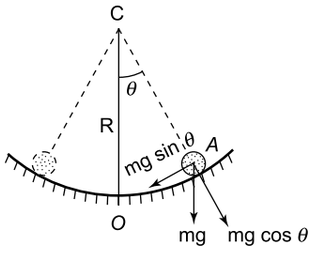


Fig. 13.10

Place a small steel ball at A, a little away from the centre O of a concave mirror of radius of curvature R (= $OC = AC$) as shown in Fig. 13.10. Let $\angle ACO = \theta$. If m is the mass of the ball, its weight mg acts vertically downwards at A. This force is resolved into two rectangular components: $mg\cos\theta$ (which is balanced by the reaction of the mirror) and $mg\sin\theta$ (which provides the restoring force F). Thus

$$F = -mg\sin\theta$$

$$= -mg\,\theta$$

(since θ is small, R being very large)

$$= -\frac{mg\,x}{R}$$

($\therefore\ x = R\theta$; x being the arc OA)

$$= -Kx$$

where force constant $K = mg/R$. Thus the motion is harmonic and the angular frequency is given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{R}}$$

$$\therefore \qquad T = 2\pi\sqrt{\frac{R}{g}}$$

(2) Oscillation of a Liquid in a U-tube

The column of the liquid is displaced through y by gently blowing into the tube (Fig. 13.11). The columns exhibit vertical oscillations. Let L, A and ρ be respectively, the length of the liquid column, area of cross-section of the tube and density of the liquid. We shall neglect viscous effects. Since the right-hand side column is higher by $2y$, with respect to the column on the left-hand side, the mass of this column of liquid is $m = 2A\rho\, y$. The restoring force (which is a gravitational force) is given by

$$F = -mg = -2A\rho g y = -Ky$$

where the force constant $K = 2A\rho g$. The angular frequency of the harmonic oscillation is

$$\omega = \sqrt{\frac{K}{M}}$$

where $M = \rho AL$ is the total mass of the liquid in oscillation. Thus

$$\omega = \sqrt{\frac{2A\rho g}{\rho AL}} = \sqrt{\frac{2g}{L}}$$

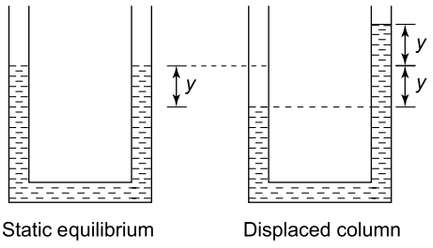


Fig. 13.11 Oscillations of a liquid column

The time period of oscillation is

$$T = 2\pi\sqrt{\frac{L}{2g}}$$

It is interesting to note that the period of oscillation does not depend on the density of the liquid or the area of cross-section of the tube.

(3) Oscillation of Floating Vertical Cylindrical Body

A cylindrical piece of cork of height h and density ρ_c floats in a liquid of density ρ_l. The cork is depressed slightly and released.

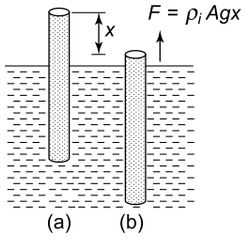


Fig. 13.12

Let A be the cross-sectional area of the cork and M its mass. Fig. 13.12 (a) shows the static equilibrium, the weight of the cork being balanced by the weight of the liquid it displaces.

If the cork is depressed through a distance x, as shown in Fig. 13.12 (b), the buoyant force on it increases by $\rho_l Agx$, because $\rho_l Ax$ is the mass of the liquid displaced by dipping, g being the acceleration due to gravity. If viscous effects are neglected, the restoring force on the cork is given by

$$F = -\rho_l Agx = -Kx$$

where $K = \rho_l Ag$. Since $F \propto -x$, the motion of the cork is simple harmonic. The time period of the motion is

$$T = 2\pi\sqrt{\frac{M}{K}}$$

where M is the mass of the cork $= Ah\rho_c$. Hence

$$T = 2\pi\sqrt{\frac{Ah\rho_c}{\rho_l Ag}} = 2\pi\sqrt{\frac{h\rho_c}{g\rho_l}}$$

(4) A Simple Pendulum

A simple pendulum consists of a massless inextensible string fixed at one end O and having a small bob at the other end (Fig. 13.13). When the bob is displaced from equilibrium position A to a position B and released, the component $mg\cos\theta$ of its weight balances with tension T and it returns under a restoring force

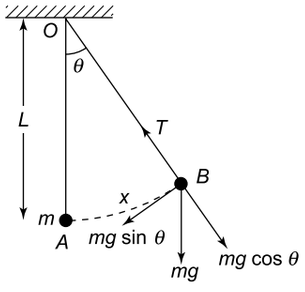


Fig. 13.13

$$F = -mg\sin\theta$$

If θ is small, $\sin\theta \simeq \theta$. Also $\theta = \frac{x}{L}$

$$\therefore \quad F = -\left(\frac{mg}{L}\right)x$$

$$\therefore \quad \text{Acceleration } a = \frac{F}{m} = -\left(\frac{g}{L}\right)x.$$

$$\text{Time period } T = 2\pi\sqrt{\frac{L}{g}}$$

NOTE

T is independent of the mass of the bob.

(5) A Compound Pendulum

A compound pendulum is a rigid body capable of oscillating about an axis. The pendulum consists of a rod of mass M and length L which is pivoted at O and carries a bob of mass m at the other end as shown in Fig. 13.14. The rod is displaced and released.

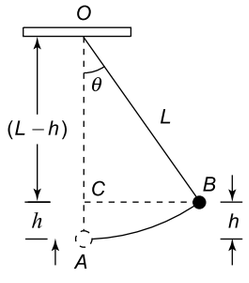


Fig. 13.14

The pendulum tends to return under the influence of a restoring torque

$$\tau = -mgL\sin\theta = -mgL\theta \qquad \text{(for small } \theta\text{)}$$

If I is the moment of inertia of the system about the axis passing through O and perpendicular to the plane of the rod, the angular acceleration is

$$\alpha = \frac{\tau}{I} = -\left(\frac{mgL}{I}\right)\theta$$

since $\alpha \propto -\theta$, the motion is simple harmonic whose angular frequency is (compare with $\alpha = -\omega^2\theta$) given by

$$\omega^2 = \frac{mgL}{I} \Rightarrow \omega = \sqrt{\frac{mgL}{I}}$$

Thus $$T = 2\pi\sqrt{\frac{I}{mgL}}$$

where $$I = \frac{ML^2}{3} + mL^2 = \frac{1}{3}(M + 3m)L^2.$$

Thus $$T = 2\pi\sqrt{\frac{(M+3m)L}{3mg}}$$

If $M << m$, $T = 2\pi\sqrt{\frac{L}{g}}$, the same as that of a simple pendulum.

(6) Horizontal Oscillations of a Cylinder-Spring System

A solid cylinder of mass M and radius R is connected to a spring of force constant k as shown in Fig. 13.15.

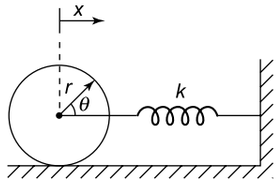


Fig. 13.15

Case (a): Cylinder slips without rolling.

The total energy of the system is translational K.E. + P.E. If x is the instantaneous displacement and v the velocity of the cylinder, the total energy is

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}kx^2 \qquad \text{(i)}$$

If friction is neglected, E = constant. Hence $\frac{dE}{dt} = 0$. Differentiating (i) w.r.t. time t and setting $\frac{dE}{dt} = 0$, we get

$$0 = Mv\frac{dv}{dt} + kx\frac{dx}{dt}$$

$$\Rightarrow \quad 0 = Mva + kxv \quad \left(\because a = \frac{dv}{dt}, v = \frac{dx}{dt}\right)$$

$$\Rightarrow \quad a = \left(-\frac{k}{M}\right)x$$

$$\therefore \quad T = 2\pi\sqrt{\frac{M}{k}}$$

Case (b): Cylinder rolls without slipping

In this case, the total energy of the system is

$$E = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 + \frac{1}{2} kx^2$$

$$= \frac{1}{2} Mv^2 \left(1 + \frac{I}{MR^2}\right) + \frac{1}{2} kx^2$$

$$\left(\because \omega = \frac{v}{R}\right)$$

Setting $\frac{dE}{dt} = 0$, we get

$$0 = Mv \frac{dv}{dt} \left(1 + \frac{I}{MR^2}\right) + kx \frac{dx}{dt}$$

$$0 = Mva \left(1 + \frac{I}{MR^2}\right) + kxv$$

$$\Rightarrow \quad a = -\left[\frac{k}{\left(M + \frac{I}{R^2}\right)}\right] x$$

$$\therefore \quad T = 2\pi \left[\frac{\left(M + \frac{I}{R^2}\right)}{k}\right]^{1\backslash 2}$$

This the general formula. For a solid cylinder, $I = \frac{1}{2} MR^2$ and then we get

$$T = 2\pi \sqrt{\frac{3M}{2k}}$$

(7) Vertical Oscillations Mass-spring System Connected by a Pulley

The block is pulled out through a small distance x and released. If M is the mass of the pulley and R its radius, the total energy of the system is [Fig. 13.16]

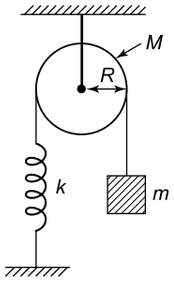


Fig. 13.16

$$E = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 + \frac{1}{2} kx^2$$

where $I = \frac{1}{2} MR^2$ and $\omega = \frac{v}{R}$. Thus

$$E = \frac{1}{2} \left(m + \frac{1}{2} M\right) v^2 + \frac{1}{2} kx^2$$

Setting $\frac{dE}{dt} = 0$, we have

$$0 = \left(m + \frac{M}{2}\right) v \frac{dv}{dt} + kx \frac{dx}{dt}$$

$$\Rightarrow \quad \text{Acceleration } a = \frac{dv}{dt} = -\frac{kx}{(m + \frac{M}{2})} \quad \left(\because v = \frac{dx}{dt}\right)$$

$$= -\omega^2 x$$

$$\Rightarrow \quad \omega = \sqrt{\frac{\left(m + \frac{M}{2}\right)}{k}}$$

$$\Rightarrow \quad T = 2\pi \sqrt{\frac{k}{\left(m + \frac{M}{2}\right)}}$$

If the pulley has a mass $M << m$, then

$$T = 2\pi \sqrt{\frac{k}{m}}$$

(8) A Block Attached to Three Springs

A block of mass m is connected to three identical springs as shown in Fig. 13.17. The block is pushed towards spring 1 through a small distance so that spring 1 is compressed by x and springs 2 and 3 are extended by $x_2 = x_3 = x \cos 45° = x/\sqrt{2}$. When the block is released, the restoring force acting on it is

$$\vec{F} = -\left(\vec{F_1} + \vec{F_2} + \vec{F_3}\right)$$

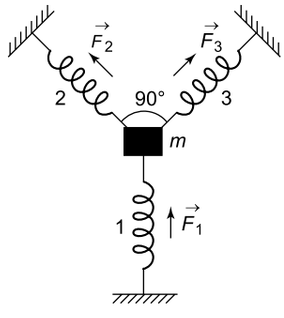


Fig. 13.17

The magnitude of $\vec{F}$ along the vertical direction is

$$F = -(F_1 + F_2 \cos 45° + F_3 \cos 45°)$$

$$= -\left(kx + kx_2 \times \frac{1}{\sqrt{2}} + kx_3 \times \frac{1}{\sqrt{2}}\right)$$

$$= -\left(kx + \frac{kx}{2} + \frac{kx}{2}\right)$$

$$= -2kx$$

$\therefore$ Acceleration $a = \dfrac{F}{m} = -\dfrac{2k}{m}x = -\omega^2 x$. Hence

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

13.7 IMPORTANT TIPS AND ADDITIONAL FORMULAE

(i) Mass m suspended from a spring of force constant k

$$T = 2\pi\sqrt{\frac{m}{k}}$$

(ii) If a spring of spring constant k is cut into n equal parts, the spring constant of each part becomes $n\,k$. If each part is loaded with a mass m, the time period of each will be

$$T = 2\pi\sqrt{\frac{m}{nk}}$$

(iii) If two springs of spring constants k_1 and k_2 are connected in parallel, the spring constant of the combination is $k = k_1 + k_2$. If the combination is loaded with mass m, then

$$T = 2\pi\left[\frac{m}{(k_1 + k_2)}\right]^{1/2}$$

(iv) If two springs of spring constants k_1 and k_2 are connected in series, the spring constant of the combination is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

in which case

$$T = \left[\frac{m(k_1 + k_2)}{(k_1 k_2)}\right]^{1/2}$$

(v) If two masses m_1 and m_2 and connected to the ends of a spring of force constant k, the time period of the oscillations of the system is

$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

where $\mu = \dfrac{m_1 m_2}{(m_1 + m_2)}$ is the reduced mass.

(vi) Simple pendulum of length l

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(vii) Liquid column of length L in a U-tube

$$T = 2\pi\sqrt{\frac{L}{2g}}$$

(viii) A pole of mass M and area of cross-section A floating in a liquid of density ρ

$$T = 2\pi\sqrt{\frac{M}{\rho A g}}$$

(ix) LC circuit consisting of inductance L and capacitance C

$$T = 2\pi\sqrt{LC}$$

(x) Simple pendulum immersed in a liquid

$$T = 2\pi\sqrt{\frac{l}{g'}} \text{ where } g' = g\left(1 - \frac{\sigma}{\rho}\right)$$

σ = density of liquid, ρ = density of bob.

(xi) Simple pendulum in a lift moving up with acceleration a

$$T = 2\pi\sqrt{\frac{l}{(g + a)}}$$

If the lift is moving down with acceleration a ($< g$)

$$T = 2\pi\sqrt{\frac{l}{(g - a)}}$$

If the lift is falling freely $a = g$ and $T = \infty$, frequency $\nu = 0$.

(xii) Simple pendulum in a trolley moving with acceleration a in horizontal direction.

$$T = 2\pi\sqrt{\frac{l}{g'}} \text{ where } g' = \sqrt{g^2 + a^2}$$

(xiii) Simple pendulum in a trolley moving down an inclined plane of inclination θ

$$T = 2\pi\sqrt{\frac{l}{g\sin\theta}}$$

(xiv) For a seconds pendulum $T = 2$s. Hence $l = 99$ cm $\simeq 1$ m.

(xv) An extended body pivoted at point P at a distance h from its centre of mass (Fig. 13.18) and oscillating in the vertical plane.

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

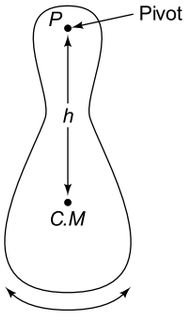


Fig. 13.18

where I = moment of inertia of the body about the pivot.

(a) If the body is a rod of mass m and length l (Fig. 13.19) pivoted at its end, then

$$h = \frac{l}{2} \text{ and } I = \frac{ml^2}{3}.$$

Hence $$T = 2\pi\sqrt{\frac{2l}{3g}}$$

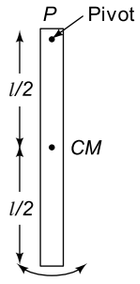


Fig. 13.19

(b) If the body is a ring of mass m and radius r pivoted at a point P on its periphery (Fig. 13.20), then

$h = r$ and $I = mr^2 + mr^2 = 2mr^2$.

Hence $$T = 2\pi\sqrt{\frac{2r}{g}}$$

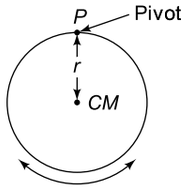


Fig. 13.20

For a disc pivoted at a point on its periphery,

$h = r$ and $I = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$.

Hence

$$T = 2\pi\sqrt{\frac{3r}{2g}}$$

(xvi) Some cases of oscillation of spring-mass-pulley system. The mass of pulley is negligible.

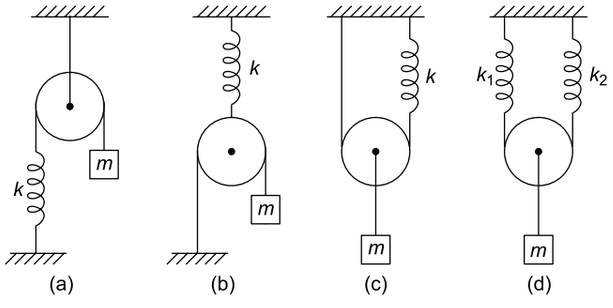


Fig. 13.21

Fig. 13.21 (a) $\Rightarrow T_a = 2\pi\sqrt{\frac{m}{k}}$

Fig. 13.21 (b) $\Rightarrow T_b = 2\pi\sqrt{\frac{4m}{k}}$

Fig. 13.21 (c) $\Rightarrow T_c = 2\pi\sqrt{\frac{m}{4k}}$

Fig. 13.21 (d) $\Rightarrow T_d = 2\pi\sqrt{\frac{m(k_1 + k_2)}{4k_1k_2}}$

(xvii) Horizontal oscillation (without slipping) of a disc, ring, cylinder or sphere of mass M and radius R rolling on a horizontal surface (Fig. 13.22).

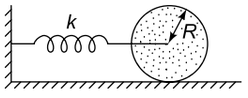


Fig. 13.22

$$T = 2\pi\sqrt{\frac{\left(M + \frac{I}{R^2}\right)}{k}}$$

13.8 DAMPED OSCILLATIONS

Oscillations under the influence of damping (or frictional) force are called damped oscillations. Due to damping the amplitude (and hence energy) of the oscillator keeps on decreasing with time and eventually the oscillator comes to rest. Damping also decreases the frequency of the oscillator.

13.9 FORCED OSCILLATIONS AND RESONANCE

The oscillations of a system under the influence of an external periodic force are called forced oscillations. The external force maintains the oscillations of a damped oscillator. The amplitude of these oscillations remains constant.

If the frequency of the externally applied force is equal to the natural frequency of the oscillator, resonance is said to occur. If damping is small, the amplitude of resonant oscillations will become very large. At resonance, the oscillator absorbs maximum energy supplied by the external force.

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. A uniform rod AB of length L is pivoted at one end A and hangs vertically. The time period of small oscillations about an axis passing through A and perpendicular to the rod is

(a) $2\pi\sqrt{\frac{L}{g}}$ (b) $2\pi\sqrt{\frac{L}{2g}}$

(c) $2\pi\sqrt{\frac{L}{3g}}$ (d) $\pi\sqrt{\frac{L}{2g}}$

2. The displacement of a particle executing simple harmonic motion is given by
$x = a \sin \omega t + a \cos \omega t$
The total energy of the particle is

(a) $\frac{1}{2} m a^2 \omega^2$ (b) $m a^2 \omega^2$

(c) $\frac{1}{4} m a^2 \omega^2$ (d) $2 m a^2 \omega^2$

3. In Q.2 above, the phase constant of the simple harmonic motion is

(a) zero (b) 30°

(c) 45° (d) 90°

4. The displacement of a particle executing simple harmonic motion varies with time t as
$x = a \sin \omega t + a \sin\left(\omega t + \frac{\pi}{3}\right)$
The amplitude of oscillation is

(a) a (b) $\sqrt{2}\, a$

(c) $\sqrt{3}\, a$ (d) $2a$

5. A block of mass m is attached to a spring of force constant k by means of a string going over a frictionless pulley as shown in Fig. 13.23. The block is held in position so that the spring is unstretched. The block is then released and it begins to oscillate with a small amplitude. The maximum velocity of the block during oscillation is

(a) $\frac{mg}{k}$ (b) $\sqrt{\frac{mg}{k}}$

(c) $g\sqrt{\frac{k}{m}}$ (d) $g\sqrt{\frac{m}{k}}$

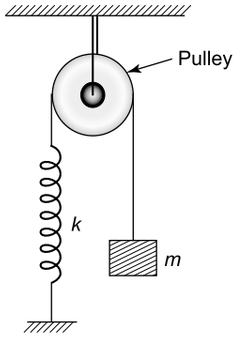


Fig. 13.23

6. A body is executing simple harmonic motion of time period T. Its kinetic energy varies periodically with time period equal to

(a) $\frac{T}{2}$ (b) T

(c) $\sqrt{2}\ T$ (d) $2\ T$

7. A particle is executing a linear simple harmonic motion. At time $t = 0$, it is at one extreme position and travels a distance 2 cm in the first second and 5 cm in the next second moving in the same direction. The amplitude of the motion is

(a) 8 cm (b) 10 cm

(c) 12 cm (d) 14 cm

8. A body of mass 200 g executing SHM has a velocity of 3 cm s^{-1} when its displacement is 4 cm and a velocity of 4 cm s^{-1} when its displacement is 3 cm. The total energy of the oscillator is

(a) 2.5×10^{-4} J (b) 2.5×10^{-2} J

(c) 2.5 J (d) 250 J

9. The percentage change in the time period of a simple pendulum if its length is increased by 2% is

(a) 4% (b) 2%

(c) 1% (d) $\sqrt{2}$ %

10. Two springs of equal lengths and equal cross-sectional areas are made of materials whose Young's modulii are in the ratio of 3 : 2. They are suspended and loaded with the same mass. When stretched and released, they will oscillate with time periods in the ratio of

(a) $\sqrt{3} : \sqrt{2}$ (b) 3 : 2

(c) $3\sqrt{3} : 2\sqrt{2}$ (d) 9 : 4

11. Two bodies A and B of equal masses are suspended from two separate springs of force constants k_1 and k_2 respectively. If the two bodies oscillate such that their maximum velocities are equal, the ratio of the amplitudes of oscillation of A and B will be

(a) k_1/k_2 (b) $\sqrt{k_1/k_2}$

(c) k_2/k_1 (d) $\sqrt{k_2/k_1}$

IIT, 1988

12. A spring of force constant k is cut into three equal pieces. If these three pieces are connected in parallel, the force constant of the combination will be

(a) $k/3$ (b) $k/9$

(c) $3k$ (d) $9k$

13. The displacement x (in centimetres) of an oscillating particle varies with time t (in seconds) as

$$x = 2 \cos \left(0.5\pi t + \frac{\pi}{3}\right)$$

The magnitude of the maximum acceleration of the particle in cms^{-2} is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi^2}{2}$ (d) $\frac{\pi^2}{4}$

14. A particle is executing linear simple harmonic motion of amplitude A. What fraction of the total energy is kinetic when the displacement is half the amplitude?

(a) $\frac{1}{4}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

15. In Q.14, at what displacement is the energy of the oscillator half potential and half kinetic?

(a) $\frac{A}{4}$ (b) $\frac{A}{2}$

(c) $\frac{A}{\sqrt{2}}$ (d) $\frac{A}{\sqrt{3}}$

16. A horizontal platform is executing simple harmonic motion in the vertical direction of frequency ν. A block of mass m is placed on the platform. What is the maximum amplitude of the platform so that the block is not detached from it?

(a) $\frac{g}{4\pi^2\nu^2}$ (b) $\frac{mg}{4\pi^2\nu^2}$

(c) $\frac{g}{2\pi^2\nu^2}$ (d) $\frac{mg}{2\pi^2\nu^2}$

17. A spring of negligible mass having a force constant k extends by an amount y when a mass m is hung from it. The mass is pulled down a little and released. The system begins to execute simple harmonic motion of amplitude A and angular frequency ω. The total energy of the mass–spring system will be

(a) $\frac{1}{2}mA^2\omega^2$ (b) $\frac{1}{2}mA^2\omega^2 + \frac{1}{2}ky^2$

(c) $\frac{1}{2}ky^2$ (d) $\frac{1}{2}mA^2\omega^2 - \frac{1}{2}ky^2$

18. A small trolley of mass 2 kg resting on a horizontal frictionless turntable is connected by a light spring to the centre of the table. The relaxed length of the spring is 35 cm. When the turntable is rotated with an angular frequency of 10 rad s^{-1}, the length of the spring becomes 40 cm. What is the force constant of the spring?

(a) 1.2×10^3 Nm^{-1} (b) 1.6×10^3 Nm^{-1}

(c) 2.0×10^3 Nm^{-1} (d) 2.4×10^3 Nm^{-1}

19. A simple pendulum of length l and bob mass m is displaced from its equilibrium position O to a position P so that the height of P above O is h. It is then released. What is the tension in the string when the bob passes through the equilibrium position O? Neglect friction.

(a) mg (b) $\frac{2mgh}{l}$

(c) $mg\left(1+\frac{h}{l}\right)$ (d) $mg\left(1+\frac{2h}{l}\right)$

20. When a mass m is hung from the lower end of a spring of negligible mass, an extension x is produced in the spring. The mass is set into vertical oscillations. The time period of oscillation is

(a) $T = 2\pi\sqrt{\frac{x}{mg}}$ (b) $T = 2\pi\sqrt{\frac{gx}{m}}$

(c) $T = 2\pi\sqrt{\frac{x}{g}}$ (d) $T = 2\pi\sqrt{\frac{x}{2g}}$

21. A small spherical steel ball is placed a little away from the centre of a large concave mirror of radius of curvature $R = 2.5$ m. The ball is then released. What is the time period of the motion? Neglect friction and take $g = 10$ ms^{-2}.

(a) $\frac{\pi}{4}$ sec (b) $\frac{\pi}{2}$ sec

(c) π sec (d) 2π sec

22. The potential energy of a particle executing simple harmonic motion at a distance x from the equilibrium position is proportional to

(a) $\sqrt{x}$ (b) x

(c) x^2 (d) x^3

23. The displacement y of a particle executing simple harmonic motion is given by

$$y = 4 \cos^2\left(\frac{t}{2}\right) \sin (1000\ t)$$

This expression may be considered to be a result of the superposition of how many simple harmonic motions?

(a) two (b) three

(c) four (d) five

IIT, 1992

24. The kinetic energy of a particle executing S.H.M. is 16 J, when it is at its mean position. If the amplitude of oscillation is 25 cm and the mass of the particle is 5.12 kg, the time period of oscillation is

(a) $\frac{\pi}{5}$ sec. (b) 2π sec.

(c) $20\ \pi$ sec. (d) $5\ \pi$ sec.

25. The time taken by a particle executing simple harmonic motion of time period T to move from the mean position to half the maximum displacement is

(a) $\frac{T}{2}$ (b) $\frac{T}{4}$

(c) $\frac{T}{8}$ (d) $\frac{T}{12}$

IIT, 1992

26. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period given by

(a) $2\pi\sqrt{\frac{m}{K}}$

(b) $2\pi\left[\frac{(YA+KL)m}{YAK}\right]^{1/2}$

(c) $2\pi\left(\frac{m\,YA}{KL}\right)$

(d) $2\pi\left(\frac{m\,L}{YA}\right)$

IIT, 1993

27. A particle is executing simple harmonic motion along the x–axis with amplitude 4 cm and time period 1.2 s. The minimum time taken by the particle to move from $x = +2$ cm to $x = +4$ cm and back again is

(a) 0.6 s (b) 0.4 s

(c) 0.3 s (d) 0.2 s

28. If a spring extends by x on loading, then the energy stored in the spring is (T is the tension in the spring and k its force constant)

(a) $\frac{T^2}{2x}$ (b) $\frac{T^2}{2k}$

(c) $\frac{2k}{T^2}$ (d) $\frac{2T^2}{k}$

29. A rigid cubical block A of mass M and side L is fixed rigidly on to another cubical block of the same dimensions and of modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small F is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes small oscillations, the time period of which is

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\frac{M\eta}{L}}$

(c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$

IIT, 1992

30. A particle free to move along the x-axis has potential energy given by

$U(x) = k[1 - \exp(-x^2)]$ for $-\infty \le x \le +\infty$

where k is a constant of appropriate dimensions. Then

(a) at points away from the origin, the particle is in unstable equilibrium

(b) for any finite nonzero value of x, there is a force directed away from the origin

(c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin

(d) for small displacements from $x = 0$, the motion is simple harmonic.

31. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by

(a) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (b) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$

(c) $2\pi\sqrt{\frac{L}{g}}$ (d) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

IIT, 2000

32. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then the longer piece will have a force constant of

(a) $\frac{2}{3}\ k$ (b) $\frac{3}{2}\ k$

(c) $3\ k$ (d) $6\ k$

IIT, 1999

33. A body of mass 1 kg is executing simple harmonic motion. Its displacement x (in cm) at time t (in second) is given by

$$x = 6\ \sin\left(100\,t + \frac{\pi}{4}\right)$$

The maximum kinetic energy of the body is
(a) 6 J (b) 18 J
(c) 24 J (d) 36 J

34. Two particles P and Q start from the origin and execute simple harmonic motion along x-axis with the same amplitude and time periods 3 s and 6 s respectively. The ratio of the velocities of P and Q when they meet is
(a) 1 : 2 (b) 2 : 1
(c) 2 : 3 (d) 3 : 2

35. A body is executing simple harmonic motion. At a displacement x, its potential energy is E_1 and at a displacement y, its potential energy is E_2. The potential energy E at a displacement $(x + y)$ is
(a) $E_1 + E_2$ (b) $\sqrt{E_1^2 + E_2^2}$
(c) $E_1 + E_2 + 2\sqrt{E_1 E_2}$ (d) $\sqrt{E_1 E_2}$

36. A body executes simple harmonic motion under the action of a force F_1 with a time period $\frac{4}{5}$ s. If the force is changed to F_2 it executes S.H.M. with time period $\frac{3}{5}$ s. If both the forces F_1 and F_2 act simultaneously in the same direction on the body, its time period in seconds is:
(a) $\frac{12}{25}$ (b) $\frac{24}{25}$
(c) $\frac{35}{24}$ (d) $\frac{15}{12}$

37. If the displacement (x) and velocity (v) of a particle executing simple harmonic motion are related through the expression $4v^2 = 25 - x^2$, then its time period is
(a) π (b) 2π
(c) 4π (d) 6π

38. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob in air is t_0. If the density of the material of the bob is $(4/3) \times 1000$ kg m^{-3}, and the viscosity of water is neglected, the relationship between t and t_0 is
(a) $t = t_0$ (b) $t = \frac{t_0}{2}$
(c) $t = 2t_0$ (d) $t = 4t_0$

39. A body at the end of a spring executes S.H.M. with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then
(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$
(c) $\frac{1}{T} = \frac{1}{t_1} + \frac{1}{t_2}$ (d) $\frac{1}{T^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$

40. A horizontal platform with an object placed on it is executing SHM in the vertical direction. The amplitude of oscillation is 2.5 cm. What must be the least period of these oscillations so that the object is not detached from the platform? Take $g = 10$ m s^{-2}.
(a) 0.1 π sec (b) 0.5 π sec
(c) π sec (d) 2 π sec

41. The ends of a rod of length l and mass m are attached to two identical springs as shown in Fig. 13.24. The rod is free to rotate about its centre O. The rod is depressed slightly at end A and released. The time period of the resulting oscillation is
(a) $2\pi\sqrt{\frac{m}{2k}}$ (b) $2\pi\sqrt{\frac{2m}{k}}$
(c) $\pi\sqrt{\frac{2m}{3k}}$ (d) $\pi\sqrt{\frac{3m}{2k}}$

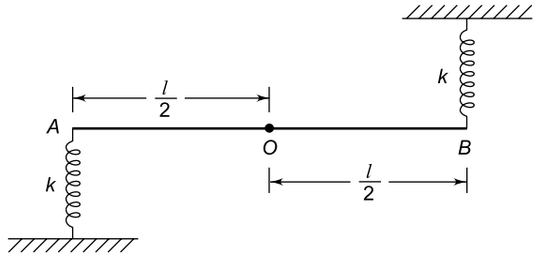


Fig. 13.24

IIT, 2009

42. A uniform metre scale of length 1 m is balanced on a fixed semi-circular cylinder of radius 30 cm as shown in Fig. 13.25. One end of the scale is slightly depressed and released. The time period (in seconds) of the resulting simple harmonic motion is (Take $g = 10$ ms^{-2})
(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

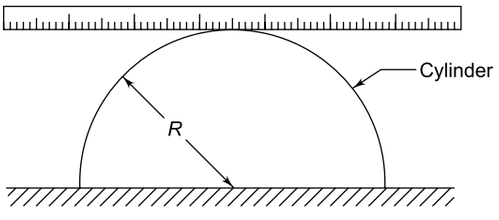


Fig. 13.25

43. A simple pendulum of length l is suspended from the ceiling of a train which is moving in the horizontal direction with a constant acceleration

a. The time period of the pendulum is given by $T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}}$ where g_{eff} is given by

(a) g (b) $(g + a)$

(c) $(g - a)$ (d) $\sqrt{g^2 + a^2}$

44. A simple pendulum of length l is suspended from the ceiling of a trolley which is moving, without friction, down an inclined plane of inclination θ. The time period of the pendulum is given by $T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}}$, where g_{eff} is given by

(a) g (b) $g \sin \theta$

(c) $g \cos \theta$ (d) $g \tan \theta$

45. One end of a massless spring of relaxed length 50 cm and spring constant k is fixed on top of a frictionless inclined plane of inclination $\theta = 30^\circ$ as shown in Fig. 13.26. When a mass $m = 1.5$ kg is attached at the other end, the spring extends by 2.5 cm. The mass is displaced slightly and released. The time period (in seconds) of the resulting oscillation will be

(a) $\frac{\pi}{7}$

(b) $\frac{2\pi}{7}$

(c) $\frac{\pi}{5}$

(d) $\frac{2\pi}{5}$

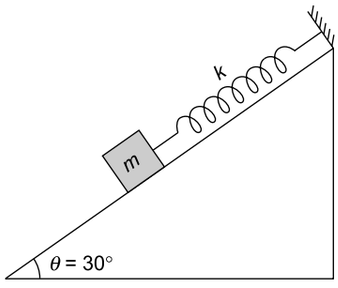


Fig. 13.26

46. Two particles are executing simple harmonic motions of the same amplitude and the same frequency along the same straight line and about the same mean position. If the maximum separation between them is $\sqrt{2}$ times the amplitude, the phase difference between them is

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

47. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then

(a) $T_1 < T_2$ (b) $T_1 > T_2$

(c) $T_1 = T_2$ (d) $T_1 = 2T_2$

IIT, 2001

48. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector is correctly shown in (see Fig. 13.27).

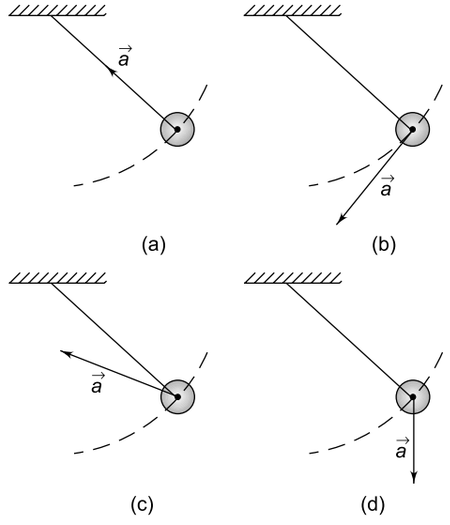


Fig. 13.27

IIT, 2002

49. For a particle executing simple harmonic motion, the displacement x is given by $x = A \cos \omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x. (see Fig. 13.28).

(a) I, III (b) II, IV

(c) II, III (d) I, IV

IIT, 2003

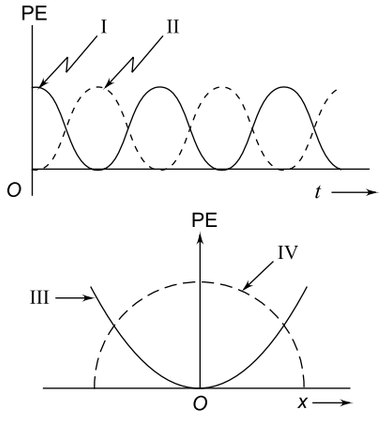


Fig. 13.28

50. The time period of a particle in simple harmonic motion is 8 seconds. At $t = 0$ it is at the mean position. The ratio of the distances travelled by it in the first and second seconds is:

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}-1}$ (d) $\frac{1}{\sqrt{3}}$

51. A particle is executing linear simple harmonic motion about the origin $x = 0$. Which of the graphs shown in Fig. 13.29 represents the variation of the potential energy function $U(x)$ versus x?

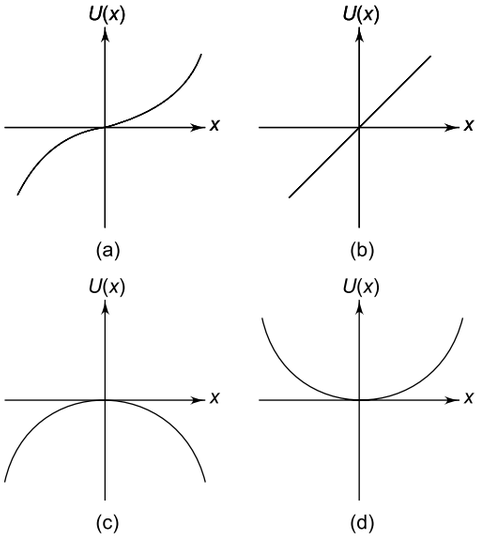


Fig. 13.29

IIT, 2004

52. A simple pendulum has a time period of 3.0 s. If the point of suspension of the pendulum starts moving vertically upward with a velocity $v = Kt$ where $K = 4.4\ \text{ms}^{-2}$, the new time period will be (Take $g = 10\ \text{ms}^{-2}$)

(a) $\frac{9}{4}$ s (b) $\frac{5}{3}$ s

(c) 2.5 s (d) 4.4 s

IIT, 2005

53. A simple pendulum is moving simple harmonically with a period of 6 s between two extreme positions B and C as shown in Fig. 13.30. If the angular distance between B and C is 10 cm, how long will the pendulum take to move from position C to position D exactly midway between O and C.

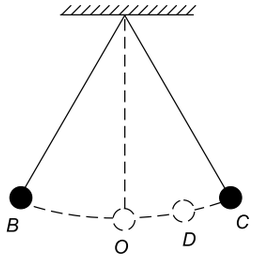


Fig. 13.30

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s

54. A block is kept on a horizontal table. The table is executing simple harmonic motion of time period T in the horizontal plane. The coefficient of static friction between the block and the table is μ. The maximum amplitude of the table for which the block does not slip on the surface of the table is

(a) $\frac{\mu g T}{2\pi}$ (b) $\frac{\mu g T^2}{2\pi^2}$

(c) $\frac{\mu g T^2}{4\pi^2}$ (d) $\mu g T^2$

55. A particle executes a linear simple harmonic motion of amplitude 25 cm and time period 3 s. What is the minimum time required for the particle to move between two points located at 12.5 cm on either side of equilibrium position?

(a) 0.5 s (b) 1.0 s

(c) 1.5 s (d) 2.0 s

56. One end of a light spring of force constant k is fixed to a block A of mass M placed on a horizontal frictionless table; the other end of the spring is fixed to a wall (Fig. 13.31). A smaller block B of mass m is placed on block A. The system is displaced by a small amount and released. What is the maximum amplitude of the resulting simple harmonic motion of the system so that the upper block does not slip over the lower block? The coefficient of static friction between the two blocks is μ.

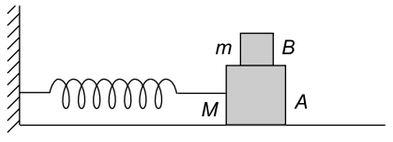


Fig. 13.31

(a) $A_{\max} = \frac{\mu Mg}{k}$ (b) $A_{\max} = \frac{\mu mg}{k}$

(c) $A_{\max} = \frac{\mu(M+m)g}{k}$ (d) None of these

57. A block A of mass m is placed on a frictionless horizontal surface. Another block B of the same mass is kept on A and connected to the wall with the help of a spring of force constant k, as shown in Fig. 13.32. The coefficient of friction between blocks A and B is μ. The blocks move together executing simple harmonic motion of amplitude a. The maximum value of frictional force between A and B is

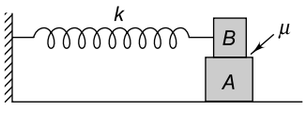


Fig. 13.32

(a) ka (b) $ka/2$
(c) zero (d) μmg

IIT, 2005

58. A simple pendulum attached to the ceiling of a stationary lift has a time period T. The distance y covered by the lift moving upwards varies with time t as $y = t^2$ where y is in metre and t in second. If $g = 10$ ms^{-2}, the time period of the pendulum will be

(a) $\sqrt{\frac{4}{5}}\,T$ (b) $\sqrt{\frac{5}{6}}\,T$

(c) $\sqrt{\frac{5}{4}}\,T$ (d) $\sqrt{\frac{6}{5}}\,T$

IIT, 2007

59. The mass M shown in Fig. 13.33 oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is

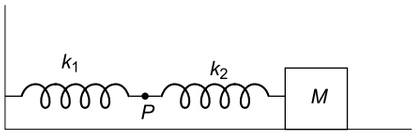


Fig. 13.33

(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$

(c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$

IIT, 2009

60. A wooden block performs SHM on a frictionless surface with frequency, v_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown in Fig. 13.34, then the SHM of the block will be

(a) of the same frequency and with shifted mean position.
(b) of the same frequency and with the same mean position
(c) of changed frequency and with shifted mean position.
(d) of changed frequency and with the same mean position.

IIT, 2011

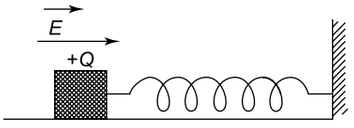


Fig. 13.34

ANSWERS

1. (c)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (d)	**6.** (a)
7. (a)	**8.** (a)	**9.** (c)	**10.** (a)	**11.** (d)	**12.** (d)
13. (c)	**14.** (d)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (b)
19. (d)	**20.** (c)	**21.** (c)	**22.** (c)	**23.** (b)	**24.** (a)
25. (d)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (d)	**30.** (d)
31. (a)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (a)
37. (c)	**38.** (c)	**39.** (b)	**40.** (a)	**41.** (c)	**42.** (c)
43. (d)	**44.** (b)	**45.** (a)	**46.** (b)	**47.** (a)	**48.** (b)
49. (a)	**50.** (c)	**51.** (d)	**52.** (c)	**53.** (a)	**54.** (c)
55. (a)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (d)	**60.** (a)

SOLUTIONS

1. $T = 2\pi\sqrt{\frac{I}{MgL}}$

where M is mass of the rod and I is its moment of inertia about the axis passing through its end and perpendicular to its length.

$$I = \frac{1}{3}\,ML^2$$

$\therefore\ T = 2\pi\sqrt{\frac{L}{3g}}$, which is choice (c).

2. Given $x = a \sin \omega t + a \cos \omega t$ (1)
The displacement equation of a simple harmonic motion is

$$x = A \sin (\omega t + \phi)$$

where A = amplitude and ϕ = phase constant.

$$x = A \sin\omega t \cos\phi + A \cos \omega t \sin \phi \quad (2)$$

Comparing (1) with (2) we get

$$A \cos \phi = a \quad (3)$$

and $A \sin \phi = a \quad (4)$

Squaring and adding Eqs. (3) and (4) we get

$$A^2 = a^2 + a^2 \Rightarrow A^2 = 2a^2$$

$$\therefore \text{Energy} = \frac{1}{2} m \omega^2 A^2 = m \omega^2 a^2$$

So the correct choice is (b).

3. Dividing Eq. (4) by Eq. (3) we get
$\tan \phi = 1 \Rightarrow \phi = 45°$, which is choice (c).

4. Given $x = a \sin \omega t + a \sin \left(\omega t + \frac{\pi}{3}\right)$

$$= a \sin \omega t + a \sin \omega t \cos \frac{\pi}{3} + a \cos \omega t \sin \frac{\pi}{3}$$

$$\Rightarrow x = \frac{3a}{2} \sin \omega t + \frac{\sqrt{3}a}{2} \cos \omega t \quad (1)$$

For a simple harmonic motion

$$x = A \sin (\omega t + \phi)$$

$$\Rightarrow x = A \sin \omega t \cos \phi + A \cos \omega t \sin \phi \quad (2)$$

Comparing (1) and (2) we get

$$A \cos \phi = \frac{3a}{2}$$

and $A \sin \phi = \frac{\sqrt{3}a}{2}$

Hence $A^2 = \left(\frac{3a}{2}\right)^2 + \left(\frac{\sqrt{3}a}{2}\right)^2 = 3a^2$

$\Rightarrow A = \sqrt{3}\, a$, which is choice (c).

5. When the block is released, the spring extends by an amount x given by

$kx = mg \Rightarrow x = \frac{mg}{k}$, which is also the amplitude of the oscillation, i.e. $A = \frac{mg}{k}$

$$v_{max} = A\omega = \frac{mg}{k} \times \sqrt{\frac{k}{m}} = g\sqrt{\frac{m}{k}}$$

6. Kinetic energy $= \frac{1}{2} m A^2 \omega^2 \cos^2 (\omega t + \phi)$

$$= \frac{1}{4} m A^2 \omega^2 [1 + \cos (2\omega t + 2\phi)]$$

$\therefore$ Time period of kinetic energy $= \frac{2\pi}{2\omega} = \frac{T}{2}$

$\left(\because \omega = \frac{2\pi}{T}\right)$

So the correct choice is (a).

7. Since the particle is at extreme position at time $t = 0$,

$$x = A \cos \omega t$$

At $t = 0$, $x_0 = A$

At $t = 1$ s, $x_1 = A \cos \omega$

At $t = 2$ s, $x_2 = A \cos 2\omega$

If A is in cm, $2 = A - A \cos \omega \rightarrow \cos \omega = 1 - \frac{2}{A}$

and $2 + 5 = A - A \cos 2\omega$

$$\Rightarrow \quad 7 = A [1 - (2 \cos^2 \omega - 1)]$$

$$= 2 A (1 - \cos^2 \omega)$$

$$= 2 A \left[1 - \left(1 - \frac{2}{A}\right)^2\right]$$

$$\Rightarrow \quad 7 = 2A \left[1 - 1 + \frac{4}{A} - \frac{4}{A^2}\right] = 8 - \frac{8}{A}$$

which gives $A = 8$ cm.

8. $V = \omega^2 (A^2 - x^2)$

$$\therefore \quad 9 = \omega^2 (A^2 - 16) \quad (1)$$

and $16 = \omega^2 (A^2 - 9) \quad (2)$

Solving Eqs. (1) and (2), we get $A = 5$ cm and $\omega = 1$ rad s^{-1}.

$$E = \frac{1}{2} m A^2 \omega^2$$

$$= \frac{1}{2} \times (200 \times 10^{-3}) \times (5 \times 10^{-2})^2 \times (1)^2$$

$$= 2.5 \times 10^{-4} \text{ J}$$

9. $T = 2\pi \sqrt{\frac{L}{g}} \quad \Rightarrow \quad T^2 = 4\pi^2 \frac{L}{g}$

$$\therefore \quad \frac{2\Delta T}{T} = \frac{\Delta L}{L}$$

or $\quad \frac{\Delta T}{T} = \frac{1}{2} \times \frac{\Delta L}{L} = \frac{1}{2} \times 2\% = 1\%$

10. Young's modulus $Y = \frac{F}{A} \cdot \frac{L}{l}$

Force constant $\quad k = \frac{F}{l} = \frac{YA}{L}$

where l is the extension in the spring of original length L and cross-sectional area A when a force $F = Mg$ is applied. Now, the time period of vertical oscillations is given by

$$T = 2\pi \sqrt{\frac{M}{k}} = 2\pi \sqrt{\frac{ML}{YA}}$$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{Y_2}{Y_1}} = \sqrt{\frac{3}{2}}$$

Hence the correct choice is (a).

11. The velocity of an oscillating body is maximum when it is at the equilibrium position where the potential energy is zero and the energy is entirely kinetic. At the extreme positions, the kinetic energy is zero and the energy is entirely potential. Therefore, the kinetic energy at equilibrium position = potential energy at extreme positions = total energy. Since the maximum velocities (i.e. velocities at equilibrium position) are equal for the two equal masses, their kinetic energies are also equal = their potential energies at extreme positions where the displacement is maximum = amplitude. If x_1 and x_2 are amplitudes of bodies A and B, we have

$$\frac{1}{2}\ k_1 x_1^2 = \frac{1}{2}\ k_2 x_2^2 \text{ or } \frac{x_1}{x_2} = \sqrt{\frac{k_2}{k_1}}$$

Hence the correct choice is (d).

12. If a force F is applied to a spring of force contant k and the spring extends by an amount x, then

$$F = kx$$

The extension x produced in a spring is proportional to its length. Thus, if the spring is cut into three equal pieces, the same force F will produce an extension $x/3$ in a piece. If k' is the force constant of the piece, we have

$$F = k'\ x/3$$

Therefore $\frac{k'}{3} = k$ or $k' = 3k$. Thus, the force constant of each piece is $3k$. When springs are connected in parallel, the force constant of the combination is equal to the sum of the individual force constants of the springs so connected. Therefore, the force constant of the combination = $3k + 3k + 3k = 9k$. Hence the correct choice is (d).

13. Given $x = 2 \cos\left(0.5\pi t + \frac{\pi}{3}\right)$

$\therefore$ Velocity $V = \frac{dx}{dt}$

$$= -\ 2 \times 0.5\ \pi \sin\left(0.5\pi t + \frac{\pi}{3}\right)$$

$\therefore$ Acceleration $a = \frac{dV}{dt}$

$$= -\ 2 \times 0.5\ \pi\ \times 0.5\ \pi \cos\left(0.5\pi t + \frac{\pi}{3}\right)$$

$\therefore$ Maximum acceleration,

$$a_{max} = -\ 2 \times 0.5\ \pi\ \times 0.5\ \pi = -\ \frac{\pi^2}{2}\ \text{cms}^{-2}$$

$\therefore$ Magnitude is $|a_{max}| = \left|-\frac{\pi^2}{2}\right| = \frac{\pi^2}{2}\ \text{cms}^{-2}$

Hence the correct choice is (c).

14. Kinetic energy (KE) = $\frac{1}{2}\ m\omega^2(A^2 - x^2)$

Potential energy (PE) = $\frac{1}{2}\ m\omega^2 x^2$

Total energy (E) = $\frac{1}{2}\ m\omega^2 A^2$

When $x = A/2$,

$$\text{KE} = \frac{1}{2}\ m\omega^2 \left(A^2 - \frac{A^2}{4}\right)$$

$$= \frac{3}{8}\ m\omega^2 A^2$$

$$E = \frac{1}{2}\ m\omega^2 A^2 \ \therefore\ \frac{\text{KE}}{E} = \frac{3}{4}$$

15. The energy will be half kinetic and half potential at a value of x when KE = PE, i.e.

$$\frac{1}{2} m\omega^2(A^2 - x^2) = \frac{1}{2}\ m\omega^2 x^2$$

or $$A^2 - x^2 = x^2 \text{ or } x = \frac{A}{\sqrt{2}}$$

16. The block will not be detached from the platform, if the amplitude of the platform's SHM is such that the maximum acceleration equals the acceleration due to gravity, i.e.

$$\omega^2 A_{max} = g \text{ or } A_{max} = \frac{g}{\omega^2} = \frac{g}{4\pi^2 v^2}$$

Hence the correct choice is (a). Notice that A_{max} is independent of the mass of the block.

17. Let L be the relaxed length of the spring and y the extension produced in it due to force mg so that

$$ky = mg \qquad \text{(i)}$$

The displacement of the mass during oscillation is given by

$$x = A \sin(\omega t + \phi) \qquad \text{(ii)}$$

At the instant when the displacement is x

$$\text{KE of mass} = \frac{1}{2}\ mV^2 = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$$

$$= \frac{1}{2}\ mA^2\ \omega^2 \cos^2(\omega t + \phi) \qquad \text{(iii)}$$

$$\text{PE of spring} = \frac{1}{2}\ k(y + x)^2$$

$$= \frac{1}{2} k\ (y^2 + 2yx + x^2)$$

$$= \frac{1}{2} ky^2 + kyx + \frac{1}{2} kx^2$$

Using (i) and (ii) and $\omega = \sqrt{\frac{k}{m}}$, we have

PE of spring $= \frac{1}{2} ky^2 + mgx + \frac{1}{2} m\omega^2 A^2 \times \sin^2 (\omega t + \phi)$ (iv)

Taking gravitational PE at the mean position to be zero,

Gravitational PE at $x = -mg\,x$ (v)

Adding (iii), (iv) and (v), we get

Total energy of mass–spring system

$$= \frac{1}{2} mA^2\omega^2 \cos^2 (\omega t + \phi) + \frac{1}{2} ky^2 + mgx + \frac{1}{2} mA^2 \omega^2 \sin^2 (\omega t + \phi) - mgx$$

$$= \frac{1}{2} mA^2\omega^2 + \frac{1}{2} ky^2$$

18. The radius of the circle along which the trolley moves is

$$r = 40 \text{ cm} = 0.4 \text{ m}$$

When the table is rotated, the tension in the spring is equal to the centripetal force, i.e.

$$F = \frac{mv^2}{r} = mr\omega^2$$

$$= 2 \times 0.4 \times (10)^2 = 80 \text{ N}$$

The extension in the spring is $x = 40 - 35 = 5$ cm $= 0.05$ m

$$\therefore \quad \text{Force constant } k = \frac{F}{x}$$

$$= \frac{80}{0.05} = 1.6 \times 10^3 \text{ N m}^{-1}$$

Hence the correct choice is (b).

19. P.E. at point $P = mgh$. If friction is neglected, the potential energy is completely converted into kinetic energy when the bob reaches the equilibrium position O (see Fig. 13.35). If V is the velocity of the bob at O, then

$$\frac{1}{2} mV^2 = mgh$$

or $$V^2 = 2gh$$

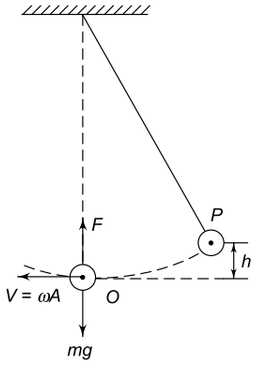


Fig. 13.35

At position O, the tension F in the string is given by

$$F - mg = \text{centripetal force} = \frac{mV^2}{l}$$

or $$F = mg + \frac{mV^2}{l}$$

$$= mg + \frac{2mgh}{l} \quad (\because V^2 = 2gh)$$

or $$F = mg\left(1 + \frac{2h}{l}\right)$$

Hence the correct choices is (d).

20. If k is the force constant, we have

$$mg = kx$$

or $$\frac{m}{k} = \frac{x}{g}$$

$$\therefore \quad T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{x}{g}}$$

Hence the correct choice is (c).

21. Let the steel ball be placed at A, a little away from the centre O of a concave mirror of radius of curvature R ($= OC = AC$), as shown in Fig. 13.36. Let $\angle ACO = \theta$. If m is the mass of the ball, its weight mg acts vertically downwards at A. This force is resolved into two rectangular components: $mg \cos \theta$ (which is balanced by the normal reaction N of the mirror) and $mg \sin \theta$ (which provides the restoring force F). Thus

$$F = -mg \sin \theta$$

$$= -mg\,\theta$$

($\because \theta$ is small; x/R being very small)

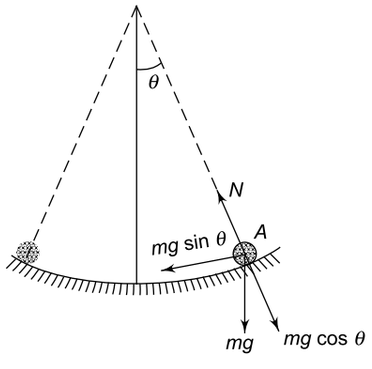


Fig. 13.36

$$= -mg\,\frac{x}{R}$$

($\because x = R\,\theta$; x being the arc OA)

or $$F = -kx$$

where $$k = \frac{mg}{R}$$ is the force constant.

Hence $$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{R}{g}}$$

$$= 2\pi \times \sqrt{\frac{2.5}{10}} = \pi \text{ seconds.}$$

Hence the correct choice is (c).

22. The potential energy of a particle of mass m executing simple harmonic motion of angular frequency ω at a distance x from the equilibrium position is given by

$$\text{P.E.} = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} k x^2$$

where $k = m\omega^2$ is constant. Hence the correct choice is (c).

23. We can write, $4\cos^2\left(\frac{t}{2}\right) = 2 \times 2\cos^2\left(\frac{t}{2}\right) = 2 \times (1 + \cos t)$. Therefore,

$y = 2 \times (1 + \cos t) \times \sin(1000\, t)$

$= 2 \sin(1000\, t) + 2 \cos t \sin(1000\, t)$

$= 2 \sin(1000\, t) + \sin(1001\, t) + \sin(999\, t)$

Thus y is a superposition of three simple harmonic motions of angular frequencies 999, 1000 and 1001 rad s^{-1}. Hence the correct choice is (b). But a superposition of two or more simple harmonic motions of different frequency does not produce a simple harmonic motion. The statement of the question is incorrect.

24. At the mean position, the velocity of the particle is $v = A\omega$. Therefore

$$\text{K.E.} = \frac{1}{2} mA^2\omega^2 = \frac{1}{2} mA^2 \left(\frac{2\pi}{T}\right)^2$$

$$= \frac{2\pi^2 m A^2}{T^2}$$

or $$T = \pi A \sqrt{\frac{2m}{\text{K.E.}}}$$

$$= \pi \times 0.25 \times \left(\frac{2 \times 5.12}{16}\right)^{1/2}$$

$$= 0.2\, \pi \text{ second}$$

Hence the correct choice is (a).

25. Let the displacement of the particle be given by

$$x = A \sin \omega t = A \sin\left(\frac{2\pi t}{T}\right)$$

i.e. when $x = 0$, $t_0 = 0$. When $x = A/2$, the value of t is given by

$$\frac{A}{2} = A \sin\left(\frac{2\pi t_1}{T}\right)$$

or $$\sin\left(\frac{2\pi t_1}{T}\right) = \frac{1}{2}$$

or $$\frac{2\pi t_1}{T} = \frac{\pi}{6} \text{ or } t_1 = \frac{T}{12}$$

$\therefore$ $$t_1 - t_0 = \frac{T}{12} - 0 = \frac{T}{12}.$$

Hence the correct choice is (d).

26. If the wire extends by an amount x when a force F is applied to it, then

$$Y = \frac{F/A}{x/L} = \frac{FL}{Ax}$$

or $$F = \left(\frac{YA}{L}\right)x = kx;\ k = \frac{YA}{L}$$

Thus the force constant of the wire is k. If K is the force constant of the spring, then the force constant of the series combination of the wire and the spring is given by

$$\frac{1}{k'} = \frac{1}{k} + \frac{1}{K}$$

or $$k' = \frac{kK}{(k+K)} = \frac{\frac{YA}{L} \times K}{\left(\frac{YA}{L} + K\right)}$$

$$= \frac{YAK}{(YA + KL)} \qquad \text{(i)}$$

The time period of the combination is

$$T' = 2\pi\sqrt{\frac{m}{k'}} \qquad \text{(ii)}$$

Using (i) in (ii) we find that the correct choice is (b).

27. Let the displacement of the particle be given by

$$x = A \sin\left(\frac{2\pi t}{T}\right)$$

where $A = 4$ cm and $T = 1.2$ s. If t_1 is the time taken by the particle to move from $x = 0$ to $x = 2$ cm, then

$$2 = 4 \sin\left(\frac{2\pi t_1}{T}\right)$$

which gives $t_1 = T/12$. If t_2 is the time taken to move from $x = 0$ to $x = 4$ cm, then

$$4 = 4 \sin\left(\frac{2\pi t_2}{T}\right)$$

which gives $t_2 = \frac{T}{4}$. Therefore, time taken to move from $x = 2$ cm to $x = 4$ cm is $t_2 - t_1 = \frac{T}{4} - \frac{T}{12} = \frac{T}{6} = \frac{1.2\text{s}}{6} = 0.2$ s. Therefore, time taken by the particle to move from $x = 2$ cm to $x = 4$ cm and back = 0.4 s. Hence the correct choice is (b).

28. $T = kx$. Therefore $x = T/k$. Now energy stored in the spring $= \frac{1}{2}\ kx^2 = \frac{1}{2}k\left(\frac{T}{k}\right)^2 = \frac{T^2}{2k}$, which is choice (b).

29. Let the force F produce a deformation x. When this force is withdrawn, the force that tries to restore block A to its equilibrium position is proportional to x and is given by

$$f = -\ \eta\ L\ x$$

$$\therefore \text{ Acceleration } \frac{d^2x}{dt^2} = \frac{f}{M} = -\frac{\eta L}{M}x = -\omega^2 x$$

where $\omega = \sqrt{\frac{\eta L}{M}}$. The angular frequency of this simple harmonic frequency is ω. Now $T = \frac{2\pi}{\omega}$. Therefore

$$T = 2\pi\sqrt{\frac{M}{\eta L}}$$

30. Figure 13.37 shows the plot of $U(x)$ versus x. At $x = 0$, potential energy $U(0) = k[1 - \exp(0)] = k(1 - 1) = 0$ and it has a maximum value $= k$ at $x = \pm\infty$ since

$$U(\pm\infty) = k[1 - \exp(-\pm\infty)^2]$$
$$= k\ (1 - 0) = k$$

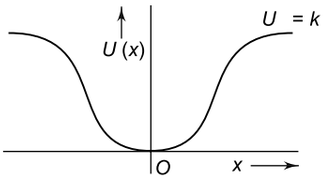


Fig. 13.37

Since the total mechanical energy has a constant value $= (k/2)$, the kinetic energy will be maximum at $x = 0$ and minimum at $x = \pm\infty$. At $x = 0$

$$\left(\frac{dU}{dx}\right)_{x=0} = [2\ kx\ \exp\ (-x^2)]_{\text{at } x\,=\,0} = 0$$

Hence the particle is in stable equilibrium at $x = 0$ (origin) and would oscillate about $x = 0$ (for small displacements) simple harmonically. Hence (d) is the only correct choice.

31. The acceleration of the vehicle down the plane is $g \sin \alpha$. The reaction force acting on the pendulum bob gives it an acceleration $a = g \sin \alpha$ up the plane. This acceleration has two rectangular components $a_x = a \cos \alpha = g \sin \alpha \cos \alpha$ and $\alpha_y = a \sin \alpha = g \sin^2 \alpha$ as shown in Fig. 13.38.

The effective acceleration due to gravity acting on the bob is given by

$$g_{\text{eff}}^2 = a_x^2 + (g - a_y)^2$$
$$= a_x^2 + g^2 + a_y^2 - 2ga_y$$
$$= g^2 \sin^2 \alpha \cos^2 \alpha + g^2 + g^2 \sin^4 \alpha - 2g^2 \sin^2 \alpha$$

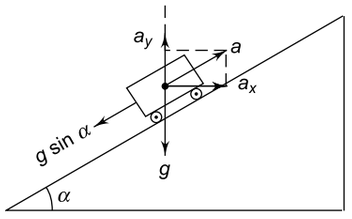


Fig. 13.38

$$= g^2 \sin^2 \alpha\ (\cos^2 \alpha + \sin^2 \alpha) + g^2 - 2g^2 \sin^2 \alpha$$
$$= g^2\ (1 - \sin^2 \alpha) = g^2 \cos^2 \alpha$$

or $\quad g_{\text{eff}} = g \cos \alpha$

Now $\quad T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$

Hence the correct choice is (*a*).

32. The force constant of a spring is inversely proportional to its length. If a spring of length L is cut into two pieces of lengths x and $(L - x)$, such that

$$x = 2\ (L - x) \text{ or } x = \frac{2L}{3},$$

then the force constant of the spring of length x is related to the force constant k of the complete spring of length L as

$$\frac{k_1}{k} = \frac{L}{x} = \frac{L}{2L/3} = \frac{3}{2} \text{ or } k_1 = \frac{3}{2}\ k$$

which is choice (b).

33. Velocity of the body at time t is

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[6\sin\left(100t + \frac{\pi}{4}\right)\right]$$
$$= 600 \cos\left(100t + \frac{\pi}{4}\right) \text{cm s}^{-1}$$

$$\therefore \quad v_{\max} = 600 \text{ cm s}^{-1} = 6 \text{ ms}^{-1}$$

Maximum K.E. $= \frac{1}{2}mv_{\max}^2 = \frac{1}{2} \times 1 \times (6)^2 = 18$ J

Hence the correct choice is (b).

34. Since the particles start from $x = 0$ and have the same amplitude but different time periods, they will meet again at $x = 0$ where their velocities are maximum equal to $a\omega_1$ and $a\omega_2$, i.e

$$\frac{v_1}{v_2} = \frac{\omega_1}{\omega_2} = \frac{2\pi}{T_1} \times \frac{T_2}{2\pi} = \frac{T_2}{T_1} = \frac{6}{3} = 2$$

Hence the correct choice is (b).

35. $E_1 = \frac{1}{2} m\omega^2 x^2$

or $\sqrt{E_1} = x\sqrt{\frac{1}{2} m\omega^2}$ (1)

$$E_2 = \frac{1}{2} m\omega^2 y^2$$

or $\sqrt{E_2} = y\sqrt{\frac{1}{2} m\omega^2}$ (2)

$$E = \frac{1}{2} m\omega^2 (x + y)^2$$

or $\sqrt{E} = (x + y)\sqrt{\frac{1}{2} m\omega^2}$ (3)

From (1), (2) and (3), it follows that

$$\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$$

or $E = E_1 + E_2 + 2\sqrt{E_1 E_2}$

which is choice (c).

36. Let the body be displaced by a distance x. If the restoring force is F_1, then the angular frequency of the resulting simple harmonic motion is given by

$$\omega_1^2 = \frac{K}{m} = \frac{Kx}{mx} = \frac{F_1}{mx} \quad \text{(i)}$$

where m is the mass of the body. For force F_2, we have

$$\omega_2^2 = \frac{F_2}{mx} \quad \text{(ii)}$$

$$\omega^2 = \frac{F_1 + F_2}{mx} \quad \text{(iii)}$$

From (i), (ii) and (iii) we get

$$\omega^2 = \omega_1^2 + \omega_2^2$$

or $\left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{T_1}\right)^2 + \left(\frac{2\pi}{T_2}\right)^2$

or $\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2} = \frac{1}{\left(\frac{4}{5}\right)^2} + \frac{1}{\left(\frac{3}{5}\right)^2}$

or $\frac{1}{T^2} = \frac{25}{16} + \frac{25}{9} = \frac{25 \times 25}{16 \times 9} = \frac{25 \times 25}{144}$

or $T = \sqrt{\frac{144}{25 \times 25}} = \frac{12}{25}$ s,

which is choice (a).

37. Given $4v^2 = 25 - x^2$. Differentiating with respect to time t, we have

$$8v \frac{dv}{dt} = 0 - 2x \frac{dx}{dt}$$

or $8va = -2xv \qquad \left(\because a = \frac{dv}{dt}, v = \frac{dx}{dt}\right)$

where a is the acceleration. Thus

$$a = -\left(\frac{1}{4}\right) x \quad \text{(i)}$$

For a simple harmonic motion

$$a = -\omega^2 x \quad \text{(ii)}$$

Comparing (i) and (ii), we have

$$\omega = \frac{1}{2}$$

or $\frac{2\pi}{T} = \frac{1}{2}$ or $T = 4\pi$, which is choice (c).

38. Since the density of water is 3/4 of the density of the bob, the effective acceleration due to gravity when the bob is in water decreases (due to buoyancy) from g to $g' = g - \frac{3g}{4} = \frac{g}{4}$. Now

$$t_0 = 2\pi\sqrt{\frac{l}{g}}$$

and $t = 2\pi\sqrt{\frac{l}{g'}}$

Dividing, we get

$$\frac{t}{t_0} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{g/4}} = 2 \text{ or } t = 2t_0$$

Hence the correct choice is (c).

39. Let m be the mass of the body and k_1 and k_2 the force constants of the two springs. Then

$$t_1 = 2\pi\sqrt{\frac{m}{k_1}} \quad (1)$$

and $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$ (2)

If the two springs are connected in series, the force constant of the combination is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

or $k = \frac{k_1 k_2}{(k_1 + k_2)}$

$\therefore$ $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$ (3)

Squaring and adding (1) and (2), we get

$$t_1^2 + t_2^2 = 4\pi^2\ m\ \left(\frac{1}{k_1}+\frac{1}{k_2}\right)$$

$$= 4\pi^2\ m\ \frac{(k_1+k_2)}{k_1k_2}$$

$$= T^2 \qquad \text{[Use Eq. (3)]}$$

40. The object will not detach from the platform, if the angular frequency ω is such that, during the downward motion, the maximum acceleration equals the acceleration due to gravity, i.e.,

$$\omega_{\max}^2\ A = g$$

or $$\omega_{\max} = \sqrt{\frac{g}{A}}$$

or $$T_{\min} = \frac{2\pi}{\omega_{\max}} = 2\pi\sqrt{\frac{A}{g}}$$

Now A = 2.5 cm = 2.5×10^{-2} m and g = 10 ms^{-2}. Substituting these values we get, $T = 0.1\ \pi$ sec. Hence the correct choice is (a).

41. Let the rod be depressed by a small amount x (Fig. 13.39). Both the springs are compressed by x. When the rod is released, the restoring torque is given by

$$\tau = (kx) \times \frac{l}{2} + (kx) \times \frac{l}{2} = (kx)l$$

Now $\sin\theta = \dfrac{x}{l/2} = \dfrac{2x}{l}$. Since θ is small, $\sin\theta \simeq \theta$, where θ is expressed in radian. Thus $\theta = 2x/l$ or $x = \theta l/2$. Hence

$$\tau = k\left(\frac{\theta l}{2}\right) \times l = \frac{k\theta l^2}{2}$$

If I is the moment of inertia of the rod about O, then

$$I\frac{d^2\theta}{dt^2} = -\left(\frac{kl^2}{2}\right)\theta$$

or $$\frac{d^2\theta}{dt^2} = -\left(\frac{kl^2}{2I}\right)\theta$$

Since $\dfrac{d^2\theta}{dt^2} \propto (-\theta)$, the motion is simple harmonic whose angular frequency is given by

$$\omega = \sqrt{\frac{kl^2}{2I}}$$

Now $\omega = \dfrac{2\pi}{T}$ and $I = \dfrac{ml^2}{12}$. Therefore, we have

$$\frac{2\pi}{T} = \sqrt{\frac{kl^2}{2}\times\frac{12}{ml^2}} = \sqrt{\frac{6k}{m}}$$

or $T = \pi\sqrt{\dfrac{2m}{3k}}$, which is choice (c).

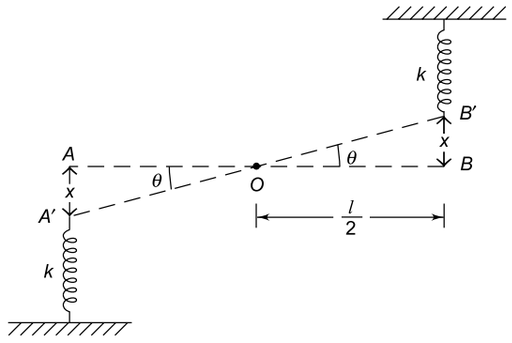


Fig. 13.39

42. Refer to Fig. 13.40. The magnitude of the restoring torque = force × perpendicular distance

$$= mg \times AB = mg \times R\sin\theta$$

Since θ is small, $\sin\theta \simeq \theta$. here θ is expressed in radian. The equation of motion of the scale is

$$I\frac{d^2\theta}{dt^2} = -mgR\theta$$

or $$\frac{d^2\theta}{dt^2} = \left(-\frac{mgR}{I}\right)\theta$$

$\therefore\ \omega = \sqrt{\dfrac{mgR}{I}}$ or $\dfrac{2\pi}{T} = \sqrt{\dfrac{mgR}{I}}$ or $T = 2\pi\sqrt{\dfrac{I}{mgR}}$.

Now $I = \dfrac{mL^2}{12}$. Hence

$$T = \frac{\pi L}{\sqrt{3gR}}$$

Using the values L = 1 m, g = 10 ms^{-2} and R = 0.3 m, we get $T = \pi/3$ second. Hence the correct choice is (c).

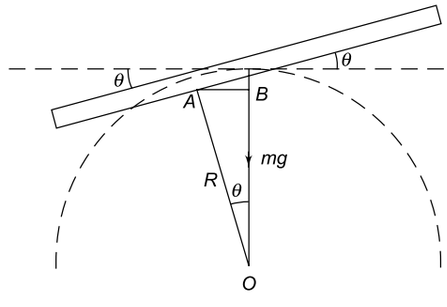


Fig. 13.40

43. There are two perpendicular acceleration vectors; g acting vertically downwards and a acting horizontally. The resultant acceleration is given by

$$g_{\text{eff}} = \sqrt{g^2 + a^2}$$

Hence the correct choice is (d).

44. The correct choice is (b) since the component of the acceleration due to gravity along the plane is $g \sin \theta$.

45. The force which increases the length of the spring by $x = 2.5$ cm is $F = mg \sin \theta$. Therefore, the spring constant is

$$k = \frac{F}{x} = \frac{mg \sin \theta}{x}$$

Now time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg \sin \theta / x}}$

$$= 2\pi\sqrt{\frac{x}{g \sin \theta}}$$

Putting $x = 2.5$ cm $= 2.5 \times 10^{-2}$ m, $g = 9.8$ ms^{-2} and $\theta = 30^\circ$, we get $T = \pi/7$ second, which is choice (a).

46. Let the motions be given by $x_1 = A \sin \omega t$ and $x_2 = A \sin (\omega t + \phi)$. Separation between the particles at time t is given by

$$x = x_2 - x_1 = A \sin (\omega t + \phi) - A \sin \omega t \qquad (1)$$

The separation will be maximum when $\frac{dx}{dt} = 0$, i.e. when

$A\omega \cos (\omega t + \phi) - A\omega \cos \omega t = 0$

or $\cos (\omega t + \phi) = \cos \omega t$

or $\omega t + \phi = \pm \omega t$

[$\because \cos (-\theta) = \cos \theta$]

The plus sign is not possible because then $\phi = 0$ which implies that the separation x is always zero. Therefore, $\omega t + \phi = -\omega t$ or $\omega t = -\phi/2$. Using this value in (1), we have

$$x_{\max} = A \sin\left(-\frac{\phi}{2} + \phi\right) - A \sin\left(-\frac{\phi}{2}\right) = 2A \sin\left(\frac{\phi}{2}\right)$$

Given $x_{\max} = \sqrt{2}\ A$. Hence

$$\sqrt{2}\ A = 2A \sin\left(\frac{\phi}{2}\right)$$

or $$\sin\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}} \text{ or } \frac{\phi}{2} = \frac{\pi}{4} \text{ or } \phi = \frac{\pi}{2}.$$

Hence the correct choice is (b).

47. In simple harmonic motion, the speed of the particle is the maximum at the mean position $x = 0$, decreases as it moves towards the extreme position becoming zero at the extreme position $x = A$. Hence the particle will take shorter time to move from $x = 0$ to $x = \frac{A}{2}$ than to move from $x = \frac{A}{2}$ to $x = A$. Thus, the correct choice is (a).

48. In the simple harmonic motion of a pendulum, the restoring force vector (and hence the acceleration vector) is tangential to the path of the bob and is directed towards the mean position. Hence the correct choice is (b).

49. Given $x = A \cos \omega t$. As a function of x, the PE is given by

$$\text{PE} = \frac{1}{2}\ m\omega^2\ x^2$$

At $x = 0$, PE = 0. Hence the correct graph is III. As a function of t, the PE is given by

$$\text{PE} = \frac{1}{2}\ m\omega^2\ (A \cos\ \omega t)^2$$

$$= \frac{1}{2}\ m\omega^2\ A^2 \cos^2\ \omega t$$

At $t = 0$, PE is maximum equal to $\frac{1}{2}\ m\omega^2\ A^2$. Hence the correct graph is I. Thus, the correct choice is (a).

50. Since the particle is at the mean position at $t = 0$, the phase of the motion is zero. Therefore, the displacement of the particle is given by

$$x = A \sin\ (\omega t) = A \sin\left(\frac{2\pi t}{T}\right)$$

$$= A \sin\left(\frac{\pi t}{4}\right) \qquad (\because T = 8\text{s}) \qquad \text{(i)}$$

The distance travelled in the first second is [put $t =$ 1s in (i)]

$$x_1 = A \sin\left(\frac{\pi}{4}\right) = \frac{A}{\sqrt{2}}$$

The distance travelled in first two seconds is [put t = 2 s in (i)]

$$x' = A \sin\left(\frac{\pi}{2}\right) = A$$

$\therefore$ Distance travelled in the IInd second is

$$x_2 = x' - x_1 = A - \frac{A}{\sqrt{2}} = A\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\therefore \quad \frac{x_1}{x_2} = \frac{\frac{A}{\sqrt{2}}}{A\left(1 - \frac{1}{\sqrt{2}}\right)} = \frac{1}{\sqrt{2} - 1}$$

Hence the correct choice is (c).

51. In simple harmonic motion, the force acting on the particle (restoring force) is given by

$$F = -kx$$

where k is a positive constant. Now

$$F = -\frac{dU}{dx}$$

Therefore, $-kx = -\frac{dU}{dx}$

or $$dU = kdx$$

$$\therefore \quad U(x) = \int_0^x kdx = \frac{1}{2}kx^2 + c$$

where c is a constant of integration.

In simple harmonic motion, the potential energy of the oscillator is zero at the mean position, i.e. $U(0) = 0$. Hence $c = 0$. Therefore

$$U(x) = \frac{1}{2}kx^2$$

which is the equation of a parabola. Since $U(x)$ is positive for all values (positive and negative) of x, the correct graph is (d).

52. Original time period is

$$T_1 = 2\pi\sqrt{\frac{l}{g}} \qquad (1)$$

When the pendulum is moving upwards, the effective value of g is

$$g_{eff} = g + a$$

where a is the acceleration of the pendulum which is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(Kt) = K = 4.4 \text{ ms}^{-2}$$

$$\therefore \quad g_{eff} = g + a = 10 + 4.4 = 14.4 \text{ ms}^{-2}.$$

Therefore, the new time period is

$$T_2 = 2\pi\sqrt{\frac{l}{g_{eff}}} \qquad (2)$$

From (1) and (2), we get

$$\frac{T_2}{T_1} = \sqrt{\frac{g}{g_{eff}}} = \sqrt{\frac{10}{14.4}} = \frac{1}{1.2}$$

or $$T_2 = \frac{T_1}{1.2} = \frac{3}{1.2} = 2.5 \text{ s}$$

Hence the correct choice is (c).

53. Amplitude $= OC = OB = \frac{1}{2}BC$

$$= \frac{10}{2} = 5 \text{ cm}$$

$$\therefore \quad OD = 2.5 \text{ cm}$$

Let the displacement of the pendulum be given by

$$x = A\sin(\omega t + \phi)$$

where $A = 5$ cm and $\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$ rad s^{-1}

Let us suppose that at $t = 0$, the pendulum is at C, i.e. at $t = 0$, $x = A$,

so that

$$A = A\sin(\omega \times 0 + \phi)$$

or $$A = A\sin\phi \text{ or } \sin\phi = 1 \text{ or } \phi = \frac{\pi}{2}$$

Thus the motion of the pendulum is given by $\left(\text{putting } \phi = \frac{\pi}{2}\right)$

$$x = A\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= A\cos\omega t = 5\cos\omega t$$

The value of t for which $x = 2.5$ cm is given by

$$2.5 = 5\cos\omega t$$

or $$\cos\omega t = \frac{1}{2} \text{ or } \omega t = \frac{\pi}{3}$$

or $$\frac{2\pi t}{T} = \frac{\pi}{3}, \text{ or } t = \frac{T}{6} = 1 \text{ s.}$$

54. Refer to Fig. 13.41. Let the mass of the block be m and let, at a certain instant of time, the direction of acceleration a of the table (executing simple harmonic motion) be along the positive x-direction. As a result, the block will experience a force ma directed along the negative x-axis. Consequently, the force of friction μmg will act along the positive x-axis. The weight mg of the block will be balanced by the normal reaction R. The block will not slip on the surface of the table, if the acceleration a of the motion of the table is such that

$$\mu mg \geq ma \text{ or } \mu g \geq a$$

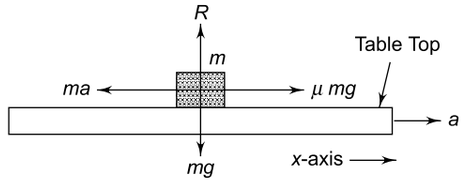


Fig. 13.41

Therefore, for no slipping, the table can have a maximum acceleration $a_{max} = \mu g$. We know that, for a simple harmonic motion, $a_{max} = \omega^2 A$, where ω is the angular frequency and A the amplitude of the motion of the table. Therefore, the maximum amplitude is given by

$$\omega^2 A_{max} = \mu g$$

or $$A_{max} = \frac{\mu g}{\omega^2} = \frac{\mu g T^2}{4\pi^2},$$

which is choice (c).

55. Let A and B be the two extreme positions of the particle with O as the equilibrium position. Displacements to the right of O are take as positive, while to the left of O are taken as negative (Fig. 13.42)

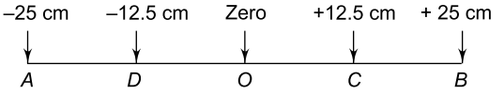


Fig. 13.42

Let the displacement of the particle in SHM be given by

$$x(t) = A \sin(\omega t + \phi) \quad \text{(i)}$$

where $A = 25$ cm and $\omega = \frac{2\pi}{T} = \frac{2\pi}{3}$ rad s^{-1}

Let us suppose that at time $t = 0$, the particle is at extreme position B. Setting $x = A$ at $t = 0$ in Eq. (i) we have

$$A = A \sin\phi \text{ giving } \phi = \pi/2$$

Putting $\phi = \pi/2$ in Eq. (i), we get

$$x(t) = A \cos \omega t \quad \text{(ii)}$$

where $A = 25$ cm.

Now let us say that the particle reaches point C at $t = t_1$ and point D at $t = t_2$. At C, the displacement $x(t_1) = +12.5$ cm and at D, it is $x(t_2) = -12.5$ cm (see Fig. 13.12). So from (ii) we have

$$+12.5 = 25 \cos \omega t_1 \text{ and}$$

$$-12.5 = 25 \cos \omega t_2$$

or $$\cos \omega t_1 = +0.5 \text{ or } \omega t_1 = \pi/3$$

and $$\cos \omega t_2 = -0.5 \text{ or } \omega t_2 = \frac{2\pi}{3}$$

Hence $$\omega(t_2 - t_1) = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$$

$$\therefore \quad t_2 - t_1 = \frac{\pi}{3\omega} = \frac{T}{6} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

or $$(t_2 - t_1)_{min} = \frac{3}{6} = 0.5 \text{ s}$$

Notice that $\cos \omega t_2 = -0.5$ even for $\omega t_2 = \frac{4\pi}{3}$ which gives $t_2 = 2T/3 = 4$ s which does not correspond to minimum value of $(t_2 - t_1)$. Thus the correct choice is (a).

56. The angular frequency of the system is

$$\omega = \left[\frac{k}{(M+m)}\right]^{1/2} \quad (1)$$

The upper block of mass m will not slip over the lower block of mass M if the maximum force on the upper block f_{max} does not exceed the frictional force μmg between the two blocks. Now

$$f_{max} = m a_{max} = m\omega^2 A_{max} \quad (2)$$

where a_{max} is the maximum acceleration and A_{max} is the maximum amplitude. Using (1) in (2), we get

$$f_{max} = \frac{mk\, A_{max}}{(M+m)}$$

For no slipping, $f_{max} = \mu mg$

or $$\frac{mk\, A_{max}}{(M+m)} = \mu mg \text{ or } A_{max} = \frac{\mu(M+m)g}{k},$$

which is choice (c).

57. The blocks will move together as long as the frictional force of block B = mass of block $B \times$ maximum acceleration of its S.H.M., i.e.

$$f = m\omega^2 a$$

where $$\omega = \sqrt{\frac{k}{(m+m)}} = \sqrt{\frac{k}{2m}}$$

Thus $$f = m \times \frac{k}{2m} \times a$$

$$= ka/2, \text{ which is choice (b).}$$

58. Given $y = t^2$. The velocity of the lift varies with t as

$$v = \frac{dy}{dt} = 2t$$

$\therefore$ Acceleration $a = \frac{dv}{dt} = 2$ ms^{-2}, directed upwards, Hence

$$T' = 2\pi\sqrt{\frac{l}{g+a}}$$

and $$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{10}{(10+2)}} = \sqrt{\frac{5}{6}}$$

The correct choice is (b)

59. If a force F is applied to M, say to the right, let A be the distance moved by M. If the system is released, it executes simple harmonic motion of amplitude A. If A_1 and A_2 are the extensions in springs k_1 and k_2 then $A = (A_1 + A_2)$ and

$$F = k_1 A_1 = k_2 A_2$$

$$\Rightarrow \qquad A_1 = \frac{F}{k_1} \text{ and } A_2 = \frac{F}{k_2}$$

$$\therefore \qquad A = A_1 + A_2 = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= \frac{F(k_1 + k_2)}{k_1 k_2}$$

$$\Rightarrow \qquad F = \frac{k_1 k_2 A}{(k_1 + k_2)}$$

The amplitude of point P = amplitude of oscillations of spring k_1 which is

$$A_1 = \frac{F}{k_1} = \frac{k_2 A}{(k_1 + k_2)}$$

60. The force exerted on charge $+Q$ by the electric field $\vec{E}$ is

$$\vec{F} = Q\vec{E}$$

in the direction of $\vec{E}$. Since $\vec{F}$ is constant, a constant force is added to the applied force. Hence only the mean position will change and the frequency of oscillation will remain the same.

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Which of the following expressions represent simple harmonic motion?
(a) $x = a \sin(\omega t + \phi)$
(b) $x = a \cos(\omega t + \delta)$
(c) $x = a \sin \omega t + b \cos \omega t$
(d) $x = a \sin \omega t \cos \omega t$

2. Choose the correct statements from the following in which k is a real, positive constant.
(a) Function $f(t) = \sin kt + \cos kt$ is simple harmonic having a period $2\pi / k$.
(b) Function $f(t) = \sin \pi t + 2 \cos 2\pi t + 3 \sin 3\pi t$ is periodic but not simple harmonic having a period of 2 s.
(c) Function $f(t) = \cos kt + 2 \sin^2 kt$ is simple harmonic having a period $2\pi / k$.
(d) Function $f(t) = e^{-kt}$ is not periodic.

3. A simple pendulum of length l and bob mass m is displaced from its equilibrium position O to a position P so that the height of P above O is h. It is then released. What is the tension in the string when the bob passes through the equilibrium position O? Neglect friction. V is the velocity of the bob at O.
(a) $m\left(g + \frac{V^2}{l}\right)$ (b) $\frac{2mgh}{l}$
(c) $mg\left(1 + \frac{h}{l}\right)$ (d) $mg\left(1 + \frac{2h}{l}\right)$

4. A trolley of mass m is connected to two identical springs, each of force constant k, as shown in Fig. 13.43. The trolley is displaced from its equilibrium position by a distance x and released. The trolley executes simple harmonic motion of period T. After some time it comes to rest due to friction. The total energy dissipated as heat is (assume the damping force to be weak)
(a) $\frac{1}{2} kx^2$ (b) kx^2
(c) $\frac{2\pi^2 mx^2}{T^2}$ (d) $\frac{mx^2}{T^2}$

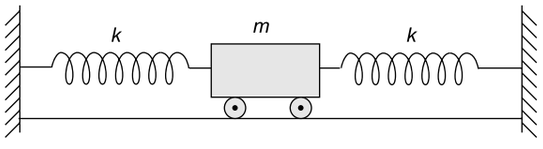


Fig. 13.43

5. In order to execute simple harmonic motion, a system must have
(a) inertia (b) moment of inertia
(c) elasticity (d) buoyancy

6. The time period of a system executing simple harmonic motion depends upon
(a) mass of the system
(b) force constant of the system
(c) amplitude of the oscillator
(d) phase constant of the oscillator.

7. The amplitude of a particle executing simple harmonic motion depends upon

(a) initial displacement
(b) initial velocity
(c) initial acceleration
(d) initial phase.

8. The displacement (x) of a particle as a function of time (t) is given by

$$x = a \sin (bt + c)$$

where a, b and c are constants of motion. Choose the correct statements from the following:

(a) The motion repeats itself in a time interval $2\pi/b$
(b) The energy of the particle remains constant
(c) The velocity of the particle is zero at $x = \pm a$
(d) The acceleration of the particle is zero at $x = \pm a$

9. Figure 13.44 (a) shows a spring of force constant k fixed at one end and carrying a mass m at the other end placed on a horizontal frictionless surface. The spring is stretched by a force F. Figure 13.44 (b) shows the same spring with both ends free and a mass m fixed at each free end. Each spring is stretched by the same force F. The mass in case (a) and the masses in case (b) are then released.
Which of the following statements are true?

(a) While oscillating, the maximum extension of the spring is more in case (a) than in case (b).
(b) The maximum extension of the spring is the same in both cases.
(c) The time period of oscillation is the same in both cases.
(d) The time period of oscillation in case (a) is $\sqrt{2}$ times that in case (b).

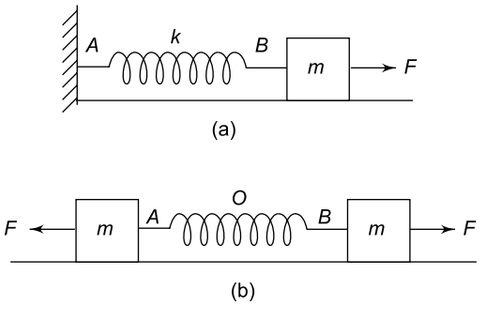


Fig. 13.44

10. A simple pendulum is oscillating between extreme positions P and Q about the mean position O. Which of the following statements are correct about the motion of the pendulum?

(a) At point O, the acceleration of the bob is different from zero
(b) The acceleration of the bob is constant throughout the oscillation
(c) The tension in the string is constant throughout the oscillation
(d) The tension is the maximum at O and the minimum at A or B.

11. Two springs A and B have force constants k_1 and k_2 respectively. The ratio of the work done on A to that done on B in increasing their lengths by the same amount is α and the ratio of the work done on A to that done on B when they are stretched with the same force is β. Then

(a) $\alpha = \dfrac{k_1}{k_2}$ (b) $\alpha = \dfrac{k_2}{k_1}$

(c) $\beta = \dfrac{k_1}{k_2}$ (d) $\beta = \dfrac{k_2}{k_1}$

12. A body of mass 50 g executing linear simple harmonic motion has a velocity of 3 cms^{-1} when its displacement is 4 cm and a velocity of 4 cms^{-1} when its displacement is 3 cm.

(a) The amplitude of oscillation is 5 cm.
(b) The angular frequency of oscillation is 1 rad s^{-1}.
(c) The maximum kinetic energy of the oscillator is 6.25×10^{-5} J.
(d) The maximum potential energy of the oscillator is 6.25×10^{-5} J.

13. The displacement of an oscillating particle is given by

$$x = a \sin(ct) + b \cos(ct)$$

where a, b and c are constants. If A is the amplitude of the oscillations and T their time period, then

(a) $A = a + b$ (b) $A = \sqrt{a^2 + b^2}$

(c) $T = \dfrac{2\pi}{c}$ (d) $T = \dfrac{2\pi a}{bc}$

14. All the springs shown in Fig. 13.45 (a), (b) and (c) are identical, each having a force constant k. If T_a, T_b and T_c are the time periods of oscillations of the three systems respectively, then

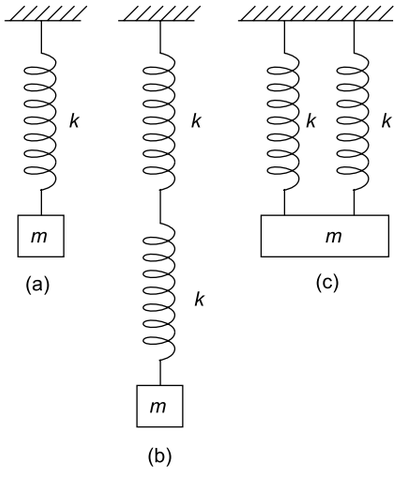


Fig. 13.45

(a) $T_a = \dfrac{T_b}{\sqrt{2}}$ (b) $T_b = 2T_c$

(c) $T_a = \sqrt{2}\, T_c$ (d) $T_a = 2T_b = \dfrac{T_c}{2}$

15. Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k (see Fig. 13.46). When the masses are in equilibrium, m_1 is removed without disturbing the system. If ω is the angular frequency of oscillation and x_2 the amplitude of oscillation of mass m_2, then

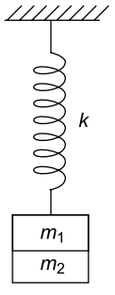


Fig. 13.46

(a) $\omega = \sqrt{\dfrac{k}{m_2}}$

(b) $\omega = \sqrt{\dfrac{k}{(m_2 - m_1)}}$

(c) $x_2 = \dfrac{m_1 g}{k}$

(d) $x_2 = \dfrac{m_2 g}{k}$

IIT, 1981

16. A simple pendulum of length L and mass m is suspended from the ceiling of the compartment of a train that is travelling at a constant speed v around a circular track of radius R. The tension in the string is T. The time period of the pendulum is

(a) $2\pi \sqrt{\dfrac{mL}{T}}$

(b) $2\pi \sqrt{\dfrac{L}{g}}$

(c) $2\pi \left[\dfrac{L}{\left(g^2 + \dfrac{v^4}{R^2}\right)^{1/2}}\right]^{1/2}$

(d) $2\pi \left[\dfrac{L}{\left(g + \dfrac{v^2}{R}\right)}\right]^{1/2}$

17. A solid sphere of density ρ and radius R is floating in a liquid of density σ with half its volume submerged. When the sphere pressed down slightly and released, it executes simple harmonic motion of time period T. If viscous effect is ignored, then

(a) $\sigma = 2\rho$ (b) $\rho = 2\sigma$

(c) $T = 2\pi \sqrt{\dfrac{2R}{3g}}$ (d) $T = 2\pi \sqrt{\dfrac{3R}{2g}}$

IIT, 2004

18. Two simple harmonic motions are represented by the following equations

$$y_1 = 10 \sin \frac{\pi}{4} (12t + 1)$$

and $$y_2 = 5 (\sin 3\pi t + \sqrt{3} \cos 3\pi t)$$

The ratio of their amplitudes is m and the ratio of their time periods is n. Then

(a) $m = \dfrac{2}{3}$ (b) $m = 1$

(c) $n = \dfrac{3}{2}$ (d) $n = 1$

IIT, 1986

19. A solid cylinder of mass m and radius R is attached to a massless spring of force constant k as shown in Fig. 13.47. The cylinder is pushed to the right a little and released. It executes simple harmonic motion. The cylinder rolls on the horizontal surface without slipping.

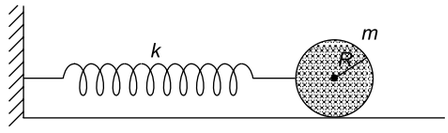


Fig. 13.47

The time period of oscillation is T. When the instantaneous displacement of the cylinder from the mean position is x, the total energy of the system is E. Then

(a) $T = 2\pi \sqrt{\dfrac{2k}{3m}}$ (b) $T = 2\pi \sqrt{\dfrac{m}{k}}$

(c) $E = \dfrac{3}{4} m v^2 + \dfrac{1}{2} k x^2$ (d) $E = \dfrac{1}{2} m v^2 + \dfrac{1}{2} k x^2$

20. A man is standing on a weighing machine placed on a horizontal platform that is executing vertical simple harmonic motion of angular frequency ω. The maximum and minimum readings of the machine are m_1 kg and m_2 kg respectively. If the true mass of the man is m and A is the amplitude of the motion then

(a) $m = \frac{1}{2}(m_1 + m_2)$ (b) $m = \frac{m_1 m_2}{(m_1 + m_2)}$

(c) $A = \frac{(m_1 + m_2)}{(m_1 - m_2)} \frac{g}{\omega^2}$ (d) $A = \frac{(m_1 - m_2)}{(m_1 + m_2)} \frac{g}{\omega^2}$

21. The displacement x of a particle varies with time t as

$$x = A \sin^2\omega t + B \cos^2\omega t + C \sin\omega t \cos\omega t$$

For what values of A, B and C is the motion simple harmonic?

(a) All values of A, B and C with $C \neq 0$.
(b) $A = B$, $C = 2B$
(c) $A = -B$, $C = 2B$
(d) $A = B$, $C = 0$

IIT, 2007

ANSWERS AND SOLUTIONS

1. Simple harmonic motion is represented by a sine function or a cosine function or a linear combination of them. Hence the correct choices are (a), (b) and (c). The choice (d) which is a product of the two functions does not represent a simple harmonic motion.

2. Statement (a) is correct. The function $f(t) = \sin kt + \cos kt$ can be written as $f(t) = \sqrt{2} \sin\left(kt + \frac{\pi}{4}\right)$ or $\sqrt{2} \cos\left(kt - \frac{\pi}{4}\right)$ both of which are simple harmonic. The coefficient of time t in the argument of the sine or cosine function $= 2\pi/T$ where T is the period. Hence $k = 2\pi/T$ or $T = 2\pi/k$.

Statement (b) is also correct. Each term represents simple harmonic motion. The period T of term $\sin \pi t$ is $\pi = 2\pi/T$ or $T = 2$s. The period of term $2 \cos 2\pi t$ is 1s, i.e. $T/2$ and the period of term $3 \sin 3\pi t$ is 2/3 s, i.e. $T/3$. The sum of two or more simple harmonic motions of different periods is not simple harmonic. The sum, however, is periodic. By the time the first term completes one cycle, the second term completes two cycles and the third term completes three cycles. Thus the sum has a period of 2s.

Statement (c) is incorrect. We can write

$$f(t) = \cos kt + 2 \sin^2 kt$$

as $f(t) = \cos kt + (1 - \cos 2kt)$

$$= 1 + \cos kt - \cos 2kt$$

The period of $\cos kt$ is $T = 2\pi/k$ and of $\cos 2kt$ is π/k which is $T/2$. As explained above, the period of the two terms together is $T = 2\pi/k$. The term 1 is a constant independent of time.

Statement (d) is correct. Function e^{-kt} decreases monotonically to zero at $t \to \infty$; it never becomes negative. Hence it is non-periodic.

3. P.E. at point $P = mgh$. If friction is neglected, the potential is completely converted into kinetic energy when the bob reaches the equilibrium position O (see Fig 13.48). If V is the velocity of the bob at O, then

$$\frac{1}{2}mV^2 = mgh$$

or $V^2 = 2gh$

At position O, the tension F in the string is given by

$F - mg$ = centripetal force

$$= \frac{mV^2}{l}$$

or $F = mg + \frac{mV^2}{l}$

$$= mg + \frac{2mgh}{l} \quad (\because V^2 = 2gh)$$

or $F = mg\left(1 + \frac{2h}{l}\right)$

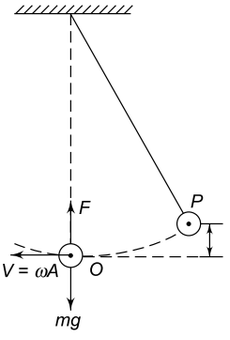


Fig. 13.48

Hence the correct choices are (a) and (d).

4. In general, the motion of a damped oscillator is not simple harmonic. If the damping forces are weak, the motion is very nearly simple harmonic and all formulae of SHM apply. The amplitude $A = x$. The time period T is

$$T = 2\pi\sqrt{\frac{m}{2k}} \quad \text{(i)}$$

If the trolley eventually comes to rest, the entire energy of oscillation is dissipated as heat due to friction. Hence, the total energy dissipated as heat is

$$E = \frac{1}{2} mA^2\omega^2 = \frac{1}{2} mx^2 \left(\frac{2\pi}{T}\right)^2 = \frac{2\pi^2 mx^2}{T^2} \quad \text{(ii)}$$

which is choice (c). Using (i) in (ii) we get

$$E = kx^2$$

Hence choice (b) is also correct.

5. The correct choices are (a) and (c).

6. The correct choices are (a) and (b).

7. The correct choices are (a) and (b).

8. The motion of the particle is simple harmonic. The displacement at time t is

$$x = a \sin (bt + c)$$

$\therefore$ Displacement at time $\left(t + \frac{2\pi}{\text{b}}\right)$ is

$$x\left(at\, t + \frac{2\pi}{b}\right) = a \sin \left[b\left(t + \frac{2\pi}{b}\right) + c\right]$$

$$= a \sin (bt + c + 2\pi)$$

$$= a \sin (bt + c)$$

$$= x \text{ at time } t$$

Hence statement (a) is correct. Statement (b) is also correct since the same displacement is recovered after a time interval of $2\pi/b$. Statement (c) is correct because the velocity is zero when the displacement = ± amplitude a, i.e. at the extreme ends of the motion. Statement (d) is incorrect, the acceleration is maximum (in magnitude) at $x = \pm a$.

9. The maximum extension x produced in the spring in Fig. 13.25 a is given by

$$F = kx$$

$$\text{or } x = F/k$$

The time period of oscillation is

$$T = 2\pi \sqrt{\frac{\text{mass}}{\text{force constant}}} = 2\pi \sqrt{\frac{m}{k}}$$

In case (a) one end A of the spring is fixed to the wall. When a force F is applied to the free end B in the direction shown in Fig. 13.44a the spring is stretched exerting a force on the wall which in turn exerts an equal and opposite reaction force on the spring, as a result of which every coil of the spring is elongated producing a total extension x. In case (b) shown in Fig. 13.44b, both ends of the spring are free. Therefore, the reaction force is absent, as a result of which every coil of the spring is not elongated when force F is applied at each end in opposite directions. The coil at point O in the middle of the spring is not elongated. This situation can be visualized as two springs each of length $l/2$ (where l is the length of the complete spring) are joined to each other at point O. Since extension is proportional to the length of the spring, the force F applied at end B produces an extension $x/2$ in the part OB of the spring and the force F applied at A produces an extension $x/2$ in the part OA. The total extension in the spring is $\frac{x}{2} + \frac{x}{2} = x$.

Thus, the maximum extension produced in the spring in cases (a) and (b) is the same.

Now, the force constant of half the spring is twice that of the complete spring. In case (b) the force constant = $2k$. Hence the time period of oscillation will be

$$T' = 2\pi \sqrt{\frac{m}{2k}}$$

$$\therefore \frac{T}{T'} = \sqrt{2}$$

Hence the correct choices are (b) and (d).

10. Statement (a) is correct. At any position between O and P or between O and Q, there are two accelerations–a tangential acceleration $g \sin \alpha$ and a centripetal acceleration v^2/l (because the pendulum moves along the arc of a circle of radius l) where l is the length of the pendulum and v its speed at that position. When the bob is at the mean position O, the angle $\alpha = 0$, therefore $\sin \alpha = 0$; hence the tangential acceleration is zero. But at O, speed v is maximum and the centripetal acceleration v^2/l is directed radially towards the point of support. When the bob is at the end points P and Q, the speed v is zero, hence the centripetal acceleration is zero at the end points, but the tangential acceleration is maximum and is directed along the tangent to the curve at P and Q. The tension in the string is not constant throughout the oscillation. At any position between O and end points P or Q, the tension in the string is given by $T = mg \cos \alpha$. At the end points P or Q, the value of α is the greatest, hence the tension is the least. At the mean position O, $\alpha = 0$ and $\cos \alpha = 1$ which is the greatest; hence tension is the greatest at the mean position.

11. $F_1 = k_1 x$, $F_2 = k_2 x$.

Work done $W_1 = \frac{1}{2} k_1 x^2$ and $W_2 = \frac{1}{2} k_2 x^2$

$$\therefore \quad \alpha = \frac{W_1}{W_2} = \frac{k_1}{k_2}$$

When the springs are stretched by the same force F, the extensions in springs A and B are x_1 and x_2 respectively which are given by

$$F = k_1 x_1 = k_2 x_2 \quad \text{or} \quad \frac{x_1}{x_2} = \frac{k_2}{k_1} \qquad \text{(i)}$$

Work done $W_1 = \frac{1}{2} k_1 x_1^2$ and $W_2 = \frac{1}{2} k_2 x_2^2$

$$\therefore \quad \frac{W_1}{W_2} = \frac{k_1}{k_2} \cdot \frac{x_1^2}{x_2^2} \qquad \text{(ii)}$$

Using (i) and (ii) we get

$$\beta = \frac{W_1}{W_2} = \frac{k_1}{k_2} \cdot \frac{k_2^2}{k_1^2} = \frac{k_2}{k_1}$$

Hence the correct choices are (a) and (d).

12. In SHM, the velocity V at a displacement x is given by

$$V = \omega (A^2 - x^2)^{1/2}$$

or $V^2 = \omega^2 (A^2 - x^2)$

Now $V = 3$ cm s^{-1} when $x = 4$ cm. Therefore,

$$9 = \omega^2 (A^2 - 16) \qquad \text{(i)}$$

Also $V = 4$ cm s^{-1} when $x = 3$ cm. Therefore,

$$16 = \omega^2 (A^2 - 9) \qquad \text{(ii)}$$

Simultaneous solution of Eqs. (i) and (ii) gives

Amplitude $A = 5$ cm

Putting $A = 5$ cm in Eq. (i) above we get $\omega = 1$ rad s^{-1}.

Maximum kinetic energy = maximum potential energy = total energy which is $E = \frac{1}{2} mA^2\omega^2$

$$= \frac{1}{2} \times (50 \times 10^{-3}) \times (5 \times 10^{-2})^2 \times (1)^2$$
$$= 6.25 \times 10^{-5} \text{ J}$$

Hence all the four choices are correct.

13. The correct choices are (b) and (c).

14. The correct choices are (a), (b) and (c). In case (b), the equivalent force constant of the series combination is $k_s = k/2$ and in case (b), the equivalent force constant of the parallel combination is $k_p = 2k$.

15. Let x_1 be the extension produced in the spring when it is loaded with mass m_2 alone and x_2 be the further extension when mass m_1 is added to mass m_2 so that $x = x_1 + x_2$ is the total extension produced by $m_1 + m_2$ (see Fig. 13.49). Thus we have,

For equilibrium state of m_2

$$m_2 g = kx_1 \qquad (1)$$

For equilibrium state of $(m_1 + m_2)$

$$(m_1 + m_2)g = k(x) = k(x_1 + x_2) \qquad (2)$$

When the mass m_1 is removed, the mass m_2 will move upwards under the unbalanced force $= m_1 g$. Hence

Restoring force (F) on $m_2 = -m_1 g$

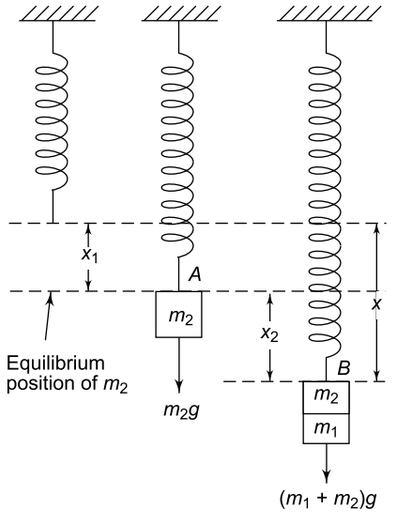


Fig. 13.49

Subtracting (1) and (2) we have

$$m_1 g = kx_2 \qquad (3)$$

Hence, Restoring force on $m_2 = -kx_2$

$$\therefore \quad \text{accelerating of } m_2 = \frac{F}{m_2} = -\frac{k}{m_2} x_2$$

i.e., acceleration $\propto$ – displacement

Angular frequency is $\omega = \sqrt{\frac{k}{m_2}}$

$\therefore$ Frequency of oscillation is

$$n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m_2}}$$

It is clear that A is the equilibrium position of m_2 and B its maximum displacement position. Hence $AB = x_2$ is the amplitude of oscillation of m_2 which from Eq. (3) is given by

$$\text{Amplitude} = x_2 = \frac{m_1 g}{k}$$

Thus the correct choices are (a) and (d).

16. Since the train is moving in a circle, it is in an accelerated (non-inertial) frame of reference. Hence, a fictitious (centrifugal) force mv^2/R is to be introduced as shown in Fig. 13.50. This force is horizontal. Consequently, the equilibrium position of the pendulum will not be vertical; it will be inclined at an angle θ with the vertical. If T is the tension in the string in this position, it follows from the figure that

$$T \cos \theta = mg \text{ and } T \sin \theta = \frac{mv^2}{R}$$

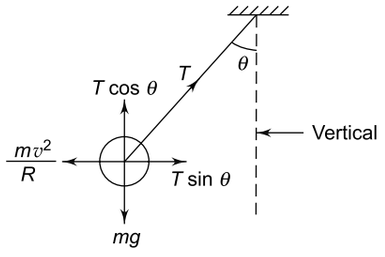


Fig. 13.50

Squaring and adding we get

$$T^2 = (mg)^2 + \left(\frac{mv^2}{R}\right)^2$$

or $$T = m\left(g^2 + \frac{v^4}{R^2}\right)^{1/2} = mg_e$$

where the effective value of acceleration due to gravity is

$$g_e = \left(g^2 + \frac{v^4}{R^2}\right)^{1/2}$$

Time period $= 2\pi\left(\frac{L}{g_e}\right)^{1/2} = 2\pi\sqrt{\frac{mL}{T}}$

$$= 2\pi\left[\frac{L}{\left(g^2 + \frac{v^4}{R^2}\right)^{1/2}}\right]^{1/2}$$

The correct choices are (a) and (c).

17. Mass of sphere is

$$m = \frac{4}{3}\pi R^3 \rho = V\rho \qquad (1)$$

where V is the volume of the sphere. The volume of sphere under water = volume of water displaced $= \frac{V}{2}$. If σ is the density of water, the upthrust is

$$U = \frac{1}{2}(V\sigma g)$$

From the law of floatation, upthrust = weight of the sphere, i.e. $U = mg$ or

$$\frac{1}{2}(V\sigma g) = V\rho g$$

or $$\sigma = 2\rho \qquad (2)$$

If the sphere is pressed down through a small distance x (see Fig. 13.51), the volume of water displaced due to this pressing = volume of a disc of radius R (since the half the sphere is submerged) and thickness x which is $\pi R^2 x$. Hence, upthrust due to this pressing is $\pi R^2 x\sigma g$, which provides the restoring force. Hence the restoring force acting on the sphere when it is released is given by

$$F = -\pi R^2 \sigma g x$$

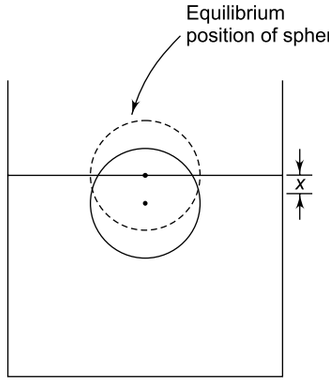


Fig. 13.51

Therefore, the acceleration of the sphere is [use Eq. (1)]

$$a = \frac{F}{m} = -\frac{\pi R^2 \sigma g x}{\frac{4}{3}\pi R^3 \rho} = -\frac{3g}{4}\left(\frac{\sigma}{\rho}\right)\frac{x}{R}$$

Using Eq. (2) we get

$$a = -\left(\frac{3g}{2R}\right)x = -\omega^2 x \qquad (3)$$

which gives $T = 2\pi\sqrt{\frac{2R}{3g}}$. Hence the correct choices are (a) and (c).

18. The two simple harmonic motions are represented by equations

$$y_1 = 10 \sin\left(\frac{12\pi t}{4} + \frac{\pi}{4}\right)$$

$$= 10 \sin\left(3\pi t + \frac{\pi}{4}\right) \qquad (1)$$

and $$y_2 = 5 \sin 3\pi t + 5\sqrt{3} \cos 3\pi t \qquad (2)$$

Refer to the solution of Q.13 of this section. The correct choices are (b) and (d). The amplitude of each = 10 units and the time period of each $= \frac{2}{3}$ second.

19. Total energy = translational K.E. + rotational K.E. + P.E. stored in spring

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \qquad E = \frac{3}{4}mv^2 + \frac{1}{2}kx^2 \qquad (1)$$

Now, the total energy of the system must remain constant, i.e. $\frac{dE}{dt} = 0$.

Differentiating Eq. (1) with respect to time t and setting $\frac{dE}{dt} = 0$, we have

$$\frac{dE}{dt} = 0 = \frac{3}{4} m\left(2v\frac{dv}{dt}\right) + \frac{1}{2} k\left(2x\frac{dx}{dt}\right)$$

Now acceleration $a = \frac{dv}{dt}$ and velocity $v = \frac{dx}{dt}$.

Therefore, $\frac{3}{2} mva + kvx = 0$

or $v\left(\frac{3}{2}ma + kx\right) = 0$

Since $v \neq 0$, we have

$$\frac{3}{2} ma + kx = 0$$

or $$a = -\left(\frac{2}{3}\frac{k}{m}\right)x = -\omega^2 x \qquad (2)$$

where $\omega = \sqrt{\frac{2k}{3m}}$. Hence $T = 2\pi\sqrt{\frac{2k}{m}}$

The correct choices are (a) and (c).

20. $mg + m\omega^2 A = m_1 g$ and $mg - m\omega^2 A = m_2 g$. Form these equations, we find that the correct choices are (a) and (d).

21. The displacement equation can be rewritten as

$$x = \frac{A}{2}(1 - \cos 2\omega t) + \frac{B}{2}(1 + \cos 2\omega t) + \frac{C}{2}\sin 2\omega t$$

or $$x = \frac{1}{2}(A + B) + \frac{1}{2}(B - A)\cos 2\omega t + \frac{C}{2}\sin 2\omega t \qquad (1)$$

Choice (a): Equation (1) can be written as

$$x = x_0 + a\cos 2\omega t + b\sin 2\omega t \qquad (2)$$

where $x_0 = \frac{1}{2}(A + B)$, $a = \frac{1}{2}(B - A)$ and $b = \frac{C}{2}$.

Equation (2) can be recast as

$$x = x_0 + A_0 \sin(2\omega + \phi) \qquad (3)$$

where $A_0 = (a^2 + b^2)^{1/2}$ and $\tan\phi = a/b$. Equation (3) represents a simple harmonic motion of angular frequency 2ω, amplitude = $x_0 + A_0$ and phase constant ϕ.

Choice (b): For $A = B$ and $C = 2B$, Eq. (1) becomes

$$x = B + B\sin 2\omega t = B(1 + \sin 2\omega t)$$

This equation represents a simple harmonic motion of amplitude $2B$ and angular frequency 2ω.

Choice (c): For $A = -B$ and $C = 2B$, Eq. (1) becomes

$$x = B\cos 2\omega t + B\sin 2\omega t$$

which represents a simple harmonic motion of amplitude B, angular frequency 2ω and phase constant $\pi/4$.

Choice (d): For $A = B$ and $C = 0$, Eq. (1) reduces to

$$x = A$$

which does not represent simple harmonic motion. Hence the correct choices are (a), (b) and (c).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage

Passage I

Two light springs of force-constants $k_1 = 1.8\ \text{Nm}^{-1}$ and $k_2 = 3.2\ \text{Nm}^{-1}$ and a block of mass $m = 200$ g are arranged on a horizontal frictionless table as shown in Fig. 13.52. One end of each spring is fixed on rigid supports and the other end is free. The distance CD between the free ends of the springs is 60 cm and the block moves with a velocity $v = 120\ \text{cm s}^{-1}$ between the springs.

IIT, 1985

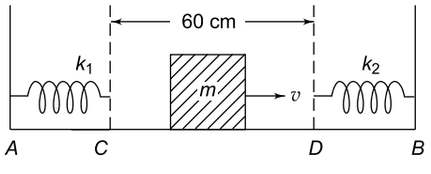


Fig. 13.52

1. When the block moves towards the spring k_2, the time taken by it to move from D upto the maximum compression of spring k_2 is (in seconds)

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

2. When the block moves towards spring k_1, the time taken by it to move from C upto the maximum compression of k_1 is (in seconds)

(a) π (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

3. The period of oscillation of the block between the springs is (in second)

(a) $\left(1+\frac{5\pi}{6}\right)$ (b) $\left(1+\frac{7\pi}{6}\right)$

(c) $\left(1+\frac{5\pi}{12}\right)$ (d) $\left(1+\frac{7\pi}{12}\right)$

SOLUTION

1. Time taken by block to move from D upto the maximum compression of spring k_2 = half the time period of oscillation of the block if it were attached to the free end of k_2, i.e.

$$t_2 = \frac{T_2}{2} = \frac{1}{2}\times 2\pi\sqrt{\frac{m}{k_2}}$$

$$= \frac{1}{2}\times 2\pi\sqrt{\frac{0.2}{3.2}} = \frac{\pi}{4} \text{ second}$$

The correct choice is (d).

2. Similarly $t_1 = \frac{T_1}{2} = \frac{1}{2}\times 2\pi\sqrt{\frac{m}{k_1}} = \frac{1}{2}\times 2\pi\sqrt{\frac{0.2}{1.8}}$

$$= \frac{\pi}{3} \text{ second}$$

3. Time period of oscillation of block is

T = time taken by the block to move freely from C to D and from D to $C = t_1 + t_2$

$$= \frac{2\times 60\,\text{cm}}{120\,\text{cms}^{-1}} + \frac{\pi}{3}+\frac{\pi}{4} = \left(1+\frac{7\pi}{12}\right),$$

which is choice (d).

Questions 4 to 6 are based on the following passage

Passage II

A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. When the cylinder is given a small downward push and released, it starts oscillating vertically with a small amplitude.

IIT, 1990

4. The extension x_0 of the spring when it is in equilibrium is

(a) $\frac{Mg}{k}$ (b) $\frac{g}{k}(M - LA\sigma)$

(c) $\frac{g}{k}\left(M-\frac{1}{2}LA\sigma\right)$ (d) $\frac{g}{k}\left(M+\frac{1}{2}LA\sigma\right)$

5. If the cylinder is given a small downward displacement x from the equilibrium position and released, the restoring force F acting on it is

(a) $-Mgx$ (b) $-(k + A\sigma g)x$

(c) $= -\left(k-\frac{1}{2}A\sigma g\right)x$ (d) $-\left(k+\frac{1}{2}A\sigma g\right)x$

6. The time period T of the vertical oscillations of the cylinder is

(a) $2\pi\sqrt{\frac{M}{k}}$

(b) $2\pi\left[\frac{M}{\left(k+\frac{1}{2}A\sigma g\right)}\right]^{1/2}$

(c) $2\pi\left[\frac{M}{(k-A\sigma g)}\right]^{1/2}$

(d) $2\pi\left[\frac{M}{k+A\sigma g}\right]^{1/2}$

SOLUTION

4. The upthrust on the cylinder with half its length submerged in the liquid is given by

U = weight of the liquid displaced by a length $L/2$ of the cylinder

$$= A \times \frac{L}{2} \times \sigma \times g = \frac{L}{2}\,(A\sigma g)$$

Let x_0 be the extension of the spring when it is in equilibrium. Then

$$kx_0 = Mg - \frac{L}{2}(A\sigma g) \qquad (1)$$

The correct choice is (c).

5. Let x be the small downward displacement given to the cylinder so that the submerged length of the cylinder is now $\left(\frac{L}{2}+x\right)$ and the extension of the spring is now $(x_0 + x)$. The upthrust now is $\left(\frac{L}{2}+x\right) A\sigma g$ and the force in the spring is $k\,(x_0 + x)$. Hence, the restoring force on the cylinder is

$$F = -\left[k(x_0+x) - \left\{Mg - \left(\frac{L}{2}+x\right)A\sigma g\right\}\right] \qquad (2)$$

Using Eq. (1) in Eq. (2), we have

$$F = -(kx + A\sigma g x)$$

or $$F = -(k + A\sigma g)x$$

Thus the correct choice is (b)

6. The acceleration of the cylinder is

$$a = \frac{F}{M} = -\left(\frac{k + A\sigma g}{M}\right)x \qquad (3)$$

Comparing Eq. (3) with $a = -\omega^2 x$, where $\omega = 2\pi/T$, we find that the correct choice is (d).

Questions 7 to 9 are based on the following passage

Passage III

One end of a light spring of force constant k is fixed to a block of mass M placed on a horizontal frictionless surface, the other end of the spring being fixed to a wall. The spring-block system is executing simple harmonic motion of amplitude A and frequency ν. When the block is passing through the equilibrium position, an object of a mass m is gently placed on the block. As a result, the frequency of the system becomes ν' and the amplitude becomes A'.

7. The ratio ν'/ν is

(a) $\left(\frac{M}{M+m}\right)^{1/2}$ (b) $\left(\frac{m}{M+m}\right)^{1/2}$

(c) $\sqrt{\frac{MA}{mA'}}$ (d) $\left[\frac{(M+m)A'}{mA}\right]^{1/2}$

8. If v and v' are the velocities before and after the object is placed on the block, then the ratio v'/v is

(a) $\frac{M}{(M+m)}$ (b) $\left(\frac{M+m}{m}\right)$

(c) $\left(\frac{M+m}{M-m}\right)\frac{A'}{A}$ (d) $\left(\frac{M-m}{M+m}\right)\frac{A}{A'}$

9. The ratio A'/A is

(a) $\left(\frac{M+m}{m}\right)^{1/2}$ (b) $\left[\frac{Am}{A'(M+m)}\right]^{1/2}$

(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left[\frac{A'(M+m)}{AM}\right]^{1/2}$

SOLUTION

7. The frequency of the system before the object is placed on the block is given by

$$\nu = \frac{1}{2\pi}\left(\frac{k}{M}\right)^{1/2}$$

After the object of mass m is placed on the block, the new frequency of the system becomes

$$\nu' = \frac{1}{2\pi}\left[\frac{k}{M+m}\right]^{1/2}$$

$$\therefore \qquad \frac{\nu'}{\nu} = \left(\frac{M}{M+m}\right)^{1/2},$$

which is choice (a)

8. From conservation of momentum, we have

$$Mv = (M + m)v'$$

or $$\frac{v'}{v} = \frac{M}{(M+m)}.$$

So the correct choice is (a).

9. From the principle of conservation of energy we know that the kinetic energy of the block when it is passing through the equilibrium position = potential energy of the spring when the displacement is equal to the amplitude.
Thus we have

$$\frac{1}{2}\ Mv^2 = \frac{1}{2}\ kA^2$$

and $\frac{1}{2}(M+m)v'^2 = \frac{1}{2}\ kA'^2$

$$\therefore \quad \frac{A'}{A} = \left(\frac{v'}{v}\right)\left[\frac{(M+m)}{M}\right]^{1/2}$$

$$= \left(\frac{M}{M+m}\right)\left(\frac{M+m}{M}\right)^{1/2}$$

$$= \left(\frac{M}{M+m}\right)^{1/2},$$

which is choice (c).

Questions 10 to 12 are based on the following passage

Passage IV

Three simple harmonic motions in the same direction having the same amplitude a and the same period are superposed. Each motion differs from the preceding motion by a phase $\phi = 45°$.

IIT, 1999

10. The amplitude of the resulting simple harmonic motion is
(a) $(1 + \sqrt{2})a$ (b) $(1 + \sqrt{3})a$
(c) $3a$ (d) $\frac{3a}{\sqrt{2}}$

11. The phase of the resultant motion relative to the first motion is
(a) 30° (b) 45°
(c) 60° (d) 90°

12. If the energy associated with each motion is E, the total energy of the resultant motion is
(a) $3E$ (b) $(1 + \sqrt{2})E$
(c) $(3 + \sqrt{2})E$ (d) $(3 + 2\sqrt{2})E$

SOLUTION

10. $x_1 = a\cos\omega t$

$x_2 = a\cos(\omega t + \phi)$

$x_3 = a\cos(\omega t + 2\phi)$

where $\phi = 45°$. From the principle of superposition, the resultant motion is given by

$$x = x_1 + x_2 + x_3$$

$$\Rightarrow \quad x = a\cos\omega t + a\cos(\omega t + \phi) + a\cos(\omega t + 2\phi)$$

It can be shown that the resultant displacement x for N collinear simple harmonic motions of the same amplitude a and differing in phase by ϕ is given by

$$x = R\cos(\omega t + \delta)$$

where $R = \frac{a\sin(N\phi/2)}{\sin(\phi/2)}$ and $\delta = \frac{(N-1)\phi}{2}$

For $N = 3$, $R = \frac{a\sin(3\phi/2)}{\sin(\phi/2)}$

$$= \frac{a(\sin\phi\cos\phi/2 + \cos\phi\sin\phi/2}{\sin\phi/2}$$

$$= a(\sin 45°\cot 22.5° + \cos 45°)$$

$$= a(\sqrt{2} + 1)$$

11. $\delta = \frac{1}{2}(3-1)\times 45° = 45°$

12. Total energy $= \frac{R^2}{a^2} \times$ energy of any one motion

$$= (\sqrt{2} + 1)^2 E = (3 + 2\sqrt{2})E$$

Questions 13 to 15 are based on the following passage

Passage V

Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along

horizontal axis and momentum is plotted along vertical axis. The phase space diagram is $x(t)$ vs. $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the Fig. 13.53. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.

IIT, 2011

13. The phase space diagram for a ball thrown vertically up from ground is

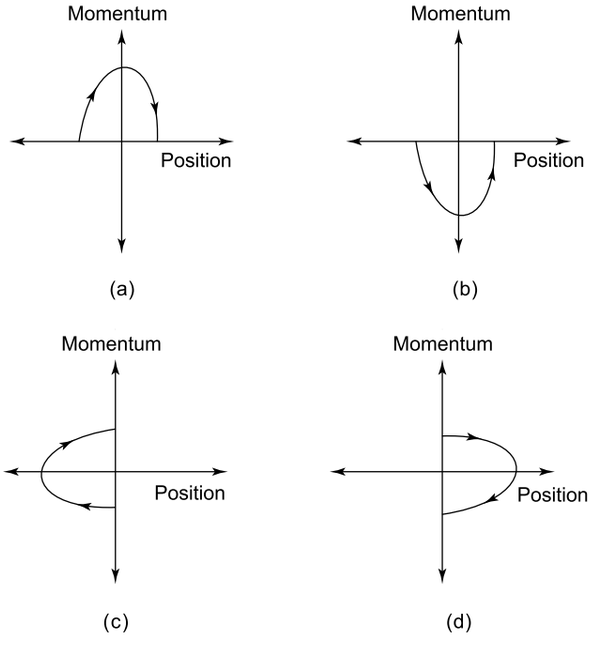


Fig. 13.53

14. The phase space diagram for simple harmonic motion is a circle centered at the origin. In Fig. 13.54, the two circles represent the same oscillator but for different initial condition, and E_1 and E_2 are the total mechanical energies respectively. Then

(a) $E_1 = \sqrt{2}E_2$

(b) $E_1 = 2E_2$

(c) $E_1 = 4E_2$

(d) $E_1 = 16E_2$

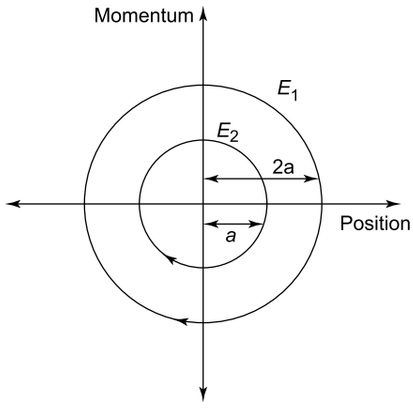


Fig. 13.54

15. Consider the spring-mass system, with the mass submerged in water, as shown in Fig. 13.55. The phase space diagram for one cycle of this system is

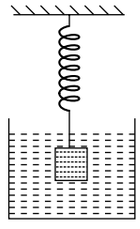


Fig. 13.55

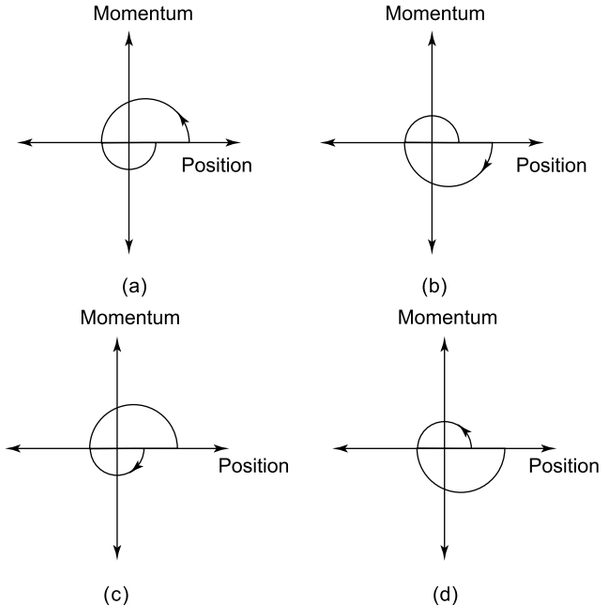


Fig. 13.56

SOLUTION

13. According to the given sign convention, position (x) remains positive and momentum is positive when the body is moving upwards and becomes zero when it reaches the highest point after which

the momentum (p) becomes negative. Hence the correct graph is (d).

14. Energy of simple harmonic oscillator is

$$E = \frac{1}{2}kA^2$$

where k is the force constant and A the amplitude of the oscillator. Since the oscillator is the same, the value of k is the same. Hence

$$E_1 = \frac{1}{2}kA_1^2 \quad \text{and} \quad E_2 = \frac{1}{2}kA_2^2$$

$$\therefore \quad \frac{E_1}{E_2} = \left(\frac{A_1}{A_2}\right)^2$$

Now A_1 = maximum value of displacement of oscillator having energy $E_1 = 2a$ and $A_2 = a$. Therefore

$$\frac{E_1}{E_2} = \left(\frac{2a}{a}\right)^2 = 4. \text{ So } E_1 = 4E_2$$

15. Due to upthrust, the spring will be compressed. Due to damping by the liquid, the final position will be smaller than the initial position. Hence choices (c) and (d) are not possible. Due to buoyancy, the block will move upwards. Hence, according to the given sign convention, position (x) is positive initially. When the system is released, x will decrease and momentum (p) will increase becoming maximum when the system reaches the mean position ($x = 0$) after which the momentum will decrease to zero when the oscillator reaches the extreme position, after which the momentum becomes negative. Hence the correct graph is (b).

SECTION IV

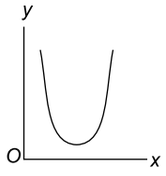
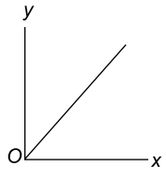
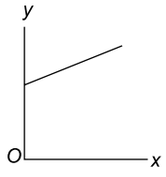
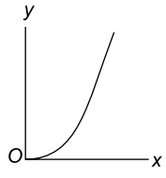
Matrix Match Type

1. Column I describes some situations in which a small object moves. **Column II** describes some characteristics of these motions. Match the situations in **Column I** with the characteristics in **Column II.**

Column I	***Column II***
(a) The object moves on the x- axis under a conservative force in such a way that its "speed" and "position" satisfy $v = c_1\sqrt{c_2 - x^2}$ where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion
(b) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction
(c) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r) The kinetic energy of the object keeps on decreasing.
(d) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s) The object can change its direction only once.

IIT, 2007

2. **Column I** gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in **Column II**. Match the set of parameters given in **Column I** with the graphs given in **Column II**.

Column I	***Column II***
(a) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)	(p) 
(b) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction	(q) 
(c) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle	(r) 
(d) The square of the time peroid (y-axis) of a simple pendulum as a function of its length (x axis).	(s) 

IIT, 2008

ANSWERS

1. (a) The velocity v of a body executing simple harmonic motion is related to displacement x as

$$v = \omega \sqrt{A^2 - x^2}$$

where ω ands A are positive constants. The given equation $v = c_1 \sqrt{c_2 - x^2}$ is similar to the above equation. Hence the body executes simple harmonic, motion, which is choice (p).

(b) It follows from equation $v = -kx$ that $v = 0$ at $x = 0$, i.e. object comes to rest at $x = 0$. Thus the object starting from a negative x value comes to rest at $x = 0$; its velocity (and hence kinetic energy) decrease with time. Hence correct choices are (q) and (r).

(c) For an observer in the reference frame of the elevator, a constant force acts on the object. This is a pseudo force. Hence the motion of the object remains simple harmonic, which is choice (p).

(d) Since the speed of the object is $\sqrt{2}$ times the escape velocity $\left(v_e = \sqrt{2GM_e/R_e}\right)$, the direction of motion of the object does not change (since the forces exerted by objects other than the earth are neglected) and the speed of the object keeps decreasing. Hence the correct choices are (q) and (r).

(a) $\rightarrow$ (p) (b) $\rightarrow$ (q), (r) (c) $\rightarrow$ (p) (d) $\rightarrow$ (q), (r)

2. (a) The potential energy of a simple pendulum is given by $U = \frac{1}{2} kx^2$ where x is the displacement and k is the force constant. U is maximum where $x = \pm A$, where A is the amplitude and minimum at $x = 0$. Hence the correct graph is (p).

(b) For one-dimensional motion with constant velocity ($a = 0$), displacement $S = vt$ (i.e. $s \propto t$) and with constant acceleration $S = ut + \frac{1}{2} at^2$, i.e. $S \propto t^2$. Hence the correct choices are (q) and (s).

(c) For a projectile, range $R \propto u^2$, where u is the speed of projection. Hence the correct graph is (s).

(d) For a simple pendulum, $T^2 = \frac{4\pi^2 l}{g}$. Hence the correct choice is (q).

(a) → (p) (b) → (q), (s) (c) → (s) (d) → (q)

Assertion-Reason Type Questions

In the following questions, Statement-1(Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1

If a spring of force constant k is cut into two equal halves, the force constant of each half is $2k$.

Statement-2

When an elastic spring is extended by an amount x, the work done is $\frac{1}{2} kx^2$.

2. Statement-1

A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from $x = 0$ to $x = A/2$ will be less than the time taken for it to go from $x = A/2$ to $x = A$.

Statement-2

In simple harmonic motion, the speed of the particle is the maximum at the mean position $x = 0$ and decreases as it moves towards the extreme position becoming zero at $x = A$.

3. Statement-1

A body is executing simple harmonic motion. At a displacement x, its potential energy is E_1 and at a displacement y, its potential energy is E_2. The potential energy at a displacement $(x + y)$ is $E = \sqrt{E_1^2 + E_2^2}$.

Statement-2

For a body executing simple harmonic motion, the potential energy is proportional to the square of its displacement from the mean position.

4. Statement-1

The time period of a simple harmonic oscillator depends upon its amplitude and force constant.

Statement-2

The frequency of a simple harmonic oscillator is determined by elasticity and inertia.

5. Statement-1

The amplitude and phase constant of a particle in SHM are determined from its initial displacement and initial velocity.

Statement-2

The amplitude and phase constant of SHM depend on the magnitude of the restoring force.

6. Statement-1

The displacement of a particle is given by

$$x = a \sin (bt + c)$$

This equation suggests that the time period of motion of $2\pi/b$.

Statement-2

The same value of x is obtained at $t = t$ and at $t' = t + 2\pi/b$.

7. Statement-1

The displacement of a simple harmonic oscillator is given by

$$x = A \sin(\omega t + \phi)$$

This equation suggests that the energy of the oscillator remains constant.

Statement-2

The same value of x is obtained after an interval equal to the time period of the oscillator.

8. Statement-1

The time period of a simple pendulum is independent of the mass of the bob.

Statement-2

The restoring force does not depend on the mass of the bob.

9. Statement-1

For small amplitudes, the motion of a simple pendulum is simple harmonic of time period $T = 2\pi\sqrt{l/g}$. For larger amplitudes, the time period is greater than $2\pi\sqrt{l/g}$.

Statement-2

For larger amplitude, the speed of the bob is greater when it passes through the mean position.

10. Statement-1

For an oscillating simple pendulum, the tension in the string is constant at $T = mg$ for all positions of the bob.

Statement-2

The tension in the string will not remain constant at $T = mg$ because the speed of the bob is different at different positions.

SOLUTION

1. The correct choice is (b). The force required to extend the spring by an amount x is given by

$$F = kx \qquad (1)$$

If the spring cut into two equal halves, the same force F will produce half the extension because the extension is directly proportional to the length of the spring. Hence

$$F = k'x' = k'\frac{x}{2} \qquad (2)$$

From (1) and (2), we get $k' = 2k$.

2. The correct choice is (a).

3. The correct choice is (d).

$$E_1 = \frac{1}{2}m\omega^2 x^2 \Rightarrow \sqrt{E_1} = x\sqrt{\frac{m\omega^2}{2}} = xk \quad (1)$$

where $k = \sqrt{\frac{m\omega^2}{2}}$

$$E_2 = \frac{1}{2}m\omega^2 y^2 \Rightarrow \sqrt{E_2} = yk \qquad (2)$$

and $E = \frac{1}{2}m\omega^2 (x+y)^2 \Rightarrow \sqrt{E} = (x+y)k \quad (3)$

From (1), (2) and (3), we get

$$\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$$

$$\Rightarrow \quad E = E_1 + E_2 + 2\sqrt{E_1 E_2}$$

4. The correct choice is (d).

5. The correct choice is (c).

6. The correct choice is (a). The value of x at $t' = t + 2\pi/b$ is

$$x' = a\sin\left[b\left(t + \frac{2\pi}{b}\right) + c\right]$$

$$= a\sin[bt + 2\pi + c]$$

$$= a\sin(bt + c) = x \quad [\because \sin(\theta + 2\pi) = \sin\theta]$$

7. The correct choice is (a).

8. The correct choice is (c). The restoring force when the string makes an angle θ with the vertical is given by $F = -mg\sin\theta$, which depends upon m.

9. The correct choice is (b). Restoring force is $F = -mg\sin\theta$ or $F = -mg_e$ where $g_e = g\sin\theta$. For small oscillations, θ is small so that $\sin\theta \simeq \theta$ (here θ is in radian) and the effective value of g is $g_e\theta$. For larger oscillations $g\sin\theta$ is less than $g\theta$ because $\sin\theta < \theta$. Hence T is greater than $2\pi\sqrt{l/g}$.

10. The correct choice is (d). The tension in the string is given by

$$T = mg\cos\theta + \frac{mv^2}{l}$$

where l = length of the pendulum, v its speed when the string makes an angle θ with the vertical.

SECTION VI

Integer Answer Type

1. A mass M attached to a spring oscillates with a period of 1s. If the mass is increased by 3 kg, the period increases by 1s. Find the value of M (in kg) assuming that Hookes' law is obeyed.

IIT, 1979

2. An object of mass 0.2 kg executes simple harmonic motion along the x-axis with a frequency of $25/\pi$ Hz. At the position $x = 0.04$ m, the object has kinetic energy of 0.5 J and potential energy of 0.4 J. Find the amplitude of oscillations in cm.

IIT, 1994

3. A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between the block and the table is 0.72. Find the maximum amplitude in cm of the table for which the block does not slip on the surface of the table. Take $g = 10\ \text{ms}^{-2}$.

IIT, 1996

4. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. It takes time t_1 to go from 0 to $A/2$ and t_2 to go from $A/2$ to A. Find the ratio T_2/T_1.

IIT, 2001

SOLUTION

1. $1 = 2\pi\sqrt{\dfrac{M}{k}}$ and $2 = 2\pi\sqrt{\dfrac{M+3}{k}}$

Dividing these equations we get

$$4 = \frac{M+3}{M} \Rightarrow M = 1 \text{ kg}$$

2. Angular frequency $\omega = 2\pi\nu = 2\pi \times \dfrac{25}{\pi} = 50 \text{ rad s}^{-1}$

Kinetic energy $K = \dfrac{1}{2} m\omega^2 (A^2 - x^2)$

Potential energy $U = \dfrac{1}{2} m\omega^2 x^2$

Dividing we get

$$\frac{K}{U} = \frac{A^2 - x^2}{x^2}$$

$$\Rightarrow \quad \frac{0.5}{0.4} = \frac{A^2 - x^2}{x^2}$$

$$\Rightarrow \quad A = \frac{3x}{2} = \frac{3 \times 0.04 \text{ m}}{2}$$

$$= 0.06 \text{ m} = 6 \text{ cm}$$

3. Let the mass of the block be m and let, at a certain instant of time, the direction of acceleration a of the table (executing simple harmonic motion) be along the positive x-direction. As a result, the block will experience a force ma directed along the negative x-axis. Consequently, the force of friction μmg will act along the positive x-axis. The weight mg of the block will be balanced by the normal reaction R. The block will not slip on the surface of the table, if the acceleration a of the motion of the table is such that [Fig. 13.57]

$$\mu mg \geq ma \text{ or } \mu g \geq a$$

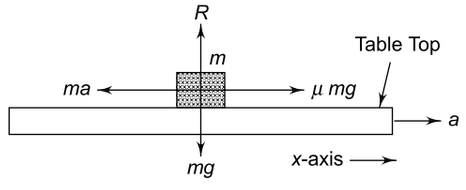


Fig. 13.57

Therefore, for no slipping, the table can have a maximum acceleration $a_{max} = \mu g$. Where $a_{max} = \omega^2 A$. Therefore, the maximum amplitude is given by

$$\omega^2 A_{max} = mg$$

or $$A_{max} = \frac{\mu g}{\omega^2} = \frac{\mu g}{4\pi^2 \nu^2} \qquad (\because \omega = 2\pi\nu)$$

$$= \frac{0.72 \times 10}{4\pi^2 \times (3)^2} = 0.02 \text{ m} = 2 \text{ cm}$$

4. From $x = A \sin \omega t$, we have

$$\frac{A}{2} = A \sin \omega t_1 \Rightarrow \omega t_1 = \frac{\pi}{6} \qquad (1)$$

Also $A = A \sin \omega (t_1 + t_2)$

$$\Rightarrow \quad \omega(t_1 + t_2) = \frac{\pi}{2} \qquad (2)$$

From (1) and (2), we get $\dfrac{t_1}{t_1 + t_2} = \dfrac{1}{3}$ which gives $\dfrac{t_2}{t_1} = 2$

14 Chapter

Waves and Doppler's Effect

REVIEW OF BASIC CONCEPTS

14.1 WAVE MOTION

Wave motion involves the transport of energy without any transport of matter. In case of mechanical waves, the disturbance is the physical displacement of particles of a medium. In case of electromagnetic waves, the disturbance is a change in electric and magnetic fields.

14.2 TYPES OF WAVES

There are two types of wave motions: (1) transverse and (2) longitudinal.

(1) ***Transverse Waves*** In transverse waves the particles of the medium vibrate at right angles to the direction in which the wave propagates. Waves on strings, surface water waves and electromagnetic waves are transverse waves.

(2) ***Longitudinal Waves*** In longitudinal waves the particles of the medium vibrate along the direction of wave propagation. Sound waves are longitudinal.

14.3 CHARACTERISTICS OF A HARMONIC WAVE

(1) ***Amplitude*** The amplitude of a wave is the maximum displacement of the particles of the medium from their mean position.

(2) ***Period*** The time period of a wave is the period of harmonic oscillations of particles of the medium. The frequency of a wave is the reciprocal of the time period.

(3) ***Wave Velocity*** Wave velocity is the distance travelled by the wave in one second.

(4) ***Wavelength*** The wavelength is defined as the distance (measured along the direction of propagation of wave) between two nearest particles which are in the same phase of vibration.

14.4 DISPLACEMENT EQUATION FOR A TRAVELLING WAVE

When a plane wave travels in a medium along the positive x-direction, the displacement y of a particle located at x at time t is given by

$$y = A \sin (\omega t - kx)$$

where A = amplitude of the wave, ω (= $2\pi\nu$) is the angular frequency (in rad s^{-1}), ν is the frequency (in Hz) and $k = \left(\frac{2\pi}{\lambda}\right)$ is the angular wave number and λ is the wavelength of the wave.

For a wave travelling along the negative x-direction,

$$y = A \sin (\omega t + kx)$$

The wave velocity is given by

$$v = \nu\lambda = \frac{\omega}{k}$$

14.5 PHASE AND PHASE DIFFERENCE

The argument of the sine (or cosine) function which represents a wave is called the phase of the wave. For a wave travelling along the positive x-direction, the phase ϕ at a space point x at time t is given by

$$\phi = \omega t - kx$$

It is clear that the phase changes with time t as well as space point x.

Phase Change with Time

The phase of a given particle (i.e. x fixed) changes with time. As time changes from t to $(t + \Delta t)$, the phase of a

particle oscillation changes from ϕ to $(\phi + \Delta\phi)$ where $\Delta\phi$ is given by

$$\Delta\phi = \{\omega\,(t + \Delta t) - kx\} - \{\omega t - kx\}$$

$$\Rightarrow \qquad \Delta\phi = \omega\,\Delta t = \frac{2\pi}{T}\Delta t$$

where T is the time period of particle oscillation. If $\Delta t = T$, $\phi = 2\pi$.

Phase Change with Position

At a given instant of time t, the phase of particles of the medium varies with position x of the particles. The phase difference at an instant t between two particles separated by x and $(x + \Delta x)$ is given by

$$\Delta\phi = \{\omega t - k(x + \Delta x)\} - (\omega t - kx)$$

$$\Rightarrow \qquad \Delta\phi = -k\,\Delta x = -\frac{2\pi}{\lambda}\Delta x$$

The minus sign indicates that, for a wave travelling along the positive x-direction, the particles located at higher values of x lag behind in phase. If $\Delta x = \lambda$, $|\,\Delta\phi\,| = 2\pi$. Hence *wavelength can be defined as the distance between two particles whose phases differ by 2π.*

EXAMPLE 14.1

When a plane wave travels in a medium, the displacements of particles are given by

$$y = 0.01\,\sin\,[2\pi\,(2t - 0.01x)]$$

where x and y are in metre and t in second. Find

(a) the amplitude, wavelength, wave velocity and frequency of the wave,

(b) the phase difference between two positions of the same particle in a time interval of 0.25s and

(c) the phase difference at a given instant of time between two particles 50 m apart.

SOLUTION

(a) The given equation is

$$y = 0.01\,\sin\,(4\pi t - 0.01 \times 2\pi x)$$

Comparing this equation with

$y = A\,\sin\,(\omega t - kx)$, we get

$$A = 0.01\ \text{m}$$

$$\omega = 4\pi \Rightarrow 2\pi\nu = 4\pi \Rightarrow \nu = 2\ \text{Hz}$$

$$k = 0.01 \times 2\pi \Rightarrow \frac{2\pi}{\lambda} = \frac{2\pi}{100}$$

$$\Rightarrow \qquad \lambda = 100\ \text{m}$$

$$v = \nu\lambda = 2 \times 100 = 200\ \text{ms}^{-1}$$

(b) $\Delta\phi = 2\pi\nu\Delta t = 2\pi \times 2 \times (0.25) = \pi = 180°$

(c) $\Delta\phi = -\frac{2\pi}{\lambda}\,\Delta x = -\frac{2\pi}{100} \times 50 = -\pi = -180°$

The negative sign indicates that the particle at $x = 50$ m lags behind the particle at $x = 0$ by a phase angle of 180°.

EXAMPLE 14.2

A travelling wave on a string is given by

$$y = 7.5\,\sin\left(0.005x + 12t + \frac{\pi}{4}\right)$$

where x and y are in cm and t in second.

(a) Find the displacement and velocity of the particle at a point $x = 1$ cm at $t = 1$s. Is this equal to the wave velocity? Given $\sin(12.7°) = 0.22$.

(b) Locate the points on the string which have the same transverse displacement and velocity as the $x = 1$ cm point has at $t = 2$s, 5s and 11s.

SOLUTION

Given $\qquad y = 7.5\,\sin\left(0.005x + 12t + \frac{\pi}{4}\right) \qquad$ (i)

Putting $x = 1$ cm and $t = 1$s in Eq. (i) we have

$$y = 7.5\,\sin\left(0.005 \times 1 + 12 \times 1 + \frac{3.142}{4}\right)$$

$$= 7.5\,\sin\,(12.79\ \text{rad})$$

$$= 7.5\,\sin\,(732.7°)$$

$$(\because\ \pi\ \text{rad} = 180°)$$

$$= 7.5\,\sin\,(4\pi + 12.7°)$$

$$= 7.5\,\sin\,(12.7°)$$

$$= 7.5 \times 0.22 = 1.65\ \text{cm}$$

Particle velocity is obtained by differentiating (i) w.r.t. time t.

$$V = \frac{dy}{dt} = 7.5 \times 12 \times \cos\left(0.005x + 12t + \frac{\pi}{4}\right)$$

Putting $\quad x = 1$ cm and $t = 1$s,

$$V = 7.5 \times 12 \times \cos\ 12.7°$$

$$= 87.8\ \text{cm s}^{-1}$$

Comparing Eq. (i) with

$$y = A\,\sin\,(kx + \omega t + \phi_0)$$

we get $k = 0.005$ rad cm^{-1} and $\omega = 12$ rad s^{-1} giving

$$v = \frac{\omega}{k} = \frac{12}{0.005} = 2400\ \text{cm s}^{-1}$$

Particle velocity V is not equal to wave velocity v.

(b) By definition of wavelength (λ), all points on the string which are located at $x = \lambda$, 2λ, ...

$= n\lambda$ ($n = 0, 1, 2, ...$) away from $x = 1$ cm have the same displacement and velocity as those at $x = 1$ cm for all values of t.

From $k = 0.005 \Rightarrow \dfrac{2\pi}{\lambda} = 0.005 \Rightarrow \lambda = 12.67$ cm. Hence the positions of the required points are $x = 1$ cm + (12.67 cm) n = (1 + 12.67 n) cm where n is an integer.

EXAMPLE 14.3

A tuning fork vibrating at 200 Hz produces a sound wave which travels in air at a speed 340 ms^{-1}. Find the distance travelled by sound during the time the fork makes 60 vibrations.

SOLUTION

$$\nu = 200 \text{ Hz}, \; v = 340 \text{ ms}^{-1}$$

$$\therefore \quad \lambda = \frac{v}{\nu} = \frac{340}{200} = 1.7 \text{ m}$$

By definition, wavelength (λ) is the distance travelled by the wave in 1 complete vibration (= time period) of the fork. Hence

$$x = 60 \times 1.7 = 102 \text{ m}$$

14.6 EXPRESSIONS FOR WAVE VELOCITY IN DIFFERENT MEDIA

1. Velocity of sound in an elastic medium is given by

$$v = \sqrt{\frac{E}{\rho}}$$

where E = modulus of elasticity of the medium and

ρ = density of the medium.

(a) *For gases*:

$$E = \gamma P$$

where $g = C_p/C_v$ is the ratio of the specific heat of the gas at constant pressure and that at constant volume and P is the pressure of the gas.

Thus

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

(b) *For solids*: $E = Y$; the Young's modulus of the solid. Thus

$$v = \sqrt{\frac{Y}{\rho}}$$

(c) *For liquids*: $E = B$, the bulk modulus of the liquid. Thus

$$v = \sqrt{\frac{B}{\rho}}$$

2. The velocity of sound in a gas is independent of the pressure but is directly proportional to the square root of the absolute temperature.

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \left(\frac{t + 273}{273}\right)^{1/2}$$

where t is the temperature in °C.

3. The velocity of sound increases with increase in humidity. Sound travels faster in moist air than in dry air at the same temperature.
4. Velocity of a transverse wave on a stretched string is given by

$$v = \sqrt{\frac{T}{m}}$$

where T = tension in the string and m = mass per unit length of the string. For a string of diameter d and density ρ, we have

$$m = \frac{\pi d^2 \rho}{4} \quad \text{Thus, } v = \frac{2}{d}\sqrt{\frac{T}{\pi\rho}}$$

EXAMPLE 14.4

Uniform rope AB of mass M and length L hangs vertically with end A fixed to a support. A transverse pulse is created at the free end A. The speed of the pulse is

(a) the same at every point of the rope

(b) maximum near end A

(c) maximum near end B

(d) zero at the mid-point of the rope.

SOLUTION

Since the rope has a finite mass, the tension is different at different points on the rope. Hence the speed of the pulse is different at different points.

Mass per unit length is $m = \dfrac{M}{L}$.

Tension at point P (Fig. 14.1) is

$$T = \frac{Mgx}{L} = mgx$$

$\therefore$ Speed of the pulse is

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{mgx}{m}} = \sqrt{gx}$$

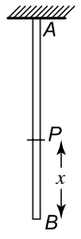


Fig. 14.1

i.e. $v \propto \sqrt{x}$. Hence the correct choice is (b).

The speed of the pulse is

EXAMPLE 14.5

A long uniform steel wire has a diameter of 2.0 mm. What should be the tension in the wire, so that the speed of the transverse wave on it equals the speed of sound at STP (= 320 m s^{-1})? The density of steel is 7800 kg m^{-3}.

SOLUTION

The volume of a wire of length L and diameter d is

$$V = \pi r^2 L = \pi d^2 L/4$$

The mass of the wire is

$$M = \text{volume} \times \text{density} = \frac{\pi d^2 L \rho}{4}$$

$$\therefore \quad \text{Mass per unit length } (m) = \frac{M}{L} = \frac{\pi d^2 \rho}{4}$$

Now $$v = \sqrt{\frac{T}{m}}$$

$$\therefore \quad T = mv^2 = \frac{\pi d^2 \rho v^2}{4}$$

Substituting the values of d, ρ and v and solving we get

$$T = 25.1 \text{ N}$$

EXAMPLE 14.6

Transverse waves are generated in two uniform steel wires A and B of diameters 10^{-3} m and 0.5×10^{-3} m respectively, by attaching their free end to a vibrating source of frequency 500 Hz. Find the ratio of the wavelengths if they are stretched with the same tension.

SOLUTION

The density of a wire of mass M, length L and diameter d is given by

$$\rho = \frac{4M}{\pi d^2 L} = \frac{4m}{\pi d^2}$$

now $$v_A = \sqrt{\frac{T}{m_A}}$$

and $$v_B = \sqrt{\frac{T}{m_B}}$$

$$\frac{v_A}{v_B} = \sqrt{\frac{m_B}{m_A}} = \frac{d_B}{d_A}$$

but $v_A = v\lambda_A$ and $v_B = v\lambda_B$, v being the frequency of the source.

Hence $$\frac{\lambda_A}{\lambda_B} = \frac{v_A}{v_B} = \frac{d_B}{d_A} = \frac{0.5 \times 10^{-3}}{10^{-3}} = 0.5$$

EXAMPLE 14.7

Compare the velocities of sound in hydrogen (H_2) and carbon dioxide (CO_2). The ratio (γ) of specific heats of H_2 and CO_2 is respectively 1.4 and 1.3.

SOLUTION

$$v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}} \quad \text{and} \quad v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{\gamma_1 \rho_2}{\gamma_2 \rho_1}}$$

Since density of a gas is proportional to its molecular weight,

$$\frac{\rho_2}{\rho_1} = \frac{44.01}{2.016} = 21.83$$

$$\therefore \quad \frac{v_1}{v_2} = \left(\frac{1.4}{1.3} \times 21.83\right)^{1/2} = 4.85$$

Velocity of sound in hydrogen is 4.85 times that in carbon dioxide.

EXAMPLE 14.8

At what temperature will sound travel in hydrogen with the same speed as in oxygen at 927°C?

SOLUTION

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1 \rho_2}{\gamma_2 \rho_1}}$$

Hydrogen and oxygen are both diatomic gases. Hence, $\gamma_1 = \gamma_2$. Also

$$\frac{\rho_2}{\rho_1} = \frac{\text{molecular mass of oxygen}}{\text{molecular mass of hydrogen}}$$

$$= \frac{32}{2} = 16$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{16} = 4$$

$$\frac{v_t}{v_0} = \left(\frac{t + 273}{273}\right)^{1/2}$$

For oxygen $$\frac{v_t}{v_2} = \left(\frac{927 + 273}{273}\right)^{1/2} = \left(\frac{1200}{273}\right)^{1/2} \quad \text{(i)}$$

For hydrogen $\frac{v_t'}{v_1} = \left(\frac{273+t}{273}\right)^{1/2}$ (ii)

Dividing (i) and (ii)

$$\frac{v_t}{v_t'} \times \frac{v_1}{v_2} = \left(\frac{1200}{273+t}\right)^{1/2}$$

$$\Rightarrow \quad \frac{v_t}{v_t'} \times 4 = \left(\frac{1200}{273+t}\right)^{1/2}$$

Given $v_t' = v_t$. Hence

$$4 = \left(\frac{1200}{273+t}\right)^{1/2}$$

$$\Rightarrow \quad 16 = \left(\frac{1200}{273+t}\right)$$

$$\Rightarrow \quad t = -198°C$$

EXAMPLE 14.9

In a laboratory experiment (room temperature being 15°C) the wavelength of a note of sound of frequency 500 Hz is found to be 0.68 m. If the density of air at STP is 1.29 kg m^{-3}, calculated the ratio of the specific heats of air.

SOLUTION

Speed of sound at 15°C is $v_1 = \nu\lambda = 500 \times 0.68 = 340$ ms^{-1}

Speed of sound at 0°C is $v_0 = 340 \times \sqrt{\frac{273}{273+15}}$ $= 331$ ms^{-1}

At STP, $\rho_0 = 1.29$ kgm^{-3} and $P_0 = 1.01 \times 10^5$ N m^{-2}

Now $v_0 = \sqrt{\frac{\gamma P_0}{\rho_0}}$; $\gamma = \frac{C_p}{C_v}$

$$\Rightarrow \quad \gamma = \frac{v_0^2 \rho_0}{P_0} = \frac{(331)^2 \times 1.29}{1.01 \times 10^5} = 1.39$$

14.7 SUPERPOSITION OF WAVES (THE SUPERPOSITION PRINCIPLE)

When a wave reaches a particle of a medium, it imparts a displacement to that particle. If two or more waves arrive at a particle, the resultant displacement of the particle is equal to the vector sum of individual displacements. In the particular case when the waves travelling in the same straight line superpose, the resultant displacement is equal to the algebraic sum of the individual displacements. This is called the principle of superposition.

$$y = y_1 + y_2 + ... + y_n$$

The following three cases of superposition are of practical importance.

14.8 INTERFERENCE

The superposition of two waves of the same frequency travelling in the same direction in a medium is called interference. Consider two waves of equal amplitude a, equal angular frequency ω and and equal angular wave number k but having a phase difference ϕ travelling along the positive x-direction. The displacements y_1 and y_2 of a particle located at x at time t are

$$y_1 = a \sin(\omega t - kx)$$

and $$y_2 = a \sin(\omega t - kx + \phi)$$

According to the superposition principle, the resultant displacement is given by

$$y = y_1 + y_2$$
$$= a[\sin(\omega t - kx) + \sin(\omega t - kx + \phi)]$$

Using $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$, we get

$$y = \left(2a\cos\frac{\phi}{2}\right)\sin\left(\omega t - kx + \frac{\phi}{2}\right)$$

$$\Rightarrow \quad y = A\sin\left(\omega t - kx + \frac{\phi}{2}\right)$$

where $A = 2a\cos\left(\frac{\phi}{2}\right)$ is the resultant amplitude. The frequency and wavelength of the resultant wave remain the same as those of individual waves.

(a) *Constructive Interference:* If A is maximum (positive or maximum), the interference is constructive, the $A_{max} = \pm 2a$. This happens if

$$\cos\left(\frac{\phi}{2}\right) = \pm 1$$

$$\Rightarrow \quad \frac{\phi}{2} = 0, \pi, 2\pi, ...$$

$$\Rightarrow \quad \phi = 2n\pi; \qquad (n = 0, 1, 2, ...)$$

(b) *Destructive Interference:* If $A = 0$, the interference is destructive. This happens if

$$\cos\left(\frac{\phi}{2}\right) = 0$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\Rightarrow \quad \phi = (2n+1)\pi; \qquad n = 0, 1, 2, ...$$

EXAMPLE 14.10

Two waves each of amplitude 2 cm and wavelength 5 cm and frequency 10 Hz have a constant phase difference of 60°. Travelling in the same direction, they superpose at a particle of the medium. What is the resultant amplitude of the oscillations of the particle? Also find the frequency and wavelength of the resultant wave.

SOLUTION

Resultant amplitude is $A = 2a\cos\left(\frac{\phi}{2}\right)$

$$= 2 \times (2\text{ cm}) \times \cos\left(\frac{60°}{2}\right)$$

$$= 2\sqrt{3}\text{ cm}$$

The frequency and wavelength of the resultant wave are 10 Hz and 5 cm respectively, the same as those of the individual interfering waves.

14.9 STANDING (OR STATIONARY) WAVES

Standing (or stationary) waves are produced when two waves of the same frequency travelling in opposite directions in a medium superpose. In actual practice, we do not send two independent waves in a medium in opposite directions. A wave is sent in a finite medium which has its boundaries, for example, a string of a finite length or a rod or a column of gas or liquid. The wave gets reflected at the boundaries and a superposition of the incident and reflected waves occurs continuously, giving rise to standing waves.

When a wave is reflected from a rigid boundary, it undergoes a reversal of amplitude (which implies a phase change of π). Consider a wave travelling in the negative x-direction towards a boundary at $x = 0$, where it is reflected. The particle displacements due to the incident and reflected waves are given by

$$y_i = a\sin(\omega t + kx)$$

and $$y_r = -a\sin(\omega t - kx)$$

From the superposition principle, the resultant displacement is

$$y = y_i + y_r$$

$$= a[\sin(\omega t + kx) - \sin(\omega t - kx)]$$

Using $\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$, we get

$$y = 2a\sin(kx)\cos(\omega t) \quad \text{(i)}$$

$$\Rightarrow \quad y = A\cos\omega t \quad \text{(ii)}$$

where $$A = 2a\sin(kx) \quad \text{(iii)}$$

Equation (i) represents a standing wave. It does not represent a travelling wave since it does not involve the combination $(\omega t \pm kx)$ in the argument of the sine or cosine function. Equation (ii) tells us each particle has a simple harmonic motion and Eq. (iii) tells us that the amplitude of motion is different for different particles (i.e. for different values of x). Such simple harmonic motions of the particles of a medium are called *normal modes*.

Nodes

There are certain points in the medium which are permanently at rest. These points are called nodes. The position of nodes is given by

$$A = 0$$

$$\Rightarrow \quad 2a\sin(kx) = 0$$

$$\Rightarrow \quad \sin kx = 0$$

$$\Rightarrow \quad kx = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \quad \frac{2\pi}{\lambda}x = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \quad x = 0, \frac{\lambda}{2}, \lambda, \ldots \text{ and so on}$$

$\therefore$ Distance between two consecutive nodes $= \frac{\lambda}{2}$.

Antinodes

There are certain points in the medium which have maximum (positive or negative) amplitude. These points are called antinodes. Their position is given by

$$A = \pm 1$$

$$\Rightarrow \quad 2a\sin(kx) = \pm 1$$

$$\Rightarrow \quad \sin kx = \pm 1$$

$$\Rightarrow \quad kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\Rightarrow \quad \frac{2\pi}{\lambda}x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\Rightarrow \quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

$\therefore$ Distance between two consecutive antinodes $= \frac{\lambda}{2}$.

NOTE

1. Exactly mid-way between two nodes is an antinode and vice versa.
2. The distance between a node and the next antinode $= \frac{\lambda}{4}$.
3. There is no transfer of energy along the medium.

EXAMPLE 14.11

Standing waves are produced by the superposition of two waves

$$y_1 = 5 \sin (3\pi t - 2\pi x)$$

and $$y_2 = 5 \sin (3\pi t + 2\pi x)$$

where y and x are in cm and t in second. Find the amplitude of the particle at $x = 2$ cm.

SOLUTION

$$y = y_1 + y_2$$
$$= 5 [\sin (3\pi t - 2\pi x) + \sin (3\pi t + 2\pi x)]$$

Using $\sin (\alpha + \beta) + \sin (\alpha - \beta) = 2 \sin \alpha \cos \beta$, we get

$$y = 10 \cos (2\pi x) \sin (3\pi t)$$

$\Rightarrow$ $y = A \sin (3\pi t)$, where $A = 10 \cos (2\pi x)$

Amplitude A at $x = 2$ cm is

$$10 \times \cos (2\pi \times 2) = 10 \cos 4\pi = 10 \text{ cm}$$

Normal Modes of a String Fixed at Both Ends

Consider a uniform string of length L stretched with a tension T and fixed rigidly at its ends at $x = 0$ and $x = L$. The string can vibrate in a number of modes. Figure 14.2 shows the first three harmonics.

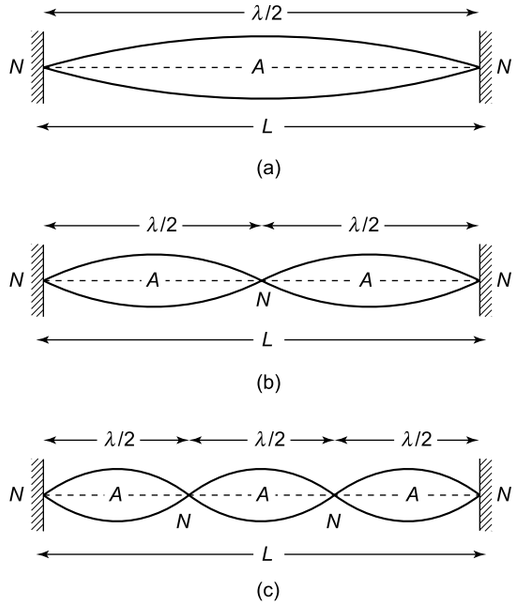


Fig. 14.2

(a) *Fundamental Mode (or First Harmonic)*

In this mode, the string vibrates in one segment. [Fig. 14.2 (a)]

$$\frac{\lambda}{2} = L \quad \Rightarrow \quad \lambda = 2L$$

The frequency of the fundamental mode is

$$\nu_1 = \frac{v}{\lambda} = \frac{v}{2L}$$

where $$v = \sqrt{\frac{T}{m}}$$

(b) *Second Harmonic*. [Fig. 14.2 (b)]

For the second harmonic, $\frac{\lambda}{2} + \frac{\lambda}{2} = L \Rightarrow \lambda = L.$

$$\nu_2 = \frac{v}{\lambda} = \frac{v}{L} = 2\nu_1$$

(c) *Third Harmonic*. [Fig.14.2(c)]

For the third harmonic, $\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{2} = L \Rightarrow \lambda = \frac{2L}{3}$

$$\nu_3 = \frac{v}{\lambda} = \frac{3v}{2L} = 3\nu_1$$

In general, for a string vibrating in the nth harmonic, the frequency of vibration is

$$\nu_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

EXAMPLE 14.12

The transverse displacement of a string fixed at both ends is given by

$$y = 0.06 \sin\left(\frac{2\pi x}{3}\right) \cos (120\pi t) \quad \text{(i)}$$

where y and x are in metre and t in second. The length of the string is 1.5 m and its mass is 3×10^{-2} kg.

(a) What is the amplitude at point $x = 0.5$ m?

(b) What is the velocity of the particle at $x = 0.75$ m at $t = 0.25$ s?

(c) Write down the equation of the component waves whose superposition gives the vibration given in Eq. (i) above. What is the wavelength, frequency and speed of each wave?

(d) Determine the tension in the string.

(e) Do all points on the string vibrate with the same (i) frequency, (ii) phase and (iii) amplitude?

SOLUTION

(a) Displacement is maximum when $\cos (120\pi t) = 1$.

Hence the amplitude $= 0.06 \sin\left(\frac{2\pi x}{3}\right)$.

At $x = 0.5$ m, the amplitude $= 0.06 \sin\left(\frac{2\pi}{3} \times 0.5\right)$

$$= 0.06 \sin\frac{\pi}{3} = 0.052 \text{ m}$$

(b) The velocity at point x at time t is obtained by differentiating Eq. (i) with respect to t.

$$V = \frac{dy}{dt}$$

$$= -(0.06 \times 120\pi) \sin\left(\frac{2\pi x}{3}\right) \sin(120\pi t)$$

Now at $t = 0.25$s, $\sin(120\pi t) = \sin(30\pi) = 0$. Hence at $t = 0.25$s, the velocity is zero for all values of x including $x = 0.75$ m.

(c) The stationary wave in the question may be written as

$$y = 2A \sin\alpha \cos\beta$$

where $A = 0.03$ m, $\alpha = \frac{2\pi x}{3}$ and $\beta = 120\pi t$.

Now $2A \sin\alpha \cos\beta = A \sin(\alpha + \beta) + A \sin(\alpha - \beta)$

$$= 0.03 \sin\left(\frac{2\pi x}{3} + 120\pi t\right)$$

$$+ 0.03 \sin\left(\frac{2\pi x}{3} - 120\pi t\right)$$

i.e. $y = y_1 + y_2$

Hence the two component waves are

$$y_1 = 0.03 \sin\left(\frac{2\pi x}{3} + 120\pi t\right)$$

and $$y_2 = 0.03 \sin\left(\frac{2\pi x}{3} - 120\pi t\right)$$

$$= -0.03 \sin\left(120\pi t - \frac{2\pi x}{3}\right)$$

Let λ be the wavelength, ν the frequency and v the speed of each wave. Then

$\frac{2\pi}{\lambda}$ = coefficient of x in the argument of the sine function = $\frac{2\pi}{3}$

or $\lambda = 3$ m.

Also $\omega = 2\pi\nu$ = coefficient of t in the argument of the sine function

$= 120\pi$

which gives $\nu = 60$ Hz.

Hence $v = \nu\lambda = 60 \times 3 = 180$ m s^{-1}

(d) Mass per unit length $(m) = \frac{3.0 \times 10^{-2}}{1.5}$

$= 2.0 \times 10^{-2}$ kg m^{-1}

We know that $v = \sqrt{\frac{T}{m}}$, where T is tension in the string.

$\therefore \quad T = mv^2 = 2.0 \times 10^{-2} \times (180)^2 = 648$ N

(e) In a stationary wave on a string, all points on the string vibrate with the same frequency and the same phase, but the amplitude is different at different points (i.e. different values of x). The amplitude is zero at nodes and maximum at antinodes.

EXAMPLE 14.13

A wire of density 9 g cm^{-3} is stretched between two clamps 100 cm apart while being subjected to an extension of 0.05 cm.. What is the lowest frequency of transverse vibrations of the wire, assuming that the Young's modulus of the material of the wire $= 0.9 \times 10^{11}$ N m^{-2}.

SOLUTION

Young's modulus

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{l/L} = \frac{TL}{Al}$$

where T = tension, L = original length = 100 cm, l = extension = 0.05 cm
and A = area of cross-section of the wire.

Hence, $$\frac{T}{A} = \frac{Yl}{L} = \frac{0.9 \times 10^{11} \times 0.05}{100}$$

$$= 4.5 \times 10^7$$

Now mass per unit length

$$m = \frac{\text{mass}}{\text{length}}$$

$$= \frac{\text{volume} \times \text{density}}{\text{length}}$$

$$= \frac{\text{area} \times \text{length} \times \text{density}}{\text{length}}$$

$$= \text{area} \times \text{density}$$

$$= A\rho$$

Density $\rho = 9$ g cm$^{-3} = 9000$ kg m^{-3}. The lowest frequency is the frequency of the fundamental mode.

$$\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{T}{A\rho}}$$

$$= \frac{1}{2 \times 1} \times \sqrt{\frac{4.5 \times 10^7}{9000}}$$

$$= 35.3 \text{ Hz}$$

EXAMPLE 14.14

A wire having a linear density of 0.05 g cm^{-1} is stretched between two rigid supports with a tension of 4.5×10^2 N. It is observed that the wire resonates at a frequency of 420 Hz. The next higher frequency at which the wire resonates is 490 Hz. Determine the length of the wire.

SOLUTION

Let 420 Hz be the nth harmonic, then 490 Hz is the $(n + 1)$th harmonic. Therefore

$$420 = \frac{n}{2L}\sqrt{\frac{T}{m}} \qquad \text{(i)}$$

and $$490 = \frac{(n+1)}{2L}\sqrt{\frac{T}{m}} \qquad \text{(ii)}$$

Dividing (i) and (ii) we get $n = 6$. Putting $n = 6$ in (i) we get

$$420 = \frac{6}{2L}\sqrt{\frac{4.5\times10^2}{0.05\times10^{-1}}} \Rightarrow L = 2.14 \text{ m}$$

Normal Modes in Air Columns in a Pipe

A gas column in a pipe can oscillate in a number of modes

Case 1: Closed Pipe

Consider a pipe of length L open at one end and closed at the other. The closed end is a node and the open end is an antinode. Figure 14.3 shows the first three modes of a closed pipe.

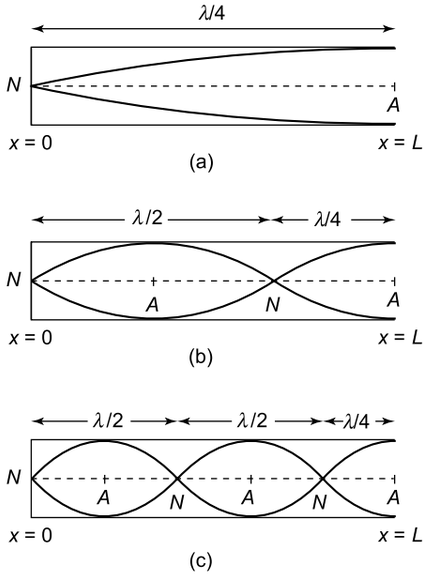


Fig. 14.3

(a) Fundamental Mode (or First Harmonic) [Fig. 14.3 (a)]

$$\frac{\lambda}{4} = L \Rightarrow L = 4\lambda$$

$$\therefore \quad \nu_1 = \frac{v}{\lambda} = \frac{v}{4L}$$

where $v = \sqrt{\frac{\gamma P}{\rho}}$ is the speed of sound in the gas.

(b) *Third Harmonic (or First Overtone)* [Fig. 14.3 (b)]

$$\frac{\lambda}{2} + \frac{\lambda}{4} = L \Rightarrow \lambda = \frac{4L}{3}$$

$$\therefore \quad \nu_3 = \frac{v}{\lambda} = \frac{3v}{4L} = 3\nu_1$$

(c) *Fifth Harmonic (or Second Overtone)* [Fig. 14.3 (c)]

$$\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4} = L \Rightarrow \lambda = \frac{4L}{5}$$

$$\therefore \quad \nu_5 = \frac{v}{\lambda} = \frac{5v}{4L} = 5\nu_1$$

In general, for nth harmonic

$$\nu_n = \frac{nv}{4L} = \frac{n}{4L}\sqrt{\frac{\gamma P}{\rho}}$$

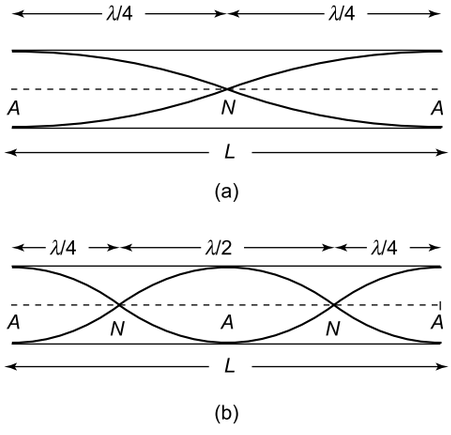
where $n = 1, 3, 5, \ldots$ etc.

NOTE

In a closed pipe only odd harmonics are present; all the even harmonics are absent.

Case 2: Open Pipe

Figure 14.4 shows the first three modes of an open pipe



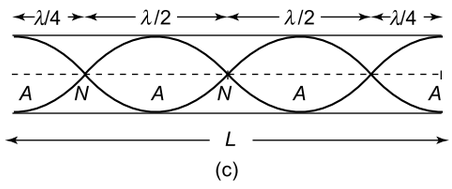


Fig. 14.4

(a) Fundamental Mode (or First Harmonic) [Fig. 14.4 (a)]

$$\frac{\lambda}{4}+\frac{\lambda}{4}=L \quad \Rightarrow \quad \lambda = 2L$$

$$\therefore \qquad \nu_1 = \frac{v}{\lambda} = \frac{v}{2L}$$

(b) Second Harmonic (or First Overtone) [Fig. 14.4 (b)]

$$\frac{\lambda}{4}+\frac{\lambda}{2}+\frac{\lambda}{4}=L \quad \Rightarrow \quad \lambda = L$$

$$\therefore \qquad \nu_2 = \frac{v}{\lambda} = \frac{v}{2L} = 2\nu_1$$

(c) Third Harmonic (or Second Overtone) [Fig. 14.4 (c)]

$$\frac{\lambda}{4}+\frac{\lambda}{2}+\frac{\lambda}{2}+\frac{\lambda}{4}=L \quad \Rightarrow \quad \lambda = \frac{2L}{3}$$

$$\therefore \qquad \nu_3 = \frac{v}{\lambda} = \frac{3v}{2L} = 3\nu_1$$

For nth harmonic

$$\nu_n = n\nu_1; \qquad n = 1, 2, 3, \ldots$$

NOTE

In an open pipe, all harmonics (even as well as odd) are present.

End Correction

We have taken the open end of a pipe to be an antinode. This is not strictly true. In fact, the particles of air just at the open end are not perfectly free because of the restriction imposed by the pipe. The true antinode is slightly away from the open end as shown in Fig. 14.5.

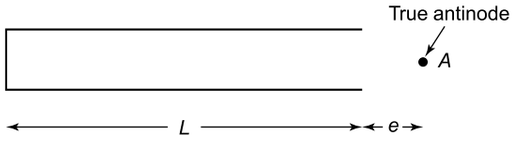


Fig. 14.5

The distance e is called the *end correction*. The effective length of the pipe is $(L + e)$.

EXAMPLE 14.15

A pipe of length 20 cm is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? Speed of sound = 340 ms^{-1}.

SOLUTION

Let N be the frequency of the source and ν_m that of the mth harmonic of the closed pipe, where m = 1, 3, 5, ... Resonance will occur if

$$N = \nu_m = \frac{mv}{4L}$$

$$\Rightarrow \qquad 430 = \frac{m \times 340}{4 \times 0.2}$$

which gives $m = 1.01 \simeq 1$. The source of frequency 430 Hz will resonantly excite the first harmonic (i.e. fundamental mode) of the closed pipe.

For an open pipe, the condition of resonance with the same source will be

$$N = \nu_n = \frac{nv}{2L}, \quad \text{where } n = 1, 2, 3, \ldots$$

$$\Rightarrow \qquad n = \frac{N \times 2L}{v} = \frac{430 \times 2 \times 0.2}{340} = 0.5$$

which is not an integer. Hence the source will not be in resonance with any harmonic of the open pipe.

EXAMPLE 14.16

A half-metre long tube open at one end, with a movable piston shows resonance with a tuning fork of frequency 512 Hz when the tube length is 16.0 cm and 49.0 cm. Calculate the speed of sound at the temperature of the experiment and determine the end-correction.

SOLUTION

The tuning fork is in resonance at two lengths of the pipe, viz. 16 cm and 49 cm. Since the second resonance length is about three times the first, it is clear that the fork is in resonance with the first harmonic (i.e. the fundamental mode) when L_1 = 16 cm and with the third harmonic when L_2 = 49 cm. Thus we have

$$N = \nu_1 = \frac{v}{4(L_1 + e)} \qquad \text{(i)}$$

Also

$$N = \nu_3 = \frac{3v}{4(L_2 + e)} \qquad \text{(ii)}$$

where N is the frequency of tuning fork and e is the end-correction. Dividing (ii) by (i) we get

$$3(16.0 + e) = 49.0 + e \quad \text{or} \quad e = 0.5 \text{ cm}$$

Then the speed of sound as obtained from (i) is

$$\begin{aligned} v &= 4N(L_1 + e) \\ &= 4 \times 512 \times (16.0 + 0.5) \\ &= 33792 \text{ cm s}^{-1} = 337.92 \text{ m s}^{-1} \\ &\simeq 338 \text{ m s}^{-1} \end{aligned}$$

14.10 BEATS

The periodic rise and fall of intensity of the wave resulting from the superposition of two waves of different frequencies is called the phenomenon of beats.

Consider two waves of angular frequencies ω_1 and ω_2. For simplicity, we assume that they have equal amplitude a and that the observation point is at $x = 0$. Then

$$y_1 = a \sin \omega_1 t; \quad \omega_1 = 2\pi\nu_1$$

and

$$y_2 = a \sin \omega_2 t; \quad \omega_2 = 2\pi\nu_2$$

Using the superposition principle,

$$\begin{aligned} y &= y_1 + y_2 \\ &= a (\sin \omega_1 t + \sin \omega_2 t) \\ &= 2a \cos\left(\frac{\omega_1 - \omega_2}{2}\right) t \sin\left(\frac{\omega_1 + \omega_2}{2}\right) t \end{aligned}$$

$$\Rightarrow \quad y = A \sin(\omega_{av} t); \quad \omega_{av} = \frac{1}{2}(\omega_1 + \omega_2)$$

where

$$A = 2a \cos\left(\frac{\omega_1 - \omega_2}{2}\right) t$$

Now, intensity is proportional to A^2. Therefore, the resultant intensity is maximum when

$$\cos\left(\frac{\omega_1 - \omega_2}{2}\right) t = \pm 1$$

$$\Rightarrow \quad \left(\frac{\omega_1 - \omega_2}{2}\right) t = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \quad 2\pi\left(\frac{\nu_1 - \nu_2}{2}\right) t = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \quad t = 0, \frac{1}{(\nu_1 - \nu_2)}, \frac{2}{(\nu_1 - \nu_2)}, \cdots$$

The time interval between two consecutive maxima is

$$t_b = \frac{1}{\nu_1 - \nu_2}$$

Therefore frequency of maxima is

$$\nu_b = \frac{1}{t_b} = \nu_1 - \nu_2$$

Similarly, we can show that the frequency of minima = ν_b.

Hence the frequency of beats is

$$\nu_b = \nu_1 - \nu_2$$

Thus

Beat frequency = difference between the frequencies of interfering waves.

EXAMPLE 14.17

Two tuning forks A and B produce 10 beats per second when sounded together. On loading fork A with a little wax it is observed that 5 beats per second are produced. If the frequency of fork B is 480 Hz, find the frequency of fork A (a) before loading and (b) after loading.

SOLUTION

There are two possibilities (i) $\nu_A < \nu_B$ or

(ii) $\nu_A > \nu_B$.

Case (i) $\nu_A < \nu_B$; $\nu_B - \nu_A = \nu_b \Rightarrow 480 - \nu_A = 10$ $\Rightarrow \nu_A = 470$ Hz.

On loading with a little wax, the frequency of a fork decreases slightly, i.e. ν_A becomes slightly less than 470 Hz. Hence the number of beats per second must increase. But ν_b decreases to 5. Hence ν_A cannot be less than ν_B.

Case (ii) $\nu_A > \nu_B$. In this case $\nu_A = \nu_B + \nu_b = 480 + 10 = 490$ Hz. On loading A, ν_A decreases. Hence $\nu_b = \nu_A - \nu_B$ will decrease. Since ν_b is observed to decrease to 5, ν_A must be greater than ν_B.

(a) Hence before loading, $\nu_A = 490$ Hz

(b) After loading $\nu_A = 490 - 5 = 485$ Hz

EXAMPLE 14.18

A metal wire of diameter 1.5 mm is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire vibrating with its fundamental frequency and a vibrating tuning fork together produce 5 beats per second. The tension in the wire is then reduced to 81 N. When the two are excited, beats are heard at the same rate. Calculate (a) the frequency of the fork, and (b) the density of the material of the wire.

SOLUTION

(a) Let N be the frequency of the tuning fork. Then, the frequency of the wire, when the tension is 100 N will be $(N + 5)$ and when the tension is 81 N, it is $(N - 5)$; since in each case 5 beats are heard per second. Hence

$$N + 5 = \frac{1}{2L}\sqrt{\frac{T_1}{m}} = \frac{1}{2\times 0.5}\sqrt{\frac{100}{m}} = \frac{10}{\sqrt{m}} \quad \text{(i)}$$

$$\text{and } N - 5 = \frac{1}{2L}\sqrt{\frac{T_2}{m}} = \frac{1}{2\times 0.5}\sqrt{\frac{81}{m}} = \frac{9}{\sqrt{m}} \quad \text{(ii)}$$

Subtracting (ii) from (i) we have

$$10 = \frac{1}{\sqrt{m}} \quad \text{or} \quad m = 0.01 \text{ kg m}^{-1}$$

Using this values of m in (i) or (ii) gives $N = 95$ Hz.

(b) Now $m = \pi r^2\rho = \dfrac{\pi d^2\rho}{4}$

Putting $d = 1.5 \times 10^{-3}$ m and $m = 0.01$ kg m^{-1}, we get

$$\rho = 5.7 \times 10^3 \text{ kg m}^{-3}$$

14.11 DOPPLER EFFECT IN SOUND

The apparent change in frequency of sound heard by an observer due to a relative motion between the observer and the source of sound is called the Doppler effect.

The expressions for the apparent frequencies are as follows:

1. Source approaching a stationary observer

$$\nu_1 = \nu\left(\frac{v}{v - u_s}\right),$$

where ν = real frequency
v = velocity of sound
u_s = velocity of source

2. Source receding from a stationary observer

$$\nu_2 = \nu\left(\frac{v}{v + u_s}\right)$$

3. Observer approaching a stationary source of sound

$$\nu_3 = \nu\left(\frac{v + u_0}{v}\right),$$

where u_0 = velocity of observer

4. Observer receding from a stationary source of sound

$$\nu_4 = \nu\left(\frac{v - u_0}{v}\right)$$

5. Both approaching each other

$$\nu_5 = \nu\left(\frac{v + u_0}{v - u_s}\right)$$

6. Both receding from each other

$$\nu_6 = \nu\left(\frac{v - u_0}{v + u_s}\right)$$

7. Source approaching a receding observer

$$\nu_7 = \nu\left(\frac{v - u_0}{v - u_s}\right)$$

8. Observer approaching a receding source

$$\nu_8 = \nu\left(\frac{v + u_0}{v + u_s}\right)$$

When the source of sound goes pass stationary observer, the apparent change in the frequency of sound is given by

$$\Delta\nu = \frac{2vu_s\nu}{\left(v^2 - u_s^2\right)}$$

If $u_s << v$, then $v^2 - u_s^2 \simeq v^2$, then $\Delta\nu = \dfrac{2u_s\nu}{v}$

When the observer goes past a stationary source of sound, the apparent change in frequency of sound is given by

$$\Delta\nu = \frac{2u_0\nu}{v}$$

NOTE

(1) If the source of sound moves, the apparent change in frequency is due to change in wavelength; the speed of sound remaining the same.
(2) If the observer moves, the apparent change in frequency is due to change in the speed of sound relative to observer; the wavelength of sound remaining the same.

Effect of the Motion of the Medium The velocity of material or mechanical waves is affected by the motion of the medium. If the medium is moving with a velocity u_m in the direction of propagation of sound, the effective velocity of sound is increased from v to $(v + u_m)$. In this case, v is replaced by $(v + u_m)$ in the above expressions. On the other hand, if the medium is moving with a velocity u_m in a direction opposite to the direction of wave propagation, v is replaced by $(v - u_m)$.

14.12 DOPPLER EFFECT IN LIGHT

If a star emitting light of frequency ν goes away from the earth with a speed v, the apparent frequency ν' of the light reaching the earth is given by

$$\nu' = \nu\left(\frac{c}{c + v}\right)$$

where c is the speed of light. Since $\nu = c/\lambda$ and $\nu' = c/\lambda'$, we have

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\left(\frac{c}{c+v}\right)$$

or $$\frac{\lambda'}{\lambda} = 1 + \frac{v}{c} \text{ or } \frac{\lambda' - \lambda}{\lambda} = \frac{v}{c}$$

or $$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

The apparent change in wavelength $\Delta\lambda$ is called the **Doppler shift.** If the wavelength of light reflected from a moving object decreases, the object is moving towards the observer and *vice versa*. The wavelength of light reflected from a galaxy is found to increase. This is called the **red shift** which indicates that the galaxy is receding from us. The red shift indicates that the universe is expanding.

EXAMPLE 14.19

A train standing at the outer signal of a railway station is blowing a whistle of frequency 500 Hz. The speed of sound is 340 m s^{-1}.

(i) Find the frequency of the sound of the whistle heard by a man standing on the platform when the train (a) approaches the platform with a speed of 20 ms^{-1} and (b) recedes from the platform with a speed of 20 ms^{-1}.

(ii) What is the speed of sound in each case?

(iii) What is the wavelength of sound heard by the man in each case?

SOLUTION

ν = 500 Hz, u_s = 20 m s^{-1}, v = 340 m s^{-1}

(i) (a) Train approaching

$$\nu_1 = \frac{\nu v}{v - u_s} = \frac{500 \times 340}{340 - 20} = 531 \text{ Hz}$$

(b) Train receding

$$\nu'_1 = \frac{\nu v}{v + u_s} = \frac{500 \times 340}{340 + 20} = 472 \text{ Hz}$$

(ii) Since the observer is at rest, the speed of sound relative to him is v = 340 m s^{-1} in cases (a) and (b).

(iii) Now, wavelength

$$= \frac{\text{speed of sound relative to observer}}{\text{frequency of sound heard by him}}$$

In case (a) $\lambda_1 = \frac{v}{\nu_1} = \frac{340}{531} = 0.64$ m

In case (b) $\lambda'_1 = \frac{v}{\nu'_1} = \frac{340}{472} = 0.72$ m

EXAMPLE 14.20

A train standing at a platform is blowing a whistle of frequency 500 Hz in still air. The speed of sound in still air is 340 m s^{-1}. (i) What is the frequency of the sound of the whistle heard by a man on a trolley which is moving (a) towards the engine with a speed of 20 m s^{-1} and (b) away from the engine with a speed of 20 m s^{-1}? (ii) What is the speed of sound heard by the man in each case? (iii) What is the wavelength of sound heard by the man in each case?

SOLUTION

ν = 500 Hz, u_0 = 20 m s^{-1}, v = 340 m s^{-1}

(i) (a) Observer approaching

$$\nu'_2 = \frac{\nu(v + u_0)}{v} = \frac{500 \times (340 + 20)}{340}$$

$$= 529 \text{ Hz}$$

(b) Observer receding

$$\nu_2 = \frac{\nu(v - u_0)}{v} = \frac{500 \times (340 - 20)}{340}$$

$$= 471 \text{ Hz}$$

(ii) Since the observer is moving, the speed of sound heard by him in case (a) is $(v + u_0)$ = 340 + 20 = 360 m s^{-1} and case (b) it is $(v - u_0)$ = 340 – 20 = 320 m s^{-1},

(iii) Wavelength = $\frac{\text{speed of sound relative observer}}{\text{frequency of sound heard by him}}$

In case (a) $\lambda'_2 = \frac{v + u_0}{\nu'_2} = \frac{360}{529} = 0.68$ m

In case (b) $\lambda_2 = \frac{v - u_0}{\nu_2} = \frac{320}{471} = 0.68$ m

Notice that $\lambda_2 = \lambda'_2$.

EXAMPLE 14.21

An ambulance blowing a siren of frequency 700 Hz is travelling towards a vertical reflecting wall with a speed of 2 ms^{-1}. If the speed of sound is 350 m s^{-1}, calculate the number of beats heard in one second by

(a) the driver of the ambulance,

(b) a person standing between the ambulance and the wall, and

(c) a person standing behind the ambulance.

SOLUTION

(a) The driver will hear two sounds, one coming directly from the siren and the other reflected at the wall or coming from the acoustic image of the car. ν = frequency of direct sound = 700 Hz. The reflected sound can be imagined to be coming from the mirror image (shown dotted in Fig. 14.6).

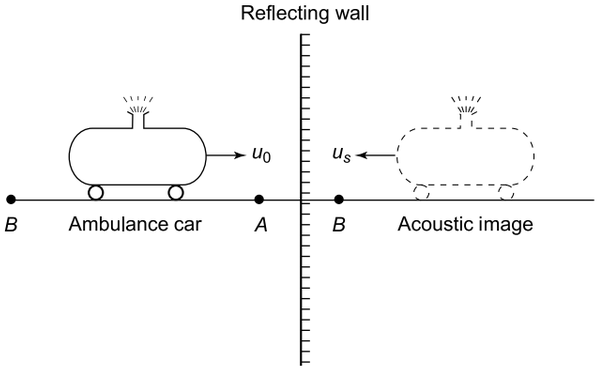


Fig. 14.6

The observer (driver) is approaching this image-source which is also approaching him with the same speed. Hence, the frequency of sound heard by him is given by ($u_0 = u_s$)

$$\nu' = \nu\frac{v+u_0}{v-u_s} = 700 \times \frac{350+2}{350-2}$$

$$= 708 \text{ Hz}$$

Number of beats heard per second = $\nu' - \nu$ = 708 – 700 = 8

$\therefore$ Beat frequency = 8 Hz

(b) A person standing at A will hear two sounds, one from the siren approaching him and the other from the mirror image of the siren also approaching him with the same speed. Hence he will hear no beats.

(c) A person standing at B will hear two sounds, one from the siren receding from him and the other from the mirror image of the siren approaching him with the same speed.

$$\nu_1 = \nu\left(\frac{v}{v+u_s}\right) = 700 \times \left(\frac{350}{350+2}\right)$$

$$= 696 \text{ Hz}$$

$$\nu_2 = \nu\left(\frac{v}{v-u_s}\right) = 700 \times \left(\frac{350}{350-2}\right)$$

$$= 704 \text{ Hz}$$

$\therefore$ Beat frequency = $\nu_2 - \nu_1$ = 704 – 696 = 8 Hz

14.13 INTENSITY, QUALITY AND PITCH OF SOUND

1. *Intensity* The intensity of a travelling wave is defined as the rate at which energy is transferred per unit area of a surface held perpendicular to the direction of propagation. Intensity I of a wave in a medium of density ρ is given by

$$I = 2\ \pi^2\ \rho\ v\ A^2\ \nu^2$$

where ν is the frequency of particle oscillation, A their amplitude and v is the velocity of the wave in that medium.

The SI unit of intensity is watt per square metre (W m^{-2}).

The intensity level of a sound wave is defined by an arbitrary scale. The zero of the scale is taken at the sound wave intensity

$$I_0 = 1 \times 10^{-12} \text{ W m}^{-2}$$

which corresponds to the faintest audible sound. The unit of intensity level is called decibel (dB). Intensity level in decibels of a sound of intensity

$$I = 10 \log \left(\frac{I}{I_0}\right)$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. When a plane harmonic wave of wavelength λ travels in a medium, the particle speed will always be less then the wave speed if the amplitude of the wave is

(a) less then $\frac{\lambda}{2\pi}$ (b) less then λ

(c) greater then $\frac{\lambda}{\pi}$ (d) greater then λ

2. An open pipe and a closed pipe have the same length L. The ratio of the frequencies of their n th overtones is

(a) $\frac{2n}{(n+1)}$ (b) $\frac{n}{(n+1)}$

(c) $\frac{2(n+1)}{(n+2)}$ (d) $\frac{2(n+1)}{(2n+1)}$

3. The frequency of the first overtone of a closed pipe is equal to that of the first overtone of an open pipe. It is also found that a vibrating source resonates with the n th harmonic of the closed pipe and with the m th harmonic of the open pipe. The ratio n/m is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\frac{3}{5}$ (d) $\frac{5}{3}$

4. A standing wave is produced due to a superposition of the incident wave and the wave reflected from a boundary. It is observed that the amplitude at antinode is 9 times that at node. The percentage of the incident intensity reflected from the boundary is

(a) 36% (b) 64%
(c) 72% (d) 81%

5. A source emitting a sound of wavelength λ is in resonance with the first overtone of a closed pipe. Another source emitting a sound of wavelength λ' is in resonance with the second overtone of the same pipe. The ratio λ/λ' is

(a) $\frac{5}{3}$ (b) $\frac{4}{3}$

(c) $\frac{3}{2}$ (d) $\frac{5}{2}$

6. The displacements of a particle located at x at time t due to two waves are given by

$$y_1 = a\sin(\omega t - kx)$$

And $$y_2 = a\ \sin\ (\omega t - kx + \phi)$$

If the amplitude of the resultant wave formed by the superposition of these two waves is a, the phase constant ϕ is equal to

(a) zero (b) $\frac{\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{4}$

7. Starting from rest, an observer moves with a constant acceleration a towards a stationary source emitting a sound of frequency ν_0. Which of the graphs shown in Fig. 14.7 correctly represents the variation of the apparent frequency ν of sound as heard by the observer with time t?

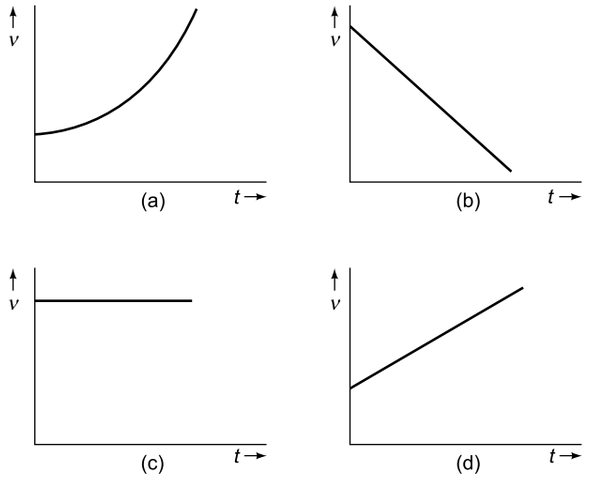


Fig. 14.7

8. A train is moving with a constant speed u along a circular track. A siren in its engine is emitting a sound of frequency ν. If $u = v/10$, where v is the speed of sound, the apparent frequency of sound as heard by a passenger at the rear end of the train is

(a) $\frac{9\nu}{10}$ (b) $\frac{11\nu}{10}$

(c) $\frac{9\nu}{11}$ (d) ν

9. A progressive wave in a medium is represented by the equation

$$y = 0.1\ \sin\left(10\pi t - \frac{5}{11}\pi x\right)$$

where y and x are in cm and t in seconds. The maximum speed of a particle of the medium due to the waves is

(a) 1cm s^{-1} (b) 10 cm s^{-1}
(c) π cm s^{-1} (d) 10 π cm s^{-1}

10. A tuning fork of frequency 340 Hz is sounded above a cylindrical tube 1 m high. Water is slowly poured into the tube. If the speed of sound is 340 ms^{-1}, at what levels of water in the tube will the sound of the fork be appreciably intensified?

(a) 25 cm, 75 cm (b) 20 cm, 80 cm
(c) 15 cm, 85 cm (d) 17 cm, 83 cm

11. A sonometer wire, 65 cm long, is in resonance with a tuning fork of frequency N. If the length of the wire is decreased by 1 cm and it is vibrated with the same tuning fork, 8 beats are heard per second. What is the value of N?

(a) 256 Hz (b) 384 Hz
(c) 480 Hz (d) 512 Hz

12. Two organ pipes, each closed at one end, give 5 beats per second when emitting their fundamental notes. If their lengths are in the ratio of 50 : 51, their fundamental frequencies (in Hz) are
(a) 250, 255 (b) 255, 260
(c) 260, 265 (d) 265, 270

13. Two sources A and B are sounding notes of frequency 680 Hz. A listener moves from A to B with a constant velocity u. If the speed of sound is 340 ms^{-1}, what must be the value of u so that he hears 10 beats per second?
(a) 2.0 ms^{-1} (b) 2.5 ms^{-1}
(c) 3.0 ms^{-1} (d) 3.5 ms^{-1}

14. Standing waves are produced by the superposition of two waves

$$y_1 = 0.05 \sin (3\pi t - 2x)$$

and $$y_2 = 0.05 \sin (3\pi t + 2x)$$

where x and y are expressed in metres and t is in seconds. What is the amplitude of a particle at x = 0.5 m. Given cos (57.3°) = 0.54.
(a) 2.7 cm (b) 5.4 cm
(c) 8.1 cm (d) 10.8 cm

15. The transverse displacement of a string fixed at both ends is given by

$$y = 0.06 \sin \left(\frac{2\pi x}{3}\right) \cos (120\ \pi t)$$

where x and y are in metres and t is in seconds. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg. What is the tension in the string?
(a) 648 N (b) 724 N
(c) 832 N (d) 980 N

16 . A pipe of length 20 cm is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 425 Hz source? The speed of sound = 340 ms^{-1}.
(a) First harmonic (b) Second harmonic
(c) Third harmonic (d) Fourth harmonic

17. A pipe of length 20 cm is open at both ends. Which harmonic mode of the pipe is resonantly excited by a 1700 Hz source? The speed of sound = 340 ms^{-1}.
(a) First harmonic (b) Second harmonic
(c) Third harmonic (d) Fourth harmonic

18. Two sitar strings A and B are slightly out of tune and produce beats of frequency 6 Hz. When the tension in string A is slightly decreased, the beat frequency is found to be reduced to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?
(a) 318 Hz (b) 321 Hz
(c) 327 Hz (d) 330 Hz

19. A metal wire of diameter 1 mm is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire vibrating with its fundamental frequency and a vibrating tuning fork together produce 5 beats per second. The tension in the wire is then reduced to 81 N. When the two are excited, beats are heard at the same rate. What is the frequency of the fork?
(a) 90 Hz (b) 95 Hz
(c) 100 Hz (d) 105 Hz

20. A submarine is fitted with a SONAR (Sound Navigation and Ranging) system which operates at an ultrasonic frequency of 42 kHz. An enemy submarine is moving towards the SONAR with a speed of 200 ms^{-1}. If the speed of sound in seawater is 1400 ms^{-1}, what is the frequency of sound received back at the SONAR fitted submarine after reflection from the enemy submarine?
(a) 36 kHz (b) 42 kHz
(c) 48 kHz (d) 56 kHz

21. A machine gun is mounted on a tank moving at a speed of 20 ms^{-1} towards a target with the gun pointing in the direction of motion of the tank. The muzzle speed of the bullet equals the speed of sound = 340 ms^{-1}. If, at the time of firing, the target is 500 m away from the tank, then
(a) the sound arrives at the target later than the bullet
(b) the sound arrives at the target earlier than the bullet
(c) both sound and bullet arrive at the target at the same time
(d) the bullet will never arrive at the target.

22. Out of the four choices given in Q. 21 above, choose the correct choice, if the gun points in a direction opposite to the direction of motion of the tank.

23. Three sound waves of equal amplitudes have frequencies $(\nu - 1)$, (ν) and $(\nu + 1)$. They superpose to give beats. The number of beats produced per second will be
(a) ν (b) $\frac{\nu}{2}$
(c) 2 (d) 1

24. The speed of sound in hydrogen at STP is v. The speed of sound in a mixture containing 3 parts of hydrogen and 2 parts of oxygen at STP will be

(a) $\frac{v}{2}$ (b) $\frac{v}{\sqrt{5}}$

(c) $\sqrt{7}\ v$ (d) $\frac{v}{\sqrt{7}}$

25. The speed of sound in hydrogen at STP is v. What is the speed of sound in helium at STP?

(a) $\frac{v}{\sqrt{2}}$ (b) $\frac{v}{2}$

(c) $\sqrt{2}\ v$ (d) $2\ v$

26. Nine tuning forks are arranged in order of increasing frequency. Each tuning fork produces 4 beats per second when sounded with either of its neighbours. If the frequency of the 9th tuning fork is twice that of the first, what is the frequency of the first tuning fork?

(a) 32 Hz (b) 40 Hz

(c) 48 Hz (d) 56 Hz

27. A sonometer wire of length 120 cm is divided into three segments of lengths in the ratio of 1 : 2 : 3. What is the ratio of their fundamental frequencies?

(a) 3 : 2 : 1 (b) 4 : 2 : 1

(c) 5 : 3 : 2 (d) 6 : 3 : 2

28. A tuning fork produces 4 beats per second when sounded with a sonometer wire of vibrating length 48 cm. It produces 4 beats per second also when the vibrating length is 50 cm. What is the frequency of the tuning fork?

(a) 196 Hz (b) 284 Hz

(c) 375 Hz (d) 460 Hz

29. Two identical strings of a stringed musical instrument are in unison when stretched with the same tension. When the tension in one string is increased by 1%, the musician hears 4 beats per second. What was the frequency of the note when the strings were in unison?

(a) 796 Hz (b) 800 Hz

(c) 804 Hz (d) 808 Hz

30. Two identical flutes produce fundamental notes of frequency 300 Hz at 27°C. If the temperature of the air in one flute is increased to 31°C, the number of beats heard per second will be

(a) 1 (b) 2

(c) 3 (d) 4

31. The wavelength of light of a particular wavelength received from a galaxy is measured on earth and is found to be 5% more that its wavelength. It follows that the galaxy is

(a) approaching the earth with a speed 3×10^7 ms^{-1}

(b) going away from the earth with a speed 3×10^7 ms^{-1}

(c) approaching the earth with a speed 1.5×10^7 ms^{-1}

(d) going away from the earth with a speed 1.5×10^7 ms^{-1}

32. Radiowaves of frequency 600 MHz are sent by a radar towards an enemy aircraft. The frequency of the radiowaves reflected from the aircraft as measured at the radar station is found to increase by 6 kHz. It follows that the aircraft is

(a) approaching the radar station with a speed 1.5 kms^{-1}

(b) going away from the radar station with a speed 1.5 kms^{-1}

(c) approaching the radar station with a speed 3 kms^{-1}

(d) going away from the radar station with a speed 3 kms^{-1}.

33. An observer moves towards a stationary source of sound with a velocity one-tenth the velocity of sound. The apparent increase in frequency is

(a) zero (b) 5%

(c) 10% (d) 0.1%

34. A wave represented by the equation $y = a \cos (kx - \omega t)$ is superposed with another wave to form a stationary wave such that the point $x = 0$ is a node. The equation of the other wave is

(a) $y' = a \sin (kx + \omega t)$

(b) $y' = -a \cos (kx - \omega t)$

(c) $y' = -a \cos (kx + \omega t)$

(d) $y' = -a \sin (kx - \omega t)$

IIT, 1983

35. Two wires of the same material and radii r and $2r$ are welded together end to end. The combination is used as a sonometer wire and kept under tension T. The welded point is mid-way between the two bridges. When stationary waves are set up in the composite wire, the joint is a node. Then the ratio of the number of loops formed in the thinner to thicker wire is

(a) 2 : 3 (b) 1 : 2

(c) 2 : 1 (d) 5 : 4

36. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced in the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope?

(a) 0.06 m (b) 0.03 m
(c) 0.12 m (d) 0.09 m

IIT, 1984

37. A tube closed at one end containing air, produces, when excited, the fundamental note of frequency 512 Hz. If the tube is open at both ends, the fundamental frequency that can be excited is (in Hz)
(a) 1024 (b) 512
(c) 256 (c) 128

IIT, 1986

38. Two sound waves of equal intensity I produce beats. The maximum intensity of sound produced in beats will be
(a) I (b) $4I$
(c) $2I$ (d) $I/2$

39. Two travelling waves

$$y_1 = A \sin [k(x + ct)]$$

and $$y_2 = A \sin [k(x - ct)]$$

are superposed on a string. The distance between adjacent nodes is

(a) $\frac{ct}{\pi}$ (b) $\frac{ct}{2\pi}$

(c) $\frac{\pi}{2k}$ (d) $\frac{\pi}{k}$

IIT, 1992

40. Three waves of equal frequency having amplitudes 10 μm, 4 μm and 7 μm arrive at a given point with a successive phase difference of $\pi/2$. The amplitude of the resulting wave in μm is given by
(a) 7 (b) 6
(c) 5 (d) 4

41. A transverse wave is represented by

$$y = y_0 \sin \left\{\frac{2\pi}{\lambda}(vt - x)\right\}$$

For what value of λ is the maximum particle velocity equal to twice the wave velocity?

(a) $\lambda = 2\ \pi y_0$ (b) $\lambda = \frac{2\pi y_0}{3}$

(c) $\lambda = \frac{\pi y_0}{2}$ (d) $\lambda = \pi y_0$

42. A train blowing its whistle moves with a constant velocity u away from the observer on the ground. The ratio of the actual frequency of the whistle to that measured by the observer is found to be 1.2. If the train is at rest and the observer moves away from it at the same velocity, the ratio would be given by
(a) 0.51 (b) 1.25
(c) 1.52 (d) 2.05

IIT, 1993

43. An organ pipe P_1, closed at one end vibrating in its first harmonic and another pipe P_2, open at both ends vibrating in its third harmonic, are in resonance with a given tuning fork. The ratio of the lengths of P_1 and P_2 is

(a) $\frac{8}{3}$ (b) $\frac{1}{6}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

IIT, 1988

44. The extension in a string, obeying Hookes' law, is x. The speed of the wave in the stretched string is v. If the extension in the string is increased to 1.5 x, the speed of the wave in the string will be
(a) 1.22 v (b) 0.61 v
(c) 1.50 v (d) 0.75 v

IIT, 1996

45. A travelling wave in a stretched string is described by the equation

$$y = A \sin (kx - \omega t)$$

The maximum particle velocity is

(a) $A\omega$ (b) $\frac{\omega}{k}$

(c) $\frac{d\omega}{dk}$ (d) $\frac{x}{t}$

IIT, 1997

46. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed at any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ ms^{-1}, then λ and f are given by

(a) $\lambda = 2\pi \times 10^{-2}$ m, $f = \frac{10^3}{2\pi}$ Hz
(b) $\lambda = 10^{-2}$ m, $f = 10^3$ Hz
(c) $\lambda = 10^{-3}$ m, $f = 10^4$ Hz
(d) $\lambda = \frac{10^{-2}}{2\pi}$ m, $f = 2\pi \times 10^3$ Hz

IIT, 1998

47. The ratio of the speed of sound in nitrogen gas to that in helium gas at 300 K is

(a) $\sqrt{\frac{2}{7}}$ (b) $\sqrt{\frac{1}{7}}$

(c) $\frac{\sqrt{3}}{5}$ (d) $\frac{\sqrt{6}}{5}$

IIT, 1999

48. Two vibrating strings of the same material but lengths L and $2L$ have radii $2r$ and r, respectively. They are stretched under the same tension. Both the

strings vibrate in their fundamental modes, the one of length L with frequency ν_1 and the other with frequency ν_2. The ratio ν_1/ν_2 is given by

(a) 2 (b) 4

(c) 8 (d) 1

IIT, 2000

49. Two sounds of wavelengths 5 m and 6 m, travelling in a medium produce 10 beats per second. The speed of sound in the medium is

(a) 300 ms^{-1} (b) 320 ms^{-1}

(c) 350 ms^{-1} (d) 1200 ms^{-1}

50. The frequencies of tuning forks A and B are respectively 3% more and 2% less than the frequency of fork C. When A and B are simultaneously excited 5 beats per second are produced. The frequency (in Hz) of fork A is

(a) 98 (b) 100

(c) 103 (d) 105

51. A metallic wire with tension T and at temperature 30ºC vibrates with its fundamental frequency of 1 kHz. The same wire with the same tension but at 10ºC temperature vibrates with a fundamental frequency of 1.001 kHz. The coefficient of linear expansion of the wire is

(a) 2×10^{-4} $ºC^{-1}$ (b) 1.5×10^{-4} $ºC^{-1}$

(c) 1×10^{-4} $ºC^{-1}$ (d) 0.5×10^{-4} $ºC^{-1}$

52. A knife-edge divides a sonometer wire into two parts. The fundamental frequencies of the two parts are ν_1 and ν_2. The fundamental frequency of the sonometer wire when the knife-edge is removed will be

(a) $\nu_1 + \nu_2$ (b) $\frac{1}{2}(\nu_1 + \nu_2)$

(c) $\sqrt{\nu_1 \nu_2}$ (d) $\frac{\nu_1 \nu_2}{\nu_1 + \nu_2}$

53. A sonometer wire is stretched by a hanging metal bob. Its fundamental frequency is ν_1. When the bob is completely immersed in water, the frequency becomes ν_2. The relative density of the metal is

(a) $\frac{\nu_1^2}{\nu_1^2 - \nu_2^2}$ (b) $\frac{\nu_2^2}{\nu_1^2 - \nu_2^2}$

(c) $\frac{\nu_1}{\nu_1 - \nu_2}$ (d) $\frac{\nu_2}{\nu_1 - \nu_2}$

54. Two pulses in a stretched string whose centers are initially 8 cm apart are moving towards each other as shown in Fig. 14.8. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be

(a) zero

(b) purely kinetic

(c) purely potential

(d) partly kinetic and partly potential

IIT, 2001

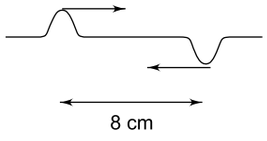


Fig. 14.8

55. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is

(a) 242/252 (b) 2

(c) 5/6 (d) 11/6

IIT, 2002

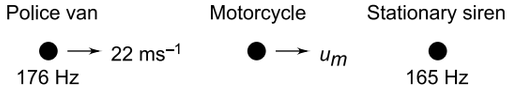
56. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is

(a) 25 kg (b) 5 kg

(c) 12.5 kg (d) (1/25) kg

IIT, 2002

57. A police van, moving at 22 ms^{-1}, chases a motorcyclist. The policeman sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz, as shown in the figure.



If the motorcyclist does not observe any beats, his speed must be (take the speed of sound = 330 ms^{-1})

(a) 33 ms^{-1} (b) 22 ms^{-1}

(c) zero (d) 11 ms^{-1}

IIT, 2003

58. In the experiment for the determination of the speed of sound in air using the resonance column, it is observed that 0.1 m of air column resonates with a tuning fork in the fundamental mode. When the

length of the air column is changed to 0.35 m, the same tuning fork resonates with the first overtone. What is the end correction?

(a) 0.0125 m (b) 0.025 m
(c) 0.05 m (d) 0.075 m

IIT, 2003

59. Transverse waves are generated in two uniform steel wires A and B of diameters 10^{-3} m and 0.5×10^{-3} m respectively, by attaching their free end to a vibrating source of frequency 500 Hz. The ratio of the wavelengths if they are stretched with the same tension is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) 2 (d) $\sqrt{2}$

60. A wire is stretched between two rigid supports with a certain tension. It is observed that the wire resonates in the nth harmonic at a frequency of 420 Hz. The next higher frequency at which the wire resonates is 490 Hz. The value of n is

(a) 2 (b) 4
(c) 6 (d) 8

61. The fundamental frequency of a sonometer wire increases by 6 Hz if its tension is increased by 44%, keeping the length constant. The frequency of the wire is

(a) 24 Hz (b) 30 Hz
(c) 36 Hz (d) 42 Hz

62. Which of the following functons represents a travelling wave? Here a, b and c are constants.

(a) $y = a \cos(bx) \sin(ct)$
(b) $y = a \sin(bx) \cos(ct)$
(c) $y = a \sin(bx + ct) - a \sin(bx - ct)$
(d) $y = a \sin(bx + ct)$

63. A sonometer wire is vibrating with a frequency of 30 Hz in the fundamental mode. If the length of the wire is increased by 20%, the change in the frequency of the fundamental mode is

(a) 5 Hz (b) 10 Hz
(c) 15 Hz (d) 20 Hz

64. An organ pipe P_1, closed at one end and containing a gas of density ρ_1 is vibrating in its first harmonic. Another organ pipe P_2, open at both ends and containing a gas of density ρ_2 is vibrating in its third harmonic. Both the pipes are in resonance with a given tuning fork. If the compressibility of gases is equal in both pipes, the ratio of the lengths of P_1 and P_2 is

(a) $\frac{1}{3}$ (b) 3
(c) $\frac{1}{6}\sqrt{\frac{\rho_1}{\rho_2}}$ (d) $\frac{1}{6}\sqrt{\frac{\rho_2}{\rho_1}}$

IIT, 2004

65. A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 ms^{-1} and in air it is 300 ms^{-1}. The frequency and wavelength of sound recorded by an observer who is standing in air respectively are

(a) 600 Hz, 0.5 m (b) 600 Hz, 2.5 m
(c) 3000 Hz, 0.4 m (d) 120 Hz, 2 m

IIT, 2004

66. In the resonance tube experiment for determining the speed of sound in air using a tuning fork of frequency 480 Hz, the first resonance was observed at 17.7 cm of air column and the second at 53.1 cm. The maximum possible error in the speed of sound in air is

(a) 192 cm s^{-1} (b) 96 cm s^{-1}
(c) 64 cm s^{-1} (d) 48 cm s^{-1}

IIT, 2005

67. A rod AB of length L is hung from two identical wires 1 and 2. A block of mass m is hung at point O of the rod as shown in Fig. 14.9. The value of x so that a tuning fork excites the fundamental mode in wire 1 and the second harmonic in wire 2 is

(a) $\frac{L}{5}$ (b) $\frac{L}{4}$
(c) $\frac{L}{3}$ (d) $\frac{2L}{3}$

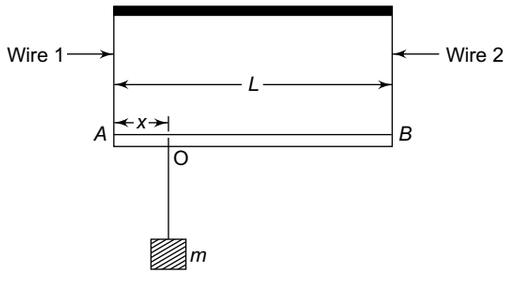


Fig. 14.9

68. Two tuning forks, each of frequency ν, move relative to a stationary observer. One fork moves away from the observer with a speed u while the other fork moves towards him at the same speed. The speed of sound is v. If $u \ll v$, the observer hears beats of frequency

(a) zero (b) $\frac{vu}{v}$

(c) $\frac{2vu}{v}$ (d) $\frac{vu}{2v}$

69. The vibrations of a string of length 60 cm fixed at both ends are represented by

$$y = 4 \sin\left(\frac{\pi x}{15}\right)\cos(96\ \pi t)$$

where x and y are in cm and t in second. The particle velocity at $x = 7.5$ cm and $t = 0.25$ s is

(a) zero (b) 10 cm s^{-1}

(c) 100 cm s^{-1} (d) (4×96) cm s^{-1}

IIT, 1985

70. A uniform rope of mass M hangs vertically from a rigid support. A block of mass m is attached to the free end of the rope. A transverse pulse of wavelength λ is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is

(a) $\left(\frac{m+M}{m}\right)\lambda$ (b) $\left(\frac{m+M}{m}\right)^{1/2}\lambda$

(c) $\left(\frac{m\lambda}{M}\right)$ (d) λ

IIT, 1984

71. A band playing music at a frequency f is moving towards a wall at a speed u. A motorist is following the band with the same speed u. If v is the speed of sound, the beat frequency heard by the motorist is

(a) f (b) $\frac{2fu}{(v+u)}$

(c) $\frac{f(u+v)}{(v-u)}$ (d) $\frac{2fu}{(v-u)}$

72. A vibrating string of certain length L under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slighty increased the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is

(a) 344 (b) 336

(c) 117.3 (d) 109.3

IIT, 2008

73. A transverse sinusoidal wave moves along a string in the postive x-direction at a speed of 10 cm s^{-1}. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in Fig. 14.10. The velocity of point P when its displacement is 5 cm is

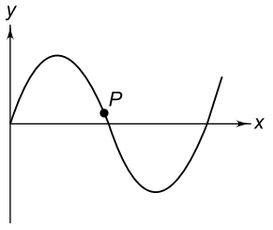


Fig. 14.10

(a) $\frac{\pi\sqrt{3}}{50}\hat{j}\,\text{ms}^{-1}$ (b) $-\frac{\pi\sqrt{3}}{50}\hat{j}\,\text{ms}^{-1}$

(c) $\frac{\pi\sqrt{3}}{50}\hat{i}\,\text{ms}^{-1}$ (d) $-\frac{\pi\sqrt{3}}{50}\hat{i}\,\text{ms}^{-1}$

IIT, 2009

74. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

(a) $\sqrt{2}A, \frac{3\pi}{4}$ (b) $A, \frac{4\pi}{3}$

(c) $\sqrt{3}A, \frac{5\pi}{6}$ (d) $A, \frac{\pi}{3}$

IIT, 2011

75. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

(a) 8.50 kHz (b) 8.25 kHz

(c) 7.75 kHz (d) 7.50 kHz

IIT, 2011

ANSWERS

1. (a)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (c)
7. (d)	**8.** (d)	**9.** (c)	**10.** (a)	**11.** (d)	**12.** (a)
13. (b)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (a)
19. (b)	**20.** (d)	**21.** (a)	**22.** (b)	**23.** (d)	**24.** (d)
25. (a)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (b)	**30.** (b)
31. (d)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (b)
37. (a)	**38.** (b)	**39.** (d)	**40.** (c)	**41.** (d)	**42.** (b)
43. (b)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (c)	**48.** (d)
49. (a)	**50.** (c)	**51.** (d)	**52.** (d)	**53.** (a)	**54.** (b)
55. (b)	**56.** (a)	**57.** (b)	**58.** (b)	**59.** (a)	**60.** (c)
61. (b)	**62.** (d)	**63.** (a)	**64.** (d)	**65.** (a)	**66.** (a)
67. (a)	**68.** (c)	**69.** (a)	**70.** (b)	**71.** (d)	**72.** (a)
73. (a)	**74.** (b)	**75.** (a)			

SOLUTION

1. For a plane harmonic wave, the particle displacement is given by

$$y = A \sin(\omega t - kx)$$

Particle speed $V = \dfrac{dy}{dt} = A\omega \cos(\omega t - kx)$

$\therefore \quad V_{\max} = A\omega = 2\pi A\nu$

Wave speed $v = \nu\lambda$

Particle speed will always less than wave speed if

$$2\pi A\nu < \nu\lambda$$

or $$A < \frac{\lambda}{2\pi}$$

2. In an open pipe, all harmonics (even as well as odd) are present, but in a closed pipe, only odd harmonics are present. Thus, in an open pipe, the second harmonic is the first overtone, the third harmonic is the second overtone and so on. In a closed pipe, the third harmonic is the first overtone, the fifth harmonic is the second overtone and so on. If v is the speed of sound, the frequency of the n th overtone in an open pipe is

$$\nu_0 = (n + 1)\frac{v}{2L}$$

In a closed pipe, the frequency of the nth overtone is

$$\nu_c = (2n + 1)\frac{v}{4L}$$

$$\therefore \quad \frac{\nu_0}{\nu_c} = \frac{2(n+1)}{(2n+1)}$$

3. Let L_c be the length of the closed pipe and L_o of the open pipe. It is given that

$$3\left(\frac{v}{4L_c}\right) = 2\left(\frac{v}{2L_o}\right)$$

$$\Rightarrow \quad \frac{L_c}{L_o} = \frac{3}{4}$$

Further, the frequency of the nth harmonic of the closed pipe = frequency of the mth harmonic of the open pipe, i.e,

$$\frac{nv}{4L_c} = \frac{mv}{2L_o} \; ; \; \begin{matrix} n = 1,3,5,\cdots \text{etc.} \\ m = 1,2,3,\cdots \text{etc.} \end{matrix}$$

$$\Rightarrow \quad \frac{n}{m} = \frac{2L_c}{L_o} = 2 \times \frac{3}{4} = \frac{3}{2}$$

4. Let A_i and A_r be the amplitudes of the incident and reflected waves. Then

Amplitude at antinode = $A_i + A_r$

Amplitude at node = $A_i - A_r$

Given

$$\frac{A_i + A_r}{A_i - A_r} = 9 \quad \Rightarrow \quad \frac{A_r}{A_i} = 0.8$$

Intensity is proportional to the square of the amplitude.

Hence

$$\frac{I_r}{I_i} = \left(\frac{A_r}{A_i}\right)^2 = (0.8)^2 = 0.64 = 64\%$$

5. Let L be the length of the pipe.

For first overtone; $L = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{2L}{3}$

For second overtone; $L = \frac{5\lambda'}{2} \Rightarrow \lambda = \frac{2L}{5}$

$\therefore \quad \frac{\lambda}{\lambda'} = \frac{5}{3}$, which is choice (a)

6. The resultant amplitude is given by

$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi \qquad (1)$

Given $A = a_1 = a_2 = a$. Substituting these in Eq (1),

We have

$$a^2 = a^2 + a^2 + 2a^2 \cos\phi$$

$$\Rightarrow \qquad \cos \phi = -\frac{1}{2} \Rightarrow \phi = \frac{2\pi}{3}$$

So the correct chose is (c).

7. Speed of observer at time t is $u_0 = 0 + at = at$

Apparent frequency $\nu = \nu_0 \left(\frac{v + u_0}{v}\right)$

$$= \nu_0 \left(\frac{v + at}{v}\right)$$

$$= \nu_0 + \frac{a\nu_0}{v} t$$

This is the equation of a straight line with a positive slope $m = \frac{a\nu_0}{v}$ and a positive intercept $c = \nu_0$

Hence the correct choice is (d).

8. Refer to Fig. 14.11.

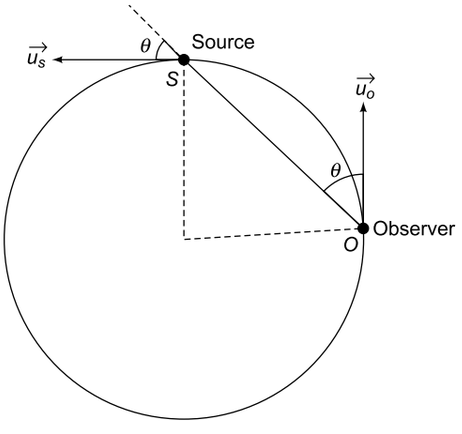


Fig. 14.11

The components of $\vec{u}_o$ and $\vec{u}_s$ along the direction of the velocity of sound are $u_o \cos\theta$ and $u_s \cos \theta$ given $u_0 = u_s = u$. Thus, the source is receding from the observer with a speed $u \cos \theta$ and the observer is approaching the source with the same speed $u \cos\theta$

$\therefore$ Apparent frequency $\nu' = \nu \left(\frac{v + u\cos\theta}{v + u\cos\theta}\right) = \nu$

Hence the correct chose is (d)

9. The speed of particle is given by

$$V = \frac{dy}{dt} = 0.1 \times 10\ \pi \cos\left(10\pi t - \frac{5}{11}\pi x\right)$$

$\therefore \quad V_{\max} = 0.1 \times 10\ \pi = \pi\ \text{cms}^{-1}$.

Hence the correct choice is (c).

10. Frequency of sound emitted by a closed pipe of length L in the fundamental mode is $\nu = \frac{v}{4L}$. For resonance $\nu = \nu'$ where ν' is the frequency of the tuning fork. Thus $340 = \frac{340}{4L}$ or $4L = 1$ m or $L = 25$ cm. The next resonance occurs at $\frac{4}{3} L = 1$ m or $L = 75$ cm.

Hence the correct choice is (a).

11. $N' = \frac{1}{2(65-1)} \cdot \sqrt{\frac{T}{m}}$. Given $N = \frac{1}{2\times 65} \cdot \sqrt{\frac{T}{m}}$.

Therefore

$$\frac{N'}{N} = \frac{2\times 65}{2(65-1)} = \frac{65}{64} \quad \text{Also } N' - N = 8$$

or $\qquad N' = N + 8.$

$\therefore \quad \frac{N+8}{N} = \frac{65}{64}$ which gives $N = 512$ Hz.

Hence the correct choice is (d).

12. $\nu_1 = \frac{1}{4L_1}\sqrt{\frac{\gamma p}{\rho}}$, $\nu_2 = \frac{1}{4L_2}\sqrt{\frac{\gamma p}{\rho}}$. Therefore

$\frac{\nu_1}{\nu_2} = \frac{L_2}{L_1} = \frac{50}{51}$. It is given that $\nu_2 = \nu_1 + 5$.

Therefore $\frac{\nu_1}{\nu_1 + 5} = \frac{50}{51}$ which gives $\nu_1 = 250$ Hz.

Hence $\nu_2 = 250 + 5 = 255$ Hz which is choice (a).

13. The listener moves away from A and approaches B. Hence the apparent frequencies are

$$\nu_1 = \nu\left(1 - \frac{u}{v}\right)$$

and $\qquad \nu_2 = \nu\left(1 + \frac{u}{v}\right)$

$\therefore \quad \nu_2 - \nu_1 = 2\ \nu\ u/v$. It is given that $\nu_2 - \nu_1 = 10$, $v = 340\ \text{ms}^{-1}$ and $\nu = 680$ Hz. Substituting these values we get

$$10 = \frac{2 \times 680 \times u}{340} \quad \text{or} \quad u = 2.5\ \text{ms}^{-1}.$$

Hence the correct choice is (b).

14. The resultant displacement is given by

$$y = y_1 + y_2$$
$$= 0.05\ \{\sin(3\pi t - 2x) + \sin(3\pi t + 2x)\}$$

Using the trigonometric relation

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin\alpha \cos\beta$$

we have $y = 0.1 \cos 2x \cdot \sin 3\pi t$

or $y = R \sin 3\pi t$

where R, the amplitude of standing waves, is given by $R = 0.1 \cos 2x$ with

$$x = 0.5 \text{ m}$$

$\therefore \quad \cos 2x = \cos(2 \times 0.5 \text{ rad})$
$= \cos(1 \text{ rad}) = \cos 57.3°$
$= 0.54$

$\therefore \quad$ Amplitude R at $x = 0.5$ is 0.1×0.54
$= 0.054 \text{ m} = 5.4 \text{ cm}$

15. Let λ be the wavelength, ν the frequency and v the speed of each wave. Then

$\frac{2\pi}{\lambda}$ = coefficient of x in the argument of the sine function $= \frac{2\pi}{3}$

or $\lambda = 3$ m.

Also $\omega = 2\pi\nu$ = coefficient of t in the argument of the sine function
$= 120\ \pi$

which gives $\nu = 60$ Hz.

Hence $v = \nu\lambda = 60 \times 3 = 180\ \text{m s}^{-1}$

Mass per unit length $(m) = \frac{3.0 \times 10^{-2}}{1.5}$
$= 2.0 \times 10^{-2}\ \text{kg m}^{-1}$

We know that $v = \sqrt{\frac{T}{m}}$, where T is tension in the string.

$\therefore \quad T = mv^2 = 2.0 \times 10^{-2} \times (180)^2$
$= 648$ N

16. Let N be the frequency of the source and ν_p that of the pth harmonic of a closed pipe. The source will resonantly excite that harmonic mode of the pipe for which

$$N = \nu_p$$

for any value of $p = 1, 3, 5, \ldots$. Now for a closed pipe, we know that

$$\nu_p = \frac{pv}{4L}$$

Therefore, for resonance,

$$N = \frac{pv}{4L}$$

or $$p = \frac{4NL}{v} = \frac{4 \times 425 \times 0.2}{340} = 1$$

Hence the correct choice is (a).

17. In an open pipe, the condition of resonance is

$$N = \nu_p = \frac{pv}{2L}\ ;\ p = 1, 2, 3, \ldots$$

or $$p = \frac{2NL}{v} = \frac{2 \times 1700 \times 0.2}{340} = 2$$

Hence, the correct choice is (b).

18. $\nu_A = 324$ Hz, $\nu_b = 6$ Hz.

The frequency of string B is $\nu_B = \nu_A \pm \nu_B = 324 \pm 6 = 330$ or 318 Hz

Now, the frequency of a string is proportional to the square root of tension. Hence, if the tension in A is slightly decreased, its frequency will be slightly reduced, i.e. it will become less than 324 Hz. If the frequency of string B is 330 Hz, the beat frequency would increase to a value greater than 6 Hz if the tension in A is reduced. But the beat frequency is found to decrease to 3 Hz. Hence, the frequency of B cannot be 330 Hz; it is, therefore 318 Hz. When the tension in A is reduced, its frequency becomes $324 - 3 = 321$ Hz which will produce beats of frequency 3 Hz with string B of frequency 318 Hz. Hence the correct choice is (a).

19. Let N be the frequency of the tuning fork. Then, the frequency of the wire, when the tension is 100 N will be $(N + 5)$ and when the tension is 81 N, it is $(N - 5)$; since in each case 5 beats are heard per second. Hence

$$N + 5 = \frac{1}{2L}\sqrt{\frac{T_1}{m}} = \frac{1}{2 \times 0.5}\sqrt{\frac{100}{m}}$$
$$= \frac{10}{\sqrt{m}} \qquad \text{(i)}$$

and $$N - 5 = \frac{1}{2L}\sqrt{\frac{T_2}{m}} = \frac{1}{2 \times 0.5}\sqrt{\frac{81}{m}}$$
$$= \frac{9}{\sqrt{m}} \qquad \text{(ii)}$$

Subtracting (ii) from (i) we have

$$10 = \frac{1}{\sqrt{m}}$$

or $$m = 0.01 \text{ kg m}^{-1}$$

Using this value of m in (i) or (ii) gives $N = 95$ Hz. Hence the correct choice is (b).

20. The frequency of ultrasonic (sound) waves sent out from the SONAR undergoes a change in two steps. (i) Before reflection, the frequency of sound received by the enemy submarine which is approaching the SONAR with a speed $u_0 = 200 \text{ ms}^{-1}$ is given by

$$\nu' = \frac{\nu(v + u_0)}{v}$$

$$= \frac{42 \times 10^3 \times (1400 + 200)}{1400}$$

$$= 48 \times 10^3 \text{ Hz}$$

(ii) The wave of frequency ν' is reflected from the enemy submarine, which acts as a virtual image source of frequency ν', approaching the SONAR with a speed $u_s = 200 \text{ ms}^{-1}$. Hence the frequency of sound received back at the SONAR will be (observer stationary, source approaching)

$$\nu'' = \frac{v\nu'}{(v - u_s)}$$

$$= \frac{1400 \times 48 \times 10^3}{(1400 - 200)}$$

$$= 56 \times 10^3 \text{ Hz} = 56 \text{ kHz}$$

Hence the correct choice is (d).

21. Since only the source of sound, i.e. the gun is in motion, the speed of sound remains unchanged at 340 ms^{-1}. Therefore, the time taken by the sound of firing to arrive at the target 500 m away is

$$t_s = \frac{500}{340} = \frac{25}{17} \text{ s}$$

If the gun points in the direction of motion of the tank, the effective speed of the bullet = 340 + 20 = 360 ms^{-1}. Therefore, the time taken by the bullet to reach the target is

$$t_b = \frac{500}{360} = \frac{25}{18} \text{ s}$$

Since $t_s > t_b$, the correct choice is (a).

22. In this case, the effective speed of the bullet = 340 – 20 = 320 ms^{-1}. Therefore,

$$t_b = \frac{500}{320} = \frac{25}{16} \text{ s}$$

Thus $t_s < t_b$. Hence the correct choice is (b).

23. When the three waves superpose at a point, then from the superposition principle, the resultant particle displacement at that point is given by

$$y = y_1 + y_2 + y_3$$

$$= a \sin \{2\pi (\nu - 1)t\} + a \sin (2\pi \nu t) + a \sin \{2\pi (\nu + 1) t\}$$

Now $\sin \{2\pi (\nu - 1) t\} + \sin \{2\pi (\nu + 1) t\} = 2 \cos 2\pi t \sin 2\pi \nu t$

Therefore,

$$y = y_1 + y_2 + y_3$$

$$y = a (1 + 2 \cos 2\pi t) \sin 2\pi \nu t$$

or $$y = A \sin 2\pi \nu t$$

where $A = a (1 + 2 \cos 2\pi t)$ is the resultant amplitude.

Now, the resultant intensity $\propto A^2$. Now A^2 will be maximum when

$$\cos 2\pi t = +1$$

or $$2\pi t = 0, 2\pi, 4\pi, \text{ ... etc.}$$

or $$t = 0, 1\text{s}, 2\text{s}, \text{... etc.}$$

$\therefore$ Time period of beats = time interval between two consecutive maxima = 1 s. Hence the beat frequency is 1 Hz, i.e. one beat is heard per second which is choice (d).

24. Let the density of hydrogen be $\rho' = 2\rho$, then the density of oxygen will be $32\ \rho$. The density of the mixture will be

$$\rho'' = \frac{3}{5} \times 2\rho + \frac{2}{5} \times 32\ \rho = 14\ \rho$$

Since the pressure is the same, we have

$$\frac{v''}{v} = \sqrt{\frac{\rho'}{\rho''}} = \sqrt{\frac{2\rho}{14\rho}} = \frac{1}{\sqrt{7}}$$

$\therefore\ \nu'' = \frac{v}{\sqrt{7}}$. Hence the correct choice is (d).

25. The density of helium at STP = 2 times the density of hydrogen at STP. Since $v \propto 1/\sqrt{\rho}$, the speed of sound in helium will be $v/\sqrt{2}$. Hence the correct choice is (a).

26. Let the frequency of the first tuning fork be ν. The frequency of the second will be $(\nu + 4)$ and of the third will be $(\nu + 8)$ and so on. Now $\nu + 8 = \nu + (3 - 1) \times 4$. Therefore, the frequency of the 9th tuning fork $= \nu + (9 - 1) \times 4 = \nu + 32$. It is given that $\nu + 32 = 2\nu$. Hence $\nu = 32$ Hz which is choice (a).

27. The frequency of the fundamental mode is given by

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

$\therefore\ \nu \propto \frac{1}{L}$. Hence $\nu_1 : \nu_2 : \nu_3 = \frac{1}{L_1} : \frac{1}{L_2} : \frac{1}{L_3}$.

Now $L_1 = 20$ cm, $L_2 = 40$ cm and $L_3 = 60$ cm. Therefore,

$$\nu_1 : \nu_2 : \nu_3 = \frac{1}{20} : \frac{1}{40} : \frac{1}{60} = 6 : 3 : 2$$

Hence, the correct choice is (d).

28. Let ν_1 be the frequency of the wire when its vibrating length is $L_1 = 48$ cm and ν_2 when $L_2 = 50$ cm. Since $\nu \propto \frac{1}{L}$; $\nu_1 > \nu_2$. If ν is the frequency of the tuning fork, then

$$\nu_1 = \nu + 4 \text{ and } \nu_2 = \nu - 4$$

$$\therefore \quad \frac{\nu_1}{\nu_2} = \frac{\nu+4}{\nu-4}$$

But $$\frac{\nu_1}{\nu_2} = \frac{L_2}{L_1} = \frac{50}{48} = \frac{25}{24}$$

Thus $$\frac{\nu+4}{\nu-4} = \frac{25}{24}$$

which gives $\nu = 196$ Hz. Hence the correct choice is (a).

29. Let T_1 be the tension in each string when they are in unison. Let T_2 be the tension in each string when they are not in unison; then since $\nu \propto \sqrt{T}$; $\nu_2 > \nu_1$ such that

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{T_2}{T_1}}$$

Now $T_2 = 1.01\ T_1$. Therefore

$$\frac{\nu_2}{\nu_1} = \sqrt{1.01} = \left(1 + \frac{1}{100}\right)^{1/2}$$

Expanding binomially, we have

$$\left(1 + \frac{1}{100}\right)^{1/2} \simeq 1 + \frac{1}{200} = \frac{201}{200}. \text{ Thus}$$

$$\frac{\nu_2}{\nu_1} = \frac{201}{200}$$

Also, $\nu_2 - \nu_1 = 4$

or $\nu_2 = \nu_1 + 4$. Therefore, we have

$$\frac{\nu_1 + 4}{\nu_1} = \frac{201}{200}$$

which gives $\nu_1 = 800$ Hz. Hence the correct choice is (b).

30. Let v_1 be the speed of sound at 27°C and v_2 at 31°C. Then

$$\frac{v_2}{v_1} = \left(\frac{273+31}{273+27}\right)^{1/2} = \left(\frac{304}{300}\right)^{1/2} = \left(1 + \frac{4}{300}\right)^{1/2}$$

$$\simeq 1 + \frac{1}{2} \times \frac{4}{300} = \frac{151}{150}$$

Now, frequency $\propto$ speed of sound. Hence

$$\frac{\nu_2}{\nu_1} = \frac{v_2}{v_1} = \frac{151}{150}$$

$$\therefore \quad \nu_2 = \nu_1 \times \frac{151}{150} = \frac{300 \times 151}{150} = 302 \text{ Hz.}$$

Hence beat frequency = 302 – 300 = 2

Thus the correct choice is (b).

31. If a source emitting light of wavelength λ goes away from the earth, the apparent wavelength λ' of the light reaching the earth is given by

$$\frac{\lambda'}{\lambda} = 1 + \frac{v}{c}$$

where v is the speed of the source of light and c the speed of light. The increase in wavelength $\Delta\lambda = \lambda' - \lambda$ is given by

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

Here $\frac{\Delta\lambda}{\lambda} = 5\% = \frac{5}{100}$ and $c = 3 \times 10^8$ ms^{-1}.

Therefore,

$$v = 3 \times 10^8 \times \frac{5}{100} = 1.5 \times 10^7 \text{ ms}^{-1}$$

Hence the correct choice is (d).

32. If the aircraft is approaching the radar station with a speed u, the apparent frequency of radiowaves received by the radar after reflection from the aircraft is given by

$$\nu' = \nu\left(1 + \frac{2u}{c}\right)$$

$\therefore$ Apparent increase in frequency is

$$\Delta\nu = \nu' - \nu = \frac{2u\nu}{c}$$

Given, $\nu = 600$ MHz $= 600 \times 10^6$ Hz

and $\Delta\nu = 6$ kHz $= 6 \times 10^3$ Hz. Thus

$$u = \frac{c\Delta\nu}{2\nu} = \frac{3 \times 10^8 \times 6 \times 10^3}{2 \times 600 \times 10^6}$$

$$= 1.5 \times 10^3 \text{ ms}^{-1} = 1.5 \text{ kms}^{-1}$$

Hence, the correct choice is (a).

33. $\nu' = \nu\left(1 + \frac{u_0}{v}\right)$. Given $u_0/v = \frac{1}{10}$. Therefore

$$\nu' = \frac{11}{10}\ \nu$$

or $\nu' - \nu = \frac{11}{10}\ \nu - \nu = \frac{\nu}{10}$.

The percentage increase in ν is

$\frac{\nu' - \nu}{\nu} \times 100 = 10\%$ which is choice (c).

34. To form a stationary wave, waves y and y' must travel in opposite directions. Wave $y = a \cos (kx - \omega t)$ travels along the positive x-direction. Waves $y' = -a \cos (kx - \omega t)$ and $y' = -a \sin (kx - \omega t)$ in choices (b) and (d) travel along positive x-direction. Hence choices (b) and (d) are not possible. Choice (a) is also incorrect because at $x = 0$

$$y' = a \sin \omega t$$

and $$y = a \cos (-\omega t) = a \cos \omega t$$

Therefore, the resultant displacement at $x = 0$ which is $y + y' = a \sin \omega t + a \cos \omega t$ is not zero, i.e. these waves do not produce a node at $x = 0$. Choice (c) is correct because at $x = 0$, $y + y' = 0$.

35. The frequency of a string of length L, mass m per unit length, stretched with a tension T and vibrating in p segments, is given by

$$\nu = \frac{p}{2L}\sqrt{\frac{T}{m}}$$

If the radius of the wire is r and ρ its density, then

$$m = \frac{\text{mass}}{\text{length}} = \frac{\pi r^2 L \rho}{L} = \pi r^2 \rho.$$

$$\therefore \quad \nu = \frac{p}{2Lr}\sqrt{\frac{T}{\pi \rho}}$$

Since ν, T, L and ρ are the same for both wires, $\frac{p}{r}$ = constant or $p \propto r$. Hence the number of loops formed in the thicker wire will be two times that in the thinner wire. Hence the correct choice is (b).

36. Because the rope has a finite mass, the tension in the rope is different at different points on the rope. At the top where the rope is rigidly fixed, the tension = weight of the rope + the weight attached to the free end of the rope = 6 kg + 2 kg = 8 kg wt. Tension at the free end of the rope = 2 kg wt. Since $\nu \propto \sqrt{T}$, if the tension becomes 4 times, the frequency is doubled. Since $\nu = \frac{v}{\lambda}$; $\lambda \propto \frac{1}{\nu}$. Hence the wavelength is halved. Thus the correct choice is (b).

37. For a closed tube $\nu = \frac{v}{4L}$. For an open tube $\nu' = \frac{v}{2L}$. Hence $\nu' = 2\nu = 2 \times 512 = 1024$ Hz. Thus the correct choice is (a).

38. When two waves of amplitudes a_1 and a_2 superpose to produce beats, the resultant amplitude of the maxima of intensity is

$$A = a_1 + a_2$$

Now, intensity $\propto$ (amplitude)^2. Since the two waves have the same intensity, their amplitudes are equal, i.e. $a_1 = a_2 = a$. Thus $A = 2a$. Therefore, $A^2 = 4a^2$ or $I_{max} = 4I$. Hence the correct choice is (b).

39. Distance between adjacent nodes (or antinodes) = $\lambda/2$. Also

$$\frac{2\pi}{\lambda} = \text{coefficient of } x \text{ in the argument of the sine function} = k$$

or $$\lambda = \frac{2\pi}{k}.$$

Hence, the distance between adjacent nodes

$$= \frac{\pi}{k}.$$

40. The amplitude of the three waves are $a_1 = 10$ μm, $a_2 = 4$ μm and $a_3 = 7$ μm. Let the phase of the first wave be zero. Then the wave of the second wave $= \pi/2$ and of third wave $= \frac{\pi}{2} + \frac{\pi}{2} = \pi$. Therefore, the phase difference between the first wave and third wave is π. Hence their resultant is

$$a = a_1 - a_3 = 10 - 7 = 3 \text{ μm}$$

The phase difference between this resultant (of first and third waves) and the second wave $= \pi/2$. Therefore, the resultant of a and a_3 is

$$A = (a^2 + a_2^2)^{1/2} = (3^2 + 4^2)^{1/2} = 5 \text{ μm}.$$

Hence the correct choice is (c).

41. Wave velocity = v. Particle velocity is

$$V = \frac{dy}{dt}$$

$$= y_0 \left(\frac{2\pi v}{\lambda}\right) \cos\left\{\frac{2\pi}{\lambda}(vt - x)\right\}$$

$$\therefore \quad V_{max} = y_0 \left(\frac{2\pi v}{\lambda}\right)$$

Now, $V_{max} = 2v$, if $y_0 \left(\frac{2\pi v}{\lambda}\right) = 2v$, which gives $\lambda = \pi y_0$. Hence the correct choice is (d).

42. If the train is going away from the observer, the apparent frequency is

$$\nu_1 = \frac{\nu v}{v + u} = \frac{\nu}{1 + \frac{u}{v}} \qquad \text{(i)}$$

It is observed that $\nu = 1.2\ \nu_1$. In the second case, the apparent frequency is

$$\nu_2 = \frac{\nu(v - u)}{v} = \nu\left(1 - \frac{u}{v}\right)$$

or $$\frac{\nu}{\nu_2} = \frac{1}{\left(1 - \frac{u}{v}\right)} \qquad \text{(ii)}$$

Now, from (i) we have $\frac{v}{v_1} = 1 + \frac{u}{v}$ or $1.2 = 1 + \frac{u}{v}$, or $u = 0.2\ v$ or $\frac{u}{v} = 0.2$. Using this in (ii), we get $\frac{v}{v_2} = \frac{5}{4} = 1.25$. Hence the correct choice is (b).

43. For pipe P_1 : $\nu_1 = \frac{v}{4L_1}$

For pipe P_2 : $\nu_3 = \frac{3v}{2L_2}$

It is given that $\nu_1 = \nu_3$. Therefore, $L_1/L_2 = 1/6$ which is choice (b).

44. The speed of the wave in the string is given by

$$v = \sqrt{\frac{T}{m}}$$

According to Hookes' law, tension $(T) \propto$ extension (x). Hence $v \propto \sqrt{x}$. Therefore

$$\frac{v'}{v} = \frac{\sqrt{1.5x}}{\sqrt{x}} = \sqrt{1.5} = 1.22$$

Hence the correct choice is (a).

45. Particle velocity $V = \frac{dy}{dt} = \frac{d}{dt}\ [A \sin (kx - \omega t)]$

$$= -A\omega \cos (kx - \omega t)$$

Hence $V_{max} = A\omega$

46. Standing waves are formed on the string. Particle displacements are given by

$$y = a \sin \left(\frac{2\pi x}{\lambda}\right) \cos (2\ \pi f t)$$

Particle velocity

$$V = \frac{dy}{dt}$$

$$= (-2\ \pi f a) \sin \left(\frac{2\pi x}{\lambda}\right) \sin (2\ \pi f t)$$

$\therefore \quad (V)_{max} = 2\ \pi f a$

Given $(V)_{max} = \frac{v}{10}$. Therefore,

$$2\ \pi f a = \frac{v}{10}$$

or $\quad f = \frac{v}{20\pi a} = \frac{10 \text{ms}^{-1}}{20\pi \times 10^{-3} \text{m}} = \frac{10^3}{2\pi}$ Hz

Now $\quad \lambda = \frac{v}{f} = \frac{10 \text{ms}^{-1}}{10^3/2\pi \text{ Hz}} = 2\pi \times 10^{-2}$ m

Hence the correct choice is (a).

47. For a gas, adiabatic elasticity $E = \gamma P$ where $\gamma = C_p/C_v$ and P is the pressure. The speed of sound in the gas is given by

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

where M is the molecular mass and R is the gas constant. Thus

$$\frac{v_N}{v_{He}} = \sqrt{\frac{M_{He}}{M_N}} = \sqrt{\frac{4}{28}} = \sqrt{\frac{1}{7}}$$

48. Frequency of the fundamental mode is given by

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}};$$

T = tension and m = mass per unit length

$$\therefore \quad \frac{\nu_1}{\nu_2} = \frac{(1/2L)\sqrt{T/m_1}}{(1/4L)\sqrt{T/m_2}} = 2\sqrt{\frac{m_2}{m_1}}$$

Now $\quad m_1 = \frac{M_1}{L} = \frac{\pi (2r)^2 L\rho}{L} = 4\ \pi r^2 \rho$

and $\quad m_2 = \frac{M_2}{2L} = \frac{\pi (r^2)(2L)\rho}{2L} = \pi r^2 \rho$

Hence $\quad \frac{\nu_1}{\nu_2} = 2 \times \sqrt{\frac{\pi r^2 \rho}{4\pi r^2 \rho}} = 1$

49. Beat frequency $\nu_0 = \frac{30}{3} = 10$ per second. Now

$$\nu_b = \nu_1 - \nu_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2}$$

or $\quad v = \frac{\lambda_1 \lambda_2 \nu_b}{(\lambda_2 - \lambda_1)} = \frac{5 \times 6 \times 10}{(6-5)}$

$$= 300 \text{ ms}^{-1}$$

Hence the correct choice is (a).

50. Let the frequency of fork C be n. Then $n_A = n + 0.03\ n = 1.03n$ and $n_B = n - 0.02\ n = 0.98\ n$. The beat frequency is

$$n_b = n_A - n_B$$

or $\quad 5 = 1.03\ n - 0.98\ n = 0.05\ n$

which gives $n = 100$ Hz. Hence $n_A = 1.03 \times 100 = 103$ Hz which is choice (c).

51. Given $t_1 = 10°C$, $n_1 = 1.001$ kHz, $t_2 = 30°C$, $n_2 = 1$ kHz. Let l_1 and l_2 be the vibrating lengths of the wire at 10°C and 30°C respectively. Since tension T is kept constant.

$$n_1 l_1 = n_2 l_2 \qquad \text{(i)}$$

$$\frac{l_2}{l_1} = \frac{n_1}{n_2} = \frac{1.001}{1} = 1.001$$

If α is the coefficient of linear expansion, then

$$l_2 = l_1\ (1 + \alpha\Delta t)$$

or $$\frac{l_2}{l_1} = 1 + a\Delta t = 1 + a\ (30–10)$$

$$= 1+ 20a \qquad \text{(ii)}$$

From (i) and (ii) we have

$$1 + 20\alpha = 1.001$$

which gives $\alpha = 0.5 \times 10^{-4}$ per ºC, which is choice (d).

52. Let L_1 and L_2 be the lengths of the two parts of sonometer wire.

Given $$\nu_1 = \frac{1}{2L_1}\sqrt{\frac{T}{m}} \text{ and } \nu_2 = \frac{1}{2L_2}\sqrt{\frac{T}{m}}$$

or $\nu_1 L_1 = \nu_2 L_2 = \frac{1}{2}\sqrt{\frac{T}{m}}$ = constant, say k. Thus

$$L_1 = \frac{k}{\nu_1} \text{ and } L_2 = \frac{k}{\nu_2}$$

The fundamental frequency of the complete sonometer wire is

$$\nu = \frac{1}{2(L_1 + L_2)}\sqrt{\frac{T}{m}} = \frac{k}{(L_1 + L_2)}$$

or $$\frac{1}{\nu} = \frac{L_1}{k} + \frac{L_2}{k} = \frac{1}{\nu_1} + \frac{1}{\nu_2}$$

or $$\nu = \frac{\nu_1 \nu_2}{\nu_1 + \nu_2}$$, which is choice (d).

53. Let W_1 be the weight of the bob in air and W_2 when it is immersed in water. Given

$$\nu_1 = \frac{1}{2L}\sqrt{\frac{W_1}{m}} \text{ and } \nu_2 = \frac{1}{2L}\sqrt{\frac{W_2}{m}}$$

$$\therefore \quad \frac{W_1}{W_2} = \frac{\nu_1^2}{\nu_2^2}$$

$$\text{Relative density} = \frac{\text{weight in air}}{\text{loss of weight in water}}$$

$$= \frac{W_1}{W_1 - W_2} = \frac{\nu_1^2}{\nu_1^2 - \nu_2^2}$$

Hence the correct choice is (a).

54. After 2 seconds, both the pulses will be at the same location on the string and will superpose on each other. Since their amplitudes are equal and opposite, they cancel each other and the string becomes straight. Hence the string has no potential energy, i.e. the total energy is purely kinetic. Thus the correct choice is (b).

55. For train A: $\nu_A = \nu\left(\frac{v + u_A}{v}\right)$ or $5.5 = 5\left(\frac{v + u_A}{v}\right)$

$\Rightarrow u_A = 0.1\ v$

For train B: $\nu_B = \nu\left(\frac{v + u_B}{v}\right)$ or $6.0 = 5\left(\frac{v + u_B}{v}\right)$

$\Rightarrow u_B = v/5 = 0.2\ v$

$$\therefore \quad \frac{u_B}{u_A} = \frac{0.2v}{0.1v} = 2,$$ which is choice (b).

56. Let L be the length of the wire between the bridges and let m be the mass per unit length of the wire. Five antinodes on a length L implies that $L = \frac{5}{2}\ \lambda_1$ or $\lambda_1 = \frac{2L}{5}$. Thus in this case we have

$$\nu_1 = \frac{v_1}{\lambda_1} = \frac{5}{2L}\sqrt{\frac{T_1}{m}}, \text{ where } T_1 = M_1\ g$$

Three antinodes on a length L implies that $L = \frac{3}{2}\ \lambda_2$ or $\lambda_2 = \frac{2L}{3}$. In this case, we have

$$\nu_2 = \frac{v_2}{\lambda_2} = \frac{3}{2L}\sqrt{\frac{T_2}{m}}, \text{ where } T_2 = M_2\ g$$

Given $\nu_1 = \nu_2$, $M_1 = 9$ kg and $M_2 = M$. Therefore, we have

$$\frac{5}{2L}\sqrt{\frac{9g}{m}} = \frac{3}{2L}\sqrt{\frac{Mg}{m}}$$

which gives $M = 25$ kg. Hence the correct choice is (a).

57. Given $v = 330\ \text{ms}^{-1}$ and $u_p = 22\ \text{ms}^{-1}$. The apparent frequency of the police man's horn of frequency 176 Hz as heard by the motorcyclist is given by

$$\nu_1 = 176\left(\frac{330 - u_m}{330 - 22}\right) = \frac{176}{308}\ (330 - u_m) \qquad \text{(i)}$$

The apparent frequency of the stationary siren of frequency 165 Hz as heard by the motorcyclist is given by

$$\nu_2 = 165\left(\frac{330 + u_m}{330}\right) \qquad \text{(ii)}$$

Since the motorcyclist does not observe any beats, $\nu_1 = \nu_2$. Equating (i) and (ii) and solving for u_m we get $u_m = 22\ \text{ms}^{-1}$.

Hence the correct choice is (b).

58. Let ν be the frequency of the tuning fork and e the end correction. Given $L_1 = 0.1$ m and $L_2 = 0.35$ m

$$\nu = \nu_1 = \frac{1}{4(L_1 + e)}\sqrt{\frac{\gamma P}{\rho}} \qquad \text{(i)}$$

$$\nu = \nu_3 = \frac{3}{4(L_2 + e)}\sqrt{\frac{\gamma P}{\rho}} \qquad \text{(ii)}$$

Equating (i) and (ii), we get

$$\frac{1}{L_1 + e} = \frac{3}{L_2 + e}$$

or $$\frac{1}{0.1 + e} = \frac{3}{0.35 + e}$$

which gives $e = 0.025$ m. Hence the correct choice is (b).

59. The density ρ of a wire of mass M, length L and diameter d is given by

$$\rho = \frac{4M}{\pi d^2 L} = \frac{4m}{\pi d^2}$$

Now $$v_A = \sqrt{\frac{T}{m_A}} \quad \text{and} \quad v_B = \sqrt{\frac{T}{m_B}}$$

$$\therefore \quad \frac{v_A}{v_B} = \sqrt{\frac{m_B}{m_A}} = \frac{d_B}{d_A}$$

but $v_A = n\lambda_A$ and $v_B = n\lambda_B$, n being the frequency of the source.

Hence $$\frac{\lambda_A}{\lambda_B} = \frac{v_A}{v_B} = \frac{d_B}{d_A} = \frac{0.5 \times 10^{-3}}{10^{-3}} = 0.5,$$

which is choice (a).

60. The frequency of the nth hormonic is 420 Hz. The frequency of the $(n+1)$th harmonic is 490 Hz. Therefore,

$$420 = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

and $$490 = \frac{(n+1)}{2L}\sqrt{\frac{T}{m}}$$

Dividing, we have

$$\frac{490}{420} = \frac{(n+1)}{n} \text{ which gives } n = 6.$$

61. The fundamental frequency of the vibration of a wire of length L, mass m per unit length and under tension T is given by

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} \qquad (1)$$

If the tension is increased by 44%, the new tension is

$$T' = T + \frac{44}{100} \times T = T + 0.44\ T = 1.44\ T$$

Since L is kept constant, the new fundamental frequency is

$$\nu' = \frac{1}{2L}\sqrt{\frac{T'}{m}} = \frac{1}{2L}\sqrt{\frac{1.44T}{m}} = \frac{1.2}{2L}\sqrt{\frac{T}{m}}$$

Comparing this with Eq. (1), we have

$$\nu' = 1.2\ \nu$$

Given $\nu' - \nu = 6$ Hz. Hence $\nu + 6 = 1.2\ \nu$

which gives $\nu = \frac{6}{0.2} = 30$ Hz, which is choice (b).

62. A travelling wave is characterized by wave functions of the type $y = f(vt + x)$ or $y = f(vt - x)$ where f stands for sine or cosine function. Hence the correct choice is (d). Choices (a), (b) and (c) represent a stationary or standing wave.

63. If the length is increased by 20%, the new length is

$$L' = L + \frac{20L}{100} = 1.2\ L$$

The original frequency is

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

Since T and m are the same, the new frequency will be

$$\nu' = \frac{1}{2L'}\sqrt{\frac{T}{m}} = \frac{1}{1.2 \times 2L}\sqrt{\frac{T}{m}} = \frac{\nu}{1.2}$$

Now $\nu = 30$ Hz. Therefore, $\nu' = \frac{30}{1.2} = 25$ Hz. Thus, the frequency decreases by 5 Hz. Hence the correct choice is (a).

64. For a closed pipe, $\nu_n = \frac{nv}{4L}$; $n = 1, 3, 5, \ldots$ etc

For an open pipe, $\nu'_n = \frac{nv}{2L}$; $n = 1, 2, 3, \ldots$ etc

where $v = \sqrt{\frac{\gamma P}{\rho}}$.

For the closed pipe vibrating in the first harmonic ($n = 1$), we have

$$\nu_1 = \frac{1}{4L_1}\sqrt{\frac{\gamma P}{\rho_1}} \qquad (1)$$

For the open pipe vibrating in the third harmonic ($n = 3$), we have

$$\nu'_3 = \frac{3}{2L_2}\sqrt{\frac{\gamma P}{\rho_2}} \qquad (2)$$

Given $\nu_1 = \nu'_3$. Equating (1) and (2), we get

$$\frac{L_1}{L_2} = \frac{1}{6}\sqrt{\frac{\rho_2}{\rho_1}}.$$

Hence the correct choice is (d).

65. The frequency of sound is a characteristic of its source. Hence frequency of sound is the same in air as is water. Therefore, the observer in air will receive a sound of frequency 600 Hz. If λ_a and λ_w are the wavelengths in air and water resptively, then

$$n = \frac{v_a}{\lambda_a} = \frac{v_w}{\lambda_w}$$

which gives $\lambda_a = \dfrac{v_a}{\nu} = \dfrac{300}{600} = 0.5$ m

Hence the correct choice is (a). The wavelength is water is

$$\lambda_w = \frac{v_w}{\nu} = \frac{1500}{600} = 2.5 \text{ m}$$

66. Refer to Fig. 14.12.

$$L_1 = \frac{\lambda}{4} \text{ and } L_2 = \frac{3\lambda}{4}$$

$$N = \nu_1 = \frac{\mathbf{v}}{4(L_1 + e)} \quad (1)$$

and $$N = \nu_3 = \frac{3v}{4(L_2 + e)} \quad (2)$$

where e is the end correction. Eliminating e from Eqs (1) and (2), we get

$$v = 2N(L_2 - L_1) \quad (3)$$

Lengths L_1 and L_2 are measured with a metre scale whose least count is 0.1 cm. Thus L_1 = (17.7 ± 0.1) cm and L_2 = (53.1 ± 0.1) cm. The maximum error in $(L_2 - L_1)$ is ± 0.2 (for maximum error, the errors in individual measurements add up). Thus $L_2 - L_1$ = (53.1 – 17.7) = 35.4 cm. Hence $L_2 - L_1$ = (35.4 ± 0.2) cm. Using this in Eq. (3), we have

$v = 2 \times 480 \times (35.4 \pm 0.2) = (33984 \pm 192)$ cms^{-1}

Hence maximum error = 192 cm s^{-1}, which is choice (a).

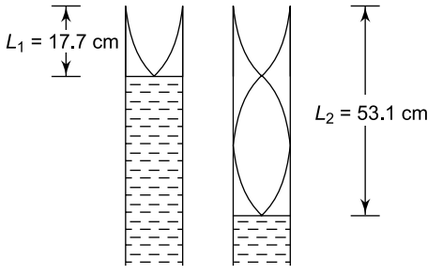


Fig. 14.12

67. Let T_1 and T_2 be the tensions in wires 1 and 2 respectively. Let m be the mass per unit length of each wire and let l be the length of each wire. Given

$$\nu = \frac{1}{2l}\sqrt{\frac{T_1}{2m}} = \frac{2}{2l}\sqrt{\frac{T_2}{m}}$$

which gives $\dfrac{T_1}{T_2} = 4$

For rotational equilibrium of the rod about O, we have

$T_1 \times AO = T_2 \times BO$ or $T_1 \times x = T_2(L - x)$,

which gives $\dfrac{T_1}{T_2} = \dfrac{(L-x)}{x}$. But $\dfrac{T_1}{T_2} = 4$. Hence

$$4 = \frac{(L-x)}{x}$$

which gives $x = \dfrac{L}{5}$. Hence the correct choice is (a).

68. When the source moves towards from the stationary observer, its apparent frequency is

$$\nu' = \nu \times \frac{v}{(v-u)} \quad (1)$$

When the source moves away from the stationary observer, its apparent frequency is

$$\nu'' = \nu \times \frac{v}{(v+u)} \quad (2)$$

When both the forks are moving relative to stationary observer, the number of beats heard by him per second = $\nu' - \nu''$

Since $u_s << v$, Eqs (1) and (2) may be simplified as follows.

$$\nu' = \nu\left(1 - \frac{u}{v}\right)^{-1} \cong \nu\left(1 + \frac{u}{v}\right)$$

$$\nu'' = \nu\left(1 + \frac{u}{v}\right)^{-1} \cong \nu\left(1 - \frac{u}{v}\right)$$

where terms of order u^2/v^2 have been neglected in the binomial expansion. Thus,

$$\nu' - \nu'' = \nu\left(1 + \frac{u}{v}\right) - \nu\left(1 - \frac{u}{v}\right) = 2\nu\frac{u}{v}$$

Thus the correct choice is (c)

69. Particle velocity

$$V = \frac{dy}{dt} = \frac{d}{dt}\left[4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)\right]$$

$$= -4 \times 96 \sin\left(\frac{\pi x}{15}\right) \sin(96\pi t)$$

At x = 7.5 cm and t = 0.25 s, the particle velocity is

$$V = -4 \times 96 \sin\left(\frac{\pi \times 7.5}{15}\right) \sin(96\pi \times 0.25)$$

$$= -(4 \times 96) \sin(0.5\pi) \sin(24\pi)$$

$$= 0 \qquad [\because \sin(24\pi) = 0]$$

The correct choice is (a).

70. The tension at the lower end of the rope is $T = mg$. The speed of pulse at this point is $v = \sqrt{\frac{T}{\mu}}$, μ = mass per unit length of the rope. If ν is the frequency of the pulse, then

$$\lambda = \frac{v}{\nu} = \frac{1}{\nu}\sqrt{\frac{T}{\mu}} \qquad (1)$$

The tension at the upper end of the rope is $T' = (m + M)\, g$. Let λ' be the wavelength of the pulse when it reaches the upper end of the rope. Then, since frequency ν of the pulse remains the same,

$$\lambda' = \frac{1}{\nu}\sqrt{\frac{T'}{\mu}} \qquad (2)$$

From Eqs. (1) and (2), we have

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{T'}{T}} = \sqrt{\frac{(m+M)g}{mg}} = \sqrt{\frac{M+m}{m}}$$

Hence the correct choice is (b).

71. Let u_m = speed of motorist and u_b = speed of the band. The motorist will hear two sounds–one coming directly from the band and the other reflected from the wall. The apparent frequency of the direct sound is given by (the observer is approaching a receding source of sound)

$$f_1 = f\left(\frac{v+u_m}{v+u_b}\right) = f \qquad (\because u_m = u_b = u)$$

The reflected sound can be regarded as coming from the mirror image which is approaching the motorist with a speed v_b. Hence, the apparent frequency of the reflected sound is given by (the observer and the source of sound are both approaching each other)

$$f_2 = f = \left(\frac{v+u_m}{v-u_b}\right) = f\left(\frac{v+u}{v-u}\right)$$

$$\therefore \quad \text{Beat frequency } (= f_2 - f_1) = f\left(\frac{v+u}{v-u}\right) - f$$

$$= \left(\frac{2fu}{v-u}\right)$$

Thus the correct choice is (d).

72. The frequency of the third harmonic of a closed pipe is

$$f = \frac{3v}{4L} = \frac{3\times 340}{4\times 0.75} = 340 \text{ Hz}.$$

Beat frequency $f_b = 4$. Therefore $n = f \pm f_b = 340 \pm 4$ or $n = 336$ Hz or 344 Hz. The frequency of string $\propto \sqrt{T}$. Hence if tension T is slightly increased, the frequency will slightly increase and become greater than 340. If $n = 336$ Hz, this will result in a greater number of beats per second. But the number of beats decreases to 2 per second. Hence n is not 336 Hz; it is 344 Hz. So the correct choice is (a).

73. In a transverse wave, the particle displacement and particle velocity are perpendicular to the direction of propagation of the wave. Hence choices (c) and (d) are wrong. The particle displacements is given by

$$y = a\sin\left[\frac{2\pi}{\lambda}(vt - x)\right] \qquad (1)$$

Putting $y = 5$ cm and $a = 10$ cm in Eq. (1), we get

$$\sin\left[\frac{2\pi}{\lambda}(vt - x)\right] = \frac{1}{2}$$

$$\Rightarrow \quad \frac{2\pi}{\lambda}(vt - x) = \frac{\pi}{6}$$

Particle velocity is

$$V = \frac{dy}{dt} = a\left(\frac{2\pi}{\lambda}v\right)\cos\left[\frac{2\pi}{\lambda}(vt - x)\right] \qquad (2)$$

Given $a = 0.1$ m, $v = 0.1$ ms^{-1}, $a = 0.1$ m and $\lambda = 0.5$ m. Also $\frac{2\pi}{\lambda}(vt - x) = \frac{\pi}{6}$. Putting these values in Eq. (2), we get

$$V = \frac{\sqrt{3}\pi}{50} \text{ ms}^{-1} \text{ along the } y\text{-axis.}$$

74. Since the mass is brought to rest, the total displacement is zero, i.e.

$$x_1(t) + x_2(t) + x_3(t) = 0$$

$$\Rightarrow A\sin\omega t + A\sin\left(\omega t + \frac{2\pi}{3}\right) + B\sin(\omega t + \phi) = 0$$

Using $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$, we get

$$2A\sin\left(\omega t + \frac{4\pi}{3}\right)\cos\frac{4\pi}{3} + B\sin(\omega t + \phi) = 0$$

$$\Rightarrow \quad -A\sin\left(\omega t + \frac{4\pi}{3}\right) + B\sin(\omega t + \phi) + 0 \qquad \left(\because \cos\frac{4\pi}{3} = -\frac{1}{2}\right)$$

$$\Rightarrow \quad B\sin(\omega t + \phi) = A\sin\left(\omega t + \frac{4\pi}{3}\right)$$

which gives $B = A$ and $\phi = \frac{4\pi}{3}$.

75. $u = 36$ km h^{-1} $= 10$ ms^{-1}, $v = 320$ ms^{-1}, $\nu = 8$ kHz

The sound reflected from the building may be imagined to be coming from the mirror image. The driver is approaching the image-source which is also approaching him with the same speed. Hence the frequency of sound heard by the driver is

$$\nu' = \nu\left(\frac{v+u}{v-u}\right)$$

$$= 8 \text{ kHz} \times \left(\frac{320+10}{320-10}\right) = 8.5 \text{ kHz}$$

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. When a wave is reflected from a boundary of a denser medium, which of the following will change?
(a) Amplitude (b) Frequency
(c) Wavelength (d) Phase

2. When a wave is refracted into another medium, which of the following will change?
(a) Amplitude (b) Velocity
(c) Frequency (d) Phase

3. Choose the correct statements from the following.
(a) Any function of the form $y(x, t) = f(vt + x)$ represents a travelling wave.
(b) The velocity, wavelength and frequency of a wave do not undergo any change when it is reflected from a surface.
(c) When an ultrasonic wave travels from air into water, it bends towards the normal to the air–water interface.
(d) The velocity of sound is generally greater in solids than in gases at STP.

4. Which of the following statements are correct?
(a) The decrease in the speed of sound at high altitudes is due to a fall is pressure.
(b) The standing wave on a string under tension, fixed at its ends, does not have well–defined nodes.
(c) The phenomenon of beats is not observed in the case of visible light waves.
(d) The apparent frequency is ν_1 when a source of sound approaches a stationary observer with a speed u and is ν_2 when the observer approaches the same stationary source with the same speed. Then $\nu_2 < \nu_1$, if $u < v$, where v is the speed of sound.

5. Which of the following functions represent a stationary wave? Here a, b and c are constants.
(a) $y = a \cos(bx) \sin(ct)$
(b) $y = a \sin(bx) \cos(ct)$
(c) $y = a \sin(bx + ct)$
(d) $y = a \sin(bx + ct) + a \sin(bx - ct)$
IIT, 1987

6. When a wave travels in a medium, the particle displacements are given by

$$y = a \sin 2\pi(bt - cx)$$

where a, b and c are constants.
(a) The wavelength of wave is c.
(b) The velocity of the wave is $\frac{b}{c}$.
(c) The maximum particle velocity is twice the wave velocity if $c = \frac{1}{\pi a}$
(d) The frequency of the wave is b.

7. Two persons A and B, each carrying a source of sound of frequency 90 Hz are standing a few metres apart in a quiet field. A starts moving towards B with a speed $u = v/10$, where v is the speed of sound. Then
(a) A will hear 9 beats per second
(b) A will hear 6 beats per second
(c) B will hear 12 beats per second
(d) B will hear 10 beats per second.

8. A wire of length L having linear density of 1.0×10^{-3} kg/m is stretched between two rigid supports with a tension of 40 N. It is observed that the wire, vibrating in p segments resonates at a frequency of 420 Hz. The next higher frequency at which the wire resonates is 490 Hz. The values of p and L are
(a) $p = 6$ (b) $p = 7$
(c) $L = (60/49)$ m (d) $L = (10/7)$ m

9. The first overtone of an open organ pipe (of length L_o) beats with the first overtone of a closed organ pipe (of length L_c) with a beat frequency of 10 Hz. The fundamental frequency of the closed pipe is 110 Hz. If the speed of sound is 330 ms^{-1}, then

(a) L_c = 0.75 m
(b) L_o = (33/34) m or (33/35) m
(c) L_o = (33/32) m or 1 m
(d) L_o= (33/32) m or (33/34) m

IIT, 1997

10. In a resonance tube experiment, a tuning fork of frequency 480 Hz resonates in the fundamental mode with an air column of length 16 cm in a tube closed at one end. If the speed of sound in air is 336 ms^{-1}, the diameter of the tube is
(a) 5.6 cm
(b) 5.0 cm
(c) either 5.0 cm or 5.6 cm
(d) neither 5.0 cm nor 5.6 cm.

IIT, 2003

11. In a standing wave on a string fixed at both ends, all points on the string vibrate with
(a) the same frequency and the same phase but different amplitude.
(b) the same frequency and the same amplitude but different phase.
(c) the same frequency but different phase and different amplitude.
(d) the same frequency, the same phase and the same amplitude.

12. Sound waves travel from location A at absolute temperature T_1 to location B at absolute temperature T_2 in time t. The air temperature increases linearly form T_1 to T_2. The speed of sound varies with absolute temperature T as $v = k\sqrt{T}$ where k is a positive constant. The distance between A and B is L.
(a) The temperature gradient $\frac{dT}{dx} = \frac{T_2 - T_1}{L}$.
(b) The temperature gradient $\frac{dT}{dx} = \frac{1}{2L}(T_2 - T_1)$.
(c) $L = \frac{kt}{2}\left(\sqrt{T_2} + \sqrt{T_1}\right)$
(d) $L = \frac{kt}{2}\left(\sqrt{T_2} - \sqrt{T_1}\right)$

13. A whistle emitting a sound of frequency 440 Hz is tied to a string of length 1.5 m and rotated with an angular velocity of 20 rad s^{-1} in the horizontal plane. An observer is stationed at a large distance from the whistle. If the speed of sound is 330 ms^{-1},
(a) he will hear two sounds of frequencies 403 Hz and 440 Hz.
(b) he will hear two sounds of frequencies 484 Hz and 440 Hz.
(c) he will hear two sounds of frequencies 403 Hz and 480 Hz.
(d) he will hear a range of frequencies between 403 Hz and 480 Hz.

14. A wave is represented by the equation

$$y = A \sin (10\pi x + 15\pi t + \pi/3)$$

where x and y are in metre and t in second. The expression represents a wave
(a) travelling in the positive x-direction
(b) travelling in the negative x-direction
(c) of wavelength 0.2 m
(d) of velocity 1.5 ms^{-1}.

IIT, 1990

15. Two identical wires as a stretched with tensions T_1 and T_2 with $T_1 > T_2$. They produce 6 beats per second when vibrated. If the tension in one of them is changed slightly, it is observed that the beat frequency remains unchanged. Which of the following is/are possible?
(a) T_1 was increased (b) T_1 was decreased
(c) T_2 was increased (d) T_2 was decreased

IIT, 1991

16. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero propagates in a non-absorbing medium along the negative x-direction with a velocity of 300 ms^{-1}. Then
(a) Wavelength of the wave is 12 m
(b) The phase difference between the oscillations at two points 6 m apart is π
(c) The corresponding amplitude difference is 2.5×10^{-5} m
(d) The corresponding amplitude difference is zero.

IIT, 1997

17. The (x, y) co-ordinates of the corners of a square plate are (0, 0), (L, 0), (L, L) and (0, L). The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x, y)$ denotes the displacement of the plate at the point (x, y) at a certain instant of time, the possible expression(s) for $u(x, y)$ is/are (a = positive constant)
(a) $u(x, y) = a \cos\left(\frac{\pi x}{2L}\right) \cos\left(\frac{\pi y}{2L}\right)$
(b) $u(x, y) = a \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right)$
(c) $u(x, y) = a \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{2\pi y}{L}\right)$
(d) $u(x, y) = a \cos\left(\frac{2\pi x}{L}\right) \sin\left(\frac{\pi y}{2L}\right)$

IIT, 1998

18. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ ms^{-1}, then
(a) $\lambda = 2\pi \times 10^{-2}$ m (b) $\lambda = 10^{-3}$ m
(c) $f = \dfrac{10^3}{2\pi}$ Hz (d) $f = 10^4$ Hz

IIT, 1998

19. As a wave propagates in a non-absorbing medium,
(a) the wave intensity remains constant for a plane wave
(b) the wave intensity decreases as the inverse of the distance from the source for a spherical wave
(c) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(d) the total intensity of a spherical wave over a spherical surface centred at the source remains constant at all times.

IIT, 1999

20. A moving pulse is represented by the expression

$$y(x, t) = \frac{0.8}{[(4x + 5t)^2 + 5]}$$

where x and y are in metre and t in second. Then
(a) the pulse is moving in the + x direction
(b) in 2s it will travel a distance of 2.5 m
(c) its maximum displacement is 0.16 m
(d) it is a symmetric pulse.

IIT, 1999

21. In a wave motion $y = a \sin (kx - \omega t)$, y can represent
(a) electric field (b) magnetic field
(c) displacement (d) pressure

IIT, 1999

22. Standing waves can be produced
(a) on a string clamped at both the ends
(b) on a string clamped at one end and free at the other
(c) when an incident wave gets reflected from a wall
(d) when two incident waves with a phase difference of π are moving in the same direction.

IIT, 1999

23. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then,
(a) the intensity of the sound heard at the first resonance was more than that at the second resonance
(b) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube
(c) the amplitude of vibration of the ends of the prongs is typically around 1 cm
(d) the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air

IIT, 2009

ANSWERS AND SOLUTIONS

1. When a wave travelling in a medium falls on the boundary of another medium, it is partly reflected back into the first medium and partly refracted into the second medium. Therefore, the intensity (and hence the amplitude) of the reflected and refracted waves will be less than that of the incident wave.

The velocity of a wave depends upon the medium in which it travels. Hence the velocity of the reflected wave will be the same as that of the incident wave. But the velocity of the refracted wave will be different from that of the incident wave. The frequency of the reflected and refracted waves is always the same as that of the incident wave. From $v = \nu\lambda$, we have.

$$\lambda = \frac{v}{\nu}$$

Hence the wavelength of the reflected wave is the same as that of the incident wave. But the wavelength of the refracted wave will be different from that of the incident wave.

Furthermore, when a wave travelling in a rarer medium is reflected from the boundary of a denser medium, it undergoes a phase change of π or 180°. But if a wave is reflected from the boundary of a rarer medium, it does not undergo any phase change. The refracted wave, in both cases, does not undergo any phase change. Thus the correct choice are (a) and (d).

2. The correct choices are (a) and (b).

3. Statement (a) is correct. Let us write

$$y(x, t) = f(vt + x) = f(z)$$

Differentiating with respect to t, we have

$$\frac{\partial y}{\partial t} = \frac{\partial f}{\partial z}\frac{\partial z}{\partial t} = v\,\frac{\partial f}{\partial z}$$

Differentiating again w.r.t time t we have

$$\frac{\partial^2 y}{\partial t^2} = v^2\,\frac{\partial^2 f}{\partial z^2}$$

Similarly differentiating twice with respect to x we have

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 f}{\partial x^2}$$

Hence,

$$\frac{\partial^2 y}{\partial t^2} = v^2\,\frac{\partial^2 y}{\partial x^2}$$

which is the standard equation (in differential form) of a travelling wave.

Statement (b) is also correct. Because the wave is reflected back into the same medium, the velocity remains unchanged. The wavelength cannot change because the frequency cannot change by reflection.

Statement (c) is incorrect. The ultrasonic wave bends away from the normal because the speed of the wave (being a sound wave) is greater in water than in air.

Statement (d) is correct. The reason is that solids have a much higher modulus of elasticity than gases at STP.

4. Statement (a) is incorrect. A change in pressure has no effect on the speed of sound. The decrease in the speed of sound at high altitudes is due to a fall in temperature. Statement (b) is correct. Standing waves are produced due to superposition of the incident waves and the waves reflected from the fixed ends of the string. Since the ends are never perfectly rigidly fixed, the amplitude of the reflected wave is always less than that of the incident wave. Consequently, the resultant amplitude at nodes is not exactly zero. Thus the nodes are not well-defined.

Statement (c) is also correct. To observe beats the difference between the two interfering frequencies must be less than about 10–16 Hz. Since visible light waves have very high frequencies, beats are not observed due to persistence of vision.

Statement (d) is correct. We know that

$$\nu_1 = \frac{\nu}{1 - \frac{u}{v}} \qquad \text{(i)}$$

and

$$\nu_2 = \nu\left(1 + \frac{u}{v}\right) \qquad \text{(ii)}$$

Expression (i) may be written as

$$\nu_1 = \nu\left(1 - \frac{u}{v}\right)^{-1}$$

Expanding binomially and retaining terms upto order u^2/v^2, we have

$$\nu_1 = \nu\left(1 + \frac{u}{v} + \frac{u^2}{v^2}\right) \qquad \text{(iii)}$$

Comparing (ii) and (iii) we find that $\nu_1 > \nu_2$.

5. A stationary wave is characterized by a function of the type $y = f(t)\,g(x)$. Hence choices (a) and (b) represent a stationary wave. Choice (d) is a superposition of two oppositely travelling waves of the same amplitude and the same frequency which gives rise to a stationary wave. Hence choice (d) also represents a stationary wave.

6. Comparing $y = a \sin 2\pi\,(bt - cx)$ with

$$y = a \sin\left\{\frac{2\pi}{\lambda}(vt - x)\right\}$$

we have

$$2\pi b = \frac{2\pi v}{\lambda} \text{ and } 2\pi c = \frac{2\pi}{\lambda}$$

which give $v = b\lambda$ and $\lambda = \frac{1}{c}$. Thus $v = b/c$.

Particle velocity is

$$V = \frac{dy}{dt} = \frac{d}{dt}\,[a \sin 2\pi\,(bt - cx)]$$

$$= 2\pi ab \cos 2\pi\,(bt - cx)$$

$\therefore$ $V_{\max} = 2\pi ab$. Now $V_{\max} = 2v$, if

$$2\pi ab = \frac{2b}{c}$$

which gives $c = \frac{1}{\pi a}$.

Frequency $\nu = \frac{v}{\lambda} = \frac{b/c}{1/c} = b$

Thus the correct choices are (b), (c) and (d).

7. Person A hears the sound of his own source whose frequency is ν. He also hears the sound of the source carried by person B, towards whom he is moving with a speed u. The apparent frequency of this sound is given by

$$\nu' = \nu\left(1 + \frac{u}{v}\right) \text{ or } \nu' - \nu = \frac{\nu u}{v}$$

$\therefore$ Beat frequency $\nu_b = \nu' - \nu = \frac{\nu u}{v} = \frac{90v}{10v}$

$= 9$ Hz $\quad (\because u = v/10)$.

Person B hears the sound of his own source of frequency in v. He also hears the sound of the source carried by person A, who is approaching with a speed u. The apparent frequency of this sound is given by

$$v'' = \frac{v}{1 - \frac{u}{v}} = \frac{vv}{v \pm u}$$

or $$v'' - v = v\left[\frac{v}{v-u} - 1\right] = \frac{vu}{v-u}$$

$\therefore$ Beat frequency $v_b = v'' - v = \dfrac{vu}{v-u}$

$$= \frac{90 \times v/10}{v - v/10} = 10 \text{ Hz}$$

Hence the correct choices are (a) and (d).

8. $$420 = \frac{P}{2L}\sqrt{\frac{T}{m}} \quad (1)$$

and $$490 = \frac{p+1}{2L}\sqrt{\frac{T}{m}} \quad (2)$$

Dividing (1) and (2) we have

$$\frac{490}{220} = \frac{p+1}{p} \text{ giving } p = 6.$$

Substituting this value of p in Eq. (1) we get

$$420 = \frac{p}{2L}\sqrt{\frac{T}{m}} = \frac{600}{L}$$

which gives $L = \dfrac{10}{7}$ m. Hence the correct choices are (a) and (d).

9. The fundamental frequencies of the open and closed pipes respectively are

$$v_0 = \frac{v}{2L_0} \quad (1)$$

and $$v_c = \frac{v}{4L_c} \quad (2)$$

where v is the speed of sound.

In an open pipe, all harmonics are present. Hence the frequencies of the overtones are 2, 3, 4, ... etc. times the fundamental frequency. Hence the frequency of the first overtone in the open pipe is

$$v_1 = 2v_0 = \frac{2v}{2L_0} = \frac{v}{L_0} \quad (3)$$

In a closed pipe, only odd harmonics are present. Hence, the frequencies of the overtones are 3, 5, 7, ... etc. times the fundamental frequency. Hence the frequency of the first overtone in the closed pipe is

$$v_2 = 3v_c = \frac{3v}{4L_c} \quad (4)$$

Now $$110 = \frac{330}{4L_c} \quad \Rightarrow \quad L_c = 0.75 \text{ m}.$$

Given $v_1 - v_2 = \pm 10$. There are the following two possibilities.

Case a : $v_1 - v_2 = 10$. Thus

$$\frac{v}{L_o} - \frac{3v}{4L_c} = 10$$

Putting $v = 330 \text{ ms}^{-1}$ and $L_c = 0.75$ m, we get

$$L_o = \left(\frac{33}{34}\right) \text{ m}$$

Case (b) : $v_1 - v_2 = -10$. In this case, we get

$$L = \left(\frac{33}{32}\right) \text{ m}$$

Hence the correct choices are (a) and (d).

10. Including end correction, we have for the fundamental mode,

$$\frac{\lambda}{4} = L + 0.3\,D \quad \text{or} \quad \lambda = 4\,(L + 0.3\,D)$$

where D is the diameter of the tube. Now $v = \nu\lambda$

$\therefore$ $$\lambda = \frac{v}{\nu}$$

or $$4\,(L + 0.3\,D) = \frac{v}{\nu} \quad \text{(i)}$$

Given $L = 16$ cm $= 0.16$ m, $v = 336 \text{ ms}^{-1}$ and $\nu = 480$ Hz. Using these values in (i) and solving we get $D = 5 \times 10^{-2}$ m $= 5$ cm. The correct choice is (b).

11. The correct choice is (a).

12. Since $v = \dfrac{dx}{dt}$, $dx = vdt = k\sqrt{T}\ dt$. As the temperature increases linearly, the rate of change of temperature with distance is given by

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

where L is the distance between locations A and B. Thus

$$dx = \frac{LdT}{T_2 - T_1}$$

But $dx = k\sqrt{T}\ dt$. Therefore

$$k\sqrt{T}\ dt = \frac{LdT}{T_2 - T_1}$$

or $$dt = \frac{LdT}{k(T_2 - T_1)\sqrt{T}}$$

Integrating from $T = T_1$ to $T = T_2$, we have

$$t = \frac{L}{k(T_2 - T_1)} \int_{T_1}^{T_2} T^{-1/2}$$

$$t = \frac{L}{k(T_2 - T_1)} \left| \frac{T^{1/2}}{1/2} \right|_{T_1}^{T_2}$$

$$= \frac{2L}{k(T_2 - T_1)} \left(\sqrt{T_2} - \sqrt{T_1} \right)$$

$$= \frac{2L}{k\left(\sqrt{T_2} + \sqrt{T_1} \right)}$$

Thus the correct choices are (a) and (c).

13. Linear velocity of source (whistle) is $u_s = r\omega = 1.5 \times 20 = 30$ ms^{-1}. The observer will hear a range of frequencies lying between a minimum value ν_{min} and a maximum value ν_{max}, which are given by

$$\nu_{min} = \nu \left(\frac{v}{v + u_s} \right); \text{ source receding}$$

and $$\nu_{max} = \nu \left(\frac{v}{v - u_s} \right); \text{ source approaching}$$

Substituting $\nu = 440$ Hz, $v = 330$ ms^{-1} and $u_s = 30$ ms^{-1} and solving we get $\nu_{min} = 403.3$ Hz and $\nu_{max} = 484$ Hz. The correct choice is (d).

14. The standard equation of a wave travelling in the negative x-direction is

$$y = A \sin(\omega t + kx + \phi_0)$$

where $\omega = \frac{2\pi v}{\lambda}$, $k = \frac{2\pi}{\lambda}$ and ϕ_0 is the phase at $x = 0$ and $t = 0$.

Comparing the given equation with this equation, we have

$$k = 10\pi \Rightarrow \frac{2\pi}{\lambda} = 10\pi \Rightarrow \lambda = 0.2 \text{ m}$$

and $$\omega = 15\pi \Rightarrow \frac{2\pi v}{\lambda} = 15\pi$$

$$\Rightarrow \quad v = \frac{15\lambda}{2} = \frac{15 \times 0.2}{2} = 1.5 \text{ ms}^{-1}$$

Thus the correct choices are (b), (c) and (d).

15. $\nu = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$. Since the wires are identical, $\nu \propto \sqrt{T}$. Since $T_1 > T_2$; $\nu_1 > \nu_2$. Also $\nu_1 - \nu_2 = 6$.

If T_1 is kept constant and T_2 is increased such that the new value ν_2 becomes greater than ν_1 by 6, again 6 beats will be heard per second. Hence choice (c) is correct.

If T_2 is kept constant and T_1 is decreased such that the new value of ν_1 becomes less than ν_2 by 6, again 6 beats will be heard per second. Hence choice (b) is also correct.

16. $\lambda = \frac{v}{\nu} = \frac{300}{25} = 12$ m

Phase difference $\Delta\phi = \frac{2\pi}{\lambda} \times$ path difference

$$= \frac{2\pi}{12} \times 6 = \pi$$

In a non-absorbing medium, the amplitude of the wave remains constant as the wave propagates. Thus the correct choices are (a), (b) and (d).

17. The expression for $u(x, y)$ must satisfy the following boundary conditions:

(i) $u = 0$ at $x = 0$ and at $y = 0$

(ii) $u = 0$ at $x = L$ and at $y = L$.

The choices (b) and (c) satisfy these conditions.

18. $y = a \sin(\omega t - kx)$

Speed at a point on the string is

$$V = \frac{dy}{dt} = a\omega \cos(\omega t - kx)$$

$\therefore$ $V_{max} = a\omega = 2\pi af$. Given $V_{max} = \frac{v}{10}$.

Hence

$$\frac{v}{10} = 2\pi af \Rightarrow f \frac{v}{20\pi a} = \frac{10}{20\pi \times 10^{-3}}$$

$$= \frac{10^3}{2\pi} \text{ Hz}$$

$$\lambda = \frac{v}{f} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2} \text{ m}$$

So the correct choices are (a) and (c).

19. The amplitude (and hence intensity) of a plane wave remains constant as the wave propagates in a non-absorbing medium. For a spherical wave, the same energy crosses a spherical surface of area $4\pi r^2$ where r is the distance from the source. The wave intensity is defined as the energy crossing per unit area per second. Hence, for a spherical wave, the intensity decreases as $1/r^2$. But the total intensity spread over the spherical surface is the same at all times. Hence the correct choices are (a), (c) and (d).

20. The displacement $y(x, t)$ is maximum when the denominator is minimum, i.e.

$[(4x + 5t)^2 + 5]$ is minimum. Its minimum value is 5 when $(4x + 5t)^2 = 0$ or $4x + 5t = 0$

or $$\frac{x}{t} = -\frac{5}{4} \Rightarrow v = -\frac{5}{4} \text{ ms}^{-1}$$

$$\therefore \quad y_{max} = \frac{0.8}{5} = 0.16 \text{ m}$$

Distance travelled by pulse in $t = 2$ s is

$$vt = -\frac{5}{4} \times 2 = -2.5 \text{ m}$$

The negative sign shows that the pulse is travelling in negative x-direction. Since y is not a symmetric function of x, the form of the pulses changes as it travels. Hence the correct choices are (b) and (c).

21. In an electromagnetic wave, y represents electric field and magnetic field. These fields oscillate perpendicular to each other as well as perpendicular to the direction of propagation of the wave. In a mechanical wave, y represents displacement. In a sound wave, y represents pressure. Thus all the four choices are correct.

22. Standing waves are produced due to a superposition of an incident wave and a reflected wave. In case (a) the incident wave is reflected from the clamped end of the string. In case (c) the incident wave is reflected from the wall. Two waves traveling in the same direction cannot produce standing waves; they give rise to interference. Thus the correct choices are (a), (b) and (c).

23. At first resonance, the frequency of the fundamental mode (first harmonic) equals the frequency of the tuning fork. At second resonance, the frequency of the third harmonic equals the frequency of the tuning fork. As the amplitude of oscillation of the fundamental mode is the highest, the intensity of sound heard at the first resonance is the highest. Hence choice (a) is correct. Choice (b) is wrong, the prongs are kept in a vertical plane. Choice (c) is also incorrect as the amplitude of vibration of the ends of the prongs is typically around 1 mm. Choice (d) is correct. Due to end-correction (= e), $(L_1 + e) = \lambda/4$ at first resonance. Thus the correct choices are (a) and (d).

SECTION III

Multiple Choice Questions Based On Passage

Questions 1 to 3 are based on the following passage

Passage I

When two sound waves travel in the same direction in a medium, the displacements of a particle located at x at time t is given by

$$y_1 = 0.05 \cos (0.50 \pi x - 100 \pi t)$$

and $$y_2 = 0.05 \cos (0.46 \pi x - 92 \pi t)$$

where y_1, y_2 and x are in metre and t in second

IIT, 2006

1. What is the speed of sound in the medium?
(a) 332 ms^{-1} (b) 100 ms^{-1}
(c) 92 ms^{-1} (d) 200 ms^{-1}

2. How many times per second does an observer hear the sound of maximum intensity?
(a) 4 (b) 8
(c) 12 (d) 16

3. At $x = 0$, how many times between $t = 0$ and $t = 1$ s does the resultant displacement become zero?
(a) 46 (b) 50
(c) 92 (d) 100

SOLUTION

1. The two displacements can be written as

$$y_1 = A \cos (k_1 x - \omega_1 t) \quad (1)$$

and $$y_2 = A \cos (k_2 x - \omega_2 t) \quad (2)$$

where $A = 0.05$ m, $k_1 = \frac{2\pi}{\lambda_1} = 0.50\pi \text{ m}^{-1}$,

$\omega_1 = 2\pi \nu_1 = 100\pi \text{ rad s}^1$, $k_2 = \frac{2\pi}{\lambda_2} = 0.46\pi \text{ ms}^{-1}$ and $\omega_2 = 2\pi\nu_2 = 92\ \pi \text{ rad s}^{-1}$. The speed of either wave is

$$v_1 = \nu_1\lambda_1 = 2\pi\nu_1 \times \frac{\lambda_1}{2\pi} = \frac{\omega_1}{k_1}$$

$$= \frac{100\pi}{0.50\pi} = 200 \text{ ms}^{-1}$$

or $v_2 = \nu_2\lambda_2 = \dfrac{\omega_2}{k_2} = \dfrac{92\pi}{0.46\pi} = 200 \text{ ms}^{-1}$.

Hence the correct choice is (d).

2. Beat frequency = $\nu_1 - \nu_2$. Now

$$\nu_1 = \frac{\omega_1}{2\pi} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

and $$\nu_2 = \frac{\omega_2}{2\pi} = \frac{92\pi}{2\pi} = 46 \text{ Hz}$$

$\therefore$ Beat frequency = 50 – 46 = 4 Hz. Hence the correct choice is (a).

3. The resultant displacement is given by

$$y = y_1 + y_2$$

$$= A\cos(k_1x - \omega_1t) + A\cos(k_2x - \omega_2t)$$

For $x = 0$, we have

$$y = A\cos\omega_1t + A\cos\omega_2t$$

$$= 2A\cos\left\{\frac{1}{2}(\omega_1 + \omega_2)t\right\}\times \cos\left\{\frac{1}{2}(\omega_1 - \omega_2)t\right\}$$

$\therefore$ $y = 0.10\cos(96\,\pi t)\cos(4\,\pi t)$

Between $t = 0$ and $t = 1$ s, $\cos(96\,\pi t)$ becomes zero 96 times and $\cos(4\pi t)$ becomes zero 4 times. Hence the resultant displacement y at $x = 0$ becomes zero 100 times between $t = 0$ and $t = 1$ s. The correct choice (d).

Questions 4 to 6 are based on the following passage

Passage II

A string 25 cm long and having a mass of 2.5 g is under of tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second heard. It is observed that decreasing the tension in the string decreases the beat frequency. The speed of sound in air is 320 ms^{-1}.

IIT, 1982

4. The frequency of the fundamental mode of the closed pipe is
(a) 100 Hz (b) 200 Hz
(c) 300 Hz (d) 400 Hz

5. The frequency of the string vibrating in its first overtone is
(a) 92 Hz (b) 108 Hz
(c) 192 Hz (d) 208 Hz

6. The tension in the string is very nearly equal to
(a) 25 N (b) 27 N
(c) 28 N (d) 30 N

SOLUTION

4. Frequency of fundamental mode of the closed pipe is

$$n_p = \frac{v}{4L} = \frac{320}{4\times 0.40} = 200 \text{ Hz}$$

The correct choice is (b).

5. Since the beat frequency is 8, the frequency of the string vibrating in its first overtone is

$n_s = n_p \pm 8 = 200 \pm 8 = 192$ Hz or 208 Hz

where for first overtone, $n_s = \dfrac{1}{l}\sqrt{\dfrac{T}{m}}$ (1)

It is given that the beat frequency decreases if the tension in the string is decreased. As the frequency decreases with decrease of tension, it is obvious that $n_s > n_p$. Hence

$n_s = 208$ Hz and not 192 Hz.

Thus the correct choice is (d).

6. Substituting the values of l, m and n_s in Eq. (1), we get $T = 27.04$ N. Hence the correct choice is (b).

Questions 7 to 11 are based on the following passage

Passage III

The vibrations of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\,\pi t)$$

where x and y are in cm and t in second.

IIT, 1985

7. The frequency of vibrations of the string is
(a) 48 Hz (b) 50 Hz
(c) 96 Hz (d) 100 Hz

8. The maximum displacement of a point at $x = 10$ cm is
(a) 2 cm (b) 4 cm
(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

9. How many nodes are formed on the string?
(a) 2 (b) 3
(c) 4 (d) 5

10. In which harmonic mode is the string vibrating?
(a) Fundamental (b) third
(c) fourth (d) fifth

11. The velocity of the particle at $x = 7.5$ cm at $t = 0.25$ s is
(a) zero (b) 320 ms^{-1}
(c) 60 ms^{-1} (d) 96 ms^{-1}

SOLUTION

7. $2\pi\nu = 96\pi \rightarrow \nu = 48$ Hz. Thus the correct choice is (a)

8. Displacement is maximum when $\cos(96\pi t) = 1$.

At $x = 10$ cm, $y_{max} = 4 \sin\left(\frac{\pi \times 10}{15}\right) = 4 \sin\left(\frac{2\pi}{3}\right)$

$= 2\sqrt{3}$ cm

The correct choice is (c).

9. At nodes the displacement is always zero. Hence nodes are located at values of x given by

$$\sin\left(\frac{\pi x}{15}\right) = 0 \text{ or } \frac{\pi x}{15} = p\pi$$

where $p = 0, 1, 2, 3, \ldots$ etc.
Thus $x = 15\ p = 0, 15, 30, 45$ and 60 cm.
Thus the correct choice is (d).

10. The correct choice is (c) because 5 nodes are formed on the string.

11. The velocity of the string at a point x at time t is obtained by differentiating

$$y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\pi t)$$

which respect to t.

$$\text{Velocity } \frac{dy}{dt} = -(4 \times 96\pi) \times \sin\left(\frac{\pi x}{15}\right) \sin(96\pi t)$$

At $x = 7.5$ cm and $t = 0.25$ s, the velocity is zero because at $t = 0.25$ s, $\sin(96\pi t) = \sin(24\pi) = 0$.
Hence the correct choice is (a).

Questions 12 to 15 are based on the following passage

Passage IV

The displacement of the medium in a sound wave is given by

$$y_1 = A \cos(ax + bt)$$

where A, a and b are positive constants. The wave is reflected by an obstacle situated at $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave.

IIT, 1991

12. The wavelength and frequency of the incident wave respectively are

(a) $\frac{2\pi}{a}, \frac{b}{2\pi}$ (b) $\frac{a}{2\pi}, \frac{2\pi}{b}$

(c) $\frac{2}{a}, \frac{b}{2}$ (d) $a, \frac{1}{b}$

13. The equation for the reflected wave is
(a) $y_2 = 0.8\ A \cos(-ax + bt)$
(b) $y_2 = -A \cos(-ax + bt)$
(c) $y_2 = -0.64\ A \cos(-ax + bt)$
(d) $y_2 = -0.8\ A \cos(-ax + bt)$

14. In the standing wave formed due to the superposition of the incident and reflected waves, the maximum values of the particle speed in the medium is
(a) Ab (b) 1.64 Ab
(c) 1.8 Ab (d) 2 Ab

15. The minimum value of the particle speed in the medium is
(a) zero (b) 0.2 Ab
(c) 0.64 Ab (d) 0.8 Ab

SOLUTION

12. The incident wave is given by

$$y_1 = A \cos(ax + bt) \quad (1)$$

The standard wave equation is

$$y = A \cos(kx + \omega t) \quad (2)$$

where k is the wave number and ω, the angular frequency. Comparing (1) and (2) we get $k = a$ and $\omega = b$. Hence wavelength $\lambda = \frac{2\pi}{k} = \frac{2\pi}{a}$ and frequency $\nu = \frac{\omega}{2\pi} = \frac{b}{2\pi}$.

The correct choice is (a).

13. Since the intensity of the reflected wave is 0.64 times that of the incident wave, the amplitude A_r of

the reflected wave will be $\sqrt{0.64} = 0.8$ times that of the incident wave, i.e.

$$A_r = 0.8\,A$$

Now, when a wave is reflected by an obstacle, it suffers a reversal of amplitude (which implies a phase change of π radian), i.e. $A_r = -0.8\,A$. Since the incident wave is travelling in the negative x direction, the reflected wave will travel in the positive x direction. Therefore, the equation of the reflected wave can be obtained from Eq. (1) by replacing A by $A_r = -0.8\,A$ and x by $-x$. Thus, the reflected wave is given by

$$y_2 = -0.8\,A\cos(-ax + bt) \qquad (3)$$

Thus the correct choice is (d).

14. Differentiating Eq. (1) will respect to time t, we get the expression for the particle speed in the medium due to the incident wave, which is

$$V_1 = \frac{dy_1}{dt} = \frac{d}{dt}\{A\cos(ax + bt)\}$$

$$= -Ab\,(ax + bt) \qquad (4)$$

$\therefore$ Maximum particle speed due to the incident wave is

$$(V_1)_{max} = Ab$$

Differentiating Eq. (3) with respect to t, the particle speed due to the reflected wave is given by

$$V_2 = \frac{dy_2}{dt}$$

$$= \frac{d}{dt}\,[-0.8\,A\cos(bt - ax)]$$

$$= 0.8\,Ab\sin(bt - ax)$$

$$\therefore \quad (V_2)_{max} = 0.8\,Ab$$

From the superposition principle, the maximum particle speed in the medium due to both the incident and reflected waves is given by the algebraic sum of the individual maximum particle speeds, i.e.

$$V_{max} = (V_1)_{max} + (V_2)_{max}$$

$$= Ab + 0.8\,Ab = 1.8\,Ab$$

So the correct choice is (c).

15. Since A and b are positive constants, the minimum particle speed is

$$V_{min} = (V_1)_{max} - (V_2)_{max}$$

$$= Ab - 0.8\,Ab = 0.2\,Ab$$

The correct choice is (b).

Questions 16 to 19 are based on the following passage

Passage V

The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 400 Hz. The speed of sound in air is 320 ms^{-1}. The end correction may be neglected. Let P_0 denote the mean pressure at any point in the pipe and ΔP_0 the maximum amplitude of pressure variation.

IIT, 1998

16. The length L of the air column is

(a) 20 cm (b) 60 cm

(c) 1.0 m (d) 1.4 m

17. The amplitude of pressure variation at the middle of the air column is

(a) ΔP_0 (b) $2\Delta P_0$

(c) $\sqrt{2}\,\Delta P_0$ (d) $\frac{\Delta P_0}{\sqrt{2}}$

18. The maximum and minimum pressures at the open end of the pipe respectively are

(a) $P_0 + \Delta P_0,\ P_0 - \Delta P_0$

(b) $P_0 + \Delta P_0,\ P_0$

(c) $P_0,\ P_0 - \Delta P_0$

(d) $P_0,\ P_0$

19. The maximum and minimum pressures at the closed end of the pipe respectively are

(a) $P_0 + \Delta P_0,\ P_0 - \Delta P_0$

(b) $P_0 + \Delta P_0,\ P_0$

(c) $P_0,\ P_0 - \Delta P_0$

(d) $P_0,\ P_0$

SOLUTION

16. Figure 14.13 shows the longitudinal displacement y as a function of x for x lying between $x = 0$ and $x = L$, where L is the length of the pipe.

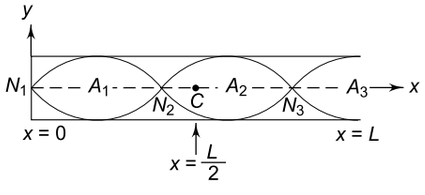


Fig. 14.13

The fundamental frequency of a closed pipe is given by

$$\nu = \frac{v}{4L}$$

In a closed pipe, only odd harmonics are present, i.e. the frequency ν_1 of the first overtone is 3 times the fundamental frequency, that of the second overtone ν_2 is 5 times the fundamental frequency and so on. Thus

$$\nu_2 = 5\nu = \frac{5v}{4L}$$

$$\Rightarrow \qquad L = \frac{5v}{4\nu_2} = \frac{5\times 320}{4\times 400} = 1.0 \text{ m}$$

The correct choice is (c).

17. Since the distance between two consecutive nodes is $\frac{\lambda}{2}$ and that between a node and the next antinode is $\frac{\lambda}{4}$, it follows from Fig. 14.7 that

$$L = \frac{\lambda}{2}+\frac{\lambda}{2}+\frac{\lambda}{4} = \frac{5\lambda}{4}$$

We know that the pressure variation (excess pressure) is maximum at a node and minimum (equal to zero) at an antinode. Therefore, the pressure variation at a distance x from a node is given by

$$\Delta P = \Delta P_0 \cos\left(\frac{2\pi x}{\lambda}\right) \qquad (1)$$

The centre C of the tube is at $x = \frac{L}{2}$ or $x = \frac{5\lambda}{8}$ $\left(\text{since } L = \frac{5\lambda}{4}\right)$. The second node N_2 is at $x = \frac{\lambda}{2}$. Therefore, the distance of C from N_2 is $x = \frac{5\lambda}{8} - \frac{\lambda}{2} = \frac{\lambda}{8}$. Using this value of x in Eq. (1) we have

$$\Delta P \text{ at } C = \Delta P_0 \cos\left(\frac{2\pi}{\lambda}\times\frac{\lambda}{8}\right)$$

$$= \Delta P_0 \cos\frac{\pi}{4} = \frac{\Delta P_0}{\sqrt{2}}$$

Thus the correct choice is (d).

18. At an antinode, the pressure variation is zero, i.e. $\Delta P_0 = 0$. Hence, at an antinode

$$P_{max} = P_{min} = P_0$$

So the correct choice is (d).

19. At a node, the pressure variation is maximum equal to ΔP_0. Hence, at a node

$$P_{max} = P_0 + \Delta P_0 \text{ and}$$

$$P_{min} = P_0 - \Delta P_0$$

Thus the correct choice is (a).

Questions 20 to 22 are based on the following passage

Passage VI

A wire of mass 9.8×10^{-3} kg per metre passes over a frictionless pulley fixed at the top of an inclined frictionless plane which makes an angle of 30° with the horizontal. Two masses M_1 and M_2 are tied at the two ends of the wire. Mass M_1 rests on the inclined plane and mass M_2 hangs freely vertically downwards. The whole system is in equilibrium. Now a transverse wave propagates along the wire with a speed of 100 ms^{-1}.

20. The tension T in the wire is

(a) 0.98 N (b) 9.8 N
(c) 98 N (d) 980 N

21. The value of mass M_1 is

(a) 2 kg (b) 5 kg
(c) 10 kg (d) 20 kg

22. The value of mass M_2 is

(a) 5 kg (b) 10 kg
(c) 15 kg (d) 20 kg

SOLUTION

20. Refer to Fig. 14.14. Let T be the tension in the string when the system is in equilibrium.

It follows from the figure that, at the equilibrium position, the component $M_1 g \cos\theta$ of weight $M_1 g$ balances with the normal reaction N and the other component $M_1 g \sin\theta$ will balance with tension T in the string. Also weight $M_2 g$ of mass M_2 will balance with tension T. Thus

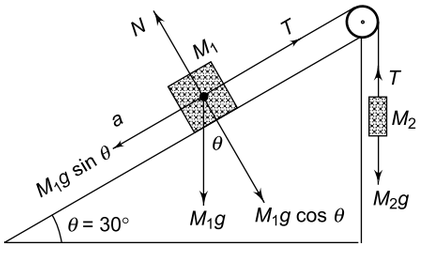


Fig. 14.14

$$M_1 g \sin\theta = T \qquad (1)$$

and $$M_2 g = T \qquad (2)$$

Now, the speed of a transverse wave in a wire of mass m per unit length and stretched with a tension T is given by

$$v = \sqrt{\frac{T}{m}}$$

or $$T = v^2 m \qquad (3)$$

Given $m = 9.8 \times 10^{-3}$ kg m^{-1} and $v = 100$ ms^{-1}. Using these values in Eq. (3), we have

$$T = (100)^2 \times 9.8 \times 10^{-3} = 98 \text{ N}$$

21. From Eq. (1), we get

$$M_1 = \frac{T}{g \sin\theta} = \frac{98}{9.8 \times \sin 30°} = 20 \text{ kg}$$

Thus the correct choice is (d).

22. From Eq. (2), we have

$$M_2 = \frac{T}{g} = \frac{98}{9.8} = 10 \text{ kg, which is choice (b).}$$

Questions 23 to 25 are based on the following passage

Passage VII

A source of sound of frequency 90 Hz is moving towards a wall with a speed $u = v/10$, where v is the speed of sound in air.

IIT, 1981

23. The beat frequency of the sound heard by an observer between the wall and the source is

(a) 20 Hz (b) 10 Hz
(c) 5 Hz (d) zero

24. The beat frequency of the sound heard by an observer behind the source is

(a) $\frac{200}{9}$ Hz (b) 20 Hz

(c) $\frac{200}{11}$ Hz (d) zero

25. The beat frequency of the sound heard by the observer moving with the source is

(a) 11 Hz (b) 9.9 Hz
(c) 10 Hz (d) 20 Hz

SOLUTION

23. The observer hears two sounds–one coming directly from the approaching source and the other sound after reflection from the wall (which can be considered as coming from the mirror image of the source). The apparent frequency of the approaching source is

$$\nu' = \nu\left(\frac{v}{v-u_s}\right) = 90 \times \left(\frac{v}{v-v/10}\right) = 100 \text{ Hz}$$

When the observer is between the wall and the source, the apparent frequency of the sound reflected from the wall is also ν'. Therefore, frequency of beats = $\nu' - \nu' = 0$. The observer will not hear any beats.

So the correct choice is (d).

24. When the observer is behind the source, i.e. when the source is between the wall and the observer, the apparent frequency of the sound coming directly from the receding source is

$$\nu'' = \nu\left(\frac{v}{v+u_s}\right) = 90 \times \left(\frac{v}{v+v/10}\right) = \frac{900}{11} \text{ Hz}$$

$$\therefore \quad \text{Beat frequency} = \nu' - \nu'' = 100 - \frac{900}{11}$$

$$= \frac{200}{11} \text{ Hz.}$$

Hence the correct choice is (c).

25. If the observer is moving with the source, the frequency of the direct sound is $\nu = 100$ Hz. The apparent frequency of the reflected sound is

$$\nu''' = \nu\left(\frac{v+u_s}{v-u_s}\right)$$

$$= 90 \times \left(\frac{v+v/10}{v-v/10}\right) = 110 \text{ Hz}$$

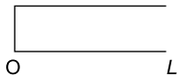
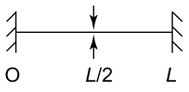
$\therefore$ Beat frequency = $\nu''' - \nu = 110 - 90 = 20$ Hz

So the correct choice is (d).

SECTION IV

Matrix Match Type

43. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves.

Column I	***Column II***
(a) Pipe closed at one end 	(p) Longitudinal waves
(b) Pipe open at both ends	(q) Transverse waves
(c) Stretched wire clamped at both ends	(r) $\lambda_f = L$
(d) Stretched wire clampeed at both ends and at mid-point 	(s) $\lambda_f = 2L$
	(t) $\lambda_f = 4L$

IIT, 2011

SOLUTION

(a) The closed end of a pipe is a node and the open end is an antinode. The distance between a node and the next antinode is $\lambda/4$. Hence $\lambda_f = 4L$. In pipes, the standing waves are due to superposition of oppositely travelling sound (longitudinal) waves.
$\therefore$ (a) $\rightarrow$ (p, t)

(b) Here $L = \dfrac{\lambda_f}{2} \Rightarrow \lambda_f = 2L$ ($\because$ distance between consecutive antinodes = $\lambda/2$)
(b) $\rightarrow$ (p, s)

(c) For a string fixed at both ends, each end is a node. Since the distance between two consecutive nodes is $\lambda/2$, $\lambda_f = 2L$. Also standing waves on a string are due to superposition of transverse waves.
$\therefore$ (c) $\rightarrow$ (q, s)

(d) Here the mid point is a node. Hence $\dfrac{L}{2} = \dfrac{\lambda_f}{2} \Rightarrow \lambda_f = L$
$\therefore$ (d) $\rightarrow$ (q, r)

ANSWERS

(a) $\rightarrow$ (p, t) (b) $\rightarrow$ (p, s) (c) $\rightarrow$ (q, s) (d) $\rightarrow$ (q, r)

SECTION V

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**
   Only longitudinal mechanical waves can propagate in gases.
   Statement-2
   Gases have only bulk modulus.
2. **Statement-1**
   Two sound waves of equal intensity I produced beats. The maximum intensity of sound produced in beats is $4I$.
   Statement-2
   If two waves of amplitudes a_1 and a_2 superpose, the maximum amplitude of the resultant wave = $a_1 + a_2$.
3. **Statement-1**
   A medium must possess elasticity in order to support wave motion.
   Statement-2
   Restoring force does not exist in a medium which does not have elasticity.
4. **Statement-1**
   Solids can support both longitudinal and transverse mechanical waves but only longitudinal mechanical waves can propagate in gases.
   Statement-2
   Gases do not have shear modulus.
5. **Statement-1**
   In standing sound waves, a displacement node is a pressure antinode and vice versa.
   Statement-2
   In a standing wave, the restoring force is the maximum at a node and minimum at an antinode.
6. **Statement-1**
   Our ears cannot distinguish two notes, one produced by a violin and other by a sitar, if they have exactly the same intensity and the same frequency.
   Statement-2
   When a musical instrument is played, it produces a fundamental note which is accompanied by a number of overtones called harmonics.
7. **Statement-1**
   Doppler's effect does not occur in case of a supersonic source.
   Statement-2
   A supersonic source produceds a shock wave.
8. **Statement-1**
   If a source of sound moves always from a stationary observer, the apparent frequency of sound as heard by the observer is greater than the actual frequency.
   Statement-2
   The cause of the apparent change in frequency is the change in the wavelength brought about by the motion of the source.
9. **Statement-1**
   If an observer moves towards a stationary source of sound, the frequency of the sound as heard by him is greater than the actual frequency.
   Statement-2
   The apparent increase in frequency is due to the fact that the observer intercepts more waves per second when the moves towards the source.
10. **Statement-1**
    If a source of sound is in motion and the observer is stationary, the speed of sound relative to him remains unchanged.
    Statement-2
    The apparent change in frequency is due to the change in the wavelength brought about by the motion of the source.
11. **Statement-1**
    If the observer is in motion and the source of sound is stationary, the speed of sound relative to him is changed.

Statement-2

The wavelength of sound received by the observer does not change due to his motion.

12. Statement-1

The apparent frequency is not the same in the following two cases– (i) source approaching a stationary observer with a certain velocity and (ii) observer approaching a stationary source of sound with the same velocity.

Statement-2

The cause of the apparent change in the frequency is different in the two cases.

SOLUTION

1. The correct choice is (a). Gases cannot withstand a shearing stress or longitudinal stress. Hence they do not have shear modulus and Young's modulus; they have only bulk modulus.

2. The correct choice is (a). When two waves of amplitudes a_1 and a_2 superpose to produce beats, the resultant amplitude of the maximum of intensity is

$$A = a_1 + a_2$$

Now, intensity $\propto$ (amplitude)^2. Since the two waves have the same intensity, their amplitudes are equal, i.e. $a_1 = a_2 = a$. Thus $A = 2a$. Therefore, $A^2 = 4a^2$ or $I_{max} = 4I$.

3. The correct choice is (a).

4. The correct choice is (a). Gases cannot withstand a shearing stress. Hence gases do not have any shear modulus; they have only bulk modulus. Solids have Young's modulus, bulk modulus and shear modulus. Therefore, solids can support both transverse and longitudinal waves.

5. The correct choice is (c).

6. The correct choice is (d). When a musical instrument is played, it produced a fundamental note which is accompanied by a number of overtones called harmonics. The number of harmonics is not the same for all instruments. It is the number of harmonics which distinguishes the note produced by a sitar and that produced by a violin.

7. The correct choice is (a). If the source of sound is moving at a speed greater than the speed of sound, then in a given time the source advances more than the wave. The resultant wave motion is a conical wave called a shock wave which produces a sudden and violent sound.

8. The correct choice is (d).

9. The correct choice is (a).

10. The correct choice is (a).

11. The correct choice is (a).

12. The correct choice is (a). In case (i) the speed of sound relative to the observer remains unchanged; the change in frequency is due to a change in wavelength brought about by the motion of the source. In case (ii) the wavelength of sound remains unchanged; the change in frequency is due to a change in the speed of sound relative to the observer.

SECTION VI

Integer Answer Type

1. An ambulance sounding a horn of frequency 256 Hz is moving towards a vertical wall with a velocity of 5 ms^{-1}. If the speed of sound is 330 ms^{-1}, how many beats per second will be heard by an observer standing a few metres behind the ambulance?

IIT, 1981

2. A steel wire of length 1 m, mass 0.1 kg and uniform cross-sectional area 10^{-7} m^2 is rigidly fixed at both ends. The temperature of the wire is decreased by 80/3°C. If transverse waves are set up in the wire, find the frequency of the fundamental mode of vibration in Hz. The Young's modulus of steel $= 2 \times 10^{11}$ Nm^{-2} and coefficient of linear expansion of steel $= 1.2 \times 10^{-5}$ per °C.

IIT, 1981

3. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (**in cm**) between the successive nodes on the string.

IIT, 2009

SOLUTIONS

1. The observer will hear a sound of the source moving away from him and another sound after reflection from the wall. The apparent frequencies of these sounds are

$$\nu_1 = \frac{\nu v}{v+u} = \frac{256\times 330}{(330+5)} = 252 \text{ Hz}$$

and $$\nu_2 = \frac{\nu v}{v-u} = \frac{256\times 330}{(330-5)} = 260 \text{ Hz}$$

$\therefore$ No. of beats per second (beat frequency) = 260 – 252 = 8

2. Contraction $\Delta L = \alpha L \Delta\theta$

Young's modulus $Y = \frac{T L}{A \Delta L}$; T = tension

$$\therefore \quad T = \frac{Y A \Delta L}{L} = YA\alpha\Delta\theta$$

$$= 2\times 10^{11} \times 10^{-7} \times 1.2 \times 10^{-5} \times \frac{80}{3}$$

$$= 6.4 \text{ N}$$

Frequency of fundamental mode is

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{\mu}}\ ;\ \mu = \text{mass per unit length of wire}$$

$$= \frac{1}{2\times 1}\sqrt{\frac{6.4}{0.1}} = 4 \text{ Hz}$$

3. Mass per unit length of the string is

$$m = \frac{1.0\times 10^{-3}}{20\times 10^{-2}} = 5 \times 10^{-3} \text{ kg m}^{-1}$$

Speed of waves in the string is

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{0.5}{5\times 10^{-3}}} = 10 \text{ m s}^{-1}$$

Now $$v = \nu\lambda \Rightarrow \lambda = \frac{v}{\nu} = \frac{10}{100} = 0.1 \text{ m}$$

$$= 10 \text{ cm}$$

$\therefore$ Separation between successive nodes

$$= \frac{\lambda}{2} = 5 \text{ cm}$$

15 Chapter Thermal Expansion

REVIEW OF BASIC CONCEPTS

15.1 THERMAL EXPANSION

If the temperature of a body is increased, its length, surface area and volume all increase.

(i) *Coefficient of Linear Expansion* If the body is in the form of a rod, the increase ΔL in its length when the temperature is increased by ΔT is proportional to (*a*) the original length L and (*b*) increase in temperature ΔT, i.e.

$$\Delta L \propto L\Delta T \quad \Rightarrow \quad \Delta L = \alpha L\Delta T$$

Thus $$\alpha = \frac{\Delta L}{L\Delta T}$$

where α is the coefficient of linear expansion of the material of the body. Thus

$$L = L_0\ (1 + \alpha\Delta T)$$

(ii) *Coefficient of Area Expansion*

$$\text{Similarly } \beta = \frac{\Delta A}{A\Delta T}$$

where β is called the coefficient of area expansion

$$A = A_0\ (1 + \beta\Delta T)$$

(iii) *Coefficient of Volume Expansion*

$$\gamma = \frac{\Delta V}{V\Delta T}$$

$$V = V_0\ (1 + \gamma\Delta T)$$

The SI unit of α, β and γ is $(°C)^{-1}$ or K^{-1}.

(iv) *Relation between α, β and γ is*

$\alpha : \beta : \gamma = 1 : 2 : 3$

i.e. $\gamma = 2\beta = 3\alpha$.

15.2 APPLICATIONS

(i) *Variation of Density with Temperature*

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \quad \text{or} \quad \rho = \frac{m}{V}$$

Since mass m remains constant, ρV = constant. Thus

$$\rho V = \rho_0\ V_0$$

$$\Rightarrow \qquad \rho = \frac{p_0 V_0}{V} = \frac{\rho_0 V_0}{V_0\,(1 + \gamma\Delta T)} = \frac{\rho_0}{(1 + \gamma\,\Delta T)}$$

Thus density of a substance decreases with increase in temperature.

(ii) *Thermal Stress*

If a rod is held between two rigid supports and its temperature is increased or decreased, the rigid supports prevent the rod from expanding or contracting. As a result, a stress (called thermal stress) is developed in the rod. The change in length of the rod is

$$\Delta L = \alpha L\Delta T$$

$$\therefore \qquad \text{Strain } \frac{\Delta L}{L} = \alpha\Delta T$$

Now $$\text{Young's modulus } (Y) = \frac{\text{stress}}{\text{strain}}$$

$$\therefore \qquad \text{Thermal stress} = \frac{Y\Delta L}{L} = Y\alpha\Delta T$$

(iii) *Heating and Cooling of a metallic scale*

A linear metallic scale expands when heated and contracts when cooled.

A reading of 1 unit of a heated scale is equivalent to a an actual length of 1 unit $\times (1 + \alpha\Delta T)$ where ΔT is the

rise in temperature. If the reading of the heated scale is x units, the actual length = $x(1 + \alpha\Delta T)$ units.
If the temperature is decreased by ΔT, the actual reading = $x(1 - \alpha\Delta T)$ units.

(iv) Consider two metal rods of lengths L_1 and L_2. Let L_1' and L_2' be their lengths when their temperature is increased by ΔT, then

$$L_1' = L_1(1 + \alpha_1\Delta T))$$

and $$L_2' = L_2(1 + \alpha_2\Delta T)$$

where α_1 and α_2 are their respective coefficients of linear expansion. The difference of their lengths is

$$L_2' - L_1' = L_2(1 + \alpha_2\Delta T) - L_1(1 + \alpha_1\Delta T)$$

Their difference in lengths will remain constant if

$$L_2' - L_1' = L_2 - L_1$$

i.e. if $L_1\alpha_1 = L_2\alpha_2$

(v) *Loss or gain of time of a metallic pendulum clock due to rise or fall of room temperature*
The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

If the room temperature rises by ΔT, the length of the pendulum increases. Hence the time period increases which implies that a metallic pendulum clock slows down. If L' is the length of heated clock, then its time period becomes

$$T' = 2\pi\sqrt{\frac{L'}{g}}$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L(1+\alpha\Delta T)}{L}}$$

$$= (1 + \alpha\Delta T)^{1/2}$$

$$= 1 + \frac{1}{2}\alpha\Delta T \ (\because \ \alpha\Delta T \text{ is small})$$

$$\Rightarrow \quad \frac{T' - T}{T} = \frac{1}{2}\alpha\Delta T$$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta T$$

This gives the time lost per second.

$$\therefore \text{ Time lost in one day} = \left(\frac{1}{2}\alpha\Delta T\right) \times (60 \times 60 \times 24)$$

$$= \left(\frac{1}{2}\alpha\Delta T\right) \times 86400 \text{ s}$$

If the room temperature falls by ΔT, then

$$\text{Time gained in one day} = \left(\frac{1}{2}\alpha\Delta T\right) \times 86400 \text{ s}$$

EXAMPLE 15.1

At 20°C, a brass rod has a length 50.0 cm. It is joined to a steel rod of the same length and the same diameter at the same temperature. Find the change in the length of the composite rod when it is heated to 220°C. For brass $\alpha = 2 \times 10^{-5}$ K^{-1} and for steel $\alpha = 1 \times 10^{-5}$ K^{-1}.

SOLUTION

For brass:

$$(\Delta L)_b = \alpha_b \, L_b \, \Delta T$$

$$= (2 \times 10^{-5}) \times 0.5 \times (220 - 20)$$

$$= 2 \times 10^{-3} \text{ m}$$

$$= 2 \text{ mm}$$

For steel:

$$(\Delta L)_s = \alpha_s L_s \Delta T$$

$$= (1 \times 10^{-5}) \times 0.5 \times (220 - 20)$$

$$= 1 \text{ mm}$$

$$\therefore \quad \Delta L = (\Delta L)_b + (\Delta L)_s = 3 \text{ mm}$$

EXAMPLE 15.2

A metal wire of cross-sectional area 5×10^{-6} m^2 is held taut at 30°C between two rigid supports with negligible tension in it. Find the tension developed in the wire if it is cooled to –20°C. Given α of metal = 2×10^{-5} K^{-1} and $Y = 1 \times 10^{11}$ N m^{-2}.

SOLUTION

Thermal stress = $Y\alpha\Delta T$

Also $$\text{stress} = \frac{F}{A}$$

$$\therefore \quad F = YA\alpha\Delta T$$

$$= (1 \times 10^{11}) \times (5 \times 10^{-6})$$

$$\times (2 \times 10^{-5}) \times [30 - (-20)]$$

$$= 500 \text{ N}$$

EXAMPLE 15.3

The coefficient of volume expansion of a liquid is 5×10^{-4} K^{-1}. If its temperature is increased by 30°C, find the percentage change in its density.

SOLUTION

$$\rho = \frac{\rho_0}{(1 + \gamma\Delta T)}$$

$$\Rightarrow \quad \rho(1 + \gamma\Delta T) = \rho_0 \quad \Rightarrow \quad \rho - \rho_0 = -\rho_0 \, \gamma\Delta T$$

or $$\frac{\rho - \rho_0}{\rho_0} = -\gamma \Delta T$$
$$= -5 \times 10^{-4} \times 30 = -1.5 \times 10^{-2}.$$

The negative sign indicates that the density decreases with increase in temperature.

$\therefore$ Percentage change in density = $(1.5 \times 10^{-2}) \times 100 = 1.5\%$

EXAMPLE 15.4

A block of mass 248 g and volume 205 cm^3 floats in a liquid contained in a vessel. The density of the liquid at 0°C is 1.248 g cm^{-3}. It is found that the block just sinks in the liquid if the temperature is raised to 50°C. Find the coefficient of volume expansion of the liquid upto appropriate significant figures. The volume expansion of the block is negligible compared to that of the liquid.

SOLUTION

The block will just sink in the liquid if its density becomes equal to the density of the liquid at 50°C.

Density of block

$$\rho = \frac{248\,\text{g}}{205\,\text{cm}^3} = 1.210 \text{ g cm}^{-3}$$

$$\rho_0 = \rho(1 + \gamma \Delta T)$$
$$1.248 = 1.210\ (1 + \gamma \times 50)$$
$$\Rightarrow \quad \gamma = 6.28 \times 10^{-4}\ (°C)^{-1} \text{ or } K^{-1}$$

The quantity ΔT has only two significant figures. Hence the value of γ must be rounded off to two significant figures. Thus

$$\gamma = 6.3 \times 10^{-4}\ K^{-1}$$

EXAMPLE 15.5

A large steel wheel is to be fitted on to a shaft of the same material. At 25°C, the diameter of the shaft is 8.00 cm and the diameter of the hole is 7.99 cm. The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip into the shaft. Give α for steel = $1 \times 10^{-5}\ K^{-1}$.

SOLUTION

Decrease in temperature

$$\Delta T = \frac{\Delta L}{\alpha L} = \frac{-0.01\,\text{cm}}{1 \times 10^{-5} \times 8.00\,\text{cm}} = -125°C$$

$\therefore \quad T_2 - T_1 = -125 \quad \Rightarrow \quad T_2 = -125 + T_1 = -125 + 25 = -100°C$

EXAMPLE 15.6

A steel tape gives correct readings at a temperature of 15°C. On a day when the temperature is 40°C, this tape measures the distance between two points as 3152 cm. What is the actual distance between the two points? Given α for steel = $1.2 \times 10^{-5}\ K^{-1}$.

SOLUTION

$$\Delta T = 40 - 15 = 25°C$$
$$\text{Actual distance} = \text{measured distance} \times (1 + \alpha \Delta T)$$
$$= 3152 \times (1 + 1.2 \times 10^{-5} \times 25)$$
$$= 3152.9 \text{ cm} \simeq 3153 \text{ cm}$$

EXAMPLE 15.7

A metal pendulum clock gives correct time at a temperature of 20°C. How much time does it lose in a day if the room temperature is 35°C. Given α of metal = $1.25 \times 10^{-5}\ K^{-1}$.

SOLUTION

$$\text{Time lost per second} = \frac{1}{2} \alpha \Delta T$$

$$\therefore \text{ Time lost in a day} = \left(\frac{1}{2} \alpha \Delta T\right) \times (24 \times 60 \times 60)\text{s}$$
$$= \left(\frac{1}{2} \times 1.2 \times 10^{-5} \times 15\right) \times (24 \times 60 \times 60)$$
$$= 8.1 \text{ s}$$

EXAMPLE 15.8

A litre glass bottle is completely filled with water at 20°C. Find the volume of water that overflows if the bottle and water are heated to 80°C if

(a) the expansion of bottle is neglected

(b) the expansion of bottle is not neglected

γ of water = $6.0 \times 10^{-4}(°C)^{-1}$, α of glass = $0.6 \times 10^{-5}(°C)^{-1}$.

SOLUTION

Volume of bottle = volume of water = 1 litre = 1000 cm^3

Volume of water that overflows = (final volume of water – final volume of bottle)

(a) If the expansion of bottle is neglected, the volume of water that overflows

$$= 1000 \times (1 + \gamma_w \Delta T) - 1000$$
$$= 1000 \times (1 + 6.0 \times 10^{-4} \times 60) - 1000$$
$$= 1036 - 1000 = 36 \text{ cm}^3$$

(b) $\gamma_g = 3\alpha_g = 3 \times 0.6 \times 10^{-5} = 1.8 \times 10^{-5}(°C)^{-1}$

If the expansion of bottle is not neglected, the volume of water that overflows

$= 1000 \times (1 + \gamma_w \Delta T) - 1000\ (1 + \gamma_g \Delta T)$

$= 1000 \times (\gamma_w - \gamma_g) \times \Delta T$

$= 1000 \times (6.0 \times 10^{-4} - 1.8 \times 10^{-5}) \times 60$

$= 34.9\ cm^3$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. When a solid metallic sphere is heated, the largest percentage increase occurs in its

(a) diameter (b) surface area

(c) volume (d) density

2. A steel scale measures the length of a copper rod as L cm when both are at 20°C, the calibration temperature for the scale. If the coefficients of linear expansion for steel and copper are α_s and α_c respectively, what would be the scale reading (in cm) when both are at 21°C?

(a) $L\ \dfrac{(1+\alpha_c)}{(1+\alpha_s)}$ (b) $L\ \dfrac{\alpha_c}{\alpha_s}$

(c) $L\ \dfrac{\alpha_s}{\alpha_c}$ (d) L

3. Two uniform brass rods A and B of lengths l and $2l$ and radii $2r$ and r respectively are heated to the same temperature. The ratio of the increase in the length of A to that of B is

(a) 1 : 1 (b) 1 : 2

(c) 1 : 4 (d) 2 : 1

4. Two rods of different materials having coefficients of thermal expansion α_1 and α_2 and Young's modulii Y_1 and Y_2 are fixed between two rigid and massive walls. The rods are heated to the same temperature. If there is no bending of the rods, the thermal stresses developed in them are equal provided

(a) $\dfrac{Y_1}{Y_2} = \sqrt{\dfrac{\alpha_1}{\alpha_2}}$ (b) $\dfrac{Y_1}{Y_2} = \sqrt{\dfrac{\alpha_2}{\alpha_1}}$

(c) $\dfrac{Y_1}{Y_2} = \dfrac{\alpha_1}{\alpha_2}$ (d) $\dfrac{Y_1}{Y_2} = \dfrac{\alpha_2}{\alpha_1}$

IIT, 1989

5. A piece of metal floats on mercury. The coefficients of volume expansion of the metal and mercury are γ_1 and γ_2 respectively. If their temperature is increased by ΔT, the fraction of the volume of metal submerged in mercury changes by a factor

(a) $\left(\dfrac{1+\gamma_2 \Delta T}{1+\gamma_1 \Delta T}\right)$ (b) $\left(\dfrac{1+\gamma_2 \Delta T}{1-\gamma_1 \Delta T}\right)$

(c) $\left(\dfrac{1-\gamma_2 \Delta T}{1+\gamma_1 \Delta T}\right)$ (d) $\dfrac{\gamma_2}{\gamma_1}$

IIT, 1991

6. A thin copper wire of length L increases its length by 1% when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions $2L \times L$ is heated from T_1 to T_2?

(a) 1% (b) 2%

(c) 3% (d) 4%

7. The coefficient of expansion of a crystal in one direction (x-axis) is $2.0 \times 10^{-6}\ K^{-1}$ and that in the other two perpendicular (y-and z-axes) directions is $1.6 \times 10^{-6}\ K^{-1}$. What is the coefficient of cubical expansion of the crystal?

(a) $1.6 \times 10^{-6}\ K^{-1}$ (b) $1.8 \times 10^{-6}\ K^{-1}$

(c) $2.0 \times 10^{-6}\ K^{-1}$ (d) $5.2 \times 10^{-6}\ K^{-1}$

8. A metal ball immersed in alcohol weighs W_1 at 0°C and W_2 at 59°C. The coefficient of cubical expansion of metal is less than that of alcohol. If the density of the metal is large compared to that of alcohol, then

(a) $W_1 > W_2$ (b) $W_1 = W_2$

(c) $W_1 < W_2$ (d) $W_2 = \dfrac{W_1}{2}$

IIT, 1980

9. A rod of length 20 cm made of a metal A expands by 0.075 cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal B having the same length expands by 0.045 cm for the same change in temperature. A third rod of the same length is composed of two parts, one of metal A and the other of metal B. This rod expands by 0.060 cm for the same change in temperature. The portion made of metal A has length

(a) 20 cm (b) 10 cm
(c) 15 cm (d) 18 cm

10. A metallic circular disc having a circular hole at its centre rotates about an axis passing through its centre and perpendicular to its plane. When the disc is heated, its
(a) moment of inertia increases but angular speed decreases
(b) moment of inertia decreases but angular speed increases
(c) moment of inertia and angular speed both increase
(d) moment of inertia and angular speed both decrease.

11. A uniform metal rod of length L and mass M is rotating with angular speed ω about an axis passing through one of the ends and perpendicular to the rod. If the temperature increases by t ºC, then the change in its angular speed is proportional to

(a) $\sqrt{\omega}$ (b) ω
(c) ω^2 (d) $\frac{1}{\omega}$

12. A steel metre scale is to be ruled so that the millimetre intervals are accurate to about 5×10^{-5} m at a certain temperature. The maximum temperature variation allowed during ruling is (the coefficient of linear expansion of steel = 10×10^{-6} K^{-1})
(a) 2 ºC (b) 5 ºC
(c) 7 ºC (d) 10 ºC

13. When the temperature of a rod increases from t to $t + \Delta t$, the moment of inertia of the rod increases from I to $I + \Delta I$. If the coefficient of linear expansion of the rod is α, the ratio $\frac{\Delta I}{I}$ is

(a) $\frac{\Delta t}{t}$ (b) $\frac{2\,\Delta t}{t}$
(c) $\alpha\,\Delta\,t$ (d) $2\,\alpha\,\Delta\,t$

14. Two spheres made of the same material have the same diameter. One sphere is hollow and the other is solid. If they are heated through the same range of temperature,
(a) the hollow sphere will expand more than the solid sphere
(b) the solid sphere will expand more than the hollow sphere
(c) both spheres will expand equally
(d) the hollow sphere will not expand at all.

15. Two rods of lengths L_1 and L_2 are welded together to make a composite rod of length $(L_1 + L_2)$. If the coefficients of linear expansion of the rods are α_1 and α_2 respectively, the effective coefficient of linear expansion of the composite rod will be

(a) $\frac{1}{2}\,(\alpha_1 + \alpha_2)$ (b) $\sqrt{\alpha_1 \alpha_2}$
(c) $\frac{L_1 \alpha_1 + L_2 \alpha_2}{L_1 + L_2}$ (d) $\frac{\sqrt{L_1 L_2 \alpha_1 \alpha_2}}{L_1 + L_2}$

16. The coefficient of linear expansion of an inhomogeneous rod changes linearly from α_1 to α_2 from one end to the other end of the rod. The effective coefficient of linear expansion of the rod is

(a) $(\alpha_1 + \alpha_2)$ (b) $\frac{1}{2}\,(\alpha_1 + \alpha_2)$
(c) $\sqrt{\alpha_1 \alpha_2}$ (d) $(\alpha_1 - \alpha_2)$

17. Three rods of the same length are arranged to form an equilateral triangle. Two rods are made of the same material of coefficient of linear expansion α_1 and the third rod which forms the base of the triangle has coefficient of expansion α_2. The altitude of the triangle will remain the same at all temperatures if the α_1/α_2 is nearly

(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) 4

18. A cylindrical block of wood floats vertically with 80% of its volume immersed in a liquid at 0ºC. When the temperature of the liquid is raised to 62.5ºC, the block just sinks in the liquid. The coefficient of cubical expansion of liquid is

(a) 1×10^{-3} K^{-1} (b) 2×10^{-3} K^{-1}
(c) 3×10^{-3} K^{-1} (d) 4×10^{-3} K^{-1}

19. When a block of iron floats in mercury at 0°C, a fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then the ratio k_1/k_2 can be expressed as

(a) $\frac{1 + 60\gamma_{Fe}}{1 + 60\gamma_{Hg}}$ (b) $\frac{1 - 60\gamma_{Fe}}{1 + 60\gamma_{Hg}}$
(c) $\frac{1 + 60\gamma_{Fe}}{1 - 60\gamma_{Hg}}$ (d) $\frac{1 + 60\gamma_{Hg}}{1 + 60\gamma_{Fe}}$

IIT, 2001

20. Two rods, one made of aluminium and the other made of steel, having initial lengths l_1 and l_2 respectively are connected together to form a single rod of length $(l_1 + l_2)$. The coefficients of linear expansion for aluminium and steel are α_1 and α_2

respectively. If the length of each rod increases by the same amount when their temperature is raised by t°C, then the ratio $l_1/(l_1 + l_2)$

(a) α_1/α_2 (b) α_2/α_1

(c) $\alpha_2/(\alpha_1 + \alpha_2)$ (d) $\alpha_1/(\alpha_1 + \alpha_2)$

IIT, 2003

21. The coefficient of real expansion of mercury is 18×10^{-5}/°C. A thermometer has a bulb of volume 10^{-6} m^3 and the cross-sectional area of the stem is 0.002 cm^2. The bulb is completely filled with mercury when the temperature is 0°C. When the temperature rises to 100°C, the length of the mercury column in the stem will be

(a) 9 cm (b) 18 cm

(c) 9 mm (d) 18 mm

22. The coefficient of apparent expansion of a liquid when determined using two different vessels A and B are γ_1 and γ_2 respectively. If the coefficient of linear expansion of vessel A is α, the coefficient of linear expansion of vessel B is

(a) $\dfrac{\alpha\gamma_1\gamma_2}{\gamma_1+\gamma_2}$ (b) $\dfrac{\gamma_1-\gamma_2}{2\alpha}$

(c) $\dfrac{\gamma_1-\gamma_2+\alpha}{3}$ (d) $\dfrac{\gamma_1-\gamma_2}{3}+\alpha$

23. A flask of volume V contains some mercury. It is found that at different temperatures, the volume of air inside the flask remains the same. If γ_g and γ_m are the coefficients of cubical expansion of glass and mercury respectively, the volume of mercury in the flask is

(a) $\dfrac{\gamma_g V}{\gamma_m}$ (b) $\dfrac{\gamma_m V}{\gamma_g}$

(c) $\left(1-\dfrac{\gamma_g}{\gamma_m}\right)V$ (d) $\left(1-\dfrac{\gamma_m}{\gamma_g}\right)V$

24. A steel rod and a copper rod have lengths L_s and L_c respectively at a certain temperature. It is found that the difference between their lengths remains constants at all temperatures. If α_s and α_c are their respective coefficients of linear expansion, the ratio L_s/L_c is given by

(a) $\left(1+\dfrac{\alpha_c}{\alpha_s}\right)$ (b) $\left(1+\dfrac{\alpha_s}{\alpha_c}\right)$

(c) $\dfrac{\alpha_s}{\alpha_c}$ (d) $\dfrac{\alpha_c}{\alpha_s}$

25. A uniform metallic circular disc, mounted on frictionless bearings, is rotating at an angular frequency ω about an axis passing through its centre and perpendicular to its plane. The coefficient of linear expansion of the metal is α. If the temperature of the disc is increased by Δt, the angular frequency of rotation of the disc will

(a) remain unchanged

(b) increase by $\alpha\omega\Delta t$

(c) increase by $2\alpha\omega\Delta t$

(d) decrease by $2\alpha\omega\Delta t$

26. A vertical glass tube, closed at the bottom, contains a mercury column of length L_0 at 0°C. If γ is the coefficient of cubical expansion of mercury α the coefficient of linear expansion of glass, the length of the mercury column when the temperature rises to t°C is (assuming that t not more than 100°C)

(a) $L_t = L_0[1 + (\gamma - 3\alpha)t]$

(b) $L_t = L_0[1 + (\gamma + 3\alpha)t]$

(c) $L_t = L_0[1 + (\gamma + 2\alpha)t]$

(d) $L_t = L_0[1 + (\gamma - 2\alpha)t]$

27. The brass scale of a barometer gives correct reading at 0°C. The coefficient of linear expansion of brass is 20×10^{-6} per °C. The barometer reads 75 cm at 40°C. The atmospheric pressure at 40°C is

(a) 75 cm of Hg (b) 74.94 cm of Hg

(c) 75.06 cm of Hg (d) none of these

28. A metal cube of coefficient of linear expansion α is floating in a beaker containing a liquid of coefficient of volume expansion γ. When the temperature is raised by ΔT, the depth upto which the cube is submerged in the liquid remains unchanged. If the expansion of the beaker is ignored, the relation between α and γ is

(a) $\alpha = \dfrac{\gamma}{3}$ (b) $\alpha = \dfrac{\gamma}{2}$

(c) $\alpha = 3\ \gamma$ (d) $\alpha = 2\ \gamma$

IIT, 2004

29. A copper wire of length L and cross-sectional area A is held at the ends by two rigid supports. At temperature T the wire is just taut with negligible tension. If the temperature reduces to $(T - \Delta T)$, the speed of transverse waves in the wire is (here Y is Young's modulus, ρ density and α coefficient of linear expansion of copper)

(a) $\sqrt{\dfrac{Y\alpha\Delta T}{\rho}}$ (b) $\sqrt{\dfrac{YA\alpha\Delta T}{L\rho}}$

(c) $\sqrt{\dfrac{YL\alpha\Delta T}{A\rho}}$ (d) $\sqrt{\dfrac{Y\rho L\Delta T}{\alpha}}$

IIT, 1979

30. A metal ball immersed in alcohol weighs W_1 at 0°C and W_2 at 59°C. The coefficient of cubical expansion of metal is less than that of alcohol. If the density of the metal is large compared to that of alcohol, then

(a) $W_1 > W_2$ (b) $W_1 = W_2$

(c) $W_1 < W_2$ (d) $W_2 = \frac{W_1}{2}$

IIT, 1980

ANSWERS

1. (c)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (b)
7. (d)	**8.** (b)	**9.** (b)	**10.** (a)	**11.** (b)	**12.** (b)
13. (d)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (d)
19. (a)	**20.** (c)	**21.** (a)	**22.** (d)	**23.** (a)	**24.** (d)
25. (d)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (b)

SOLUTIONS

1. On heating, the diameter, surface area and volume of the sphere will all increase. Since the mass remains unchanged, the density decreases. The percentage increase in the volume is the largest because the coefficient of volume expansion is greater than the coefficients of area expansion and linear expansion. Hence the correct choice is (c).

2. The length of 1 cm division of the steel scale at 21°C is

$$(1 \text{ cm}) \times [1 + \alpha_s(21 - 20)] = (1 + \alpha_s) \text{ cm}$$

Length of copper rod at 21°C will be

$$(L \text{ cm}) \times [1 + \alpha_c (21 - 20)] = L (1 + \alpha_c) \text{ cm}$$

$$\therefore \text{ Scale reading at } 21°\text{C} = \frac{L(1+\alpha_c)}{(1+\alpha_s)}.$$

Hence the correct choice is (a).

3. The increase in length due to heating is independent of the radius of the rod. The increase in the length of rod A is

$$\Delta l = \alpha l \Delta t$$

and of rod B is $\Delta l' = \alpha l' \Delta t = 2\alpha l \Delta t \quad (\because l' = 2l)$

$$\therefore \quad \frac{\Delta l}{\Delta l'} = \frac{1}{2}$$

Hence the correct choice is (b).

4. If a rod of length L and coefficient of thermal expansion α is heated to a temperature t, the increase in the length is given by

$$l = \alpha Lt$$

$$\therefore \quad \text{Strain} = \frac{l}{L} = \alpha t$$

and $\quad \text{stress} = Y \times \text{strain} = Y \alpha t$

Since the value of t is the same for the two rods, the stresses in them will be equal if

$$Y_1 \alpha_1 t = Y_2 \alpha_2 t \text{ or } \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1}$$

Hence the correct choice is (d).

5. The correct choice is (a). It follows from the fact that the volumes of metal and mercury increase to $V_0 (1 + \gamma_1 \Delta T)$ and $V_0 (1 + \gamma_2 \Delta T)$ respectively; V_0 being the initial volume.

6. Length of wire at temperature T_2 is

$$L_t = L \left(1+\frac{1}{100}\right)$$

$$\therefore \quad 2L_t^2 = 2L^2 \left(1+\frac{1}{100}\right)^2$$

Now $2L_t^2$ = area of the plate at temperature T_2 and $2L^2$ = area of the plate at temperature T_1. Therefore,

$$A_t = A \left(1+\frac{1}{100}\right)^2$$

$$= A \left(1+\frac{2}{100}\right) = \frac{102A}{100}$$

Thus the area increases by 2%, which is choice (b).

7. Coefficient of cubical expansion is

$$\gamma = \alpha_x + \alpha_y + \alpha_z = \alpha_x + 2\alpha_y \quad (\because \alpha_y = \alpha_z)$$

$$= 2.0 \times 10^{-6} + 2 \times 1.6 \times 10^{-6}$$

$$= 5.2 \times 10^{-6} \text{ K}^{-1}$$

Hence the correct choice is (d).

8. Let V_0 be the volume of the metal at 0°C and V_t its volume at t°C. At temperature t the upthrust is

$$U_t = V_t \rho_t g$$

where ρ_t is the density of alcohol at temperature t. Now

$$V_t = V_0\ (1 + \gamma t)$$

where γ is the coefficeint of cubical expansion of alcohol and V_0 is the volume of alcohol displaced at temperature $t = 0°C$. Now the density of alcohol at temperature t is

$$\rho_t = \frac{\rho_0}{1+\gamma t}$$

where ρ_0 is the density of alcohol at $t = 0°C$. Hence

$$U_t = V_0\ (1 + \gamma t) \times \frac{\rho_0\, g}{(1+\gamma t)}$$

$$= V_0\ \rho_0\ g = U_0$$

where U_0 is the upthrust at 0°C. Since the upthrust is independent of temperature, $W_1 = W_2$. Hence the correct choice is (b).

9. Here

$$(\Delta l)_1 = L\ \alpha_1\ \Delta t$$
$$(\Delta l)_2 = L\ \alpha_2\ \Delta t$$
$$(\Delta l)_3 = x\ \alpha_1\ \Delta t + (L - x)\ \alpha_2\ \Delta t$$

where x is the length of metal A and $(L - x)$ that of metal B in rod C. Now $(\Delta l)_1 = 0.075$ cm, $(\Delta l)_2 = 0.045$ cm and $(\Delta l)_3 = 0.060$ cm. Notice that $(\Delta l)_3 = \frac{1}{2}\ [(\Delta l)_1 + (\Delta l)_2]$. This is possible only if $x = \frac{L}{2} = \frac{20\text{ cm}}{2} = 10$ cm. Hence the correct choice is (b).

10. Due to thermal expansion, the diameter of the disc as well as that of the hole will increase. Therefore, the moment of inertia will increase resulting in a decrease in the angular speed. Hence the correct choice is (a).

11. At t ºC, the length of the rod becomes $L' = L\ (1 + \alpha t)$, where α is the coefficeint of linear expansion. From the law of conservation of angular momentum, we have

$$I\omega = I'\omega'$$

or $$\frac{1}{3} ML^2\omega = \frac{1}{3} ML'^2\omega'$$

or $$\frac{\omega'}{\omega} = \left(\frac{L}{L'}\right)^2 = \frac{1}{(1+\alpha t)^2}$$

Now, for a given value of t, $(1 + \alpha t)^{-2}$ is constant, say k.

$\because$ $$\frac{\omega'}{\omega} = k$$

or $$\frac{\omega' - \omega}{\omega} = k - 1$$

or $$\omega' - \omega = (k - 1)\ \omega$$

i.e. $(\omega' - \omega) \propto \omega$. Hence the corect choice is (b).

12. The maximum temperature variation ΔT allowed is given by $\Delta L = L\ \alpha\ \Delta T$, which gives

$$\Delta T = \frac{\Delta L}{L\alpha} = \frac{5\times10^{-5}}{1\times10\times10^{-6}} = 5°C$$

Hence the correct choice is (b).

13. The moment of inertia of a rod of mass M and length L is given by

$$I = k\ ML^2 \qquad (1)$$

where $k = \frac{1}{12}$ if the axis of rotation is through the centre and $k = \frac{1}{3}$ if the axis of rotation is through the end of the rod. Partially differentiating (1), we have

$$\Delta I = 2k\ ML\ \Delta L$$

Now $\Delta L = L\alpha\Delta t$. Therefore,

$$\Delta I = 2kML \times L\alpha\Delta t$$

$$= 2\ (kML^2) \times \alpha\Delta t = 2I\alpha\Delta t$$

or $$\frac{\Delta I}{I} = 2\alpha\Delta t$$, which is choice (d).

14. The correct choice is (c).

15. Total length of the composite rod at 0°C is $L_0 = L_1 + L_2$. When the composite rod is heated to t°C, its length at t°C will be $L = L_1\ (1 + \alpha_1\ t) + L_2\ (1 + \alpha_2\ t) = (L_1 + L_2) + L_1\ \alpha_1\ t + L_2\ \alpha_2\ t$, or

$$L = L_0 + (L_1\alpha_1 + L_2\alpha_2)t$$

$\therefore$ Effective coefficient of expansion of the composite rod is

$$\alpha = \frac{L - L_0}{L_0 t} = \frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2},$$

which is choice (c).

16. Consider a small element of length dx at a distance x from one end of the rod. Let L be the length of the rod. Now the increase in the coefficient of linear expansion by unit length of the rod is $(\alpha_2 - \alpha_1)/L$. Therefore, the value of α at the element located at x is

$$\alpha_x = \alpha_1 + \left(\frac{\alpha_2 - \alpha_1}{L}\right)x$$

$\therefore$ Increase in the length of the element = $\alpha_x\ dx\ \Delta t$, where ΔT is the rise in temperature. Therefore, the increase in the length of rod is

$$\Delta L = \int_0^L \alpha_x \, dx \, \Delta T$$

$$= \Delta T \int_0^L \left\{\alpha_1 + \left(\frac{\alpha_2 - \alpha_1}{L}\right)x\right\} dx$$

$$= \Delta T \left| \alpha_1 x + \left(\frac{\alpha_2 - \alpha_1}{L}\right)\frac{x^2}{2}\right|_0^L$$

$$= \Delta T \left[\alpha_1 L + \left(\frac{\alpha_2 - \alpha_1}{2}\right)L\right]$$

$$= \left(\frac{\alpha_1 + \alpha_2}{2}\right) L\Delta T = \alpha_{\text{eff}} \, L\Delta T$$

where $\alpha_{\text{eff}} = \frac{1}{2}(\alpha_1 + \alpha_2)$. Hence the correct choice is (b).

17. Let L be the length of each rod at 0°C and h_0 be the altitude AD at 0°C. Then (Fig. 15.1),

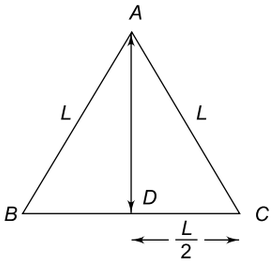


Fig. 15.1

$$h_0 = \left(L^2 - \frac{L^2}{4}\right)^{1/2} \quad (1)$$

When the rods are heated to a temperature t, the altitude becomes

$$h = \left[L^2(1+\alpha_1 t)^2 - \frac{L^2}{4}(1+\alpha_2 t)^2\right]^{1/2}$$

Since α_1 and α_2 are much less than unity, we can write $(1 + \alpha_1 t)^2 \simeq 1 + 2\alpha_1 t$ and $(1 + \alpha_2 t)^2 \simeq 1 + 2\alpha_2 t$. Thus

$$h = \left[L^2(1+2\alpha_1 t) - \frac{L^2}{4}(1+2\alpha_2 t)\right]^{1/2} \quad (2)$$

Equating (1) and (2), we get

$$\left(L^2 - \frac{L^2}{4}\right)^{1/2} = \left[L^2(1+2\alpha_1 t) - \frac{L^2}{4}(1+2\alpha_2 t)\right]^{1/2}$$

which simplifies to give $\frac{\alpha_1}{\alpha_2} = \frac{1}{4}$, which is choice (c).

18. Weight of cylinder = weight of the liquid displaced, i.e.

$$Ah\rho_b = A \times (0.8\ h) \times \rho_0$$

or $$\rho_0 = \frac{\rho_b}{0.8} = \frac{5\rho_b}{4} \quad (1)$$

Here ρ_0 is the density of the liquid at 0°C. For the block to sink in the liquid at t°C, the density of the liquid must change from ρ_0 to ρ at t°C. Now

$$\rho = \frac{\rho_0}{1+\gamma t} \quad (2)$$

For the block to sink $\rho = \rho_b$. Using (1) and (2), we have

$$\rho_b = \frac{5\rho_b/4}{1+\gamma t}$$

or $$1 + \gamma t = \frac{5}{4}$$

Given t = 62.5°C. Using this value, we get $\gamma = 4 \times 10^{-3}$ (°C)$^{-1}$ or K^{-1}, which is choice (d).

19. Let V_0 be the total volume of the block of iron at 0°C and V its total volume at 60°C. Then

$$V = V_0 (1 + 60\ \gamma_{\text{Fe}}) \quad (1)$$

Let v_0 be the volume of the block submerged in mercury at 0°C and v the volume submerged at 60°C. Then

$$v = v_0 (1 + 60\ \gamma_{\text{Hg}}) \quad (2)$$

Dividing (1) by (2), we have

$$\frac{V/v}{V_0/v_0} = \frac{1+60\gamma_{\text{Fe}}}{1+60\gamma_{\text{Hg}}} \quad (3)$$

Given $\frac{v_0}{V_0} = k_1$ and $\frac{v}{V} = k_2$. Using these in (3), we find that the correct choice is (a).

20. The length of each rod increases by the same amount if $l_1\ \alpha_1 = l_2\ \alpha_2$ or

$$\frac{l_2}{l_1} = \frac{\alpha_1}{\alpha_2}$$

or $$\frac{l_2}{l_1} + 1 = \frac{\alpha_1}{\alpha_2} + 1$$

or $$\frac{l_1 + l_2}{l_1} = \frac{\alpha_1 + \alpha_2}{\alpha_2}$$

or $$\frac{l_1}{l_1 + l_2} = \frac{\alpha_2}{\alpha_1 + \alpha_2}$$, which is choice (c).

21. Increase in volume of mercury when the temperature increases by 100°C is

$$v = \gamma_r V \Delta T = (18 \times 10^{-5}) \times (10^{-6}) \times 100$$
$$= 18 \times 10^{-9} \text{ m}^3$$

Now $v = Ah$ or $h = \dfrac{v}{A}$

$$= \frac{(18 \times 10^{-9} \text{ m}^3)}{(0.002 \times 10^{-4} \text{ m}^2)} = 0.09 \text{ m} = 9 \text{ cm}$$

Hence the correct choice is (a).

22. Coefficient of real expansion of a liquid = coefficient of apparent expansion of the liquid + coefficient of volume (cubical) expansion of the vessel, i.e.

$$\gamma_r = \gamma_a + \gamma_v = \gamma_a + 3\alpha_v \qquad (\because \gamma_v = 3\alpha_v)$$

For vessel A: $\gamma_r = \gamma_1 + 3\alpha_1 = \gamma_1 + 3\alpha$ ($\because$ $\alpha_1 = \alpha$; given)

For vessel B: $\gamma_r = \gamma_2 + 3\alpha_2$

Since the liquid is the same, γ_r is the same. Hence

$$\gamma_1 + 3\alpha = \gamma_2 + 3\alpha_2$$

which gives $\alpha_2 = \dfrac{\gamma_1 - \gamma_2}{3} + \alpha$, which is choice (d).

23. Let x be the volume of mercury in the flask. Since the volume of air in the flask remains constant, the volume expansion of the flask must be exactly equal to the expansion of mercury in the flask.

$$\therefore \qquad V\gamma_g \Delta t = x\, \gamma_m \Delta t$$

or $$x = \frac{V\gamma_g}{\gamma_m}$$, which is choice (a).

24. When the rods are heated by Δt °C, the increase in length of steel rod and copper rod are

$$\Delta L_s = L_s\, \alpha_s\, \Delta t$$
$$\Delta L_c = L_c\, \alpha_c\, \Delta t$$

The difference between their lengths will remain constant at all temperatures if $\Delta L_s = \Delta L_c$, i.e. if

$$L_s\, \alpha_s\, \Delta t = L_c\, \alpha_c\, \Delta t$$

or $$\frac{L_s}{L_c} = \frac{\alpha_c}{\alpha_s}$$

Hence the correct choice is (d).

25. The moment of inertia of the disc about the given axis of rotation is

$$I = \frac{1}{2} MR^2 \qquad (1)$$

where M is the mass of the disc and R its radius.

If the disc is heated, it expands. Hence R increases. The resulting increase in I is obtained by partially differentiating (1).

$$\Delta I = \frac{1}{2} \times M \times 2\, R \Delta R \qquad (\because M = \text{constant})$$

or $$\Delta I = MR\Delta R$$

But $\Delta R = R\alpha\Delta t$. Therefore,

$$\Delta I = MR^2\alpha\Delta t \qquad (2)$$

Now, the angular momentum of the disc is given by

$$J = I\omega \qquad (3)$$

Since no external torque acts, J remains constant. Partially differentiating (3), we have

$$I\Delta\omega + \omega\Delta I = 0$$

or $$\Delta\omega = -\frac{\omega\Delta I}{I} \qquad (4)$$

Using (1) and (2) in (4) we get

$$\Delta\omega = -2\, \alpha\omega\Delta t$$

The negative sign indicates that the angular frequency decreases due to increase in temperature. Hence the correct choice is (d).

26. If V_0 is the volume of mercury at 0°C and A_0 the cross-sectional area of the tube at 0°C, then the length of the mercury column at 0°C is

$$L_0 = \frac{V_0}{A_0} \qquad (1)$$

The cross-sectional area at t°C is given by

$$A_t = A_0\, (1 + \beta t)$$

where β is the coefficient of superficial expansion of glass. Since $\beta = 2\alpha$, we have

$$A_t = A_0\, (1 + 2\, \alpha t)$$

If V_t is the volume of mercury at t°C, the length of the mercury column at t°C is

$$L_t = \frac{V_t}{A_t} = \frac{V_0 (1 + \gamma t)}{A_0 (1 + 2\alpha t)}$$

Using (1) we have

$$L_t = L_0\, (1 + \gamma t)\, (1 + 2\, \alpha t)^{-1}$$

Since t is not too high (~ 100°C) and γ and α are of the order of 10^{-5}, γt and αt will be very small compared to unity. Hence we can expand $(1 + 2\alpha t)^{-1}$ binomially and retain terms upto order αt. Thus

$$(1 + 2\, \alpha t)^{-1} = (1 - 2\, \alpha t)$$

$$\therefore \qquad L_t = L_0\, (1 + \gamma t)\, (1 - 2\, \alpha t)$$

or $$L_t = L_0\, [1 + (\gamma - 2\alpha)\, t]$$

where we have neglected term of $\alpha\gamma t^2$ which is negligibly small. Thus the correct choice is (d).

27. Due to rise in temperature, the brass scale expands. It will give lower readings because the graduations on the scale will be farther apart. If H is the barometric height at 0°C, the error in the reading of the scale at 40°C is

$$\Delta H = H\alpha\ \Delta t = 75 \times 20 \times 10^{-6} \times 40$$
$$= 0.06 \text{ cm}$$

$\therefore$ Atmospheric pressure at 27°C = $H + \Delta H = 75 + 0.06 = 75.06$ cm of Hg.

Thus the correct choice is (c).

28. Let the initial temperature be T and let M be the mass of the cube. Let A_0, ρ_0 and d_0 respectively be the base area of the cube, the density of the material of the cube and the depth upto which it is submerged in the liquid, the upthrust = $A_0\, d_0\, \rho_0\, g$. From the principle of floatation, weight = upthrust, i.e.

$$Mg = A_0 d_0 \rho_0 g$$

or
$$M = A_0 d_0 \rho_0 \quad (1)$$

When the temperature is raised to $(T + \Delta T)$, let A, ρ and d be the base area, density and depth at this temperature. Now, the coefficient of superficial (area) expansion is $\beta = 2\alpha$.

Hence

$$A = A_0(1 + \beta\ \Delta T)$$
$$= A_0\ (1 + 2\ \alpha\Delta T) \quad (2)$$

Also
$$\rho = \frac{\rho_0}{(1+\gamma\ \Delta T)} \quad (3)$$

The upthrust at temperature $(T + \mathrm{D}T) = Ad\rho g$. From the principle of floatation we have

$$Mg = Ad\rho g$$

or
$$M = Ad\rho \quad (4)$$

From (1) and (4), we get

$$Ad\rho = A_0 d_0 \rho_0$$

or
$$A\rho = A_0\rho_0 \qquad (\because\ d = d_0;\ \text{given})$$

Using (2) and (3) we have

$$A_0\ (1 + 2\ a\Delta T) \times \frac{\rho_0}{(1+\gamma\ \Delta T)} = A_0\rho_0$$

which gives $2\alpha = \gamma$. Thus the correct choice is (b).

29. $\Delta L = L\ \alpha\ \Delta T$. Since the wire is free to contract, the tension F must increase to produce the same change in length. Now

$$Y = \frac{F/A}{\Delta L/L}$$

$$\Rightarrow \qquad F = AY\ \frac{\Delta L}{L} = AY\ \alpha\ \Delta T$$

Speed of transverse waves in the wire is

$$v = \sqrt{\frac{F}{\mu}};\ \mu = \text{mass per unit length} = \rho\ AL/L = \rho A$$

$$= \sqrt{\frac{F}{\rho A}} = \sqrt{\frac{AY\alpha\,\Delta T}{\rho A}} = \sqrt{\frac{Y\alpha\,\Delta T}{\rho}}$$

30. Let V_0 be the volume of the metal at 0°C and V_t its volume at t°C. At temperature t the upthrust is

$$U_t = V_t\ \rho_t\ g$$

where ρ_t is the density of alcohol at temperature t. Now

$$V_t = V_0\ (1 + \gamma t)$$

where γ is the coefficeint of cubical expansion of alcohol and V_0 is the volume of alcohol displaced at temperature $t = 0$°C. Now the density of alcohol at temperature t is

$$\rho_t = \frac{\rho_0}{1+\gamma t}$$

where ρ_0 is the density of alcohol at $t = 0$°C. Hence

$$U_t = V_0\ (1 + \gamma t) \times \frac{\rho_0\, g}{(1+\gamma t)}$$
$$= V_0\ \rho_0\ g = U_0$$

where U_0 is the upthrust at 0°C. Since the upthrust is independent of temperature, $W_1 = W_2$.

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Choose the wrong statements from the following. A metallic circular disc having a circular hole at its centre rotates about an axis passing through its centre and perpendicular to its plane. When the disc is heated.

(a) its speed will decrease
(b) its diameter will decrease
(c) its moment of inertia will increase
(d) its speed will increase

2. Choose the wrong statements from the following. Two spheres made of the same material have the same diameter. One sphere is hollow and the other is solid. If they are heated through the same range of temperature,
(a) the hollow sphere will expand more than the solid sphere
(b) the solid sphere will expand more than the hollow sphere
(c) both spheres will expand equally
(d) the hollow sphere will not expand at all.

3. Two rods A and B of different metals have lengths L_A and L_B at a certain temperature. It is observed that rod A is 5 cm longer than rod B at all temperatures. If $\alpha_A = 1.0 \times 10^{-5}$ per °C and $\alpha_B = 1.5 \times 10^{-5}$ per °C, then
(a) L_A = 10 cm (b) L_A = 15 cm
(c) L_B = 10 cm (d) L_B = 5 cm

4. A uniform metallic circular disc of mass M and radius R, mounted on frictionless bearings, is rotating an angular frequency ω about an axis passing through its centre and perpendicular to its plane. The temperature of the disc is then increased by Δt. If α is the coefficient of linear expansion of the metal,
(a) the moment of inertia increases by $MR^2\alpha\Delta t$.
(b) the moment of inertia remains unchanged.
(c) the angular frequency increases by $2\alpha\omega\Delta t$.
(d) the angular frequency decreases by $2\alpha\omega\Delta t$.

5. A clock with a metallic pendulum gains 6 seconds each day when the temperature is 20°C and loses 12 seconds when the temperature is 40°C. Then
(a) the clock will keep correct time at temperature $\frac{80}{3}$ °C
(b) the clock will keep correct time at temperature $\frac{100}{3}$ °C.
(c) the coefficient of linear expansion of metal is 1.2×10^{-5} per °C.
(d) the coefficient of linear expansion of metal is 2.1×10^{-5} per °C.

6. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. When the temperature of the strip is increased by ΔT, it bends to form an arc of radius of curvature R. Then R is
(a) proportional to ΔT
(b) inversely proportional to ΔT
(c) proportional to $(\alpha_C - \alpha_B)$
(d) inversely proportional to $(\alpha_C - \alpha_B)$.

IIT, 1999

ANSWERS AND SOLUTIONS

1. Due to thermal expansion, the diameter of the disc as well as that of the hole will increase. Therefore, the moment of inertia will increase resulting in an increase in the angular speed. Hence the correct choices are (a) and (c).

2. Statements (a), (b) and (d) are wrong.

3. When the rods are heated by Δt °C, the increase in the length of the rods is

$$\Delta L_A = L_A \alpha_A \Delta t$$

and $$\Delta L_B = L_B \alpha_B \Delta t$$

$(L_A - L_B)$ will remain the same for all t, if $\Delta L_A = \Delta L_B$, i.e.

$$L_A \alpha A \Delta t = L_B \alpha_B \Delta t$$

$$\Rightarrow \quad \frac{L_A}{L_B} = \frac{\alpha_A}{\alpha_B} = \frac{1.5 \times 10}{1.0 \times 10^{-5}} = 1.5$$

$\Rightarrow L_A = 1.5\, L_B$. Also $(L_A - L_B) = 5$ cm. Thus

$1.5\, L_B - L_B = 5$ cm $\Rightarrow L_B = 10$ cm, and $L_A = 15$ cm. The correct choices are (b) and (c).

4. The moment of inertia of the disc about the given axis of rotation is

$$I = \frac{1}{2} MR^2 \qquad (1)$$

If the disc is heated, it expands. Hence R increases. The resulting increase in I is obtained by partially differentiating (1).

$$\Delta I = \frac{1}{2} \times M \times 2\, R \Delta R \qquad (\because M = \text{constant})$$

or $$\Delta I = MR\Delta R$$

But $\Delta R = R\alpha\Delta t$. Therefore,

$$\Delta I = MR^2\alpha\Delta t, \text{ which is choice (a)} \qquad (2)$$

Now, the angular momentum of the disc is given by

$$L = I\omega \qquad (3)$$

Since no external torque acts, L remains constant. Partially differentiating (3), we have

$$I\Delta\omega + \omega\Delta I = 0$$

or $$\Delta\omega = -\frac{\omega \Delta I}{I} \quad (4)$$

Using (1) and (2) in (4) we get

$$\Delta\omega = -2\,\alpha\omega\Delta t$$

The negative sign indicates that the angular frequency decreases due to increase in temperature. Thus the correct choices are (a) and (d).

5. Time taken for one oscillation of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

or $$T^2 = 4\pi^2 \times \frac{L}{g} \quad (1)$$

Partially differentiating, we get

$$2T\Delta T = 4\pi^2 \frac{\Delta L}{g} \quad (2)$$

Dividing (2) by (1), we get

$$\frac{\Delta T}{T} = \frac{\Delta L}{2L} = \frac{\alpha L \Delta t}{2L} = \frac{1}{2}\,\alpha\Delta t \qquad (\therefore\ \Delta L = \alpha L \Delta t)$$

where Δt is the change in temperature. Now, one day = 24 hours = 86400 s. Therefore, gain or loss of time in one day is

$$\Delta T = \frac{1}{2}\,\alpha\Delta t \times 86400 \text{ seconds}$$

Let t be the temperature at which the clock keeps correct time.

At 20°C, the gain in time is

$$6 = \frac{1}{2} \times \alpha \times (t - 20) \times 86400 \quad (3)$$

At 40°C, the loss in time is

$$12 = \frac{1}{2} \times \alpha \times (40 - t) \times 86400 \quad (4)$$

Dividing (4) by (3), we have

$$\frac{12}{6} = \frac{40 - t}{t - 20}$$

which gives $t = \frac{80}{3}$ °C. Using this value in Eq. (3), we have

$$6 = \frac{1}{2} \times \alpha \times \left(\frac{80}{3} - 20\right) \times 86400$$

which gives $\alpha = 2.1 \times 10^{-5}$ per °C.

Thus the correct choices are (a) and (d).

6. $L_C = L_0\,(1 + \alpha_C\,\Delta T)$ and $L_B = L_0\,(1 + \alpha_B\,\Delta T)$

$\therefore\quad L_C - L_B = L_0\,(\alpha_C - \alpha_B)\,\Delta T.$

Initially the bimetallic strip is straight, i.e. R is infinity. On heating, it bends and the value of R decreases. The amount of bending is proportional to $(\alpha_C - \alpha_B)$ and ΔT. Greater the bending, the smaller is the value of R. Hence the correct choices are (b) and (d).

SECTION III

Multiple Choice Questions Based On Passage

Question 1 to 3 are based on the following passage

Passage I

20,000 J of heat energy is supplied to a metal block of mass 500 g at atmospheric pressure. The initial temperature of the block is 30°C. Given specific heat of metal = 400 J kg^{-1} °C^{-1}, relative density of metal = 8.0, coefficient of volume expansion of metal = 8×10^{-5} °C^{-1} and atmospheric pressure = 10^5 Pa.

IIT, 2005

1. The final temperature of the block is

(a) 120°C (b) 130°C
(c) 140°C (d) 150°C

2. Work done by the block on the surroundings is

(a) 0.05 J (b) 0.1 J
(c) 1.0 J (d) 10 J

3. The change in internal energy is

(a) zero
(b) equal to 20,000 J
(c) slightly greater than zero
(d) slightly less than 20,000 J.

SOLUTION

1. $\Delta Q = ms\Delta T$. Therefore,

$$\Delta T = \frac{\Delta Q}{ms} = \frac{20{,}000\,\text{J}}{(0.5\,\text{kg}) \times (400\,\text{J kg}^{-1}\,\text{°C}^{-1})} = 100\text{°C}$$

$\therefore$ Final temperature = 100 + 30 = 130°C, which is choice (b).

2. Density of metal (ρ) = 8000 kg m^{-3}. Volume of the block is

$$V = \frac{m}{\rho} = \frac{0.5\,\text{kg}}{8000\,\text{kg m}^{-3}} = \frac{1}{16} \times 10^{-3}\ \text{m}^3$$

$\therefore$ Increase in volume = $\Delta V = \gamma V \Delta T$

$$= (8 \times 10^{-5}) \times \left(\frac{1}{16} \times 10^{-3}\right) \times 100$$

$$= \frac{1}{2} \times 10^{-6}\ \text{m}^3$$

Work done $\Delta W = P\Delta V = (10^5) \times \left(\frac{1}{2} \times 10^{-6}\right)$

= 0.05 J, which is choice (a).

3. Change in internal energy $\Delta U = \Delta Q - \Delta W$
= 20,000 – 0.05 = 19999.95 J
Thus the correct choice is (d).

Questions 4 to 6 are based on the following passage

Passage II

A rod of metal X of length 50.0 cm elongates by 0.10 cm when it is heated from 0°C to 100°C. Another rod of metal Y of length 80.0 cm elongates by 0.08 cm for the same rise in temperature. A third rod of length 50 cm, made by welding pieces of rods X and Y placed end to end, elongates by 0.03 cm when its temperature is raised from 0°C to 50°C.

4. The coefficients of linear expansion of metal X and of metal Y are in the ratio of

(a) 2 : 1 (b) 3 : 2
(c) 3 : 1 (d) 4 : 3

5. The length of the rod of metal X in the composite piece is

(a) 10 cm (b) 20 cm
(c) 30 cm (d) 40 cm

6. The length of the rod of metal Y in the composite piece is

(a) 10 cm (b) 20 cm
(c) 30 cm (d) 40 cm

SOLUTION

4. For rod X, $\alpha_x = \frac{\Delta L}{L\Delta t} = \frac{0.10}{50.0 \times 100}$

$= 2.0 \times 10^{-6}$ per °C.

For rod Y, we get $\alpha_y = 1.0 \times 10^{-6}$ per °C. So the correct choice is (a).

5. In the composite rod, $L_x + L_y = 50.0$ cm. When this rod is heated by 50°C, let the new lengths be L'_x and L'_y. Given $L'_x + L'_y = 50.0 + 0.03 = 50.03$ cm. Here $L'_x = L_x\,(1 + \alpha_x \times 50)$ and $L'_y = L_y(1 + \alpha_y \times 50)$. Therefore,

$$50.03 = L_x + L_y + (L_x\,\alpha_x + L_y\,\alpha_y)\,50$$
$$= 50 + (L_x\,\alpha_x + L_y\,\alpha_y)\,50$$

Substituting the values of α_x and α_y and noting that $L_x + L_y = 50$ cm, we get $L_x = 10$ cm, which is choice (a).

6. $L_y = 50 - L_x = 50 - 10 = 40$ cm, which is choice (d).

SECTION IV

Integer Answer Type

1. A composite rod is made by joining a copper rod, end to end, with a second rod of a different material but of the same cross-section. At 25°C, the composite rod is 1 m in length of which the length of the copper rod is 30 cm. At 125°C the length of the composite rod increases by 1.91 mm. The coefficient of linear expansion of copper is $\alpha = 1.7 \times 10^{-5}$ per °C and that of the second rod is $\beta = n \times 10^{-5}$ per °C. Find the value of n.

IIT, 1979

SOLUTION

1. Length of the second rod at 25°C = 70 cm.
Length of copper rod at 125°C

$= 30 \times (1 + 1.7 \times 10^{-5} \times 100)$

$= 30.051$ cm

$\therefore$ Increase in the length of copper rod = 0.051 cm

Increase in the length of second rod = $70 \times \beta \times 100$
= $7000\,\beta$ cm
Total increase in length = 0.051 cm + $7000\,\beta$ cm
= 0.191 cm (given) which gives $\beta = 2 \times 10^{-5}$ per °C.
Thus the value of $n = 2$.

16 Chapter Measurement of Heat

REVIEW OF BASIC CONCEPTS

16.1 HEAT ENERGY

Heat is a form of energy. It is, therefore, measured in energy units. The SI unit of heat is joule (J). Another unit commonly used is the calorie. A calorie is the amount of heat required to raise the temperature of 1 g of water through 1°C. Experiments have shown that 4.18 J of mechanical work produce one calorie of heat.
Thus

$$1 \text{ calorie} = 4.18 \text{ joules}$$

or $$1 \text{ cal} = 4.18 \text{ J}$$

The ratio $$J = \frac{\text{work done}}{\text{heat produced}} = 4.18 \text{ J per cal}$$

is called the mechanical equivalent of heat.

16.2 CALORIMETRY

If two substances having different temperatures are brought into thermal contact, heat energy will flow from the hotter to the colder substance. The flow of heat energy will continue till the temperatures are equalized. The common temperature at thermal equilibrium is called the equilibrium temperature. If no heat energy is allowed to escape to the surroundings, the amount of heat energy gained by the initially colder body is equal to the amount of heat energy lost by the initially hotter body, i.e.

Heat gained by one body = heat lost by the other body.
This is the basic principle of calorimetry.

The heat energy Q needed to raise the temperature through ΔT of a mass m of a substance of specific heat capacity s is given by

$$Q = ms\,\Delta T$$

Specific Heat Capacity

If $m = 1$ unit and $\Delta T = 1$ unit, then $s = Q$. Hence the specific heat of a substance is the amount of heat required to raise the temperature of a unit mass of the substance through a unit degree. A commonly used unit of s is cal g^{-1} °C^{-1}. In the SI system s is expressed in J kg^{-1} K^{-1}. The two units are related as

$$1 \text{ cal g}^{-1} \text{ °C}^{-1} = 4.18 \text{ Jg}^{-1} \text{ °C}^{-1}$$

($\because$ 1 cal = 4.18 J)

or $$1\text{cal g}^{-1} \text{ °C}^{-1} = 4180 \text{ J kg}^{-1} \text{ °C}^{-1}$$

Since the size of a degree on the celsius scale is equal to that on the kelvin scale, a temperature *difference* of, say, 1 °C is equal to a temperature difference of 1 K. Thus

$$1 \text{ cal g}^{-1} \text{ °C}^{-1} = 4180 \text{ J kg}^{-1} \text{ K}^{-1}$$

With our definition of a calorie, the specific heat capacity of water = 1 cal g^{-1} °C^{-1} or 4180 J kg^{-1} K^{-1}.

Molar Specific Heat

The molar specific heat C of a substance is the amount of heat energy required to raise the temperature of 1 mole of the substance through 1 K. It is expressed in J mol^{-1} K^{-1}.

Specific heat (s) and molar specific heat (C) are related as

$$s = \frac{C}{m}$$

where m is the number of kilograms per mole in the substance.

The molar specific heat of a gas at constant volume (C_v) is the amount of heat energy required to raise the temperature of 1 mole of the gas through 1 K when its volume is kept constant. The molar specific heat of a gas at constant pressure (C_p) is the amount of heat energy required to raise the temperature of 1 mole of the gas through 1 K when its pressure is kept constant.

The two specific heats of an ideal gas are related as

$$C_p - C_v = R$$

where R is the universal gas constant and its value is

$$R = 8.315 \text{ J mol}^{-1} \text{ K}^{-1}$$

16.3 LATENT HEAT

The heat energy supplied to a substance to change from solid to liquid state or from liquid to gaseous state is not registered by a thermometer as the heat energy is used up in bringing about a change of state. Hence it is called latent (or hidden) heat. A substance has two latent heats.

1. The latent heat of fusion (or melting) of a substance is the heat energy required to convert a unit mass of a substance from the solid to the liquid state, without change of temperature. The latent heat of fusion of ice is 3.36×10^5 J kg^{-1} or 80 cal g^{-1}.

2. The latent heat of vaporisation (or boiling) of a substance is the heat energy required to convert a unit mass of the substance from the liquid to the gaseous state, without change of temperature. The latent heat of vaporisation of steam is 2.26×10^6 J kg^{-1} or 540 cal g^{-1}.

EXAMPLE 16.1

A 12 kW drilling machine is used to drill a hole in a metal block of mass 10 kg. Assuming that 25% power is lost in the machine, calculate the rise in temperature of the block in 2 minutes. Specific heat capacity of metal = 0.4 J g^{-1} K^{-1}.

SOLUTION

Useful power available = 75% of 12 kW $= \frac{3}{4} \times 12$ kW = 9 kW = 9000 W

Heat energy consumed in 2 minutes (= 120 s) is

$$Q = (9000 \times 120) \text{ J}$$

Now $s = 0.4 \text{ J g}^{-1} \text{ K}^{-1} = 0.4 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$

$\therefore$ Rise in temperature is $\Delta T = \frac{Q}{ms}$

$$= \frac{9000 \times 120}{10 \times (0.4 \times 10^3)} = 270°\text{C}$$

EXAMPLE 16.2

A copper block of mass 3.35 kg is heated to 500°C and then placed on a large block of ice. What is the maximum mass of ice that can melt? Specific heat capacity of copper = 390 J kg^{-1} K^{-1} and latent heat of fusion of water = 335 J g^{-1}.

SOLUTION

The amount of heat energy in copper block at 500°C is

$$Q = ms\Delta T$$
$$= (3.35 \times 390 \times 500) \text{ J}$$

Now $L = 335 \text{ J g}^{-1} = 335 \times 10^3 \text{ J kg}^{-1}$.

The maximum mass of ice that can melt is

$$m = \frac{Q}{L} = \frac{3.35 \times 390 \times 500}{335 \times 10^3}$$
$$= 1.95 \text{ kg}$$

EXAMPLE 16.3

16 g of oxygen is heated at constant volume from 25°C to 35°C. Find the amount of heat energy required. Given $C_v = 20$ J mol^{-1} K^{-1}.

SOLUTION

Mass of 1 mole of oxygen = 32 g

$\therefore$ Number of moles in 16 g of oxygen is

$$n = \frac{16}{32} = \frac{1}{2}$$

Heat energy required is

$$Q = n \, C_v \, \Delta T$$
$$= \frac{1}{2} \times 20 \times (35 - 25) = 100 \text{ J}$$

EXAMPLE 16.4

290 J of heat energy is required to raise the temperature of 7 g of nitrogen by 40°C. Find molar specific heat of nitrogen at constant pressure.

SOLUTION

Number of moles in 7 g of oxygen is $n = \frac{7}{28} = \frac{1}{4}$

$\therefore$ $Q = n \, C_p \, \Delta T$

$\Rightarrow$ $C_p = \frac{Q}{n\Delta T} = \frac{290}{\frac{1}{4} \times 40} = 29 \text{ J mol}^{-1} \text{ K}^{-1}$

EXAMPLE 16.5

200 g of water at 25°C is added to 75 g of ice at 0°C in an insulated vessel. What is the final temperature of the mixture?

SOLUTION

Heat energy of 200 g of water at 25°C is

$$Q_1 = ms\Delta T = 200 \times 1 \times 25$$
$$= 5000 \text{ cal}$$

Heat energy required to melt 75 g of ice at 0°C is

$$Q_2 = mL = 75 \times 80 = 6000 \text{ cal}$$

Since $Q_1 < Q_2$, the whole of ice will not melt. Hence, the final temperature of mixture is 0°C.

NOTES

If 1 kg of ice at 0°C is mixed with 1 kg of steam at 100°C, the equilibrium temperature is 100°C, 0.665 kg of steam will be left and 1.335 kg of water will be formed.

Water of mass m_w at t°C is mixed with ice of mass m_i at 0°C.

(a) If $m_w = \dfrac{L_f m_i}{t}$, whole of ice will melt and the final temperature = 0°C.

(b) If $m_w < \dfrac{L_f m_i}{t}$, the whole of ice will not melt, final temperature = 0°C and mass of ice melted is $m'_i = \dfrac{m_w t}{L_f}$. Amount of ice left = $m_i - m'_i$.

(c) If $m_w > \dfrac{L_f m_i}{t}$, the whole of ice will melt and final temperature = $\dfrac{m_w t - L_f m_i}{m_w + m_i} > 0$°C

EXAMPLE 16.6

According to the theory of specific heat of solids at very low temperatures (close to absolute zero), the specific heat s of a solid varies with absolute temperature T as

$$s = kT^3$$

where k is a constant whose value depends upon the material of the solid. Find the heat energy required to raise the temperature of 200 g of the solid from 1 K to 4 K.

SOLUTION

The amount of heat energy required to raise the temperature of mass m of the solid by dT kelvin is given by

$$dQ = msdT$$

Heat energy required to raise the temperature of the solid from $T_1 = 1$ K to $T_2 = 4$ K is

$$Q = \int_{T_1}^{T_2} msdT = mk\int_{T_1}^{T_2} T^3 dT$$
$$= \frac{mk}{4}\left[T_2^4 - T_1^4\right]$$
$$= \frac{0.2 \times k}{4}\left[(4)^4 - (1)^4\right]$$
$$= 12.75\ k \text{ joule}$$

EXAMPLE 16.7

Three liquids 1, 2 and 3 of masses $m_1 = m$, $m_2 = 2m$ and $m_3 = 3m$ are at temperatures of 10°C, 18°C and 30°C. When liquids 1 and 2 are mixed, the equilibrium temperature is 16°C. When liquids 2 and 3 are mixed, the equilibrium temperature is 22°C. Find the equilibrium temperature when liquids 1 and 3 are mixed. Assume that there is no loss of heat to the surroundings.

SOLUTION

Let s_1, s_2 and s_3 be the specific heat capacities of liquids 1, 2 and 3 respectively.

When liquids 1 and 2 are mixed,

heat gained by 1 = heat lost by 2

$$\Rightarrow \quad m_1 s_1(16 - 10) = m_2 s_2(18 - 16)$$
$$\Rightarrow \quad ms_1(6) = 2ms_2(2)$$
$$\Rightarrow \quad s_1 = \frac{2}{3}\ s_2 \qquad (1)$$

When liquids 2 and 3 are mixed

heat gained by 2 = heat lost by (3)

$$\Rightarrow \quad 2ms_2(22 - 18) = 3ms_3(30 - 22)$$
$$\Rightarrow \quad s_3 = \frac{s_2}{3} \qquad (2)$$

Let T be the equilibrium temperature. When liquids 1 and 3 are mixed.

Heat gained by 1 = heat lost by 3

$$\Rightarrow \quad ms_1(T - 10) = 3ms_3(30 - T) \qquad (3)$$

Using (1) and (2) in (3), we have

$$\frac{2}{3} \times s_2(T - 10) = 3 \times \frac{s_2}{3}\ (30 - T)$$

$$\Rightarrow \quad T = 22°\text{C}$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. 300 g of water at 25°C is added to 100 g of ice at 0°C. The final temperature of the mixture is

(a) $-\frac{5}{3}$ °C (b) $-\frac{5}{2}$ °C

(c) – 5 °C (d) 0 °C

IIT, 1989

2. 100 g of ice at 0°C is mixed with 100 g of water 80°C. The final temperature of the mixture will be

(a) 0°C (b) 20°C

(c) 40°C (d) 60°C

3. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter rises to 80°C. The mass of steam condensed in kilogram is

(a) 0.13 (b) 0.065

(c) 0.260 (d) 0.135

IIT, 1986

4. A geyser, operating on LPG (liquified petroleum gas) heats water flowing at the rate of 3.0 litres per minute, from 27°C to 77°C. If the heat of combustion of LPG is 4.0×10^4 J g^{-1}, how much fuel in grams is consumed per minute?

(a) 15.25 (b) 15.5

(c) 15.75 (d) 16

5. A copper block of mass 2 kg is heated to a temperature of 500°C and then placed in a large block of ice at 0°C. What is the maximum amount of ice that can melt? The specific heat of copper is 400 J kg^{-1} $°C^{-1}$ and latent heat of fusion of water is 3.5×10^5 J kg^{-1}.

IIT, 2005

(a) $\frac{4}{3}$ kg (b) $\frac{6}{5}$ kg

(c) $\frac{8}{7}$ kg (d) $\frac{10}{9}$ kg

6. How much heat energy is joules must be supplied to 14 grams of nitrogen at room temperature to raise its temperature by 40°C at constant pressure. Molar mass of nitrogen = 28 and R J K^{-1} mol^{-1} is the gas constant.

(a) 50 R (b) 60 R

(c) 70 R (d) 80 R

7. The temperature of a liquid does not increase during boiling. The heat energy supplied during this process,

(a) increases the kinetic energy of the molecules of the liquid

(b) increases the potential energy of the molecules

(c) increases both the kinetic and potential energy of the molecules

(d) is merely wasted since no increase occurs in the total energy of the molecules.

8. A block of ice at – 10°C is slowly heated and converted to steam at 100°C. Which of the following curves represents the phenomenon qualitatively?

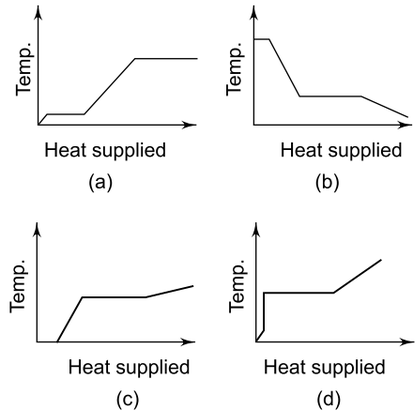


Fig. 16.1

IIT, 2000

9. 2 kg of ice at –20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water in the vessel. It is given that the specific heats of water and ice are 1 kcal/kg/°C and 0.5 kcal/kg/°C re-spectively and the latent heat of fusion of ice is 80 kcal/kg.

(a) 7 kg (b) 6 kg

(c) 4 kg (d) 2 kg

IIT, 2003

10. A metal sphere of radius r and specific heat S is rotated about an axis passing through its centre at a speed of n rotations per second. It is suddenly stopped and 50% of its energy is used in increasing

its temperature. Then the rise in temperature of the sphere is:

(a) $\frac{2\pi^2 n^2 r^2}{5S}$ (b) $\frac{\pi^2 n^2}{10 r^2 S}$

(c) $\frac{7}{8}\ \pi r^2\ n^2\ S$ (d) $\frac{5(\pi r n)^2}{14S}$

11. If there are no heat losses, the heat released by the condensation of x grams of steam at 100°C into water at 100°C converts y grams of ice at 0°C into water at 100°C. The ratio y/x is

(a) 1 (b) 2

(c) 3 (d) 4

12. 5 g of water at 30°C and 5 g of ice at –20°C are mixed together in a calorimeter. What is the final temperature of the mixture. Given, specific heat of ice = 0.5 cal g^{-1} $(°C)^{-1}$ and latent heat of fusion of ice = 80 cal g^{-1}.

(a) –5°C (b) 0°C

(c) + 5°C (d) + 10°C

13. A metal block of mass 10 kg is dragged on a horizontal rough road with a constant speed of 5 ms^{-1}. If the coefficient of friction between the block and the road is 0.5, the rate at which heat is generated in (J s^{-1}) is

(a) 100 (b) 245

(c) 9.8 (d) 10

14. Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represents the veriation of temperature with time? (see Fig. 16.2).

IIT, 2004

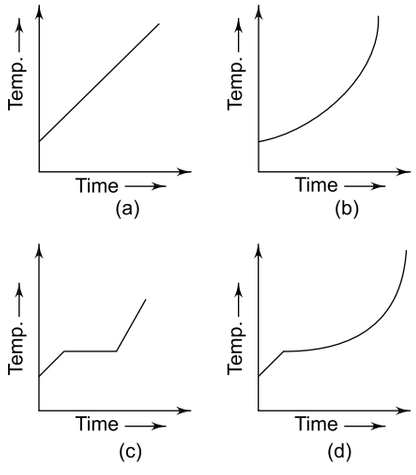


Fig. 16.2

15. A 1 kW electric kettle contains 2 litre water at 27°C. The kettle is operated for 10 minutes. If heat is lost to the surroundings at a constant rate of 160 J/sec, the temperature attained by water in 10 minutes will be (specific heat of water = 4.2 kJ/kg°C)

(a) 57°C (b) 67°C

(c) 77°C (d) 87°C

IIT, 2005

16. In an industrial process 10 kg of water per hour is to be heated from 20°C to 80°C. To do this, steam at 150°C is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at 90°C. How many kg of steam are required per hour? Specific heat of steam = 1 kilo cal kg^{-1} $°C^{-1}$. Latent heat of steam = 540 kilo cal kg^{-1}.

(a) 1 kg (b) 2 kg

(c) 3 kg (d) 4 kg

17. With what velocity should a lead bullet at an initial temperature of 30°C strike a target so that it just melts? Assume that 84% of the heat produced is absorbed by the bullet. Specific of lead = 0.03 kilo cal kg^{-1} $(°C)^{-1}$, latent heat of lead = 6 kilo cal kg^{-1} and melting point of lead = 330°C.
(1 kilo calorie = 4.2×10^3 joule)

(a) $100\sqrt{5}$ ms^{-1} (b) $100\sqrt{2}$ ms^{-1}

(c) $100\sqrt{10}$ ms^{-1} (d) $100\sqrt{15}$ ms^{-1}

18. According to the theory of specific heats of solids at extremely low temperature (close to absolute zero), the specific heat s of a solid varies with absolute temperature T as

$$s = cT^3$$

where c is a constant depending on the material of the solid. The heat energy required to raise the temperature of 0.1 kg of the solid from 0 K to 4 K is.

(a) 4.2 c joule (b) 6.4 c joule

(c) 8.4 c joule (d) 12.6 c joule

19. 400 g of ice at 253 K is mixed with 0.05 kg of steam at 100°C. Latent heat of vaporisation of steam = 540 cal/g. Latent heat of fusion of ice = 80 cal/g. Specific heat of ice = 0.5 cal/g °C. The resultant temperature of the mixture is

(a) 273 K (b) 300 K

(c) 330 K (d) 373 K

IIT, 2007

20. Two litres of water (density = 1 g/ml) in an open-lid insulated kettle is heated by an electric heater of power 1 kW. The heat is lost from the lid at the rate of 160 J/s. The time taken for heating water (specific heat capacity 4.2 kJ kg^{-1} K^{-1}) from 20°C to 75°C is

(a) 340 s (b) 550 s
(c) 620 s (d) 760 s

IIT, 2005

ANSWERS

1. (d)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (c)
7. (b)	**8.** (a)	**9.** (b)	**10.** (a)	**11.** (c)	**12.** (b)
13. (b)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (d)	**18.** (b)
19. (a)	**20.** (b)				

SOLUTION

1. Let the temperature of the mixture be t°C. Heat lost by water in calories = $300 \times 1 \times (25 - t) = 7500 - 300t$
Heat in calories required to melt 100 g of ice

$$= 100 \times 80 = 8000$$

Now Heat lost = heat gained

or $7500 - 300t = 8000$ or $t = -\frac{5}{3}$ °C

Since t is negative, the water at 25°C cools to 0°C and melts a part of ice at 0°C.

$\therefore$ Heat lost = $300 \times 1 \times (25 - 0) = 7500$ cal. Hence only a part of the ice melts and resulting temperature is 0°C. Hence the correct choice is (d).

2. The amount of heat required to convert 100 g of ice at 0°C into water at 0°C = $100 \times 80 = 8000$ calories. This is precisely the amount of heat lost by 100 g of water at 80°C to bring its temperature down to 0°C. Therefore, the temperature of the mixture remains 0°C. Hence the correct choice is (a).

3. Let the mass of steam condensed be m kg. The latent of vaporisation of water = 556 k cal/kg. Therefore, heat lost by steam = $556\ m + m\ (100 - 80) = 576\ m$ kcal. Heat gained by calorimeter and water = $(1.1 + 0.02)\ (80 - 15) = 72.8$ kcal. Now, heat lost = heat gained, i.e.

$$576\ m = 72.8 \text{ or } m = 0.13 \text{ kg}$$

4. Volume of water flowing per minute = 3.0 litres min^{-1} = 3000 cm^3 min^{-1}
Density of water = 1 g cm^{-3}.

$\therefore$ Mass of water flowing per minute

$$= 3000 \text{ g min}^{-1} = 3.0 \text{ kg min}^{-1}$$

Rise in temperature = 77 – 27 = 50°C

Specific heat of water = 4200 J kg^{-1} $°C^{-1}$

$\therefore$ Heat energy supplied to the geyser per minute

$$= 3.0 \times 4200 \times 50$$

$$= 63 \times 10^4 \text{ J min}^{-1}$$

Now, heat of combustion of LPG

$$= 4.0 \times 10^4 \text{ J g}^{-1} = 4.0 \times 10^7 \text{ J kg}^{-1}$$

$\therefore$ Amount of fuel consumed per minute

$$= \frac{63 \times 10^4 \text{J min}^{-1}}{4.0 \times 10^7 \text{ J kg}^{-1}}$$

$$= 15.75 \times 10^{-3} \text{ kg min}^{-1}$$

$$= 15.75 \text{ g min}^{-1}$$

5. Heat energy in copper block = $2 \times 400 \times 500 = 4 \times 10^5$ J. The amount of ice that melts will be maximum if the entire heat energy of the copper block is used up in melting ice. Now, 3.5×10^5 J of heat energy is needed to melt 1 kg of ice into water. Therefore, the amount of ice melted by 4×10^5 J of heat energy is

$$\frac{4 \times 10^5 \text{J}}{3.5 \times 10^5 \text{J kg}^{-1}} = \frac{8}{7} \text{ kg}$$

Hence the correct choice is (c).

6. Number of moles in 14 grams of nitrogen (n) = $14/28 = \frac{1}{2}$. Since nitrogen is diatomic $C_p = 7R/2$.
Therefore, amount of heat energy supplied

$$= n \times C_p \times \Delta\theta = \frac{1}{2} \times \frac{7R}{2} \times 40 = 70R \text{ joules}$$

Hence the correct choice (c)

7. The correct choice is (b).

8. When heat is supplied, firstly the temperature of ice increases from – 10°C to 0°C. At 0°C, ice starts melting at a constant temperature. When the whole

of ice has melted into water, the temperature of water will increase from 0°C to 100°C. At 100°C, again the temperature becomes constant due to the conversion of liquid water into water vapour (steam) at 100°C. Hence the correct graph is (a).

9. Let m kg be the mass of ice melted into water. Heat lost by 5 kg of water = 5 kg × 1 kcal/kg/°C × 20°C = 100 kcal. Heat gained = m kg × 80 kcal/kg + 2 kg × 0.5 kcal/kg/°C × 20°C = 80 m kcal + 20 kcal. Now, heat gained = heat lost. Therefore,

$$80\ m + 20 = 100$$

or m = 1 kg. Therefore, final mass of water = 5 kg + 1 kg = 6 kg. Hence the correct choice is (b).

10. Moment of inertia of the sphere $I = \frac{2}{5}\ Mr^2$. Given ω = n rotations per second = $2\pi n$ rad s^{-1}. The kinetic energy is

$$\text{KE} = \frac{1}{2}\ I\omega^2 = \frac{1}{2} \times \frac{2}{5}\ Mr^2 \times (2\pi n)^2$$

$$= \frac{4}{5}\ M\pi^2 r^2 n^2$$

Since half of KE is converted into heat energy, we have

$$dQ = \frac{1}{2} \times \text{KE} = \frac{2}{5}\ M\pi^2\ r^2\ n^2$$

Now $dQ = MSdT$ which gives

$$dT = \frac{dQ}{MS} = \frac{\frac{2}{5} M\pi^2\ r^2\ n^2}{MS} = \frac{2\pi^2\ r^2\ n^2}{5S}$$

Hence the correct choice is (a).

11. The latent heat of vaporisation of water is very nearly 540 calories per gram. Therefore heat released in the condensation of x gram of steam = 540 x calories. The latent heat of fusion of ice is very nearly 80 calories. Therefore, heat required to convert y gram of ice at 0°C to water at 100°C = 80 y + 100 y = 180 y calories. Thus

$$180\ y = 540\ x$$

or

$$\frac{y}{x} = 3$$

Hence the correct choice is (c).

12. Let us first calculate the heat required to (i) raise the temperature of ice from – 20°C to 0°C and (ii) melt the ice at 0°C into water at 0°C.

Heat required for step (i) = 5 × 0.5 × {0 – (–20)} = 50 cal

Heat required for step (ii) = 5 × 80 = 400 cal

When water at 30°C is allowed to cool to 0°C, the heat given out

$$= 5 \times 1 \times (30 - 0) = 150 \text{ cal}$$

Thus it is clear that all the ice cannot melt and the system will remain at 0°C. Since only (150 – 50) = 100 cal are available for melting ice, the mass of ice melted = 100/80 = 1.25 g.

Thus finally the mixture consists of (5 + 1.25) = 6.25 g of water at 0°C and (5 – 1.25) = 3.75 g of ice at 0°C. Hence the final temperature is 0°C.

Hence the correct choice is (b).

13. Heat energy generated per second = work done per second against friction = force of friction × distance moved in 1s

$$= \mu mgx$$

$$= 0.5 \times 10 \times 9.8 \times 5 = 245 \text{ J s}^{-1}$$

Hence the correct choice is (b).

14. Liquid oxygen will undergo a change of phase when heated from 50 K to 300 K. During the phase change the temperature will not change. When all the liquid oxygen has changed to gaseous state, the temperature will increase because rate of heating is constant (see Fig.16.3). Hence the correct graph is (c).

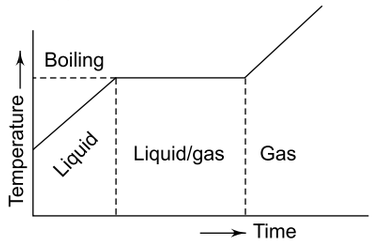


Fig. 16.3

15. Power (P) = 1 kW = 1000 W, mass of water (m) = 2 kg, specific heat of water (s) = 4200 J/kg °C. Time t = 10 min = 600 s. Initial temperature T_1 = 27°C. Heat energy supplied by kettle in 600 s = Pt = 1000 × 600 = 600,000 J. Heat energy lost in 600 s = 160 × 600 = 96,000 J. Therefore, heat energy used up in heating the water is

$$H = 600{,}000 - 96{,}000 = 504{,}000 \text{ J}$$

Now $H = ms\ \Delta T = ms\ (T_2 - T_1) = 2 \times 4200 \times (T_2 - 27)$, where T_2 is the final temperature.

$$\therefore \quad 2 \times 4200 \times (T_2 - 27) = 504{,}000$$

which gives T_2 = 87°C, which is choice (d).

16. Let the mass of steam required per hour be m kg. Heat gained by water in boiler per hour is

$$= 10 \text{ kg} \times 1 \text{ kilo cal kg}^{-1}\ ^\circ\text{C}^{-1} \times (80 - 20)^\circ\text{C} = 600 \text{ kilo cal} \quad (1)$$

Heat lost by steam per hour is
= heat needed to cool m kg of steam from 150°C to 100°C + heat needed to convert m kg of steam at 100°C into water at 100°C + heat needed to cool m kg of water from 100°C to 90°C

$$= m \times 1 \times (150 - 100) + m \times 540 + m \times 1 \times (100 - 90)$$

$$= 50\ m + 540\ m + 10\ m$$

$$= 600\ m \text{ kilo cal} \quad (2)$$

Heat lost = heat gained. Equating (1) and (2) we have

$$600\ m = 600$$

or $m = 1$ kg, which is choice (a).

17. Let the mass of the bullet be m kg and its velocity be v ms^{-1}. Before striking the target, the kinetic energy of the bullet is $\frac{1}{2}\ mv^2$ joule which is converted into heat energy when the bullet strikes the target. Thus heat energy produced is

$$Q = \frac{1}{2}\ mv^2 \text{ joule}$$

$$= \frac{\frac{1}{2}mv^2 \text{ joule}}{4.2 \times 10^3 \text{ joule/kilo cal}}$$

$$= \frac{mv^2}{8.4 \times 10^3} \text{ kilo cal}$$

Heat energy absorbed by the bullet is

$$Q' = 84\% \text{ of } Q = 0.84\ Q = \frac{0.84 \times mv^2}{8 \cdot 4 \times 10^3} \text{ kilo cal}$$

In order that heat energy Q' melts the bullet, it should be sufficient to raise the temperature of the bullet from 30°C upto its melting point (330°C) and then to supply the latent for melting. Hence

$$\frac{0.84 \times mv^2}{8.4 \times 10^3} = m \times 0.03 \times (330 - 30) + m \times 6$$

$$= 9\ m + 6\ m = 15\ m$$

which gives $v = 100\sqrt{15}$ ms^{-1}

Thus the correct choice is (d).

18. The specific heat varies with temperature; it is not constant. The amount of heat energy required to raise the temperature of the solid of mass m through dT kelvin is

$$dQ = msdT$$

$\therefore$ Heat energy needed to raise the temperature of the solid from 0 K to 4 K is

$$Q = \int_0^4 msdT = m \int_0^4 cT^3 dT$$

$$= mc \int_0^4 T^3 dT = mc \left| \frac{T^4}{4} \right|_0^4$$

$$= mc \left(\frac{4^4}{4} \right) = 0.1 \times c \times (4)^3$$

$$= 6.4\ c \text{ joule.}$$

So the correct choice is (b).

19. Heat required to melt the whole of ice is

$$Q_1 = ms\Delta T + mL$$

$$= 400 \times 0.5 \times 20 + 400 \times 80 \quad (\therefore\ 253 \text{ K} = -20^\circ\text{C})$$

$$= 4000 + 32000 = 36000 \text{ cal}$$

The maximum heat released by steam when the whole of it (= 50 g) is converted into water at 0°C is

$$Q_2 = ms\Delta T + mL$$

$$= 50 \times 1 \times 100 + 540 \times 50$$

$$= 5000 + 27000 = 32000 \text{ cal}$$

Since Q_2 is less than Q_1, the whole of ice will not melt. Hence the final temperature of the mixture will be 0°C or 273 K.

20. Mass of 2 litres of water = 2 kg. Heat energy needed to raise the temperature of 2 kg of water from 20°C to 75°C is

$$Q = 2 \times (4.2 \times 10^3) \times 55 = 4.62 \times 10^5 \text{ J}$$

If t is the time taken, heat energy supplied by the heater in time t is

$$Q_1 = (\text{power} \times \text{time}) = 1000\ t \text{ joule}$$

Heat energy lost in time t is

$$Q_2 = 160\ t \text{ joule}$$

Heat energy available for heating water is

$$Q' = Q_1 - Q_2 = 840 \text{ J}$$

Equating $Q = Q'$, we get $t \simeq 550$ s.

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. A source of heat supplies heat at a constant rate to a solid cube. The variation of the temperature of the cube with heat supplied is shown in Fig. 16.4.
   (a) Portion *BC* of the graph represents conversion of solid into liquid.
   (b) Portion *BC* of the graph represents conversion of solid into vapour.
   (c) Portion *DE* of the graph represents conversion of vapour into liquid.
   (d) Portion *DE* of the graph represents conversion of liquid into vapour

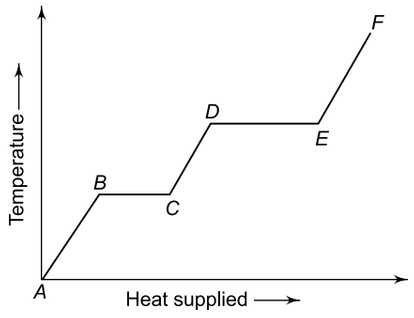


Fig. 16.4

2. In Q.1 above, the slope of portion *CD* of the graph shown in Fig. 16.4 gives
   (a) latent heat of fusion
   (b) latent heat of vaporisation
   (c) thermal capacity of liquid
   (d) thermal capacity of vapour

3. In Q.1 above, the slope of the portion *EF* of graph shown in Fig. 16.4 gives
   (a) specific heat of the vapour
   (b) specific heat of the liquid
   (c) thermal capacity of liquid
   (d) thermal capacity of vapour.

4. In Fig. 16.4, it is observed that $DE = 3\ BC$. This means that
   (a) the thermal capacity of the vapour is 3 times that of the liquid
   (b) the specific heat of the vapour is 3 times that of the liquid
   (c) the latent heat of vaporisation of the liquid is 3 times the latent heat of fusion of the solid.
   (d) the latent heat of fusion of the solid is 3 times the latent heat of vaporisation of the liquid.

5. A bullet of mass 50g and specific heat capacity 800 J kg^{-1} K^{-1} is initially at a temperature 20°C. It is fired vertically upwards with a speed of 200 ms^{-1} and on returning to the starting point strikes a lump of ice at 0°C and gets embedded in it. Assume that all the energy of the bullet is used up in melting. Neglect the friction of air. Latent heat of fusion of ice = 3.36×10^5 Jkg^{-1}.
   (a) Energy of bullet used in melting is 1000 J.
   (b) The mass of ice method = 5 g
   (c) The mass of ice melted is slightly greater than 5 g.
   (d) The mass of ice melted is less than 5 g.

6. A metal ball of mass 1 kg and specific heat capacity *s* falls from a height of 10 cm and bounces to a height of 4 m. If all dissipated energy *(Q)* is absorbed by the ball as heat, its temperature rises by 0.075 K. Take $g = 10\ ms^{-2}$.
   (a) $Q = 60$ J (b) $Q = 100$ J
   (c) $s = 800$ J kg^{-1} K^{-1} (d) $s = 1333$ J kg^{-1} K^{-1}

ANSWERS AND SOLUTIONS

1. The correct choices are (a) and (d). The heat supplied is the latent heat.

2. The slope of portion *CD* of the graph gives the amount of heat supplied by unit rise in temperature of the liquid. This, by definition, is the thermal capacity of the liquid. Hence the correct choice is (c).

3. The correct choice is (d).

4. The portion *BC* of the graph represents the conversion of solid into liquid, temperature remaining the same. The portion *DE* represents the conversion of liquid into vapour at the same temperature. The heat supplied in the two cases is latent (hidden). Hence the correct choice is (c).

5. If the friction offered by air is neglected, the speed of the bullet on returning to the starting point will be equal to its initial speed $v = 200\ ms^{-1}$. The kinetic energy of the bullet is

$$\text{K.E.} = \frac{1}{2}\, mv^2$$

$$= \frac{1}{2}\,(50 \times 10^{-3}) \times (200)^2 = 1000 \text{ J}$$

Heat lost by bullet for its temperature to fall from 20°C to 0°C = $(50 \times 10^{-3}) \times 800 \times 20 = 800$ J. If x kg is the mass of ice melted, then

$$x \times (3.36 \times 10^5) = 1000 + 800$$

$\Rightarrow \quad x = 5.3 \times 10^{-3}$ kg = 5.3 g. Hence the correct choices are (a) and (c).

6. $Q = mg\,(h - h') = 1 \times 10 \times (10 - 4) = 60$ J.

Also $\quad Q = ms\Delta T$. Hence

$$s = \frac{Q}{m\Delta T} = \frac{60}{1 \times 0.075}$$

$= 800 \text{ J kg}^{-1} \text{ K}^{-1}$.

Thus the correct choices are (a) and (c).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage

Passage I

The basic principle of calorimetry is heat gained by one body = heat lost by the other body. This follows from the principle of conservation of energy according to which the total heat energy of the two substances must remain constant. Hence heat lost by one body must be gained by the other, provided no part of heat energy is allowed to escape.

An aluminium container of mass 100 g contains 200 g of ice at – 20°C. Heat is added to the system at the rate of 420 J per second. Specific heat capacity of ice = 2100 J kg^{-1} K^{-1}, specific heat capacity of aluminium = 840 J kg^{-1} K^{-1} and latent heat of fusion of ice = 3.36×10^5 J kg^{-1}.

1. The time taken to raise the temperature of the container and ice from – 20°C to 0°C is

(a) 12 s (b) 24 s

(c) 36 s (d) 48 s

2. The time taken to melt ice at 0°C into water at 0°C is

(a) 40 s (b) 80 s

(c) 120 s (d) 160 s

3. The temperature of the system after 4 minutes is

(a) 15.45°C (b) 20.45°C

(c) 25.45°C (d) 30.45°C

SOLUTION

1. Heat energy needed to raise the temperature of the container and ice from – 20°C to 0°C is

$$Q_1 = (100 \times 10^{-3}) \times 840 \times 20 + (200 \times 10^{-3}) \times 2100 \times 20$$

$$= 1680 + 8400 = 10080 \text{ J}$$

$\therefore \quad$ Time needed is $t_1 = \dfrac{10080}{420} = 24$ s

So the correct choice is (b).

2. Heat energy required to melt ice at 0°C into water at 0°C is

$$Q_2 = (200 \times 10^{-3}) \times (3.36 \times 10^5) = 6.72 \times 10^4 \text{ J}$$

$\therefore$ Time needed is $t_2 = \dfrac{6.72 \times 10^4}{420} = 160$ s, which is choice (d).

3. Total time $t_1 + t_2 = 24 + 160 = 184$ s is less than 4 minutes (= 240 s). Hence heat energy supplied during (240 – 184) = 5 s will be used up in raising the temperature of the system (container + 200 g of water) from 0°C to final temperature. Heat supplied is 56 s = $420 \times 56 = 23520$ J. If t°C is the final temperature, then

$$23520 = (100 \times 10^{-3}) \times 840 \times t + (200 \times 10^{-3}) \times 4200 \times t$$

which gives $t = 25.45$°C. So the correct choice is (c).

Questions 4 to 6 are based on the following passage

Passage II

A steel drill making 180 revolutions per minute is used in during a hole in a block of steel. The mass of the steel block is 180 g. 90% of the entire mechanical energy is used up in producing heat and the rate of rise of temperature of the block is 0.5°C per second. The specific heat capacity of steel = 420 J kg^{-1} K^{-1}.

4. The rate at which heat is produced is
(a) 35.8 Js^{-1} (b) 36.8 Js^{-1}
(c) 37.8 Js^{-1} (d) 38.8 Js^{-1}

5. The power of the drill is
(a) 40 W (b) 42 W
(c) 48 W (d) 56 W

6. The torque required to drive the drill is
(a) 1.2 Nm (b) 2.2 Nm
(c) 3.2 Nm (d) 4.2 Nm

SOLUTION

4. $\frac{Q}{t}$ = mass × sp. heat × rise in temperature per second = $(180 \times 10^{-3}) \times 420 \times 0.5 = 37.8$ Js^{-1}.
So the correct choice is (c).

5. Power of drill = $\frac{37.8}{0.9}$ = 42 W, which is choice (b).

6. Number of revolutions per second = $\frac{180}{60}$
= 3 r.p.s.
Angular frequency of rotation (ω) = $2\pi \times 3$
= 6π rad s^{-1}
Now power = torque × angular frequency or $P = \tau\omega$
$\therefore \quad \tau = \frac{P}{\omega} = \frac{42}{6 \times 3.14} = 2.2$ Nm
Thus the correct choice is (b).

SECTION IV

Integer Answer Type

1. 2 kg of ice at – 20°C is mixed with 5 kg of water at 20°C in an insulating vessel having negligible heat capacity. Calculate the final mass of water (in kg) remaining in the container. Specific hearts of water and ice are kcal/kg/°C and 0.5 kcal/kg/°C and latent heat of fusion of ice is 80 kcal/kg.

IIT, 2003

SOLUTION

1. $m_i s_i [0 - (-20)] + m' L = m_w s_w (20 - 0)$
$\Rightarrow \quad 2 \times 0.5 \times 20 + m' \times 80 = 5 \times 1 \times 20$
$\Rightarrow \quad m' = 1$ kg.
$\therefore$ Mass of water in container = 5 + 1 = 6 kg

17 Chapter Thermodynamics (Isothermal and Adiabatic Processes)

REVIEW OF BASIC CONCEPTS

17.1 EQUATION OF STATE

In the case of ideal gases, the equation of state is

$$PV = RT, \text{ for one mole}$$

and $$PV = nRT, \text{ for } n \text{ moles}$$

where P, V and T are respectively the pressure, volume and temperature of the gas and R is the universal gas constant.

17.2 MOLAR SPECIFIC HEAT

The molar specific heat C of a substance is the amount of heat energy required to raise the temperature of 1 mole of the substance through 1 K. It is expressed in J mol^{-1} K^{-1}.

Specific heat (s) and molar specific heat (C) are related as

$$s = \frac{C}{m}$$

where m is the number of kilograms per mole in the substance.

The molar specific heat of a gas at constant volume (C_v) is the amount of heat energy required to raise the temperature of 1 mole of the gas through 1 K when its volume is kept constant. The molar specific heat of a gas at constant pressure (C_p) is the amount of heat energy required to raise the temperature of 1 mole of the gas through 1 K when its pressure is kept constant.

The two specific heat of an ideal gas are related as

$$C_p - C_v = R$$

when R is the universal gas constant and its value is

$$R = 8.315 \text{ J mol}^{-1} \text{ K}^{-1}$$

17.3 ZEROTH LAW OF THERMODYNAMICS

The Zeroth law of thermodynamics which states that *if two systems A and B are separately in thermal equilibrium with a third system C, then the systems A and B are in thermal equilibrium with each other.*

Thus if $T_A = T_C$ and $T_C = T_B$, then $T_A = T_B$.

17.4 INTERNAL ENERGY

Every system (solid, liquid or gas) possesses a certain amount of energy. This energy is called the *internal energy* and is usually denoted by the symbol U. The internal energy of solid, liquid or gas consists of two parts: (i) kinetic energy due to the motion (translational, rotational and vibrational) of the molecules, and (ii) potential energy due to the configuration (separation) of the molecules.

If the intermolecular forces are extremely weak or absent, then the change in internal energy is given by

$$\Delta U = mc_v \Delta T$$

where m is the mass of the gas, c_v its specific heat and ΔT is the change in temperature. In terms of molar specific heat C_v (= Mc_v where M is the molecular mass), we have, for n moles of an ideal gas

$$\Delta U = nC_v \Delta T = \frac{m}{M} C_v \Delta T$$

17.5 FIRST LAW OF THERMODYNAMICS

When heat energy is supplied to a system, a part of this energy is used up in raising the temperature of the system (i.e. in increasing the internal energy of the system) and the rest is used up in doing external work against the

surroundings. Thus, if ΔQ is the heat energy supplied to a gas and if ΔW is the work done by it, then from the law of conservation of energy, the increase ΔU in the internal energy of the gas must be equal to $(\Delta Q - \Delta W)$ or

$$\Delta Q = \Delta U + \Delta W$$

Here all quantities are measured in energy units (joule). This equation is the mathematical statement of the first law of thermodynamics which may be stated in words as '*if energy is supplied to a system which is capable of doing work, then the quantity of heat energy absorbed by the system will be equal to the sum of the increase in the internal energy of the system and the external work done by it.*'

Sign convention for ΔQ, ΔW and ΔU

1. ΔQ is positive if heat is supplied to the system and negative if heat is taken out of the system.
2. ΔW is positive if work is done by the system and negative if work is done on the system.
3. ΔU is positive if the temperature of the system increases and negative if the temperature of the system decreases.

17.6 THERMODYNAMIC PROCESSES

1. Isothermal Process

The process in which the temperature remains constant is called isothermal process. From $PV = nRT$, it follows that for a given mass of a gas (n = constant), the relation between P and V is

$$PV = \text{constant} \quad \Rightarrow \quad P_1V_1 = P_2V_2$$

(for an isothermal process T = constant)

If T is kept constant $\Delta T = 0$. Hence $\Delta U = 0$, i.e. for an isothermal process, the internal energy of the gas remains constant. From first law of thermodynamics, it follows that

$$\Delta Q = \Delta W$$

i.e. all the heat energy supplied to the gas is used up in doing work.

(a) If the gas expands isothermally, work is done by the gas. Hence both ΔW and ΔQ are positive.
(b) If the gas is compressed isothermally, work is done on the gas. Hence both ΔW and ΔQ are negative.

2. Adiabatic Process

The process in which no heat energy enters the system or leaves the system is called adiabatic process. For an adiabatic process, the realtion between P and V is

$$PV^{\gamma} = \text{constant} \quad \Rightarrow \quad P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

where $\gamma = \dfrac{C_p}{C_v}$.

For an adiabatic process, $\Delta Q = 0$. From the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$, it follows that

$$\Delta U + \Delta W = 0 \text{ or } \Delta W = -\Delta U$$

(a) If the gas expands adiabatically, ΔW is positive. hence ΔU is negative, i.e. the temperature of the gas falls.
(b) If the gas is compresssed adiabatically ΔW is negative. Hence ΔU is positive, i.e. the temperature of the gas increases.

3. Isochoric Process

A process in which the volumee is kept constant is called an isochoric process. The relation between P and T is

$$P \propto T \quad \Rightarrow \quad \frac{P}{T} = \text{constant} \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

The work done in a process is $\Delta W = P\Delta V$. For an isochoric process, $\Delta V = 0$. Hence $\Delta W = 0$. From the first law of thermodynamics it follows that

$$\Delta Q = \Delta U$$

(a) If heat is supplied to the gas, ΔQ is positive. Hence ΔU is positive, i.e. the temperautre of the gas rises. Hence pressure also increases.
(b) If heat is taken out of the gas, ΔQ is negative. Hence ΔU is negative, i.e. the temperature of the gas falls. Henc pressure also falls.

4. Isoboric Process

A process in which the pressure is kept constant is called an isobaric process. The relation between V and T is

$$V \propto T \quad \Rightarrow \quad \frac{V}{T} = \text{constant} \Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

(a) If the gas expands at constant pressure, it follows from $V \propto T$, that the temperature rises. Hence ΔW and ΔU are positive. It follows from the first law of thermodynamics, that ΔQ is also positive. Hence ΔQ, ΔU and ΔW are all positive in isobaric expansion.
(b) If the gas is compressed at constant pressure; ΔQ, ΔU and ΔW are all negative.

17.7 WORK DONE IN A PROCESS

The work done by a gas in a finite but slow expansion from volume V_1 to volume V_2 is given by $W = \int_{V_1}^{V_2} PdV$

1. **Work Done in an Isothermal Process** When an ideal gas is allowed to expand isothermally (i.e.

at constant temperature) work is done by it. For an isothermal process, the equation of state for n moles of ideal gas is

$$PV = nRT \quad (1)$$

where T is the constant absolute temperature and R is the universal gas constant. The value of R for all gases is $R = 8.31\ \text{J K}^{-1}\ \text{mol}^{-1}$

From Eq. (1)

$$P = \frac{nRT}{V}$$

The work done

$$W = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln \left(\frac{V_2}{V_1}\right); \quad (\ln = \log_e)$$

or $$W = 2.303\ nRT \log \left(\frac{V_2}{V_1}\right) \quad (2)$$

where V_1 is the initial volume and V_2, the final volume of gas. In terms of initial and final pressures, P_1 and P_2, Eq. (2) can be written as (since $P_1V_1 = P_2V_2$)

$$W = 2.303\ nRT \log \left(\frac{P_1}{P_2}\right)$$

2. **Work Done in Adiabatic Process** When a gas undergoes an adiabatic change, the pressure-volume changes obey the relation

$$PV^{\gamma} = \text{constant} = C \quad (3)$$

where $\gamma = C_p/C_v$ is the ratio of specific heats of the gas at constant pressure to that at constant volume.

From Eq. (3), we have

$$P = \frac{C}{V^{\gamma}}, \text{ where } C \text{ is a constant}$$

The work done is given by

$$W = \left(\frac{1}{1-\gamma}\right)(P_2V_2 - P_1V_1)$$

$$= \frac{1}{(\gamma - 1)}(P_1V_1 - P_2V_2)$$

$$= \frac{nR}{(\gamma - 1)}(T_1 - T_2)$$

since $P_1V_1 = nRT_1$ and $P_2V_2 = nRT_2$, T_1 and T_2 being the absolute temperatures before and after the adiabatic change.

3. **Work Done in an Isobaric Process** For an isobaric process, pressure P is constant. Therefore, the work done is given by

$$W = P \int_{V_1}^{V_2} dV = P(V_2 - V_1)$$

4. **Work Done in an Isochoric Process** In an isochoric process, volume V is constant, i.e. $dV = 0$. Hence, work done $W = 0$.

17.8 INDICATOR DIAGRAM

Indicator diagram (or $P - V$ diagram) is a graph in which pressure (P) is plotted on the y-axis and volume (V) on the x-axis.

Figure 17.1 shows indicator diagrams for expansion, compression and for a closed cyclic process. The initial state (P_1, V_1) is represented by point A and the final state (P_2, V_2) by point B. The intermediate states are represented by points between A and B on the curve AB.

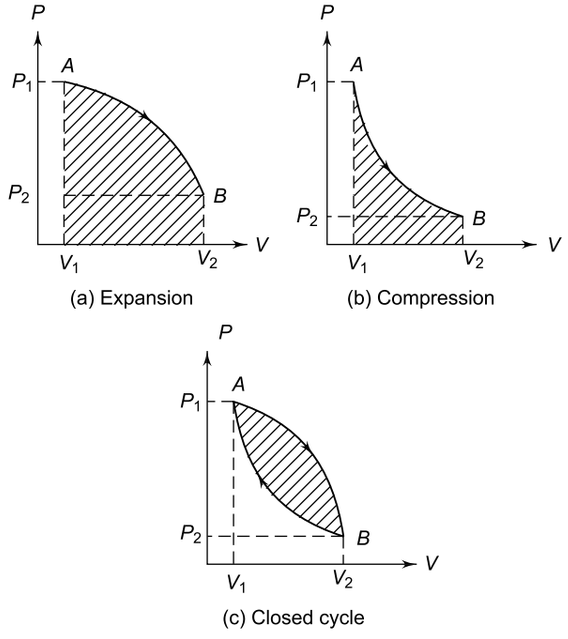


Fig. 17.1

The work done in a process is given by

W = area enclosed by $P - V$ curve and volume axis. Thus, the work done is given by the area of the shaded portion in Fig. 17.1.

17.9 EFFICIENCY OF AN IDEAL HEAT ENGINE

The efficiency (η) of an ideal reversible heat engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_1 = absolute temperature of the source which supplies heat

and T_2 = absolute temperature of the sink which takes in the part of heat not converted into useful work,

T_1 is always greater than T_2. If $T_1 = T_2$; $\eta = 0$ implying that an engine working under isothermal conditions can produce no useful work. Complete conversion of heat into work (i.e. $\eta = 1$) is possible only if $T_2 = 0$, i.e. the sink is at absolute zero, which is unattainable.

17.10 C_v, C_p AND $\gamma = C_p/C_v$ FOR AN IDEAL GAS

1. For a monoatomic gas,
$$C_v = \frac{3R}{2},\ C_p = \frac{5R}{2} \text{ and } \gamma = \frac{5}{3} = 1.67$$
2. For a diatomic gas,
$$C_v = \frac{5R}{2},\ C_p = \frac{7R}{2} \text{ and } \gamma = \frac{7}{5} = 1.4$$
3. For a triatomic or polyatomic gas
$$C_v = 3R,\ C_p = 4R \text{ and } \gamma = \frac{4}{3} = 1.33$$

17.11 RELATION BETWEEN C_p, C_v AND γ

$$C_p - C_v = R \quad (4)$$

and $$\frac{C_P}{C_v} = \gamma$$

or $$C_p = \gamma C_v \quad (5)$$

Using (5) in (4), we have

$$C_v = \frac{R}{\gamma - 1} \text{ and } C_p = \frac{\gamma R}{\gamma - 1}$$

EXAMPLE 17.1

5 moles of an ideal gas are compressed to half the initial volume at a constant temperature of 27.0°C. Calculate the work done in the process. Given R = 8.3 J K^{-1} mol^{-1}. Write your result up to appropriate significant figures.

SOLUTION

Given T = 27.0°C = 300 K. Since T is constant, the process is isothermal

$$\therefore \quad W = nRT \log_e \left(\frac{V_2}{V_1}\right)$$
$$= 5 \times 8.3 \times 300 \times \log_e \left(\frac{1}{2}\right)$$
$$= 5 \times 8.3 \times 300 \times (-0.693)$$
$$= -8628 \text{ J}$$

Since the value of R has two significant figures (*s.f.*), the value of W must be rounded off to 2 *s.f.* as

$$W = -8.6 \times 10^3 \text{ J}$$

The negative sign indicates that work is done on the gas.

EXAMPLE 17.2

Two moles of a diatomic gas (γ = 1.4) at 127°C are expanded adiabatically to twice the original volume. Calculate (a) the final temperature and (b) the work done in the process. Given R = 8.3 J K^{-1} mol^{-1} and $(0.5)^{0.4} = 0.76$.

SOLUTION

(a) $T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$

$$\Rightarrow \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = (127 + 273) \times (0.5)^{0.4}$$
$$= 400 \times 0.76 = 304 \text{ K}$$

(b) $$W = \frac{nR}{(\gamma - 1)}(T_1 - T_2)$$
$$= \frac{2 \times 8.3}{1.4 - 1} \times (400 - 304) = 3984 \text{ J}$$

EXAMPLE 17.3

A Carnot's engine working between 0°C and 100°C takes up 746 J of heat from the high temperature reservoir in one cycle. Calculate (a) the work done by the engine, (b) heat-rejected to the sink and (c) the efficiency of the engine.

SOLUTION

T_1 = 100°C = 373 K, T_2 = 0°C = 273 K, Q_1 = 746 J

(a) $$\frac{W}{Q_1} = 1 - \frac{T_2}{T_1} \quad \Rightarrow \quad W = Q_1\left(\frac{T_1 - T_2}{T_1}\right)$$
$$= 746 \times \left(\frac{373 - 273}{373}\right)$$
$$= 200 \text{ J}$$

(b) $Q_2 = Q_1 - W = 746 - 200 = 546$ J

(c) $$\eta = \frac{W}{Q_1} = \frac{200}{746} = 0.268 = 26.8\%$$

NOTE

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273}{373} = 0.268$$

EXAMPLE 17.4

Figure 17.2 shows the P-V diagram of a cyclic process $ABCA$. Calculate the work done in process

(a) A to B (b) B to C

(c) C to A (d) $ABCA$

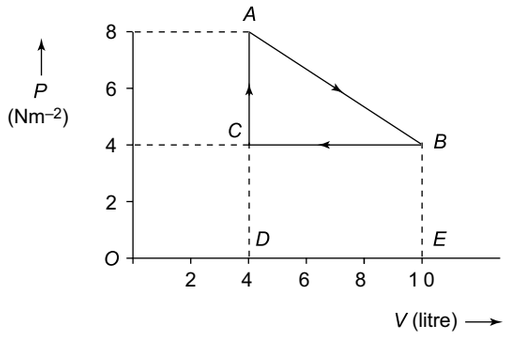


Fig. 17.2

SOLUTION

(a) $W_{A \to B}$ = Area of $ABED$

= Area of ΔABC + area of rectangle $BEDC$

$$= \frac{1}{2} AC \times BC + CD \times DE$$

$$= \frac{1}{2} \times 4 \times 6 \times 10^{-3} + 4 \times 6 \times 10^{-3}$$

($\because$ 1 litre = 10^{-3} m^3)

$$= 36 \times 10^{-3} \text{ J}$$

(b) $W_{B \to C}$ = Area of $BEDC = -24 \times 10^{-3}$ J

Negative sign shows that the work is done on the gas.

(c) $W_{C \to A} = 0$ ($\because$ volume is constant)

(d) $W_{ABCA} = 36 \times 10^{-3} - 24 \times 10^{-3} = 12 \times 10^{-3}$ J

which is equal to area enclosed by the closed loop $ABCA$.

EXAMPLE 17.5

5 moles of an ideal diatomic gas ($\gamma = 1.4$) are heated at constant pressure. If 280 J of heat energy is supplied to the gas, find (a) the change in internal energy of the gas and (b) the work done by the gas.

SOLUTION

(a) $$\Delta U = n C_v \Delta T = n \frac{C_v}{C_p} \times C_p \Delta T = \frac{n C_p \Delta T}{\gamma}$$

Given $\Delta Q = n C_p \Delta T = 280$ J. Hence

$$\Delta U = \frac{2.80}{1.4} = 200 \text{ J}$$

(b) From first law of thermodynamics

$$\Delta W = \Delta Q - \Delta U = 280 - 200 = 80 \text{ J}$$

EXAMPLE 17.6

5 moles of an ideal diatomic gas ($\gamma = 1.4$) are heated at constant volume. If 280 J of heat energy is supplied to the gas, find the change in internal energy of the gas and the work done.

SOLUTION

Since $\Delta V = 0$, $\Delta W = 0$. Hence from first law of thermodynamics,

$$dU = dQ = 280 \text{ J}$$

EXAMPLE 17.7

The pressure P of an ideal gas varies with volume V as $P = kV$ where k is a constant. The volume of n moles of the gas is increased from V to mV. Find the work done and the change in internal energy.

SOLUTION

$$W = \int_V^{mV} PdV = \int_V^{mV} kVdV = k\left|\frac{V^2}{2}\right|_V^{mV} = \frac{kV^2}{2}(m^2 - 1)$$

From first law of thermodynamics

$$\Delta Q = \Delta U + W$$

$$\Rightarrow \quad nC_p \Delta T = \Delta U + W$$

$$\Rightarrow \quad n\gamma C_v \Delta T = \Delta U + W \qquad \left(\because \gamma = \frac{C_p}{C_v}\right)$$

$$\Rightarrow \quad \gamma \Delta U = \Delta U + W \qquad (\because \Delta U = n C_v \Delta T)$$

$$\Rightarrow \quad \Delta U = \frac{W}{(\gamma - 1)} = \frac{kV^2(m^2 - 1)}{2(\gamma - 1)}$$

EXAMPLE 17.8

Two moles of a monoatomic gas undergo a cyclic process $ABCA$ as shown in Fig. 17.3.

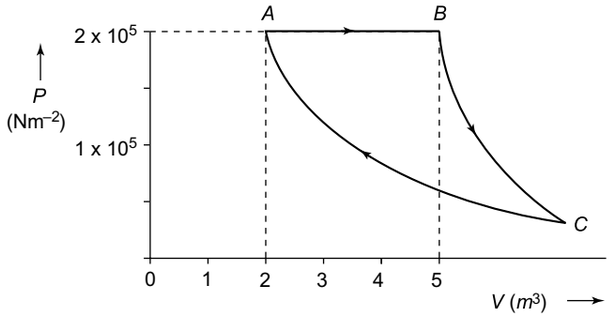


Fig. 17.3

Process $A \to B$ is isobaric, process $B \to C$ is adiabatic and process $C \to A$ is isothermal. Find

(a) pressure and volume at state C

(b) total work done in the process $ABCA$.

SOLUTION

$P_A = P_B = 2 \times 10^5$ Nm^{-2}

$V_A = 2$ m^3, $V_B = 5$ m^3 and $\gamma = 1.67$ (monoatomic gas)

(a) For adiabatic process $B \rightarrow C$

$$P_B V_B^{\gamma} = P_C V_C^{\gamma} \quad (1)$$

For isothermal process $C \rightarrow A$

$$P_A V_A = P_C V_C \quad (2)$$

From (1) and (2), we get $(\because P_A = P_B)$

$$V_C = \left(\frac{V_B^{\gamma}}{V_A}\right)^{\frac{1}{\gamma - 1}}$$

Substituting the values of V_A, V_B and γ, we get $V_C = 19.5$ m^3

Also $$P_C = \frac{P_A V_A}{V_C} = 0.2 \times 10^5 \text{ N m}^{-2}$$

(b) $W = W_{A \rightarrow B} + W_{B \rightarrow C} + W_{C \rightarrow A}$

$$= P_A (V_B - V_A) + \frac{1}{(\gamma - 1)} (P_B V_B - P_C V_C) + P_A V_A \log_e \left(\frac{V_A}{V_C}\right)$$

Substituting the values, we get

$$W = 6 \times 10^5 \text{ N m}^{-2}$$

EXAMPLE 17.9

Show that the slope of P-V curve for an adiabatic process is greater than that for an isothermal process.

SOLUTION

For an isothermal process, $PV =$ constant. Differentiating w.r.t. volume V we have

$$PdV + VdP = 0 \Rightarrow \left(\frac{dP}{dV}\right)_{\text{iso}} = -\frac{P}{V} \quad \text{(i)}$$

For an adiabatic process, $PV^{\gamma} =$ constant,

where $$\gamma = \frac{C_p}{C_v}.$$

Differentiating w.r.t. volume V, we have

$$\gamma P V^{\gamma - 1} dV + V^{\gamma} dP = 0$$

$$\Rightarrow \left(\frac{dP}{dV}\right)_{\text{adia}} = -\gamma \frac{P}{V} \quad \text{(ii)}$$

Since $\gamma > 1$, $\left(\frac{dP}{dV}\right)_{\text{adia}} > \left(\frac{dP}{dV}\right)_{\text{iso}}$.

Hence the sope of P-V graph for an adiabatic process is greater than for an isothermal process. Figure 17.4 shows two P-V curves, one for adiabatic expansion and the other for isothermal expansion.

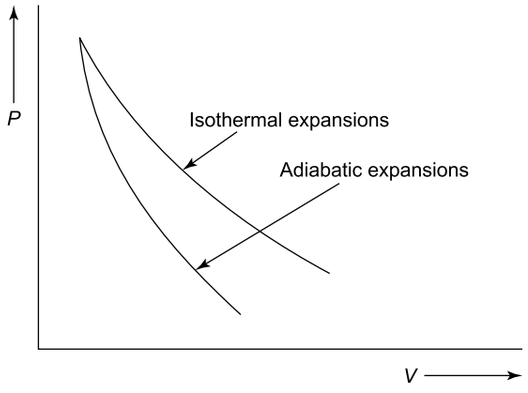


Fig. 17.4

EXAMPLE 17.10

Two moles of a diatomic ideal gas, initially at pressure 5.0×10^4 Pa and temperature 300 K are expanded isothermally until the volume of the gas is doubled and then adiabatically expanded until the volume is again doubled. Find

(a) the pressure and temperature of the gas at the end of the complete proces

(b) total work in the complete process and

(c) the change in internal energy in the complete process.

(d) draw the P-V graph for the complete process.

Given $R = 8.3$ JK^{-1} mol^{-1}.

SOLUTION

Let P_1, V_1, T_1 be the initial pressure, volume and temperature of the gas

For isothermal process

$$P_1 V_1 = P_2 V_2 \text{ where } V_2 = 2V_1$$

$$\therefore \quad P_2 = P_1 \left(\frac{V_1}{V_2}\right) = 5.0 \times 10^4 \times \frac{1}{2} = 2.5 \times 10^4 \text{ Pa}$$

In an isothermal process, the temperature remains constant. Hence $T_2 = T_1 = 300$ K. Therefore change in internal energy $(\Delta U)_1 = 0$.

Work done is

$$W_1 = nRT_1 \ln\left(\frac{V_2}{V_1}\right) = 2 \times 8.3 \times 300 \times \ln(2)$$

$$= 3.45 \times 10^3 \text{ J}$$

For adiabatic process

$$P_2V_2^{\gamma} = P_3V_3^{\gamma}$$

$$\Rightarrow \quad P_3 = P_2\left(\frac{V_2}{V_3}\right)^{\gamma} = 2.5 \times 10^4 \times \left(\frac{1}{2}\right)^{1.4}$$

$$= 9.5 \times 10^3 \text{ Pa}$$

Also $T_2V_2^{(\gamma-1)} = T_3V_3^{(\gamma-1)}$

$$\Rightarrow \quad T_3 = T_2\left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

$$= 300 \times \left(\frac{1}{2}\right)^{0.4} = 227 \text{ K}$$

Work done is

$$W_2 = \frac{nR}{(\gamma-1)}(T_2 - T_3)$$

$$= \frac{2 \times 8.3}{(1.4-1)} \times (300 - 227)$$

$$= 3.03 \times 10^3 \text{ J}$$

Change in internal energy is

$$(dU)_2 = nC_v\Delta T = n \times \frac{5R}{2} \times \Delta T$$

$$= 2 \times \frac{5}{2} \times 8.3 \times (227 - 300)$$

$$(\because C_v = \frac{5R}{2} \text{ for diatomic gas})$$

$$= -3.03 \times 10^3 \text{ J}$$

(a) $P_3 = 9.5 \times 10^3$ Pa, $T_3 = 227$ K

(b) $W = W_1 + W_2 = 3.45 \times 10^3 + 3.03 \times 10^3 = 6.48 \times 10^3$ J

(c) $\Delta U = (\Delta U)_1 + (\Delta U)_2 = 0 - 3.03 \times 10^3 = -3.03 \times 10^3$ J. The negative sign shows that there is a decrease in internal energy.

(d) *P-V* graph for the complete process is shown in Fig. 17.5.

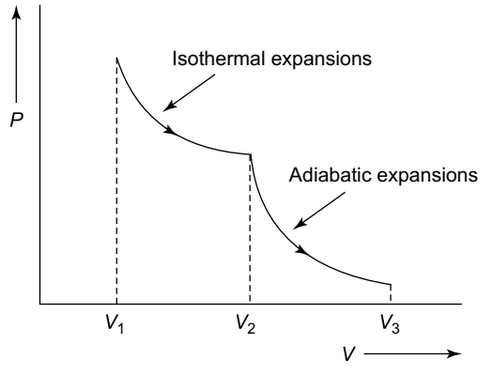


Fig. 17.5

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. If 2 moles of an ideal monoatomic gas at temperature T are mixed with 3 moles of another monoatomic gas at temperature $2T$, the temperature of the mixture will be

(a) $\frac{8T}{5}$ (b) $\frac{6T}{5}$

(c) $\frac{4T}{3}$ (d) $\frac{3T}{2}$

2. If heat energy ΔQ is supplied to an ideal diatomic gas, the increase in internal energy is ΔU and the work done by the gas is ΔW. The ratio $\Delta Q : \Delta U : \Delta W$ is

(a) 5 : 3 : 2 (b) 5 : 2 : 3

(c) 7 : 5 : 2 (d) 7 : 2 : 5

3. Figure 17.6 shows a cyclic process. When a given mass of a gas is expanded from state A to state B, it absorbs 30 J of heat energy. When the gas is adiabatically compressed from state B to state A, the work done on the gas is 50 J. The change in internal energy of the gas in the process $A \rightarrow B$ is

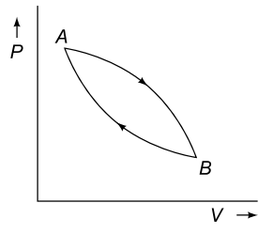


Fig. 17.6

(a) 80 J (b) 20 J

(c) –20 J (d) –50 J

4. Figure 17.7 shows a cyclic process $ABCA$ for an ideal diatomic gas. The ratio of the heat energy absorbed in the process $A \rightarrow B$ to the work done on the gas in the process $B \rightarrow C$ is

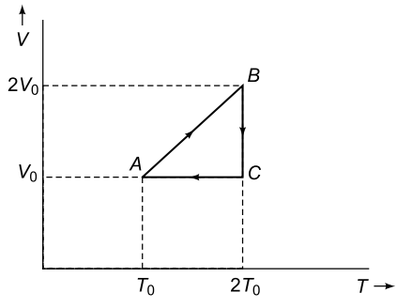


Fig. 17.7

(a) $\frac{7}{4\ln(2)}$ (b) $\frac{7}{2\ln(2)}$

(c) $\frac{5\ln(2)}{4}$ (d) $\frac{5\ln(2)}{2}$

5. The internal energy of 3 moles of hydrogen at temperature T is equal to the internal energy of n moles of helium at temperature $T/2$. The value of n is (assume hydrogen and helium to behave like ideal gases)

(a) 5 (b) 10

(c) $\frac{3}{2}$ (d) 6

6. The temperature of n moles of an ideal gas is increased from T to $3T$ in a process in which the temperature changes with volume as $T = kV^2$ where k is a constant. The work done by the gas in this process is

(a) nRT (b) $2nRT$

(c) $\frac{3}{2}nRT$ (d) $3nRT$

7. In a certain process, pressure P, volume V and temperature T of a gas are related as $PV = kT^n$ where k and n are constants. The work done by the gas when the pressure is kept constant, is proportional to

(a) $(T)^{1/n}$ (b) $(T)^n$

(c) $T^{(n+1)}$ (d) $T^{(n-1)}$

8. If the pressure of an ideal gas in a closed container is increased by 2%, the temperature of the gas increases by 5°C. The initial temperature of the gas is

(a) 100 K (b) 150 K

(c) 200 K (d) 250 K

9. A balloon is filled with a mixture of ideal gases. The pressure P in the balloon is related to the volume V as $PV^{2/3} = k$, where k is a constant. If T is the temperature of the mixture, volume V is proportional to

(a) T (b) T^2

(c) T^3 (d) T^4

10. For a thermodynamic process, the P-V graph for a monoatomic gas is a straight line passing through the origin and having a positive slope. The molar heat capacity of the gas in this process is

(a) R (b) $2R$

(c) $\frac{3}{2}R$ (d) $3R$

11. A Carnot's engine working between 300 K and 600 K has a work output of 800 J per cycle. How much heat energy is supplied to the engine from the source in each cycle?

(a) 1400 J (b) 1500 J

(c) 1600 J (d) 1700 J

12. A Carnot's engine whose sink is at a temperature of 300 K has an efficiency of 40%. By how much should the temperature of the source be increased so as to increase the efficiency to 60%?

(a) 250 K (b) 275 K

(c) 300 K (d) 325 K

13. A thermodynamic process is shown in Fig. 17.8. The pressures and volumes corresponding to some points in the figure are, $P_A = 3 \times 10^4$ Pa, $V_A = 2 \times 10^{-3}$ m^3, $P_B = 8 \times 10^4$ Pa, $V_D = 5 \times 10^{-3}$ m^3. In process AB, 600 J of heat and in process BC, 200 J of heat is added to the system. The change in the internal energy in process AC would be

(a) 560 J (b) 800 J

(c) 600 J (d) 640 J

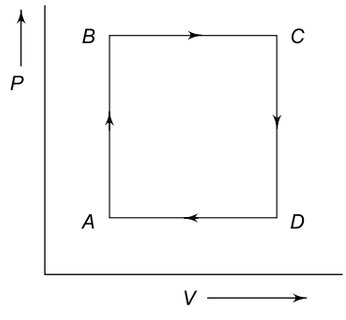


Fig. 17.8

14. One mole of an ideal gas requires 207 J heat to raise its temperature by 10 K when heated at constant pressure. If the same gas is heated at constant volume to raise the temperature by the same 10 K, the heat required will be (R, the gas constant = 8.3 JK^{-1} mol^{-1})

(a) 198.7 J (b) 29 J

(c) 215.3 J (d) 124 J

15. An ideal monoatomic gas is taken round the cycle $ABCA$ as shown in the Fig. 17.9. The work done during the cycle is

(a) zero (b) $3\ PV$

(c) $6\ PV$ (d) $9\ PV$

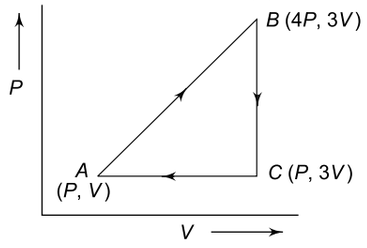


Fig. 17.9

16. The equation of state corresponding to 8 g of O_2 is (assume O_2 to be an ideal gas)

(a) $PV = 8\ RT$ (b) $PV = \dfrac{RT}{4}$

(c) $PV = RT$ (d) $PV = \dfrac{RT}{2}$

17. The equation of state of a gas is

$$\left(P + \frac{aT^2}{V}\right) \times V^c = (RT + b)$$

where a, b, c and R are constants. The isotherms can be represented by

$$P = AV^m - BV^n$$

where A and B depend only on temperature and

(a) $m = -c,\ n = -1$ (b) $m = c,\ n = 1$

(c) $m = -c,\ n = 1$ (d) $m = c,\ n = -1$

18. When an ideal monoatomic gas is heated at constant pressure, the fraction of heat energy supplied which increases the internal energy of the gas is

(a) $\dfrac{2}{5}$ (b) $\dfrac{3}{5}$

(c) $\dfrac{3}{7}$ (d) $\dfrac{3}{4}$

19. For a gas, the difference between the two specific heats is 4150 J kg^{-1} K^{-1} and the ratio of the two specific heats is 1.4. What is the specific heat of the gas at constant volume in units of J kg^{-1} K^{-1}?

(a) 8475 (b) 5186

(c) 1660 (d) 10375

20. A given mass of a gas expands from state A to B by three different paths 1, 2 and 3 as shown in Fig. 17.10. If W_1, W_2 and W_3 respectively be the work done by the gas along the three paths, then

(a) $W_1 > W_2 > W_3$ (b) $W_1 < W_2 < W_3$

(c) $W_1 = W_2 = W_3$ (d) $W_1 < W_2$; $W_1 < W_3$

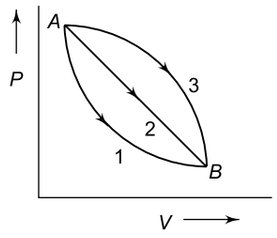


Fig. 17.10

21. In rising from the bottom of a lake to the top, the temperature of an air bubble remains unchanged, but its diameter is doubled. If h is the barometric height (expressed in metres of mercury of relative density ρ) at the surface of the lake, the depth of the lake is (in metres)

(a) $8\ \rho h$ (b) $4\ \rho h$

(c) $7\ \rho h$ (d) $2\ \rho h$

22. Entropy of a thermodynamic system does not change when the system is used for

(a) conduction of heat from a hot reservoir to a cold reservoir

(b) conversion of heat into work adiabatically

(c) conversion of heat into internal energy isochorically

(d) conversion of work into heat isothermally.

23. Heat energy absorbed by a system in going through a cyclic process shown in Fig.17.11 is

(a) $10^7\ \pi$ joule (b) $10^4\ \pi$ joule

(c) $10^2\ \pi$ joule (d) $10^{-3}\ \pi$ joule

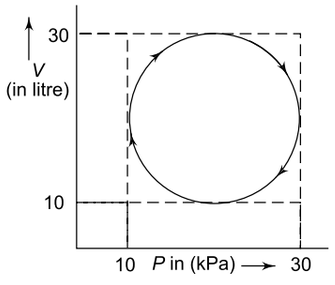


Fig. 17.11

24. One mole of a monoatomic gas ($\gamma = 5/3$) is mixed with one mole of a diatomic gas ($\gamma = 7/5$). What will be value of γ for the mixture?

(a) 1.5 (b) 1.54

(c) 1.4 (d) 1.45

25. If the ratio $C_p/C_v = \gamma$, the change in internal energy of the mass of a gas, when the volume changes from V to $2V$ at constant pressure P is

(a) $\dfrac{R}{(\gamma - 1)}$ (b) PV

(c) $\frac{PV}{(\gamma-1)}$ (d) $\frac{\gamma PV}{(\gamma-1)}$

26. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is

(a) 30 K (b) 18 K

(c) 50 K (d) 42 K

IIT, 1998

27. Two identical containers A and B fitted with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume $2V$. The changes in pressure in A and B are found to be ΔP and 1.5 ΔP respectively. Then

(a) $4\ m_A = 9\ m_B$ (b) $2\ m_A = 3\ m_B$

(c) $3\ m_A = 2\ m_B$ (d) $9\ m_A = 4\ m_B$

IIT, 1999

28. Two monoatomic ideal gases 1 and 2 of molecular masses M_1 and M_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is

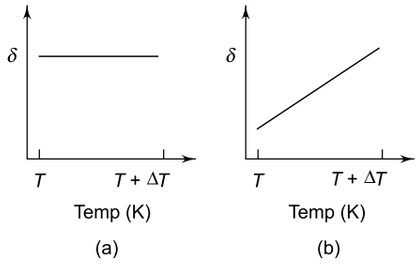
(a) $\sqrt{\frac{M_1}{M_2}}$ (b) $\sqrt{\frac{M_2}{M_1}}$

(c) $\frac{M_1}{M_2}$ (d) $\frac{M_2}{M_1}$

IIT, 2000

29. An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta V/(V\,\Delta T)$ varies with temperature as (see Figs. 17.12)

IIT, 2000



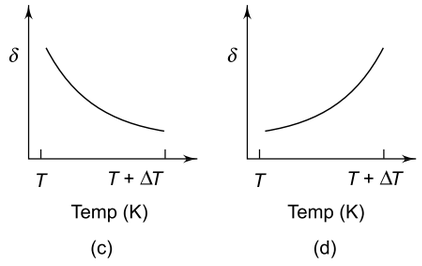


Fig. 17.12

30. A monoatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the pison suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$ (b) $\frac{L_1}{L_2}$

(c) $\frac{L_2}{L_1}$ (d) $\left(\frac{L_2}{L_1}\right)^{2/3}$

IIT, 2000

31. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic. Then

(a) $W_2 > W_1 > W_3$ (b) $W_2 > W_3 > W_1$

(c) $W_1 > W_2 > W_3$ (d) $W_1 > W_3 > W_2$

IIT, 2000

32. A gas does 4.5 J of external work during adiabatic expansion. If its temperature falls by 2K, its internal energy will

(a) increase by 4.5 J (b) decrease by 4.5 J

(c) decrease by 2.25 J (d) increase by 9.0 J

33. 5 moles of Hydrogen $\left(\gamma = \frac{7}{5}\right)$ initially at S.T.P. are compressed adiabatically so that its temperature becomes 400ºC. The increase in the internal energy of the gas in kilo-joules is:
(R = 8.30 J mol^{-1} K^{-1})

(a) 21.55 (b) 41.50

(c) 65.55 (d) 80.55

34. During an adiabatic process, the pressure of a gas is proportional to the cube of its absolute temperature. The value of C_p/C_v for that gas is:

(a) $\frac{3}{5}$ (b) $\frac{4}{3}$

(c) $\frac{5}{3}$ (d) $\frac{3}{2}$

35. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature $2T$ has a pressure of

(a) $\frac{P}{8}$ (b) P

(c) $2P$ (d) $8P$

IIT, 1997

36. In a given process on an ideal gas, $dW = 0$ and $dQ < 0$. Then for the gas

(a) the temperature will decrease

(b) the volume will increase

(c) the pressure will remain constant

(d) the temperature will increase

IIT, 2001

37. P-V plots for two gases during adiabatic processes are shown in Fig. 17.13. Plots 1 and 2 should correspond respectively to

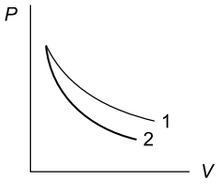


Fig. 17.13

(a) He and O_2

(b) O_2 and He

(c) He and Ar

(d) O_2 and N_2

IIT, 2001

38. An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in Fig. 17.14. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \rightarrow A$ is

(a) – 5 J (b) – 10 J

(c) – 15 J (d) – 20 J

IIT, 2002

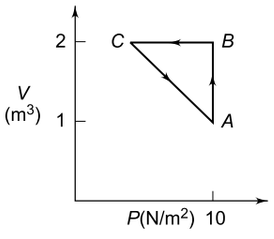


Fig. 17.14

39. Which of the graphs shown in Fig. 17.15 correctly represents the variation of $\beta = -(dV/dp)/V$ with p for an ideal gas at constant temperature?

IIT, 2002

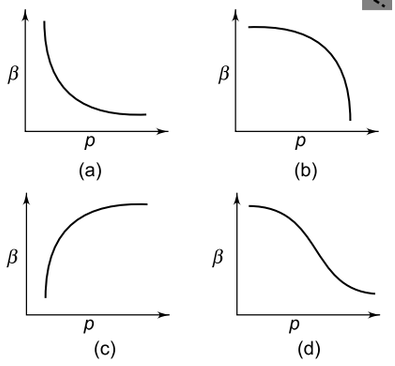


Fig. 17.15

40. Figure 17.16 shows the P-V diagram for a fixed mass of an ideal gas undergoing cyclic process. AB represents isothermal process and CA represents adiabatic process. Which of the graphs shown in Fig. 17.17 represents the P-T diagram of the cyclic procee?

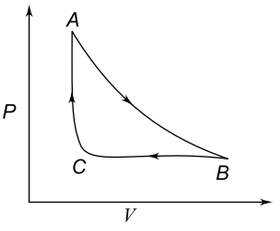


Fig. 17.16

IIT, 2003

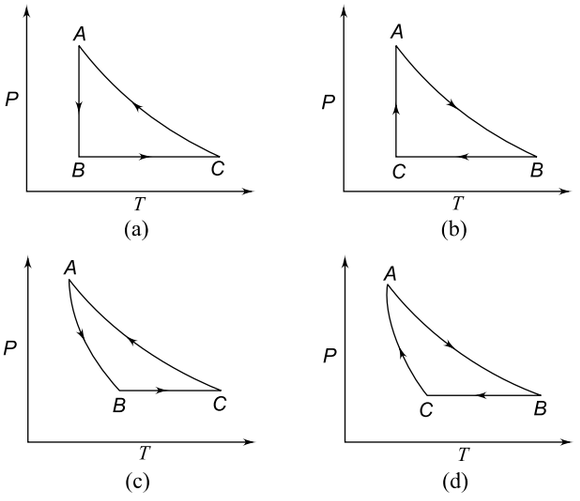


Fig. 17.17

41. The pressure and density of a diatomic gas ($\gamma = 7/5$) change adiabatically form (p_1, d_1) to (p_2, d_2). If $\frac{d_2}{d_1} = 32$, then $\frac{p_2}{p_1}$ is

(a) $\frac{1}{128}$ (b) 32

(c) 128 (d) 256

42. One mole of an ideal gas ($\gamma = 1.4$) is adiabatically compressed so that its temperature rises from 27°C to 35°C. The change in the internal energy of the gas is (given $R = 8.3$ J/mole/K)

(a) – 166 J (b) 166 J

(c) – 168 J (d) 168 J

43. A sample of an ideal gas has volume V, pressure P and temperature T. The mass of each molecule of the gas is m. The density of the gas is (k is the Boltzmann's constant)

(a) mkT (b) $\frac{P}{kT}$

(c) $\frac{P}{kVT}$ (d) $\frac{mP}{kT}$

44. A certain amount of heat energy is supplied to a monoatomic ideal gas which expands at constant pressure. What fraction of the heat energy is converted into work?

(a) 1 (b) $\frac{2}{3}$

(c) $\frac{2}{5}$ (d) $\frac{5}{7}$

45. A closed hollow insulated cylinder is filled with gas at 0°C and also contains an insulated piston of negligible weight and negligible thickness at the middle point. The gas on one side is heated to 100°C. If the piston moves through 5 cm, the length of the hollow cylinder is

(a) 13.65 cm (b) 27.3 cm

(c) 38.6 cm (d) 64.6 cm

46. An ideal gas is expanded isothermally from a volume V_1 to volume V_2 and then compressed adiabatically to original volume V_1. The initial pressure is P_1 and the final pressure is P_3. If the net work done is W, then

(a) $P_3 > P_1$, $W > 0$ (b) $P_3 < P_1$, $W < 0$

(c) $P_3 > P_1$, $W < 0$ (d) $P_3 = P_1$, $W = 0$

IIT, 2004

47. A thermally insulated rigid container contains an ideal gas at 27ºC. It is fitted with a heating coil of resistance 50 Ω. A current is passed through the coil for 10 minutes by connecting it to a d.c. source of 10 V. The change in the internal energy is

(a) zero (b) 300 J

(c) 600 J (d) 1200 J

48. Two different adiabatic paths for the same gas intersect two isothermals at T_1 and T_2 as shown in the P–V diagram (Fig. 17.18). Then

(a) $\frac{V_a}{V_c} = \frac{V_b}{V_d}$ (b) $\frac{V_a}{V_b} = \frac{T_2}{T_1}$

(c) $\frac{V_a}{V_b} = \frac{V_d}{V_c}$ (b) $\frac{V_a}{V_d} = \frac{T_1}{T_2}$

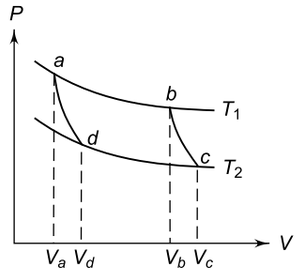


Fig. 17.18

49. Figure 17.19 shows a cyclic process $ABCA$ in the $V - T$ diagram. Which of the diagrams shown in Fig. 17.20 shows the same process on a $P - V$ diagram.

IIT, 1981

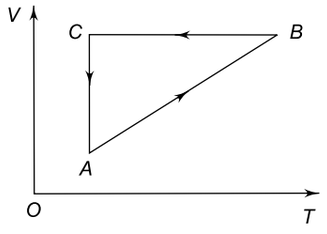


Fig. 17.19

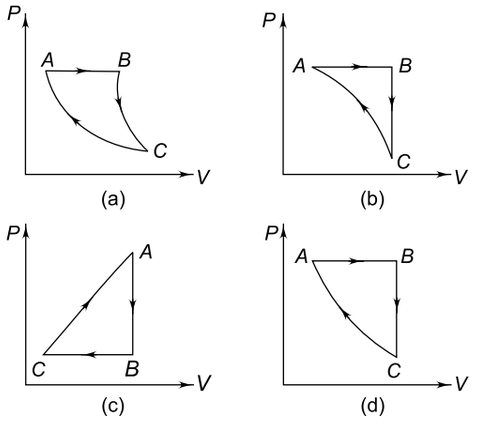


Fig. 17.20

50. An ideal gas ($\gamma = 1.4$) expands from 5×10^{-3} m^3 to 25×10^{-3} m^3 at a constant pressure of 1×10^5 Pa. The heat energy supplied to the gas in this process is

(a) 7 J (b) 70 J

(c) 700 J (d) 7000 J

51. Three moles of an ideal gas are taken through a cyclic process *ABCA* as shown on *T–V* diagram in Fig. 17.21. The gas loses 2510 J of heat in the complete cycle. If T_A = 100 K and T_B = 200 K, The work done by the gas during the process *BC* is

(a) 5000 J (b) – 5000 J

(c) 4000 J (d) – 2500 J

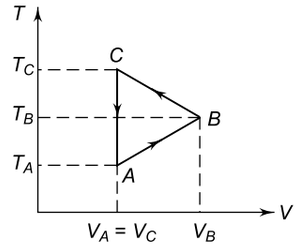


Fig. 17.21

52. Liquid oxygen at 50 K is heated at 300 K at constant pressure of 1 atmosphere. The rate of heating is constant. Which of the graphs shown in Fig. 17.22 represent the variation of temperature (*T*) with time (*t*)?

IIT, 2004

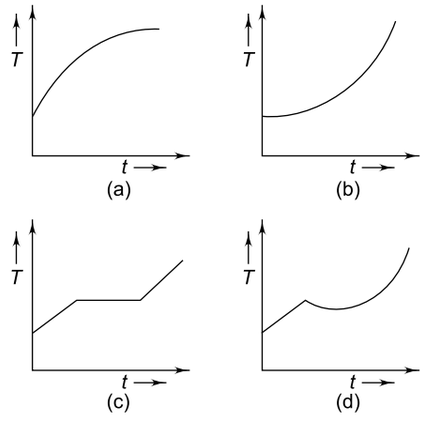


Fig. 17.22

53. One mole of a monoatomic ideal gas is contained in a insulated and rigid container. It is heated by passing a current of 2 A for 10 minutes through a filament of resistance 100 Ω. The change in the internal energy of the gas is

(a) 30 kJ (b) 60 kJ

(c) 120 kJ (d) 240 kJ

IIT, 2005

54. The *P–V* diagram for *n* moles of an ideal gas undergoing a process A → B is shown in Fig. 17.23. The maximum temperature of the gas during the process is

(a) $\frac{9P_0V_0}{4nR}$ (b) $\frac{3P_0V_0}{nR}$

(c) $\frac{2P_0V_0}{nR}$ (d) $\frac{3P_0V_0}{2nR}$

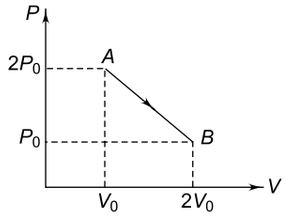


Fig. 17.23

55. Two litres of water (density = 1 g/ml) in an open-lid insulated kettle is heated by an electric heater of power 1 kW. The heat is lost from the lid at the rate of 160 J/s. The time taken for heating water (specific heat capacity 4.2 kJ kg^{-1} K^{-1}) from 20°C to 75°C is

(a) 340 s (b) 550 s

(c) 620 s (d) 760 s

IIT, 2005

56. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(a) $\frac{1}{T}$ (b) $\frac{2}{T}$

(c) $\frac{3}{T}$ (d) $\frac{4}{T}$

IIT, 2008

ANSWERS

1. (a)	**2.** (c)	**3** (d)	**4.** (a)	**5.** (b)	**6.** (a)
7. (b)	**8.** (d)	**9.** (c)	**10.** (b)	**11.** (c)	**12.** (a)
13. (a)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (b)
19. (d)	**20.** (b)	**21.** (c)	**22.** (d)	**23.** (c)	**24.** (a)
25. (c)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (d)

31. (a)	**32.** (b)	**33.** (b)	**34.** (d)	**35.** (c)	**36.** (a)
37. (b)	**38.** (a)	**39.** (a)	**40.** (a)	**41.** (c)	**42.** (b)
43. (d)	**44.** (c)	**45.** (d)	**46.** (c)	**47.** (d)	**48.** (c)
49. (d)	**50.** (d)	**51.** (b)	**52.** (c)	**53.** (d)	**54.** (a)
55. (c)	**56.** (c)				

SOLUTIONS

1. Let T_0 be the temperature of the mixture. Since the total internal energy remains unchanged, we have

U of mixture $= U_1 + U_2$

$\Rightarrow (n_1 + n_2)\, C_v\, T_0 = n_1\, C_v\, T_1 + n_2\, C_v\, T_2$

$\Rightarrow (n_1 + n_2)\, T_0 = n_1\, T_1 + n_2\, T_2$

$\Rightarrow (2 + 3)\, T_0 = 2T + 3 \times (2T) = 8T$

Which gives $T_0 = \dfrac{8T}{5}$

2. For a diatomic gas $C_p = \dfrac{7R}{2}$ and $C_v = \dfrac{5R}{2}$

$$\Delta Q = n\, C_p\, \Delta T = \frac{7}{2}\, nR\Delta T$$

$$\Delta U = n\, C_v\, \Delta T = \frac{5}{2}\, nR\Delta T$$

From the first law of thermodynamics

$$\Delta W = \Delta Q - \Delta U = \frac{7}{2}\, nR\Delta T - \frac{5}{2}\, nR\Delta T$$

$$= nR\Delta T$$

$\therefore \Delta Q : \Delta U : \Delta W = 7 : 5 : 2$

3. In the process $B \rightarrow A$, work is done on the gas. Hence $(\Delta W)_{B \rightarrow A} = -50$ J. Since this process is adiabatic, $(\Delta Q)_{B \rightarrow A} = 0$. From the first law of thermodynamics, the change in internal energy in this process is

$$(\Delta U)_{B \rightarrow A} = (\Delta Q)_{B \rightarrow A} - (\Delta W)_{B \rightarrow A}$$

$$= 0 - (-50) = +50 \text{ J}$$

Since the process is cyclic, there is no net change in internal energy. Hence

$(\Delta U)_{A \rightarrow B} = -(\Delta U)_{B \rightarrow A} = -50$ J

4. In the process $A \rightarrow B$, V is proportional to T. Hence pressure P remains constant. Therefore, heat energy absorbed in this process is ($\because C_p = 7R/2$ for a diatomic gas)

$$(Q)_{A \rightarrow B} = n\, C_p\, \Delta T = n \times \frac{7R}{2} \times (2T_0 - T_0)$$

$$= \frac{7}{2}\, nRT_0$$

Process $B \rightarrow C$ is isothermal in which the gas is compressed. Hence work done on the gas in this process is

$$(W)_{B \rightarrow C} = -nR\,(2T_0) \ln\left(\frac{V_0}{2V_0}\right)$$

$$= -2\, nRT_0 \ln\left(\frac{1}{2}\right)$$

$$= 2\, nRT_0 \ln(2)$$

$\therefore \dfrac{(Q)_{A \rightarrow B}}{(W)_{B \rightarrow C}} = \dfrac{7}{4 \ln(2)}$, which is choice (a).

5. The internal energy of n moles of an ideal gas at temperature T is given by

$$U = \frac{f}{2}\, nRT$$

where f = number of degrees of freedom.

For hydrogen, $f = 5$. Therefore

$$U_1 = \frac{5}{2} \times 3 \times RT = \frac{15}{2}\, RT$$

For helium, $f = 3$. Therefore

$$U_2 = \frac{3}{2}\, nR(T/2) = \frac{3}{4}\, nRT$$

Given $U_1 = U_2$, i.e.

$$\frac{15}{2}\, RT = \frac{3}{4}\, nRT$$

which gives $n = 10$.

6. Given $T = kV^2$. Therefore $dT = 2\, kV\, dV$ or

$$dV = \frac{dT}{2kV}$$

Also $PV = nRT \Rightarrow P = \dfrac{nRT}{V}$

$\therefore$ Work done $W = \displaystyle\int_T^{3T} PdV$

$$= \int_T^{3T} \left(\frac{nRT}{V}\right) \times \left(\frac{dT}{2kV}\right)$$

$$= \int_T^{3T} \frac{nRT\, dT}{2kV^2}$$

$$= \frac{nR}{2} \int_T^{3T} dT \qquad (\because kV^2 = T)$$

$$= nRT$$

So the correct choice is (a).

7. Given $V = \dfrac{kT^n}{P}$. Since P = constant,

$$dV = \frac{kn}{P}\ T^{(n-1)}\ dT$$

Work done $W = \int P\,dV = kn \int T^{(n-1)}\,dT$

$$= kT^n + c$$

where c = constant of integration. Hence the correct choice is (b).

8. Since the volume of the gas is constant,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \qquad (1)$$

Now $P_2 = P_1 + 0.02\ P_1 = 1.02\ P_1$ and $T_2 = T_1 + 5$. Using these values in Eq. (1), we have

$$\frac{P_1}{1.02\,P_1} = \frac{T_1}{T_1 + 5} \Rightarrow T_1 = 250\ \text{K}$$

9. $PV^{2/3} = k$ (1)

Equation of state is $PV = nRT \Rightarrow P = \frac{nRT}{V}$. Using this in Eq. (1) we get

$$\frac{nRT}{V} \times V^{2/3} = k$$

or $\quad TV^{-1/3} = \frac{k}{nR} = \text{constant}$

Hence $V \propto T^3$, which is choice (c).

10. Since the $P - V$ graph is a straight line with a positive slope, $P \propto V$

or $\quad PV^{-1} = \text{constant}$

For a process in which PV^n = constant, the molar heat capacity is given by

$$C = \frac{R}{(\gamma - 1)} + \frac{R}{(1-n)}$$

Putting $n = -1$ and $\gamma = \frac{5}{3}$ (for a monoatomic gas), we have

$$C = \frac{R}{\left(\frac{5}{3} - 1\right)} + \frac{R}{(1+1)} = \frac{3R}{2} + \frac{R}{2} = 2R$$

11. $\frac{W}{Q} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{600} = 0.5$. Therefore $Q = 2W$ $= 2 \times 800 = 1600$ J. Hence the correct choice is (c).

12. $T_2 = 300$ K. Now $\eta = 1 - T_2/T_1$. When $\eta = 40\% = 0.4$, the value of T_1 is given by

$$\frac{T_2}{T_1} = 1 - 0.4 = 0.6$$

or $\quad T_1 = \frac{T_2}{0.6} = \frac{300}{0.6} = 500$ K. When $\eta = 60\%$ $= 0.6$, the value of T_2 should be

$$T_2' = \frac{300}{0.4} = 750\ \text{K}$$

$\therefore\ T_2' - T_2 = 750 - 500 = 250$ K, which is choice (a).

13. Process AB is isochoric, i.e. the volume remains constant. Thus $\Delta V = 0$. Hence work done $P\Delta V = 0$. Process BC is isobaric, i.e. the pressure remains constant and external work has to be done. The work done $= P_B \times (V_D - V_A) = 8 \times 10^4 \times (5 \times 10^{-3} - 2 \times 10^{-3}) = 240$ J. Therefore, change in internal energy is

$$dU = dQ - dW = 800 - 240 = 560\ \text{J}$$

Hence the correct choice is (a).

14. Heat energy required to raise the temperature of n moles of a gas by ΔT at constant pressure is

$$Q_p = n\ C_p\ \Delta t$$

Heat energy required to raise the temperature n moles of a gas by ΔT at constant volume is

$$Q_v = n\ C_v\ \Delta T,\ \therefore\ \frac{Q_v}{Q_p} = \frac{C_v}{C_p}$$

or $\quad Q_v = \frac{C_v}{C_p} \times Q_p = \frac{3R/2}{5R/2} \times Q_p$

$$= \frac{3}{5} \times 207 = 124.2 \simeq 124\ \text{J}$$

Hence the correct choice is (d).

15. Work done = area enclosed by the indicator diagram ABC

$$= \frac{1}{2} \times BC \times AC$$

$$= \frac{1}{2} \times (4P - P) \times (3V - V)$$

$$= 3\ PV$$

Hence the correct choice is (b).

16. Number of moles in 8 g of oxygen $(n) = \frac{1}{4}$. Now the equation of state for n moles of an ideal gas is

$$PV = n\ R\ T = \frac{1}{4} \times RT = \frac{RT}{4}$$

Hence the correct choice is (b).

17. Expanding the equation of state we have

$$PV^c + aT^2\ V^{c-1} = RT + b$$

or $\quad P = -\ aT^2\ V^{-1} + RT\ V^{-c} + bV^{-c}$

or $\quad P = AV^{-c} - BV^{-1} \qquad (i)$

where $\quad A = RT + b$ and $B = aT^2$. We are given that

$$P = AV^m - BV^n \qquad (ii)$$

Comparing the powers of V in (i) and (ii) we get $m = -c$ and $n = -1$. Hence the correct choice is (a).

18. Now $Q_p = n\,C_p\,\Delta T$ and $Q_v = n\,C_v\,\Delta T$. But Q_v gives the heat energy which increases the internal energy of the gas. Thus the required fraction is

$$\frac{Q_v}{Q_p} = \frac{C_v}{C_p} = \frac{1}{\gamma} = \frac{1}{5/3} = \frac{3}{5}$$

$$\left(\because \text{ For monoatomic gas } \gamma = \frac{5}{3}\right)$$

Hence the correct choice is (b).

19. Given $C_p - C_v = 4150$ and $C_p/C_v = 1.4$ or $C_p = 1.4\,C_v$. Therefore,

$1.4\,C_v - C_v = 4150$

or $$C_v = \frac{4150}{0.4} = 10375 \text{ J kg}^{-1} \text{ K}^{-1}$$

Hence the correct choice is (d).

20. Work done ($P\Delta V$) = area under the (P-V) curve, which is the largest for curve 3 and the smallest for curve 1. Hence the correct choice is (b).

21. Volume $\propto$ (diameter)^3. Since the diameter of the bubble is doubled in rising from the bottom to the top of the lake, its volume becomes 8 times. Now PV = constant. Therefore, the pressure at the bottom of lake = 8 times that at the top. Let H be the depth of the lake.

$$H\,\rho_w\,g = (8h - h)\,\rho_m g$$

or $$H = 7h\,\frac{\rho_m}{\rho_w} = 7\,h\rho \qquad \left(\because\ \rho = \frac{\rho_m}{\rho_w}\right)$$

Hence the correct choice is (c).

22. When work is converted into heat at a constant temperature, the entropy of the system remains constant. Hence the correct choice is (d).

23. Heat energy absorbed = work done = area of the loop

$$= \pi r^2 = \pi d^2/4 = \frac{\pi}{4}\,(30 - 10)^2 = 10^2\,\pi \text{ joule}$$

which is choice (c).

24. For a monoatomic gas, $C_v = 3R/2$ and for a diatomic gas, $C_v = 5R/2$. Since one mole of each gas is mixed together, the C_v of the mixture will

$$C_v = \frac{1}{2}\left[\frac{3R}{2} + \frac{5R}{2}\right] = 2R$$

Now $C_p - C_v = R$. Therefore, for the mixture, $C_p = R + C_v = R + 2R = 3R$. Hence, the ratio of the specific heats of the mixture is

$$\gamma = \frac{C_p}{C_v} = \frac{3R}{2R} = \frac{3}{2} = 1.5$$

Thus the correct choice is (a).

25. Let ΔT be the increase in temperature when the volume of the gas is changed by ΔV at constant pressure. The change in internal energy of n moles of a gas is given by

$$\Delta U = n\,C_v\,\Delta T \qquad \text{(i)}$$

We know that $C_p - C_v = R$

or $$\frac{C_p}{C_v} = 1 + \frac{R}{C_v}.\ \text{But}\ \frac{C_p}{C_v} = \gamma.$$

Therefore, $$\gamma = 1 + \frac{R}{C_v}, \text{ which gives}$$

$$C_v = \frac{R}{(\gamma - 1)} \qquad \text{(ii)}$$

Also $PV = n\,RT$. At constant pressure, when volume changes by ΔV, the change in temperature ΔT is given by

$$P\Delta V = nR\Delta T$$

or $$\Delta T = \frac{P\Delta V}{nR} = \frac{PV}{nR}$$

$$(\because\ \Delta V = 2V - V = V) \qquad \text{(iii)}$$

Using (ii) and (iii) in (i) we have

$$\Delta U = n \times \frac{R}{(\gamma - 1)} \times \frac{PV}{nR} = \frac{PV}{(\gamma - 1)}$$

Hence the correct choice is (c).

26. Heat is given to the gas in cylinder A at constant pressure while the same amount of heat is given to the gas in cylinder B at constant volume. Heat given to gas in A is

$$Q_A = n\,C_p\,\Delta T_A$$

Heat given to gas in B is $Q_B = nC_v\,\Delta T_B$

Since $Q_A = Q_B$, we have

$$nC_p\,\Delta T_A = nC_v\,\Delta T_B$$

or $$\Delta T_B = \frac{C_p}{C_v}\,\Delta T_A = \frac{7}{5} \times 30 \text{ K} = 42 \text{ K}$$

$$(\because \text{ for a diatomic gas, } C_p/C_v = 7/5)$$

27. The equation of state for an ideal gas of mass m and molecular mass M is

$$PV = \frac{m}{M}\,RT \qquad \text{(i)}$$

For an isothermal process, T = constant. Differentiating (i) partially at constant T, we get

$P\Delta V + V\Delta P = 0$

or $\Delta P = -P\,\dfrac{\Delta V}{V}$ (ii)

From (i), $P = \dfrac{mRT}{MV}$. Using this in (ii), we get

$$\Delta P = -\frac{mRT}{MV} \quad (\because \Delta V = 2V - V = V)$$

$$\therefore \quad \Delta P_A = -\frac{m_A RT}{MV} \text{ and } \Delta P_B = -\frac{m_B RT}{MV}$$

Hence $\dfrac{\Delta P_A}{\Delta P_B} = \dfrac{m_A}{m_B}$

Given $\Delta P_B = 1.5\ \Delta P_A$. Therefore, $\dfrac{1}{1\cdot 5} = \dfrac{m_A}{m_B}$

or $3\ m_A = 2\ m_B$.

28. The speed of sound in a gas of bulk modulus B and density ρ is given by

$$v = \sqrt{\frac{B}{\rho}}$$

Bulk modulus B is given by $B = -\dfrac{V\Delta P}{\Delta V}$

Now, for a perfect gas, $PV = nRT$. Differentiating at constant T, we get

$P\Delta V + V\Delta P = 0$ or $\dfrac{V\Delta P}{\Delta V} = -P$

Hence $v = \sqrt{\dfrac{P}{\rho}}$

If m is the mass of the gas and M its molecular mass, then

$$PV = \frac{m}{M}RT \text{ or } PM = \frac{mRT}{V} = \rho RT$$

or $\dfrac{P}{\rho} = \dfrac{RT}{M}$ or $v^2 = \dfrac{RT}{M}$

or $v = \sqrt{\dfrac{RT}{M}}$

Hence $v_1 = \sqrt{\dfrac{RT}{M_1}}$ and $v_2 = \sqrt{\dfrac{RT}{M_2}}$ which

give $\dfrac{v_1}{v_2} = \sqrt{\dfrac{M_2}{M_1}}$

29. For an ideal gas, $PV = nRT$. Since pressure P is kept constant,

$$P\Delta V = nR\Delta T$$

or $\dfrac{\Delta V}{\Delta T} = \dfrac{nR}{P} = \dfrac{nRV}{nRT} = \dfrac{V}{T} \quad \left(\because P = \dfrac{nRT}{V}\right)$

or $\dfrac{1}{V}\dfrac{\Delta V}{\Delta T} = \dfrac{1}{T}$ or $\delta = \dfrac{1}{T}$

Thus, the value of δ decreases as T is increased. Hence the correct choice is (c).

30. For adiabatic process, $T_1\,V_1^{(\gamma-1)} = T_2\,V_2^{(\gamma-1)}$. Thus

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{(\gamma-1)}$$

For a monoatomic gas, $\gamma = 5/3$. Also $V_2/V_1 = L_2/L_1$. Hence

$$\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{\left(\frac{5}{3}-1\right)} = \left(\frac{L_2}{L_1}\right)^{2/3}$$

31. Since the slope of the P–V graph for adiabatic expansion is γ times that for isothermal expansion, curves AB and AC in Fig. 17.24(a) respectively represent isothermal and adiabatic expansions of the gas from initial volume V_1 to final volume V_2.

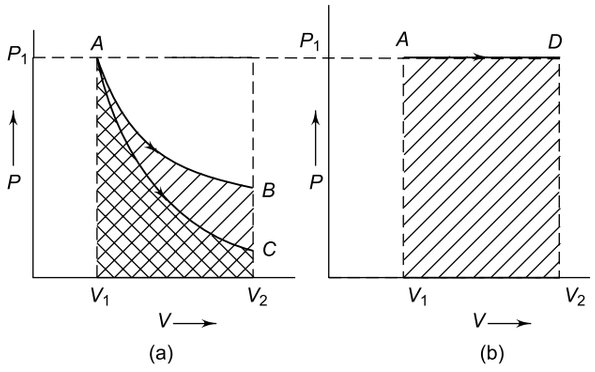


Fig. 17.24

As the area under curve AB between volumes V_1 and V_2 is greater than the area under curve AC between V_1 and V_2, it follows that $W_1 > W_3$.

Figure 17.24 (b) shows the P–V graph for isobaric (at constant pressure) expansion from initial volume V_1 and pressure P_1 to final volume V_2; the pressure remaining unchanged at P_1. Comparing figures (a) and (b) we find that the area under AD between volumes V_1 and V_2 is greater than the area under curves AB and AC. Hence W_2 is greater than W_1 and W_3. Hence the correct choice is (a).

32. $dQ = dU + dW$. In an adiabatic process, $dQ = 0$. Hence $dU = -dW = -4.5$ J. Hence the correct choice is (b).

33. Given $T_1 = 0$ºC $= 273$ K, $T_2 = 400$ºC $= 673$ K

Work done $W = \dfrac{nR}{(\gamma-1)}(T_2 - T_1) = \dfrac{5\times 8.3\times 400}{\left(\frac{7}{5}-1\right)}$

$$= 41500 \text{ J} = 41.5 \text{ kJ}$$

By convention, the work done on the gas is taken to be negative, i.e. $W = -41.5$ kJ. From the first law of thermodynamics $dQ = dU + dW$. For an adiabatic process, $dQ = 0$. Hence $dU = -dW = -(-41.5) = 41.5$ kJ. The positive sign of dU implies that the internal energy increases. Hence the correct choice is (b).

34. For an adiabatic process

$$TP^n = k$$

where $n = \dfrac{(1-\gamma)}{\gamma}$, $\gamma = \dfrac{C_p}{C_v}$ and k is a constant.

Therefore,

$$P = \left(\frac{k}{T}\right)^{1/n}$$

Since n = constant for a given gas,

$$P \propto T^{-1/n}$$

Given $P \propto T^3$. Hence $-\dfrac{1}{n} = 3$ or $-\dfrac{\gamma}{1-\gamma} = 3$, which gives $\gamma = \dfrac{3}{2}$. Hence the correct choice is (d).

35. For a gas, $PV = nRT$. Hence

$$(P)_{O2} = \frac{(1 \text{ mole})\, RT}{V}$$

and $$(P)_{He} = \frac{(1 \text{ mole})\, R(2T)}{V}$$

$$\therefore \quad \frac{(P)_{He}}{(P)_{O_2}} = 2$$

or $(P)_{He} = 2(P)_{O2}$, which is choice (c).

36. From the first law of thermodynamics, we have

$$dU = dQ - dW$$

Given $dW = 0$ and $dQ < 0$. Hence the change in internal energy $dU < 0$. Now, for an ideal gas, the internal energy can decrease only by decrease in temperature. Hence the correct choice is (a).

37. For an adiabatic process, PV^γ = constant, Differentiating, we have

$$\gamma PV^{\gamma-1} + \frac{dP}{dV} V^\gamma = 0 \text{ or } \frac{dP}{dV} = -\frac{\gamma P}{V}$$

Since at any instant PV = constant, $\dfrac{dP}{dV} \propto \gamma$, i.e. the slope of P-V curve is proportional to γ. Now, for a diatomic gas, γ(= 7/5) is than that for a monoatomic gas for which γ= 5/3. Therefore, the slope of the P-V curve is less for a diatomic gas than for a monoatomic gas. Hence curve 1 corresponds to diatomic gas and curve 2 to monoatomic gas. Thus the correct choice is (b).

38. Process $A \rightarrow B$ occurs at constant pressure. Hence the work done in this process is (see figure of Q. 38)

$$W_{AB} = PdV = P(V_2 - V_1) = 10 \times (2 - 1) = 10 \text{ J}$$

Process $B \rightarrow C$, occurs at constant volume. Hence $W_{BC} = 0$. Given $Q = 5$ J, i.e. total work done is $W_t = 5$ J. Therefore, we have

$$W_t = W_{AB} + W_{BC} + W_{CA}$$

or $$5 = 10 + 0 + W_{CA}$$

or $W_{CA} = -5$ J, which is choice (a).

39. For an ideal gas, $pV = nRT$. Differentiating, we have since T is kept constant

$$p\frac{dV}{dp} + V = 0 \text{ or } \frac{dV}{dp} = -\frac{V}{p}$$

Hence $$\beta = -\frac{1}{V}\left(\frac{dV}{dp}\right) = \frac{1}{p}$$

Therefore, choice (a) correctly represents the graph of β versus p.

40. Since AB is an isothermal process, the temperature of the gas remains constant between AB. Hence the P-T diagram must be perpendicular to the T-axis between A and B. Hence the correct choice is (a).

41. pV^γ = constant. For a given mass of gas, $V \propto \dfrac{1}{d}$. Hence

$$\frac{p}{d^\gamma} = \text{constant}$$

$$\therefore \quad \frac{p_1}{d_1^\gamma} = \frac{p_2}{d_2^\gamma}$$

$$\text{or } \frac{p_2}{p_1} = \left(\frac{d_2}{d_1}\right)^\gamma = (32)^{7/5} = (2)^7 = 128$$

Hence the correct choice is (c).

42. $\Delta U = C_v\, \Delta T$. Now $C_p - C_v = R$ or $\dfrac{C_p}{C_v} - 1 = \dfrac{R}{C_v}$

or $$C_v = \frac{R}{\gamma - 1}, \text{ where } \gamma = \frac{C_p}{C_v}. \text{ Hence}$$

$$\Delta U = \frac{R\Delta T}{(\gamma - 1)} = \frac{8.3 \times 8}{(1.4 - 1)} = 166 \text{ J}$$

Hence the correct choice (b).

43. $PV = \frac{m}{M} RT$. Therefore, the density of the gas is

$$\rho = \frac{m}{V} = \frac{PM}{RT} = \frac{PmN}{RT} = \frac{mP}{kT}$$

Hence the correct choice is (d).

44. Heat energy supplied $dQ = C_p\, dT$. Change in internal energy $dU = C_v\, dT$. Therefore, work done $dW = dQ - dU = (C_p - C_v)dT$.

$$\therefore \quad \frac{dW}{dQ} = \frac{(C_p - C_v)dT}{C_p dT} = 1 - \frac{1}{\gamma}$$

$$= 1 - \frac{1}{5/3} = \frac{2}{5}$$

($\because$ $\gamma = 5/3$ for a monoatomic gas)

Hence the correct choice is (c).

45. Let L be the length (in cm) of the hollow cylinder and r its radius. Since the mass of the gas remains unchanged and the pressures of the gas in both sides are equal, we have, from Charles' law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad (1)$$

Given $V_1 = \left(\frac{L}{2} - 5\right) \pi r^2$, $V_2 = \left(\frac{L}{2} + 5\right) \pi r^2$, $T_1 = 0°\text{C} = 273$ K and $T_2 = 100°\text{C} = 373$ K. Using these values in (1), we get

$$\frac{\frac{L}{2} - 5}{273} = \frac{\frac{L}{2} + 5}{373}$$

which gives $L = 64.6$ cm.

46. For an isothermal process: PV = constant and for an adiabatic process: PV^{γ} = constant, where γ is the ratio of the two specific heats (C_p/Cv) of the gas. When a gas is compressed from a volume V to a volume $(V - \Delta V)$, the increase in pressure is

$(\Delta P)_{\text{adia}} = \frac{\gamma \Delta V P}{V}$ for an adiabatic compression

and $(\Delta P)_{\text{iso}} = \frac{\Delta V P}{V}$ for an isothermal compression.

Hence P_3 will be greater than P_1. Therefore, the P–V diagrams of isothermal expansion from V_1, P_1 to V_2, P_2 and adiabatic compression for V_2, P_2 to V_1, P_3 are as shown in Fig. 17.25. Let W_1 and W_2 be the work done in isothermal expansion and adiabatic compression respectively. Therefore, net work done is

$$W = W_1 + (-W_2) = W_1 - W_2$$

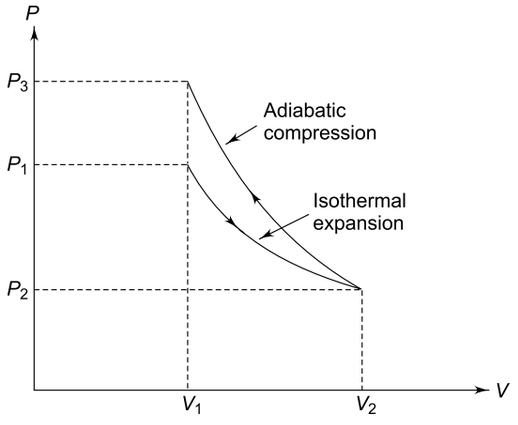


Fig. 17.25

Now, the area under the adiabatic curve is more than that under the isothermal curve. Hence $W_2 > W_1$. Therefore, $W < 0$. Hence the correct choice is (c).

47. Heat produced is given by

$$dQ = \frac{V^2 t}{R} = \frac{(10)^2 \times (10 \times 60)}{50} = 1200 \text{ J}$$

Since the container is rigid, the change in volume $dV = 0$. Hence work done $dQ = PdV = 0$. From the first law of thermodynamics, the change in internal energy is $dU = dQ - dW = dQ = 1200$ J. Hence the correct choice is (d).

48. The two adiabatic paths ad and bc for the gas intersect the two isothermals ab and cd at temperatures T_1 and T_2 (see Fig. 17.18). Since points a and d lie on the same adiabatic path, we have

$$T_1 V_a^{(\gamma - 1)} = T_2 V_d^{(\gamma - 1)}$$

or $$\left(\frac{V_a}{V_d}\right)^{(\gamma - 1)} = \frac{T_2}{T_1} \qquad (1)$$

Since points b and c also lie on the same adiabatic path,

$$T_1 (V_b)^{(\gamma - 1)} = T_2 V_c^{(\gamma - 1)}$$

or $$\left(\frac{V_b}{V_c}\right)^{(\gamma - 1)} = \frac{T_2}{T_1} \qquad (2)$$

From (1) and (2), we get

$$\left(\frac{V_a}{V_d}\right)^{(\gamma - 1)} = \left(\frac{V_b}{V_c}\right)^{(\gamma - 1)}$$

or $$\frac{V_a}{V_d} = \frac{V_b}{V_c}$$

Thus the correct choice is (c).

49. Since the temperature T remains constant along the path CA, P will be inversely proportional to V along this path. Hence, as P increases, V must decrease in a nonlinear fashion. This is represented by the curve CA in Fig. 17.26.

Along the path BC, the volume V is constant. Hence the graph of P against V is a straight line perpendicular to the V-axis. On a $P - V$ diagram, the corresponding path is BC as shown in Fig. 17.26.

For the path AB, V is directly proportional to T pressure remaining constant. The corresponding path AB is, therefore a straight line parallel to the V-axis. Thus the cyclic process on a P–V diagram is represented by choice (d) in Fig. 17.20.

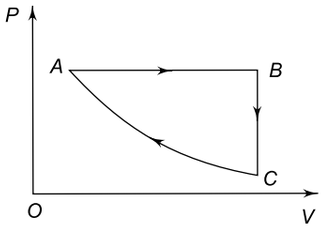


Fig. 17.26

50. Work done on the gas is

$$\Delta W = P\Delta V = P(V_f - V_i)$$
$$= 1 \times 10^5 \times (25 - 5) \times 10^{-3}$$
$$= 2000 \text{ J}$$

The internal energy is given by $U = \dfrac{PV}{(\gamma - 1)}$

$$\therefore \quad U_i = \frac{PV_i}{(\gamma - 1)}, \; U_f = \frac{PV_f}{(\gamma - 1)}.$$

Therefore, change in internal energy is

$$\Delta U = U_f - U_i = \frac{P}{(\gamma - 1)} (V_f - V_i)$$

$$= \frac{1 \times 10^5 \times (25 - 5) \times 10^{-3}}{(1.4 - 1)} = 5000 \text{ J}$$

From the first law of thermodynamics, the heat energy supplied to the gas is

$$\Delta Q = \Delta W + \Delta U = 2000 + 5000$$
$$= 7000 \text{ J}$$

which is choice (d)

51. In process AB, the volume V increases linearly with temperature T. Hence process AB is isobaric (constant pressure). Therefore, work done in this process is

$$W_{AB} = P\Delta V = nR\Delta T \qquad (\because PV = nRT)$$
$$= nR(T_B - T_A)$$
$$= 3 \times 8.3 \times (200 - 100) = 2490 \text{ J}$$

Process CA is isochoric (constant volume). Hence work done in this process $W_{CA} = 0$. Since the whole process $ABCA$ is cyclic, the change in internal energy in the complete cycle is zero, i.e. $\Delta U = 0$. Now, from the first law of thermodynamics, (Given $Q = -2510$ J)

$$Q = \Delta U + W = \Delta U + W_{AB} + W_{BC} + W_{CA}$$

or $$-2510 = 0 + 2490 + W_{BC} + 0$$

or $$W_{BC} = -2510 - 2490 = -5000 \text{ J}$$

The negative sign shows that the work is done by the gas. Thus the correct choice is (b).

52. At atmospheric pressure, the boiling point of liquid oxygen is greater than 50 K. Therefore, between 50 K and 300 K, liquid oxygen undergoes a change of state. Hence the correct choice is (c).

53. The heat energy supplied is

$$\Delta Q = I^2 R t$$
$$= (2)^2 \times 100 \times (10 \times 60)$$
$$= 240 \times 10^3 \text{ J} = 240 \text{ kJ}$$

Since $\Delta V = 0$; work done $\Delta W = 0$. From first law of thermodynamics, $\Delta U = \Delta Q = 240$ kJ. Thus the correct choice is (d).

54. The equation of straight line AB is

$$P = mV + c \qquad (1)$$

where m is the slope and c is the intercept. For points A and B, we have

$$2P_0 = mV_0 + c$$

and $$P_0 = m(2V_0) + c$$

These equations give $m = -\dfrac{P_0}{V_0}$ and $c = 3P_0$.

Now $PV = nRT \Rightarrow P = \dfrac{nRT}{V}$. Using this in Eq. (1), we have

$$T = \frac{1}{nR}(mV^2 + cV) \qquad (2)$$

T will be maximum if $\dfrac{dT}{dV} = 0$ and $\dfrac{d^2T}{dV^2} < 0$.

Differentiating Eq. (2) w.r.t to V and putting $\dfrac{dT}{dV} = 0$, we get

$$2mV + c = 0$$

which gives $$V = -\frac{c}{2m}$$

$$T_{\max} = -\frac{c^2}{4nRm} = -\frac{1}{4nR} \times \frac{(3P_0)^2}{-P_0/V_0} = \frac{9P_0V_0}{4nR}$$

Thus the correct choice is (a).

55. Mass of 2 litres of water = 2 kg. Heat energy needed to raise the temperature of 2 kg of water from 20°C to 75°C is

$Q = 2 \times (4.2 \times 10^3) \times 55 = 4.62 \times 10^5$ J

If t is the time taken, heat energy supplied by the heater in time t is

Q_1 = (power × time) = 1000 t joule

Heat energy lost in time t is

$$Q_2 = 160\ t \text{ joule}$$

Heat energy available for heating water is

$$Q' = Q_1 - Q_2 = 840\ t \text{ J}$$

Equating $Q = Q'$, we get $t \simeq 550$ s. Thus the correct choice is (b).

56. Given $PT^2 = k$ (constant). From $PV = nRT$, we have $P = \dfrac{nRT}{V}$. Hence

$$nRT^3 = kV$$

Differentiating we have

$$3nRT^2\Delta T = k\Delta V \quad \Rightarrow \quad \frac{\Delta V}{\Delta T} = \frac{3nRT^2}{k}$$

Coefficient of volume expansion is

$$\gamma = \frac{\Delta V}{V\Delta T} = \frac{3nRT^2}{kV} \qquad (1)$$

Using $V = \dfrac{nRT}{P}$ and $PT^2 = k$ in Eq. (1),

we get $\gamma = \dfrac{3}{T}$.

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Figure 17.27 is the P-V diagram for a Carnot cycle. In this diagram,

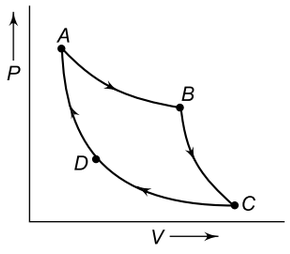


Fig. 17.27

(a) curve AB represents isothermal process and BC adiabatic process

(b) curve AB represents adiabatic process and BC isothermal process

(c) curve CD represents isothermal process and DA adiabatic process

(d) curve CD represents adiabatic process and DA isothermal process.

2. Figure 17.28 shows the P-V diagram of a cyclic process. If dQ is the heat energy supplied to the system, dU is the internal energy of the system and dW is the work done by the system, then which of the following relations is/are correct

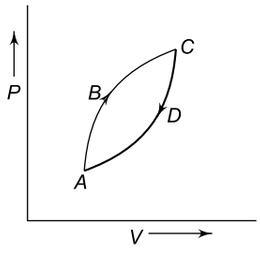


Fig. 17.28

(a) $dQ = dU dW$

(b) $dU = 0$

(c) $dQ = dW$

(d) $dQ = -dW$

3. If ΔQ represents the heat energy supplied to a system, ΔU the increase in internal energy and ΔW the work done by the system, then which of the following are correct?

(a) $\Delta Q = \Delta W$ for an isothermal process

(b) $\Delta U = -\Delta W$ for an adiabatic process

(c) $\Delta U = \Delta Q$ for an isochoric process

(d) $\Delta U = -\Delta Q$ for an isobaric process.

4. The initial state of n moles of an ideal gas is represented by P_1, V_1, T_1 and the final state by P_2, V_2, T_2. W_i is the work done by the gas in an isothermal process ($T_1 = T_2 = T$) and W_a in an adiabatic process, then

(a) $W_i = nRT \log_e \left(\dfrac{V_2}{V_1}\right)$

(b) $W_i = nRT \log_e \left(\dfrac{P_1}{P_2}\right)$

(c) $W_a = \dfrac{1}{(\gamma - 1)} (P_1V_1 - P_2V_2)$

(d) $W_a = \dfrac{nR}{(\gamma - 1)} (T_1 - T_2)$

5. An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to $T/2$. If f is the number of degrees of freedom of gas molecules and W is the work done by the gas during the expansion, then

(a) $f = 3$ (b) $f = 5$

(c) $W = \dfrac{5PV}{4}$ (d) $W = \dfrac{3PV}{2}$

IIT, 1990

6. An ideal gas is taken through a cyclic thermodynamic process involving four steps. The amounts of heat involved in these steps are $Q_1 = 5960$ J, $Q_2 = -5585$ J, $Q_3 = -2980$ J and $Q_4 = 3645$ J respectively. The corresponding amounts of work done are $W_1 = 2200$ J, $W_2 = -825$ J and $W_3 = -1100$ J and W_4 respectively. The efficiency of the cycle is η. Then

(a) $W_4 = 765$ J (b) $W_4 = 275$ J

(c) $\eta \simeq 11\%$ (d) $\eta \simeq 16\%$

IIT, 1994

7. An ideal gas has pressure P, volume V and temperature T. The ratio $C_P/C_v = \gamma$ and U is the internal energy. If R is the gas constant, then

(a) $C_v = \dfrac{R}{\gamma - 1}$ (b) $U = nC_v T$

(c) $U = \dfrac{PV}{(\gamma - 1)}$ (d) $U = nC_pT$

8. A carnot's engine whose sink is at 27°C has an efficiency of 25%.

(a) The temperature of the source is 400 K.

(b) To increase efficiency by 20%, the temperature of the source should be increased by 28.6°C.

(c) To increase efficiency by 20%, the temperature of the sink should be decreased by 28.6 °C.

(d) If the heat energy supplied to the engine is 800 J per cycle, the work output per cycle is 200 J.

9. A refrigerator freezes 1 kg of water at 0°C in 3 minutes. The room temperature is 27°C. The latent heat of fusion of ice is 80 cal/g.

(a) Heat extracted from water is 3.36×10^5 J.

(b) The coefficient of performance is nearly equal to 10.

(c) The work done by the motor is nearly 3.36×10^4 J.

(d) The power of the momter is nearly 20 kW.

10. Figure 17.29 shows the P–V diagram of a cyclic process $ABCA$

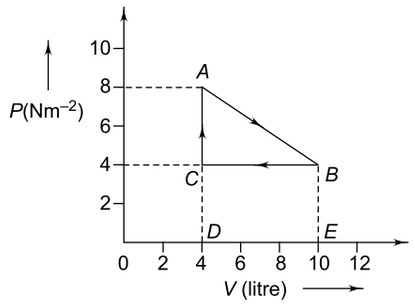


Fig. 17.29

(a) Work done in process $A \rightarrow B$ is 0.036 J.

(b) Work done in process $B \rightarrow C$ is – 0.024 J.

(c) Work done in process $C \rightarrow A$ is zero.

(d) Work done in cycle $ABCA$ is 0.06 J.

11. Figure 17.30 shows the P–V diagram for an ideal gas. From the graph, we conclude that

(a) the process is isothermal.

(b) the internal energy of the gas remains constant.

(c) The work done in the process is positive.

(d) the heat energy absorbed in the process is zero.

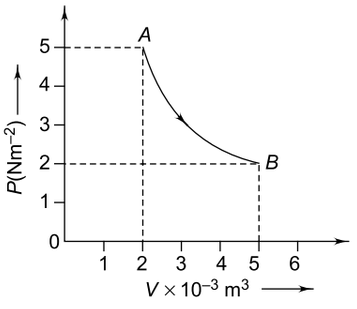


Fig. 17.30

12. Figure 17.31 shows the $P - V$ diagram for an ideal gas. From the graph we conclude that

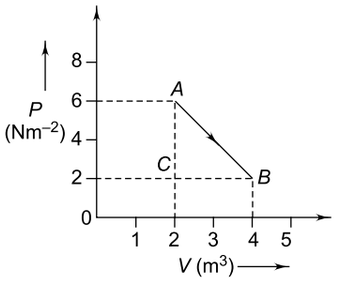


Fig. 17.31

(a) the process $A \rightarrow B$ is adiabatic.

(b) the internal energy of the gas increases in this process.

(c) the work done by the gas = area of triangle *ABC*.

(d) the heat energy absorbed by the gas is zero in the process.

13. n moles of an ideal monoatomic gas is kept in a closed vessel of volume 0.0083 m^3 at a temperature of 300 K and a pressure of 1.6×10^6 Pa. Heat energy of 2.49×10^4 J is supplied to the gas. Given $R = 8.3$ J mol^{-1} K^{-1}.

(a) The value of $n = 5$

(b) For the gas $C_p = 20.75$ J mol^{-1} K^{-1}

(c) The Final temperature of the gas is 402°C

(d) The final pressure of the gas is 3.6×10^6 Pa

IIT, 1987

14. For an ideal gas

(a) the change in internal energy at constant pressure when the temperature of n moles of the gas changes by ΔT is $n\ C_v\ \Delta T$.

(b) the change in internal energy of the gas in an adiabatic process is equal in magnitude to the work done by the gas.

(c) the internal energy does not change in an isothermal process.

(d) no heat is added or removed in an adiabatic process.

IIT, 1989

15. An ideal gas is taken from state A (pressure P, volume V) to state B (pressure $P/2$, Volume $2V$) along a straight line in the P-V diagram as shown in Fig. 17.32. Then

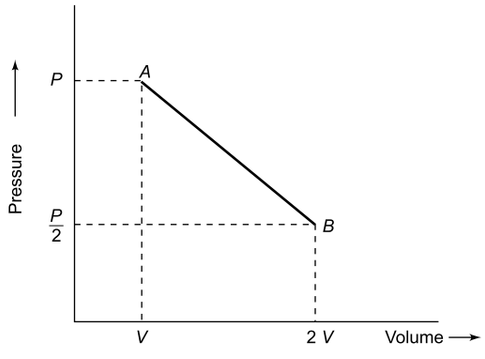


Fig. 17.32

(a) the work done by the gas in the process A to B exceeds the work that would be done by it if the system were taken from A to B along the isotherm.

(b) in the T-V diagram, the path AB becomes a part of a parabola.

(c) in the P-T diagram, the path AB becomes a part of a hyperbola.

(d) in going from A to B, the temperature T of the gas first increases to a maximum and then decreases.

IIT, 1993

16. One mole of oxygen at 27 °C is enclosed in a vessel which is thermally insulated. The vessel is moved with a constant speed v and is then suddenly stopped. The process results in a rise of temperature of the gas by 1 °C. Then, if M = molecular mass of oxygen.

(a) $\gamma\ (= C_p/C_v) = \dfrac{5}{3}$ (b) $\gamma = \dfrac{7}{5}$

(c) $v = \sqrt{\dfrac{R}{M(\gamma+1)}}$ (d) $v = \sqrt{\dfrac{2R}{M(\gamma-1)}}$

IIT, 1996

17. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then

(a) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(b) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(c) C_p/C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas

(d) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

IIT, 2009

18. Figure 17.33 shows the P-V plot of an ideal gas taken through a cycle $ABCDA$. The part ABC is a semi-circle and CDA is half of an ellipse. Then,

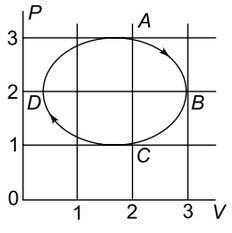


Fig. 17.33

(a) the process during the path $A \rightarrow B$ is isothermal

(b) heat flows out of the gas during the path $B \rightarrow C \rightarrow D$

(c) work done during the path $A \rightarrow B \rightarrow C$ is zero

(d) positive work is done by the gas in the cycle $ABCDA$

IIT, 2009

ANSWERS AND SOLUTIONS

1. For adiabatic process, PV^{γ} = constant. Differentiating w.r.t V we get

$$\frac{dP}{dV} V^{\gamma} + P\gamma\, V^{\gamma-1} = 0$$

or $$\frac{dP}{dV} = -\frac{\gamma P}{V}$$

For isothermal process, PV = constant. Hence

$$\frac{dP}{dV} = -\frac{P}{V}$$

Now, dP/dV is the slope of the (P-V) graph. Thus, the slope of the (P-V) graph for an adiabatic process is γ times that for an isothermal process. Hence curves BC and DA both represent adiabatic process and curves AB and CD both represent isothermal process. Thus the correct choices are (a) and (c).

2. In a cyclic process, the system returns to its initial state. Hence the change in internal energy $dU = 0$. Therefore, choice (b) is correct. From the first law of thermodynamics,

$$dQ = dU + dW = dW \qquad (\because\ dU = 0)$$

Hence choice (c) is also correct.

3. $\Delta W = P\Delta V$ and $\Delta U = nC_v\,\Delta T$ and $\gamma = C_p/C_v$. Also $\Delta Q = \Delta U + \Delta W$.

For an isothermal process, $\Delta T = 0$. Therefore $\Delta U = 0$. Hence $\Delta Q = \Delta W$, which is choice (a). For an adiabatic process $\Delta Q = 0$. So $\Delta U = -\Delta W$, which is choice (b). For an isochoric process, $\Delta W = 0$, so $\Delta Q = \Delta U$, which is choice (c). For an isobaric process, $\Delta Q = \Delta U + \Delta W$. So choice (d) is wrong.

4. All the four choices are correct.

5. For an adiabatic change the relation between T and V is

$$TV^{(\gamma-1)} = \text{constant};\ \gamma = \frac{C_p}{C_v}$$

$$TV^{(\gamma-1)} = T'\,V'^{(\gamma-1)} \text{ or } \left(\frac{V'}{V}\right)^{(\gamma-1)} = \frac{T}{T'}$$

Given $V' = 5.66\ V$ and $T' = \dfrac{T}{2}$. Therefore,

$$(5.66)^{(\gamma-1)} = 2$$

Taking logarithm of both sides, we have

$$(\gamma - 1)\log(5.66) = \log(2)$$

or $$\gamma = 1 + \frac{\log(2)}{\log(5.66)} = 1 + \frac{0.3010}{0.7528}$$

$$= 1 + 0.4 = 1.4$$

Since $\gamma = 1.4$, the gas is diatomic. For a diatomic gas, the number of degrees of freedom of the molecules = 5.

We know that the work done by the gas during adiabatic expansion is given by

$$W = \frac{1}{(\gamma-1)}\,(PV - P'V') \qquad (1)$$

where pressure P' after expansion is obtained from the relation

$$\frac{P'V'}{T'} = \frac{PV}{T}$$

or $$P' = P \times \frac{V}{V'} \times \frac{T'}{T}$$

$$= P \times \frac{V}{5.66V} \times \frac{T/2}{T} = \frac{P}{11.32}$$

Putting $\gamma = 1.4$, $V' = 5.66\ V$ and $P' = \dfrac{P}{11.32}$ in Eq. (1), We have

$$W = \frac{1}{(1.4-1)}\left(PV - \frac{P}{11.32} \times 5.66V\right)$$

$$= \frac{1}{0.4}\left(PV - \frac{1}{2}PV\right) = 1.25\ PV$$

So the correct choices are (b) and (c).

6. Since W_2 and W_3 are negative, it means that the work is done on the gas. Hence Q_2 and Q_3 are negative which implies that heat is evolved in processes 2 and 3. Since Q_1 and Q_4 are positive, heat is absorbed by the gas in processes 1 and 4. As $(Q_1 + Q_4)$ is greater than $(Q_2 + Q_3)$, the gas absorbs a net amount of heat energy in a complete cycle, which is given by

$$\Delta Q = Q_1 + Q_2 + Q_3 + Q_4$$
$$= 5960 - 5585 - 2980 + 3645$$
$$= 1040 \text{ joule}$$

The net work done by the gas is

$$\Delta W = W_1 + W_2 + W_3 + W_4$$
$$= 2200 - 825 - 1100 + W_4$$
$$= (275 + W_4) \text{ joule}$$

Since the process is cyclic, the change in internal energy $\Delta U = 0$. From the first law of thermodynamics,

we have

$$\Delta W = \Delta Q - \Delta U = \Delta Q$$

or $275 + W_4 = 1040$ or $W_4 = 1040 - 275$

$$= 765 \text{ J}$$

Efficiency of the cycle is defined as

$$h = \frac{\text{net work done by the gas}}{\text{total heat absorbed by the gas}}$$

$$= \frac{\Delta W}{Q_1 + Q_4} = \frac{275 + 765}{5960 + 3645}$$

$$= \frac{1040}{9605} = 0.1083 = 10.83\%$$

$$\simeq 11\%$$

Thus the correct choices are (a) and (c).

7. The internal energy of n moles of an ideal gas at absolute temperature T is given by

$$U = nC_vT \qquad (1)$$

where C_v is the molar specific heat at constant volume. We know that

$$C_p - C_v = R \text{ or } \frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

$$\text{or } \gamma - 1 = \frac{R}{C_v} \text{ or } C_v = \frac{R}{\gamma - 1} \qquad (2)$$

Now, the ideal gas equation for n nodes is

$$PV = nRT \text{ or } n = \frac{PV}{RT} \qquad (3)$$

Using (2) and (3) in (1), we have

$$U = \frac{PV}{RT} \times \frac{R}{\gamma - 1} \times T = \frac{PV}{(\gamma - 1)}$$

Hence the correct choices are (a), (b) and (c).

8. (a) $T_2 = 27°C = 300$ K, efficiency $\eta = 0.25$.

$\eta = 1 - \frac{T_2}{T_1} \Rightarrow 0.25 = 1 - \frac{300}{T_1}$ which gives $T_1 = 400$ K. So choice (a) is correct.

(b) Increase in efficiency = 20% of 0.25 = 0.05. So new efficiency is $\eta' = 0.25 + 0.05 = 0.30$. The new temperature of the source should be T_1' so that

$$0.30 = 1 - \frac{T_2}{T_1'} = 1 - \frac{300}{T_1'}$$

which gives $T_1' = 428.6$ K. So, increase in temperature of the source $= T_1' - T_1 = 428.6 - 400 = 28.6$ K or °C. So choice (b) is also correct.

(c) The new temperature T_2' of the sink should be such that

$$0.30 = 1 - \frac{T_2'}{T_1} = 1 - \frac{T_2'}{400}$$

which gives $T_2' = 280$ K = 7°C. Decrease in temperature of the sink is 27°C – 7°C = 20°C, choice (c) is wrong.

(d) Work output $= Q\eta = 800 \times 0.25 = 200$ J. So choice (d) is correct.

9. (a) Heat extracted from water is $Q_2 = mL = 10^3 \times 80 = 80{,}000$ J $= 80{,}000 \times 4.2 = 3.36 \times 10^5$ J. So choice (a) is correct.

(b) $\beta = \frac{T_2}{T_1 - T_2} = \frac{273}{300 - 273} \simeq 10$. Choice (b) is correct.

(c) $W = \frac{Q_2}{\beta} = \frac{3.36 \times 10^5}{10} \simeq 3.36 \times 10^4$ J, which is choice (c)

(d) $Q_1 = Q_2 + W = 3.36 \times 10^5 + 3.36 \times 10^4$

$\simeq 37 \times 10^4$ J

Power $= \frac{Q_1}{t} = \frac{37 \times 10^4 \text{ J}}{180 \text{ s}} \simeq 20$ kW. So choice (d) is also correct.

10. $W_{A \to B}$ = Area of $ABED = \frac{1}{2} BC \times AC + CD \times DE$

$= \frac{1}{2} (6 \times 10^{-3} \text{ m}^3) \times 4 \text{ Nm}^{-2} + 4 \text{ Nm}^{-2} \times 6 \times 10^{-3} \text{m}^3$

$= 0.012 + 0.024 = 0.036$ J

$W_{B \to C}$ = Area of $BCDE = -0.024$ J. The negative sign shows that the work is done on the gas.

$W_{C \to A} = P\Delta V = 0$ because $\Delta V = 0$.

Work done in complete cycle $= 0.036 - 0.024 + 0$

$= 0.012$ J

Hence the correct choices are (a), (b) and (c).

11. For point A, $PV = 5 \times (2 \times 10^{-3}) = 10 \times 10^{-3}$ J

For point B, $PV = 2 \times (5 \times 10^{-3}) = 10 \times 10^{-3}$ J

Since PV = constant, the process is isothermal. For an isothermal process, $\Delta U = 0$. Since the gas undergoes expansion ΔW is positive and $\Delta Q = \Delta W$. Hence the correct choices are (a), (b) and (c).

12. The P–V graph for an adiabatic process is not a straight-line. Hence choice (a) is wrong. $P_A V_A = n R T_A$ and $P_B V_B = n R T_B$. Therefore

$$\frac{P_A V_A}{P_B V_B} = \frac{T_A}{T_B} \Rightarrow \frac{6 \times 1}{2 \times 4} \Rightarrow \frac{T_A}{T_B} = \frac{3}{4},$$

i.e. $T_B > T_A$. Hence the internal energy increases.

Work done = area under AB upto the volume axis. Heat energy is absorbed in the process. Hence the only correct choice is (b).

13. (a) $PV = nRT \Rightarrow n = \frac{PV}{RT}$

$$= \frac{(1.6 \times 10^6) \times 0.0083}{8.3 \times 300} = \frac{16}{3}$$

(b) For a monoatomic gas $C_p = \frac{5R}{2} = \frac{5}{2} \times 8.3$
$= 20.75$ J mol^{-1} K^{-1}

(c) $\Delta Q = n\ C_v\ \Delta T$; $C_v = \frac{3R}{2}$

$$\therefore\ \Delta T = \frac{\Delta Q}{nC_v} = \frac{2\Delta Q}{3nR} = \frac{2 \times 2.49 \times 10^4}{3 \times \frac{16}{3} \times 8.3}$$
$$= 375 \text{ K}$$

$\therefore$ Final temperature = 300 + 375 = 675 K = 402°C.

(d) $\frac{P_2}{T_2} = \frac{P_1}{T_1} \Rightarrow P_2 = \frac{P_1 \times T_2}{T_1} = \frac{1.6 \times 10^6 \times 675}{300}$
$= 3.6 \times 10^6$ Pa

Thus the correct choices are (b), (c) and (d).

14. (a) $\Delta U = n\ C_v\ \Delta T$

(b) $\Delta Q = \Delta U + \Delta W$. For an adiabatic process, $\Delta Q = 0$. Hence $0 = \Delta U + \Delta W$ or $|\Delta U| = |\Delta W|$.

(c) For an isothermal process, $\Delta T = 0$, Hence $\Delta U = 0$.

(d) For an adiabatic process, $\Delta Q = 0$.

All the four choices are correct.

15. (a) Work done in process A to B is (see Fig. 17.34)

$$W_1 = \text{area of trapezium } ABCD$$
$$= \left(P + \frac{P}{2}\right) V = \frac{3PV}{2}$$
$$= \frac{3RT}{2} \text{ (for 1 mole)}$$

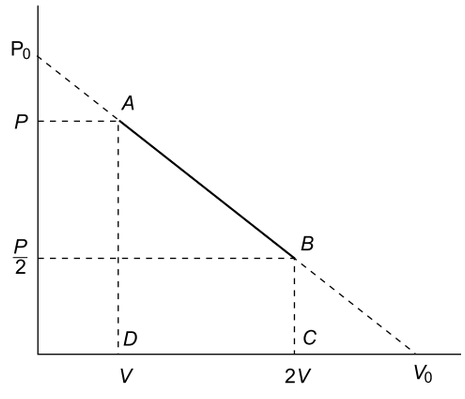


Fig. 17.34

If the process A to B were isothermal, the work done would be

$$W_2 = RT \log_e\left(\frac{V_2}{V_1}\right) = RT \log_e(2) = 0.69\ RT$$

Thus $W_1 > W_2$. So choice (a) is correct.

(b) Let P_0 and V_0 be the intercepts on the P and V axes. The equation of straight line AB is

$$P = -\frac{P_0}{V_0}(V - V_0)$$
$$\Rightarrow \quad \frac{P}{P_0} + \frac{V}{V_0} = 1 \qquad (1)$$

Since $P = \frac{RT}{V}$, Eq. (1) becomes

$$\frac{RT}{VP_0} + \frac{V}{V_0} = 1 \Rightarrow T = \frac{P_0 V}{R} - \frac{P_0 V^2}{RV_0}$$

which represents a parabola on the T-V graph. So choice (b) is also correct.

(c) Since $V = \frac{RT}{P}$, Eq. (1) becomes

$$\frac{P}{P_0} + \frac{RT}{PV_0} = 1 \Rightarrow \text{T} = V_0\ P - \frac{V_0 P^2}{R P_0} \qquad (2)$$

which does not represent a hyperbola. So choice (c) is incorrect.

(d) If follows from Eq. (2) above that choice (d) is correct.

16. Oxygen is diatomic; it has 5 degrees of freedom. Therefore, $C_v = 5\ R/2$ and $C_p = 7\ R/2$. So $\gamma = C_p/C_v = 7/5$. The kinetic energy of oxygen molecules with a velocity $v_0 = \frac{1}{2}\ M\ v^2$, where M = molecular weight of oxygen.

Now heat energy = $C_v\ dT = C_v \times 1 = C_v$

But $C_p - C_v = R$ or $\frac{C_p}{C_v} - 1 = \frac{R}{C_v}$

or $(\gamma - 1) = \frac{R}{C_v}$ or $C_v = \frac{R}{(\gamma - 1)}$

Therefore, $\frac{1}{2}\ M v^2 = \frac{R}{(\gamma - 1)}$

or $v = \sqrt{\frac{2R}{M(\gamma - 1)}}$

So the correct choices are (b) and (d).

17. $(C_p + C_v)$ for diatomic gas $= \frac{7R}{2} + \frac{5R}{2} = 6\ R$

$(C_p + C_v)$ for monoatomic gas $= \frac{5R}{2} + \frac{3R}{2} = 4\ R$

$C_p \cdot C_v$ for diatomic gas $= \frac{35R^2}{4}$

$C_p \cdot C_v$ for monoatomic gas $= \frac{15R^2}{4}$

$\frac{C_p}{C_v} = \gamma = 1 + \frac{2}{f}$ is smaller for diatomic gas than for monoatomic gas.

$C_p - C_v = R$.

Hence the correct choices are (b) and (d).

18. Choice (a) is incorrect as the slope of the graph for process $A \rightarrow B$ increases with V. Choice (b) is correct. For the process $B \rightarrow C \rightarrow D$, volume is decreasing. Hence work done is negative ($W < 0$). Also

$\Delta U = \frac{P(V_D - V_B)}{\gamma - 1}$ is negative because $V_D < V_B$.

From $\Delta Q = \Delta U + W$, we find that ΔQ is negative, i.e. heat flows out of the gas in process $B \rightarrow C \rightarrow D$. Choice (c) is wrong because area under the graph $A \rightarrow B \rightarrow C$ is not zero. Choice (d) is correct because the cyclic process is clockwise. Hence the correct choices are (b) and (d).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage

Passage I

Three moles of an ideal gas $\left(C_p = \frac{7R}{2}\right)$ at pressure P_A and temperature T_A are isothermally expanded to twice the original volume. The gas is then compressed at constant pressure to its original volume. Finally the gas is heated at constant volume to its original pressure P_A.

IIT, 1991

1. Which of the graphs shown in Fig. 17.35 represents the $P - V$ diagram for the complete process?

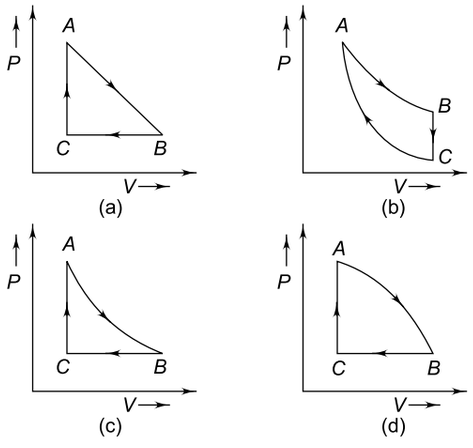


Fig. 17.35

2. Which of the graphs shown in Fig. 17.36 represents the $P - T$ diagram for the complete process?

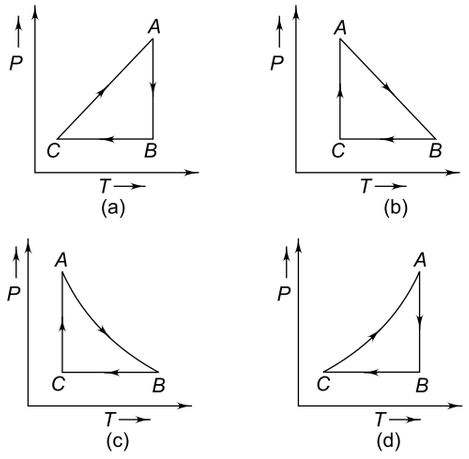


Fig. 17.36

3. The net work done ΔW by the gas during the complete process is (R is the gas constant).

(a) $0.5\ RT_A$ (b) $0.58\ RT_A$

(c) $0.64\ RT_A$ (d) RT_A

4. The net heat energy supplied to the gas in the complete process is

(a) zero (b) equal to ΔW

(c) less than ΔW (d) greater than ΔW

SOLUTION

1. During isothermal process $A \rightarrow B$, $P \propto \frac{1}{V}$. During isobaric process ($B \rightarrow C$), P = constant and during isochoric process ($C \rightarrow A$), V = constant. Hence the correct $P - V$ diagram of the complete process is (c).

2. During isothermal process ($A \rightarrow B$), T = constant. During isobaric process ($B \rightarrow C$), P = constant and during isochoric process ($C \rightarrow A$), $P \propto T$. Hence the correct $P - T$ diagram for the complete process is (a).

3. For process $A \to B$, $P_A V_A = P_B V_B$ which gives $P_B = \frac{P_A}{2}$ because $V_B = 2V_A$ (given).

For process $B \to C$, $\frac{V_B}{T_B} = \frac{V_C}{T_C}$. Since $V_B = 2V_A$ and $V_C = V_A$ and $T_B = T_A$, we get $T_C = \frac{T_A}{2}$. Also $P_C = P_B = \frac{P_A}{2}$.

For process $C \to A$, $\frac{P_C}{T_C} = \frac{P_A}{T_A}$. Since $P_C = \frac{P_A}{2}$; $T_A = 2T_C$.

Work done is isothermal process $A \to B$ is

$$W_{AB} = nRT \log_e\left(\frac{V_B}{V_A}\right) = 3\, RT_A \log_e(2) \qquad (2)$$

Work done in isobaric process $B \to C$ is

$$W_{BC} = P_B(V_C - V_B) = \frac{P_A}{2}(V_A - 2V_A)$$

$$= -\frac{P_A V_A}{2} = -\frac{3}{2}\, RT_A .$$

Work done is isochoric process $C \to A$ is $W_{CA} = 0$

$\therefore$ Total work done

$$\Delta W = 3\, RT_A \log_e(2) - \frac{3}{2}\, RT_A$$

$$= 3\, RT_A(0.693 - 0.5) = 0.58\, RT_A$$

Thus the correct choice is (b).

4. Since the process is cyclic, $\Delta U = 0$. From $\Delta Q = \Delta U + \Delta W$, we get $\Delta Q = \Delta W$. So the correct choice is (b).

Questions 5 to 9 are based on the following passage

Passage II

Two moles of an ideal gas at volume V, pressure $2P$ and temperature T undergo a cyclic process $ABCDA$ as shown in Fig. 17.37.

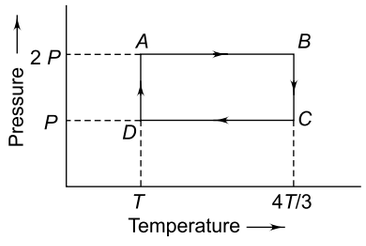


Fig. 17.37

IIT, 1992

5. The volume (V_B) of the gas in state B is

(a) $\frac{V}{3}$ (b) $\frac{2V}{3}$

(c) V (d) $\frac{4V}{3}$

6. The volume (V_C) of the gas in state C is

(a) $\frac{8V}{3}$ (b) $2\, V$

(c) $\frac{4V}{3}$ (d) $\frac{2V}{3}$

7. The volume (V_D) of the gas in state D is

(a) V (b) $2\, V$

(c) $3\, V$ (d) $4\, V$

8. The net work done (ΔW) in the complete cycle is

(a) $2\, RT$ (b) $2\, RT \log_e(2)$

(c) $\frac{4}{3}\, RT$ (d) $\frac{2}{3}\, RT \log_e(2)$

9. The net change (ΔQ) in the heat energy in the complete process is

(a) zero (b) greater than ΔW

(c) less than ΔW (d) equal to ΔW.

SOLUTION

5. For isobaric process $A \to B$,

$$\frac{V_A}{T_A} = \frac{V_B}{T_B} \Rightarrow V_B = \frac{V_A T_B}{T_A} = \frac{V \times 4T/3}{T} = \frac{4V}{3}$$

So the correct choice is (d).

6. For isothermal process $B \to C$,

$$P_B V_B = P_C V_C \Rightarrow V_C = \frac{P_B V_B}{P_C} = \frac{2P \times 4V/3}{P} = \frac{8V}{3},$$

which is choice (a).

7. For isobaric process $C \to D$,

$$\frac{V_C}{T_C} = \frac{V_D}{T_D} \Rightarrow V_D = \frac{V_D T_D}{T_C} = \frac{8V/3 \times T}{4T/3} = 2V$$

Hence the correct choice is (b).

8. $W_{AB} = P_A(V_B - V_A) = 2P \times \left(\frac{4V}{3} - V\right) = \frac{2PV}{3}$

$$W_{BC} = nRT_B \log_e\left(\frac{V_C}{V_B}\right)$$

$$= 2 \times R \times \frac{4T}{3} \log_e \left(\frac{8V/3}{4V/3} \right) = \frac{8}{3} RT \log_e(2)$$

$$W_{CD} = P_C(V_D - V_C) = P \times \left(2V - \frac{8V}{3} \right) = - \frac{2PV}{3}$$

$$W_{DA} = nRT_D \log_e \left(\frac{V_A}{V_D} \right)$$

$$= 2 \times R \times T \log_e \left(\frac{V}{2V} \right)$$

$$= 2\, RT \log_e \frac{1}{2} = - 2\, RT \log_e(2)$$

Total work done is

$$\Delta W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= \frac{2PV}{3} + \frac{8}{3} RT \log_e(2) - \frac{2PV}{3} - 2\, RT \log_e(2)$$

$$= \frac{2}{3} RT \log_e(2), \text{ which is choice (d).}$$

9. For a cyclic process, $\Delta U = 0$. Hence $\Delta Q = \Delta W$. So the correct choice is (d).

Questions 10 to 12 are based on the following passage

Passage III

A gaseous mixture enclosed in a vessel of volume V consists of one gram mole of a gas A with $\gamma (= C_p/C_v) = \frac{5}{3}$ and another gas B with $\gamma = \frac{7}{5}$ at a certain temperature T. The gram molecular weights of gases A and B are 4 and 32 respectively. The gases do not react with each other and are assumed to be ideal. The gaseous mixture follows the relation $PV^{-19/13}$ = constant in an adiabatic process.

IIT, 1995

10. The number of moles of gas B in the gaseous mixture is

(a) 1 (b) 2

(c) 3 (d) 4

11. The adiabatic compressibility of the mixture at pressure P is

(a) $\frac{3}{5P}$ (b) $\frac{5}{7P}$

(c) $\frac{13}{19P}$ (d) $\frac{7}{9P}$

12. If the temperature T of the mixture is raised from 300 K to 301 K, the percentage change in the speed of sound in the gaseous mixture is

(a) $\frac{1}{3}\%$ (b) $\frac{1}{6}\%$

(c) $\frac{1}{2}\%$ (d) 1%

SOLUTION

10. Let n_A and n_B be the number of moles of gases A and B respectively and let $(C_v)_A$ and $(C_v)_B$ be their respective molar specific heats at constant volume. Since the gases do not react with each other and are assumed to be ideal, the internal molecular kinetic energy before and after the gases are mixed must be the same. Hence

$$n_A (C_v)_A T + n_B (C_v)_B T = (n_A + n_B) (C_v)_m T$$

where $(C_v)_m$ is the molar specific heat of the mixture at constant volume. Now

$$(C_v)_A = \frac{R}{(\gamma_A - 1)}, (C_v)_B = \frac{R}{(\gamma_B - 1)}$$

and $(C_v)_m = \frac{R}{(\gamma_m - 1)}$

Therefore,

$$\frac{n_A}{(\gamma_A - 1)} + \frac{n_B}{(\gamma_B - 1)} = \frac{(n_A + n_B)}{(\gamma_m - 1)} \qquad (1)$$

Given $n_A = 1$, $\gamma_A = \frac{5}{3}$, $\gamma_B = \frac{7}{5}$ and γ_m is given by the adiabatic relation for the mixture,

$$PV^{\gamma_m} = \text{constant}$$

Given $PV^{19/13}$ = constant. Hence $\gamma_m = \frac{19}{13}$. Using these values in Eq. (1), we have

$$\frac{1}{\left(\frac{5}{3} - 1\right)} + \frac{n_B}{\left(\frac{7}{5} - 1\right)} = \frac{(1 + n_B)}{\left(\frac{19}{13} - 1\right)}$$

or $$\frac{3}{2} + \frac{5n_B}{2} = \frac{13(1 + n_B)}{6}$$

or $$9 + 15n_B = 13 + 13\, n_B$$

or $$2\, n_B = 4 \text{ or } n_B = 2$$

Thus the mixture contains 2 moles of gas B.
so the correct choice is (b).

11. Adiabatic compressibility β of the mixture is defined as

$$\beta = -\frac{1}{V} \cdot \frac{\Delta V}{\Delta P} \qquad (2)$$

We are given that

$$PV^{\gamma_m} = \text{constant}$$

where $\gamma_m = \frac{19}{13}$. Partially differentiating, we have

$$\Delta P V^{\gamma_m} + \gamma_m \, PV^{\gamma_m - 1} \Delta V = 0$$

or $$\frac{\Delta V}{\Delta P} = -\frac{V^{\gamma_m}}{\gamma_m P V^{\gamma_m - 1}} = -\frac{V}{\gamma_m P}$$

or $$-\frac{1}{V}\frac{\Delta V}{\Delta P} = \frac{1}{\gamma_m P} \qquad (3)$$

Using (3) in (2), we get

$$\beta = \frac{1}{\gamma_m P} = \frac{13}{19P},$$

which is choice (c).

12. We know that the speed of sound in a gas is proportional to the square root of absolute temperature, i.e.

$$v = kT^{1/2};\ k = \text{constant}$$

Taking logarithm, $\log v = \log k + \frac{1}{2} \log T$

Differentiating, $\frac{\delta v}{v} = 0 + \frac{1}{2}\frac{\delta T}{T}$

$$= \frac{1}{2} \times \frac{1\text{K}}{300\,\text{K}} = \frac{1}{600}$$

$\therefore$ Percentage change

$$\frac{\delta v}{v} \times 100 = \frac{100}{600} = \frac{1}{6}\%$$

So the correct choice is (b).

Questions 13 to 15 are based on the following passage

Passage IV

Two moles of a monoatomic ideal gas occupy a volume V at 27°C. The gas is expanded adiabatically to volume $2\sqrt{2}\,V$. Gas constant $R = 8.3\ \text{JK}^{-1}\ \text{mol}^{-1}$.

IIT, 1996

13. The final temperature of the gas is

(a) $\frac{150}{\sqrt{2}}$ K (b) 150 K

(c) $150\sqrt{2}$ K (d) 13.6°C

14. The change in the internal energy of the gas in this process is

(a) – 3735 J (b) 1245 J

(c) – 2490 J (d) + 3735 J

15. The work done by the gas during the process is

(a) 1245 J (b) 1660 J

(c) 2490 J (d) 3735 J

SOLUTION

13. $T_1 = 300$ K, $V_1 = V$, $V_2 = 2\sqrt{2}\,V$.

Let T_2 be the final temperature of the gas. T_2 is obtained from the adiabatic relation

$$T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$$

or $$T_2 = T_1 \times \left(\frac{V_1}{V_2}\right)^{(\gamma-1)}$$

For a monoatomic gas $\gamma = \frac{5}{3}$. Therefore,

$$T_2 = 300 \times \left(\frac{1}{2\sqrt{2}}\right)^{2/3} = 150\ \text{K}$$

So the correct choice is (b).

14. $\Delta U = nC_v(T_2 - T_1)$

For a monoatomic gas $C_v = \frac{3R}{2}$. Therefore,

$$\Delta U = 2 \times \frac{3}{2} \times 8.3 \times (150 - 300) = -3735\ \text{J}$$

The negative sign implies that the internal energy decreases in this process. So the correct choice is (a).

15. For a adiabatic process, $\Delta Q = 0$. Hence $\Delta W = -\Delta U = 3735$ J, which is choice (d).

Questions 16 to 21 are based on the following passage

Passage V

A sample of 2 kg of monoatomic helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC as shown in Fig. 17.38. The molecular mass = 4 and $R = 8.3\ \text{J K}^{-1}\ \text{mol}^{-1}$.

IIT, 1997

16. The temperature of state A is

(a) 100 K (b) 200 K

(c) 300 K (d) 415 K

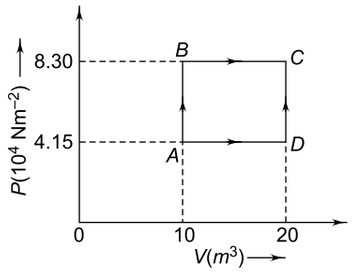


Fig. 17.38

17. The temperature state B is
(a) 150 K (b) 200 K
(c) 415 K (d) 830 K

18. The temperature of state C is
(a) 150 K (b) 300 K
(c) 400 K (d) 415 K

19. The temperature of state D is
(a) 830 K (b) 415 K
(c) 300 K (d) 200 K

20. The work done in process ABC is
(a) 4.15×10^4 J (b) 8.30×10^4 J
(c) 4.15×10^3 J (d) 8.30×10^3 J

21. The heat supplied in process ADC is nearly equal to
(a) 1.8×10^6 J (b) 1.9×10^6 J
(c) 2.0×10^6 J (d) 2.1×10^6 J

SOLUTION

16. Number of moles of helium is

$$n = \frac{\text{mass in gram}}{\text{molecular mass}} = \frac{2000}{4} = 500$$

From equation state at A,

$$P_A\ V_A = n\ R\ T_A \Rightarrow T_A = \frac{P_A V_A}{nR}$$

$$= \frac{4.15\times10^4\times10}{500\times8.3} = 100\ \text{K}$$

So the correct choice is (a)

17. For isochoric process $A \rightarrow B$,

$$\frac{T_B}{T_A} = \frac{P_B}{P_A} = \frac{8.3\times10^4}{4.15\times10^4} = 2.$$

Thus $T_B = 2\ T_A = 200$ K, which is choice (b).

18. For isobaric process $B \rightarrow C$,

$$\frac{T_C}{T_B} = \frac{V_C}{V_B} = \frac{20}{10} = 2$$

Thus $T_C = 2\ T_B = 400$ K, which is choice (c).

19. For isobaric process $A \rightarrow D$,

$$\frac{T_D}{T_A} = \frac{P_D}{P_A} = \frac{20}{10} = 2$$

which gives $T_D = 2\ T_A = 200$ K. So the correct choice is (d).

20. Work done in process ABC is

$$W = W_{AB} + W_{BC} = 0 + P_B(V_C - V_B)$$

$$= 8.3 \times 10^4\ (20 - 10) = 8.3 \times 10^3\ \text{J}$$

Thus the correct choice is (d).

21. From $\Delta U = n\ Cv\ \Delta T$, heat energy in process ADC is ($\therefore$ for a monoatomic gas $C_v = 3\ R/2$)

$$\Delta Q = (\Delta U)ADC + (\Delta W)ADC$$

$$= n\ C_v(T_C - T_A) + P_A(V_D - V_A)$$

$$= 500 \times \left(\frac{3}{2}\times83\right)\times(400-100) + 4.15 \times 10^4\ (20\ -\ 10)$$

$$= 1.87 \times 10^6\ \text{J}, \text{ which is choice (b).}$$

Questions 22 to 25 are based on the following passage

Passage VI

One mole of an ideal monoatomic gas is taken round the cyclic process $ABCA$ as shown in Fig. 17.39.

IIT, 1998

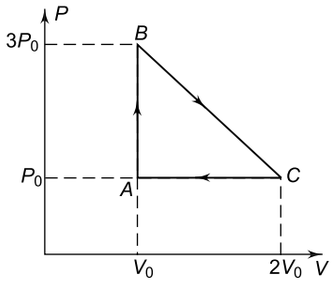


Fig. 17.39

22. The work done by the gas is
(a) P_0V_0 (b) $2P_0V_0$
(c) $3P_0V_0$ (d) $4P_0V_0$

23. The heat energy rejected by the gas in the process $C \rightarrow A$ is
(a) $-\frac{P_0V_0}{2}$ (b) $-\frac{3P_0V_0}{2}$
(c) $-\frac{5P_0V_0}{2}$ (d) $-3\ P_0V_0$

24. The heat energy absorbed by the gas in the process $A \rightarrow B$ is
(a) $P_0\ V_0$ (b) $\frac{3P_0V_0}{2}$

(c) $\frac{5P_0V_0}{2}$ (d) $3\ P_0\ V_0$

25. The net heat energy rejected or absorbed by the gas in the process $B \rightarrow C$ is

(a) $\frac{P_0V_0}{2}$ (b) $-2\ P_0\ V_0$

(c) $-\frac{5P_0V_0}{2}$ (d) $3\ P_0\ V_0$

SOLUTION

22. Work done by the gas in the cyclic process $ABCA$ is

W = area enclosed in the P–V diagram

$$= \text{area of triangle } ABC = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2}\ (3P_0 - P_0) \times (2V_0 - V_0) = P_0V_0,$$

which is choice (a).

23. In the isobaric process CA, the heat energy rejected by the gas is given by

$$Q_{CA} = nC_p\ \Delta T = nC_p\ (T_A - T_C) \quad (1)$$

Using ideal gas equation $PV = nRT$ for points A and C, we have

$$P_A\ V_A = nRT_A \text{ or } P_0\ V_0 = nRT_A \quad (2)$$

and $$P_C\ V_C = nRT_C \text{ or } P_0\ (2V_0) = nRT_C \quad (3)$$

Subtracting (2) from (3), we have

$$P_0V_0 = nR\ (T_C - T_A)$$

or $$T_A - T_C = -\frac{P_0V_0}{nR} \quad (4)$$

Using (4) in (1), we get

$$Q_{CA} = -\frac{C_p\ P_0\ V_0}{R}$$

Now, for a monoatomic gas, $C_p = \frac{5R}{2}$. Hence

$$Q_{CA} = -\frac{5P_0V_0}{2} \quad (5)$$

24. The heat energy absorbed in the isochoric process AB is given by

$$Q_{AB} = nC_v\ \Delta T = nC_v\ (T_B - T_A) \quad (6)$$

Using the ideal gas equation for points A and B, we have

$$P_A\ V_A = nRT_A \text{ or } P_0\ V_0 = nRT_A$$

and $$P_B\ V_B = nRT_B \text{ or } (3\ P_0)V_0 = nRT_B$$

which give $$T_B - T_A = \frac{2P_0V_0}{nR} \quad (7)$$

Using (7) in (6), we get

$$Q_{AB} = \frac{2C_v\ P_0V_0}{R}$$

Now, for a monoatomic gas, $C_v = \frac{3R}{2}$. Therefore

$$Q_{AB} = 3\ P_0\ V_0 \quad (8)$$

25. From the first law of thermodynamics, the heat energy absorbed by the gas is given by

$$\Delta Q = \Delta U + \Delta W$$

where $\Delta W = W$ = work done by the gas $= P_0V_0$. Since the process is cyclic, there is no change in the internal energy of the gas, i.e. $\Delta U = 0$. Hence

$$\Delta Q = \Delta W = P_0V_0 \quad (9)$$

If Q_{BC} is the heat absorbed in the process BC, then

$$\Delta Q = Q_{AB} + Q_{BC} + Q_{CA} \quad (10)$$

Using (5), (8) and (9) in (10), we have

$$P_0V_0 = 3P_0V_0 + Q_{BC} - \frac{5P_0V_0}{2}$$

or $$Q_{BC} = \frac{P_0V_0}{2}$$

Questions 26 to 30 are based on the following passage

Passage VII

Two moles of an ideal monoatomic gas, initially at pressure $P_1 = P$ and volume $V_1 = 2\sqrt{2}\ V$, undergo an adiabatic compression until its volume is $V_2 = V$ and the pressure is P_2. Then the gas is given heat energy Q at constant volume V_2.

IIT, 1999

26. Which of the graphs shown in Fig. 17.40 represents the P–V diagram of the complete process?

27. Pressure P_2 is

(a) $\sqrt{2}\ P$ (b) $2P$

(c) $2\sqrt{2}\ P$ (d) none of these

28. The total work done by the gas is

(a) $-PV$ (b) $-2\sqrt{2}\ PV$

(c) $-3\sqrt{2}\ PV$ (d) $-3\ PV$

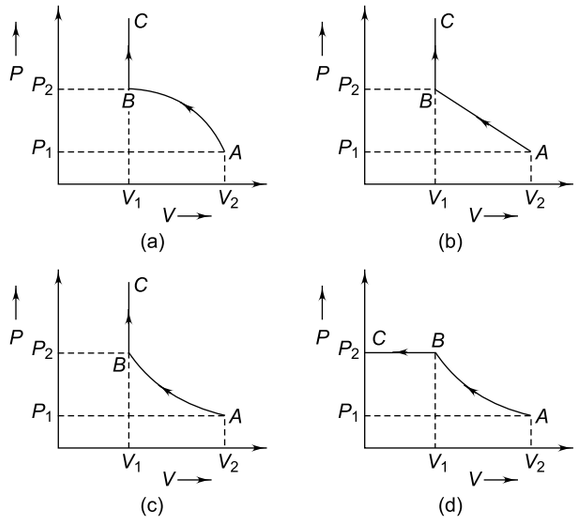


Fig. 17.40

29. The change in internal energy due to adiabatic compression is

(a) $3\ PV$ (b) $3\sqrt{2}\ PV$

(c) $2\sqrt{2}\ PV$ (d) $2\ PV$

30. The temperature T_2 of the gas after it is adiabatically compressed is (here R is the gas constant).

(a) $\dfrac{PV}{R}$ (b) $\dfrac{\sqrt{2}PV}{R}$

(c) $\dfrac{2PV}{R}$ (d) $\dfrac{2\sqrt{2}PV}{R}$

SOLUTION

26. The correct choice is (c).

27. For an adiabatic change, $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$. Therefore,

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$$

For a monoatomic gas, $\gamma = 5/3$. Hence

$P_2 = P\left(\dfrac{2\sqrt{2}V}{V}\right)^{5/3} = P\left(2\sqrt{2}\right)^{5/3}$. So the correct choice is (d).

28. In an adiabatic process, heat $Q = 0$. Hence from the first law of thermodynamics, we have

$$W_1 = -\Delta U_1 = -n\,C_v\,\Delta T$$
$$= -2\ C_v \times (T_2 - T_1)\ (\because\ n = 2)$$

Now for 2 moles $P_1\ V_1 = 2\ RT_1$ and $P_2 V_2 = 2\ R\ T_2$. Thus

$$T_1 = \frac{P_1 V_1}{2R} \text{ and } T_2 = \frac{P_2 V_2}{2R}$$

Using these, we have

$$W = -\frac{C_v}{R}\ (P_2\ V_2 - P_1\ V_1)$$
$$= -\frac{C_v}{R}\left(\frac{P_1 V_1^{\gamma}}{V_2^{\gamma}} \times V_2 - P_1 V_1\right)$$
$$= -\frac{C_v}{R} \times P_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{\gamma - 1} - 1\right]$$
$$= -\frac{3P_1 V_1}{2}\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$
$$= -\frac{3P \times 2\sqrt{2}V}{2}\left[(2\sqrt{2})^{2/3} - 1\right]$$
$$= -3\sqrt{2}\ PV.$$

The work done at contant volume = 0. Hence the correct choice is (c).

29. In adiabatic compression, $Q = 0$. Hence $\Delta U = -W$, So the correct choice is (b).

30. Since $\Delta U = nC_v\,\Delta T = 2\ C_v\ (T_2 - T_1)$

we have $T_2 = T_1 + \dfrac{\Delta U}{2C_v}$

Using $T_1 = \dfrac{P_1 V_1}{2R}$ and $C_v = \dfrac{3R}{2}$, we get

$$T_2 = \frac{P_1 V_1}{2R} + \frac{3P_1 V_1}{2 \times 2 \times 3R/2}\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$
$$= \frac{P_1 V_1}{2R}\left[\left(\frac{V_1}{V_2}\right)^{2/3}\right] \qquad (4)$$
$$= \frac{P \times 2\sqrt{2}V}{R} = \frac{2\sqrt{2}PV}{R}$$

So the correct choice is (d).

Questions 31 to 35 are based on the following passage

Passage VIII

Two moles of an ideal mono-atomic gas is taken through a cycle $ABCA$ as shown in the P–T diagram (Fig. 17.41). During the process AB, pressure and temperature of the gas vary such that $PT = K$, where K is a constant.

IIT, 2000

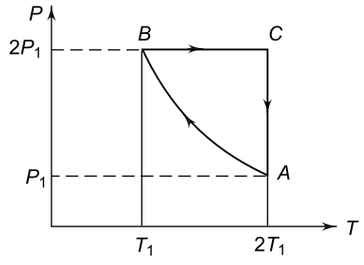


Fig. 17.41

31. Constant K is given by

(a) $\frac{P_1T_1}{2}$ (b) P_1T_1

(c) $2\ P_1T_1$ (d) $\sqrt{2}\ P_1T_1$

32. The work done in process $A \to B$ is (R is the gas constant)

(a) $-RT_1$ (b) $-2RT_1$

(c) $-3RT_1$ (d) $-4RT_1$

33. The heat energy released in process $A \to B$ is

(a) $-3RT_1$ (b) $-5RT_1$

(c) $-7RT_1$ (d) $-9RT_1$

34. The heat energy absorbed in process $B \to C$ is

(a) RT_1 (b) $3RT_1$

(c) $5RT_1$ (d) $7RT_1$

35. The heat energy absorbed in process $C \to A$ is

(a) $\log_e(2)\ RT_1$ (b) $2\log_e(2)\ RT_1$

(c) $3\log_e(2)\ RT_1$ (d) $4\log_e(2)\ RT_1$

SOLUTION

31. Using the ideal gas equation $PV = nRT$, the volumes of the gas in states A, B and C are

$$V_A = \frac{nRT_A}{P_A} = \frac{nR(2T_1)}{P_1} = \frac{2nRT_1}{P_1} \quad (1)$$

$$V_B = \frac{nRT_B}{P_B} = \frac{nR(T_1)}{2P_1} = \frac{1}{2}\frac{nRT_1}{P_1} \quad (2)$$

and $$V_C = \frac{nRT_C}{P_C} = \frac{nR(2T_1)}{2P_1} = \frac{nRT_1}{P_1} \quad (3)$$

It is given that in the process $A \to B$, the pressure and temperature of the gas vary such that

$$PT = K$$

where K is a constant. Thus for point A, we have

$$K = P_A\ T_A = P_1\ (2\ T_1)$$
$$= 2\ P_1T_1 \quad (4)$$

So the correct choice is (c).

32. For the process $A \to B$, we have

$$PV = nRT \quad (5)$$

and $$PT = K \quad (6)$$

Eliminating T from (5) and (6) we get

$$PV = nR \times \frac{K}{P} \text{ or } P^2V = nRK \quad (7)$$

or $$P = \left(\frac{nRK}{V}\right)^{1/2} \quad (8)$$

The work done in process $A \to B$ is given by

$$W_{AB} = \int_{V_A}^{V_B} PdV = \int_{V_A}^{V_B} \left(\frac{nRK}{V}\right)^{1/2} dV$$

$$= \sqrt{nRK} \int_{V_A}^{V_B} \frac{dV}{\sqrt{V}}$$

$$= 2\sqrt{nRK}\ \left(\sqrt{V_B} - \sqrt{V_A}\right)$$

Using (1), (2) and (4), we get

$$W_{AB} = 2\sqrt{nR(2P_1T_1)}\ \left[\sqrt{\frac{1}{2}\frac{nRT_1}{P_1}} - \sqrt{\frac{2nRT_1}{P_1}}\right]$$

$$= 2\sqrt{2}\ nRT_1\left(\frac{1}{\sqrt{2}} - \sqrt{2}\right) = -2\ nRT_1$$

$$= -4\ RT_1.$$

The negative sign indicates that heat is absorbed in this process. The correct choice is (d).

33. For a monoatomic gas $C_v = 3R/2$. The internal energy of the gas in the process $A \to B$ is

$$(\Delta U)_{AB} = n\ C_v\ \Delta T = n\ C_v\ (T_B - T_A)$$

$$= 2 \times \frac{3R}{2} \times (T_1 - 2\ T_1)$$

$$= 3R \times (-\ T_1) = -3RT_1$$

From the first law of thermodynamics,

$$Q_{AB} = (\Delta U)_{AB} + W_{AB}$$

$= -3\ RT_1 - 4\ RT_1 = -7\ RT_1$ which is choice (c).

34. The process $B \rightarrow C$ takes place at a constant pressure $P = 2\ P_1$. Therefore, the work done in this process is

$$W_{BC} = P\Delta V = (2P_1)\ (V_C - V_B)$$

Using (2) and (3), we get

$$W_{BC} = 2P_1 \left(\frac{nRT_1}{P_1} - \frac{1}{2} \frac{nRT_1}{P_1} \right)$$

$$= nRT_1 = \ 2\ RT_1$$

The change in the internal energy is process $B \rightarrow C$ is

$$(\Delta U)_{BC} = nC_v\ \Delta T = nC_v\ (T_C - T_B)$$

$$= nC_v\ (2\ T_1 - T_1)$$

$$= nC_v\ T_1 = 2 \times \frac{3R}{2} \times T_1 = 3RT_1$$

$\therefore \quad Q_{BC} = (\Delta U)_{BC} + W_{BC}$

$= 3\ RT_1 + 2\ RT_1 = 5\ RT_1$, which is choice (c).

35. The process $C \rightarrow A$ takes place at a constant temperature $T = 2\ T_1$. Therefore, the work done in this process is ($\because\ P = nRT/V$)

$$W_{CA} = \int_{V_C}^{V_A} PdV = \int_{V_C}^{V_A} \frac{nRT}{V}\ dV$$

$$= nRT\ \log_e \left(\frac{V_A}{V_C} \right)$$

Now, from (1) and (3), $\frac{V_A}{V_C} = 2$. Therefore,

$$W_{CA} = 2 \times R \times 2T_1 \times \log_e\ (2)$$

$$= 4RT_1 \log_e(2) (\because\ T = 2\ T_1)$$

Now $(\Delta U)_{CA} = 0$ as the temperature is constant. Therefore

$$Q_{CA} = (\Delta U)_{CA} + W_{CA}$$

So the correct choice is (d).

Questions 36 to 38 are based on the following passage

Passage IX

A monoatomic ideal gas of 2 moles is taken through a cyclic process starting from A as shown in Fig. 17.42.

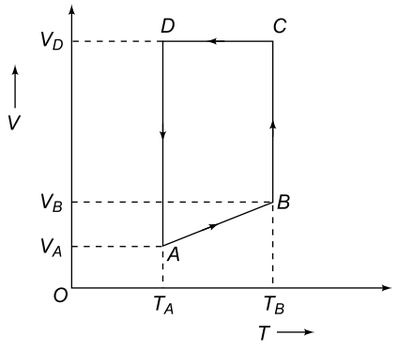


Fig. 17.42

Given $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$. The temperature $T_A = 27°C$. R is the gas constant.

IIT, 2001

36. The temperature of the gas at point B is
(a) 400 K (b) 500 K
(c) 600 K (d) 800 K

37. The total heat absorbed in the complete cycle is
(a) 600 R (b) 900 R
(c) 1200 R (d) 1500 R

38. The total work done by the gas in the complete cycle is
(a) zero (b) 300 R
(c) 450 R (d) 600 R

SOLUTION

36. For process $A \rightarrow B$, the plot of V versus T is linear. Hence

$$\frac{V_A}{T_A} = \frac{V_B}{T_B}$$

$$\Rightarrow \quad T_B = \left(\frac{V_B}{V_A} \right) T_A = 2 \times 300\ K = 600\ K$$

37. Process $A \rightarrow B$ occurs at constant pressure $\because\ V \propto T$. Hence

$$Q_{A \rightarrow B} = 2 \times \frac{5R}{2} \times (T_B - T_A) = 1500\ R$$

Process $B \rightarrow C$ occurs at constant temperature. From first law of thermodynamics $\Delta Q = \Delta U + \Delta W$.

At constant temperature $\Delta U = 0$. Hence $\Delta Q = \Delta W$.

$$W_{B \rightarrow C} = nRT_B\ \log_e \left(\frac{V_C}{V_B} \right)$$

$$= nRT_B\ \log_e \left(\frac{V_D}{V_B} \right) \quad (\because\ V_D = V_C)$$

$$= nRT_B\ \log_e \left(\frac{V_D}{V_A} \times \frac{V_A}{V_B} \right)$$

$$= 2 \times R \times 600\ \log_e \left(4 \times \frac{1}{2} \right)$$

$$= 1200\ \log_e (2)\ R$$

$\therefore \quad Q_{B \rightarrow C} = 1200\ \log_e (2)\ R$

Process $C \rightarrow D$ occurs at constant volume. Hence

$$Q_{C \rightarrow D} = nC_v\,(T_A - T_B)$$
$$= 2 \times \frac{3R}{2} \times (300 - 600)$$
$$= -\,900\,R$$

Process $D \rightarrow A$ occurs at constant temperature. Hence

$$Q_{D \rightarrow A} = W_{D \rightarrow A} = nRT_A \log_e \left(\frac{V_A}{V_D}\right)$$
$$= 2 \times R \times 300 \times \log_e \left(\frac{1}{4}\right)$$
$$= -\,1200\,\log_e(2)R$$

$\therefore$ Total heat absorbed in the complete cycle is

$Q = 1500R + 1200\,\log_e(2)R - 900R - 1200\,\log_e(2)R = 600R$

38. In a cyclic process $\Delta U = 0$. Hence $\Delta W = \Delta Q = 600R$

Questions 39 to 41 are based on the following paragraph

Passage X

A small spherical monoatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_l (see Fig. 17.43). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension).

IIT, 2008

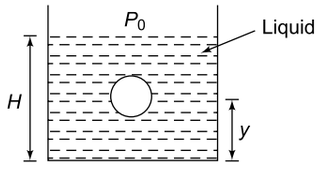


Fig. 17.43

39. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it

(a) Only the force of gravity

(b) The force due to gravity and the force due to the pressure of the liquid

(c) The force due to gravity, the force due to the pressure of the liquid and force due to viscosity of the liquid

(d) The force due to the gravity and the force due to viscosity of the liquid

40. When the gas bubble is at height y from the bottom, its temperature is

(a) $T_0 \left(\frac{P_0 + \rho_\ell gh}{P_0 + \rho_\ell gy}\right)^{2/5}$

(b) $T_0 \left(\frac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell gH}\right)^{2/5}$

(c) $T_0 \left(\frac{P_0 + \rho_\ell gH}{P_0 + \rho_\ell gy}\right)^{3/5}$

(d) $T_0 \left(\frac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell gH}\right)^{3/5}$

41. The buoyancy force acting on the gas bubble is (R is the universal gas constant)

(a) $\rho_l nRgT_0 \frac{(P_0 + \rho_\ell gH)^{2/5}}{(P_0 + \rho_\ell gy)^{7/5}}$

(b) $\frac{\rho_\ell nRgT_0}{(P_0 + \rho_\ell gH)^{2/5}[P_0 + \rho_\ell g(H-y)]^{3/5}}$

(c) $\rho_\ell nRgT_0 \frac{(P_0 + \rho_\ell gH)^{3/5}}{(P_0 + \rho_\ell gy)^{8/5}}$

(d) $\frac{\rho_l nRgT_0}{(P_0 + \rho_\ell gH)^{3/5}[P_0 + \rho_\ell g(H-y)]^{2/5}}$

SOLUTION

39. The correct choice is (d).

40. For an adiabatic process $TP^{\frac{1-\gamma}{\gamma}} = \text{constant}$.

For $\gamma = 5/3$, $\frac{(1-\gamma)}{\gamma} = -\frac{2}{5}$. Therefore,

$$T_0[P_0 + \rho_\ell gH]^{-2/5} = T[P_0 + \rho_\ell g(H-y)]^{-2/5}$$

Which gives $T = T_0 \left[\frac{P_0 + \rho_\ell g(H-y)}{(P_0 + \rho_\ell gH)}\right]^{2/5}$

41. Buoyant force F = weight of liquid displaced $= \rho_\ell Vg$, V = volume of the bubble.

From $PV = nRT$, we have $V = \frac{nRT}{P}$ Therefore,

$$F = \frac{nRT\rho_\ell g}{P}$$
$$= \frac{nR\rho_\ell g}{\left[P_0 + \rho_\ell g(H-y)\right]} \times T_0 \left[\frac{P_0 + \rho_\ell g(H-y}{P_0 + \rho_\ell gh}\right]^{2/5}$$
$$= \frac{nR\rho_\ell g}{(P_0 + \rho_\ell gH)^{2/5}[P_0 + \rho_\ell g(H-y)]^{3/5}}$$

SECTION IV

Matching

1. The pressure-volume ($P - V$) graph of a cyclic process $ABCDA$ of an ideal gas is shown in Fig. 17.44. Match the process listed in column I with the consequences listed in Column II

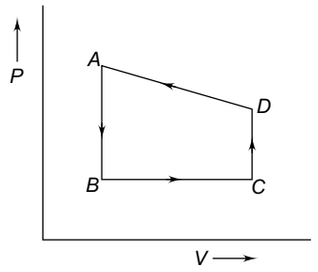


Fig. 17.44

Column I	***Column II***
(a) Process $A \to B$	(p) $dQ > 0$
(b) Process $B \to C$	(q) $dQ < 0$
(c) Process $C \to D$	(r) $dW > 0$
(d) Process $D \to A$	(s) $dW < 0$

IIT, 2006

SOLUTION

In process $A \to B$, the volume remains constant. Hence $dW = PdV = 0$. Also $P \propto T$. Since the pressure decreases, temperature T decreases. Hence the gas loses heat energy, i.e., $dQ < 0$. In process $B \to C$, the pressure remains constant and volume increases. Now $V \propto T$. Hence T increases which means that $dQ > 0$. Also $dW = PdV$ is positive. In process $C \to D$, the volume remains constant but pressure increases. Hence $dQ > 0$ and $dW = 0$. In process $D \to A$, volume decreases and pressure increases. Hence $dW < 0$ and $dQ < 0$.

(a) $\to$ (q) (b) $\to$ (p), (r) (c) $\to$ (r) (d) $\to$ (q), (s)

2. **Column I** contains a list of processes involving expansion of an ideal gas. Match this with **Column II** describing the thermodynamic change during this process.

Column I	***Column II***
(a) An insulated containar has two chambers separated by a valve. Chamber I contains an ideal gas and the chamber II has vacuum. The valve is opened. (see Fig. 17.45)	(p) The temperature of the gas decreases
(b) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^2}$. where V is the volume of the gas.	(q) The temperature of the gas increases or remains constant
(c) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^{4/3}}$. where V is its volume	(r) The gas loses heat
(d) An ideal monoatamic gas expands such that its pressure P and volume V follow the behaviour shown in the graph in Fig. 17.46.	(s) The gas gains heat

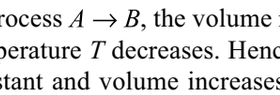


Fig. 17.45

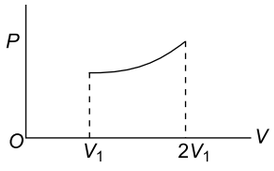


Fig. 17.46

IIT, 2008

SOLUTION

(a) Since the gas expands freely, it does no work, i.e. $dW = 0$. Since the system is completely insulated, no heat can enter or leave the sysytem, i.e. $dQ = 0$. From the first law of thermodyramics $dU = dQ - dW = 0$, i.e.there is no change in internal energy. Hence the temperatue remains unchanged. So the correct choice is (q).

(b) Given PV^2 = constant. $PV = n\,RT$ or $P = n\,RT/V$. Therefore, $PV^2 = \frac{nRT}{V} \times V^2 = nRTV$. Hence $T \propto \frac{1}{V}$. Thus if V is increased, T will decrease, i.e. dT is negative.

For PV^x, the work done $dW = \frac{nR.dT}{(1-x)}$. Hence

$$dQ = nC_v\,dT + \frac{nRdT}{(1-x)}$$

$$\Rightarrow \qquad nC_p\,dT = n\,C_v\,dT + \frac{nR}{(1-x)}dT$$

$$\Rightarrow \qquad C_p = C_v + \frac{R}{1-x} \qquad (1)$$

For monoatomic gas $C_v = \frac{3R}{2}$ and for $x = 2$

$$C_p = \frac{3R}{2} + \frac{R}{1-2} = \frac{3R}{2} - R = \frac{R}{2}$$

Now $dQ = nC_p\,dT$. Since dT is negative and C_p is positive, dQ is negative, i.e. the gas loses heat. Hence the correct choices are (p) and (r)

(c) If $PV^{4/3}$ = constant, Then $T \propto \frac{1}{V^{1/3}}$. Thus if V is increased, T will decrease, i.e. dT is negative.

Putting $x = 4/3$ in Eq. (1) we get

$C_p = C_v + \frac{R}{(1-4/3)} = \frac{3R}{2} - 3R = -\frac{3R}{2}$, which is negative. Now $dQ = nC_p\,dT$. Since C_p and dT are both negative, dQ will be positive, i.e. the gas gains heat. Hence the correct choices are (p) and (s).

(d) If follows from the graph that if the volume of the gas increases, its pressure also increases. Hence the temperature of the gas increases. Therefore, dU is positive. Also, work is done by the gas. Hence dW is positive. Now $dQ = dU + dW$. Since both dU and dW are positive, dQ will be positive, i.e. the gas gains heat. Hence the correct choices are (q) and (s).

(a) $\rightarrow$ (q) (b) $\rightarrow$ (p), (r) (c) $\rightarrow$ (p), (s) (d) $\rightarrow$ (q), (s)

3. One mole of a monatomic gas is taken through a cycle $ABCDA$ as shown in the P-V diagram (Fig. 17.47). Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I.

IIT, 2011

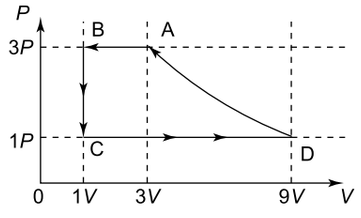


Fig. 17.47

Column I	***Column II***
(a) Process $A \rightarrow B$	(p) Internal energy decreases
(b) Process $B \rightarrow C$	(q) Internal energy increases

(c) Process $C \rightarrow D$
(d) Process $D \rightarrow A$

(r) Heat is lost
(s) Heat is gained
(t) Work is done on the gas

SOLUTION

(a) Process $A \rightarrow B$ is isobaric. Hence $V \propto T$. Therefore $T_A > T_B$. $\Delta U = nC_v\Delta T = nC_v(T_B - T_A)$. Since $T_B < T_A$, ΔU is negative, i.e. internal energy decreases.
$\Delta Q = nC_p\Delta T$ is also negative. Hence heat is lost.
$\Delta W = 3P(V_B - V_A) = -6PV$. Which is negative. Hence work is done on the gas.
$\therefore$ (a) $\rightarrow$ (p, r, t)

(b) Process $B \rightarrow C$ is isochoric. Hence $P \propto T$. Therefore $T_B > T_C$. $\Delta U = nC_v\Delta T = nC_v(T_C - T_B)$ is negative, i.e. internal energy decreases.
$\Delta W = P\Delta V = 0 \quad (\because \Delta V = 0)$
From first law of thermodynamics ($\Delta Q = \Delta U + \Delta W$), $\Delta Q = \Delta U$. Since ΔU is negative, heat is lost.
$\therefore$ (b) $\rightarrow$ (p, r)

(c) Process $C \rightarrow D$ is isobaric, i.e. $V \propto T$. Hence $T_D > T_C$. $\Delta U = nC_v(T_D - T_C)$ is positive. Hence internal energy increases.
$\Delta Q = nC_p(T_D - T_C)$ is positive. Hence heat is gained by the gas.
$\Delta W = P\Delta V = P(9V - V) = 8\ PV$ which is positive. So work is done by the gas.
$\therefore$ (c) $\rightarrow$ (q, s)

(d) In process $D \rightarrow A$, the gas is returned to the initial state A. Hence $\Delta U = 0$. Therefore $\Delta Q = \Delta W$. Since the gas is compressed, work is done on the gas, i.e. ΔW is negative. Hence ΔQ is negative. Hence heat is lost by the gas.
$\therefore$ (d) $\rightarrow$ (r, t)

ANSWER

(a) $\rightarrow$ (p, r, t) (b) $\rightarrow$ (p, r) (c) $\rightarrow$ (q, s) (d) $\rightarrow$ (r, t)

SECTION V

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-1 is true.

1. Statement-1

Figure 17.48 shows $\frac{PV}{T}$ versus P graph for a certain mass of oxygen gas at two temperatures T_1 and T_2. It follows from the graph that $T_1 > T_2$.

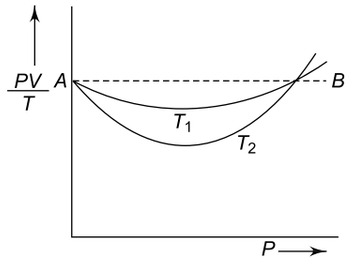


Fig. 17.48

Statement-2
At higher temperatures, real gas behaves more like an ideal gas.

IIT, 1989

2. Statement-1
If two bodies of equal mass and made of the same material at different temperature T_1 and T_2 are

brought in thermal contact, the temperature of each body will be $(T_1 + T_2)/2$ when thermal equilibrium is attained.

Statement-2

They have the same thermal capacity.

3. **Statement-1**

Two vessels A and B of equal capacity are connected to each other by a stopcock. Vessel A contains a gas at 0°C and 1 atmosphere pressure and vessel B is evacuated. If the stopcock is suddenly opened, the final pressure in A and B will be 0.5 atmosphere.

Statement-2

If the temperature is kept constant, the pressure of a gas is inversely proportional to its volume.

4. **Statement-1**

Two vessels A and B are connected to each other by a stopcock. Vessel A contains a gas at 0°C and 1 atmosphere pressure and vessel B is evaluated. The two vessels are thermally insulated from the surroundings. If the stopcock is suddenly opened, there will be no change in the internal energy of the gas.

Statement-2

No transfer of heat energy takes place between the system and the surroundings.

5. **Statement-1**

Two vessels A and B are connected to each other by a stopcock. Vessel A contains a gas at 300 K and 1 atmosphere pressure and vessel B is evacuated. The two vessels are thermally insulated from the surroundings. If the stopcock is suddenly opened, the expanding gas does no work.

Statement-2

Since $\Delta Q = 0$ and $\Delta U = 0$, it follows from the first law of thermodynamics that $\Delta W = 0$.

6. **Statement-1**

Heating system based on circulation of steam are more efficient in warming a house than those based or circulation of hot water.

Statement-2

The latent heat of steam is high.

7. **Statement-1**

Figure 17.49 shows the V-T graphs of a certain mass of an ideal gas at two pressures P_1 and P_2. It follows from the graphs that P_1 is greater than P_2.

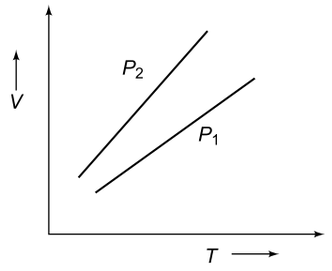


Fig. 17.49

Statement-2

The slope of V-T graph for an ideal gas is directly proportional to pressure.

IIT, 1982

8. **Statement-1**

The curves A and B in Fig. 17.50 show P-V graphs for an isothermal and an adiabatic process for an ideal gas. The isothermal process is represented by curve A.

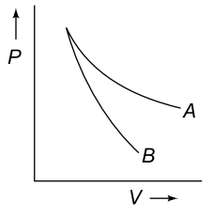


Fig. 17.50

Statement-2

The slope of the P-V graph is less for an isothermal process than for an adiabatic process.

IIT, 1985

SOLUTION

1. The correct choice is (a). The line AB is parallel to the P-axis. This means that PV/T is a constant, independent of pressure. Hence line AB corresponds to an ideal gas for which PV/T = constant. At higher temperatures, a real gas behaves more like an ideal gas. Hence T_1 is greater than T_2.
2. The correct choice is (a). Statement-1 is true only if the two bodies have the same thermal capacity which is equal to mass of the body × its specific heat capacity. Since the two bodies have the same mass and are made of the same material, they have the same thermal capacity.
3. The correct choice is (a). Since the two vessels are of equal capacity, the volume occupied by the gas is doubled when the stopclock is opened. Hence, pressure becomes half.
4. The correct choice is (a)
5. The correct choice is (a). Since the system is thermally insulated from the surroundings, no heat flows into the system or out of it, i.e. $\Delta Q = 0$. Since $\Delta U = 0$; $\Delta W = 0$.

6. The correct choice is (a).

7. From $PV = RT$, we have

$$\frac{PV}{T} = \text{constant } (R)$$

$$\Rightarrow \quad \frac{V}{T} = \frac{R}{P}, \text{ i.e. } \frac{V}{T} \propto \frac{1}{P}$$

Hence Statement-1 is true but Statement-2 is false.

8. For an isothermal process PV = constant. Differentiating we get $PdV + VdP = 0 \Rightarrow \frac{dP}{dV} = -\frac{P}{V}$. For an adiabatic process PV^{γ} = constant. Differenting, we get

$$P\gamma V^{\gamma - 1}\, dV + dP\, V^{\gamma} = 0$$

which gives $\frac{dP}{dV} = -\frac{\gamma P}{V}$; where $\gamma = C_p/C_v$

Since $\gamma > 1$, the slope of the P-V curve for an adiabatic process is greater than that for an isothermal process. Thus both the statements are true and statement-2 is the correct explanation for statement-1.

SECTION VI

Integer Answer Type

1. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.]

IIT, 2009

SOLUTION

1. If P_0 is the external pressure, then

$$P_A = P_0 + \frac{4\sigma}{r_A} = 8 + \frac{4 \times 0.04}{2 \times 10^{-2}} = 16 \text{ N m}^{-2}$$

$$P_B = P_0 + \frac{4\sigma}{r_B} = 8 + \frac{4 \times 0.04}{4 \times 10^{-2}} = 12 \text{ N m}^{-2}$$

Using $P_A V_A = n_A RT$ and $P_B V_B = n_B RT$, we have

$$\frac{n_B}{n_A} = \frac{P_B}{P_A} \times \frac{V_B}{V_A} = \frac{P_B}{P_A} \times \left(\frac{r_B}{r_A}\right)^3 \quad \left(\because V = \frac{4\pi}{3} r^3\right)$$

$$= \left(\frac{12}{16}\right) \times \left(\frac{4}{2}\right)^3 = 6$$

18 Chapter

Kinetic Theory of Gases

REVIEW OF BASIC CONCEPTS

18.1 PRESSURE EXERTED BY AN IDEAL GAS

The pressure of a gas in a container is a result of the continuous bombardment of the gas molecules against the walls of the container and is given by

$$P = \frac{1}{3}\frac{mn v_{\text{rms}}^2}{V} = \frac{1}{3}\frac{M v_{\text{rms}}^2}{V} = \frac{1}{3}\rho v_{\text{rms}}^2 = \frac{2}{3}\frac{U}{V}$$

where m = mass of each molecule, n = number of molecules in the container, v_{rms} = root mean square speed of molecules, V = volume of container, M = mass of gas in the container, ρ = density of the gas and U = internal energy of the gas.

18.2 ROOT MEAN SQUARE SPEED

The root mean square (rms) speed is defined as

$$v_{\text{rms}} = \left[\frac{1}{n}(v_1^2 + v_2^2 + v_3^2 + \cdots + v_n^2)\right]^{1/2}$$

where $v_1, v_2, v_3, \cdots v_n$ are the speeds of the molecules 1, 2, 3, $\cdots$ n respectively. In terms of P and ρ, v_{rms} is given by

$$v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

where m is the mass of each molecule, T the absolute temperature of the gas and k a constant called *Boltzman's constant*. Its value is

$$k = 1.38 \times 10^{-23} \text{ J K}^{-1} \text{ per molecule}$$

R is the universal gas constant and its value is

$$R = 8.315 \text{ J K}^{-1} \text{ mol}^{-1}$$

18.3 MEAN TRANSLATIONAL KINETIC ENERGY

The mean translational kinetic energy of a molecule of a gas is given by

$$E = \frac{1}{2}mv_{\text{rms}}^2$$

In terms of E, the pressure of the gas is given by

$$P = \frac{1}{3}\rho v_{\text{rms}}^2 = \frac{2}{3}\frac{nE}{V} = \frac{2}{3}\frac{U}{V}$$

where $U = nE$ is the total translational kinetic energy of all the n molecules of the gas. It is also called the internal energy of the gas.

Boltzman's constant $k = \dfrac{R}{N_0}$

where N_0 is Avagdro number.

18.4 EQUATION OF STATE OF AN IDEAL GAS

The relationship between pressure P, volume V and absolute temperature T of an ideal gas is called the equation of state. For n moles of a gas, this relation is

$$PV = nRT$$

where R is the molar gas constant.

From Avogadro's law, it follows that one mole of all gases, at the same temperature and pressure, occupies equal volume. Experiments confirm that one mole of any gas occupies 22.4 litres at STP. Consequently, for on mole the ratio $\dfrac{PV}{T}$ is constant for all gases. This constant is the molar gas constant and can be evaluated as follows:

At STP, $V = 22.4 \text{ litre} = 22.4 \times 10^{-3} \text{ m}^3$

$$P = 0.76 \text{ m Hg} = 1.013 \times 10^5 \text{ Nm}^{-2}$$
$$T = 273 \text{ K}$$

$$\therefore \quad R = \frac{PV}{T} = \frac{1.013 \times 10^5 \times 22.4 \times 10^{-3}}{273}$$
$$= 8.315 \text{ J mol}^{-1} \text{ K}^{-1}$$

18.5 VAN DER WAAL'S EQUATION OF STATE

According to Van der Waal the true pressure exerted by a gas is greater than P by an amount a/V^2 (where a is a constant) due to attractive forces between molecules and the true volume of the gas is less than V by an amount b (where b is another constant) because molecules themselves occupy a finite space. The Van der Waal's equation of state is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

At high pressures, when the molecules are too many and too close together, the correction factors a and b both become important. But at low pressures, when they are not too many and not too close together, a gas behaves like an ideal gas and obeys the equation $PV = RT$.

18.6 DEGREES OF FREEDOM AND EQUIPARTITION OF ENERGY

The total number of coordinates or independent quantities required to completely specify the position or configuration of a dynamical system is called the degrees of freedom of the system.

The molecules of a monoatomic gas consist of single atoms. Therefore, the molecules of a monoatomic gas have *three* degrees of freedom corresponding to translational motion. The molecules of a diatomic gas have *five* degrees of freedom-three corresponding to translational motion and two for rotational motion. A polyatomic molecule has *six* degrees of freedom including one of vibrational motion.

The law of equipartition of energy is stated as follows. In any dynamical system with a uniform absolute temperature T, the total energy is distributed equally among all the degrees of freedom and the average energy per degree of freedom per molecule equals $\frac{1}{2}kT$, where $k = 1.38 \times 10^{-23}$ J K^{-1}.

If the molecules of a gas have f degrees of freedom, then kinetic energy per molecule $= f \times \frac{1}{2}kT$. Therefore, kinetic energy per mole is

$$U = Nf \times \frac{1}{2}kT = \frac{f}{2}RT$$

Now $$C_v = \frac{\Delta U}{\Delta T} = \frac{f}{2}R$$

and $$C_p = C_v + R = \frac{f}{2}R + R = \left(\frac{f}{2} + 1\right)R$$

$$\therefore \quad \gamma = \frac{C_p}{C_v} = \frac{\left(\frac{f}{2} + 1\right)R}{\frac{f}{2}R} = 1 + \frac{2}{f}$$

Thus $\gamma = 1 + \frac{2}{3} = \frac{5}{3}$ for a monoatomic gas $= 1 + \frac{2}{5} = \frac{7}{5}$ for a diatomic gas $= 1 + \frac{2}{6} = \frac{4}{3}$ for a triatomic or polyatomic gas

Relation between C_p, C_v and R

For a monoatomic gas; $C_v = \frac{3R}{2}$ and $C_p = \frac{5R}{2}$

For a diatomic gas; $C_v = \frac{5R}{2}$ and $C_p = \frac{7R}{2}$

For a polyatomic gas; $C_v = 3R$ and $C_p = 4R$

Relation between C_p, C_v and γ

$$C_p - C_v = R$$

$$\gamma = \frac{C_p}{C_v} \quad \Rightarrow \quad C_p = \gamma C_v$$

$$\therefore \quad \gamma C_v - C_v = R \quad \Rightarrow \quad C_v = \frac{R}{\gamma - 1}$$

$$C_p = C_v + R = \frac{R}{\gamma - 1} + R = \frac{\gamma R}{\gamma - 1}$$

EXAMPLE 18.1

Calculate the root mean square speed of the molecules of hydrogen gas at S.T.P. Density of hydrogen at S.T.P. is 9×10^{-2} kg m^{-3}.

SOLUTION

At S.T.P., pressure $P = 1.01 \times 10^5$ Pa and density $\rho = 9 \times 10^{-2}$ kg m^{-3}.

$$v_{\text{rms}} = \sqrt{\frac{3P}{\rho}}$$

$$= \sqrt{\frac{3 \times 1.01 \times 10^5}{9 \times 10^{-2}}} = 1840 \text{ ms}^{-1}$$

EXAMPLE 18.2

Calculate the temperature (in kelvin) at which the root mean square speed of a gas molecule is half its value at 0°C.

SOLUTION

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

$$\therefore \quad \frac{v'_{rms}}{v_{rms}} = \sqrt{\frac{T'}{T}} = \sqrt{\frac{T'}{273}}$$

$$\Rightarrow \quad \frac{1}{2} = \sqrt{\frac{T'}{273}} \quad \Rightarrow \quad T' = 68.25 \text{ K}$$

EXAMPLE 18.3

Find the mean translational kinetic energy of an oxygen molecule at 0°C. Given Avogadro number $N = 6.03 \times 10^{23}$ per mole and $R = 8.3$ JK^{-1} mol^{-1}.

SOLUTION

$$E = \frac{1}{2} m v^2_{rms} = \frac{3}{2} kT = \frac{3}{2} \frac{RT}{N}$$

$$\therefore \quad E = \frac{3 \times 8.3 \times 273}{2 \times 6.03 \times 10^{23}} = 5.64 \times 10^{-21} \text{ J}$$

EXAMPLE 18.4

Calculate the mean translational kinetic energy of 1 mole of hydrogen at S.T.P. Density of hydrogen at S.T.P. is 0.09 kg m^{-3}.

SOLUTION

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{0.09}}$$

$$= 1.84 \times 10^3 \text{ ms}^{-1}$$

Mass of 1 mole is

$$m = 22.4 \times 10^{-3} \text{ m}^3 \times 0.09 \text{ kg m}^{-3}$$

$$= 2.016 \times 10^{-3} \text{ kg}$$

$$\therefore \quad \text{K.E.} = \frac{1}{2} m v^2_{rms}$$

$$= \frac{1}{2} \times (2.016 \times 10^{-3}) \times (1.84 \times 10^3)^2$$

$$= 3.41 \times 10^3 \text{ J}$$

EXAMPLE 18.5

Calculate (a) the average translational kinetic energy of the molecules of an ideal gas at 0°C and at 100°C and (b) the energy per mole of the gas at 0°C and 100°C. Given Avogadro's number $N = 6.02 \times 10^{23}$ and Boltzmann's constant $k = 1.38 \times 10^{-23}$ JK^{-1}.

SOLUTION

(a) Average translational K.E. of a molecule of an ideal gas is

$$E = \frac{3}{2} kT, \quad \text{where } T = \text{temperature in kelvin}$$

At $T = 0°\text{C} = 273$ K,

$$E = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 273$$

$$= 5.65 \times 10^{-21} \text{ J}$$

At $T = 100°\text{C} = 373$ K,

$$E = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 373$$

$$= 7.72 \times 10^{-21} \text{ J}$$

(b) Number of molecules in 1 mole of a gas is $N = 6.02 \times 10^{23}$

$\therefore$ K.E. of 1 mole at 273 K = $(5.65 \times 10^{-21}) \times (6.02 \times 10^{23}) = 3.40$ J

K.E. of 1 mole at 373 K = $(7.72 \times 10^{-21}) \times (6.02 \times 10^{23}) = 4.65$ J

EXAMPLE 18.6

The speed of sound in a gas at S.T.P. is 330 ms^{-1} and the density of the gas is 1.3 kg m^{-3}. Find the number of degrees of freedom of a molecule of the gas.

SOLUTION

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$\therefore \quad \gamma = \frac{v^2 \rho}{P} = \frac{(330)^2 \times 1.3}{1.01 \times 10^5} = 1.4$$

If f is the number of degrees of freedom, then

$$f = \frac{2}{\gamma - 1} = \frac{2}{(1.4 - 1)} = 5$$

EXAMPLE 18.7

The volume of 2 moles of a diatomic ideal gas at 300 K is doubled keeping its pressure constant. Find the change in the internal energy of the gas. Given $R = 8.3$ JK^{-1} mol^{-1}.

SOLUTION

According to kinetic theory, there are no internal forces of interaction between the molecules of an ideal gas. This implies that the potential energy is

zero. Hence, for an ideal gas, the internal energy is only due to kinetic energy of the molecules.

For n moles of an ideal gas at absolute temperature T, the internal energy is

$$U = \frac{f}{2} nRT$$

where f is the number of degrees of freedom. For a diatomic gas $f = 5$. Therefore

$$U = \frac{5}{2} nRT$$

Since pressure is kept constant, $V \propto T$ (Charles' law), i.e.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow \quad T_2 = \left(\frac{V_2}{V_1}\right) T_1 = 2T_1 = 600 \text{ K } (\because V_2 = V_1)$$

$$\therefore \quad \Delta U = U_2 - U_1 = \frac{5}{2} nR(T_2 - T_1)$$

$$= \frac{5}{2} \times 2 \times 8.3 \times (600 - 300)$$

$$= 1.245 \times 10^4 \text{ J}$$

Alternative Method

$$\Delta U = nC_v \Delta T$$

For diatomic gas, $C_v = \frac{5R}{2}$. Therefore

$$\Delta U = 2 \times \frac{5R}{2} \times (600 - 300)$$

$$= 1.245 \times 10^4 \text{ J}$$

EXAMPLE 18.8

Hydrogen gas is contained in a vessel of volume 10 litres at 27°C. The gas pressure is 10^6 Pa. Find

(a) total translational kinetic energy of hydrogen molecules,

(b) mean (average) kinetic energy of the molecules and

(c) total kinetic energy of the molecules.

SOLUTION

Volume of gas = 10 litres = 10×10^{-3} m^3 = 10^{-2} m^3)

(a) Total K.E. of molecules = $\frac{3}{2} PV = \frac{3}{2} \times 10^6 \times 10^{-2} = 1.5 \times 10^4$ J

(b) Average K.E. of molecules

$$= \frac{5}{2} kT \quad (\because \text{Hydrogen is diatomic})$$

$$= \frac{5}{2} \times 1.38 \times 10^{-23} \times (273 + 27)$$

$$= 1.035 \times 10^{-20} \text{ J}$$

(c) Total K.E. = $\frac{5}{2} PV = \frac{5}{2} \times 10^6 \times 10^{-2} = 2.5 \times 10^4$ J

EXAMPLE 18.9

n_1 moles of a monoatomic gas are contained in a vessel A of volume V_1 at pressure P_1 and temperature T_1. n_2 moles of the same gas are contained in a vessel B of volume V_2 at pressure P_2 and temperature T_2. The two vessels are now connected by a tube. Obtain the expression for (a) common temperature T and (b) common pressure P in the Vessels.

SOLUTION

If f = number of degrees of freedom,

Initial total energy = final total energy

$$\Rightarrow \quad \frac{f}{2} n_1 RT_1 + \frac{f}{2} n_2 RT_2 = \frac{f}{2} (n_1 + n_2) RT$$

Here $f = 3$ (monoatomic gas)

Equation (1) gives

$$T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

Final pressure is

$$P = \frac{(n_1 + n_2) RT}{V_1 + V_2} = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2}$$

18.7 IDEAL GAS LAWS

Boyle's Law

At any given temperature, the volume of a given mass of a gas is inversely proportional to pressure, i.e.

$$V \propto \frac{1}{P} \quad (T = \text{constant and } n = \text{constant})$$

$$\Rightarrow \quad PV = \text{constant}$$

$$\Rightarrow \quad P_1 V_1 = P_2 V_2$$

Charle's Law

If the pressure of a gas is kept constant, the volume of a given mass of the gas is directly proportional to its absolute temperature, i.e.

$$V \propto T \ (P = \text{constant and } n = \text{constant})$$

$$\Rightarrow \quad \frac{V}{T} = \text{constant}$$

$$\Rightarrow \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

If the volume of a gas is kept constant, the pressure of a given mass of the gas is directly proportional to its absolute temperature, i.e.

$$\frac{P}{T} = \text{constant } (V = \text{constant and } n = \text{constant})$$

$$\Rightarrow \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Avogadro Law

Equal volumes of all gasses at the same temperature and pressure contain an equal number of molecules. The number of molecules in one mole of any gas is $N_0 = 6.02 \times 10^{23}$. N_0 is called the Avogadro number. If volume V_1 of one gas contains N_1 molecules and volume V_2 of another gas contains N_2 molecules at the same temperature and pressure, then

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

Equation of State in terms of Boltzmann constant (k)

$$k = \frac{\text{universal gas constant}}{\text{Avogadro number}} = \frac{R}{N_0}$$

$$PV = nRT = nN_0 \frac{R}{N_0} T = nN_0kT$$

or $$PV = NkT$$

where $$N = nN_0 = \text{number of molecules}$$

EXAMPLE 18.10

An electric bulb of volume 250 cm^3 was sealed during manufacture at a pressure of 10^{-3} mm of Hg at 27°C. Find the number of air molecules in the bulb.

SOLUTION

Let N be the number of air molecules in the bulb. It is given that $P = 10^{-3}$ mm of Hg = 10^{-4} cm of Hg, $V = 250$ cm^3 and $T = 273 + 27 = 300$ K.

Now $$PV = NkT \qquad (1)$$

At STP, one mole of a gas occupies a volume $V_0 = 22400$ cm^3 and contains $N_0 = 6.02 \times 10^{23}$ molecules, $P_0 = 76$ cm of Hg and $T_0 = 273$ K.

$$P_0V_0 = N_0kT_0 \qquad (2)$$

Dividing (1) by (2), we get

$$N = N_0 \times \frac{T_0}{T} \times \frac{P}{P_0} \times \frac{V}{V_0}$$

$$= (6.02 \times 10^{23}) \times \left(\frac{273}{300}\right) \times \left(\frac{10^{-4}}{76}\right) \times \left(\frac{250}{22400}\right)$$

$$= 8.04 \times 10^{15} \text{ molecules}$$

EXAMPLE 18.11

A cylinder of volume 30 litres contains oxygen at a gauge pressure of 15 atm at 27°C. When some oxygen is ejected out from the cylinder, the gauge pressure falls to 11 atm and temperature falls to 17°C. Find the mass of oxygen ejected. $R = 8.3$ J K^{-1} mol^{-1} and molecular mass of oxygen = 32.

SOLUTION

Let N_1 be the number of molecules, P_1 be the pressure and T_1 be the temperature of oxygen before some oxygen is ejected and N_2, P_2 and T_2 be their values after the gas is ejected. Since the volume V remains unchanged (= volume of cylinder),

$$P_1V = N_1kT_1 \qquad (1)$$

$$P_2V = N_2kT_2 \qquad (2)$$

where $k = \frac{R}{N_0} = \frac{8.3}{6.02 \times 10^{23}} = 1.38 \times 10^{-23}$ JK^{-1} per molecules is Boltzmann constant. Given, $V = 30$ litres $= 30 \times 10^{-3}$ m^3, $P_1 = 15$ atm $= 15 \times 1.01 \times 10^5$ Pa, $T_1 = 273 + 27 = 300$ k, $P_2 = 11$ atm $= 11 \times 1.01 \times 10^5$ Pa and $T_2 = 273 + 17 = 290$ K. Substituting these values in Eqs. (1) and (2), we get

$$N_1 = 10.87 \times 10^{24} \text{ and } N_2 = 8.25 \times 10^{24}$$

$\therefore$ Number of molecules of oxygen ejected = $N_1 - N_2 = 2.62 \times 10^{24}$. The molecular mass of oxygen = 32, i.e. one mole of oxygen (which contains 6.02×10^{23} molecules) has a mass of 32 g = 0.032 kg. Hence 2.62×10^{24} molecules will have a mass of

$$\frac{0.032 \times 2.62 \times 10^{24}}{6.02 \times 10^{23}} = 0.14 \text{ kg}$$

EXAMPLE 18.12

A narrow glass tube of length 100 cm is closed at both ends. It lies horizontally with 20 cm of mercury column in the middle dividing the tube into two compartments I and II of equal length. The air in each compartment is at standard temperature and pressure. The tube is then turned to a vertical position. By what distance will the mercury column be displaced?

SOLUTION

Let A cm^2 be the cross-sectional area of the tube. Then $P_0 = 76$ cm of Hg and $V_0 = 40\ A$ cm^3 [Fig. 18.1(a)]. When the tube is turned to vertical position [Fig. 18.1(b)], Let x cm be the displacement of mercury column and let P_1, V_1 be the air pressure in compartment I and P_2, V_2 those in compartment II.

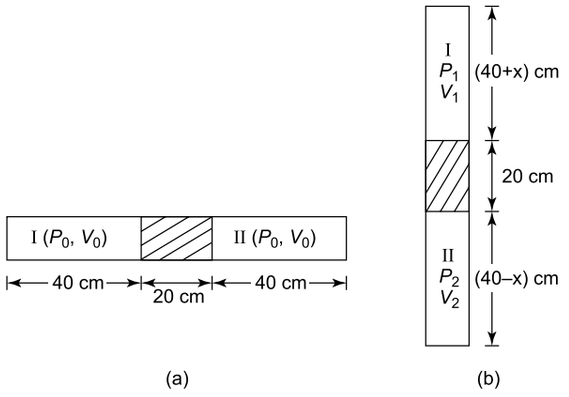


Fig. 18.1

Using Boyle's Law, we have

$$P_0V_0 = P_1V_1 \text{ and } P_0V_0 = P_2V_2$$

$$\Rightarrow \quad P_1 = \frac{P_0V_0}{V_1} = \frac{P_0 \times 40A}{(40+x)A} = \frac{40\,P_0}{(40+x)}$$

and $$P_2 = \frac{P_0V_0}{V_2} = \frac{P_0 \times 40A}{(40-x)A} = \frac{40\,P_0}{(40-x)}$$

The mercury column will be in equilibrium if P_2 (in cm of Hg) + 10 cm of Hg = P_1(in cm of Hg), i.e. if

$$\frac{40\,P_0}{40+x} + 10 = \frac{40\,P_0}{(40-x)}$$

where P_0 = 76 cm of Hg. Thus

$$\Rightarrow \frac{40 \times 76}{(40+x)} + 10 = \frac{40 \times 76}{(40-x)}$$

$$\Rightarrow \quad x^2 + 608x - 1600 = 0$$

Solving for positive root, we get x = 2.6 cm

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. In an adiabatic process, the root mean square speed of the molecules of a monoatomic gas becomes twice its initial value. The ratio of the initial volume of the gas to the final volume is
(a) 2 (b) $2^{3/2}$
(c) 4 (d) 8

2. The root mean square speed of hydrogen molecules at a certain temperature is v. If the temperature is doubled and the hydrogen gas dissociates into atomic hydrogen, the rms speed will become
(a) $\frac{v}{4}$ (b) $\frac{v}{2}$
(c) $2v$ (d) $4v$

3. A gas in a closed container has temperature T and pressure P. If the molecules of the gas undergo inelastic collisions with the walls of the container, then
(a) both P and T will decrease
(b) P decreases and T increases
(c) P increases and T decreases
(d) both P and T remain the same

4. Two moles of hydrogen are mixed with n moles of helium. The root mean square speed of the gas molecules in the mixture is $\sqrt{2}$ times the speed of sound in the mixture. The value of n is
(a) 1 (b) 2
(c) 3 (d) 4

5. A vessel contains 4 moles of oxygen and 2 moles of argon at absolute temperature T. The total internal energy of the gas mixture is
(a) 6 RT (b) 9 RT
(c) 11 RT (d) 13 RT

6. The average translational kinetic energy of a molecule of a gas at absolute temperature T is proportional to
(a) $\frac{1}{T}$ (b) $\sqrt{T}$
(c) T (d) T^2

7. The root mean square speed of the molecules of an enclosed gas is v. What will be the root mean square speed if the pressure is doubled, the temperature remaining the same?
(a) $v/2$ (b) v
(c) 2 v (d) 4 v

8. The mass of an oxygen molecule is about 16 times that of a hydrogen molecule. At room temperature the rms speed of oxygen molecules is v. The rms speed of the hydrogen molecule at the same temperature will be
(a) $v/16$ (b) $v/4$
(c) 4 v (d) 16 v

9. The average kinetic energy of hydrogen molecules at 300 K is E. At the same temperature, the average kinetic energy of oxygen molecules will be

(a) $E/16$ (b) $E/4$
(c) E (d) $4E$

10. At room temperature (27°C) the rms speed of the molecules of a certain diatomic gas is found to be 1920 ms^{-1}. The gas is
(a) H_2 (b) F_2
(c) O_2 (d) Cl_2

IIT, 1984

11. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K, the root mean square speed of the gas molecules is v, then at 480 K it will be
(a) $4\ v$ (b) $2\ v$
(c) $\frac{v}{2}$ (d) $\frac{v}{4}$

IIT, 1996

12. Three closed vessels A, B and C are at the same temperature. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of O_2 molecules in vessel A is v_1, that of N_2 molecules in vessel B is v_2, the average speed of O_2 molecules is vessel C is
(a) $\frac{1}{2}\ (v_1 + v_2)$ (b) v_1
(c) $\sqrt{v_1 v_2}$ (d) $\sqrt{\frac{3kT}{M}}$

IIT, 1992

13. The average translational energy and the rms speed of molecules of a sample of oxygen gas at 300 K are 6.21×10^{-21} J and 484 ms^{-1} respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour)
(a) 12.42×10^{-21} J, 968 ms^{-1}
(b) 8.78×10^{-21} J, 684 ms^{-1}
(c) 6.21×10^{-21} J, 968 ms^{-1}
(d) 12.42×10^{-21} J, 684 ms^{-1}

IIT, 1997

14. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature $2T$ has a pressure of
(a) $\frac{P}{8}$ (b) P
(c) $2P$ (d) $8P$

IIT, 1997

15. A vessel contains a mixture of 1 mole of oxygen and two moles of nitrogen at 300 K. The ratio of the rotational kinetic energy per O_2 molecule to that per N_2 molecule is
(a) 1 : 1
(b) 1 : 2
(c) 2 : 1
(d) depends on the moment of inertia of the two molecules.

IIT, 1998

16. Two thermally insulated vessels 1 and 2 are filled with air at temperature (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be
(a) $T_1 + T_2$
(b) $(T_1 + T_2)/2$
(c) $T_1 T_2\ (P_1V_1 + P_2V_2)/(P_1V_1T_2 + P_2V_2\ T_1)$
(d) $T_1 T_2\ (P_1V_1 + P_2V_2)/(P_1V_1T_1 + P_2V_2T_2)$

17. If the temperature of a gas is increased from 27°C to 927°C, the root mean square speed of its molecules
(a) becomes half
(b) is doubled
(c) becomes 4 times
(d) remains unchanged

18. At what temperature will oxygen molecules have the same root mean square speed as hydrogen molecules at 200 K?
(a) 527°C (b) 1327°C
(c) 2127°C (d) 2927°C

19. An enclosure of volume V contains a mixture of 8 g of oxygen, 14 g of nitrogen and 22 g of carbon dioxide at absolute temperature T. The pressure of the mixture of gases is (R is universal gas constant)
(a) $\frac{RT}{V}$ (b) $\frac{3RT}{2V}$
(c) $\frac{5RT}{4V}$ (d) $\frac{7RT}{5V}$

20. At what absolute temperature T is the root mean square speed of a hydrogen molecule equal to its escape velocity from the surface of the moon? The radius of moon is R, g is the acceleration due to gravity on moon's surface, m is the mass of a hydrogen molecule and k is the Boltzmann constant
(a) $\frac{mgR}{2k}$ (b) $\frac{2mgR}{k}$
(c) $\frac{3mgR}{2k}$ (d) $\frac{2mgR}{3k}$

21. Two perfect gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy in this process. If n_1 and n_2 are the respective number of molecules of the gases, the temperature of the mixture will be

(a) $\frac{n_1T_1 + n_2T_2}{n_1 + n_2}$ (b) $\frac{n_2T_1 + n_1T_2}{n_1 + n_2}$

(c) $T_1 + \frac{n_2}{n_1}T_2$ (d) $T_2 + \frac{n_1}{n_2}T_1$

22. An insulated box containing a diatomic gas of molar mass M is moving with a velocity v. The box is suddenly stopped. The resulting change in temperature is (R is the gas constant)

(a) $\frac{Mv^2}{2R}$ (b) $\frac{Mv^2}{3R}$

(c) $\frac{Mv^2}{5R}$ (d) $\frac{2Mv^2}{5R}$

IIT, 2003

23. 0.014 kg of nitrogen is enclosed in a vessel at a temperature of 27 °C. The amount of heat energy to be supplied to the gas to double the rms speed of its molecules is approximately equal to

(a) 6350 J (b) 7350 J

(c) 8350 J (d) 9350 J

24. A vessel of volume V contains an ideal gas at absolute temperature T and pressure P. The gas is allowed to leak till its pressure falls to P'. Assuming that the temperature remains constant during leakage, the number of moles of the gas that have leaked is

(a) $\frac{V}{RT}(P + P')$ (b) $\frac{V}{2RT}(P + P')$

(c) $\frac{V}{RT}(P - P')$ (d) $\frac{V}{2RT}(P - P')$

ANSWERS

1. (d)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (c)
7. (b)	**8.** (c)	**9.** (c)	**10.** (a)	**11.** (b)	**12.** (b)
13. (d)	**14.** (c)	**15.** (a)	**16.** (c)	**17.** (b)	**18.** (d)
19. (c)	**20.** (d)	**21.** (a)	**22.** (c)	**23.** (d)	**24.** (c)

SOLUTIONS

1. Let T_1 be the initial temperature. Since $v_{rms} \propto \sqrt{T}$, the final temperature $T_2 = 4\ T_1$. For an adiabatic process,

$$T_1\ V_1^{(\gamma - 1)} = T_2\ V_2^{(\gamma - 1)}$$

$$\Rightarrow \quad \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = \frac{T_2}{T_1} = 4$$

$$\Rightarrow \quad \left(\frac{V_1}{V_2}\right)^{\frac{5}{3} - 1} = 4$$

$$\Rightarrow \quad \frac{V_1}{V_2} = (4)^{3/2} = 8$$

2. $v_{rms} = \sqrt{\frac{3RT}{M}}$; M = molecular mass. Now T is doubled and M is halved. Hence the rms speed will become twice, which is choice (c).

3. Since the temperature of the molecules is the same as that of the walls, there is no exchange of energy in a collision. Hence the molecules rebound with the same average speed whether the collision is elastic or inelastic. Therefore, both P and T will not change.

4. $v_{rms} = \sqrt{\frac{3RT}{M}}$; M = molecular mass of mixture

The speed of sound in the mixture is

$v = \sqrt{\frac{\gamma RT}{M}}$; $\gamma = \frac{C_p}{C_v}$ of the mixture

Given $v_{rms} = \sqrt{2}\ v$. Therefore

$$\sqrt{\frac{3RT}{M}} = \sqrt{2}\sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow \quad \gamma = \frac{3}{2}$$

For the mixture,

$$C_v = \frac{n_1(C_v)_1 + n_2(C_v)_2}{n_1 + n_2}$$

and $$C_p = \frac{n_1(C_p)_1 + n_2(C_p)_2}{n_1 + n_2}$$

$$\therefore \quad \gamma = \frac{C_p}{C_v} = \frac{n_1(C_p)_1 + n_2(C_p)_2}{n_1(C_v)_1 + n_2(C_v)_2} \qquad (1)$$

For hydrogen, $n_1 = 2$, $(C_v)_1 = \frac{3R}{2}$ and $(C_p)_1 = \frac{5R}{2}$

For helium, $n_2 = n$, $(C_v)_2 = \frac{5R}{2}$ and $(C_p)_2 = \frac{7R}{2}$

Using these values and $\gamma = 3/2$ in Eq. (1) and solving, we get $n = 2$.

5. The internal energy of n moles of a gas temperature T is given by

$$U = \frac{f}{2}(nRT)$$

where f = number of degrees of freedom.
For oxygen $f = 5$ and for argon $f = 3$. Hence

$$U = U_1 + U_2$$
$$= \frac{5}{2} \times (4RT) + \frac{3}{2} \times (2RT)$$
$$= 13RT$$

6. $\bar{E} = \frac{3}{2}kT$. So the correct choice is (c).

7. The rms speed = $\sqrt{3kT/m}$ which is independent of pressure. Hence the correct choice is (b).

8. $v = \sqrt{3kT/m}$. Therefore, $v_0 = \sqrt{3kT/m_0}$ and $v_h = \sqrt{3kT/m_h}$.

Thus $$\frac{v_0}{v_h} = \sqrt{\frac{m_h}{m_0}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

or $$v_h = 4\ v_0.$$

Hence the correct choice is (c).

9. The correct choice is (c) as explained above.

10. We know that $v_{rms} = \sqrt{\frac{3RT}{M}}$ which gives

$$M = \frac{3RT}{v_{rms}^2} = \frac{3 \times 8.3 \times 300}{(1920)^2}$$
$$= 2 \times 10^{-3}\ \text{kg} = 2\ \text{g}$$

Since $M = 2$, the gas is hydrogen.

11. $v_{rms} = \sqrt{\frac{3RT}{M}}$ or $v_{rms} \propto \sqrt{T}$. Hence the correct choice is (b).

12. For a given M, $v_{av} \propto \sqrt{T}$ only. Therefore, the correct choice is v_1 as the temperatures of vessels A and C are the same.

13. $E \propto T$ and $v_{rms} \propto \sqrt{T}$. Hence at 600 K,

$$E = 6.21 \times 10^{-21} \times \frac{600}{300}$$
$$= 12.42 \times 10^{-21}\ \text{J}$$

and $$v_{rms} = 484 \times \sqrt{\frac{600}{300}} \simeq 684\ \text{ms}^{-1}$$

which is choice (d).

14. For a gas, $PV = nRT$. Hence

$$(P)_{O2} = \frac{(1\ \text{mole})\,RT}{V} \quad \text{and}$$

$$(P)_{He} = \frac{(1\ \text{mole})\,R(2T)}{V}$$

$$\therefore \quad \frac{(P)_{He}}{(P)_{O_2}} = 2 \text{ or } (P)_{He} = 2\ (P)_{O_2}$$

15. Since both the gases are diatomic, each has two degrees of freedom associated with rotational motion. According to the law of equipartition of energy, the rotational kinetic energy per degree of freedom is $(1/2)kT$. Since the temperatures of the two gases are equal, their rotational kinetic energies will be equal. Hence the correct choice is (a).

16. According to the kinetic theory, the average kinetic energy (KE) per molecule of a gas = $\frac{3}{2}\ kT$. Let n_1 and n_2 be the number of moles of air in vessels 1 and 2 respectively.
Before mixing, the total KE of molecules in the two vessels is

$$E_1 = \frac{3}{2}n_1kT_1 + \frac{3}{2}n_2\ kT_2$$
$$= \frac{3}{2}k(n_1T_1 + n_2T_2)$$

After mixing, the total KE of molecules is

$$E_2 = \frac{3}{2}(n_1 + n_2)\ kT$$

where T is the temperature when equilibrium is established. Since there is no loss of energy (because the vessels are insulated), $E_2 = E_1$ or

$$\frac{3}{2}(n_1 + n_2)kT = \frac{3}{2}k\ (n_1T_1 + n_2T_2)$$

or $$T = \frac{n_1T_1 + n_2T_2}{(n_1 + n_2)} \qquad (1)$$

Now $P_1V_1 = n_1RT_1$ and $P_2V_2 = n_2RT_2$ which give

$$n_1 = \frac{P_1V_1}{RT_1} \text{ and } n_2 = \frac{P_2V_2}{RT_2}$$

Using these in Eq. (1) and simplifying, we get

$$T = \frac{T_1T_2(P_1V_1 + P_2V_2)}{(P_1V_1T_2 + P_2V_2T_1)}$$

17. The root mean square speed is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

i.e. $v_{\text{rms}} \propto T$. Initial temperature $T_1 = 27 + 273 = 300$ K. Final temperature $T_2 = 927 + 273 = 1200$ K. Since temperature is increased by 4 times, the speed is doubled. Hence the correct choice is (b).

18. For oxygen : $v_{\text{rms}} = \sqrt{\frac{3kT_0}{m_0}}$

For hydrogen : $v_{\text{rms}} = \sqrt{\frac{3kT_h}{m_h}}$

The value of v_{rms} will be the same if

$$\frac{T_0}{m_0} = \frac{T_h}{m_h}$$

or $T_0 = \frac{m_0}{m_h} \times T_h = 16 \times 200 = 3200$ K $= 2927°$C

Hence the correct choice is (d).

19. The pressure exerted by a gas is given by

$$\text{P} = \frac{nRT}{V}$$

$$= \frac{\text{mass}}{\text{molecular weight}} \times \frac{RT}{V}$$

Pressure exerted by oxygen $P_1 = \frac{8}{32}\frac{RT}{V} = \frac{1}{4}\frac{RT}{V}$

Pressure exerted by oxygen $P_2 = \frac{14}{28}\frac{RT}{V} = \frac{1}{2}\frac{RT}{V}$

Pressure exerted by carbon dioxide

$$P_3 = \frac{22}{44}\frac{RT}{V} = \frac{1}{2}\frac{RT}{V}$$

From Dalton's law of partial pressures, the total pressure exerted by the mixture is given by

$$P = P_1 + P_2 + P_3$$

$$= \frac{1}{4}\frac{RT}{V} + \frac{1}{2}\frac{RT}{V} + \frac{1}{2}\frac{RT}{V}$$

$$= \frac{5RT}{V}, \text{ which is choice (c).}$$

20. The root mean square speed is given by

$$v_{\text{rms}} = \frac{\sqrt{3kT}}{m}$$

The escape velocity is given by $v_e = \sqrt{2gR}$

For $v_{\text{rms}} = v_e$, we require

$$\sqrt{\frac{3kT}{m}} = \sqrt{2gR} \text{ or } T = \frac{2mgR}{3k}, \text{ which is choice (d).}$$

21. Average kinetic energy per molecule of a perfect gas $= \frac{3}{2}kT$.

$\therefore$ Average KE of molecules of the first gas $= \frac{3}{2}n_1kT_1$

Average KE of molecules of the second gas $= \frac{3}{2}n_2kT_2$

$\therefore$ Total KE of the molecules of the two gases before they are mixed is

$$K = \frac{3}{2}n_1kT_1 + \frac{3}{2}n_2kT_2$$

$$= \frac{3}{2}(n_1T_1 + n_2T_2)k \quad (1)$$

If T is the temperature of the mixture, the kinetic energy of the molecules $(n_1 + n_2)$ in the mixture is

$$K' = \frac{3}{2}(n_1 + n_2)kT \quad (2)$$

Since there is no loss of energy $K = K'$. Equating (1) and (2) we get

$$n_1T_1 + n_2T_2 = (n_1 + n_2)T \text{ or } T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$$

Hence the correct choice is (a).

22. Let n be the number of moles of the gas in the box. The kinetic energy of the gas $= n\left(\frac{1}{2}Mv^2\right)$. When the box is suddenly stopped, this energy is used up in changing the internal energy, as a result of which the temperature of the gas rises. The change in internal energy is given by

$$\Delta U = nC_v\Delta T = n \times \frac{5}{2}R \times \Delta T$$

For a diatomic gas $C_v = \frac{5}{2}R$.

Hence $n \times \frac{5R}{2} \times \Delta T = n \times \frac{1}{2}Mv^2$

or $\Delta T = \frac{Mv^2}{5R}$, which is choice (c)

23. The root mean square speed is related to absolute temperature T as

$$c_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

For a given gas, m is fixed. Therefore, $c_{\text{rms}} \propto \sqrt{T}$. Hence in order to double the root mean square speed, the absolute temperature must be increased to four times the initial value. Initial temperature $T_1 = 273 + 27 = 300$ K. Therefore, final temperature $T_2 = 4\ T_1 = 1200$ K.

Since the volume of the vessel is fixed, $\Delta V = 0$. Hence the heat energy supplied to the gas does no work on the gas; it only increases the internal energy of the molecules.

The increase in internal energy is

$$\Delta U = n\ C_v\ \Delta T$$

Since nitrogen is diatomic $C_v = \frac{5R}{2}$. The number of moles of nitrogen in the vessel is

$$n = \frac{\text{mass in kg}}{\text{molecular mass}} = \frac{0.014 \times 10^3}{28} = 0.5$$

$$\therefore \quad \Delta U = 0.5 \times \frac{5R}{2} \times (1200 - 300)$$

$$= \frac{0.5 \times 5 \times 8.314 \times 900}{2} = 9353 \text{ J}.$$

So the correct choice is (d)

24. Number of moles present initially is $n = \frac{PV}{RT}$. Let n' be the number of moles of the gas that leaked till the pressure falls to P'. Since volume V of the vessel cannot change and temperature T remains constant during leakage, we have

$$n' = \frac{P'V}{RT}$$

$\therefore$ Number of moles that leaked is

$$\Delta n = n - n' = \frac{PV}{RT} - \frac{P'V}{RT} = \frac{V}{RT}\ (P - P')$$

So the correct choice is (c).

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. The root mean square speed of the molecules of a gas depends upon
(a) the pressure of the gas
(b) the density of the gas
(c) the temperature of the gas
(d) the mass of a molecule of the gas

2. The average translational kinetic energy of a molecule of a gas at absolute temperature T is E and the root mean square speed of the molecules is v. Then
(a) $E \propto T$ (b) $E \propto \sqrt{T}$
(c) $v \propto T$ (d) $v \propto \sqrt{T}$

3. The root mean square speeds of the molecules of hydrogen, oxygen and carbon dioxide at the same temperature are v_h, v_o and v_c respectively. Then
(a) $v_h = v_o = v_c$ (b) $v_h > v_o$
(c) $v_o > v_c$ (d) $v_h < v_o < v_c$

4. One mole of oxygen at 27 °C is enclosed in a vessel which is thermally insulated. The vessel is moved with a constant speed v and is then suddenly stopped. The process results in a rise of temperature of the gas by 1 °C. Then, if M = molecular mass of oxygen.

(a) $\gamma\ (= C_p/C_v) = \frac{5}{3}$ (b) $\gamma = \frac{7}{5}$

(c) $v = \sqrt{\frac{R}{M(\gamma + 1)}}$ (d) $v = \sqrt{\frac{2R}{M(\gamma - 1)}}$

IIT, 1983, 1996

5. In a vessel a gas at temperature T has a pressure P. The density of the gas is ρ and v_{rms} is the average root-mean-square speed of a molecule. If N is the number of molecules per unit volume and m the mass of a molecule, then (k = Boltzmann constant)

(a) $P = \frac{1}{3}\rho v_{\text{rms}}^2$ (b) $P = \frac{2}{3}\rho v_{\text{rms}}^2$

(c) $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$ (d) $P = N\ kT$

6. N molecules of a gas, each of mass m, strike per second a fixed wall of a container of area A at an angle θ to the vertical and rebound with a speed v.

The collisions are assumed to be elastic.

(a) The magnitude of change in momentum is $|\Delta p| = 2mv \sin\theta$

(b) $|\Delta p| = 2mv \cos\theta$

(c) The pressure exerted on the wall is $P = \dfrac{|\Delta p|}{NA}$

(d) $P = \dfrac{N|\Delta p|}{A}$

7. The pressure P of n moles of a gas varies with volume V as

$$P = a - bV^2$$

where a and b are positive constants. The highest absolute temperature to which the gas can be heated is T_{max}.

(a) $T_{max} = \dfrac{a^{3/2}}{nR\sqrt{3b}}$; R = gas constant

(b) $T_{max} = \dfrac{2a^{3/2}}{3nR\sqrt{3b}}$

(c) At T_{max}, $V = \left(\dfrac{a}{b}\right)^{1/2}$

(d) At T_{max}, $V = \left(\dfrac{a}{3b}\right)^{1/2}$

8. A cylinder, fitted with piston, contains a diatomic gas at temperature $T_1 = 27$ °C. Initially the height of the piston from the base of the cylinder is $h_1 = 20$ cm as shown in Fig. 18.2. When the temperature is raised to $T_2 = 177$ °C, the new height of the piston above the base becomes h_2. The system is then insulated from the surroundings and the piston is brought back to its original height. The new equilibrium temperature becomes T_3. Given $(1.5)^{0.4} = 1.18$.

IIT, 2004

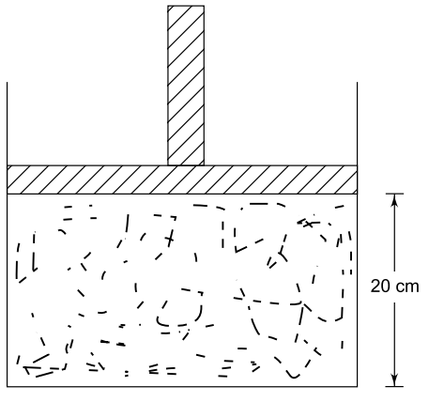


Fig. 18.2

(a) $h_2 = 30$ cm (b) $h_2 = 24.5$ cm

(c) $T_3 = 81$°C (d) $T_3 = 258$ °C

ANSWERS AND SOLUTIONS

1. $v_{rms} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3kT}{m}}$. Hence all the four choices are correct.

2. $E = \dfrac{3}{2}kT$ and $v_{rms} = \sqrt{\dfrac{3kT}{m}}$. Hence the correct choices are (a) and (d).

3. The rms speed is the maximum for the gas which is the lightest. Hence the correct choices are (b) and (c).

4. Oxygen is diatomic; it has 5 degrees of freedom. Therefore, $C_v = 5R/2$ and $C_p = 7R/2$. So $\gamma = C_p/C_v = 7/5$.

The kinetic energy of oxygen molecules with a velocity $v_0 = \dfrac{1}{2}Mv^2$, where M = molecular weight of oxygen.

Now heat energy = $C_v\,dT = C_v \times 1 = C_v$

But $C_p - C_v = R$ or $\dfrac{C_p}{C_v} - 1 = \dfrac{R}{C_v}$

or $(\gamma - 1) = \dfrac{R}{C_v}$ or $C_v = \dfrac{R}{(\gamma - 1)}$

Therefore, $\dfrac{1}{2}Mv^2 = \dfrac{R}{(\gamma - 1)}$

or $v = \sqrt{\dfrac{2R}{M(\gamma - 1)}}$

So the correct choices are (b) and (d).

5. The average pressure of a gas is given by

$$P = \frac{1}{3}\rho v_{rms}^2$$

where density $\rho = mN$; here m is the mass of a molecule and N is the number of molecules per unit volume of the gas. Thus

$$P = \frac{1}{3}mN v_{rms}^2 \qquad (1)$$

The average kinetic energy of a molecule is given by

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT \qquad (2)$$

Using (2) in (1), we get

$$P = NkT$$

Thus the correct choices are (a), (c) and (d).

6. Refer to Fig. 18.3. Since the collision is elastic, the speed v of the molecule before collision = speed after rebound. Therefore, the magnitude of the change in momentum normal to the wall is

$|\Delta p| = mv \cos \theta - (-mv \cos \theta) = 2\ mv \cos \theta$

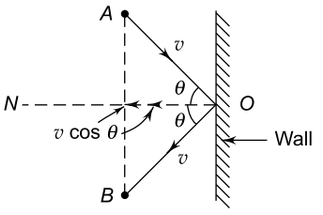


Fig. 18.3

Number of collisions per second $= N$. Therefore, time interval between successive collisions is

$$\Delta t = \frac{1}{N}$$

Force exerted on the wall by the molecules is = rate of change of momentum

$$F = \frac{|\Delta p|}{\Delta t}$$

Pressure $P = \dfrac{F}{A}$. Hence the correct choices are (b) and (c).

7. For n moles of an ideal gas, $PV = nRT$

Using the given relation between P and V, we have

$(a - bV^2)\ V = nRT$ or $aV - bV^3 = nRT$ (1)

Differentiating with respect to V, we get

$$a - 3\ bV^2 = nR\ \frac{dT}{dV}$$

or $$\frac{dT}{dV} = \frac{1}{nR}\ (a - 3\ bV^2) \qquad (2)$$

T will be maximum if $\dfrac{dT}{dV} = 0$ and $\dfrac{d^2T}{dV^2}$ is negative.

Setting $\dfrac{dT}{dV} = 0$ in Eq. (2), we get

$$a = 3\ bV^2 \quad \text{or} \quad V = \left(\frac{a}{3b}\right)^{1/2} \qquad (3)$$

Using (3) in (1) we get

$$a\left(\frac{a}{3b}\right)^{1/2} - b\left(\frac{a}{3b}\right)^{3/2} = nR\ T_{\max}$$

or $$\left(\frac{a}{3b}\right)^{1/2}\left(a - b \times \frac{a}{3b}\right) = nR\ T_{\max}$$

or $$\left(\frac{a}{3b}\right)^{1/2}\left(a - \frac{a}{3}\right) = nR\ T_{\max}$$

or $$T_{\max} = \frac{2a^{3/2}}{3\sqrt{3b}\ nR} \qquad (4)$$

It is easy to check that $\dfrac{d^2T}{dV^2}$ is indeed negative at $a = 3\ bV^2$. Hence Eq. (4) gives the maximum value of T. So the correct choices are (b) and (d).

8. Let A be the cross-sectional area of the base of the cylinder. Given $T_1 = 273 + 27 = 300$ K and $T_2 = 273 + 177 = 450$ K.

At constant pressure, we have from Charles' law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

or $$\frac{Ah_1}{T_1} = \frac{Ah_2}{T_2}$$

or $$h_2 = \frac{T_2}{T_1} \times h_1 = \frac{450\ \text{K}}{300\ \text{K}} \times 20\ \text{cm}$$

$$= 30\ \text{cm}$$

Since the system is completely insulated from the surroundings, it cannot take heat from or give heat to the surroundings. Hence the second process is adiabatic for which

$$T_2 V_2^{(\gamma-1)} = T_3 V_3^{(\gamma-1)}$$

where $V_2 = Ah_2$ and $V_3 = Ah_1$. Also, for a diatomic gas $\gamma = 1.4$. Therefore,

$$T_3 = T_2 \times \left(\frac{V_2}{V_3}\right)^{(\gamma-1)}$$

$$= 450 \times \left(\frac{30}{20}\right)^{(1.4-1)}$$

$$= 450 \times (1.5)^{0.4}.$$

$$= 450 \times 1.18 = 531\ \text{K} = 258\ ^\circ\text{C}$$

So the correct choices are (a) and (d).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 5 are based on the following passage

Passage I

Kinetic Theory of Gases

The molecules of a gas move in all directions with various speeds. The speeds of the molecules of a gas increase with rise in temperature. During its random motion, a fast molecule often strikes against the walls of the container of the gas. The collisions are assumed to be perfectly elastic, i.e. the molecule bounces back with the same speed with which it strikes the wall. Since the number of molecules is very large, billions of molecules strike against the walls of the container every second. These molecules exert a sizeable force on the wall. The force exerted per unit area is the pressure exerted by the gas on the walls. According to the kinetic theory, the pressure of a gas of density ρ at absolute temperature T is given by

$$P = \frac{1}{3}\rho v_{\text{rms}}^2$$

where v_{rms} is the root mean square speed of the gas molecule and is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

where m is the mass of a molecule and k is Boltzmann constant.

1. From the relation $P = \frac{1}{3}\rho v_{\text{rms}}^2$, we find that the dimensions of pressure are

(a) $ML^{-1}T^{-1}$ (b) $ML^{-1}T^{-2}$

(c) $ML^{-2}T^{-1}$ (d) $ML^{-2}T^{-2}$.

2. From the relation $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$, it follows that the constant k should be expressed in units of

(a) newton per metre per kelvin

(b) newton per kelvin

(c) joule per kelvin

(d) joule per kilogram per kelvin

3. Choose the only correct statement from the following.

(a) The pressure of a gas is equal to the total kinetic energy of its molecules per unit volume of the gas.

(b) The product of pressure and volume of a gas is always constant.

(c) The average kinetic energy of the molecules of a gas is proportional to its absolute temperature.

(d) The root mean square speed of a molecule is proportional to the absolute temperature of the gas.

4. If the temperature of a gas is increased from 27°C to 927°C, the root mean square speed of its molecules

(a) becomes half (b) is doubled

(c) becomes four times (d) remains unchanged

5. The root mean square speed of oxygen gas molecule at $T = 320$ K is very nearly equal to (the molar mass of oxygen is $M = 0.0320$ kg per mole and gas constant $R = 8.31$ J mol^{-1} K^{-1})

(a) 300 ms^{-1} (b) 500 ms^{-1}

(c) 700 ms^{-1} (d) 900 ms^{-1}

ANSWERS AND SOLUTIONS

1. Dimensions of P = dimensions of density × dimensions of (velocity)^2

$$= ML^{-3} \times (LT^{-1})^2 = ML^{-1}T^{-2}$$

Hence the correct choice is (b).

2. Squaring we have

$$k = \frac{mv_{\text{rms}}^2}{3T} = \frac{\text{unit of energy}}{\text{unit of temperature}}$$

= joule per kelvin

3. The correct choice is (c)

4. The correct choice is (b) since v_{rms} is proportional to the square root of absolute temperature

5. $v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 320}{0.032}}$

$$= 499.3 \text{ ms}^{-1}$$

Hence the correct choice is (b)

Questions 6 to 12 are based on the following passage

Passage II

Van der Waals Equation of State

The equation of state $PV = nRT$ holds for an ideal gas. The behaviour of real gases shows departures from an ideal gas behaviour especially at high pressures. The model of an ideal gas is based on a number of assumptions. Van der Waals modified the ideal gas equation $PV = nRT$ by taking into account two of those assumptions which may not be valid. He argued that (i) the volume of the molecules may not be negligible compared to the volume V occupied by the gas and (ii) the attractive forces between the molecules may not be negligible. He said that pressure P in equation $PV = nRT$ is less than the true pressure by an amount p because of attractive forces between the molecules. According to him, the pressure 'defect' p is inversely proportional to the square of volume, i.e.

$$p \propto \frac{1}{V^2}$$

or

$$p = \frac{a}{V^2}$$

where a is constant depending on the nature of the gas.

Thus the true pressure of the gas is $P' = P + p = P + \frac{a}{V^2}$.

He further argued that V is not the true volume of the gas because the molecules themselves occupy a finite volume. According to him, the true volume of the gas is $V' = (V - b)$ where b is a factor depending on the actual volume of the molecules themselves. Thus Van der Waals' equation for real gases is $P'V' = nRT$, i.e.

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

At high pressures, when the molecules are too many and too close together, the correction factors a and b become important.

6. In Van der Waals' equation of state for real gases, the product PV has the same dimensions as those of
(a) bP (b) $\frac{a}{V}$
(c) $\frac{ab}{V^2}$ (d) nRT

7. The dimensions of a are the same as those of
(a) PV (b) PV^2
(c) P^2V (d) P/V

8. The dimensions of b are the same as those of
(a) P (b) V
(c) PV (d) nRT

9. The dimensions of $\frac{a}{b}$ are the same as those of
(a) work (b) force
(c) pressure (d) power

10. The dimensional formula for ab is
(a) ML^2T^{-2} (b) ML^4T^{-2}
(c) ML^6T^{-2} (d) ML^8T^{-2}

11. The correction factors a and b depend upon
(a) the pressure of the gas
(b) the volume of the gas
(c) the temperature of the gas
(d) the nature of the gas

12. The equation of state $PV = nRT$ holds if the gas has
(a) low pressure and low density
(b) low pressure and high density
(c) high pressure and low density
(d) high pressure and high density.

ANSWERS AND SOLUTIONS

6. From Van der Waals' equation, we have

$$PV - bP + \frac{a}{V} - \frac{ab}{V^2} = nRT$$

From the principle of homogeneity of dimensions, we find that all the four choices are correct.

7. Dimensions of $\frac{a}{V^2}$ = dimensions of P

$\therefore$ Dimensions of a = dimensions of PV^2.

Hence the correct choice is (b).

8. Dimensions of b = dimensions of V, which is choice (b)

9. Dimensions of $\frac{a}{b} = \frac{\text{dimensions of } PV^2}{\text{dimensions of } V}$

= dimensions of PV

= dimensions of work

Hence the correct choice is (a).

10. Dimensions of ab = dimensions of $PV^2 \times$ dimensions of V

= dimensions of (PV^3)

= $ML^{-1}T^{-2} \times (L^3)^3 = ML^8T^{-2}$

Hence the correct choice is (d).

11. The correct choice is (d).

12. The correct choice is (a).

Questions 13 and 14 are based on the following passage

Passage III

Mean Free Path

During their random motion, the molecules of a gas often come close to each other. When the distance between two molecules is comparable with the diameter of a molecule, the forces between them become very strong. As a result, their individual momenta before and after the encounter are different. When this happens a 'collision' is said to have occurred. The average distance a molecule travels before it suffers a collision with another molecule is called the mean free path (l_c), which can be estimated as follows. Suppose the average speed of a molecule of diameter d is $\bar{v}$. In one second, this molecule sweeps out a volume $\pi d^2 \bar{v}$, If it finds any other molecules in this volume, it will suffer collisions with them. If n is the number density (i.e. number per unit volume) of the molecules, then the number of molecules in this volume $= \pi d^2 \bar{v} n$. The number of collisions per second $= \nu_c = \pi d^2 \bar{v} n$. Therefore, the average time between two collisions (called collision period T_c) is

$$T_c = \frac{1}{\nu_c} = \frac{1}{\pi d^2 \bar{v} n}$$

Hence the mean free path (i.e. the average distance the molecule travels between two successive collisions) is

$$l_c = \bar{v}\, T_c = \frac{1}{\pi d^2 n}$$

For air at S.T.P. the value of $l_c \simeq 3 \times 10^{-7}$ m.

13. The mean free path of a molecule of a gas depends
- (a) only on its diameter (d)
- (b) only on the number density (n) of the molecules
- (c) on both d and n
- (d) neither on d nor on n.

14. The average collision period in a gas
- (a) increases if the pressure is increased
- (b) decreases if the pressure is increased
- (c) increases if the temperature of the gas is increased.
- (d) decreases if the temperature of the gas is increased.

ANSWERS AND SOLUTIONS

13. The mean free path is given by

$$l_c = \frac{1}{\pi d^2 n}$$

Hence the correct choice is (c).

14. The collision period is given by

$$T_c = \frac{1}{\pi d^2 \bar{v} n}$$

If the pressure is increased, the volume of the gas decreases. Hence number density n increases. Therefore, T_c will decrease. If the temperature of the gas is increased, the average speed $\bar{v}$ of the molecules increases. Hence T_c will decrease. Thus the correct choices are (b) and (d).

19 Chapter

Transmission of Heat

REVIEW OF BASIC CONCEPTS

19.1 THERMAL CONDUCTIVITY

If a steady temperature difference $(T_1 - T_2)$ is to be maintained between the ends of a rod, heat must be supplied at a steady rate at one end and the same must be taken out at the other end. Suppose an amount of heat Q flows through the rod in time t so that Q/t is the rate of heat flow. Then

(i) Q/t will be proportional to the area A of the cross-section of the rod.

(ii) Q/t will be proportional to $(T_1 - T_2)$.

(iii) Q/t will be inversely proportional to l, the distance between ends of the rod.

Thus we find that

$$H = \frac{Q}{t} \propto \frac{A(T_1 - T_2)}{l} \quad \text{or} \quad \frac{Q}{t} = \frac{k\,A(T_1 - T_2)}{l}$$

where k is a constant of proportionality called the *thermal conductivity* of the substance. It is a measure of how quickly heat energy can conduct (flow) through the substance. In the SI system, k is expressed in $Js^{-1}\ m^{-1}\ °C^{-1}$ or $J\ s^{-1}\ m^{-1}\ K^{-1}$ or $Wm^{-1}\ K^{-1}$.

$$[k] = [MLT^{-3}K^{-1}]$$

19.2 CONDUCTION THROUGH A COMPOSITE SLAB

Case 1. Two slabs placed one on top of the other

Suppose we have a composite slab made up of two different slabs of materials of thermal conductivities k_1 and k_2, and cross-sectional areas A_1 and A_2 but of equal length L placed one on top of the other as shown in Fig. 19.1.

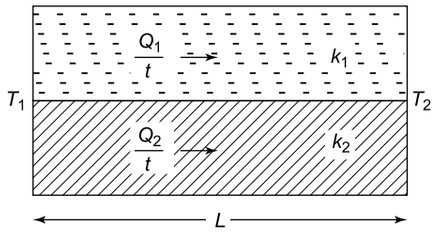


Fig. 19.1

The ends of the composite slab are maintained at temperatures T_1 and T_2 $(T_1 > T_2)$. The rates of heat flow through each slab in the steady state are

$$\frac{Q_1}{t} = \frac{k_1 A_1 (T_1 - T_2)}{L} \tag{1}$$

and

$$\frac{Q_2}{t} = \frac{k_2 A_2 (T_1 - T_2)}{L} \tag{2}$$

The cross-sectional area of the composite slab is $(A_1 + A_2)$ but its length is L. If k_{eq} is the equivalent thermal conductivity, then the rate of heat flow through the composite slab is

$$\frac{Q}{t} = \frac{k_{eq}\,(A_1 + A_2)\,(T_1 - T_2)}{L} \tag{3}$$

Since $\frac{Q}{t} = \frac{Q_1}{t} + \frac{Q_2}{t}$, we get from Eqs. (1), (2) and (3)

$$k_{eq}\,(A_1 + A_2) = k_1\,A_1 + k_2\,A_2$$

$$\Rightarrow \quad k_{eq} = \frac{k_1 A_1 + k_2 A_2}{A_1 + A_2} \tag{4}$$

If $A_1 = A_2$, then $k_{eq} = \frac{1}{2}(k_1 + k_2)$.

Case 2. Two slabs placed in contact one after the other

Suppose we have two slabs of different lengths L_1 and L_2 but of the same cross-sectional area A placed in contact as shown in Fig. 19.2.

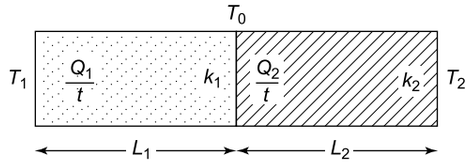


Fig. 19.2

The ends of the composite slab are maintained at temperatures T_1 and T_2 ($T_1 > T_2$). Let T_0 be the temperature of the junction. In the steady state, $\frac{Q_1}{t} = \frac{Q_2}{t}$, i.e.

$$\frac{k_1 A(T_1 - T_0)}{L_1} = \frac{k_2 A(T_0 - T_2)}{L_2}$$

$$\Rightarrow \quad \frac{k_1}{L_1}(T_1 - T_0) = \frac{k_2}{L_2}(T_0 - T_2)$$

$$\Rightarrow \quad T_0 = \frac{\frac{k_1 T_1}{L_1} + \frac{k_2 T_2}{L_2}}{\frac{k_1}{L_1} + \frac{k_2}{L_2}} \tag{5}$$

The rate of flow of heat through the composite slab is

$$\frac{Q}{t} = \frac{Q_1}{t} \quad \text{or} \quad \frac{Q_2}{t}$$

$$\Rightarrow \quad \frac{Q}{t} = \frac{k_1 A(T_1 - T_0)}{L_1} \tag{6}$$

Using (5) in (6) and simplifying, we get

$$\frac{Q}{t} = \frac{k_1 k_2 A(T_1 - T_2)}{k_1 L_2 + k_2 L_1} \tag{7}$$

The length of the composite slab is $(L_1 + L_2)$ but its cross-sectional area is A. If k_{eq} is the equivalent thermal conductivity of the composite slab,

$$\frac{Q}{t} = \frac{k_{eq} A (T_1 - T_2)}{(L_1 + L_2)} \tag{8}$$

From Eqs. (7) and (8), we get

$$k_{eq} = \frac{k_1 k_2 (L_1 + L_2)}{k_1 L_2 + k_2 L_1} \tag{9}$$

If $L_1 = L_2$, $k_{eq} = \frac{2k_1 k_2}{(k_1 + k_2)}$ and

$$T_0 = \frac{(k_1 T_1 + k_2 T_2)}{(k_1 + k_2)}$$

19.3 THERMAL RESISTANCE

Finding the equivalent thermal conductivity of a composite slab becomes much easier if we use the concept of thermal resistance. Just as electrical resistance is defined as

$$R = \frac{V}{I} = \frac{\text{potential difference}}{\text{rate of flow of charge}},$$

thermal resistance is defined as

$$R = \frac{\text{temperature difference}}{\text{rate of flow of heat}} = \frac{T_1 - T_2}{Q/t}$$

$$\Rightarrow \quad R = \frac{(T_1 - T_2)L}{k A(T_1 - T_2)} = \frac{L}{k A}$$

(a) If two slabs are joined in series as shown in Fig. 19.2, the equivalent thermal resistance of the composite slab is

$$R_{eq} = R_1 + R_2$$

$$\Rightarrow \quad \frac{L_1 + L_2}{k_{eq} A} = \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A}$$

$$\Rightarrow \quad k_{eq} = \frac{k_1 k_2 (L_1 + L_2)}{(L_1 k_2 + L_2 k_1)}, \text{ which is Eq. (9).}$$

(b) If the two slabs are joined in parallel as shown in Fig. 19.1, then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \quad \frac{k_{eq}(A_1 + A_2)}{L} = \frac{k_1 A_1}{L} + \frac{k_2 A_2}{L}$$

$$\Rightarrow \quad k_{eq} = \frac{(k_1 A_1 + k_2 A_2)}{(A_1 + A_2)}, \text{ which is Eq. (4).}$$

EXAMPLE 19.1

A metal rod of length 30 cm and radius 1 cm is covered with an insulating material. One end of the rod is maintained at 100°C and the other end is immersed in ice at 0°C. It is found that 31.4 g of ice melts in 5 minutes. Calculate the thermal conductivity of metal. Latent heat of fusion of ice = 80 cal g^{-1}.

SOLUTION

Given $L = 0.3$ m, $A = \pi r^2 = \pi \times (10^{-2})^2 = \pi \times 10^{-4}$ m^2, $T_1 = 100°C$ and $T_2 = 0°C$. Now

$$Q = mL_f = 31.4 \text{ g} \times 80 \text{ cal g}^{-1}$$
$$= 31.4 \times 80 \text{ cal}$$
$$= 31.4 \times 80 \times 4.2 \text{ J} \ (\because \ 1 \text{ cal} = 4.2 \text{ J})$$

Also $$\frac{Q}{t} = \frac{k A(T_1 - T_2)}{L}$$

$$\Rightarrow \quad k = \frac{QL}{t A(T_1 - T_2)}$$
$$= \frac{(31.4 \times 80 \times 4.2) \times 0.3}{(5 \times 60) \times (\pi \times 10^{-4}) \times (100 - 0)}$$
$$= 336 \text{ W m}^{-1} \text{ K}^{-1}$$

EXAMPLE 19.2

A cylindrical metal boiler of radius 10 cm and thickness 3.14 cm is filled with water and placed on an electric heater. If the water boils at the rate of 50 g s^{-1}, estimate the temperature of the filament. Thermal conductivity of metal = 1.13×10^2 W m^{-1} K^{-1} and latent heat of vaporisation = 2.26×10^3 J g^{-1}.

SOLUTION

$$Q = mL_v$$

$$\therefore \quad \frac{Q}{t} = (50 \text{ g s}^{-1}) \times (2.26 \times 10^3 \text{ J g}^{-1})$$
$$= 50 \times 2.26 \times 10^3 \text{ Js}^{-1}$$

Base area of boiler $A = \pi r^2 = \pi \times (0.1)^2 = \pi \times 10^{-2}$ m^2

Thickness of metal $L = 3.14$ cm $= 3.14 \times 10^{-2}$ m

$$\frac{Q}{t} = \frac{k A(T_f - T_w)}{L}$$

$$\Rightarrow \quad 50 \times 2.26 \times 10^3 = \frac{(1.13 \times 10^2) \times (\pi \times 10^{-2}) \times (T_f - T_w)}{3.14 \times 10^{-2}}$$

$$\Rightarrow \quad T_f - T_w = 1000°\text{C}$$

$$\therefore \quad T_f = 1000 + T_w = 1000 + 100$$
$$= 1100°\text{C}$$

EXAMPLE 19.3

A steel rod ($L_1 = 10$ cm, $A_1 = 0.02$ m^2 and $k_1 = 50$ J s^{-1} m^{-1} K^{-1}) is welded to a silver rod ($L_2 = 20$ cm, $A_2 = 0.01$ m^2, $k_2 = 400$ J s^{-1} m^{-1} K^{-1}) as shown in Fig. 19.3. The ends of the composite rod are maintained at 300°C and 0°C. The rod is covered with an insulating material so that the heat loss from the sides is negligible. Compute the temperature of the steel-silver junction in the steady state.

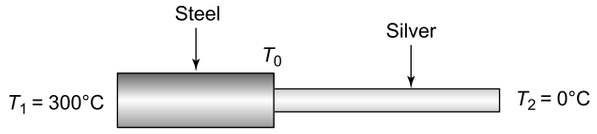


Fig. 19.3

SOLUTION

In the steady state, $\frac{Q_1}{t} = \frac{Q_2}{t}$, i.e.

$$\frac{k_1 A_1 (T_1 - T_0)}{L_1} = \frac{k_2 A_2 (T_0 - T_2)}{L_2}$$

$$\Rightarrow \quad \frac{50 \times 0.02 \times (300 - T_0)}{0.1} = \frac{400 \times 0.01 \times (T_0 - 0)}{0.2}$$

$$\Rightarrow \quad T_0 = 100°\text{C}$$

EXAMPLE 19.4

A steel rod ($L_1 = 10$ cm, $A_1 = 0.02$ m^2, $k_1 = 50$ W m^{-1} K^{-1}) and a brass ($L_2 = 10$ cm, $A_2 = 0.02$ m^2, $k_2 = 110$ W m^{-1} K^{-1}) are soldered as shown in Fig. 19.4. The ends of the composite rod are maintained at 403 K and 273 K. The sides of compound rod is covered with an insulating material. Calculate the rate of flow through the compound rod in the steady state.

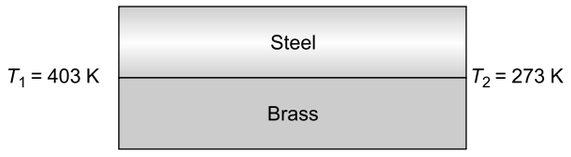


Fig. 19.4

SOLUTION

Equivalent thermal conductivity of the composite rod is

$$k_{eq} = \frac{1}{2}(k_1 + k_2) = \frac{1}{2}(50 + 110)$$
$$= 80 \text{ W m}^{-1} \text{ K}^{-1}$$

$$\therefore \quad \frac{Q}{t} = \frac{k_{eq} \times (A_1 + A_2)(T_1 - T_2)}{0.1}$$
$$= \frac{80 \times 0.04 \times (403 - 273)}{0.1}$$
$$= 4.16 \times 10^3 \text{ Js}^{-1}$$

EXAMPLE 19.5

A metal cylinder of radius r and thermal conductivity $k_1 = 2k$ is surrounded by a cylindrical metallic shell of inner radius r and outer radius $2r$ having thermal conductivity $k_2 = k$ (Fig. 19.5). The ends of this composite system are maintained at constant temperatures

T_1 and T_2 ($T_1 > T_2$). Find the equivalent thermal conductivity of the system.

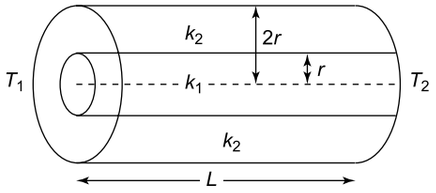


Fig. 19.5

SOLUTION

The given system is equivalent to a system consisting of a rod of length L and cross-sectional area $A_1 = \pi r^2$ of conductivity k_1 and a rod of the same length L and cross-sectional area $A_2 = \pi[(2r)^2 - r^2] = 3\pi r^2$ and conductivity k_2 placed one on top of the other (in parallel). In the steady state

$$\frac{Q}{t} = \frac{Q_1}{t} + \frac{Q_2}{t}$$

$$= \frac{k_1 A_1 (T_1 - T_2)}{L} + \frac{k_2 A_2 (T_1 - T_2)}{L}$$

$$\Rightarrow \quad \frac{Q}{t} = \frac{k_1 (\pi r^2)(T_1 - T_2)}{L} + \frac{k_2 (3\pi r^2)(T_1 - T_2)}{L} \qquad \text{(i)}$$

Cross-sectional area of the composite system is $A = \pi(2r)^2 = 4\pi r^2$. If k_{eq} is the equivalent conductivity, then

$$\frac{Q}{t} = \frac{k_{eq} (4\pi r^2)(T_1 - T_2)}{L} \qquad \text{(ii)}$$

From (i) and (ii) we find that

$$4\, k_{eq} = k_1 + 3\, k_2$$

$$\Rightarrow \quad k_{eq} = \frac{k_1 + 3k_2}{4} = \frac{2k + 3k}{4} = \frac{5k}{4}$$

EXAMPLE 19.6

A cylindrical metallic shell has inner radius $r_1 = 2$ cm and outer radius $r_2 = 4$ cm and it has a length $L = 50$ cm. The inner and outer surfaces are maintained at $T_1 = 0°C$ and $T_2 = 100°C$. The thermal conductivity of metal is 69.3 $Wm^{-1}K^{-1}$. Find the rate of flow of heat from the outer to the inner surface.

SOLUTION

Refer to Fig. 19.6.

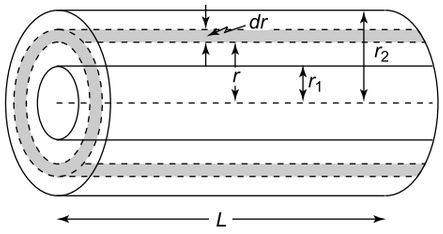


Fig. 19.6

Consider a small element of thickness dr at a distance r from the axis of the shell. Let dT be the temperature between the inner and outer surfaces of the element. If H is the rate of heat flow in the element,

$$H = \frac{k \times 2\pi r L \times dT}{dr}$$

$$\Rightarrow \quad \frac{dr}{r} = \frac{2\pi k L\, dT}{H}$$

Integrating

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k L}{H} \int_{T_1}^{T_2} dT$$

$$\Rightarrow \quad \ln\left(\frac{r_2}{r_1}\right) = \frac{2\pi k L}{H} \times (T_2 - T_1)$$

$$\Rightarrow \quad \ln\left(\frac{4\,\text{cm}}{2\,\text{cm}}\right) = \frac{2 \times 3.14 \times 69.3 \times 0.5}{H} \times (100 - 0)$$

$$0.693 = \frac{2 \times 3.14 \times 69.3 \times 0.5 \times 100}{H}$$

$$\Rightarrow \quad H = 3.14 \times 10^4 \text{ Js}^{-1}$$

19.4 CONVECTION

In *convection heat is transferred by the physical movement of matter* (fluid). No such large-scale movement of matter is involved in conduction where heat energy is transported by molecules via collisions in their *local regions.*

19.5 RADIATION

All bodies emit heat from their surfaces at all temperatures. The heat radiated by a body is called *radiant heat* or *thermal radiation.* Thermal radiations are electromagnetic waves which travel in space with a velocity equal to that of light.

(1) Black Body A perfect black body is one which completely absorbs radiations of every wavelength incident on it. A good absorber of radiations is also a good emitter of radiations. Consequently, a black body, when heated to a suitably high temperature, will emit radiations

of all wavelengths. Such radiation is called *black body radiation.*

(2) Emissive Power The emissive power (e) of a body is the amount of heat energy emitted per second from a unit area of a radiating surface. The SI unit of e is $Js^{-1}\ m^{-2}$ or Wm^{-2}.

(3) Absorptive Power The absorptive power of a body is the ratio of radiant energy absorbed by it to the total amount of radiant energy incident on it. It is denoted by a and is a fraction. Since, by definition, a black body completely absorbs all radiations, $a = 1$ for a black body.

(4) Kirchhoff's Law At any given temperature and for radiations of the same wavelength, the ratio of the emissive power to the absorptive power is the same for all substances, i.e.

$$\frac{e}{a} = \text{constant}$$

(5) Stefan's Law The total energy emitted per second by a unit area of a black body is proportional to the fourth power of its absolute temperature, i.e.

$$E \propto T^4 \quad \text{or} \quad E = \sigma T^4$$

where σ is a constant known as *Stefan's constant.* Its value is

$$\sigma = 5.735 \times 10^{-8}\ \text{Wm}^{-2}\ \text{K}^{-4}$$

When a black body at absolute temperature T is surround by another black body at a lower absolute temperature, T_0, then the net amount of heat energy lost by the body at a higher temperature is given by

$$E = \sigma\,(T^4 - T_0^4)$$

This is known as Stefan-Boltzmann law. If the body is not a perfect black body, then

$$E = \varepsilon\sigma\,(T^4 - T_0^4)$$

where emissivity ε is always less than unity.

Emissivity ε of a body is defined as the ratio of its emissive power to that of the black body.

(6) Newton's Law of Cooling The rate of loss of heat by a body is directly proportional to the excess of its temperature over that of its surroundings, provided this temperature difference is small, i.e.

$$\frac{dQ}{dt} \propto (T - T_0)$$

where T is the temperature of the body and T_0 that of the surroundings. If m is the mass of the body, s its specific heat and dT the change in temperature in time dt, then $dQ = msdT$. Therefore,

$$\frac{dQ}{dt} = ms\,\frac{dT}{dt} = -\,k\,(T - T_0)$$

where $-k$ is a constant of proportionality. Thus

$$\frac{dT}{(T - T_0)} = -\frac{k}{ms}\,dt$$

Integrating between limits T_1 and T_2, we have

$$\left|\log_e (T - T_0)\right|_{T_1}^{T_2} = -\frac{k}{ms}t,$$

$$\therefore \qquad t = K\,\log_e\left(\frac{T_1 - T_0}{T_2 - T_0}\right)$$

where $$K = \frac{ms}{k}$$

This expression gives the time taken by a body to cool from T_1 to T_2 when placed in a medium of temperature T_0.

An approximate formula is

$$\frac{T_1 - T_2}{t} = \frac{1}{K}\left(\frac{T_1 + T_2}{2} - T_0\right)$$

(7) Wien's Displacement Law As the temperature of a black body increases, the maximum intensity of emission shifts (or is displaced) towards shorter wavelengths. In other words,

$$\lambda_m T = b = \text{constant}$$

where λ_m is the wavelength at which maximum emission takes place at absolute temperature T. The value of constant b is

$$b = 2.89 \times 10^{-3}\ \text{mK (metre kelvin)}$$

EXAMPLE 19.7

The surface area of the skin of a person is 2 m^2. He is sitting in a room where the air temperature is 27°C. If his skin temperature is 37°C, calculate the rate at which his body loses heat. The emissivity of his skin is 0.8 and Stefan's constant $\sigma = 5.7 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-1}$.

SOLUTION

$T = 273 + 37 = 310$ K and $T_0 = 273 + 27 = 300$ K

Rate of loss of heat $= \varepsilon A\,\sigma\,(T_4 - T_0^4)$

$= 0.8 \times 2 \times (5.7 \times 10^{-8})\ [(310)^4 - (300)^4]$

$= 1.03 \times 10^2$ W

EXAMPLE 19.8

Experimental investigations show that the intensity of solar radiation is maximum for a wavelength 475.3 nm. Estimate the surface temperature of the sun. Wien's constant $b = 2.89 \times 10^{-3}$ mK.

SOLUTION

According to Wien's law, $\lambda_m T = b$.

$$\therefore \qquad T = \frac{b}{\lambda_m} = \frac{2.89 \times 10^{-3}}{475.3 \times 10^{-9}} = 6080\ \text{K}$$

EXAMPLE 19.9

A body cools from 80°C to 50°C in 6 minutes in a room where the temperature is 20°C. What is the temperature of the body at the end of next 6 minutes?

SOLUTION

Given $\log_e\left(\frac{80-20}{50-20}\right) = 6\ K$

or $\log_e(2) = 6\ K$ (i)

If the temperature at the end of next 6 minutes is T, then

$$\log_e\left(\frac{50-20}{T-20}\right) = 6\ K \quad \text{(ii)}$$

From (i) and (ii)

$$\log_e\left(\frac{30}{T-20}\right) = \log_e(2)$$

$\Rightarrow \quad 30 = 2(T-20)$

$\Rightarrow \quad T = 35°C$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. Two different metal rods of the same length have their ends kept at the same temperatures θ_1 and θ_2 with $\theta_2 > \theta_1$. If A_1 and A_2 are their cross-sectional areas and k_1 and k_2 their thermal conductivities, the rate of flow of heat in the two rods will be the same if

(a) $\frac{A_1}{A_2} = \frac{k_1}{k_2}$ (b) $\frac{A_1}{A_2} = \frac{k_2}{k_1}$

(c) $\frac{A_1}{A_2} = \frac{k_1\theta_1}{k_2\theta_2}$ (d) $\frac{A_1}{A_2} = \frac{k_2\theta_2}{k_1\theta_1}$

2. A cylinder of radius R made of a material of thermal conductivity k, is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity k_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is

(a) $k_1 + k_2$ (b) $\frac{k_1 k_2}{k_1 + k_2}$

(c) $\frac{k_1 + 3k_2}{4}$ (d) $\frac{3k_1 + k_2}{4}$

IIT, 1988

3. Two different metal rods of equal lengths and equal areas of cross-section have their ends kept at the same temperatures θ_1 and θ_2. If k_1 and k_2 are their thermal conductivities, ρ_1 and ρ_2 their densities and s_1 and s_2 their specific heats, then the rate of flow of heat in the two rods will be the same if

(a) $\frac{k_1}{k_2} = \frac{\rho_1 s_1}{\rho_2 s_2}$ (b) $\frac{k_1}{k_2} = \frac{\rho_1 s_2}{\rho_2 s_1}$

(c) $\frac{k_1}{k_2} = \frac{\theta_1}{\theta_2}$ (d) $k_1 = k_2$

4. A slab of stone of area 0.34 m^2 and thickness 10 cm is exposed on the lower face to steam at 100°C. A block of ice at 0°C rests on the upper face of the slab. In one hour, 3.6 kg of ice is melted. Assume that the heat loss from the sides is negligible. The latent heat of fusion of ice is 3.4×10^5 J kg^{-1}. What is the thermal conductivity of the stone in units of $Js^{-1}m^{-1}{}^{\circ}C^{-1}$?

(a) 1.0 (b) 1.5

(c) 2.0 (d) 2.5

5. The tungsten filament of an electric lamp has a surface area A and a power rating P. If the emissivity of the filament is ε and σ is Stefan's constant, the steady temperature of the filament will be

(a) $T = \left(\frac{P}{A\varepsilon\sigma}\right)^2$ (b) $T = \frac{P}{A\varepsilon\sigma}$

(c) $T = \left(\frac{P}{A\varepsilon\sigma}\right)^{1/2}$ (d) $T = \left(\frac{P}{A\varepsilon\sigma}\right)^{1/4}$

6. What are the dimensions of Stefan's constant?

(a) $ML^{-2}T^{-2}K^{-4}$ (b) $ML^{-1}T^{-2}K^{-4}$

(c) $MLT^{-3}K^{-4}$ (d) $ML^0T^{-3}K^{-4}$

7. The amount of energy radiated by a body depends upon

(a) the nature of its surface

(b) the area of its surface

(c) the temperature of its surface
(d) all the above factors

8. The wavelength of the radiation emitted by a body depends upon
(a) the nature of its surface
(b) the area of its surface
(c) the temperature of its surface
(d) all the above factors

9. A composite slab consists of two slabs A and B of different materials but of the same thickness placed one on top of the other. The thermal conductivities of A and B are k_1 and k_2 respectively. A steady temperature difference of 12°C is maintained across the composite slab. If $k_1 = k_2/2$, the temperature difference across slab A will be
(a) 4°C (b) 8°C
(c) 12°C (d) 16°C

10. Two cylindrical rods of lengths l_1 and l_2, radii r_1 and r_2 have thermal conductivities k_1 and k_2 respectively. The ends of the rods are maintained at the same temperature difference. If $l_1 = 2l_2$ and $r_1 = r_2/2$, the rates of heat flow in them will be the same if k_1/k_2 is
(a) 1 (b) 2
(c) 4 (d) 8

11. A solid sphere and a hollow sphere of the same material and size are heated to the same temperature and allowed to cool in the same surroundings. If the temperature difference between the surroundings and each sphere is T, then
(a) the hollow sphere will cool at a faster rate for all values of T
(b) the solid sphere will cool at a faster rate for all values of T
(c) both spheres will cool at the same rate for all values of T
(d) both spheres will cool at the same rate only for small values of T.

12. If the temperature of a black body increases from 7°C to 287°C, then the rate of energy radiation increases by a factor of
(a) $\left(\frac{287}{7}\right)^4$ (b) 16
(c) 4 (d) 2

13. A body cools from 60°C to 50°C in 10 minutes. If the room temperature is 25°C and assuming newton's law of cooling to hold good, the temperature of the body at the end of next 10 minutes will be
(a) 38.5°C (b) 40°C
(c) 42.85°C (d) 45°C

14. Two rods of the same length and material transfer a given amount of heat in 12 seconds when they are joined end to end. But when they are joined lengthswise, they will transfer the same amount of heat, in the same conditions, in
(a) 24 s (b) 3 s
(c) 1.5 s (d) 48 s

15. A body cools from 50.0°C to 49.9°C in 5s. How long will it take to cool from 40.0°C to 39.9°C? Assume the temperature of the surroundings to be 30.0°C and Newton's law of cooling to be valid.
(a) 2.5 s (b) 10 s
(c) 20 s (d) 5 s

16. Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle ABC, right angled at B. The points A and B are maintained at temperatures T and $\sqrt{2}\ T$ respectively. In the steady state, the temperature of point C is T_C. Assuming that only heat conduction takes place, the ratio T_C/T is
(a) $\frac{1}{2(\sqrt{2}-1)}$ (b) $\frac{3}{\sqrt{2}+1}$
(c) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ (d) $\frac{1}{\sqrt{2}+1}$

IIT, 1995

17. Two metallic spheres S_1 and S_2 are made of the same material and have identical surface finish. The mass of S_1 is three times that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is
(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{\sqrt{3}}{1}$ (d) $\left(\frac{1}{3}\right)^{1/3}$

IIT, 1995

18. A spherical black body of radius 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be
(a) 225 (b) 450
(c) 900 (d) 1800

IIT, 1997

19. A black body is at a temperature of 2880 K. The energy of radiation emitted by this body between

wavelengths 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien's constant $b = 2.88 \times 10^6$ nm K. Then

(a) $U_1 = 0$ (b) $U_3 = 0$
(c) $U_1 > U_2$ (d) $U_2 > U_1$

IIT, 1998

20. The plots of intensity versus wavelength of three black bodies at temperatures T_1, T_2 and T_3 respectively are shown in Fig. 19.7. Their temperatures are such that

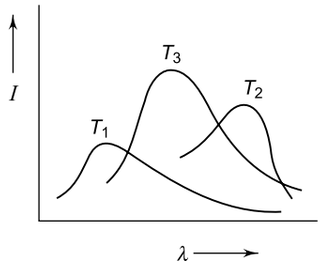


Fig. 19.7

(a) $T_1 > T_2 > T_3$ (b) $T_1 > T_3 > T_2$
(c) $T_2 > T_3 > T_1$ (d) $T_3 > T_2 > T_1$

IIT, 2000

21. When the temperature of a black body increases, it is observed that the wavelength corresponding to maximum energy changes from 0.26 μm to 0.13 μm. The ratio of the emissive powers of the body at the respective temperature is:

(a) $\frac{16}{1}$ (b) $\frac{4}{1}$
(c) $\frac{1}{4}$ (d) $\frac{1}{16}$

22. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously will be

(a) 4 (b) 16
(c) 32 (d) 64

23. The temperatures of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively, are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$, with f equal to (see Fig. 19.8)

(a) 1 (b) 1/2
(c) 2/3 (d) 1/3

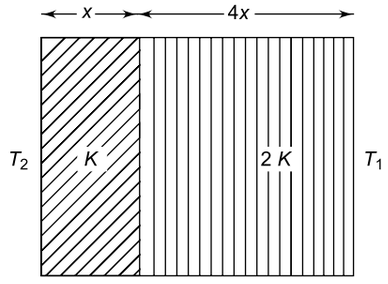


Fig. 19.8

24. Three rods made of the same material and having the same cross-section have been joined as shown in Fig. 19.9. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be

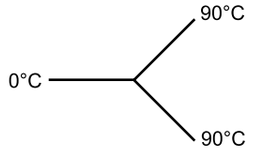


Fig. 19.9

(a) 45°C (b) 60°C
(c) 30°C (d) 20°C

IIT, 2001

25. An ideal black-body at room temperature is thrown into a furnace. It is observed that
(a) initially it is the darkest body and at later times the brightest
(b) it is the darkest body at all times
(c) it cannot be distinguished at all times
(d) initially it is the darkest body and at later times it cannot be distinguished.

IIT, 2002

26. The graph, shown in Fig. 19.10, shows the fall of temperature (T) of two bodies x and y, having the same surface area, with time (t) due to emission of radiation. Find the correct relation between emissive power (E) and absorptive power (a) of the two bodies.

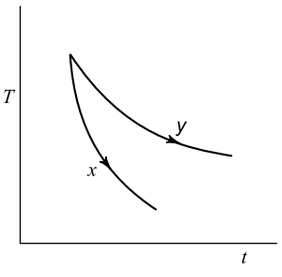


Fig. 19.10

(a) $E_x > E_y$; $a_x < a_y$ (b) $E_x < E_y$; $a_x > a_y$
(c) $E_x > E_y$; $a_x > a_y$ (d) $E_x < E_y$; $a_x < a_y$

IIT, 2003

27. A rod 1 m long and made of material of thermal conductivity 420 W/m/K has one of its ends in melting ice and the other end in boiling water. If its area of cross-section is 10 cm^2, the amount of ice that melts in 1 minute is

(a) 0.125 g (b) 75 g

(c) 7.5 g (d) 450 g

28. A body cools from 75°C to 65°C in 5 minutes in a room where the temperature is 25°C. The temperature of the body at the end of next 5 minutes will be

(a) 55°C (b) 56°C

(c) 57°C (b) 58°C

29. A liquid takes 6 minutes to cool from 80°C to 50°C. If the temperature of the surroundings is 20°C, how long will it take to cool from 60°C to 30°C?

(a) 6 min. (b) 8 min.

(c) 10 min. (d) 12 min.

30. A body initially at 80°C cools to 64°C in 5 minutes and to 52°C in 10 minutes. The temperature of the surroundings is

(a) 15°C (b) 16°C

(c) 20°C (d) 25°C

31. In Q. 30 above, the temperature of the body at the end of 15 minutes will be

(a) 41°C (b) 43°C

(c) 45°C (b) 47°C

32. Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's modulii Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to:

(a) 2 : 3 (b) 1 : 1

(c) 3 : 2 (d) 4 : 9

33. A sphere, a cube and a thin circular plate have the same mass and are made of the same material. All of them are heated to the same temperature. The rate of cooling is

(a) the maximum for the sphere and minimum for the plate.

(b) the maximum for the sphere and minimum for the cube.

(c) the maximum for the plate and minimum for the sphere.

(d) the same for all the three.

34. Experimental investigations show that the intensity of solar radiation is maximum for a wavelength 480 nm in the visible region. Estimate the surface temperature of the sun. Given Wien's constant $b = 2.88 \times 10^{-3}$ mK.

(a) 5000 K (b) 6000 K

(c) 8000 K (b) 10^6 K

35. The surface of the earth receives solar radiation at the rate of 1400 Wm^{-2}. The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7.0×10^8 m. Treating the sun as a black body, it follows from the above data that the surface temperature of the sun is about

(a) 5800 K (b) 5900 K

(c) 6000 K (d) 6100 K

36. In the first experiment, two identical conducting rods are joined one after the other and this combination is connected to two vessels, one containing water at 100°C and the other containing ice at 0°C (see Fig. 19.11). In the second experiment, the two rods are placed one on top of the other and connected to the same vessels. If q_1 and q_2 (in gram per second) are the respective rates of melting of ice in the two cases, then the ratio $\frac{q_1}{q_2}$ is

(a) $\frac{1}{2}$ (b) $\frac{2}{1}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

IIT, 2004

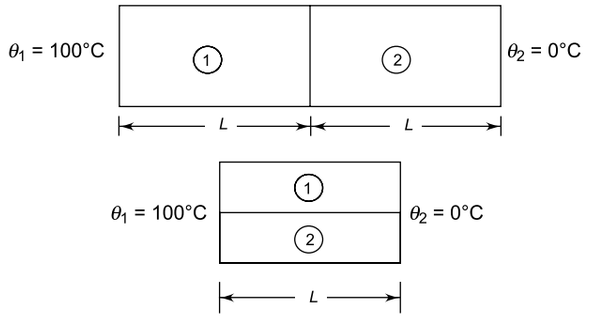


Fig. 19.11

37. Two spheres A and B having radii 3 cm and 5 cm respectively are coated with carbon black on their outer surfaces. The wavelengths of maximum intensity of emission of radiation are 300 nm and 500 nm respectively. The respective powers radiated by them are in the ratio of

(a) $\sqrt{\frac{5}{3}}$ (b) $\frac{5}{3}$

(c) $\left(\frac{5}{3}\right)^2$ (d) $\left(\frac{5}{3}\right)^4$

IIT, 2004

38. In which of the following processes is the heat transfer promarily due to radiation?
(a) Boiling of water
(b) Land and sea breezes
(c) Heating of a metal rod placed over a flame
(d) Heating of the glass surface of an electric bulb due to current in its filament.

IIT, 2005

39. A spherical body of emissivity ε, placed inside a perfectly black body (emissivity = 1), is maintained at absolute temperature T. The energy radiated by a unit area of the body per second will be (σ is Stefan's constant)
(a) σT^4 (b) $\varepsilon\sigma T^4$
(c) $(1 - \varepsilon)\sigma T^4$ (d) $(1 + \varepsilon)\sigma T^4$

IIT, 2005

40. Two spherical balls A and B made of the same material, are heated to the same temperature. They are then placed in identical surroundings. If the diameter of A is twice that of B, the ratio of rates of cooling of A and B will be
(a) 1 : 1 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

41. A cubical ice box is made of thermocole of thickness 5.4 cm. Each side of the box is 30 cm. A lump of ice of mass 500 g is placed in the box. The box is suspended in a room at a temperature of 40 °C. If the coefficient of thermal conductivity of thermocole is 1.0×10^{-2} Wm^{-1} K^{-1} and latent heat of fusion of water is 3.35×10^5 J kg^{-1}, the mass of ice left unmelted in 3 hours is very nearly equal to
(a) 370 g (b) 335 g
(c) 285 g (d) 258 g

42. A layer of ice at 0 °C of thickness x_1 is floating on a pond of water. L, ρ and k respectively are the latent heat of fusion of water, density of ice and thermal conductivity of ice. If the atmospheric temperature is $-T$ °C, the time taken for the thickness of the layer of ice to increase from x_1 to x_2 is given by
(a) $\frac{\rho L}{2kT}(x_1 + x_2)^2$ (b) $\frac{\rho L}{kT}(x_2 - x_1)^2$
(c) $\frac{\rho L}{2kT}\left(x_2^2 - x_1^2\right)$ (d) $\frac{\rho L}{kT}\left(x_2^2 - x_1^2\right)$

43. Two identical rods AB and CD, each of length L, cross-sectional area A and thermal conductivity k are connected as shown in Fig. 19.12. Ends A, C and D are maintained at temperatures T_1 = 20°C, T_2 = 30°C and T_3 = 40°C respectively. The temperature at B is
(a) 32 °C (b) 33 °C
(c) 34 °C (d) 35 °C

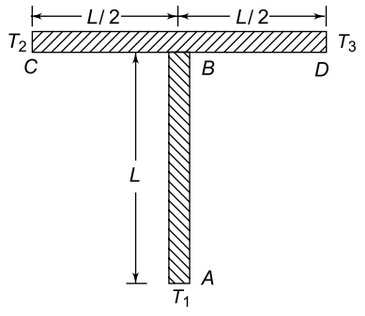


Fig. 19.12

ANSWERS

1. (b)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (d)	**6.** (d)
7. (d)	**8.** (c)	**9.** (b)	**10.** (d)	**11.** (c)	**12.** (b)
13. (c)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (d)	**18.** (d)
19. (d)	**20.** (b)	**21.** (d)	**22.** (d)	**23.** (c)	**24.** (b)
25. (a)	**26.** (c)	**27.** (c)	**28.** (c)	**29.** (d)	**30.** (b)
31. (b)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (a)	**36.** (d)
37. (c)	**38.** (d)	**39.** (b)	**40.** (c)	**41.** (a)	**42.** (c)
43. (a)					

SOLUTION

1. Rate of flow is $\frac{Q}{t} = \frac{kA(\theta_2 - \theta_1)}{l}$

Since $(\theta_2 - \theta_1)$ and l are the same for the two rods, the rate of flow Q/t will be the same if the product kA is the same for the two rods, i.e. if

$$k_1A_1 = k_2A_2 \text{ or } \frac{A_1}{A_2} = \frac{k_2}{k_1}$$

Hence the correct choice is (b).

2. Let the length of the cylinder be l and let its ends be maintained at temperatures θ_1 and θ_2. Area of

the cross-section of the inner cylinder = πR^2. Area of cross-section of outer cylinder = $\pi(2R)^2 - \pi R^2 = 3\pi R^2$.

Rate of flow of heat across inner cylinder is

$$Q_1 = \frac{k_1 \pi R^2 (\theta_1 - \theta_2)}{l} \quad \text{(i)}$$

Rate of flow of heat across the outer shell is

$$Q_2 = \frac{k_2 (3\pi R^2)(\theta_1 - \theta_2)}{l} \quad \text{(ii)}$$

Let the effective thermal conductivity of the compound cylinder be k. The rate of flow of heat across the compound cylinder is

$$Q = \frac{k(4\pi R^2)(\theta_1 - \theta_2)}{l} \quad \text{(iii)}$$

Now $Q = Q_1 + Q_2$ (iv)

Using (i), (ii) and (iii) in (iv) we get $4k = k_1 + 3k_2$

or $$k = \frac{k_1 + 3k_2}{4}$$

Hence the correct choice is (c).

3. Refer to the solution of Q. 1. The correct choice is (d).

4. Amount of heat flowing through the stone in 1 hour = mL, where m is the mass of ice melted and L its latent heat. Now time (t) = 1 hour = 3600 s and thickness d = 10 cm = 0.1 m. The rate of flow of heat is

$$Q = \frac{mL}{t}$$

$\therefore$ Thermal conductivity $k = \dfrac{Qd}{A(\theta_2 - \theta_1)}$

$$= \frac{mLd}{tA(\theta_2 - \theta_1)}$$

$$= \frac{3.6 \times 3.4 \times 10^5 \times 0.1}{3600 \times 0.34 \times (100 - 0)} = 1 \text{ Js}^{-1} \text{ m}^{-1} \text{ °C}^{-1}$$

Hence the correct choice is (a).

5. The energy radiated per second per unit area at temperature T is given

$$= \sigma \varepsilon T^4$$

Thus, the energy radiated per second (or power radiated) from the filament of area A is

$$P = A\ \sigma\ \varepsilon T^4 \text{ or } T = \left(\frac{P}{A\,\sigma\,\varepsilon}\right)^{1/4}$$

Hence the correct choice is (d).

6. The energy emitted per unit area per second by a black body is given by

$$E = \sigma T^4$$

where σ is the Stefan's constant and T is the absolute temperature.

$\therefore$ $$\sigma = \frac{E}{T^4}$$

$\therefore$ Dimensions of σ = $\dfrac{\text{dimensions of energy per unit area per second}}{\text{dimension of } T^4}$

$$= \frac{\text{ML}^2\text{T}^{-2} \times \text{L}^{-2} \times \text{T}^{-1}}{\text{K}^4} = \text{ML}^0\text{T}^{-3}\ \text{K}^{-4}$$

7. The correct choice is (d).

8. The correct choice is (c).

9. Let θ_1 and θ_2 be the temperatures at the two faces of the composite slab and let θ be the temperature at the common face of the slab. If l is the length of each slab and A the area of their face, then, in the steady state, the rate of flow of heat across A = rate of flow of heat across B, i.e.

$$\frac{k_1 A (\theta_1 - \theta)}{l} = \frac{k_2 A (\theta - \theta_2)}{l}$$

or $k_1(\theta_1 - \theta) = k_2(\theta - \theta_2)$

Now $k_2 = 2k_1$. Therefore

$$(\theta_1 - \theta) = 2(\theta - \theta_2) \quad \text{(i)}$$

Also, $\theta_1 - \theta_2 = 12°\text{C}$ or $\theta_2 = \theta_1 - 12$ (ii)

Using (ii) in (i) we have

$$(\theta_1 - \theta) = 2\{\theta - (\theta_1 - 12)\}$$

or $3(\theta_1 - \theta) = 24$ or $\theta_1 - \theta = 8°\text{C}$

Hence the correct choice is (b).

10. The rate of heat flow in rods A and B are

$$\frac{Q_1}{t} = \frac{k_1 \pi r_1^2 \Delta\theta}{l_1} \text{ and } \frac{Q_2}{t} = \frac{k_2 \pi r_2^2 \Delta\theta}{l_2}$$

$Q_1 = Q_2$, if

$$\frac{k_1 r_1^2}{l_1} = \frac{k_2 r_2^2}{l_2} \text{ or } \frac{k_1}{k_2} = \frac{l_1}{l_2} \times \frac{r_2^2}{r_1^2} = 2 \times (2)^2 = 8$$

Hence the correct choice is (d).

11. The rate at which a sphere radiates heat depends upon its material, its surface area and its temperature. Therefore, the solid and the hollow spheres, being of the same material and of the same size and at the same temperature, will cool at the same rate. Hence the correct choice is (c).

12. From Stefan's law, $E = \sigma T^4$. Therefore

$$E_1 = \sigma(273 + 7)^4$$

$$E_2 = \sigma(273 + 287)^4 \ \therefore \ \frac{E_2}{E_1} = \left(\frac{560}{280}\right)^4 = (2)^4 = 16$$

Hence the correct choice is (b).

13. According to Newton's law of cooling, the rate of loss of heat is given by

$$\frac{\Delta Q}{\Delta t} = k(T - T_0)$$

where k is a constant, T is the average temperature in time interval Δt and T_0 is the temperature of the surroundings. If m is the mass of the body and s its specific heat, then (here θ is the temperature at the end of the next 10 minutes)

$$\frac{ms\,(60-50)}{10\text{ min}} = k\left(\frac{50+60}{2} - 25\right) \text{ and}$$

$$\frac{ms\,(50-\theta)}{10\text{ min}} = k\left(\frac{50+\theta}{2} - 25\right)$$

Dividing the two equations and solving, we get $\theta = 42.85°$C. Hence the correct choice is (c).

14. Let Q be the heat transferred. If k is the thermal conductivity of each rod, their equivalent conductivity, when they are joined in series (end to end) is $2k$. If t_1 is time of transfer of heat, then

$$Q_1 = \frac{(2k)\,A\,\Delta\theta\,t_1}{l}$$

If the rods are joined in parallel (lengthwise) the equivalent conductivity is $k/2$.

Hence $$Q_2 = \frac{\left(\frac{k}{2}\right)A\Delta\theta\,t_2}{l}$$

Now $Q_1 = Q_2$ (given). Therefore

$$\frac{2kA\Delta\theta t_1}{l} = \frac{kA\Delta\theta t_2}{2l} \text{ or } t_2 = 4t_1 = 4 \times 12 = 48 \text{ s}$$

Hence the correct choice is (d).

15. According to Newton's law of cooling, the rate of cooling is proportional to the difference between the temperature of the body and that of the surroundings. In the first case, this temperature difference = 50 – 30 = 20° and in the second case the temperature difference = 40 – 30 = 10°. Since the temperature difference in the second case is half that in the first case, the rate of cooling will also be half. Hence the correct choice is (b)

16. Refer to Fig. 19.13. Since $T_B > T_A$, heat flows from B to A and from B to C. In the steady state, rate of flow of heat from B to C = rate of flow of heat from C to A, i.e. $\frac{Q_2}{t} = \frac{Q_3}{t}$

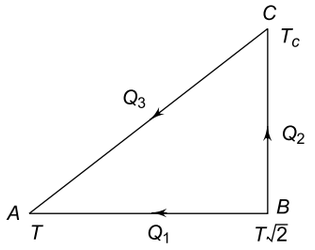


Fig. 19.13

or $\frac{kA(T_B - T_C)}{l} = \frac{kA(T_C - T)}{\sqrt{2}l}$ $(\because AC = \sqrt{2}\ BC)$

which gives $\frac{T_C}{T} = \frac{3}{\sqrt{2}+1}$ $(\because T_B = T\sqrt{2})$

17. The rate of loss of heat is given by

$$ms\ \frac{d\theta}{dt} = (4\ \pi r^2)\ \sigma T^4$$

$\therefore$ Initial rate of cooling is $\frac{d\theta}{dt} \propto \frac{r^2}{ms}$. Since $m = \frac{4}{3}\ \pi r^3 \rho$, $r \propto m^{1/3}$. Hence

$$\frac{d\theta}{dt} \propto \frac{m^{2/3}}{m} \propto \frac{1}{m^{1/3}}$$

Therefore $$\frac{\frac{d\theta}{dt}\text{ for } S_1}{\frac{d\theta}{dt}\text{ for } S_2} = \left(\frac{m \text{ of } S_2}{m \text{ of } S_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

18. Power radiated is given by

$$P = (4\pi r^2)\sigma\ T^4 \text{ or } P \propto r^2\ T^4$$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{1}\right)^4 = 4$$

or $P_2 = 4P_1 = 4 \times 450 = 1800$ W

19. From Wien's displacement law $\lambda_m T = b$, the maximum radiation is emitted at wavelength

$$\lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6 \text{ nmK}}{2880 \text{ K}} = 1000 \text{ nm}$$

Hence U_2 is the maximum. Since a black body emits radiations at all wavelengths, $U_1 \neq 0$ and $U_3 \neq 0$. Hence the correct choice is (d).

20. For black body radiations, $\lambda_m\ T$ = constant. It is clear from the figure that $\lambda_2 > \lambda_3 > \lambda_1$. Hence, it follows that $T_1 > T_3 > T_2$, which is choice (b).

21. According to Wein's displacement law, λT = constant, i.e.

$$\lambda_1\ T_1 = \lambda_2\ T_2 \text{ or } \frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1} = \frac{0.26}{0.13} = 2$$

or $T_2 = 2T_1$.

According to Stefan's law, $E = \sigma T^4$. Thus

$$E_1 = \sigma T_1^4 \text{ and } E_2 = \sigma T_2^4$$

Hence $$\frac{E_1}{E_2} = \frac{\sigma T_1^4}{\sigma T_2^4} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$(\because\ T_2 = 2T_1)$

Thus the correct choice is (d)

22. According to Stefan's law, the total energy emitted per second per unit surface area of a body is given by

$$E = \sigma T^4$$

where σ is the Stefan's constant. Therefore, the radiant energy emitted by spherical body of radius R at absolute temperature T is given by

$$Q' = \sigma \times 4\pi R^2 \times T^4$$

If R and T are both doubled, we have

$$Q = \sigma \times 4\pi (2R)^2 \times (2T)^4$$
$$= 64\ \sigma \times 4\pi R^2 \times T^4$$
$$= 64\ Q$$

Hence the correct choice is (d).

23. Let A be the area of each slab. In the steady state, the rate of heat flow through the composite slab is given by

$$\frac{Q}{t} = \frac{T_2 - T_1}{\frac{l_1}{K_1 A} + \frac{l_2}{K_2 A}} = \frac{A(T_2 - T_1)}{\frac{l_1}{K_1} + \frac{l_2}{K_2}} \qquad (1)$$

Given $l_1 = x$, $l_2 = 4x$, $K_1 = K$ and $K_2 = 2K$. Using these values in (1) we get

$$\frac{Q}{t} = \frac{A(T_2 - T_1)}{\frac{x}{K} + \frac{4x}{2K}} = \left[\frac{A(T_2 - T_1)K}{x}\right] \times \frac{1}{3}$$

Comparing this with the given rate of heat transfer, we get $f = \frac{1}{3}$. Hence the correct choice is (d).

24. Let A and l be the area of cross-section and the length of each rod. If k is the coefficient of thermal conductivity and t°C the temperature of the junction O, then the rates at which heat energy enters O from rods A and B are (Fig. 19.14)

$$Q_A = \frac{kA(90 - t)}{l}$$

and $$Q_B = \frac{kA(90 - t)}{l}$$

The rate at which heat energy flows in rod C is

$$Q_C = \frac{kA(t - 0)}{l}$$

In the steady state, rate at which heat energy enters O = rate at which heat energy leaves O, i.e.

$$Q_A + Q_B = Q_C$$

or $$\frac{kA(90 - t)}{l} + \frac{kA(90 - t)}{l} = \frac{kA(t - 0)}{l}$$

or $$(90 - t) + (90 - t) = t$$

or $$3t = 180 \text{ or } t = 60°\text{C}.$$

Hence the correct choice is (b).

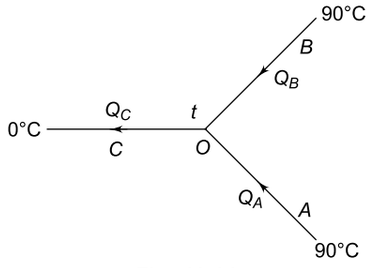


Fig. 19.14

25. The correct choice is (a) because, initially the black body will absorb radiations as the surroundings, i.e. furnace, are at a higher temperature and, therefore, is the blackest. Later it emits the radiations (having become hot) and is, therefore, the brightest.

26. If follows from the figure that the temperature of body x falls more rapidly with time than that of body y. Hence $E_x > E_y$. Also a good emitter of radiation is also a good absorber of radiation (Kirchhoff's law). Hence $a_x > a_y$. Thus the correct choice is (c).

27. Given $k = 420$ W/m/K, $A = 10\ \text{cm}^2 = 10 \times 10^{-4}\ \text{m}^2 = 10^{-3}\ \text{m}^2$, $\theta_1 = 100°\text{C}$, $\theta_2 = 0°\text{C}$, $t = 1$ minute $= 60$ s and $l = 1$ m. Using these values in

$$Q = \frac{kA(\theta_1 - \theta_2)t}{l}$$

we get $$Q = 2520\ \text{J} = \frac{2520}{4.2} = 600\ \text{cal}.$$

Therefore, mass of ice melted is

$$m = \frac{Q}{L} = \frac{600\ \text{cal}}{80\ \text{cal/g}} = 7.5\ \text{g}$$

Hence the correct choice is (c).

28. We have

$$\log_e\left(\frac{T_1 - T_0}{T_2 - T_0}\right) = Kt$$

or $$\log_e\left(\frac{75 - 25}{65 - 25}\right) = K \times 5 \text{ or } \log_e\left(\frac{50}{40}\right) = 5K$$

or $$\log_e\left(\frac{5}{4}\right) = 5K \qquad (1)$$

If the temperature at the end of next 5 minutes is T', we have

$$\log_e\left(\frac{65-25}{T'-25}\right) = 5K$$

or $$\log_e\left(\frac{40}{T'-25}\right) = 5K \quad (2)$$

From Eqs (1) and (2), we get

$$\log_e\left(\frac{40}{T'-25}\right) = \log_e\left(\frac{5}{4}\right)$$

or $$\left(\frac{40}{T'-25}\right) = \frac{5}{4}$$

which gives $T' = 57°C$.

29. Given

$$\log_e\left(\frac{80-20}{50-20}\right) = 6K \text{ or } \log_e(2) = 6K \quad (1)$$

If the body takes t minutes to cool from 60°C to 30°C, then

$$\log_e\left(\frac{60-20}{30-20}\right) = tK \text{ or } \log_e(4) = tK \quad (2)$$

Dividing (2) by (1), we have

$$\frac{t}{6} = \frac{\log_e(4)}{\log_e(2)} = \frac{\log_{10}(4)}{\log_{10}(2)} = \frac{0.602}{0.301} = 2$$

or $t = 12$ minutes.

Hence the correct choice is (d).

30. If T_0 is the temperature of the surrounding, then we have

$$\log_e\left(\frac{80-T_0}{64-T_0}\right) = 5K \quad (1)$$

and $$\log_e\left(\frac{64-T_0}{52-T_0}\right) = 5K \quad (2)$$

Equating Eqs (1) and (2), we get

$$\frac{80-T_0}{64-T_0} = \frac{64-T_0}{52-T_0}$$

which gives $T_0 = 16°C$, which is choice (b).

31. If T is the temperature after 15 minutes, then

$$\log_e\left(\frac{52-T_0}{T-T_0}\right) = 5K$$

or $$\log_e\left(\frac{52-16}{T-16}\right) = 5K \quad (3)$$

From Eqs (1) and (3), we get

$$\frac{80-16}{64-16} = \left(\frac{52-16}{T-16}\right)$$

which gives $T = 43°C$, which is choice (b).

32. Let l be the original length of the rods. Let them be heated so that the increase in temperature is t°C.
Increase in length = $\alpha\, l\, t$
where α is the coefficient of linear expansion of the rod

$$\text{Strain} = \alpha\, l\, t/l = \alpha\, t$$

$$\text{Stress} = Y \times \text{strain} = Y\, \alpha\, t$$

Stresses are equal for the two rods if

$$Y_1\, \alpha_1\, t = Y_2\, \alpha_2\, t$$

or $$\frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$$

Hence the correct choice is (c).

33. Since the material is the same the density is the same. Since the mass is the same and density is the same, all three have the same volume. For the same volume, the surface area of the plate is the largest and of the sphere the smallest. The rate of loss of heat by radiation is proportional to the surface area. Hence the correct choice is (c).

34. According to Wien's law, $\lambda_m T = b$. Hence

$$T = \frac{b}{\lambda_m} = \frac{2.88\times10^{-3}}{480\times10^{-9}} = 6000\text{K}$$

Hence the correct choice is (b).

35. Let R be the radius of the sun and let r be the radius of the earth's orbit round the sun. If the surface temperature of the sun is T (in kelvin), the energy emitted per second by the surface of the sun = $4\pi R^2\, \sigma\, T^4$, where σ is Stefan's constant whose value is 5.67×10^{-8} Wm^{-2} K^{-4}. Now the area of the spherical surface of radius r is $4\pi r^2$. Therefore, energy received per second by a unit area of the earth's surface is

$$\frac{4\pi R^2\sigma T^4}{4\pi r^2} = \frac{R^2\sigma T^4}{r^2} = 1400$$

which gives $$T^4 = \frac{1400\, r^2}{\sigma R^2}$$

$$= \frac{1400\times\left(1.5\times10^{11}\right)^2}{\left(5.67\times10^{-8}\right)\times\left(7\times10^{8}\right)^2}$$

$$= 1.1338 \times 10^{15}$$

or $$T = 5803 \text{ K}$$

Hence the correct choice is (a).

36. If a steady temperature difference $(\theta_1 - \theta_2)$ is maintained between the ends of a conducting rod of length L and cross-sectional area A, the rate of flow of heat through the rod is given by

$$q = \frac{kA(\theta_1 - \theta_2)}{L}$$

where k is the coefficient of thermal conductivity of the material of the rod. In the first experiment, the two rods are connected in series. If two rods of equal cross-sectional areas and of lengths L_1 and L_2 and conductivities k_1 and k_2 are joined in series, the equivalent conductivity k_s is given by

$$\frac{L_1 + L_2}{k_s} = \frac{L_1}{k_1} + \frac{L_2}{k_2} \quad (1)$$

For two identical rods, $L_1 = L_2 = L$ and $k_1 = k_2 = k$, in which case, Eq. (1) gives $k_s = k$. Further, when two identical rods are joined in series, the length of the composite rod is $(2L)$ but its cross-sectional area is A, the same as that of each rod. Hence the rate of flow of heat in this case is given by

$$q_1 = \frac{kA(\theta_1 - \theta_2)}{(2L)} \quad (2)$$

In the second case, the two rods are connected in parallel. If two rods of equal lengths and equal cross-section areas and having conductivities k_1 and k_2 are joined in parallel, the equivalent conductivity of the composite rod is given by

$$k_p = k_1 + k_2$$

For two identical rods, $k_1 = k_2 = k$. Hence $k_p = (2k)$. Furthermore, the cross-sectional area of the composite rod is $(2A)$. Therefore, in the second case, the rate of flow of heat is given by

$$q_2 = \frac{(2k)(2A)(\theta_1 - \theta_2)}{L} \quad (3)$$

Dividing (2) by (3), we get $\frac{q_1}{q_2} = \frac{1}{8}$. Now, the rate of melting of ice is proportional to the rate of flow of heat. Hence the correct choice is (d).

37. According to Stefan's law, the power radiated by a black body at absolute temperature T is given by

$$Q = \sigma A T^4 \quad (1)$$

where A is the surface area of the body and σ is Stefan's constant. According to Wien's displacement law,

$$\lambda_m T = b$$

where λ_m is the wavelength corresponding to maximum emission of radiation and b is Wien's constant. Thus

$$T = \frac{b}{\lambda_m} \quad (2)$$

Using (2) in (1), we get

$$Q = \sigma A \left(\frac{b}{\lambda_m}\right)^4 = \frac{\sigma b^4 A}{\lambda_m^4}$$

For a sphere of radius r, $A = 4\pi r^2$. Hence

$$Q = \frac{\sigma b^4 4\pi r^2}{\lambda_m^4} = k\frac{r^2}{\lambda_m^4} \quad (3)$$

where $k = 4\pi\sigma b^4$ is a constant. Hence

$$Q_1 = k\frac{r_1^2}{\left(\lambda_m^4\right)_1}$$

and $$Q_2 = k\frac{r_2^2}{\left(\lambda_m^4\right)_2}$$

$$\therefore \quad \frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{\left(\lambda_m^4\right)_2}{\left(\lambda_m^4\right)_1}$$

$$= \left(\frac{3\,\text{cm}}{5\,\text{cm}}\right)^2 \times \left(\frac{500\,\text{nm}}{300\,\text{nm}}\right)^4 = \left(\frac{5}{3}\right)^2$$

Hence the correct choice is (c).

38. The correct choice is (d). Heat transfer in boiling of water and in land and sea-breezes is primarily due to convection. A metal rod placed over a flame is heated primarily by conduction. Heat transfer by convection and conduction takes place through matter (or medium). The electric bulb is evacuated. Hence heat transfer in an electric bulb is not due to conduction or convection.

39. The correct choice is (b). According to Stefan's law, the energy radiated per second by a unit area of a body of emissivity ε is $\varepsilon\sigma T^4$, irrespective of the surroundings.

40. The correct choice is (c). The rate of cooling is proportional to the surface area.

41. Length of each side of the cubical box = 30 cm = 0.30 m. Since a cube has 6 faces, the total surface area of the cube exposed to air is $A = 6 \times (0.3)^2 = 0.54\ \text{m}^2$.

Thickness of thermacole $(d) = 5.4\ \text{cm}$

$$= 5.4 \times 10^{-2}\ \text{m}$$

Time of exposure $(t) = 3\ \text{hours} = 3 \times 60 \times 60$

$$= 1.08 \times 10^4\ \text{s}$$

Thermal conductivity $(k) = 1.0 \times 10^{-2}\ \text{J s}^{-1}\text{m}^{-1}\text{°C}^{-1}$

Temperature of air $(T_a) = 40°\text{C}$

Temperature of ice $(T_i) = 0°\text{C}$

Amount of heat energy entering the box in time t is

$$Q = \frac{kA(T_a - T_i)t}{d}$$

$$= \frac{1.0\times10^{-2}\times0.54\times(40-0)\times1.08\times10^{4}}{5.4\times10^{-2}}$$

$$= 4.32 \times 10^{4} \text{ J}$$

Now, heat of fusion of water, $L = 3.35 \times 10^2$ J g^{-1} $= 3.35 \times 10^5$ J kg^{-1}. This means that 3.35×10^6 J of heat energy is needed to melt 1 kg of ice into water. Therefore, mass of ice melted in 3 hours is

$$m = \frac{Q}{L} = \frac{4.32\times10^{4}}{3.35\times10^{5}} = 0.129 \text{ kg} = 129 \text{ g}$$

$\therefore$ Mass of ice left unmelted = 500 – 129 = 371 g.

Hence the correct choice is (a).

42. When the temperature of the air is less than 0°C, the cold air near the surface of the pond takes heat (latent) from the water which freezes in the form of layers. Fig. 19.15. Consequently, the thickness of the ice layer keeps increasing with time. Let x be the thickness of the ice layer at a certain time. If the thickness increases by dx in time dt, then the amount of heat flowing through the slab in time dt is given by (see Fig 19.15)

$$Q = \frac{kA[0-(-T)]dt}{x} = \frac{kATdt}{x} \qquad (1)$$

where A is the area of the layer of ice and $-T$ °C is the temperature of the surrounding air. If dm is the mass of water frozen into ice, then $Q = dm \times L$. But $dm = A\rho dx$, where ρ is the density of ice. Hence

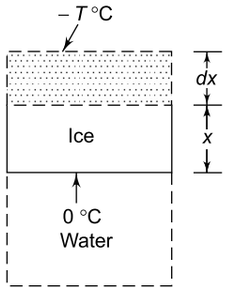


Fig. 19.15

$$Q = A\rho L dx \qquad (2)$$

Equating (1) and (2), we have

$$\frac{kATdt}{x} = A\rho L dx \text{ or } dt = \frac{\rho L}{kT} \cdot x dx$$

Integrating, we have

$$\int_0^t dt = \frac{\rho L}{kT}\int_{x_1}^{x_2} x dx$$

or $$t = \frac{\rho L}{kT}\left|\frac{x^2}{2}\right|_{x_1}^{x_2} = \frac{\rho L}{2kT}(x_2^2 - x^2{}_1),$$

which is choice (c).

43. Let T_B be the temperature at B. The rate of flow of heat from C towards B is

$$\frac{Q_1}{t} = \frac{k\,A\,(T_2 - T_B)}{L/2}$$

The rate of flow of heat from D towards B is

$$\frac{Q_2}{t} = \frac{k\,A\,(T_3 - T_B)}{L/2}$$

The rate of flow of heat from B towards A is

$$\frac{Q_3}{t} = \frac{k\,A\,(T_B - T_1)}{L}$$

In the steady state, the rate at which heat enters B = rate at which heat leaves B, i.e.

$$\frac{Q_1}{t} + \frac{Q_2}{t} = \frac{Q_3}{t}$$

or $$\frac{2k\,A\,(T_2 - T_B)}{L} + \frac{2k\,A\,(T_3 - T_B)}{L} = \frac{k\,A\,(T_B - T_1)}{L}$$

or $2(T_2 - T_B) + 2(T_3 - T_B) = T_B - T_1$

which gives $T_B = \frac{T_1 + 2T_2 + 2T_3}{5}$

$$= \frac{20 + 2\times30 + 2\times40}{5}$$

= 32 °C, which is choice (a).

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Choose the correct statements from the following

(a) All bodies emit thermal radiations at all temperatures

(b) Thermal radiations are electromagnetic waves

(c) Thermal radiations are not reflected from a mirror

(d) Thermal radiations travel in free space with a velocity of 3×10^8 ms^{-1}

2. The rate at which energy is radiated by a hot body depends upon
(a) the nature of its surface
(b) the area of its surface
(c) the temperature of its surface
(d) the temperature of the surroundings

3. The wavelength of the radiation emitted by a body does not depend upon
(a) the density of the body
(b) the nature of its surface
(c) the area of its surface
(d) the temperature of its surface

4. A star appears red if the wavelength of maximum emission is in the range 620 nm to 780 nm. The corresponding temperatures of the star for maximum emission are T_1 and T_2 respectively. If Wien's constant is $b = 2.9 \times 10^{-3}$ mK, then
(a) $T_1 = 3718$ K (b) $T_1 = 2690$ K
(c) $T_2 = 2318$ K (d) $T_2 = 4677$ K

5. Two plates A and B of equal surface area are placed one on top of the other to form a composite plate of the same surface area. The thickness of A and B are 4.0 cm and 6.0 cm respectively. The temperature of the exposed surface of plate A is – 10 °C and that of the exposed surface of plate B is 10 °C. Neglect heat loss from the edges of the composite plate, the temperature of the contact surface is T_1 if the plates A and B are made of the same material and T_2 if their thermal conductivities are in the ratio 2 : 3 then
(a) $T_1 = -4$°C (b) $T_1 = -2$°C
(c) $T_2 = -3$°C (d) $T_2 = 0$°C

6. Initially a black body at absolute temperature T is kept inside a closed chamber at absolute temperature T_0. Now the chamber is slightly opened to allow sun rays to enter. It is observed that temperatures T and T_0 remains constant. Which of the following statement is/are true?
(a) The rate of emission of energy from the black body remains the same.
(b) The rate of emission of energy from the black body increases.
(c) The rate of absorption of energy by the black body increases.
(d) The energy radiated by the black body equals the energy absorbed by it.

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7. A composite block is made of slabs A, B, C, D and E of different thermal conductivites (given in terms of a constant K) and sizes (given in terms of length, L) as shown in Fig. 19.16. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state
(a) heat flow through A and E slabs are same.
(b) heat flow through slab E is maximum.
(c) temperature difference across slab E is smallest.
(d) heat flow through C = heat flow through B + heat flow through D.

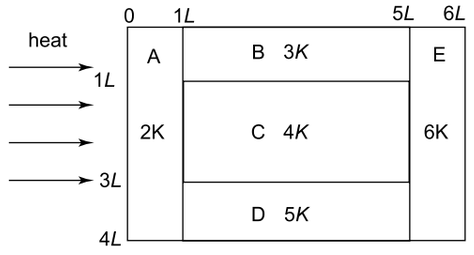

Fig. 19.16

8. Initially a black body at absolute temperature T is kept inside a closed chamber at absolute temperature T_0. Now the chamber is slightly opened to allow sun rays to enter. It is observed that temperatures T and T_0 remains constant. Which of the following statement is/are true?
(a) The rate of emission of energy from the black body remains the same.
(b) The rate of emission of energy from the black body increases.
(c) The rate of absorption of energy by the black body increases.
(d) The energy radiated by the black body equals the energy absorbed by it.

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ANSWERS AND SOLUTIONS

1. Statement (c) is incorrect. Thermal radiations are reflected from a mirror exactly as visible light.

2. All the four choices are correct.

3. From Wien's law, $\lambda_m T = b$, where b is a constant and λ_m is the wavelength at maximum intensity of emission. The value of λ_m depends only on T, the temperature of the body.

4. From Wien's law, $\lambda_{max}\, T = b$. Thus

$$T_1 = \frac{2.9\times10^{-3}}{7.8\times10^{-7}} = 3718 \text{ K}$$

$$T_2 = \frac{2.9\times10^{-3}}{6.2\times10^{-7}} = 4677 \text{ K}$$

So the correct choices are (a) and (d).

5. Let T be the temperature of the contact surface. Then, in the steady state, we have

$$\frac{Q}{t} = \frac{k_A A(-10-T)}{4.0} = \frac{k_B A(T-10)}{6.0}$$

or $$\frac{k_A(-10-T)}{4} = \frac{k_B(T-10)}{6} \qquad (1)$$

(a) If $k_A = k_B$, Eq. (1) gives $T_1 = -2°C$.

(b) If $\frac{k_A}{k_B} = \frac{2}{3}$, Eq. (1) gives $T_2 = 0°C$.

Thus the correct choices are (b) and (d).

6. From Stefan's law, the rate of emission of energy from a black body is proportional to $(T - T_0)^4$. Since T and T_0 remain constant, the rate of emission of energy remains the same. Since the temperature of the black body remains constant, the energy radiated by it = energy absorbed by it. Hence the correct choices are (a) and (d).

7. Let W be the width of each slab. The thermal resistance of a slab of length L, area A $(= LW)$ and thermal conductivity K is given by

$$R = \frac{L}{KA} = \frac{L}{K(LW)}$$

The thermal resistances of slabs A, B, C, D and E respectively are

$$R_A = \frac{L}{(2K)(4LW)} = \frac{R}{8}$$

where $$R = \frac{L}{K(LW)} = \frac{1}{KW}$$

$$R_B = \frac{4L}{3K(LW)} = \frac{4R}{3}$$

$$R_C = \frac{4L}{4K(2LW)} = \frac{R}{2}$$

$$R_D = \frac{4L}{5K(LW)} = \frac{4R}{5}$$

$$R_E = \frac{L}{6K(4LW)} = \frac{R}{24}$$

Since slabs B, C and D are in parallel and slabs A and E are in series with this parallel combination, the thermal resistances are connected as shown in the following figure.

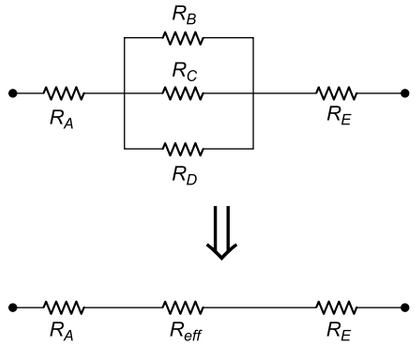


Fig. 19.17

R_{eff} is given by

$$\frac{1}{R_{eff}} = \frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_D}$$

$$= \frac{3}{4R} + \frac{2}{R} + \frac{5}{4R} = \frac{4}{R}$$

$$\Rightarrow \qquad R_{eff} = \frac{R}{4}$$

In a series combination, the rate of flow of heat in the steady is the same. Hence $Q_A = (Q)_{eff} = Q_E$. So choice (a) is correct. The rate of flow of heat is given by $Q = \frac{\Delta T}{R}$, where ΔT is the temperature difference between the ends of the slab. Also the temperature difference between the ends of slabs B, C and D is the same $= (\Delta T)_{eff}$ (say). Then

$$(\Delta T)_A = Q_A R_A = \frac{Q_A R}{8}$$

$$(\Delta T)_B = (\Delta T)_C = (\Delta T)_D = Q_A R_{eff} = \frac{Q_A R}{4}$$

and $$(\Delta T)_E = \frac{Q_A R}{24} \qquad (\because\ Q_E = Q_A)$$

Hence the temperature difference across slab E is the minimum.

Now $$Q_B = \frac{(\Delta T)_B}{R_B} = \frac{Q_A R/4}{4R/3} = \frac{3}{16}Q_A$$

$$Q_C = \frac{(\Delta T)_C}{R_C} = \frac{Q_A R/4}{R/2} = \frac{1}{2}Q_A$$

$$Q_D = \frac{(\Delta T)_D}{R_D} = \frac{Q_A R/4}{4R/5} = \frac{5}{16}Q_A$$

And $Q_E = Q_A$. Thus heat flow through slab B is the smallest. So choice (b) is wrong.

Now $Q_B + Q_D = \frac{3}{16}Q_A + \frac{5}{16}Q_A = \frac{1}{2}Q_A$ which is equal to Q_C. So choice (d) is correct. Hence the correct choices are (a), (c) and (d).

8. From Stefan's law, the rate of emission of energy from a black body is proportional to $(T - T_0)^4$. Since T and T_0 remain constant, the rate of emission of energy remains the same. Since the temperature of the black body remains constant, the energy radiated by it = energy absorbed by it. Hence the correct choices are (a) and (d).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 6 are based on the following passage

Passage I

Thermal Radiations

All bodies emit heat energy from their surfaces by virtue of their temperature. This heat energy is called radiant energy or thermal radiation. The heat that we receive from the sun is transferred to us by a process which, unlike conduction or convection, does not require the help of a medium in the intervening space which is almost free of particles. Radiant energy travels in space as electromagnetic waves in the infra-red region of the electromagnetic spectrum. Thermal radiations travel through vacuum with the speed of light. Thermal radiations obey the same laws of reflection and refraction as light does. They exhibit the phenomena of interference, diffraction and polarization as light does.

The emission of radiation from a hot body is expressed in terms of that emitted from a reference body (called the black body) at the same temperature. A black body absorbs and hence emits radiations of all wavelengths. The total energy E emitted by a unit area of a black body per second is given by

$$E = \sigma T^4$$

where T is the absolute temperature of the body and σ is a constant known as Stefan's constant. If the body is not a perfect black body, then

$$E = \varepsilon \sigma T^4$$

where ε is the emissivity of the body.

1. From stefan-Boltzman law, the dimensions of Stefan's constant σ are
(a) $ML^{-2}T^{-2}K^{-4}$ (b) $ML^{-1}T^{-2}K^{-4}$
(c) $MLT^{-3}K^{-4}$ (d) $ML^0T^{-3}K^{-4}$

2. What is the SI unit of Stefan's constant?
(a) $J\ s^{-1}K^{-4}$ (b) $W\ m^{-1}K^{-4}$
(c) $W\ m^{-2}K^{-4}$ (d) $J\ m^{-2}K^{-4}$

3. In which region of the electromagnetic spectrum do thermal radiations lie?
(a) Visible region (b) Infrared region
(c) Ultraviolet region (d) Microwave region

4. Which of the following devices is used to detect thermal radiations?
(a) Constant volume air thermometer
(b) Platinum resistance thermometer
(c) Thermostat
(d) Thermopile

5. When a body A at a higher temperature T_1 is surrounded by another body B at a lower temperature T_2, then the rate of loss of heat from body A will be proportional to
(a) T_1^4 (b) $(T_1 - T_2)^4$
(c) $(T_1 - T_2)$ (d) $(T_1^4 - T_2^4)$

6. The rate at which energy is radiated by a body depends upon
(a) the surface area of the body
(b) the temperature of the body
(c) the nature of the surface of the body
(d) the emissivity of the surface of the body

ANSWERS AND SOLUTIONS

1. Stefan-Boltzmann law states that $E = \sigma T^4$ where E stands for the total energy emitted per unit area per second. Thus the dimensions of E = dimensions of $\left(\frac{\text{energy}}{\text{area} \times \text{time}}\right) = \frac{ML^2T^{-2}}{L^2T} = MT^{-3}$. Therefore,

Dimensions of $\sigma = \frac{ML^{-3}}{K^4} = MT^{-3}K^{-4}$, which is choice (d).

2. The correct choice is (c).

3. The correct choice is (b).

4. The correct choice is (d).

5. The correct choice is (d).

6. All the four choices are correct.

Questions 7 to 13 are based on the following passage

Passage II

Stellar Spectra

Like the solar spectrum, the spectra of stars show a continuous spectrum on which dark aborption lines are superimposed. The inner layer (called the *photosphere*) of the star emits radiations of all wavelenghts, producing a continuous spectrum. When these radiations pass through the outer, relatively cooler, layer of the star, the radiations of certain wavelengths are selectively absorbed by this layer. This explains the dark lines in the spectrum of a star. The dark lines are characreristic of the substances present in the outer layer of the star.

The surface temperature T of a star can be estimated by measuring the wavelengths λ_m at which the intensity of the emitted radiation is maximum and then using Wien's displacement law which states that

$$\lambda_m \times T = b$$

where b is a constant called Wien's constant and the above relation is called Wien's Displacement Law which states that as the temperature increases, the maximum intensity of emission shifts (or is displaced) towards the shorter wavelengths. The value of constant b has been found experimentally to be 2.89×10^{-3} mK.

7. The spectrum of light received from a star is a
   (a) continuous emission spectrum
   (b) emission line spectrum
   (c) emission band spectrum
   (d) absorption line spectrum

8. The dark lines in the solar spectrum are due to the
   (a) absence of corresponding wavelengths from the light emitted by the core of the sun
   (b) absorption of corresponding wavelenghts by the outer layers of the sun
   (c) absorption of corresponding wavelengths by the prism used in the spectrograph
   (d) destructive interference between waves of certain definite wavelengths

9. The study of dark lines in the spectra of stars has revealed that the atmospheres of stars contain
   (a) oxygen (b) nitrogen
   (c) uranium (d) helium

10. The colour of a star depends upon its
   (a) surface temperature
   (b) mass
   (c) size
   (d) all the above factors

11. Wien's displacement law tells us that an extremely hot star should look
   (a) violet or indigo (b) green or yellow
   (c) orange or red (d) white

12. In Wien's displacement law the SI unit of Wien's constant b is
   (a) metre per kelvin
   (b) metre per kelvin squared
   (c) metre kelvin
   (d) metre kelvin squared

13. Which one of the curves shown in Fig. 19.18 represents the spectral distribution of energy E_λ of black body radiations where λ is the wavelength?

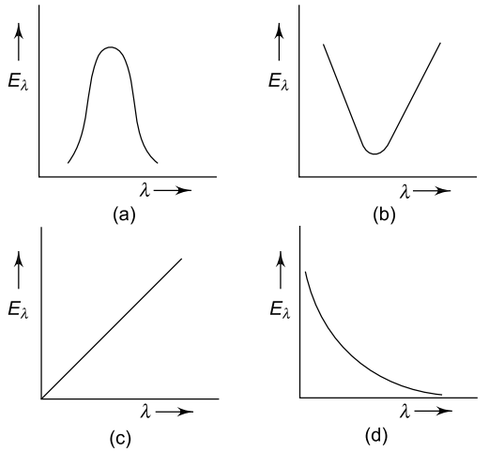


Fig. 19.18

SOLUTION

7. The correct choice is (d)

8. The correct choice is (b)

9. The correct choice is (d)

10. The correct choice is (a)

11. According to Wien's law $\lambda_m T$ = constant, if T is very high, λ_m will be very small. The shortest wavelengths in the visible region are violet and indigo. Hence the correct choice is (a).

12. From $\lambda_m T = b$, the SI unit of b = SI unit of $\lambda_m \times$ SI unit of T = metre kelvin, which is choice (c).

13. The correct choice is (a)

Questions 14 to 16 are based on the following passage

Passage III

Three cylindrical rods A, B and C of equal lengths and equal diameters are joined in series as shown in the following figure. Their thermal conductivities are 2 k, k and 0.5 k respectively (Fig. 19.19)

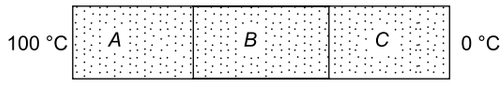


Fig. 19.19

In the steady state, the free ends of rods A and C are at 100°C and 0°C respectively. Neglect loss of heat from the curved surfaces of rods.

14. The temperature of the junction between rods *A* and *B* is
(a) 55.7°C (b) 65.7°C
(c) 75.7°C (d) 85.7°C

15. The temperature of the junction between rods *B* and *C* is
(a) 57.1°C (b) 47.1°C
(c) 37.1°C (d) 27.1°C

16. The equivalent thermal conductivity of the combination is
(a) $\frac{7k}{3}$ (b) $\frac{2k}{7}$
(c) $\frac{5k}{3}$ (d) $\frac{3k}{5}$

SOLUTION

14. In the steady state, the rate of flow of heat is the same for all rods. If T_1 and T_2 are the temperatures at the junction points between *A* and *B* and between and C respectively, then

$$\frac{Q}{t} = \frac{k_A A(100 - T_1)}{d}$$

$$= \frac{k_B A(T_1 - T_2)}{d} = \frac{k_C A(T_2 - 0)}{d}$$

Given $k_A = 2k$, $k_B = k$ and $k_C = 0.5k$. Hence

$$2(100 - T_1) = (T_1 - T_2) = 0.5(T_2 - 0)$$

which give $200 - 2T_1 = T_1 - T_2$ (1)

and $T_1 - T_2 = 0.5\ T_2$ (2)

Equations (1) and (2) give $T_1 = 85.7°C$. So the correct choice is (d).

15. Putting $T_1 = 85.7°C$ in Eq. (1) or (2), we get $T_2 = 57.1°C$, which is choice (a).

16. Since the rods have the same lengths and the same diameters, the equivalent thermal conductivity of the series combination is given by

$$\frac{1}{k_e} = \frac{1}{k_A} + \frac{1}{k_B} + \frac{1}{k_C} = \frac{1}{2k} + \frac{1}{k} + \frac{1}{0.5k}$$

which gives $k_e = \frac{2k}{7}$. So the correct choice is (b).

Questions 17 to 19 are based on the following passage

Passage IV

A double-pane window used for insulating a room thermally from outside consists of two glass sheets each of area 1 m^2 and thickness 0.01 m separated by 0.05 m thick stagnant air space. In the steady state, the room-glass interface and the glass-outdoor interface are at constant temperatures of 27°C and 0°C respectively. The thermal conductivity of glass is 0.8 and of air 0.08 W m^{-1} K^{-1}.

IIT, 1997

17. The temperature of the outer glass-air interface is
(a) 26.5°C (b) 25.5°C
(c) 24.5°C (d) 23.5°C

18. The temperatures of the inner glass-air interface is
(a) 2.5°C (b) 2.0°C
(c) 1.5°C (d) 0.5°C

19. The rate of flow of heat through the window pane is nearly equal to
(a) 1000 J s^{-1} (b) 2000 J s^{-1}
(c) 3000 J s^{-1} (d) 4000 J s^{-1}

SOLUTION

17. Refer to Fig.19.20. Let T_2 and T_3 be the temperature of the Glass 1-air interface and air-glass 2 interface.

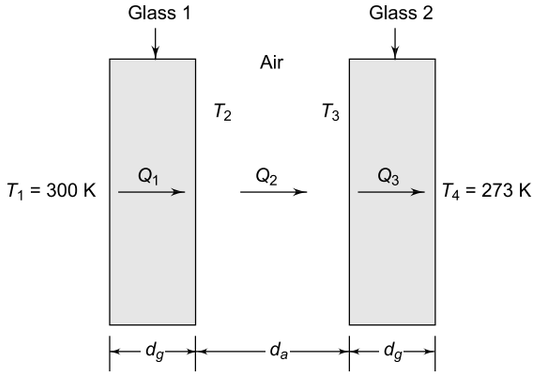


Fig. 19.20

The equsations of heat flow are:

$$\frac{dQ_1}{dt} = \frac{k_g A(T_1 - T_2)}{d_g} = \frac{0.8 \times 1 \times (300 - T_2)}{0.01} \quad (1)$$

$$\frac{dQ_2}{dt} = \frac{k_a A(T_2 - T_3)}{d_a} = \frac{0.08 \times 1 \times (T_2 - T_3)}{0.05} \quad (2)$$

$$\frac{dQ_3}{dt} = \frac{k_g A(T_3 - T_4)}{d_g} = \frac{0.8 \times 1(T_3 - 273)}{0.01} \quad (3)$$

In the steady,

$$\frac{dQ_1}{dt} = \frac{dQ_2}{dt} = \frac{dQ_3}{dt}$$

Equating (1) and (2), we have

$$50 \times (300 - T_2) = T_2 - T_3 \quad (4)$$

Equating (1) and (3), we have

$$(300 - T_2) = T_3 - 273 \qquad (5)$$

Equations (4) and (5) give $T_3 \cong 299.5$ K $\simeq 26.5°$C. So the correct choice is (a).

18. Putting $T_3 = 299.5$ K in Eq. (4) or (5), we get $T_2 = 273.5$ K $= 0.5°$C, which is choice (d).

19. Putting $T_2 = 273.5$ K and $T_3 = 299.5$ K in Eq. (1), we get

$\frac{dQ_1}{dt} \simeq 2000$ J s^{-1}, which is choice (b).

SECTION IV

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1is false, Statement-2 is true.

1. Statement 1
Good reflectors are poor emitters of thermal radiation.
Statement 2
The ratio of the emissive power (e) and absorptive power (a) is constant for all substances at any given temperature and for radiation of the same wavelength.

2. Statement 1
On a chilly day, the metallic cap of a pen feels much colder than the plastic body although both are at the same (room) temperature.
Statement 2
Metal has a higher coefficient of thermal conductivity than plastic.

3. Statement 1
If the earth did not have an atmosphere, it would become extremely cold.
Statement 2
Heat energy is transferred through air mainly by convection.

4. Statement 1
Evaporation takes place from the surface of a liquid.
Statement 2
The molecules at the surface have less attractive energy than those inside the liquid.

5. Statement 1
Radiation involves transfer of heat by electromagnetic waves.
Statement 2
Electromagnetic waves do not required any material medium for propagation.

6. Statement 1
Two spheres of the same material have radii 1 m and 4 m and termperatures 4000 K and 1000 K respectively. The energy radiated per second by the two spheres will be the same.
Statement 2
The rate at which energy is radiated from a body is directly proportional to its absolute temperature and inversely proportional to its surface area.

IIT, 1988

SOLUTION

1. The correct choice is (a). According to Kirchhoff's law,

$$\frac{e}{a} = \text{constant}$$

Thus if e is large, a must also be large, i.e. if a body is a good emitter of a radiation of a particular wavelength, it is also a food absorber of that radiation. Conversely, if a body is a poor emitter of a radiation, it is also a poor absorber (and hence a good reflector) of that radiation.

2. The correct choice is (a). On a chilly day, the room temperature is lower than our body temperature. Since metal is a better conductor of heat than plastic, when we touch the metal cap and the plastic

body of a pen, heat from our fingers will flow to the metal cap much more quickly than to the plastic body.

3. The correct choice is (c). Thermel radiation from the sun warms the earth during the day. Since air is a poor conductor of heat, the atmosphere acts as a blanket for the earth and keeps the earth warn during the night.
4. The correct choice is (a).
5. The correct choice is (b).
6. From Stefan's law, the energy radiated per second from a sphere is given by
   $E = \sigma T^4 A = \sigma T^4 \times 4\pi R^2$; R = radius of sphere
   Thus statement-1 is true but statement-2 is false.

SECTION V

Integer Answer Type

1. A composite rod is made by joining a copper rod, end to end, with a second rod of a different material but of the same cross-section. At 25°C, the composite rod is 1 m in length of which the length of the copper rod is 30 cm. At 125°C the length of the composite rod increases by 1.91 mm. The coefficient of linear expansion of copper is $\alpha = 1.7 \times 10^{-5}$ per °C and that of the second rod is $\beta = n \times 10^{-5}$ per °C. Find the value of n.

   IIT, 1979

2. A wall has two layers A and B each made of different material. Both layers have the same thickness. The thermal conductivity A is 3 times that of B. In the steady state, the temperature difference across the wall is 36°C. Find the temperature difference (in °C) across the layer A.

   IIT, 1980

3. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. If they emit total radiant energy at the same rate, find their temperature ratio T_A/T_B.

   IIT, 1994

SOLUTIONS

1. Length of the second rod at 25°C = 70 cm.
   Length of copper rod at 125°C
   $= 30 \times (1 + 1.7 \times 10^{-5} \times 100)$
   $= 30.051$ cm
   $\therefore$ Increase in the length of copper rod = 0.051 cm
   Increase in the length of second rod = $70 \times \beta \times 100 = 7000\,\beta$ cm
   Total increase in length = 0.051 cm + $7000\,\beta$ cm = 0.191 cm (given) which gives $\beta = 2 \times 10^{-5}$ per °C.
   Thus the value of $n = 2$.
2. In the steady state, the rate of flow of heat through layers A and B is the same, i.e. (a = area of cross-section of each layer) [Fig. 19.21]
   $$\frac{k_A\, a(T_1 - T_0)}{x} = \frac{k_B\, a(T_0 - T_2)}{x}$$
   Putting $k_A = 3\, k_B$, we find that
   $$3(T_1 - T_0) = T_0 - T_2 \qquad (1)$$
   Given $T_1 - T_2 = 36 \Rightarrow T_2 = T_1 - 36$ (2)
   Using (2) in (1) we get $T_1 - T_0 = 9$ °C.

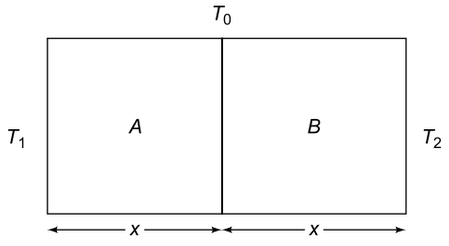


Fig. 19.21

3. $E_1 = e_A\, \sigma A\, T_A^4$, $E_2 = e_B\, \sigma\, A\, T_B^4$
   Given $E_1 = E_2$. Thus
   $$\frac{T_A}{T_B} = \left(\frac{e_B}{T_A}\right)^{1/4} = \left(\frac{0.81}{0.01}\right)^{1/4} = 3$$

20 Chapter

Electrostatic Field and Potential

REVIEW OF BASIC CONCEPTS

20.1 COULOMB'S LAW

On the basis of his measurements, Coulomb arrived at a law, known after his name as Coulomb's law, which states that the *magnitude of the electric force between two charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them,* i.e.

$$F \propto \frac{q_1 q_2}{r^2}$$

or
$$F = k \frac{q_1 q_2}{r^2}$$

In the SI system, k is written as $1/4\pi\varepsilon_0$ where ε_0 is called the permittivity of vacuum and its value is

$$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$$

Then
$$k = \frac{1}{4\pi\varepsilon_0} \simeq 9 \times 10^9\ \text{Nm}^2\ \text{C}^{-2}$$

The force F is attractive for unlike charges ($q_1 q_2 < 0$) and repulsive for like charges ($q_1 q_2 > 0$).

Coulomb's Law in vector form

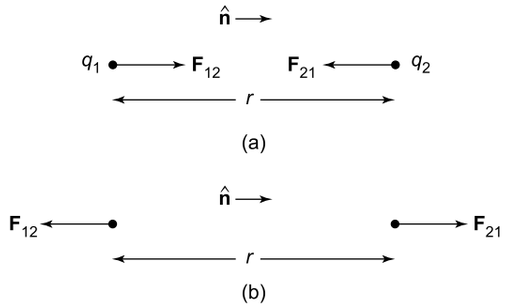


Fig. 20.1

Case (a): Unlike charges ($q_1 q_2 < 0$) [Fig. 20.1(a)]
Force exerted on q_2 by q_1 is

$$\mathbf{F}_{12} = \frac{q_1 q_2 \hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

where $\hat{\mathbf{n}}$ is a unit vector directed from q_1 to q_2.
Force exerted by q_1 on q_2 is

$$\mathbf{F}_{21} = -\frac{q_1 q_2 \hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

Case (b): Like charges ($q_1 q_2 > 0$) [Fig. 20.1(b)]

$$\mathbf{F}_{12} = -\frac{q_1 q_2 \hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

$$\mathbf{F}_{21} = \frac{q_1 q_2 \hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

20.2 RELATIVE PERMITTIVITY (OR DIELECTRIC CONSTANT)

Relative permittivity of a medium is defined as the ratio of the permittivity of the medium to permittivity of vacuum, i.e.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

ε_r is also called the dielectric constant (K) of the medium.

Thus $K = \dfrac{\varepsilon}{\varepsilon_0}$ or $\varepsilon = K\varepsilon_0$. By definition K for air = 1. If charges q_1 and q_2 are situated in a medium other than air or vacuum, the magnitude of force between them is given by

$$F = \frac{q_1 q_2}{4\pi\varepsilon r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 K r^2}$$

20.3 PRINCIPLE OF SUPERPOSITION

If many charges are present, the total force on a given charge is equal to the vector sum of the individual forces exerted on it by all other charges taken one at a time.

EXAMPLE 20.1

Two point charges $q_1 = +9\ \mu C$ and $q_2 = -1\ \mu C$ are held 10 cm apart. Where should at third charge $+Q$ be placed from q_2 on the line joining them so that charge Q does not experience any net force?

SOLUTION

Charge Q will not experience any net force if the forces exerted on it by charges q_1 and q_2 are equal and in opposite directions.

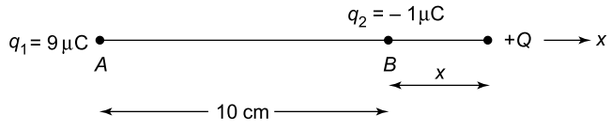


Fig. 20.2

It follows from Fig. 20.2 that charge Q will not experience forces in opposite direction if it lies at any point between AB. Let x be the distance of Q from q_2. Then forces exerted on Q by q_1 and q_2 respectively are

$$\mathbf{F}_1 = \frac{q_1 Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 (0.1+x)^2} = \frac{9\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 (0.1+x)^2}$$

and $$\mathbf{F}_2 = -\frac{q_2 Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 x^2} = -\frac{1\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 x^2}$$

Net force on $Q = \mathbf{F}_1 + \mathbf{F}_2$

Net force on $Q = 0$ if $\mathbf{F}_1 + \mathbf{F}_2 = 0$

$$\Rightarrow \quad \frac{9\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 (0.1+x)^2} - \frac{1\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 x^2} = 0$$

$$\Rightarrow \quad 9 = \frac{(0.1+x)^2}{x^2}$$

$$\Rightarrow \quad 3 = \frac{0.1+x}{x}$$

$$\Rightarrow \quad x = 0.05\ \text{m} = 5\ \text{cm}$$

EXAMPLE 20.2

Two charges, each equal to $-4\ \mu C$, are held a certain distance apart. A charge Q is placed exactly mid-way between them. Find the magnitude and sign of Q so that the system of three charges is in equilibrium.

SOLUTION

A system of charges is in equilibrium if no charge of the system experiences any net force.

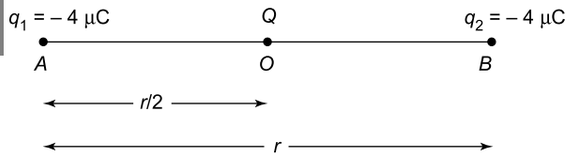


Fig. 20.3

Equilibrium of charge Q at O [See Fig. 20.3]

Since Q is at the same distance from equal charges q_1 and q_2, it will be equilibrium for any positive or negative value, because it will experience equal and opposite forces.

Equilibrium of charge q_1 at A

If Q is negative, it will repel q_1. Also q_2 is repel q_1. Hence q_1 cannot be in equilibrium if Q is negative. So Q must be positive.

Force exerted on q_1 by Q is

$$\mathbf{F} = \frac{4\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 \left(\frac{r}{2}\right)^2}$$

Force exerted on q_1 by q_2 is

$$\mathbf{F}' = -\frac{\left(4\times10^{-6}\right)\times\left(4\times10^{-6}\right)\hat{\mathbf{i}}}{4\pi\varepsilon_0 r^2}$$

Net force on q_1 will be zero if $\mathbf{F} + \mathbf{F}' = 0$, i.e. if

$$\frac{4\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 \left(\frac{r}{2}\right)^2} - \frac{\left(4\times10^{-6}\right)\times\left(4\times10^{-6}\right)\hat{\mathbf{i}}}{4\pi\varepsilon_0 r^2} = 0$$

$$\Rightarrow \quad Q = 1\times10^{-6}\ \text{C} = 1\ \mu\text{C}$$

It is easy to check that charge q_2 will also be in equilibrium. Hence the system of three charges will be in equilibrium if $Q = +1\ \mu C$.

EXAMPLE 20.3

Four point charges, each equal to $q = 4\ \mu C$, are held at the corners of a square $ABCD$ of side $a = 10$ cm. Find the magnitude and sign of a charge Q placed at the centre of the square so that the system of charges is in equilibrium.

SOLUTION

$AC\ (= r) = \sqrt{2}a$. Let us consider the equilibrium of charge q at A (Fig. 20.4)

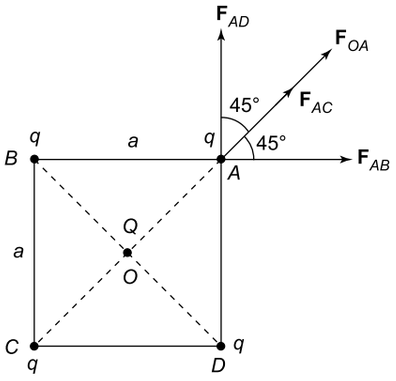


Fig. 20.4

Force exerted on charge at A by charge at B is

$$\mathbf{F}_{AB} = \frac{kq^2}{a^2}\hat{\mathbf{i}}, \quad \text{where } k = \frac{1}{4\pi\varepsilon_o}$$

Similarly

$$\mathbf{F}_{AD} = \frac{kq^2}{a^2}\hat{\mathbf{j}}$$

$$\mathbf{F}_{AC} = \frac{kq^2}{(a\sqrt{2})^2}(\cos 45°\,\hat{\mathbf{i}} + \sin 45°\,\hat{\mathbf{j}})$$

$$= \frac{kq^2}{a^2 2\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

and $$\mathbf{F}_{OA} = \frac{kqQ}{\left(\frac{a}{\sqrt{2}}\right)^2}(\cos 45°\,\hat{\mathbf{i}} + \sin 45°\,\hat{\mathbf{j}})$$

$$= \sqrt{2}\frac{kqQ}{a^2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

Net force on charge q at A in the x-direction is

$$\mathbf{F}_x = \left(\frac{kq^2}{a^2} + \frac{kq^2}{a^2 2\sqrt{2}} + \frac{\sqrt{2}\,kqQ}{a^2}\right)\hat{\mathbf{i}}$$

$$\Rightarrow \quad \mathbf{F}_x = \frac{kq}{a^2}\left(q + \frac{q}{2\sqrt{2}} + \sqrt{2}\,Q\right)\hat{\mathbf{i}} = \alpha\,\hat{\mathbf{i}}$$

where $$\alpha = \frac{kq}{a^2}\left(q + \frac{q}{2\sqrt{2}} + \sqrt{2}\,Q\right)$$

Similarly net force on charge q at A in the y-direction is

$$\mathbf{F}_y = \alpha\,\hat{\mathbf{j}}$$

$\therefore$ Resultant force on charge q at A is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\alpha^2 + \alpha^2} = \sqrt{2}\alpha$$

Charge q will be in equilibrium if $F = 0$ i.e. if

$$\alpha = 0$$

i.e. if $$\frac{kq}{a^2}\left(q + \frac{q}{2\sqrt{2}} + \sqrt{2}\,Q\right) = 0$$

$$\Rightarrow \quad Q = -\frac{q}{4}(1 + 2\sqrt{2})$$

$$= -\frac{4\,\mu C}{4}(1 + 2\sqrt{2}) = -(1 + 2\sqrt{2})\,C$$

20.4 ELECTRIC FIELD

An electric field exists at any point in the space surrounding a charge. To define the electric field, we place a small positive point charge q_0 at the point in space where the electric field is to be found and we measure the coulomb force **F** at that point. The electric field **E** is then given by

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

If a charge q is placed at a point where the electric field due to other charge or charges is **E**, then the charge q will experience a force **F** given by

$$\mathbf{F} = q\mathbf{E}$$

(1) Electric field due to an isolated point charge Electric field at a distance r from a source charge q is given by

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

For a positive charge ($+q$), vector **E** is directed radially outwards from it and for a negative charge ($-q$), **E** is directed radially inwards it. Because electric field **E** is vector quantity, the net electric field due to several charges is given by the vector sum of the electric fields due to the individual charges.

(2) Electric field due to an electric dipole A pair of equal and opposite point charges separated by a certain distance is called an electric dipole.

Case (a): Electric field at a point on the axis of a dipole

Let $2a$ be the separation between point charges $-q$ and $+q$ (Fig. 20.5).

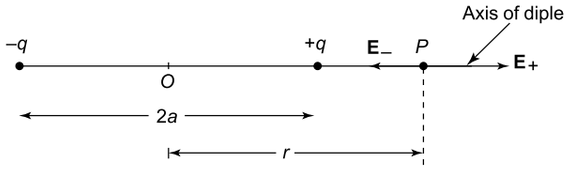


Fig. 20.5

Electric fields at P due to $+q$ and $-q$ respectively are

$$\mathbf{E}_+ = \frac{q\,\hat{\mathbf{i}}}{4\pi\varepsilon_0(r-a)^2}$$

$$\mathbf{E}_{-} = -\frac{q\hat{\mathbf{i}}}{4\pi\varepsilon_0(r+a)^2}$$

Electric field at point P is

$$\mathbf{E}_a = \mathbf{E}_{+} + \mathbf{E}_{-}$$

$$= \frac{2q(2\mathbf{a})r}{4\pi\varepsilon_0(r^2-a^2)^2}$$

$$= \frac{2\mathbf{p}r}{4\pi\varepsilon_0(r^2-a^2)^2}$$

where $\mathbf{p} = q(2\mathbf{a})$ is the dipole moment and $2\mathbf{a}$ is the vector distance between charges $-q$ and $+q$. Dipole moment $\mathbf{p}$ is a vector quantity directed from $-q$ to $+q$.

For a very short dipole ($a << r$)

$$\mathbf{E}_a = \frac{2\mathbf{p}}{4\pi\varepsilon_0 r^3}$$

Case (b): Electric field at a point on the perpendicular bisector (equatorial plane) of a dipole

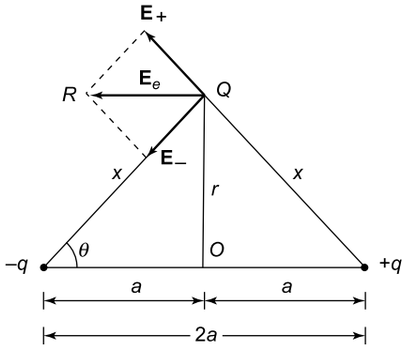


Fig. 20.6

Electric fields at point Q due to $+q$ and $-q$ are (see Fig. 20.6)

$$\mathbf{E}_{+} = \frac{q}{4\pi\varepsilon_0 x^2} \quad \text{and} \quad \mathbf{E}_{-} = \frac{q}{4\pi\varepsilon_0 x^2}$$

The magnitude of the resultant electric field at Q is

$$E_e = E_{+}\cos\theta + E_{-}\cos\theta$$

Using $\cos\theta = \frac{a}{x}$ and $x = \sqrt{r^2+a^2}$, we get

$$E_e = \frac{q(2a)}{4\pi\varepsilon_0(r^2+a^2)^{3/2}} \text{ directed from } Q \text{ to } R$$

In vector form

$$\mathbf{E}_e = -\frac{\mathbf{p}}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$$

For a very short dipole (a << r)

$$\mathbf{E}_e = -\frac{\mathbf{p}}{4\pi\varepsilon_0 r^3}$$

NOTE

(i) The direction the electric field at a point on the axial line of a dipole is along the dipole moment.

(ii) The direction of the electric field at a point on the equatorial line of a dipole is antiparallel to the dipole moment.

(3) Electric field due to a uniformly charged conducting rod A conducting rod AB of negligible thickness and length L carries a charge a charge Q uniformly distributed on it. To find the electric field at point P at a distance a from end B (Fig. 20.7), we consider a small element of length dx of the rod located at a distance x from P.

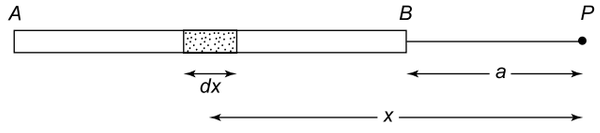


Fig. 20.7

Charge of element is $dq = \frac{Q}{L}dx = \lambda\, dx$ where $\lambda = \frac{Q}{L}$ is the linear charge density. The electric field at point P due to the element is

$$dE = \frac{dq}{4\pi\varepsilon_0 x^2} = \frac{\lambda\, dx}{4\pi\varepsilon_0 x^2}$$

Hence

$$E = \int dE = \frac{\lambda}{4\pi\varepsilon_0}\int_a^{(L+a)}\frac{dx}{x^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left|-\frac{1}{x}\right|_a^{(L+a)}$$

$$\Rightarrow \quad E = -\frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{(L+a)} - \frac{1}{a}\right]$$

$$\Rightarrow \quad E = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{L}{a(L+a)}\right]$$

If Q is positive, $\mathbf{E}$ is directed from left to right.

(4) Electric field due to a uniformly charged ring (or loop) of wire at a point on its axis Consider a ring of radius R carrying a chare Q distributed uniformly on it. To find electric field at a point P on its axis at a distance x from the centre O, consider an element of length dl (Fig. 20.8). The charge of the element is

$$dq = \frac{Q\, dl}{2\pi R}$$

The electric field due to element A is dE given by

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$

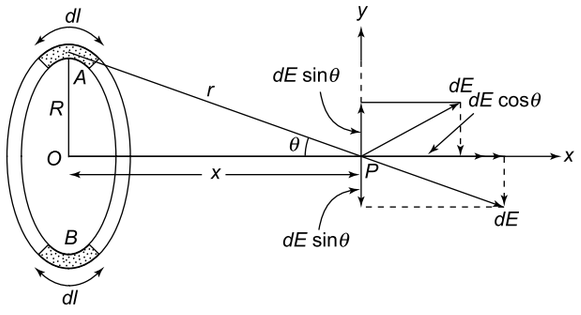


Fig. 20.8

There is a similar electric field at P due to element a diametrically opposite point B. The x components of electric fields due to these elements add up while the y components cancel. Hence

$$E = \int dE\cos\theta$$

$$= \int \frac{dq\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$= \frac{Q}{2\pi R}\times\frac{1}{4\pi\varepsilon_0}\times\frac{x}{(R^2+x^2)^{3/2}}\int dl$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qx}{(R^2+x^2)^{3/2}}\right]$$

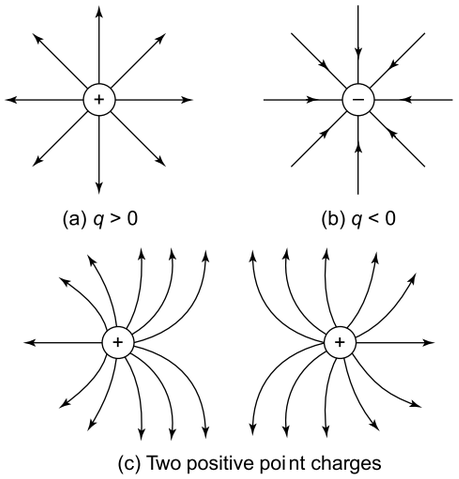
$$\left(\because \int dl = 2\pi R\right)$$

The direction of **E** is from O to P if charge Q is positive.

20.5 ELECTRIC FIELD LINES

Electric field lines of an electrostatic field give a pictorial representation of the field. An electric field line is a curve, the tangent to which at a point gives the direction of the electric field at that point.

Figure 20.9 shows field line patterns around some charge distributions.



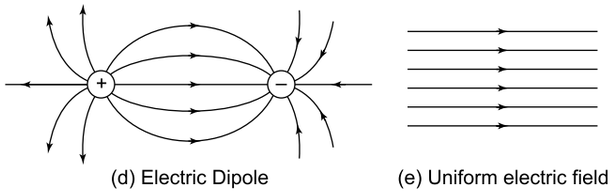


Fig. 20.9

Properties of Electric Field Lines

(i) The tangent to a field line at any point gives the direction of electric field at that point.

(ii) Field lines originate from a positive charge and terminate on a negative charge.

(iii) No two field lines intersect.

(iv) Field lines are closer together in the region where the field is stronger and farther apart where the field is weaker.

(v) The number of field lines originating or ending on a charge is proportional to the magnitude of the charge.

NOTE

Electric field line due to a charge distribution never forms a closed loop. But if the electric is induced by a time-varying magnetic field, its field line forms a closed loop.

20.6 ELECTRIC FLUX

The electric flux through a surface in an electric field is a measure of the number of electric field lines passing through the surface.

For a plane surface of surface area **S** in an electric field **E**, the electric flux ϕ is defined as

$$\phi = \mathbf{E}\cdot\mathbf{S} = ES\cos\theta$$

where **S** is called the area vector, its magnitude is S and its direction is normal to the surface and away from it. Angle θ is the angle between **E** and **S**.

For a curved surface,

$$\phi = \int \mathbf{E}\cdot\mathbf{dS} = \int (\mathbf{E}\cdot\hat{\mathbf{n}})\,dS$$

where $\hat{\mathbf{n}}$ is a unit outward normal to the surface. dS is the surface area of an element of the surface (Fig. 20.10). The SI unit of electric flux is $NC^{-1}\,m^2$ or Vm (volt metre).

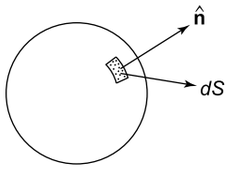


Fig. 20.10

20.7 GAUSS'S LAW IN ELECTROSTATICS

Gauss's law states that the electric flux through a closed surface **S** in an electric field **E** is equal to $\frac{q}{\varepsilon_0}$, where q is the net charge enclosed in the surface and ε_0 is electrical permittivity of vacuum.

$$\oint_S \mathbf{E}\cdot\mathbf{dS} = \frac{q}{\varepsilon_0} \quad (1)$$

Gauss's law is used to obtain the expression for the electric field due to linear, surface and volume charge distributions which are uniform and symmetric so that a proper and convenient closed surface (called the Gaussian surface) can be chosen to evaluate the surface integral in Eq. (1).

Some Important Points about Gauss's Law

(1) Gauss's law holds for any closed surface of any shape or size.
(2) The surface that we choose to evaluate electric flux [i.e. to evaluate the surface integral in Eq. (1)] is called Gaussian surface.
(3) If the Gaussian surface is so chosen that there are some charges outside and some inside the surface, then charge q on the R.H.S. of Eq. (1) is the net charge (taking into account the sign of charges) enclosed inside the surface but electric field **E** on the L.H.S. of Eq. (1) is the electric field due to all the charges both inside and outside the surface.
(4) The exact location of charges inside Gaussian surface does not affect the value of the electric flux.
(5) If Coulomb's law did not hold, Gauss's law also would not hold.

20.8 APPLICATIONS OF GAUSS'S LAW

(1) Electric field due to a thin infinitely long straight charged rod or wire Electric field at a point at a pendicular distance r from a thin, infinitely long straight rod or wire carrying a uniform linear charge density λ is given by

$$\mathbf{E} = \frac{\lambda\hat{\mathbf{n}}}{2\pi\varepsilon_0 r}$$

where $\lambda = q/L$ is the charge per unit length of the rod and $\hat{\mathbf{n}}$ is a unit vector pointing away from the rod if q is positive and towords it if q is negative.

(2) Electric field due to a thin sheet of charge Electric field at a perpendicular distance r from a thin, flat and infinite sheet carrying a uniform surface charge density σ is given by

$$\mathbf{E} = \frac{\sigma\hat{\mathbf{n}}}{2\varepsilon_0}$$

where $\sigma = q/A$ is charge per unit area and $\hat{\mathbf{n}}$ is a unit vector pointing away from the sheet if q is positive and towards it if q is negative. Notice that **E** is independent of r, the distance from the sheet.

(3) Electric field due to a thin charged spherical shell Electric field at a distance r from a spherical shell of radius R carrying a surface charge density σ $(= q/4\pi R^2)$ is given by

$$\mathbf{E} = \frac{q\hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2} = \frac{\sigma R^2\hat{\mathbf{n}}}{\varepsilon_0 r^2} \quad (\text{for } r > R)$$

$$= \frac{\sigma\hat{\mathbf{n}}}{\varepsilon_0} \quad (\text{for } r = R)$$

$$= \text{zero} \quad (\text{for } r < R)$$

where $\hat{\mathbf{n}}$ is a unit vector pointing radially outwards if q is positive and inwards if q is negative.

20.9 ELECTRIC POTENTIAL

The electric potential at a point in an electrostatic field is the work per unit charge that is done to bring a small charge in from infinity to that point along any path.

$$V = \lim_{q_0 \to 0} \frac{W}{q_0}$$

(1) Electric potential due to an isolated point charge Electric potential at a point P in the electric field of a point charge is given by

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

where r is the distance of the point P from the charge. This potential is spherically symmetric around the point, i.e. it depends only on r for a given charge q. Since potential is a scalar function, the spherical symmetry means that the potential at a point does not depend upon the direction of that point with respect to the point charge; it only depends on the distance of the point from the charge.

Notice that the potential due to a positive charge $(q > 0)$ is positive, it is negative in the neighbourhood of an isolated negative charge $(q < 0)$.

(2) Electric potential due to two point charges To find the electric potential at a point in the electric field due to two or more charges, we first calculate the potential due to each charge, assuming that all other charges are absent, and then simply add these individual contributions. Since, unlike electric field, electric potential is a scalar, the addition here is the *ordinary* sum, not a *vector* sum.

The potential at any point due to two point charges q_1 and q_2 is, therefore, simply given by

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

where r_1 and r_2 are the distances of the point in the question from charges q_1 and q_2 respectively.

(3) Electric potential due to many point charges The potential at any point due to a system of N point charges in given by

$$V = V_1 + V_2 + \ldots + V_N = \frac{1}{4\pi\varepsilon_0}\sum_{n=1}^{N}\frac{q_n}{r_n}$$

20.10 RELATION BETWEEN E AND V

Electric field is the negative gradient of potential. This means that the potential decreases along the direction of the electric field.

$$E = -\frac{dV}{dr}$$

20.11 ELECTRIC POTENTIAL ENERGY

The electric potential energy of a system of point charges is defined as the amount of work done to assemble this system of charges by bringing them in from an infinite distance. We assume that the charges were at rest when they were infinitely separated, i.e. they had no initial kinetic energy.

The electric potential energy of two point charges q_1 and q_2 separated by a distance r_{12} as shown in Fig. 20.11 (a) is given by

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}}$$

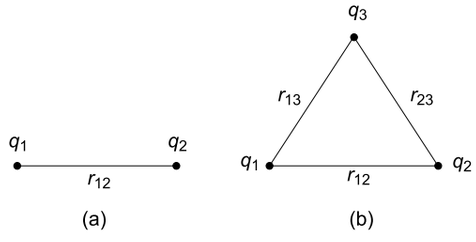


Fig. 20.11

The electric potential energy of a system of three point charges as shown in Fig. 20.11 (b) is given by

$$U = U_{12} + U_{23} + U_{31}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}\right)$$

This expression can be generalized for any number of charges.

20.12 THE ELECTRON-VOLT

The SI unit of potential energy is the *joule*. In atomic physics a more convenient unit called the *electron-volt* (written as eV) is used. *An electron-volt is the potential energy gained or lost by an electron in moving through a potential difference of 1 volt.* Since the magnitude of charge on an electron is 1.6×10^{-19} C,

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

20.13 POTENTIAL ENERGY OF AN ELECTRIC DIPOLE IN AN EXTERNAL ELECTRIC FIELD

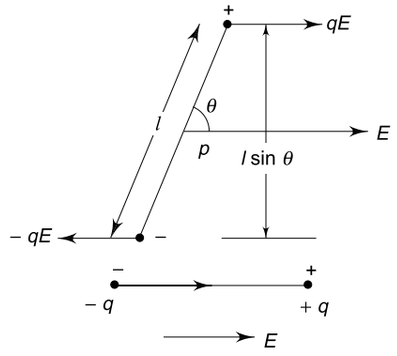


Fig. 20.12

When a dipole is placed in a uniform electric field E, as shown in Fig. 20.12, it experiences a torque given by

$$\tau = p\ E\ \sin\ \theta$$

where θ is the angle between the line joining the two charges and the electric field. In vector form

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

The torque tends to rotate the dipole to a position where $\theta = 0$, i.e, $\mathbf{p}$ is parallel to $\mathbf{E}$.

The electric potential energy of a dipole is

$$U = -\ \mathbf{p} \cdot \mathbf{E}$$

20.14 ADDITIONAL USEFUL FORMULAE

(1) Electric field and potential due to a group of charges

(i) Charge q at each vertex of an equilateral triangle of side a (Fig. 20.13).

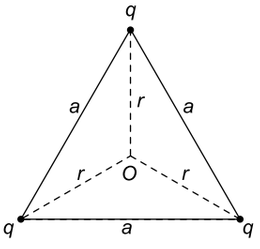


Fig. 20.13

At centroid O, $E_0 = 0$ and $V_0 = \dfrac{3q}{4\pi\varepsilon_0 r}$

where $r = \dfrac{a}{\sqrt{3}}$.

(ii) Charge q at each vertex of a square of side a (Fig. 20.14).

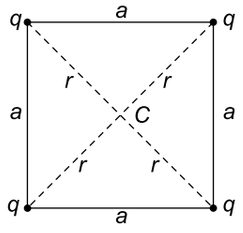


Fig. 20.14

At center C, $E_c = 0$ and $V_c = \dfrac{4q}{4\pi\varepsilon_0 r}$; $r = \dfrac{a}{\sqrt{2}}$

NOTE

In the above two cases, if one of the charges is removed from a vertex, the net electric field at O and C is $E = q/4\pi\varepsilon_0 r^2$, directed towards the empty vertex.

(iii) For Fig. 20.15,

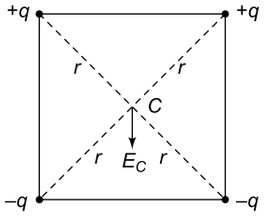


Fig. 20.15

$$V_c = 0 \quad \text{and} \quad E_c = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 r^2}$$

(iv) For Fig. 20.16,

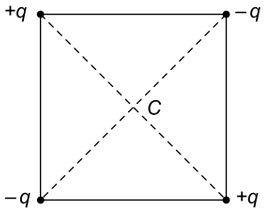


Fig. 20.16

$$V_c = 0$$

$$E_c = 0$$

(v) Infinite number of charges, each equal to q, placed on the x-axis at $x = r$, $x = 2r$, $x = 4r\ldots$ and so on. Electric field at origin O is

$$E_0 = \frac{q}{4\pi\varepsilon_0 r^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \cdots\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\left(1 + \frac{1}{4} + \frac{1}{16} + \cdots\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \times \frac{1}{\left(1 - \frac{1}{4}\right)} = \frac{q}{3\pi\varepsilon_0 r^2}$$

Potential at O is

$$V_0 = \frac{q}{4\pi\varepsilon_0 r}\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r} \times \frac{1}{\left(1 - \frac{1}{2}\right)} = \frac{q}{2\pi\varepsilon_0 r}$$

(vi) A short electric dipole of dipole moment $\vec{p}$ (Fig. 20.17)

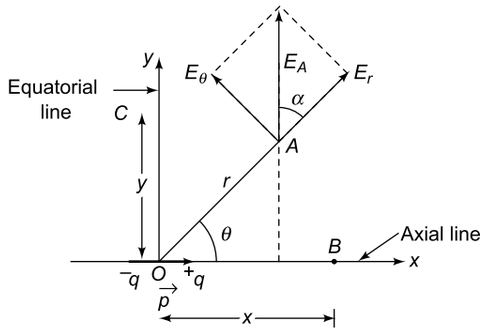


Fig. 20.17

At point A, $E_r = \dfrac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$ and $E_\theta = \dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}$

Net electric field at A is $E_A = \sqrt{E_r^2 + E_\theta^2}$

$$= \frac{p}{4\pi\varepsilon_0 r^3}(3\cos 2\theta + 1)^{1/2}$$

Also $\tan\alpha = \dfrac{1}{2}\tan\theta$. Angle between $\boldsymbol{p}$ and $\boldsymbol{E}_A$ is $(\alpha + \theta)$.

At point B on axial line, $E_B = \dfrac{2p}{4\pi\varepsilon_0 x^3}$ ($\because \theta = 0°$)

At point C on equatorial line, $E_C = \dfrac{p}{4\pi\varepsilon_0 y^3}$ ($\because \theta = 90°$)

Electric potential at A is $V_A = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

At point B; $V_B = \dfrac{p}{4\pi\varepsilon_0 r^2}$

At point C; $V_C = 0$

(2) Electric field and potential due to some charge distributions (Linear charge density $\lambda = \frac{Q}{L}$)

(i) Charge Q distributed uniformly on a rod of length L (Fig. 20.18)

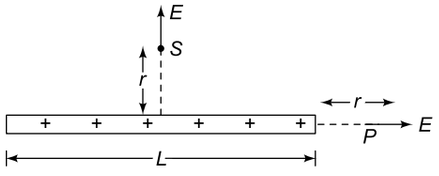


Fig. 20.18

At point P, $E_P = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r(r+L)}\right]$

$$V_P = \frac{Q}{4\pi\varepsilon_0 L}\log_e\left(\frac{r+L}{r}\right)$$

(ii) Charge Q distributed uniformly on a conducting sphere or shell of radius R (Fig. 20.19) (Surface charge density $\sigma = \frac{Q}{4\pi R^2}$)

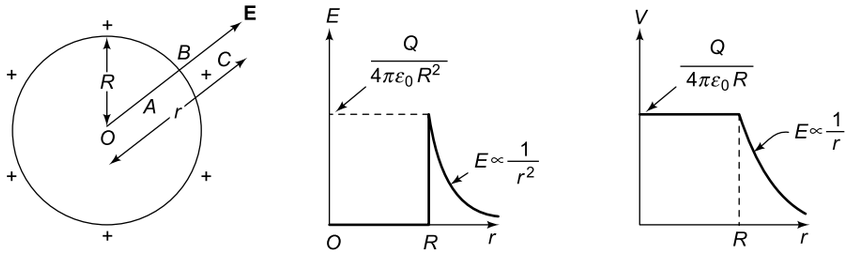


Fig. 20.19

At point C outside the sphere or shell ($r > R$),

$$E_C = \frac{Q}{4\pi\varepsilon_0 r^2}$$

At point B just outside the surface ($r = R$),

$$E_B = \frac{Q}{4\pi\varepsilon_0 R^2}$$

At point A inside the sphere or shell ($r < R$),

$E_A = 0$

Potential at center O of sphere or shell,

$$V_0 = \frac{Q}{4\pi\varepsilon_0 R}$$

At points inside ($r < R$), $V_A = \frac{Q}{4\pi\varepsilon_0 R} = V_B$ (at surface)

At point C outside ($r > R$), $V_C = \frac{Q}{4\pi\varepsilon_0 R}$

(iii) Charge Q distributed uniformly on a semicircular wire of radius R (Fig. 20.20) (Linear charge density $\lambda = \frac{Q}{\pi R}$)

At center O, $E_0 = \frac{\lambda}{4\pi\varepsilon_0 R}$ along negative y-direction

$$= \frac{Q}{4\pi^2\varepsilon_0 R^2}$$

At centre O, $V_0 = \frac{Q}{4\pi\varepsilon_0 R} = \frac{\lambda}{4\varepsilon_0}$

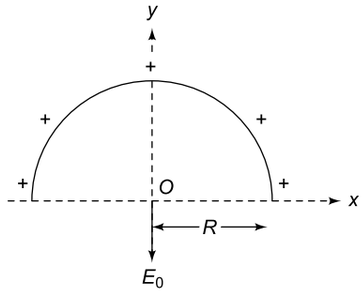


Fig. 20.20

(iv) Charge Q distributed uniformly on a ring of radius R. (Fig. 20.21). Electric field at a point P on the axis is

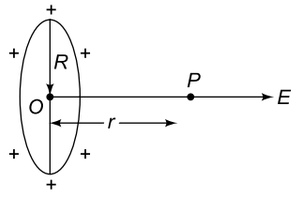


Fig. 20.21

$$E_P = \frac{Q}{4\pi\varepsilon_0} \times \frac{r}{\left(R^2 + r^2\right)^{3/2}}$$

E is maximum at $r = \pm\frac{R}{\sqrt{2}}$ and

$$E_{max} = \frac{1}{4\pi\varepsilon_0} \times \frac{2Q}{3\sqrt{3}R^2}$$

Electric potential at point P is

$$V_P = \frac{Q}{4\pi\varepsilon_0} \times \frac{1}{\left(R^2 + r^2\right)^{1/2}}$$

V is maximum at $r = 0$ (i.e. at centre O) and

$$V_{max} = \frac{Q}{4\pi\varepsilon_0 R}$$

(3) Potential energy of a system of charges

(i) Charge Q kept at each vertex of an equilateral of side a. Potential energy is (Fig. 20.22).

$$U = \frac{3Q^2}{4\pi\varepsilon_0 a}$$

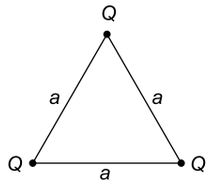


Fig. 20.22

(ii) Charge Q kept at each vertex of a square of side a (Fig. 20.23)

$$U = \frac{4 \times Q^2}{4\pi\varepsilon_0 a} + \frac{2Q^2}{4\pi\varepsilon_0 (a\sqrt{2})}$$

$$= \frac{Q^2}{4\pi\varepsilon_0 a}(4 + \sqrt{2})$$

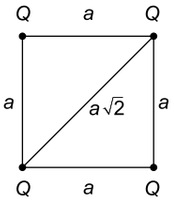


Fig. 20.23

(iii) An electric dipole of dipole moment **p** placed in uniform electric field **E** with angle θ between **p** and **E**. Torque $\boldsymbol{\tau} = \mathbf{p} \times \mathrm{E}$ and potential energy $U = -\mathbf{p}.\mathbf{E}$. The zero of potential energy is taken at $\theta = 90°$.

(a) When $\theta = 0°$, $\tau = 0$, U = minimum = $-pE$ (stable equilibrium)

(b) When $\theta = 90°$, τ = maximum = pE, $U = 0$

(c) When $\theta = 180°$, $\tau = 0$, U = maximum = pE (unstable equilibrium)

(d) Work done in turning a dipole from angle θ_1 to angle θ_2 is

$$W = pE\ (\cos\theta_1 - \cos\theta_2)$$

If $\theta_1 = 0°$ and $\theta_2 = 180°$, $W = 2pE$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. Three point charges Q, $-2Q$ and $-2Q$ are placed at the vertices of an equilateral triangle of side r. The work done to increase their separation to $2r$ is

(a) zero (b) $\frac{Q^2}{4\pi\varepsilon_0 r}$

(c) $\frac{2Q^2}{4\pi\varepsilon_0 r}$ (d) $\frac{\sqrt{2}\,Q^2}{4\pi\varepsilon_0 r}$

2. A point charge Q is placed at point P at a distance R from the centre O of a metallic spherical shell of inner radius $2R$ and outer radius $2.5R$. The electric potential at the centre of the shell will be

(a) $\frac{Q}{4\pi\varepsilon_0 R}$ (b) $\frac{1}{4\pi\varepsilon_0}\left(\frac{5Q}{6R}\right)$

(c) zero (d) $\frac{1}{4\pi\varepsilon_0}\left(\frac{9Q}{10R}\right)$

3. Two concentric metallic shells of radii R and $2R$ are given charges Q and $2Q$ respectively. If the two shells are connected by a metallic wire, the change in electric potential on the outer shell is

(a) $\frac{Q}{4\pi\varepsilon_0 R}$ (b) $\frac{Q}{4\pi\varepsilon_0 (2R)}$

(c) $\frac{3Q}{4\pi\varepsilon_0 (2R)}$ (d) zero

4. A metal sphere of radius R carries a charge Q. The electric field on its surface is E and the electric potential is V. If R is doubled keeping Q the same, the new values of E and V will be

(a) $\frac{E}{4}$ and $\frac{V}{2}$ (b) $\frac{E}{2}$ and $\frac{V}{4}$

(c) $4E$ and $2V$ (d) $2E$ and $4V$

5. A metal sphere of radius R has surface charge density σ. The electric field on its surface is E and the electric potential is V. If R is halved, keeping σ the same, the new values of E and V will be

(a) $4E$ and $2V$ (b) $2E$ and $4V$

(c) E and $\frac{V}{2}$ (d) $\frac{E}{2}$ and V

6. The electric field due to a very short electric dipole at a point P on its axial line at a distance r from it is $\vec{E}_P$ and at a point Q on its equatorial line at a distance $2r$ from it is $\vec{E}_Q$. Then

(a) $\vec{E}_P = -16\vec{E}_Q$ (b) $\vec{E}_P = -8\vec{E}_Q$

(c) $\vec{E}_P = 8\vec{E}_Q$ (d) $\vec{E}_P = -\vec{E}_Q$

7. A hollow metal sphere is charged such that the potential at its centre is V. The potential on the surface of the sphere is

(a) zero (b) V

(c) more than V (d) less than V

8. The work done in carrying a charge q once round a circle with a charge Q at the centre is W_1. The work done is W_2 if charge q is moved from one end of a diameter to the other. Then

(a) $W_1 > W_2$ (b) $W_1 < W_2$

(c) $W_1 = W_2 \neq 0$ (d) $W_1 = W_2 = 0$

9. Three point charges $4q$, Q and q are placed in a straight line of length l at points distant 0, $l/2$ and l respectively. If the net force on charge q is zero, the magnitude of the force on charge $4q$ is

(a) $\dfrac{q^2}{\pi \varepsilon_0 l^2}$ (b) $\dfrac{2q^2}{\pi \varepsilon_0 l^2}$

(c) $\dfrac{3q^2}{\pi \varepsilon_0 l^2}$ (d) $\dfrac{4q^2}{\pi \varepsilon_0 l^2}$

10. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to

(a) $-\dfrac{Q}{2}$ (b) $-\dfrac{Q}{4}$

(c) $+\dfrac{Q}{2}$ (d) $+\dfrac{Q}{4}$

IIT, 1987

11. One thousand spherical water droplets, each of radius r and each carrying a charge q, coalesce to form a single spherical drop. If v is the electrical potential of each droplet and V that of the bigger drop, then

(a) $\dfrac{V}{v} = \dfrac{1}{1000}$ (b) $\dfrac{V}{v} = \dfrac{1}{100}$

(c) $\dfrac{V}{v} = 100$ (d) $\dfrac{V}{v} = 1000$

12. Two metallic identical spheres A and B carrying equal positive charge $+q$ are a certain distance apart. The force of repulsion between them is F. A third uncharged sphere of the same size is brought in contact with sphere A and removed. It is then brought in contact with sphere B and removed. What is the new force of repulsion between A and B?

(a) F (b) $\dfrac{3F}{8}$

(c) $\dfrac{F}{2}$ (d) $\dfrac{F}{4}$

13. Two small identical balls P and Q, each of mass $\sqrt{3}/10$ gram, carry identical charges and are suspended by threads of equal lengths. At equilibrium, they position themselves as shown in Fig. 20.24. What is the charge on each ball. Given $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{Nm}^2\text{C}^{-2}$ and take $g = 10\ \text{ms}^{-2}$.

(a) 10^{-3} C (b) 10^{-5} C

(c) 10^{-7} C (d) 10^{-9} C

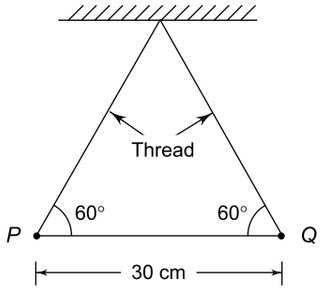


Fig. 20.24

14. Two point charges $q_1 = 2\ \mu\text{C}$ and $q_2 = 1\ \mu\text{C}$ are placed at distances $b = 1$ cm and $a = 2$ cm from the origin on the y and x axes as shown in Fig. 20.25. The electric field vector at point $P(a, b)$ will subtend an angle θ with the x-axis given by

(a) $\tan\theta = 1$ (b) $\tan\theta = 2$

(c) $\tan\theta = 3$ (d) $\tan\theta = 4$

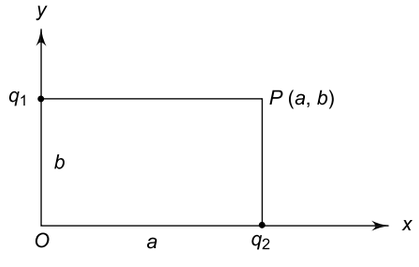


Fig. 20.25

15. An electric dipole placed with its axis in the direction of a uniform electric field experiences

(a) a force but no torque

(b) a torque but no force

(c) a force as well as a torque

(d) neither a force nor a torque.

16. An electric dipole placed with its axis inclined at an angle to the direction of a uniform electric field experiences

(a) a force but no torque
(b) a torque but no force
(c) a force as well as a torque
(d) neither a force nor a torque

17. An electric dipole placed in a non-uniform electric field experiences
(a) a force but no torque
(b) a torque but no force
(c) a force as well as a torque
(d) neither a force nor a torque.

18. A cube of side b has a charge q at each of its vertices. What is the electric potential at the centre of the cube?

(a) $\frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$ (b) $\frac{\sqrt{3}q}{\pi\varepsilon_0 b}$

(c) $\frac{2q}{\pi\varepsilon_0 b}$ (d) zero

19. In Q. 18, the electric field at the centre of the cube is

(a) $\frac{4q}{3\pi\varepsilon_0 b^2}$ (b) $\frac{3q}{\pi\varepsilon_0 b^2}$

(c) $\frac{2q}{\pi\varepsilon_0 b^2}$ (d) zero

20. Two point charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$ respectively. What is the electric potential at point $(0, 0, z)$?

(a) $\frac{qa}{4\pi\varepsilon_0 z^2}$ (b) $\frac{q}{4\pi\varepsilon_0 a}$

(c) $\frac{2qa}{4\pi\varepsilon_0\left(z^2-a^2\right)}$ (d) $\frac{2qa}{4\pi\varepsilon_0\left(z^2+a^2\right)}$

21. In Q. 20, how much work is done in moving a small test charge q_0 from point (5, 0, 0) to a point (– 7, 0, 0) along the x-axis?

(a) $\frac{5}{7}\times\frac{q_0 q}{4\pi\varepsilon_0 a}$ (b) $\frac{7}{5}\times\frac{q_0 q}{4\pi\varepsilon_0 a}$

(c) $\frac{2}{12}\times\frac{q_0 q}{4\pi\varepsilon_0 a}$ (d) zero

22. A neutral hydrogen molecule has two protons and two electrons. If one of the electrons is removed we get a hydrogen molecular ion (H_2^+). In the ground state of H_2^+ the two protons are separated by roughly 1.5 Å and the electron is roughly 1 Å from each proton. What is the potential energy of the system?

(a) – 38.4 eV (b) – 19.2 eV
(c) – 9.6 eV (d) zero

23. In a hydrogen atom, the electron and the proton are bound together at a separation of about 0.53 Å. If the zero of potential energy is taken at an infinite separation of the electron from the proton, the potential energy of the electron-proton system is

(a) – 54.4 eV (b) – 27.2 eV
(c) – 13.6 eV (d) zero

24. A positive charge $(+q)$ is located at the centre of a circle as shown in Fig. 20.26. W_1 is the work done in taking a unit positive charge from A to B and W_2 is the work done in taking the same charge from A to C. Then

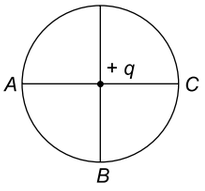


Fig. 20.26

(a) $W_1 > W_2$
(b) $W_1 < W_2$
(c) $W_1 = W_2$
(d) $W_1 = W_2 = 0$

25. Two concentric spheres of radii r_1 and r_2 carry charges q_1 and q_2 respectively. If the surface charge density (σ) is the same for both spheres, the electric potential at the common centre will be

(a) $\frac{\sigma}{\varepsilon_0}\cdot\frac{r_1}{r_2}$ (b) $\frac{\sigma}{\varepsilon_0}\cdot\frac{r_2}{r_1}$

(c) $\frac{\sigma}{\varepsilon_0}(r_1 - r_2)$ (d) $\frac{\sigma}{\varepsilon_0}(r_1 + r_2)$

26. The magnitude of the electric field on the surface of a sphere of radius r having a uniform surface charge density σ is

(a) $\frac{\sigma}{\varepsilon_0}$ (b) $\frac{\sigma}{2\varepsilon_0}$

(c) $\frac{\sigma}{\varepsilon_0 r}$ (d) $\frac{\sigma}{2\varepsilon_0 r}$

27. Two equal negative charges $-q$ are fixed at points $(0, a)$ and $(0, -a)$. A positive charge $+Q$ is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will
(a) execute SHM about the origin
(b) move to the origin and remain at rest there.
(c) move to infinity.
(d) execute oscillations but not SHM.

IIT, 1985

28. Four charges q, $2q$, $3q$ and $4q$ are placed at corners A, B, C and D of a square as shown in Fig. 20.27. The field at centre P of the square has the direction along

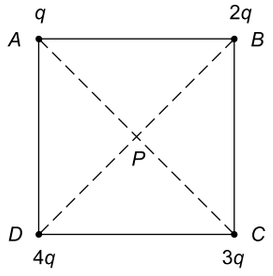


Fig. 20.27

(a) AB (b) CB
(c) AC (d) DB

29. Particle A has a charge + q and particle B has a charge + $4q$, each having the same mass m. When allowed to fall from rest through the same potential difference, the ratio of their speeds v_A/v_B will be
(a) 2 : 1 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

30. Four equal charges Q are placed at the four corners of a square of side a. The work done in removing a charge – Q from the centre of the square to infinity is
(a) zero (b) $\dfrac{\sqrt{2}\,Q^2}{4\pi\varepsilon_0 a}$
(c) $\dfrac{\sqrt{2}\,Q^2}{\pi\varepsilon_0 a}$ (d) $\dfrac{Q^2}{2\pi\varepsilon_0 a}$

31. A point charge + q is placed at the mid point of a cube of side L. The electric flux emerging from the cube is
(a) $\dfrac{q}{\varepsilon_0}$ (b) zero
(c) $\dfrac{6qL^2}{\varepsilon_0}$ (d) $\dfrac{q}{6L^2\varepsilon_0}$

32. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. The potential difference between the surface of the solid sphere and the outer surface of the hollow shell is V. If the shell is now given a charge of – $3Q$, the new potential difference between the same two surfaces is
(a) V (b) $2\ V$
(c) $4\ V$ (d) $-2\ V$

IIT, 1989

33. Two identical thin rings, each of radius R are coaxially placed a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to the centre of the other is
(a) zero
(b) $\dfrac{q}{4\pi\varepsilon_0\sqrt{2}\,R}(Q_1 - Q_2)(\sqrt{2}-1)$
(c) $\dfrac{q\sqrt{2}}{4\pi\varepsilon_0 R}(Q_1 + Q_2)$
(d) $\dfrac{(\sqrt{2}+1)\,q\,(Q_1+Q_2)}{\sqrt{2}\,4\pi\,\varepsilon_0\,R}$

IIT, 1992

34. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to
(a) 1 (b) $(m_p/m_e)^{1/2}$
(c) $(m_e/m_p)^{1/2}$ (d) 1836

IIT, 1997

35. A nonconducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed *non-uniformly* on its circumference producing an electric field E everywhere in space. The value of the line integral $\int_{l=\infty}^{l=0} -\boldsymbol{E}\cdot dl$ ($l = 0$ being centre of the ring) in volts is
(a) + 2 (b) – 1
(c) – 2 (d) zero

IIT, 1997

36. A metallic solid sphere is placed in a uniform electric field. In Fig. 20.28, which path will the lines of force follow?
(a) 1 (b) 2
(c) 3 (d) 4

IIT, 1996

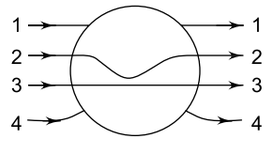


Fig. 20.28

37. A charge + q is fixed at each of the points $x = x_0$, $x = 3\,x_0$, $x = 5x_0$ … upto infinity and a charge – q is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$ … upto infinity. Here x_0 is a positive constant. The potential at the origin to this system of charges is
(a) zero (b) $\dfrac{q}{4\pi\varepsilon_0\ x_0 \ln(2)}$

(c) infinity (d) $\frac{q \ln (2)}{4\pi \varepsilon_0 x_0}$

IIT, 1998

38. Three charges Q, + q and + q are placed at the vertices of a right-angled isosceles triangle as shown in Fig. 20.29. The net electrostatic energy of the configuration is zero if Q is equal to

(a) $\frac{-q}{1+\sqrt{2}}$ (b) $\frac{-2q}{2+\sqrt{2}}$

(c) $-2q$ (d) $+q$

IIT, 2000

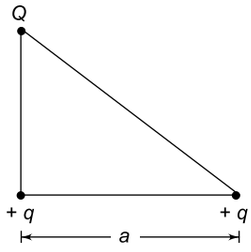


Fig. 20.29

39. Eight dipoles of charges of magnitude e are placed inside a cube. The total electric flux coming out of the cube will be

(a) $\frac{8e}{\varepsilon_0}$ (b) $\frac{16e}{\varepsilon_0}$

(c) $\frac{e}{\varepsilon_0}$ (d) zero

40. A point Q lies on the perpendicular bisector of an electrical dipole of dipole moment p. If the distance of Q from the dipole is r (much larger than the size of the dipole), then the electric field at Q is proportional to

(a) p^{-1} and r^{-1} (b) p and r^{-2}

(c) p^2 and r^{-3} (d) p and r^{-3}

41. A particle of mass m and charge q is released from rest in a uniform electric field E. The kinetic energy attained by the particle after moving a distance x is

(a) qEx^2 (b) qE^2x

(c) qEx (d) q^2Ex

42. There is a uniform field of strength 10^3 Vm^{-1} along the y-axis. A body of mass 1 g and charge 10^{-6} C is projected into the field from the origin along the positive x-axis with a velocity of 10 ms^{-1}. Its speed (in ms^{-1}) after 10 second will be (neglect gravitation)

(a) 10 (b) $5\sqrt{2}$

(c) $10\sqrt{2}$ (d) 20

43. Two identical charges are placed at the two corners of an equilateral triangle. The potential energy of the system is U. The work done in bringing an identical charge from infinity to the third vertex is

(a) U (b) 2 U

(c) 3 U (d) 4 U

44. The magnitude of electric intensity at a distance x from a charge q is E. An identical charge is placed at a distance $2x$ from it. Then the magnitude of the force it experiences is

(a) qE (b) $2qE$

(c) $\frac{qE}{2}$ (d) $\frac{qE}{4}$

45. A particle carrying a charge q is shot with a speed v towards a fixed particle carrying a charge Q. It approaches Q up to a certain distance r and then returns as shown in Fig. 20.30.

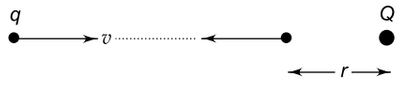


Fig. 20.30

If charge q were moving with a speed $2v$, the distance of the closest approach would be

(a) r (b) $2r$

(c) $\frac{r}{2}$ (d) $\frac{r}{4}$

46. Four charges, each equal to $-Q$, are placed at the corners of a square and a charge $+q$ is placed at its centre. If the system is in equilibrium, the value of q is

(a) $-\frac{Q}{4}\left(1+2\sqrt{2}\right)$ (b) $\frac{Q}{4}\left(1+2\sqrt{2}\right)$

(c) $-\frac{Q}{2}\left(1+2\sqrt{2}\right)$ (d) $\frac{Q}{2}\left(1+2\sqrt{2}\right)$

47. A charge having magnitude Q is divided into two parts q and $(Q - q)$ which are held a certain distance r apart. The force of repulsion between the two parts will be maximum if the ratio q/Q is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

48. A charge Q is given to a hollow metallic sphere of radius R. The electric potential at the surface of the sphere is

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^2}$ (d) $4\pi\varepsilon_0 Q/R$

49. In Q. 48, the potential at a distance r from the centre of the sphere where $r < R$ is

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{(R-r)}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R+r}$ (d) $\frac{4\pi\varepsilon_0 Q}{(R-r)}$

50. The electric potential V at any point (x, y, z) in space is given by $V = 4x^2$ volt where x, y and z are all in metre. The electric field at the point (1 m, 0, 2 m) in Vm^{-1} is

(a) 8 along negative x-axis
(b) 8 along positive x-axis
(c) 16 along negative x-axis
(d) 16 along positive x-axis

IIT, 1992

51. Two spheres of radii r and R carry charges q and Q respectively. When they are connected by a wire, there will be no loss of energy of the system if

(a) $qr = QR$ (b) $qR = Qr$
(c) $qr^2 = QR^2$ (d) $qR^2 = Qr^2$

52. Two equal point charges of 1 μC each are located at points $(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$ m and $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})$ m. What is the magnitude of electrostatic force between them?

(a) 10^{-3} N (b) 10^{-6} N
(c) 10^{-9} N (d) 10^{-12} N

53. Three equal point charges q are placed at the corners of an equilateral triangle. Another charge Q is placed at the centroid of the triangle. The system of charges will be in equilibrium if Q equals

(a) $\frac{q}{\sqrt{3}}$ (b) $-\frac{q}{\sqrt{3}}$

(c) $\frac{q}{3}$ (d) $-\frac{q}{3}$

54. A metallic sphere A of radius a carries a charge Q. It is brought in contact with an uncharged sphere B of radius b. The charge on sphere A now will be

(a) $\frac{aQ}{b}$ (b) $\frac{bQ}{a}$

(c) $\frac{bQ}{a+b}$ (d) $\frac{aQ}{a+b}$

55. Three positive charges of equal value q, are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in (see Fig. 20.31).

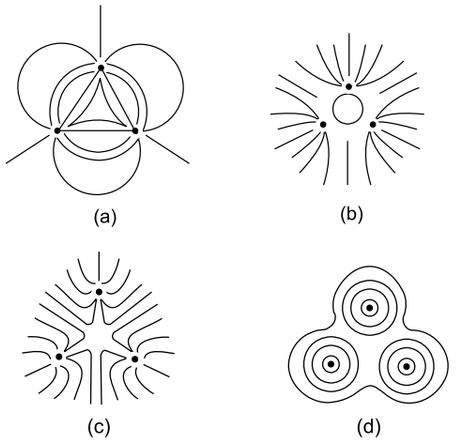


Fig. 20.31

IIT, 2001

56. A quantity X is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$ where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of

(a) resistance (b) charge
(c) voltage (d) current

IIT, 2001

57. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy:

(a) $V_A < V_B$ (b) $V_A > V_B$
(c) $V_A < V_C$ (d) $V_A > V_C$

IIT, 2001

58. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to

(a) x (b) x^2
(c) x^3 (d) $1/x$

IIT, 2002

59. A metallic shell has a point charge q kept inside its circular cavity. The charge is not exactly at the centre of the cavity. Which of the diagrams in Fig. 20.32 correctly represents the electric lines of force?

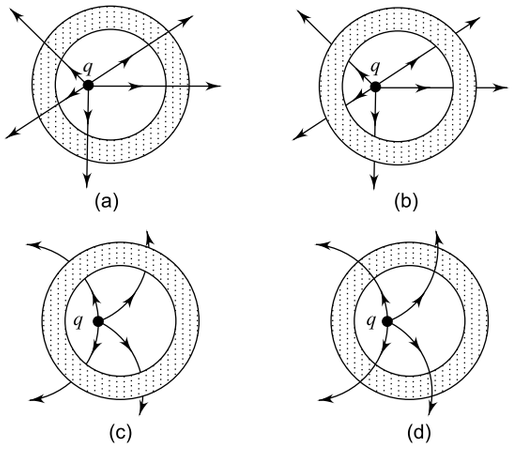


Fig. 20.32

IIT, 2003

60. Three negative point charges $-q$ each, and three positive point charges $+q$, $+q$ and $+Q$ are placed at the vertices of a regular hexagon as shown in Fig. 20.33. For what value of Q will the electric field at O to due to the five charges at A, B, D, E and F be twice the electric field at centre O due to charge Q at C alone?

(a) q (b) $\frac{q}{2}$

(c) $2q$ (d) $5q$

IIT, 2004

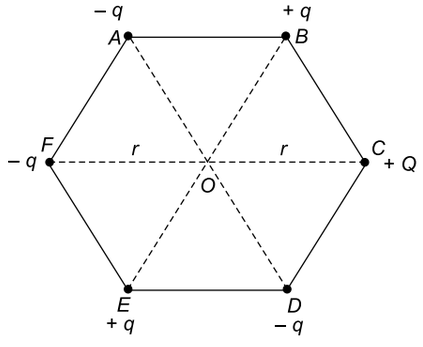


Fig. 20.33

61. Figure 20.34 shows a spherical Gaussian surface and a charge distribution. When calculating the flux of electric field through the Gaussian surface, the electric field will be due to

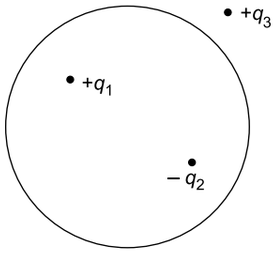


Fig. 20.34

(a) $+q_3$ alone (b) $+q_1$ and $+q_3$

(c) $+q_1$, $+q_3$ and $-q_2$ (d) $+q_1$ and $-q_2$

IIT, 2004

62. Three infinite long plane sheets carrying uniform charge densities

$\sigma_1 = -\sigma$, $\sigma_2 = +2\sigma$ and $\sigma_3 = +3\sigma$

are placed parallel to the x–z plane at $y = a$, $y = 3a$ and $y = 4a$ as shown in Fig. 20.35. The electric field at point P is

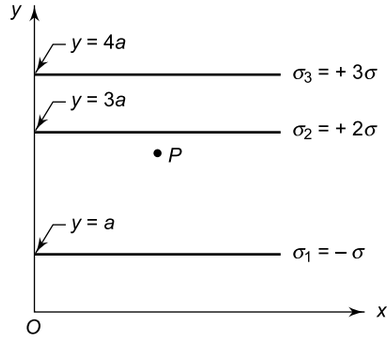


Fig. 20.35

(a) zero (b) $-\frac{2\sigma}{\varepsilon_0}\hat{\mathbf{j}}$

(c) $-\frac{3\sigma}{\varepsilon_0}\hat{\mathbf{j}}$ (d) $\frac{3\sigma}{\varepsilon_0}\hat{\mathbf{j}}$

IIT, 2005

63. A metallic sphere of radius R is charged to a potential V. The magnitude of the electric field at a distance r ($> R$) from the center of the sphere is

(a) $\frac{V}{r}$ (b) $\frac{Vr}{R^2}$

(c) $\frac{VR}{r^2}$ (d) zero

64. Two point charges $q_1 = 1\mu C$ and $q_2 = 2\mu C$ are placed at points A and B 6 cm apart as shown in Fig. 20.36. A third charge $Q = 5\mu C$ is moved from C to D along the arc of a circle of radius 8 cm as shown. The change in the potential energy of the system is

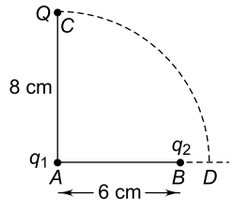


Fig. 20.36

(a) 3.0 J (b) 3.6 J

(c) 5.0 J (d) 7.2 J

65. A partical of mass m and charge $+q$ is midway between two fixed charged particles, each having a charge $+q$ and at a distance $2L$ apart. The middle charge is displaced slightly along the line joining the fixed charges and released. The time period of oscillation is proportional to.

(a) $L^{1/2}$ (b) L

(c) $L^{3/2}$ (d) L^2

66. Five point charges, each equal to $+q$, are placed at five vertices of a regular hexagon of side L. The magnitude of the force on a point charge $-q$ placed at the centre of the haxagon is

(a) $\dfrac{5q^2}{4\pi\varepsilon_0 L^2}$ (b) $\dfrac{3q^2}{4\pi\varepsilon_0 L^2}$

(c) $\dfrac{q^2}{4\pi\varepsilon_0 L^2}$ (d) zero

IIT, 1992

67. Two isolated metal spheres of radii R and $2R$ are charged such that both have the same surface charge density σ. The spheres are located far away from each other. When they are connected by a thin conducting wire, the new surface charge density on the bigger sphere will be

(a) $\dfrac{2\sigma}{3}$ (b) $\dfrac{3\sigma}{5}$

(c) $\dfrac{5\sigma}{6}$ (d) $\dfrac{\sigma}{2}$

IIT, 1996

68. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in Fig. 20.37. The electric field inside the emptied space is

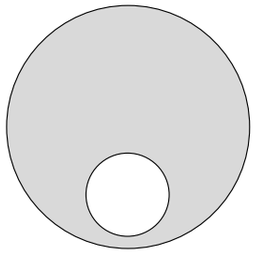


Fig. 20.37

(a) zero everywhere
(b) non-zero and uniform
(c) non-uniform
(d) zero only at its centre

IIT, 2007

69. Positive and negative point charges of equal magnitude are kept at $\left(0, 0, \dfrac{a}{2}\right)$ and $\left(0, 0, \dfrac{-a}{2}\right)$ respectively. The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$ is

(a) positive
(b) negative
(c) zero
(d) depends on the path connecting the initial and final positions

IIT, 2007

70. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral.

(a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder
(b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder
(c) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders
(d) No potential difference appears between the two cylinders when same charge density is given to both the cylinders

IIT, 2007

71. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then,

(a) negative and distributed uniformly over the surface of the sphere
(b) negative and appears only at the point on the sphere closest to the point charge
(c) negative and distributed non-uniformly over the entire surface of the sphere
(d) zero

IIT, 2008

72. Three concentric metallic spherical shells of radii R, $2R$, $3R$, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is

(a) 1 : 2 : 3 (b) 1 : 3 : 5

(c) 1 : 4 : 9 (d) 1 : 8 : 18

IIT, 2009

73. A disk of radius $a/4$ having a uniformly distributed charge 6C is placed in the x-y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8C is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges – 7C and 3C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by

six surfaces $x = \pm a/2$, $y = \pm a/2$, $z = \pm a/2$. The electric flux through this cubical surface is [see Fig. 20.38]

(a) $\frac{-2\text{C}}{\varepsilon_0}$ (b) $\frac{2\text{C}}{\varepsilon_0}$

(c) $\frac{10\text{C}}{\varepsilon_0}$ (d) $\frac{12\text{C}}{\varepsilon_0}$

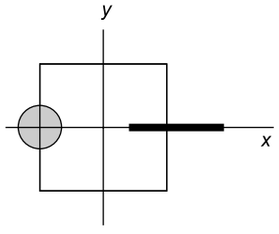


Fig. 20.38

IIT, 2009

74. Consider an electric field $\vec{E} = E_0 \hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in Fig. 20.39) due to this field is

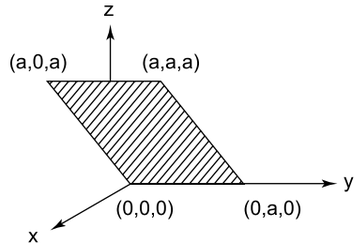


Fig. 20.39

(a) $2E_0a^2$ (b) $\sqrt{2}\,E_0a^2$

(c) E_0a^2 (d) $\frac{E_0a^2}{\sqrt{2}}$

ANSWERS

1. (a)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (a)
7. (b)	**8.** (d)	**9.** (c)	**10.** (d)	**11.** (c)	**12.** (b)
13. (c)	**14.** (b)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (a)
19. (d)	**20.** (c)	**21.** (d)	**22.** (b)	**23.** (b)	**24.** (d)
25. (d)	**26.** (a)	**27.** (d)	**28.** (b)	**29.** (b)	**30.** (c)
31. (a)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (d)	**36.** (d)
37. (d)	**38.** (b)	**39.** (d)	**40.** (d)	**41.** (c)	**42.** (c)
43. (b)	**44.** (d)	**45.** (d)	**46.** (a)	**47.** (a)	**48.** (b)
49. (a)	**50.** (a)	**51.** (b)	**52.** (a)	**53.** (b)	**54.** (d)
55. (c)	**56.** (d)	**57.** (b)	**58.** (b)	**59.** (c)	**60.** (b)
61. (c)	**62.** (c)	**63.** (c)	**64.** (b)	**65.** (c)	**66.** (c)
67. (c)	**68.** (b)	**69.** (c)	**70.** (a)	**71.** (d)	**72.** (b)
73. (a)	**74.** (c)				

SOLUTIONS

1. Work done = final P.E. – initial P.E.

or $W = U_f - U_i$

$$U_i = \frac{1}{4\pi\,\varepsilon_0\,r}\,[(q)\,(-2q) + q(-2q) + (-2q)\,(-2q)]$$

$$= 0$$

$$U_f = \frac{1}{4\pi\,\varepsilon_0\,(2r)}[(q)\,(-2q) + q(-2q) + (-2q)\,(-2q)]$$

$$= 0$$

$\therefore$ $W = 0$. So the correct choice is (a).

2. Charge Q at point P will induce a charge $-Q$ on the inner surface and a charge $+Q$ on the outer surface of the shell (Fig. 20.40). The electric potential at O is

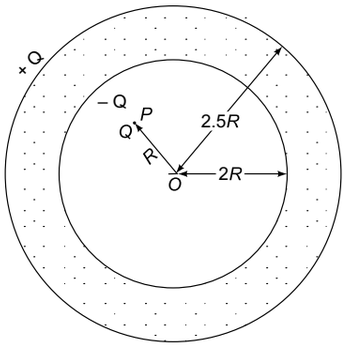


Fig. 20.40

$$V = \frac{1}{4\pi\,\varepsilon_0}\left[\frac{Q}{R} - \frac{Q}{2R} + \frac{Q}{2.5R}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{9Q}{10R}\right)$$

3. Before connection, charge Q on the inner shell induces a charge $-Q$ on the inner surface of the outer shell and a charge $+Q$ on its outer surface. Therefore, the total charge on the outer surface of the outer shell $= Q + 2Q = 3Q$. When the two shells are connected by a conducting wire, the entire charge Q on the inner shell is conducted to the outer shell. Therefore, the charge on the outer shell now is $Q + 2Q = 3Q$, the same as before. Hence there will be no change in its electric potential.

4. $E = \dfrac{Q}{4\pi\varepsilon_0 R^2}$ and $V = \dfrac{Q}{4\pi\varepsilon_0 R}$

Hence if R is doubled, E becomes $E/4$ and V becomes $V/2$. So the correct choice is (a).

5. $\sigma = \dfrac{Q}{4\pi R^2}$. In terms of σ,

$$E = \frac{\sigma}{\varepsilon_0} \text{ and } V = \frac{\sigma R}{\varepsilon_0}$$

If R is halved, E remains the same but V becomes $\dfrac{V}{2}$. So the correct choice is (c).

6. If $\vec{p}$ is the electric dipole moment, then

$$\vec{E}_P = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3}$$

$$\vec{E}_Q = -\frac{\vec{p}}{4\pi\varepsilon_0 (2r)^3}$$

$\therefore\ \vec{E}_P = -16\,E_Q$, which is choice (a).

7. The potential inside a spherical conductor is constant including that on its surface. Hence the correct choice is (b).

8. The potential at every point on the circle due to charge Q is the same. Work done = $q \times$ (potential difference). Hence the correct choice is (d).

9. Refer to Fig. 20.41.

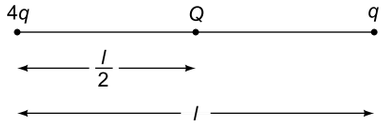


Fig. 20.41

The net force on q will be zero if

$$\frac{q(4q)}{4\pi\varepsilon_0 l^2} + \frac{qQ}{4\pi\varepsilon_0\left(\frac{l}{2}\right)^2} = 0$$

$$\Rightarrow\quad 4q^2 + 4qQ = 0 \quad\Rightarrow\quad 4q(q+Q) = 0$$

Hence $Q = -q$. The force on charge $4q$ is

$$F = \frac{4qQ}{4\pi\varepsilon_0\left(\frac{l}{2}\right)^2} + \frac{4q^2}{4\pi\varepsilon_0 l^2}$$

Putting $Q = -q$, we get

$$F = -\frac{16q^2}{4\pi\varepsilon_0 l^2} + \frac{4q^2}{4\pi\varepsilon_0 l^2} = -\frac{3q^2}{\pi\varepsilon_0 l^2}$$

$\therefore\ |F| = \dfrac{3q^2}{\pi\varepsilon_0 l^2}$, which is choice (c).

10. Refer to Fig. 20.42. The three charges will be in equilibrium if no net force acts on each charge. Charge q is in equilibrium because the forces acting on it by charge Q at A and charge Q at B are equal and opposite. Charge Q at A will be in equilibrium if

$$\frac{qQ}{4\pi\varepsilon_0 l^2} + \frac{Q^2}{4\pi\varepsilon_0 (2l)^2} = 0 \quad\Rightarrow\quad q = -\frac{Q}{4}.$$

Similarly charge Q at B will be in equilibrium if $q = -\dfrac{Q}{4}$. Hence the correct choice is (d).

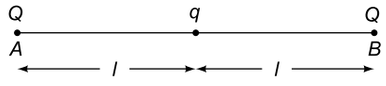


Fig. 20.42

11. If R is the radius of the big drop, we have

$$\frac{4\pi R^3}{3} = 1000 \times \frac{4\pi r^3}{3}$$

which gives $R = 10\,r$. The electrical potential of each droplet is

$$v = \frac{q}{4\pi\varepsilon_0 r}$$

and that of the big drop is

$$V = \frac{1000q}{4\pi\varepsilon_0 R}$$

$$\therefore\qquad \frac{V}{v} = \frac{1000r}{R} = 100 \qquad (\because\ R = 10r)$$

Hence the correct choice is (c).

12. When two identical metallic spheres are brought in contact, the charges on them are equalized due to the flow of free electrons. Thus when an un-charged sphere C is brought in contact with sphere A having a charge $+q$, and then removed, the total charge q

is equally shared between the two so that the charge left on A is $+q/2$ and that developed on C is $+q/2$. The sphere C carrying a charge $+q/2$ is now brought in contact with sphere B which is already carrying a charge $+q$. The total charge is $q/2 + q = +3q/2$ which must distribute equally on B and C. Thus when C is removed, B will have a charge of $+3q/4$ and C also has a charge of $+3q/4$. Hence when C is removed from both A and B,

$$\text{New charge on } A = +\frac{q}{2}$$

$$\text{New charge on } B = +\frac{3q}{4}$$

Since force is proportional to the product of the charges, it follows that the new force of repulsion between A and B is 3/8 of the earlier force (F). Hence, the new force of repulsion between A and B is $3F/8$.

13. Refer to Fig. 20.43. Let us consider forces on a ball, say, Q. Three forces act on it: (i) tension T in the thread, (ii) force mg due to gravity and (iii) force F due to Coulomb repulsion along + x-direction. For equilibrium, the sum of the x and y components of these forces must be zero, i.e.

$$T \cos 60° - F = 0$$

and

$$T \sin 60° - mg = 0$$

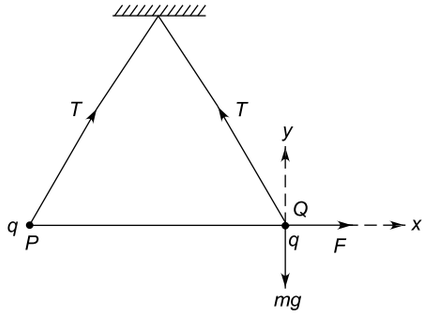


Fig. 20.43

These equations give $F = mg \cot 60° = \frac{\sqrt{3}}{10} \times 10^{-3} \times 10 \times \frac{1}{\sqrt{3}} = 10^{-3}$ N. Now

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2}$$

Putting $F = 10^{-3}$ N, $r = 0.3$ m and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$, we get $q = 10^{-7}$ coulomb.

14. Refer to Fig. 20.44. The electric field E_1 at (a, b) due to q_1 has a magnitude

$$E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{a^2}$$

and is directed along + x-axis. The electric field E_2 at (a, b) due to q_2 has a magnitude

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{b^2}$$

and is directed along + y-axis. The angel θ subtended by the resultant field E with the x-axis is given by

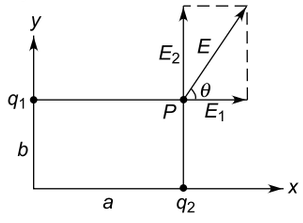


Fig. 20.44

$$\tan\theta = \frac{E_2}{E_1} = \frac{q_2}{q_1} \cdot \frac{a^2}{b^2}$$

$$= \frac{1}{2} \times \left(\frac{2}{1}\right)^2 = 2$$

Hence the correct choice is (b).

15. The correct choice is (d). The electric field E exerts a force qE on charge + q and a force – qE on charge – q of the dipole. Since these forces are equal and opposite, they add upto zero.

16. The correct choice is (b). A torque acts on the dipole which tends to align it along the field.

17. The correct choice is (c). In a non-uniform electric field, a dipole experiences a force which gives it a translational motion and a torque which gives it a rotational motion.

18. The distance of a vertex from the the centre of the cube of side b is $r = \sqrt{3}\, b/2$. Now the potential due to charge q at the centre is $q/4\,\pi\varepsilon_0 r$. Hence the potential due to the arrangement of eight charges (each of magnitude q) at the centre is

$$V = \frac{8q}{4\pi\varepsilon_0 r} = \frac{4q}{\sqrt{3}\,\pi\varepsilon_0 b}$$

19. We know that electric fields are to be added vectorially. From the symmetry of the eight charges with respect to the centre of the cube, it is evident the electric fields at the centre due to two opposite charges cancel in pairs (being equal and opposite). Hence the net electric field at the centre of the cube will be zero.

20. Refer to Fig. 20.45. The distance of point P_1 from charge $+q$ is $r_1 = z - a$ and from charge $-q$ is $r_2 = z + a$.

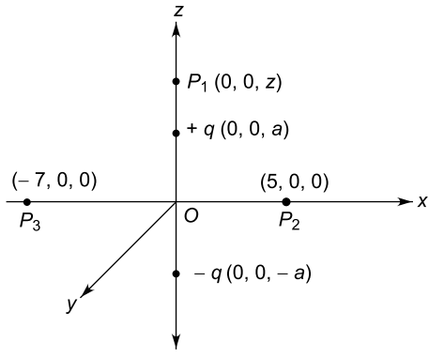


Fig. 20.45

$$\therefore \text{ Potential at } P_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{r_2 - r_1}{r_1 r_2}$$

$$= \frac{2qa}{4\pi\varepsilon_0\left(z^2 - a^2\right)},$$

which is choice (c).

21. Refer to Fig. 20.45 again. Any point on the perpendicular bisector passing through the centre of the dipole is at the same distance from the two charges. Hence the potentials at point $P_2(5, 0, 0)$ and at point $P_3(–7, 0, 0)$ are zero. Since P_2 and P_3 are at the same potential (zero), the potential difference between them is zero. Hence no work will be done in moving a charge from P_2 to P_3. The answer will not change if the path of the charge is changed because the work done is independent of the path taken.

22. Refer to Fig. 20.46. The total potential energy of the arrangement of charges is the sum of the energies of each pair of charges. The potential energy of the system comprising the three charges q_1, q_2 and q_3 is

$$U = W_1 + W_2 + W_3$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

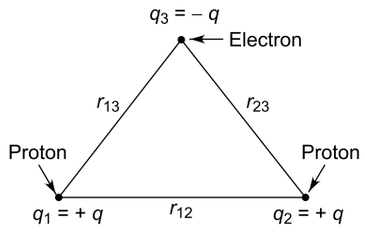


Fig. 20.46

Here $q_1 = q_2 = q = +\ 1.6 \times 10^{-19}$ C (proton), $q_3 = -q = -\ 1.6 \times 10^{-19}$ C (electron), $r_{12} = 1.5$ Å $= 1.5 \times 10^{-10}$ m, $r_{13} = r_{23} = 1$ Å $= 1 \times 10^{-10}$ m and $1/4\pi\varepsilon_0 = 9 \times 10^9$ Nm^{2}C^{-2}. Thus

$$U = -\ \frac{4}{3}\cdot\frac{q^2 \times 10^{10}}{4\pi\varepsilon_0} \text{ joule}$$

$$= -\ \frac{4}{3}\cdot\frac{q \times 10^{10}}{4\pi\varepsilon_0} \text{ eV}$$

$$= -\ \frac{4 \times 1.6 \times 10^{-19} \times 10^{10} \times 9 \times 10^9}{3}$$

$$= -\ 19.2 \text{ eV}$$

23. Charge on electron $(-e) = -\ 1.6 \times 10^{-19}$ C, charge on proton $(e) = 1.6 \times 10^{-19}$ C, separation $r = 0.53$ Å $= 0.53 \times 10^{-10}$ m. If the zero of potential energy is taken to be at infinite separation, the potential energy of the electron-proton system is

$$U = -\ \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r} \text{ joule}$$

$$= -\ \frac{1}{4\pi\varepsilon_0}\cdot\frac{e}{r} \text{ eV}$$

$$= -\ \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)}{0.53 \times 10^{-10}}$$

$$= -\ 27.2 \text{ eV}$$

Hence the correct choice is (b).

24. Points A, B and C are at the same distance from charge $+q$; hence electrical potential is the same at these points, i.e. there is no potential difference between A, B and C. Hence $W_1 = W_2 = 0$.

25. The electric potential at the common centre is

$$V = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2}$$

Now $$\sigma = \frac{q_1}{4\pi r_1^2} = \frac{q_2}{4\pi r_2^2}$$

$$\therefore \quad V = \frac{1}{\varepsilon_0}\left[\frac{q_1 r_1}{4\pi r_1^2} + \frac{q_2 r_2}{4\pi r_2^2}\right]$$

$$= \frac{\sigma}{\varepsilon_0}(r_1 + r_2)$$

Hence the correct choice is (d).

26. If q is charge on the sphere, the electric field on its surface is

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

But $$\sigma = \frac{q}{4\pi r^2}. \text{ Therefore } q = 4\pi r^2\sigma.$$

Hence $$E = \frac{4\pi r^2 \sigma}{4\pi \varepsilon_0 r^2} = \frac{\sigma}{\varepsilon_0}$$

Thus the correct choice is (a).

27. Refer to Fig. 20.47. Forces exerted by charges $-q$ at A and B on charge Q are F_1 and F_2 which are equal and have a magnitude

$$F = \frac{qQ}{4\pi \varepsilon_0 r^2}$$

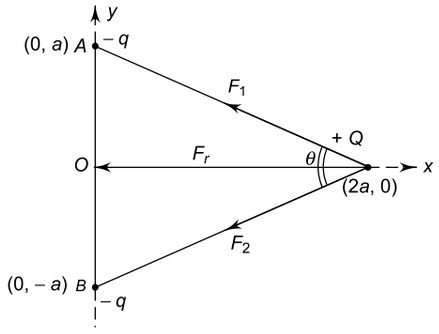


Fig. 20.47

The resultant of these equal forces equally inclined with the x-axis is along the negative x-direction towards the origin. The magnitude of the resultant is given by

$$F_r^2 = F^2 + F^2 + 2F^2 \cos\theta$$
$$= 2F^2 (1 + \cos\theta)$$

Since $F \propto \frac{1}{r^2}$; F_r is also proportional to $(1/r^2)$. Hence charge Q will move towards the origin and because of its inertia it will overshoot the origin O. Thus, charge Q will oscillate about O but its motion is not simple harmonic because the force F_r is not proportional to its instantaneous distance from the origin as $F_r \propto 1/r^2$. Hence the correct choice is (d).

28. The electric field at a distance r from a point charge Q is given by

$$E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$$

If Q is positive, the field is directed radially away from Q. Refer to Fig. 20.48. Let $PA = PB = PC = PD = r$. Then the electric field at P due to charge $2q$ at B is

$$E_1 = \frac{1}{4\pi \varepsilon_0} \frac{2q}{r^2} \text{ along } PD$$

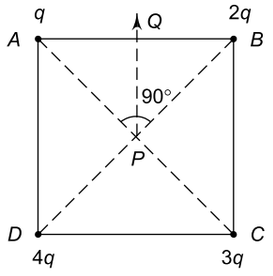


Fig. 20.48

The electric field at P due to charge $4q$ at D is

$$E_2 = \frac{1}{4\pi \varepsilon_0} \frac{4q}{r^2} \text{ along } PB$$

$\therefore$ Net field along PB is $E = E_2 - E_1 = \frac{1}{4\pi \varepsilon_0} \frac{2q}{r^2}$

Similarly, the net electric field at P due to charges q and $3q$ at A and C will be

$$E' = \frac{1}{4\pi \varepsilon_0} \frac{2q}{r^2} \text{ directed along } PA.$$

Thus $E = E'$, but they are mutually perpendicular to each other. therefore, their resultant will be along PQ (see Figure) which is parallel to CB. Hence the correct choice is (b)

29. Let the electric field be E. The force on charge q is $F_A = qE$

Therefore, its acceleration is $a_A = \frac{qE}{m}$

Similarly, the acceleration of charge $4q$ is $a_B = \frac{4qE}{m} = 4a_A$

Let s be the distance travelled by A and B to acquire speeds v_A and v_B. Then (since $u = 0$)

$$v_A^2 = 2a_A s \text{ and } v_B^2 = 2a_B s$$

$\therefore$ $\frac{v_A^2}{v_B^2} = \frac{a_A}{a_B} = \frac{1}{4}$

or $v_A/v_B = 1/2$. Hence the correct choice is (b).

30. Refer to Fig. 20.49.

$$AO = BO = CO = DO = r = \frac{a}{\sqrt{2}}$$

Since electric potential is a scalar and since the charges at corners are equal, the magnitude of the electric potential at point O due to the four charges = 4 times that due to a single charge. Thus

$$V = 4 \times \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q}{r}$$

$$= \frac{1}{\pi \varepsilon_0} \cdot \frac{Q}{a/\sqrt{2}} = \frac{\sqrt{2}Q}{\pi \varepsilon_0 a}$$

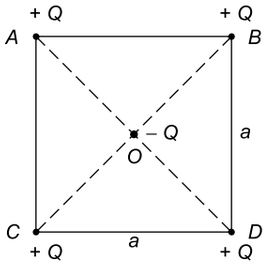


Fig. 20.49

$\therefore$ Work done = $Q\ V = \dfrac{\sqrt{2}Q^2}{\pi \varepsilon_0 a}$. Hence the correct choice is (c).

31. From Gauss's theorem, the electric flux $\int_s \mathbf{E} \cdot \boldsymbol{ds} = \dfrac{q}{\varepsilon_0}$. Hence the correct choice is (a).

32. When any additional negative charge is given to a hollow spherical shell, the potential on its surface falls, but the potential at each point within the shell also falls by the same amount. Hence the potential difference between the given surfaces remains unchanged. Thus, the correct choice is (a).

33. Refer to Fig. 20.50. Potential at C_1 is

$$V_1 = \frac{1}{4\pi \varepsilon_0} \left(\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}\ R} \right)$$

Potential at C_2 is

$$V_2 = \frac{1}{4\pi \varepsilon_0} \left(\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}\ R} \right)$$

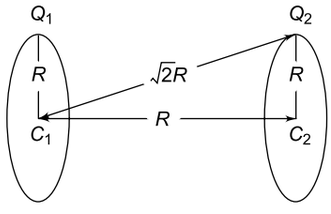


Fig. 20.50

$\therefore$ Work done

$$W = q\ (V_1 - V_2)$$

$$= \frac{q}{4\pi \varepsilon_0} \left[\left(\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}\ R} \right) - \left(\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}\ R} \right) \right]$$

$$= \frac{q}{4\pi \varepsilon_0 \sqrt{2} R}\ (Q_1 - Q_2)\ (\sqrt{2} - 1)$$

34. Force $F = qE$. Therefore, acceleration $a = qE/m$. Hence the distance travelled by the particle in time t is

$$s = \frac{1}{2}\ at^2 = \frac{1}{2} \left(\frac{qE}{m} \right) t^2$$

For electron, $s_e = \dfrac{1}{2} \left(\dfrac{qE}{m_e} \right) t_1^2$

For proton, $s_p = \dfrac{1}{2} \left(\dfrac{qE}{m_p} \right) t_2^2$

Given $s_e = s_p$. Therefore

$$\frac{t_1^2}{m_e} = \frac{t_2^2}{m_p} \quad \text{or} \quad \frac{t_2}{t_1} = \left(\frac{m_p}{m_e} \right)^{1/2}$$

35. The integral $\int_{\infty}^{0} -\boldsymbol{E} \cdot \boldsymbol{dl}$ gives the potential at the centre of the ring, which is zero.

36. The electric field is always perpendicular to the surface of a conductor. On the surface of a metallic solid sphere, the electric field is perpendicular to the surface and directed towards the centre of the sphere. Hence the correct choice is (d).

37. $$V = \frac{1}{4\pi \varepsilon_0} \left\{ \frac{q}{x_0} + \frac{q}{3x_0} + \frac{q}{5x_0} + \ldots \text{upto infinity} \right\}$$

$$+ \frac{1}{4\pi \varepsilon_0} \left\{ \frac{-q}{2x_0} + \frac{-q}{4x_0} + \frac{-q}{6x_0} + \ldots \text{upto infinity} \right\}$$

$$= \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{x_0} \left\{ 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \ldots \text{upto infinity} \right\}$$

$$= \frac{q}{4\pi \varepsilon_0 x_0}\ \log_e (1 + 1) = \frac{q \log_e (2)}{4\pi \varepsilon_0 x_0}$$

38. The net electrostatic energy is ($\because$ hypotenuse side of triangle = $\sqrt{2}\ a$)

$$U = \frac{Qq}{a} + \frac{Qq}{\sqrt{2}\ a} + \frac{qq}{a}$$

For $U = 0$, we require

$$\frac{Qq}{a} + \frac{Qq}{\sqrt{2}\ a} + \frac{q^2}{a} = 0 \text{ or } Q + \frac{Q}{\sqrt{2}} + q = 0$$

which gives $Q = -q \left(\dfrac{\sqrt{2}}{\sqrt{2}+1} \right) = \dfrac{-2q}{2+\sqrt{2}}$

39. A dipole consists of two equal and opposite charges separated by a certain distance. Hence the total charge enclosed in the cube is zero. Therefore, the electric flux is zero.

40. The electric field at a point far away on the perpendicular bisector of a dipole is given by (for $r >> a$), here p is the dipole moment

$$E = \frac{p}{4\pi\varepsilon_0 r^3}$$

Hence the correct choice is (d).

41. Initial kinetic energy of the particle is zero. The gain in kinetic energy in distance x = decrease in potential energy = work done by the electric field to move the particle through a distance x = force × distance = qEx. Hence the correct choice is (c).

42. Given $v_x = 10$ ms^{-1}. Since the electric field is directed along the y-axis, the acceleration of the body along the y-axis is

$$a_y = \frac{qE}{m} = \frac{10^{-6} \times 10^3}{10^{-3}} = 1 \text{ ms}^{-2}$$

Therefore, the velocity of the body along the y-axis at time $t = 10$ s is

$$v_y = a_y t = 1 \times 10 = 10 \text{ ms}^{-1}$$

$\therefore$ Resultant velocity $v = \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ ms}^{-1}$$

Hence the correct choice is (c).

43. Let q be the magnitude of each charge and a the length of each side of the triangle ABC. The potential energy of the system of two equal charges placed at vertices A and B is U (given). This means that U is the work done in bringing a charge q from infinity to vertex B with the charge q at vertex A. Hence the work done in bringing an identical charge q from infinity to the third vertex C = work done to overcome the force of repulsion of q placed at A placed at a distance a from it + work done to overcome the force of repulsion of q placed at B at the same distance a from it = $U + U = 2U$, which is choice (b).

44. Given $E = \frac{q}{4\pi\varepsilon_0 x^2}$. Hence the magnitude of the electric intensity at a distance $2x$ from charge q is

$$E' = \frac{q}{4\pi\varepsilon_0 (2x)^2} = \frac{q}{4\pi\varepsilon_0 x^2} \times \frac{1}{4} = \frac{E}{4}$$

Therefore, the force experienced by a similar charge q at a distance $2x$ is

$$F = qE' = \frac{qE}{4}$$

Hence the correct choice is (d).

45. Charge q will momentarily come to rest at a distance r from charge Q when all its KE is converted to PE, i.e.

$$\frac{1}{2} mv^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{r}$$

Therefore, the distance of closest approach is given by

$$r = \frac{qQ}{4\pi\varepsilon_0} \cdot \frac{2}{mv^2}$$

Thus $r \propto \frac{1}{v^2}$. Hence if v is doubled, r becomes one-fourth. Thus the correct choice is (d).

46. Let the side of the square be a. $OA = OC = r = \frac{a}{\sqrt{2}}$ (see Fig. 20.51).

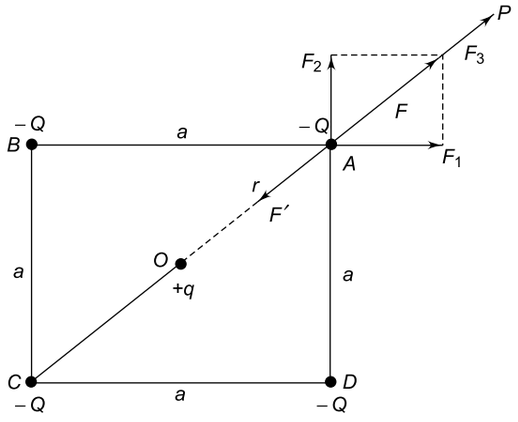


Fig. 20.51

1. Stability of charge + q at the centre

Charges $-Q$ at corners A and C will attract charge $+q$ with equal and opposite forces. Similarly charges $-Q$ at corners B and D will attract charge $+q$ with equal and opposite force. Hence no net force acts on charge $-q$.

2. Stability of charge –Q at any corner

Let us find the forces on charge $-Q$ at corner A. This charge will experience four forces:

(i) Force of repulsion F_1 due to charge $-Q$ at B

(ii) Force of repulsion F_2 due to charge $-Q$ at D

(iii) Force of repulsion F_3 due to charge $-Q$ at C

(iv) Force of attraction F' due to charge $+q$ at O.

Now $$F_1 = F_2 = \frac{Q^2}{4\pi\varepsilon_0 a^2}$$

and $$F_3 = \frac{Q^2}{4\pi\varepsilon_0 (2r)^2} = \frac{Q^2}{4\pi\varepsilon_0 (2a^2)}$$

and $$F' = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(-Q)}{r^2} = -\frac{2qQ}{4\pi\varepsilon_0 a^2}$$

The resultant of F_1 and F_2 is given by

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{2}\,F_1 = \frac{\sqrt{2}\,Q^2}{4\pi\varepsilon_0 a^2}$$

The forces F and F_3 act along AP. Hence the net force acting on charge $-Q$ at A due to charges $-Q$ at B, C and D is

$$F'' = F + F_3$$

$$= \frac{\sqrt{2}\,Q^2}{4\pi\varepsilon_0 a^2} + \frac{Q^2}{4\pi\varepsilon_0(2a^2)} = \frac{Q^2(1+2\sqrt{2})}{4\pi\varepsilon_0(2a^2)}$$

For equilibrium, $F' = F''$, i.e.

$$-\frac{2qQ}{4\pi\varepsilon_0 a^2} = \frac{Q^2(1+2\sqrt{2})}{4\pi\varepsilon_0(2a^2)}$$

or $$q = -\frac{Q}{4}\left(1+2\sqrt{2}\right)$$

Hence the correct choice is (a).

47. The force of repulsion between the two parts is given by

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(Q-q)}{r^2}$$

For F to be maximum, $\frac{dF}{dq} = 0$, i.e.

$$\frac{d}{dq}\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(Q-q)}{r^2}\right] = 0$$

Since r is fixed, we have

$$\frac{d}{dq}\,[q(Q-q)] = 0$$

which gives $$\frac{q}{Q} = \frac{1}{2}$$

Hence the correct choice is (a).

48. For points on the surface of the sphere or outside the sphere, a charged sphere behaves as if the charge is concentrated at its centre. Therefore, the potential at the surface of the sphere is given by

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$$, which is choice (b).

49. At points inside a charged metallic sphere, i.e. for $r < R$, the potential is zero. Hence the correct choice is (a).

50. $\mathbf{E} = -\frac{dV}{dx}\hat{\mathbf{i}}$ where $\hat{\mathbf{i}}$ is a unit vector along the positive x-axis. Hence $\mathbf{E}$ at a point whose x-coordinate is $x = 1$ m is

$$\mathbf{E} = -\frac{d}{dx}\left(4x^2\right)\hat{\mathbf{i}} = -8x\hat{\mathbf{i}} = -8\hat{\mathbf{i}}\ \text{Vm}^{-1}.$$

The negative sign shows that $\mathbf{E}$ is along the negative x-axis. Hence the correct choice is (a).

51. There will be no loss of energy if the potential of the spheres is the same i.e. if

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 R}$$

or $\frac{q}{r} = \frac{Q}{R}$. Hence the correct choice is (b).

52. $\mathbf{r} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ m.

The magnitude of $\mathbf{r}$ is

$$r = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1+4+4} = 3\ \text{m}$$

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}$$

$$= \frac{9\times10^9\times10^{-6}\times10^{-6}}{(3)^2} = 10^{-3}\ \text{N}$$

Hence the correct choice is (a).

53. The system will be in equilibrium if the net force on charge q at one vertex due to charges q at the other two vertices is equal and opposite to the force due to charge Q at the centroid, i.e. (here a is the side of the triangle)

$$\frac{\sqrt{3}\,q^2}{4\pi\varepsilon_0 a^2} = -\frac{Qq}{4\pi\varepsilon_0\left(\frac{a}{\sqrt{3}}\right)^2}$$

which gives $Q = -\frac{q}{\sqrt{3}}$. Hence the correct choice is (b).

54. Charge will flow from A to B until their potentials become equal. If charge q flows from A to B, then

$$\frac{Q-q}{4\pi\varepsilon_0 a} = \frac{q}{4\pi\varepsilon_0 b}$$

or $Q - q = \frac{a}{b}\,q$ which gives $q = \frac{bQ}{a+b}$. Hence charge left on $A = Q - q = Q - \frac{bQ}{a+b} = \frac{aQ}{a+b}$. Hence the correct choice is (d).

55. Since all the three charges are of the same polarity, the lines of force of the electric field in the region around them cannot be closed. Hence choices (a), (b) and (d) are not possible. Notice that in figure

(b), the line in the middle is a closed circular loop. Thus, the only correct choice is (c).

56. The capacitance of a parallel plate capacitor is given by $C = \varepsilon_0 A/d$. Hence the dimensions of $\varepsilon_0 L$ are the same as those of capacitance.

$$\therefore \text{ Dimensions of } \varepsilon_0 L \frac{\Delta V}{\Delta t}$$

$$= \frac{\text{dimension of } C \times \text{dimensions of } V}{\text{time}}$$

$$= \frac{\text{dimension of } Q}{\text{time}} \qquad (\because Q = CV)$$

$$= \frac{\text{charge}}{\text{time}} = \text{current}$$

Hence the correct choice is (d).

57. Electric field is the negative gradient of potential, i.e.

$$E = -\frac{dV}{dx}$$

Thus V decreases as dx increases in the direction of the field. This implies that $V_A > V_B$, which is choice (b).

58. Potential energy of the system when charge Q is at O is

$$U_0 = \frac{qQ}{a} + \frac{qQ}{a} = \frac{2qQ}{a}$$

When charge Q is shifted to position O', the potential energy will be (see Fig. 20.52).

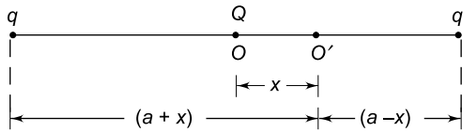


Fig. 20.52

$$U = \frac{qQ}{(a+x)} + \frac{qQ}{(a-x)} = \frac{qQ(2a)}{\left(a^2 - x^2\right)}$$

$$= \frac{2qQ}{a} \times \left(1 - \frac{x^2}{a^2}\right)^{-1}$$

$$\simeq \frac{2qQ}{a} \times \left(1 + \frac{x^2}{a^2}\right) \qquad (\because x << a)$$

$$\therefore \Delta U = U - U_0 = \frac{2qQ}{a}\left(1 + \frac{x^2}{a^2}\right) - \frac{2qQ}{a}$$

$$= \frac{2qQ}{a^3}(x^2)$$

Hence $\Delta U \propto x^2$ which is choice (b).

59. Because the charge is not located at the centre of the cavity, inside the cavity the lines of force are skewed. Hence choice (a) and (b) are incorrect. Outside the shell, the lines of force are the same as if the charge were located at the centre of the cavity. Also there can be no line of force in the metallic body of the shell. Hence choice (d) also incorrect. Thus the correct pattern is shown in (c).

60. The electric fields at centre O due to charges $-q$ at A and D are equal and opposite. Hence they cancel each other. Similarly charges $+q$ at B and E do not contribute to electric field at O. Due to charge $-q$ at F, the electric field at O will be

$$E_1 = \frac{q}{4\pi\varepsilon_0 r^2} \text{ directed from } O \text{ to } F$$

This is the net electric field at O due to the five charges at A, B, D, E and F. The electric field at O due to charge $+Q$ at C is given by

$$E_2 = \frac{Q}{4\pi\varepsilon_0 r^2} \text{ directed from } O \text{ to } F$$

For $E_1 = 2E_2$, we require

$$\frac{q}{4\pi\varepsilon_0 r^2} = \frac{2Q}{4\pi\varepsilon_0 r^2}$$

which gives $Q = \frac{q}{2}$. Hence the correct choice is (b).

61. The electric flux is given by the surface integral $\int \mathbf{E}.\mathbf{ds}$. Here the electric field $\mathbf{E}$ is due to all the charges, both inside and outside the Gaussian surface. Hence the correct choice is (c).

62. The electric field at a point P due to an infinite long plane sheet carrying a uniform charge density σ is given by

$$E = \frac{\sigma}{2\varepsilon_0}$$

It is independent of the distance of point P from the sheet and is, therefore, uniform. The direction of the electric field is away from the sheet and perpendicular to it if σ is positive and is towards the sheet and perpendicular to it if σ is negative. Hence

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right) \text{ along } -\text{ve } y\text{–direction}$$

$$E_2 = \frac{2\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right) \text{ along } -\text{ve } y\text{–direction}$$

and $$E_3 = \frac{3\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right) \text{ along } -\text{ve } y\text{–direction}$$

From the superposition principle, the net electric field at point P is

$$E = E_1 + E_2 + E_3$$
$$= \frac{\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right) + \frac{2\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right) + \frac{3\sigma}{2\varepsilon_0}\left(-\hat{\mathbf{j}}\right)$$
$$= -\frac{3\sigma}{\varepsilon_0}\hat{\mathbf{j}}, \text{ which is choice (c).}$$

63. Let the charge on the sphere be Q. Then

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

which gives $Q = 4\pi\varepsilon_0 RV$

The electric field at a distance r is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{4\pi\varepsilon_0 RV}{4\pi\varepsilon_0 r^2} = \frac{RV}{r^2}$$

Thus the correct choice is (c).

64. If charge Q is moved from C to D along the arc, the potential energy between pairs (q_1, Q) and (q_1, q_2) will not change as the distance between them remains unchanged ($\because AC = AD$). The potential energy of the pair of chages q_2 and Q will change.

Now, distance $BC = \sqrt{(8)^2 + (6)^2} = 10$ cm and $BD = 8 - 6 = 2$ cm. Therefore, change in P.E. is

$$\Delta U = \frac{q_2 Q}{4\pi\varepsilon_0}\left[\frac{1}{BD} - \frac{1}{BC}\right]$$

$$= (2\times10^{-6})\times(5\times10^{-6})\times(9\times10^{-9})\left(\frac{1}{0.02} - \frac{1}{0.1}\right)$$

$= 3.6$ J, which is choice (b).

65. If the middle charge is displaced by a distance x, the net force acting it, when it is released, is

$$F = \frac{1}{4\pi\varepsilon_0}\times\frac{q^2}{(L+x)^2} - \frac{1}{4\pi\varepsilon_0}\times\frac{q^2}{(L-x)^2}$$

$$= \frac{4q^2Lx}{4\pi\varepsilon_0(L^2 - x^2)^2}$$

For $x << L$, $F = -\frac{q^2 x}{\pi\varepsilon_0 L^3} = -kx$

where $k = \frac{q^2}{\pi\varepsilon_0 L^3}$

Now $T = 2\pi\sqrt{\frac{m}{k}}$

So, the correct choice is (c).

66. As shown in Fig. 20.53, all forces cancel in pairs, except the forces F on charge $-q$ due to charge $+q$ at vertex D (i.e. the vertex opposite to the empty vertex). The magnitude of the net force on $-q$ is

$$F = \frac{q\times q}{4\pi\varepsilon_o (OD)^2} = \frac{q^2}{4\pi\varepsilon_o L^2}$$

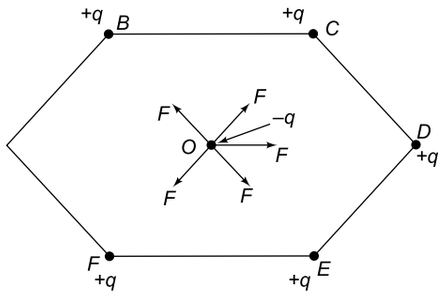


Fig. 20.53

67. Initial charge on sphere of radius R is $Q_1 = 4\pi R^2\sigma$ and on sphere of radius $2R$ is $Q_2 = 4\pi(2R)^2\sigma = 16\pi R^2\sigma$

Total initial charge is $Q = Q_1 + Q_2 = 20\pi R^2\sigma$

Initial potential of the sphere of radius R is

$$V_1 = \frac{Q_1}{4\pi\varepsilon_o R}$$

and of the sphere of radius $2R$ is

$$V_2 = \frac{Q_2}{4\pi\varepsilon_o (2R)}$$

When the spheres are connected by a thin wire, charge will flow from one sphere to the other until their potentials become equal. Let Q'_1 and Q'_2 be the new charges, then the potential of each sphere will be

$$V = \frac{Q'_1}{4\pi\varepsilon_o R} = \frac{Q'_2}{4\pi\varepsilon_o (2R)}$$

which gives $Q'_1 = \frac{Q'_2}{2}$

From conservation of charge, we have

$$Q_1 + Q_2 = Q'_1 + Q'_2$$

$$\Rightarrow 20\pi R^2\sigma = \frac{Q'_2}{2} + Q'_2 = \frac{3Q'_2}{2}$$

$$\Rightarrow \quad Q'_2 = \frac{40}{3}\pi R^2\sigma$$

$\therefore$ New surface charge density on the sphere of radius $2R$ is

$$\sigma' = \frac{Q'_2}{4\pi(2R)^2} = \frac{\frac{40}{3}\pi R^2\sigma}{16\pi R^2} = \frac{5\sigma}{6}$$

68. Let O be the centre of the sphere and Q be the centre of the cavity. Let r be the separation between

them. Let $OP = b$ and $QP = a$. Here P is a point inside the cavity. Let ρ be the charge density. From the superposition principle, the net electric field at point P is given by [See Fig. 20.54]

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{b} - \frac{\rho}{3\varepsilon_0}\vec{a} = \frac{\rho}{3\varepsilon_0}(\vec{b} - \vec{a})$$

$$= \frac{\rho\vec{r}}{3\varepsilon_0} \qquad (\because \vec{r} + \vec{a} = \vec{b})$$

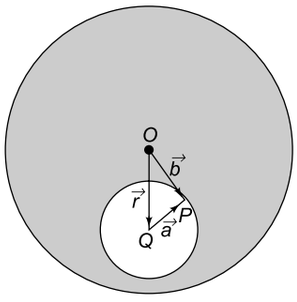


Fig. 20.54

Hence the electric field at any point inside the emptied space is finite and constant. Thus the correct choice is (b).

69. The two charges constitute an electric dipole. The points $(-a, 0, 0)$ and $(0, a, 0)$ are at the same distance from the positive and negative charges of equal magnitude. Hence the electric potential is zero at these points. Hence the work done is zero.

70. When a charge density is given to the inner cylinder, an electric fields is produced between the inner and outer cylinders. Hence a potential difference appears between the two cylinders. If a charge density is given to the outer cylinder, the same potential appears on the inner and the outer cylinders. Hence, in this case, there is no potential difference between them. Thus the correct choice is (a).

71. When a positive charge $+Q$ is placed outside a neutral conducting sphere, it will induce a negative charge $-Q$ on the side of the sphere closer to it and an equal positive charge $+Q$ on the opposite side of the sphere. Thus the net charge on the sphere will be zero, which is choice (d). [see Fig. 20.55]

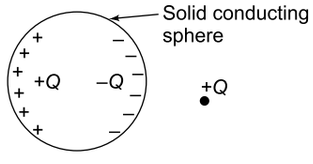


Fig. 20.55

72. Charge Q_1 on shell 1 induces a charge $-Q_1$ on the inner surface of shell 2 and a charge $+Q_1$ on its outer surface, so that the total charge on the outer surface of shell 2 is $(Q_1 + Q_2)$. This charge induces a charge $-(Q_1 + Q_2)$ on the inner surface of shell 3 and a charge $+(Q_1 + Q_2)$ on its outer surface so that the total charge on the outer surface of shell 3 is $(Q_1 + Q_2 + Q_3)$ as shown in Fig. 20.56. Given $\sigma_1 = \sigma_2 = \sigma_3$

$$\Rightarrow \frac{Q_1}{4\pi\varepsilon_0 R^2} = \frac{(Q_1 + Q_2)}{4\pi\varepsilon_0 (2R)^2} = \frac{(Q_1 + Q_2 + Q_3)}{4\pi\varepsilon_0 (3R)^2}$$

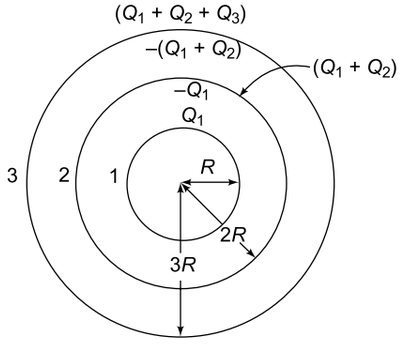


Fig. 20.56

Solving we get $Q_1 = 3Q_2 = 5Q_3$. Hence the correct choice is (b).

73. Electric flux through the cubical surface $= \frac{q}{\varepsilon_0}$ where q is the net charge enclosed in the surface. Since half the disc lies inside the surface, one-fourth of the rod lies inside the surface, point charge -7C lies inside the surface and point charge 3C lies outside the surface, the net charge enclosed in the surface is

$$q = \frac{6\text{C}}{2} + \frac{8\text{C}}{4} - 7\text{C} + 0$$

$$= 3\text{C} + 2\text{C} - 7\text{C} = -2\text{C}$$

$\therefore$ Electric flux $= \frac{-2\text{C}}{\varepsilon_0}$. So the correct choice is (a).

74. Refer to Fig. 20.57.

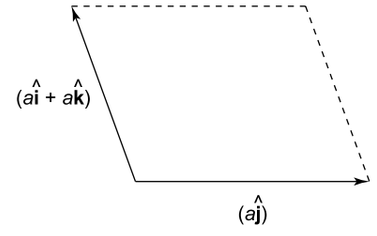


Fig. 20.57

Electric flux $\phi = \vec{E} \cdot \vec{A}$

Area vector $\vec{A} = a\hat{\mathbf{j}} \times (a\hat{\mathbf{i}} + a\hat{\mathbf{k}})$

$$\therefore \quad \phi = E_0\,\hat{\mathbf{i}}\cdot[a\,\hat{\mathbf{j}}\times(a\,\hat{\mathbf{i}}+a\,\hat{\mathbf{k}})] = E_0\,\hat{\mathbf{i}}\cdot[-a^2\,\hat{\mathbf{k}}+a^2\,\hat{\mathbf{i}}] = E_0a^2\,(-\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}+\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}) = E_0a^2\,(0+1) = E_0a^2$$

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Choose the correct statements from the following.
- (a) If the electric field is zero at a point, the electric potential must also be zero at that point.
- (b) If electric potential is constant in a given region of space, the electric field must be zero in that region.
- (c) Two different equipotential surfaces can never intersect.
- (d) Electrons move from a region of lower potential to a region of higher potential.

2. Figures 20.58 (a) and (b) show the lines of force of the electric field of a positive charge (+ q) and a negative charge (– q) respectively.

Which of the following statements are correct?
- (a) Potential at P is greater than that at Q.
- (b) Potential at A is greater than that at B.
- (c) Potential energy at P is less than that at Q.
- (d) Potential energy at A is greater than that at B.

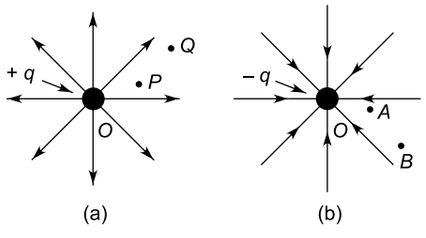


Fig. 20.58

3. The lines of force of the electric field of a positive charge (+ q) and a negative charge (– q) are shown in Figs. 20.36 (a) and (b) above. Then
- (a) the work done in moving a small positive charge (+ q_0) from Q to P will be positive
- (b) the work done in moving a small negative charge (– q_0) from B to A will be positive
- (c) in going from Q to P, the kinetic energy of a small negative charge (– q_0) increases
- (d) in going from B to A, the kinetic energy of a small negative charge (– q_0) decreases

4. Which of the following statements are correct?
- (a) The work done by the electric field of a nucleus in moving an electron around it in a complete orbit is zero irrespective of whether the orbit is circular or elliptical.
- (b) The equipotential surfaces corresponding to the electric field of an isolated point charge are concentric spheres with the point charge as the common centre.
- (c) If Coulomb's law involved $1/r^3$ dependence instead of $1/r^2$, Gauss's law would still hold good.
- (d) A single conductor cannot have any capacitance.

5. A pendulum bob of mass m carrying a charge q is at rest with its string making an angle θ with the vertical in a uniform horizontal electric field E. The tension in the string is
- (a) $\dfrac{mg}{\sin\theta}$ (b) $\dfrac{mg}{\cos\theta}$
- (c) $\dfrac{qE}{\sin\theta}$ (d) $\dfrac{qE}{\cos\theta}$

6. Four point charges + q, + q, – q and – q are placed respectively at corners A, B, C, and D of a square. Then
- (a) the potential at the centre O of the square is zero.
- (b) the electric field at the centre O of the square is zero.
- (c) If E is the mid-point of side BC, the work done in carrying an electron from O to E is zero.
- (d) If F is the mid-point of side CD, the work done in carrying an electron from O to F is zero.

7. Six charges, each equal to + q, are placed at the corners of a regular hexagon of side a. The electric potential at the point where the diagonals intersect is V and the electric field at that point is E. Then

(a) $V = 0$ (b) $V = \dfrac{6q}{4\pi\varepsilon_0 a}$

(c) $E = 0$ (d) $E = \dfrac{6q}{4\pi\varepsilon_0 a^2}$

8. Two parallel plane sheets 1 and 2 carry uniform charge densities $+\sigma$ and $-\sigma$ as shown in Fig. 20.59. The electric fields in the regions marked I, II and III are E_1, E_2 and E_3 respectively

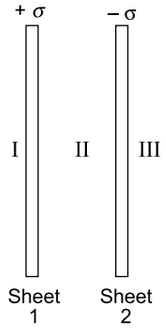


Fig. 20.59

(a) $E_1 = 0$

(b) $E_2 = \dfrac{\sigma}{\varepsilon_0}$

(c) $E_3 = 0$

(d) $E_1 = E_3 = \dfrac{2\sigma}{\varepsilon_0}$

9. Electric potential V due to a spherically symmetric charge system varies with distance r as shown in Fig. 20.60

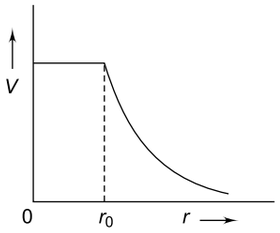


Fig. 20.60

Given $V = \dfrac{Q}{4\pi\varepsilon_0 r_0}$ for $r \le r_0$

and $V = \dfrac{Q}{4\pi\varepsilon_0 r}$ for $r > r_0$

Which of the following statements are true?

(a) Electric field due to the charge system is discontinuous at $r = r_0$.

(b) The net charge enclosed in a sphere of radius $r = 2r_0$ is Q.

(c) No charge exists at any point in a spherical region of radius $r < r_0$.

(d) Electrostatic energy inside the sphere of radius $r = r_0$ is zero.

IIT, 2006

10. A sphere of radius R is made of a non-conducting material and carries a positive charge Q. Figure 20.61 shows the variation of electric field E with distance r from the centre of the sphere.

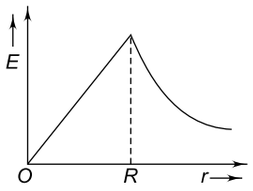


Fig. 20.61

Form the graph we conclude that

(a) the electric field is always perpendicular to the surface of the sphere.

(b) for $r > R$, the entire charge Q may be assumed to be concentrated at the centre of the sphere.

(c) the electric potential is maximum at centre of the sphere.

(d) the electric potential is zero at centre of the sphere.

11. A sphere of raduis R is made of a conducting material and carries a charge Q. Choose the correct statements from the following.

(a) Charge Q resides on the surface of the sphere.

(b) The electric field is always perpendicular to the surface of the sphere.

(c) The electric field is zero inside the sphere.

(d) No work is needed to move a charge from one point to another on the surface of the sphere.

12. A proton and an electron are placed in a uniform electric field. The magnitudes of the force experienced by proton and electron are F_p and F_e and a_p and a_e are the respective magnitudes of their accelerations. Then

(a) $F_e > F_p$ (b) $F_e = F_p$

(c) $a_e > a_p$ (d) $a_e < a_p$

13. A metal ring of radius R carries a charge Q distributed uniformly on it. A point P lies on the axis of the ring at a distance r from its center. Then

(a) The potential at the centre of the ring is zero.

(b) The electric field at the centre of the ring zero

(c) For $r >> R$, the potential varies as $1/r$.

(d) For $r >> R$, the electric field varies as $1/r^2$.

14. An infinitely long straight wire is uniformly charged. A point P lies at a perpendicular distance r from it . Then

(a) the electric field varies as $1/r$.

(b) the electric field varies as $1/r^2$.

(c) the electric field lines are parallel and equidistant straight lines prependicular to the wire.

(d) the electric field is perpendicular to the wire.

15. An infine plane sheet is uniformly charged. A point P lises at a perpendicular distance r from it. Then

(a) the electric field varies as $1/r$.

(b) the electric field varies as $1/r^2$.

(c) the electric field lines are parallel and equi distant straight lines prependicular to the plane sheet

(d) the electric field is perpendicular to the sheet.

16. In Gauss's theorem $\oint_S \mathbf{E}\cdot\mathbf{ds} = \frac{q}{\varepsilon_0}$. The surface integral is evaluated by choosing a closed surface called the Gaussian surface. Here
(a) the closed surface can be of any shape or size.
(b) q is the net charge enclosed inside the Gaussian surface; charges outside the surface are not considered.
(c) $\mathbf{E}$ is the electric field due to all the charges both inside and outside the surface.
(d) The exact location of the charges inside the surface does not affect the value of the integral.

17. Two equal point charges q each are held at $x = +a$ and $x = -a$. A third charge Q is placed at $x = 0$. The potential of the system will
(a) decrease if Q is displaced by a small distance along the x-axis
(b) increase if Q is displaced by a small distance along the x-axis
(c) decrease if Q is displaced by a small distance along the y-axis
(d) increase if Q is displaced by a small distance along the y-axis

18. The electric potential at a point P at a distance x from a point charge is given by

$$V = \frac{k}{r}$$

where k is a constant.
(a) k is dimensionsless
(b) the dimensions of k are $[ML^{-3}T^{-3}A^{-1}]$
(c) Electric field at $P = \frac{k}{r^2}$.
(d) Electric field at $P = kr^2$.

19. Two point charges $q_1 = 4\mu C$ and $q_2 = -1\mu C$ are placed at $x = 0$ and $x = 15$ cm on the x-axis.
(a) Electric potential is zero at $x = 3$ cm.
(b) Electric potential is zero at $x = 12$ cm.
(c) Electric field cannot be zero between $x = 0$ and $x = 15$ cm.
(d) Electric field is zero at $x = \pm 30$cm.

20. Choose the correct statement(s) from the following.
(a) The electric potential due to a point charge at a distance r from it varies as $1/r$
(b) Electric potential at a distance r from the centre of a charged sphere varies as $1/r$ provided r is less than the radius of the sphere
(c) Electric field at a distance r from a point charge varies as $1/r^2$
(d) Electric field inside a charged sphere is zero.

IIT, 1980

21. Two small balls, each having a charge $+Q$ are suspended by two insulating strings each of length L from a hook fixed to a stand. The whole set up is taken in a satellite orbiting the earth. In the satellite the angle between the strings is θ and the tension in each string is T. Then
(a) θ = zero (b) $\theta = 180°$
(c) $T = \frac{Q^2}{16\pi\varepsilon_0 L^2}$ (d) $T = \frac{Q^2}{4\pi\varepsilon_0 L^2}$

IIT, 1986

22. A positively charged thin metal ring of radius R is fixed in the x-y plane with its centre at origin O. A negatively charged particle P is released from rest at the point $(0, 0, z)$ where $z > 0$. Then the motion of P is
(a) periodic for all values of z satisfying $0 < z < \infty$
(b) simple harmonic for all values of z satisfying $0 < z \le \infty$
(c) approximately simple harmonic provided $z << R$.
(d) such that it crosses O and continues to move along the negative z-axis towards $z = -\infty$.

IIT, 1998

23. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre
(a) increases as r increases for $r < R$
(b) decreases as r increases for $0 < r < \infty$
(c) decreases as r increases for $R < r < \infty$
(d) is discontinuous at $r = R$.

IIT, 1998

24. An ellipsoidal cavity is made within a perfect conductor. A positive charge q is placed in the cavity and A and B are two points on the surface of the cavity as shown in Fig. 20.62. Then

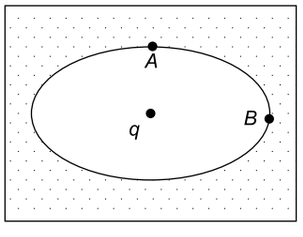


Fig. 20.62

(a) Electric field near A in the cavity = electric field near B in the cavity.
(b) Charge density at A = charge density at B
(c) Potential at A = potential at B

(d) Total electric flux through the surface of the cavity is $\frac{q}{\epsilon_0}$.

IIT, 1999

25. A positive charge q is fixed at the origin. An electric dipole with dipole moment $\vec{p}$ is placed along the x-axis far away from the origin with $\vec{p}$ pointing along the positive x-axis. The kinetic energy when it reaches a distance x from the origin is K and the magnitude of the force experienced by charge q at this moment is F. Then

(a) K varies as $1/x$ (b) K varies as $1/x^2$
(c) F varies as $1/x^2$ (d) F varies as $1/x^3$

IIT, 2003

26. Electric potential V due to a spherically symmetric charge system varies with distance r as shown in Fig. 20.63.

Given $V = \frac{Q}{4\pi\varepsilon_0 r_0}$ for $r \le r_0$

and $V = \frac{Q}{4\pi\varepsilon_0 r}$ for $r > r_0$

Which of the following statements are true?

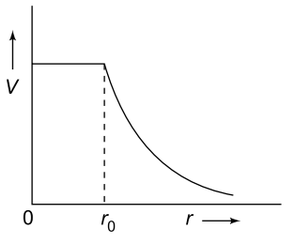


Fig. 20.63

(a) Electric field due to the charge system is discontinuous at $r = r_0$.
(b) The net charge enclosed in a sphere of radius $r = 2r_0$ is Q.
(c) No charge exists at any point in a spherical region of radius $r < r_0$.
(d) Electrostatic energy inside the sphere of radius $r = r_0$ is zero.

IIT, 2005

27. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s).

(a) The angular momentum of the charge $-q$ is constant
(b) The linear momentum of the charge $-q$ is constant
(c) The angular velocity of the charge $-q$ is constant
(d) The linear speed of the charge $-q$ is constant

IIT, 2009

28. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($< R_A$) are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then

(a) $E_A^{\text{inside}} = 0$
(b) $Q_A > Q_B$
(c) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$
(d) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

IIT, 2011

29. Which of the following statement(s) is/are correct?

(a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.
(b) The Guass law can be used to calculate the field distribution around an electric dipole
(c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
(d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

IIT, 2011

ANSWERS AND SOLUTIONS

1. Statement (a) is incorrect. If electric field is zero at a point, electric potential is not necessarily zero at that point. For example, the electric field is zero at a point exactly mid-way between two equal charges of the same sign, but the potential at this point is twice that due to a single charge. Statement (b) is correct because $\mathbf{E} = -\nabla V$. If V is constant $\nabla V = 0$. Hence $\mathbf{E} = 0$ in that region. Statement (c) is also correct. The electric field at a point on an equipotential surface is normal to the surface at that point. If two different equipotential surfaces intersect, there would be two directions for the normals to the two surfaces at the point of intersection. Hence there will be two directions of the electric field at that point, which is not possible. Statement (d) is correct. Since electron has a negative charge, it has less potential energy at a point where the potential is higher and vice versa.

Hence, in an electric field, electrons will move from the region of lower potential to the region of higher potential.

2. The potential at a point in the electric field of a charge + q is given by

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

where r is the distance of that point from the charge. Referring to Fig. 20.35 (a), the potentials at points P and Q are

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{OP} \text{ and } V_Q = \frac{1}{4\pi\varepsilon_0}\frac{q}{OQ}$$

since $OP < OQ$, $V_P > V_Q$. Hence statement (a) is correct.

The potential at a point in the electric field of a charge – q is given by

$$V = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Referring to Fig. 20.35(b), the potentials at points A and B are

$$V_A = -\frac{1}{4\pi\varepsilon_0}\frac{q}{OA} \text{ and } V_B = -\frac{1}{4\pi\varepsilon_0}\frac{q}{OB}$$

Since $OA < OB$, the potential at A is more negative than at B, i.e. $V_B > V_A$. Hence statement (b) is in-correct.

We know that the potential energy of a charge q_2 in the field of a charge q_1 at a distance r from it is given by

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r}$$

Referring to Fig. 20.21(a) the potential energy of a small negative charge ($-q_0$) in the field of a positive charge (+ q) at points P and Q is (setting $q_1 = q$ and $q_2 = -q_0$)

$$U_p = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q q_0}{OP} \text{ and } U_Q = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q q_0}{OQ}$$

Since $OP < OQ$, it is clear that $U_Q > U_P$. Hence statement (c) is correct.

Referring to Fig. 20.21(b), the potential energy of a small negative charge ($-q_0$) in the field of a negative charge ($-q$) at points A and B is (setting $q_1 = -q$ and $q_2 = -q_0$)

$$U_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q q_0}{OA} \text{ and } U_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q q_0}{OB}$$

Since $OA < OB$, $U_A > U_B$. Hence statement (d) is correct.

3. The small positive charge (+ q_0) will tend to move from P to Q due to the force of repulsion exerted on it by the charge (+ q). Thus the electric field of charge (+ q) does positive work on charge q_0 to move it from P to Q. Hence the work done by the field of charge (+ q) in moving the charge (+ q_0) from Q to P will be negative. Hence statement (a) is incorrect.

The work done by the external agency to move a negative charge ($-q_0$) from B to A is positive, since the external agency has to overcome the force of repulsion exerted by the negative charge ($-q$) on the small negative charge ($-q_0$). Hence statement (b) is correct.

In going from Q to P, a small negative charge is speeded up due to the force of attraction exerted on it by the positive charge (+ q). Hence statement (c) is correct.

In going from B to A, the small negative charge ($-q_0$) is slowed down due to the force of repulsion exerted on it by the negative charge ($-q$). Hence the kinetic energy of the small negative charge decreases. Hence statement (d) is correct.

4. The work done by an electric field E in moving a charge ($-q$) around a closed path of any shape (circular or elliptical) is given by

$$W = q \oint \mathbf{E}\cdot\mathbf{d1}$$

Now, we know that the line integral of an electrostatic field around a closed path is zero, i.e.

$$\oint \mathbf{E}\cdot\mathbf{d1} = 0$$

Hence the work done $W = 0$ irrespective of whether the path is circular or elliptical. Hence statement (a) is correct.

We know that the potential at points equidistant from a point charge is the same. Hence, the equipotential surfaces of the electric field of a point charge are concentric spheres with the point charge as the common centre. Hence statement (b) is correct.

The derivation of Gauss's law assumes $1/r^2$ dependence of distance between charges in Coulomb's law. Gauss's law will not hold if Coulomb's law involved $1/r^3$ or any other power of the distance r. Hence statement (c) is incorrect.

A single conductor can have capacitance. It is a capacitor whose one plate is at infinity. For example, a single spherical conductor of radius r has a capacitance $C = 4\pi\varepsilon_0 r$. Hence statement (d) is incorrect.

5. Refer to Fig. 20.64. Since the bob is in equilibrium the forces acting on it are as shown in the figure. For equilibrium, we have

$$mg = T\cos\theta$$

and $$qE = T\sin\theta$$

Thus $$T = \frac{mg}{\cos\theta} = \frac{qE}{\sin\theta}.$$

Hence the correct choices are (c) and (b).

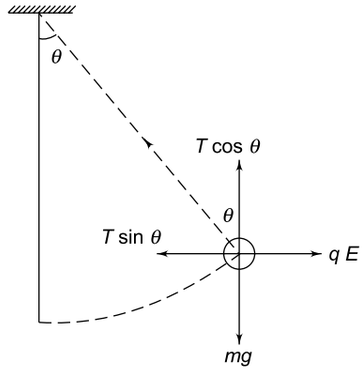


Fig. 20.64

6. Refer to Fig. 20.65. Potential at O is

$$V_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{q}{r} - \frac{q}{r} - \frac{q}{r}\right) = 0$$

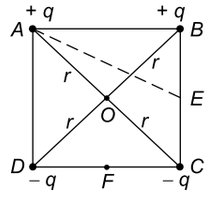


Fig. 20.65

Refer to Fig. 20.65 again. $(AE)^2 = a^2 + \left(\frac{a}{2}\right)^2$

$= \frac{5a^2}{4}$, giving $AE = \frac{\sqrt{5}a}{2}$. Similarly $DE = \frac{\sqrt{5}a}{2}$

and $BE = \frac{a}{2}$, $CE = \frac{a}{2}$. Potential at E is

$$V_E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{AE} + \frac{q}{BE} - \frac{q}{DE} - \frac{q}{CE}\right) = 0$$

Work done in carrying a charge $-e$ from O to E is

$$W = -e\,(V_E - V_O)$$
$$= -e\,(0 - 0) = 0$$

Potential at F is

$$V_F = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{AF} + \frac{q}{BF} - \frac{q}{DF} - \frac{q}{CF}\right)$$

Now, $AF = BF = \sqrt{5}\,a/2$ and $CF = DF = a/2$. Putting these values, we get

$$V_F = \frac{q}{\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{5}} - 1\right)$$

Work done in carrying a charge $-e$ from O to F is

$$W = -e\,(V_F - V_0) = -eV_F$$
$$= \frac{qe}{\pi\varepsilon_0 a}\left(1 - \frac{1}{\sqrt{5}}\right)$$

Hence the correct choice is (a) and (c).

7. The distance of the point of intersection of diagonals = side of the hexagon = a. The potential at this point due to each charge = $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a}$. Therefore, total potential = $\frac{1}{4\pi\varepsilon_0}\cdot\frac{6q}{a}$

The net electric field at the point of intersection of diagonals is zero because the electric field at this point due to equal charges at opposite corners will cancel each other in pairs.

So the correct choices are (b) and (c).

8. Refer to Fig. 20.66. Since the charge on sheet 1 is positive, it produces a field of magnitude $E = \sigma/2\varepsilon_0$ which points away from it; to the left in region I and to the right in regions II and III. Sheet 2 produces a field of magnitude $E = \sigma/2\varepsilon_0$. Since the charge of sheet 2 is negative, the direction E is towards it; i.e. to the right in regions I and II and to the left in region III.

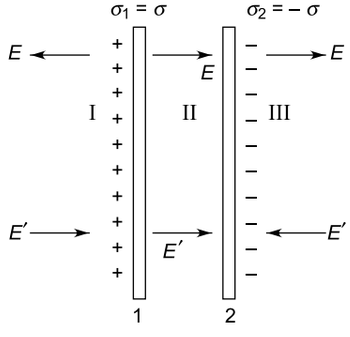


Fig. 20.66

Region I : $E_1 = E - E' = -\frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$

Region II : $E_2 = E + E' = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

directed to the right.

Region III : $E_3 = E - E' = 0$.

Thus the correct choices are (a); (b) and (c).

9. Electric field in the region $r > r_0$ is given by

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{Q}{4\pi\varepsilon_0 r}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 r^2}$$

For $r \le r_0$, $E = -\frac{d}{dr}\left(\frac{Q}{4\pi\varepsilon_0 r_0}\right) = 0$

($\because$ r_0 = constant)

Hence, the electric field is discontinuous at $r = r_0$. Therefore, statement (a) is true.

For $r < r_0$, $E = 0$. Hence the charge resides only on the spherical surface of radius $r = r_0$. No charge exists in the region for which $r < r_0$. Therefore, statement (c) is also true. Electric energy density is given by

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Since for $r < r_0$, $E = 0$; $u = 0$ for $r < r_0$. Hence statement (d) is true.

Let Q' be the net charge enclosed inside the spherical surface of radius $r = 2r_0$. Then from Gauss's theorem, we have

$$\int \mathbf{E}\cdot d\boldsymbol{s} = \frac{Q'}{\varepsilon_0}$$

or $$E \times 4\pi r^2 = \frac{Q'}{\varepsilon_0}$$

or $$\frac{Q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2 = \frac{Q'}{\varepsilon_0}$$

or $Q' = Q$, which is independent of r as long as r is greater than r_0. Hence statement (b) is also true. All the four choices (a), (b), (c) and (d) are correct

10. For $r \le R$, the electric field is given by

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}, \text{ i.e. } E \propto r$$

For $r > R$, the entire charge Q may be assumed to be concentrated at the centre of the sphere. Hence for $r > R$,

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}, \text{ i.e. } E \propto 1/r^2$$

Since the sphere is made of a non-coducting meterial, charge Q is uniformly distributed over the entire volume of the sphere. For a unifrom distribution of charge, the potential is maximum at the centre of the sphere. The electric field is perpendicular to the surface of a conductor. Hence the correct choices are (b) and (c).

11. For a conductor, all the four choices are correct.

12. Force on proton is $F_p = eE$, in the direction of **E** and force of electron is $F_e = eE$, opposite to the direction of ***E***. Also $a_p = F_p/m_p$ and $a_e = F_e/m_e$. Since $m_p > m_e$; $a_p < a_e$. So the correct choices are (b) and (c).

13. For a ring, electric potential and electric field at P are

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{(R^2 + r^2)^{1/2}}$$

and $$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{(R^2 + r^2)^{3/2}},$$

which is zero for $r = 0$.

For $r >> R$, $V = \frac{Q}{4\pi\varepsilon_0 r}$ and $E = \frac{Q}{4\pi\varepsilon_0 r^2}$. So the correct choices are (b), (c) and (d).

14. The electric field at P is given by

$$\boldsymbol{E} = \frac{\lambda\hat{\mathbf{n}}}{2\pi\varepsilon_0 r}$$

Where λ is the linear charge density and $\hat{\mathbf{n}}$ is a unit vector perpendicular to the wire. So the correct choices are (a) and (d).

15. The electric field at P is

$$\boldsymbol{E} = \frac{\sigma\hat{\mathbf{n}}}{2\varepsilon_0}$$

where $\hat{\mathbf{n}}$ is a unit vector perpendicular to the sheet. Since **E** is independent of r, the electric field is uniform. Hence the correct choices are (c) and (d).

16. All the four choices are correct.

17. Initial P.E. of the system [Fig. 20.67(a)] is

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{qQ}{a} + \frac{qQ}{a} + \frac{q^2}{2a}\right)$$

$$= \frac{q}{4\pi\varepsilon_0 a}\left(2Q + \frac{q}{2}\right) \quad (1)$$

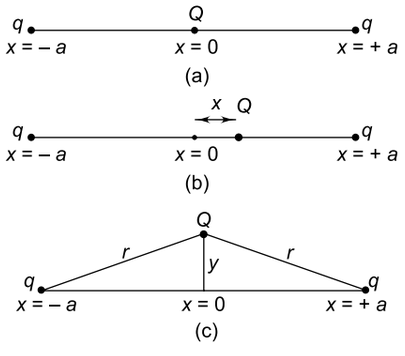


Fig. 20.67

If charge Q is given a displacemet x along the x-axis the P.E. of the system becomes [Fig. 20.67 (b)]

$$U' = \frac{1}{4\pi\varepsilon_0}\left[\frac{qQ}{(a+x)} + \frac{qQ}{(a-x)} + \frac{q^2}{2a}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 a}\left[2Q\frac{a^2}{(a^2-x^2)} + \frac{q}{2}\right] \quad (2)$$

Since $a^2 > (a^2 - x^2)$; it follows from Eqs. (1) and (2) that $U' > U$.

If charge Q is given a displacement y along the y – axis, the P.E of the system will be [Fig. 20.67(c)]

$$U'' = \frac{1}{4\pi\varepsilon_0}\left[\frac{qQ}{r} + \frac{qQ}{r} + \frac{q^2}{2a}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 a}\left[\frac{2aQ}{r} + \frac{q}{2}\right] \quad (3)$$

Since $r > a$, it follows from Eqs. (1) and (3) that $U'' < U$. So the correct choices are (b) and (c).

18. $V = \dfrac{\text{work}}{\text{charge}}$. Therefore $[V] = \dfrac{[ML^2T^{-2}]}{[AT]}$

$$= [ML^2T^{-3}A^{-1}]$$

$$\therefore [k] = [V] \times [r]$$

$$= [ML^2T^{-3}A^{-1}] \times [L]$$

$$= [ML^3T^{-3}A^{-1}]$$

Now $E = -\dfrac{dV}{dr} = -\dfrac{d}{dr}\left(\dfrac{k}{r}\right) = \dfrac{k}{r^2}$

Hence the correct choices are (b) and (c).

19. Let the electric potential be zero at a point P at a distance x from charge q_1. Then

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x} + \frac{q_2}{(r-x)}\right] = 0$$

$$\Rightarrow \quad \frac{q_1}{x} = -\frac{q_2}{(r-x)} \Rightarrow \frac{4}{x} = \frac{1}{(r-x)}$$

which gives $x = \dfrac{4r}{5} = \dfrac{4 \times 15\,\text{cm}}{5} = 12$ cm

Electric field E is zero at a value of x given by

$$\frac{|q_1|}{|q_2|} = \frac{x^2}{(r-x)^2}$$

which gives $x = \pm 30$ cm. It is easy to check that $E \neq 0$ between $x = 0$ and $x = 15$ cm. So the correct choices are (b), (c) and (d).

20. All the four statements (a), (b), (c) and (d) are correct.

21. The two charges will repel and will come to rest when $\theta = 180°$ because an orbiting satellite is in a state of weightlessness. The distance between the balls = 2 L. Hence

$$T = \frac{Q^2}{4\pi\varepsilon_0(2L)^2}$$

So the correct choices are (b) and (c).

22. If Q_1 is the charge on the ring and $-Q_2$ is the charge on particle P, the force due to the ring on particle P when it is at (0, 0, z) is

$$F = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2 z}{(R^2 + z^2)^{3/2}}$$

When $z > 0$, F is in the $-z$ direction. When $z < 0$, F is in the $+z$ direction. So the motion of P will be periodic for $0 < z < \infty$.

When $z << R$, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1 Q_2 z}{R^2}$, i.e. $F \propto -z$. Hence for $z << R$, the motion of P will be simple harmonic. So the correct choices are (a) and (c).

23. The graph shows the variation of electric field E with distance r. [Fig. 20.68]

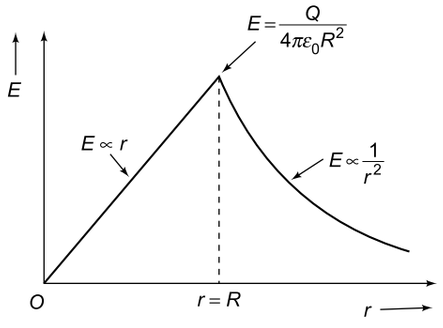


Fig. 20.68

So the correct choices are (a) and (c).

24. The surface of a perfect conductor is an equipotential surface. Charge density at B is greater than at A because curvature at B is greater than that at A. From Gauss's theorem, electric flux through a closed surface $= \dfrac{q}{\varepsilon_0}$. The electric field at the surface of the cavity is zero. Hence the correct choices are (c) and (d).

25. Potential energy at position x is

$$U = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{qp}{x^2}$$

When the dipole is far away ($x \to \infty$), $U = 0$. Hence

K = change in P.E. $= 0 - \left[-\dfrac{1}{4\pi\varepsilon_0}\dfrac{q\,p}{x^2}\right]$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,p}{x^2}$$

$$F = \frac{dU}{dx} = -\frac{2q\,p}{4\pi\varepsilon_0 x^3}$$

Hence the correct choices are (b) and (d).

26. Electric field in the region $r > r_0$ is given by

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{Q}{4\pi\varepsilon_0 r}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 r^2}$$

For $r \le r_0$, $E = -\dfrac{d}{dr}\left(\dfrac{Q}{4\pi\varepsilon_0 r_0}\right) = 0$

($\because$ r_0 = constant)

Hence, the electric field is discontinuous at $r = r_0$. Therefore, statement (a) is true.

For $r < r_0$, $E = 0$. Hence the charge resides only on the spherical surface of radius $r = r_0$. No charge exists in the region for which $r < r_0$. Therefore, statement (c) is also true. Electric energy density is given by

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Since for $r < r_0$, $E = 0$; $u = 0$ for $r < r_0$. Hence statement (d) is true.

Let Q' be the net charge enclosed inside the spherical surface of radius $r = 2r_0$. Then from Gauss's theorem, we have

$$\int \mathbf{E}\cdot d\mathbf{s} = \frac{Q'}{\varepsilon_0}$$

or $\quad E \times 4\pi r^2 = \dfrac{Q'}{\varepsilon_0}$

or $\quad \dfrac{Q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2 = \dfrac{Q'}{\varepsilon_0}$

or $Q' = Q$, which is independent of r as long as r is greater than r_0. Hence statement (b) is also true. All the four choices (a), (b), (c) and (d) are correct

27. The torque of Coulomb force (which is radial) on charge $-q$ is zero. Hence $\dfrac{\mathbf{dL}}{\mathbf{dt}} = 0 \Rightarrow \mathbf{L}$ = constant.

Hence the angular momentum of charge $-q$ is constant. So choice (a) is correct. All other choices are wrong because, for elliptical orbit, the speed of charge $-q$ is the highest when it is closest to charge Q (as in planetary motion). Hence the only correct choice is (a).

28. Let Q_A and Q_B are the charges on metal shell A and metal sphere B after they are connected by a wire. Since their electric potentials will be equal,

$$V_A = V_B$$

$$\Rightarrow \quad \frac{Q_A}{4\pi\varepsilon_0 R_A} = \frac{Q_B}{4\pi\varepsilon_0 R_B} \Rightarrow \frac{Q_A}{Q_B} = \frac{R_A}{R_B}$$

Since $R_B < R_A$, $Q_A > Q_B$. So choice (b) is correct. From Gauss's law, the electric field inside a spherical shell is zero. So choice (a) is correct.

Now $\sigma_A = \dfrac{Q_A}{4\pi R_A^2}$ and $\sigma_B = \dfrac{Q_B}{4\pi R_B^2}$

$$\therefore \quad \frac{\sigma_A}{\sigma_B} = \frac{Q_A}{Q_B} \times \left(\frac{R_B}{R_A}\right)^2 = \frac{R_A}{R_B} \times \left(\frac{R_B}{R_A}\right)^2$$

$$= \frac{R_B}{R_A}$$

Hence choice (c) is also correct.

Electric fields on the surface of shell and sphere are

$$E_A = \frac{\sigma_A}{\varepsilon_0}$$

And $\quad E_B = \dfrac{\sigma_B}{\varepsilon_0}$

$$\therefore \quad \frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B} < 1, \text{ i.e. } E_A < E_B.$$

So choice (d) is also correct. All the four choices are correct.

29. Gauss's law is valid only if Coulomb's law holds, i.e. if $E \propto r^{-2}$. Hence choice (a) is wrong. Gauss's law cannot be used to calculate a non-uniform field distribution around an electric dipole. So choice (b) is also wrong.

Choice (c) is correct becausse the directions of electric fields are opposite at a point between two similar charges.

Work done $W_{A\to B} = q(V_B - V_A) = (V_B - V_A)$

($\because$ $q = +1$ C)

Hence choice (d) is correct.

So the correct choices are (c) and (d).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage

Passage I

An infinite number of charges, equal to q, are placed along the x-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$ .... and so on.

1. The electric potential at the point $x = 0$ due to this set of charges is

(a) $\frac{q}{\pi\varepsilon_0}$ (b) $\frac{q}{2\pi\varepsilon_0}$

(c) $\frac{q}{3\pi\varepsilon_0}$ (d) $\frac{q}{4\pi\varepsilon_0}$

2. The electric field at point $x = 0$ is

(a) $\frac{q}{3\pi\varepsilon_0}$ (b) $\frac{q}{4\pi\varepsilon_0}$

(c) $\frac{q}{5\pi\varepsilon_0}$ (d) $\frac{q}{6\pi\varepsilon_0}$

3. If the consecutive charges have opposite sign, the electric potential at $x = 0$ would be

(a) $\frac{q}{3\pi\varepsilon_0}$ (b) $\frac{q}{4\pi\varepsilon_0}$

(c) $\frac{q}{5\pi\varepsilon_0}$ (d) $\frac{q}{6\pi\varepsilon_0}$

4. If the consecutive charges have opposite sign, the electric field at $x = 0$ would be

(a) $\frac{q}{3\pi\varepsilon_0}$ (b) $\frac{q}{4\pi\varepsilon_0}$

(c) $\frac{q}{5\pi\varepsilon_0}$ (d) $\frac{q}{6\pi\varepsilon_0}$

SOLUTION

1. The potential at $x = 0$ due to a set of infinite number of charges placed on the x-axis as shown in Fig. 20.69, is

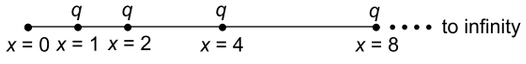


Fig. 20.69

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}+\frac{q}{2}+\frac{q}{4}+\frac{q}{8}+\cdots\text{to}\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\text{to}\infty\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1-\left(\frac{1}{2}\right)^{\infty}}{1-\frac{1}{2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\times\frac{(1-0)}{\left(\frac{1}{2}\right)} = \frac{q}{2\pi\varepsilon_0},$$

which is choice (b).

2. Since the charges are placed along the same straight line, the electric field at $x = 0$ will be directed along the x-axis and its magnitude is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2}+\frac{q}{2^2}+\frac{q}{4^2}+\frac{q}{8^2}+\cdots\text{to}\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots\text{to}\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1-\left(\frac{1}{4}\right)^{\infty}}{1-\frac{1}{4}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\times\frac{(1-0)}{\left(\frac{3}{4}\right)} = \frac{q}{3\pi\varepsilon_0},$$ which is choice (a).

3. If the consecutive charges have opposite sign, the potential at $x = 0$ is given by

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}-\frac{q}{2}+\frac{q}{4}-\frac{q}{8}+\frac{q}{16}-\frac{q}{32}\cdots\text{to}\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(1+\frac{1}{4}+\frac{1}{16}+\cdots\text{to}\infty\right)\right.$$

$$\left.-\left(\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\cdots\text{to}\infty\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(\frac{1}{1-\frac{1}{4}}\right)-\frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4}{3}-\frac{1}{2}\times\frac{4}{3}\right] = \frac{q}{6\pi\varepsilon_0}$$

Hence the correct choice is (d).

4. $E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2}-\frac{q}{2^2}+\frac{q}{4^2}-\frac{q}{(8)^2}+\frac{q}{(16)^2}-\frac{q}{(32)^2}+\cdots\text{to}\infty\right]$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(1+\frac{1}{16}+\frac{1}{256}+\cdots\text{to}\infty\right)-\left(\frac{1}{4}+\frac{1}{64}+\frac{1}{1024}+\cdots\text{to}\infty\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(\frac{1}{1-\frac{1}{16}}\right)-\frac{1}{4}\left(\frac{1}{1-\frac{1}{16}}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{16}{15}-\frac{1}{4}\times\frac{16}{15}\right] = \frac{q}{5\pi\varepsilon_0}$$

Hence the correct choice is (c).

Question 5 to 9 are based on the following passage

Passage II

A rigid insulated wire frame in the form of a right-angled triangle ABC, is set in a vertical plane as shown in Fig. 20.70. Two beads of equal masses m each and carrying charges q_1 and q_2 are conn-ected by a cord of length l and can slide without friction on the wires. The beads are stationary.

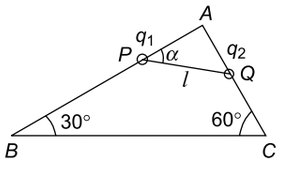


Fig. 20.70

IIT, 1978

5. The value of angle α is
(a) 30° (b) 45°
(c) 60° (d) 75°

6. The tension in the chord is
(a) mg (b) $\frac{q_1 q_2}{4\pi\varepsilon_0 l^2}$
(c) $mg + \frac{q_1 q_2}{4\pi\varepsilon_0 l^2}$ (d) zero

7. The normal reaction on bead P is
(a) $\sqrt{2}\ mg$ (b) $2\ mg$
(c) $\sqrt{3}\ mg$ (d) $3\ mg$

8. The normal reaction on bead Q is
(a) mg (b) $\sqrt{2}\ mg$
(c) $2\ mg$ (d) $\sqrt{3}\ mg$

9. If the cord is cut, the magnitude of the product $|q_1 q_2|$ of the charges for which the beads continue to remain stationary is
(a) $\frac{mgl^2}{4\pi\varepsilon_0}$ (b) $\frac{\sqrt{3}mgl^2}{4\pi\varepsilon_0}$
(c) $\sqrt{3}\,(4\pi\varepsilon_0)mgl^2$ (d) $(4\pi\varepsilon_0)mgl^2$

SOLUTION

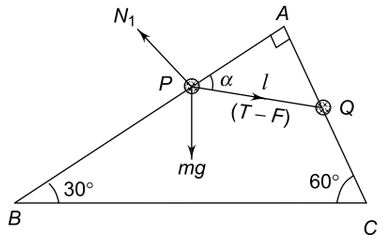


Fig. 20.71

Let us consider forces acting on bead P as shown in Fig. 20.71. These forces are:

(i) Weight mg vertically downwards

(ii) Tension T in the string

(iii) Electric force between P and Q given by

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{l^2}$$

(iv) Normal reaction N_1.

The net force along the string is $(T - F)$. Bead P will be in equilibrium, if the net force acting on it is zero. Resolving forces mg and $(T - F)$ parallel and perpendicular to plane AB, we get, when the bead P is in equilibrium,

$$mg\cos 60° = (T - F)\cos\alpha \quad (1)$$

and $$N_1 = mg\cos 30° + (T - F)\sin\alpha \quad (2)$$

For the bead at Q, we have

$$mg\sin 60° = (T - F)\sin\alpha \quad (3)$$

and $$N_2 = mg\cos 60° + (T - F)\cos\alpha \quad (4)$$

5. Dividing Eq. (3) by Eq. (1), we get
$\tan\alpha = \tan 60°$ or $\alpha = 60°$, which is choice (c).

6. Using $\alpha = 60°$ in (3), we have
$mg \sin 60° = (T - F) \sin 60°$

$$\text{or } T = F + mg = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{l^2} + mg \qquad (5)$$

So the correct choice (c).

7. From Eq. (2) we have (since $T - F = mg$)
$N_1 = mg \cos 30° + mg \sin 60°$
$= 2\ mg \cos 30° = \sqrt{3}\, mg;$
which is choice (c).

8. From Eq. (4) we have
$N_2 = mg \cos 60° + mg \cos 60° = mg$
Thus is the correct choice is (a).

9. When the string is cut, $T = 0$. Putting $T = 0$ in Eq. (5), we get

$$mg = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{l^2}$$

The right hand side of this equation should be positive which is possible if q_1 and q_2 have opposite signs. Thus, for equilibrium the beads must have unlike charges. The magnitude of the product of the charges is
$|q_1\ q_2| = (4\ \pi\varepsilon_0)\ mgl^2,$
which is choice (d).

Questions 10 to 13 are based on the following passage

Passage III

Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin.

10. The charge q_0 is given a small displaceplacement x ($<< a$) along the x-axis and then released. The restoring force acting on q_0 is ($k = 1/(4\pi\varepsilon_0)$)

(a) $-\left(\frac{kq^2}{a^3}\right)x$ (b) $-\left(\frac{2kq^2}{a^3}\right)x$

(c) $\left(\frac{kq^2}{a^2}\right)x$ (d) $\left(\frac{2kq^2}{a^2}\right)x$

11. In Q.10 the time period of oscillation of the particle is

(a) $T = 2\pi\left(\frac{ma^3}{2kq^2}\right)^{1/2}$ (b) $T = 2\pi\left(\frac{ma^3}{kq^2}\right)^{1/2}$

(c) $T = 2\pi\left(\frac{2kq^2}{ma^3}\right)^{1/2}$ (d) $T = 2\pi\left(\frac{kq^2}{ma^3}\right)^{1/2}$

12. If charge q_0 is given a small displacement y ($<< a$) along the y-axis, the net force acting on the particle is proportional to

(a) y (b) $-y$

(c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

13. In Q. 12, the particle,

(a) will execute simple harmonic motion.
(b) will execute oscillatory but not simple harmonic motion.
(c) will execute a non-periodic and non-oscillatory motion.
(d) will never come back to $x = 0$.

SOLUTION

10. Refer to Fig. 20.72. Suppose the charge $q_0 = q/2$ is given at a small displacement x from origin O. Then the force of repulsion on charge q_0 due to charge $q_1 = q$ is

$$F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_0}{(a+x)^2} = k\cdot\frac{q\times q/2}{(a+x)^2}$$

along positive x-direction
The force of repulsion on charge q_0 due to charge $q_2 = q$ is

$$F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_0}{(a-x)^2} = \frac{kq^2}{2(a-x)^2}$$

along negative x-direction

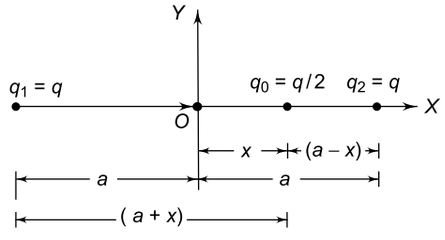


Fig. 20.72

Since $F_2 > F_1$, the net force on charge q_0 is along the negative x-direction. Hence the restoring force on charge q_0 is
$F = F_1 - F_2$

$$= k \times \frac{q^2}{2} \times \left[\left(\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2}\right)\right]$$

since $x << a$, $\dfrac{1}{(a+x)^2} = \dfrac{1}{a^2\left(1+\dfrac{x}{a}\right)^2}$

$$= \frac{\left(1+\frac{x}{a}\right)^{-2}}{a^2}$$

Expanding binomially and retaining only the terms of order $\frac{x}{a}$, we have

$$\frac{1}{(a+x)^2} = \frac{\left(1-\frac{2x}{a}\right)}{a^2} = \frac{1}{a^2} - \frac{2x}{a^3}$$

Similarly, $\dfrac{1}{(a-x)^2} = \dfrac{1}{a^2} + \dfrac{2x}{a^3}$

Hence

$$F = k \times \frac{q^2}{2} \times \left[\left(\frac{1}{a^2} - \frac{2x}{a^3}\right) - \left(\frac{1}{a^2} + \frac{2x}{a^3}\right)\right]$$

$$= -k \cdot \frac{2q^2 x}{a^3}$$

which is choice (b).

11. Since $F \propto (-x)$, the motion of the particle is simple harmonic whose time period is given by choice (a).

12. Refer to Fig. 20.73. Suppose the charge $q_0 = q/2$ is given a small displacement $OC = y$ along the y-axis, the force of repulsion on charge q_0 due to the charge $q_1 = q$ is

$$F_1 = k \cdot \frac{q\,q_0}{(AC)^2} = k \cdot \frac{q \times q/2}{\left(a^2 + y^2\right)}$$

The force of repulsion on charge q_0 due to charge $q_2 = q$ is

$$F_2 = k \cdot \frac{q\,q_0}{(BC)^2} = k \cdot \frac{q \times q/2}{\left(a^2 + y^2\right)}$$

The directions of F_1 and F_2 are shown in Fig. 20.73. Force F_1 can be resolved into two components $(F_1)_x = F_1 \sin\theta$ along positive x-direction and $(F_1)_y = F_1 \cos\theta$ along positive y-direction. Similarly, force F_2 can be resolved into two components $(F_2)_x = F_2 \sin\theta$ along negative x-direction and $(F_2)_y = F_2 \cos\theta$ along positive y-direction. Since $F_1 = F_2$, components $(F_1)_x$ and $(F_2)_x$ are equal and opposite and hence they cancel each other. The net force on charge q_0 is along the positive y-direction and is given by

$$F = (F_1)_y + (F_2)_y = F_1 \cos\theta + F_2 \cos\theta$$

$$= (F_1 + F_2)\cos\theta = 2F_1\cos\theta \quad (\because F_1 = F_2)$$

or $$F = k \cdot \frac{q^2 \cos\theta}{\left(a^2 + y^2\right)}$$

Now $\cos\theta = \dfrac{OC}{AC} = \dfrac{y}{\left(a^2 + y^2\right)^{1/2}}$. Therefore,

$$F = k \cdot \frac{q^2 y}{\left(a^2 + y^2\right)^{3/2}} \text{ along positive } y\text{-direction}$$

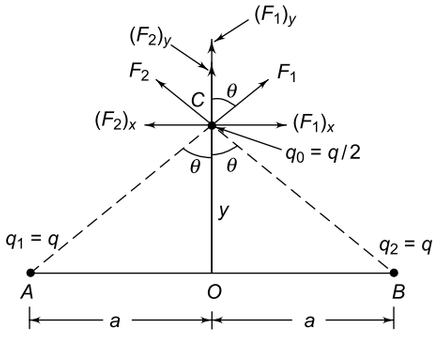


Fig. 20.73

Since $y << a$, $(a^2 + y^2)^{3/2} \approx a^3$. Thus

$$F = k \cdot \frac{q^2 y}{a^3}$$

So the correct choice is (a).

13. Since the net force on charge q_0 is along the postive y-direction, it will keep on moving in the positive y-direction away from the origin O and will never come back. So the correct choice is (d).

Questions 14 to 16 are based on the following passage

Passage IV

A point particle of mass M is attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-q$. This arrangement is held in a region of a uniform electric field **E** such that the rod makes a small angle θ (say of about 5°) with the field direction as shown in Fig. 20.74.

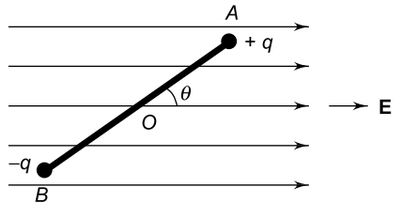


Fig. 20.74

IIT, 1989

14. The magnitude of the torque acting on the rod is
(a) $qEL \sin \theta$ (b) $qEL \cos \theta$
(c) qEL (d) zero

15. When the rod is released, it will rotate with an angular frequency ω equal to

(a) $\left(\frac{qE}{ML}\right)^{1/2}$ (b) $\left(\frac{2qE}{ML}\right)^{1/2}$

(c) $\left(\frac{qE}{2ML}\right)^{1/2}$ (d) $\frac{1}{2}\left(\frac{qE}{ML}\right)^{1/2}$

16. The minimum time taken by the rod to align itself parallel to the electric field after it is set free is given by

(a) $\frac{\pi}{2}\left(\frac{ML}{2qE}\right)^{1/2}$ (d) $2\pi\left(\frac{ML}{qE}\right)^{1/2}$

(c) $2\pi\left(\frac{2ML}{qE}\right)^{1/2}$ (d) $2\pi\left(\frac{ML}{2qE}\right)^{1/2}$

SOLUTION

14. A non-conducting rigid rod having equal and opposite charges at the ends is an electric dipole. When it is placed in a uniform electric field, it experiences a torque which tends to align it with the field lines. Referring to Fig. 20.75, the electric forces $F = qE$ each acting at A and B constitute a couple whose torque is given by

$\tau =$ force $\times$ perpendicular distance

$= F \times AC = F \times AB \sin \theta = qEL \sin \theta$

So the correct choice is (a).

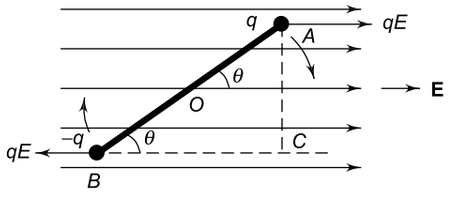


Fig. 20.75

15. Since θ is small, $\sin \theta \simeq \theta$, where θ is expressed in radian. Thus $\tau = qEL\theta$

$\therefore$ Restoring torque $\tau = -qEL\theta$ (1)

If α is the angular acceleration of the rotatory motion,

$$\tau = I\alpha$$

where I is the moment of inertia of the two masses at A and B about an axis passing through the centre O and perpendicular to the rod. Since the rod is massless,

$$I = M \times (AO)^2 + M \times (BO)^2$$

$$= M \times \left(\frac{L}{2}\right)^2 + M \times \left(\frac{L}{2}\right)^2 = \frac{ML^2}{2}$$

Thus $\tau = \frac{ML^2\alpha}{2}$ (2)

Using Eq. (2) in Eq. (1), we get

$$\alpha = -\left(\frac{2qE}{ML}\right)\theta = -\omega^2\theta$$

where $\omega = \left(\frac{2qE}{ML}\right)^{1/2}$, which is choice (b).

16. The time period of oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi\left(\frac{ML}{2qE}\right)^{1/2}$$

Rotating in the clockwise sense, the minimum time taken by the rod to align itself parallel to the electric field is the time it takes to complete one-fourth of angular oscillation, i.e.

$$t_{\min} = \frac{T}{4} = \frac{\pi}{2}\left(\frac{ML}{2qE}\right)^{1/2}$$

So the correct choice is (a).

Questions 17 to 20 are based on the following passage

Passage V

Two identical particles A and B of mass m carry a charge Q each. Initially particle A is at rest on a smooth horizontal plane and the partical B is projected with a speed v along the horizontal plane from a large distance directly towards the first particle. The distance of closest approach is x.

17. At the closest approach, speed v_1 of particle A is
(a) $v_1 = v$ (b) $v_1 = \sqrt{2}v$
(c) $v_1 = v/2$ (d) $v_1 = v/\sqrt{2}$

18. At the closest approach, speed v_2 of particle B is
(a) $v_2 = v/2$ (b) $v_2 = v/\sqrt{2}$
(c) $v_2 = v$ (d) $v_2 = 2v$

19. At the distance of closest approach, the total energy of the system is

(a) $\frac{mv^2}{2}$ (b) $\frac{mv^2}{2}+\frac{Q^2}{4\pi\varepsilon_0 x}$

(c) $mv^2+\frac{Q^2}{4\pi\varepsilon_0 x}$ (d) $\frac{mv^2}{4}+\frac{Q^2}{4\pi\varepsilon_0 x}$

20. The distance of closest approach is

(a) $x=\frac{Q^2}{\pi\varepsilon_0 mv^2}$ (b) $x=\frac{Q^2}{2\pi\varepsilon_0 mv^2}$

(c) $x=\frac{Q^2}{4\pi\varepsilon_0 mv^2}$ (d) $x=\frac{2Q^2}{\pi\varepsilon_0 mv^2}$

SOLUTION

17. Initially particle A is at rest and particle B moves towards it with a velocity v from a large distance. Therefore, the initial momentum of the system is

$$p_i = mv + 0 = mv$$

Since the initial distance is large, the particle B exerts negligible repulsive force of the first particle. But, as the distance decreases, the first particle begins to move in the same direction as the second particle under the action of the force of repulsion. The distance between them, therefore, keeps decreasing until it attains a certain minimum value. Let v_1 and v_2 be the velocities of particles 1 and 2 and let t be the time taken by them to acquire these velocities. Then the distances travelled by them will be $x_1 = v_1 t$ and $x_2 = v_2 t$. The separation between them at this time t is

$$x = x_1 - x_2 = (v_1 - v_2)t$$

This separation will be minimum if $dx/dt = 0$. Now

$$\frac{dx}{dt} = v_1 - v_2$$

$\therefore$ For closest approach, $v_1 - v_2 = 0$ or $v_1 = v_2$.

Therefore, the final momentum of the system at the closest approach is

$$p_f = mv_1 + mv_2 = m\,(v_1 + v_2)$$

From the law of conservation of momentum,

$p_i = p_f$ or $mv = m\,(v_1 + v_2)$ or $v = v_1 + v_2$

Since the particles are identical, it follows that

$v_1 = v_2 = \frac{v}{2}$.

So the correct choice (c).

18. The correct choice is (a).

19. If x is the distance of closest approach, the final energy of the system is

$$E_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$$

or $= \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$

or $E_f = \frac{mv^2}{4} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$

which is choice (d).

20. The initial energy of the system is

E_i = KE of first particle + KE of second particle + PE due to charge on each particle.

$$= 0 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{\infty}$$

or $E_i = \frac{1}{2}mv^2$

From the law of conservation of energy, $E_i = E_f$. Therefore

$$\frac{1}{2}mv^2 = \frac{1}{4}mv^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$$

which gives

$$x = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4Q^2}{mv^2} = \frac{Q^2}{\pi\varepsilon_0 mv^2}$$

so the correct choice is (a).

Questions 21 to 24 are based on the following passage

Passage VI

Charge Q is uniformly distributed within a non-conducting sphere of radius R. The volume charge density is ρ.

IIT, 1992

21. The potential energy of the spherical distribution of charge is

(a) $\frac{3Q^2}{20\pi\varepsilon_0 R}$ (b) $\frac{2Q^2}{5\pi\varepsilon_0 R}$

(c) $\frac{Q^2}{4\pi\varepsilon_0 R}$ (d) $\frac{3Q^2}{10\pi\varepsilon_0 R}$

22. The magnitude of the electric field at a distance r ($> R$) from the centre of the sphere is

(a) $\frac{3\rho R^3}{\varepsilon_0 r^2}$ (b) $\frac{3\rho R^3}{4\varepsilon_0 r^2}$

(c) $\frac{\rho R^3}{3\varepsilon_0 r^2}$ (d) $\frac{3\rho R^3}{2\varepsilon_0 r^2}$

23. If the same charge is given to a spherical conducting sphere of the same reduis R, the potential energy of the system will be

(a) zero (b) $\dfrac{Q^2}{4\pi\varepsilon_0 R}$

(c) $\dfrac{Q^2}{8\pi\varepsilon_0 R}$ (d) the same as in Q.21.

24. In Q.23 the magnitude of the electric field at a distance r ($> R$) from the centre of the sphere will be

(a) $\dfrac{Q}{4\pi\varepsilon_0 r^2}$ (b) $\dfrac{QR}{4\pi\varepsilon_0 r^3}$

(c) $\dfrac{3Q}{4\pi\varepsilon_0 r^2}$ (d) $\dfrac{3QR}{4\pi\varepsilon_0 r^3}$

SOLUTION

21. $\rho = \dfrac{Q}{V} = \dfrac{Q}{\frac{4\pi}{3}R^3} = \dfrac{3Q}{4\pi R^3}$ (1)

Refer to Fig. 20.76.

The spherical charge may be regarded as an infinitely large number of infinitesimally thin concentric spherical shells. Consider one such spherical shell of radius r and thickness dr. The charge within the volume $(4/3)\pi r^3$ is

$$q = \frac{4}{3}\ \pi r^3 \rho$$

The charge in the spherical shell of thickness dr is

$$dq = 4\ \pi r^2 dr\rho$$

Therefore, the potential energy due to charges q and dq is

$$dU = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\,dq}{r}$$

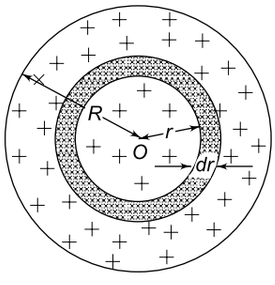


Fig. 20.76

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{\left(\frac{4\pi}{3}r^3\rho\right)\times\left(4\pi r^2\, dr\,\rho\right)}{r}$$

$$= \frac{4\pi\rho^2}{3\varepsilon_0}\times r^4 dr$$

Hence, the total potential energy of the spherical distribution of charge is

$$U = \frac{4\pi\rho^2}{3\varepsilon_0}\int_0^R r^4 dr = \frac{4\pi\rho^2}{3\varepsilon_0}\times\frac{R^5}{5}$$

Using Eq. (1) in this we have

$$U = \frac{4\pi}{3\varepsilon_0}\left(\frac{Q}{\frac{4}{3}\pi R^3}\right)^2\times\frac{R^5}{5}$$

$$= \frac{3Q^2}{20\pi\varepsilon_0 R}$$

So the correct choice is (a).

22. For points outside the sphere, the charge Q may be assumed to be concentrated at the centre. Hence (For $r > R$),

$E = \dfrac{Q}{4\pi\varepsilon_0 r^2} = \dfrac{\rho R^3}{3\pi\varepsilon_0 r^2}$, which is choice (c).

23. Any charge given to a solid conductor, cannot stay in its body; it is immediately transferred to the surface of the conductor. The electrostatic energy of a charge Q on the surface of a conducting sphere of R is given by

$$U = \frac{Q^2}{2C}$$

where $C = 4\ \pi\varepsilon_0 R$ is the capacitance of the sphere. Thus

$$U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

So the correct choice is (c).

24. For points outside the sphere, it behaves if the entire charge Q is concentrated at its centre. Hence the correct choice is (a).

Questions 25 to 26 are based on the following passage

Passage VII

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p' which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

IIT, 2011

25. Take the electronic charge as 'e' and the permittivity as 'ε_0'. Use dimensional analysis to determine the correct expression for ω_p.

(a) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$

(c) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

26. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}$ m^{-3}. Taking $\varepsilon_0 = 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

(a) 800 nm (b) 600 nm
(c) 300 nm (d) 200 nm

SOLUTION

25.

From $F = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}$, dimensions of $\left[\dfrac{e^2}{\varepsilon_0}\right] = [Fr^2]$

$= [ML^3T^{-2}]$

$[N] = [L^{-3}]$. Hence

$$\left[\frac{Ne^2}{m\varepsilon_0}\right] = [L^{-3}] \times [ML^3T^{-2}] \times [M^{-1}] = [T^{-2}]$$

$$\therefore \left[\frac{Ne^2}{m\varepsilon_0}\right]^{1/2} = [T^{-1}] = \text{dimension of } \omega_p.$$

26.

$$\omega_p = \left[\frac{Ne^2}{m\varepsilon_0}\right]^{1/2}$$

$$= \left[\frac{4\times10^{27}\times(1.6\times10^{-19})^2}{10^{-30}\times10^{-11}}\right]^{1/2}$$

$$= 3.2 \times 10^{15} \text{ rad s}^{-1}$$

$$\nu_p = \frac{\omega_p}{2\pi}. \text{ Since } c = \nu\lambda,$$

$$\lambda = \frac{c}{\nu_p} = \frac{3\times10^8\times2\pi}{3.2\times10^{15}}$$

$$\simeq 6 \times 10^{-7} \text{ m} = 600 \text{ nm}$$

SECTION IV

Assertion–Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1

Figure 20.77 shows the tracks of two charged partices A and B in a uniform electric field between two charged plates. The charge to mass ratio of B is greater than that of A. Neglect the effect of gravity.

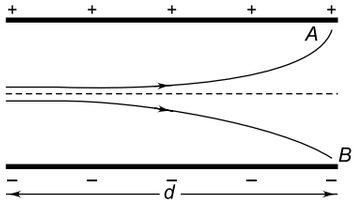


Fig. 20.77

Statement-2
The vertical acceleration of particle B is greater than that of particle A.

2. Statement-1
A positive charge + q is located at the centre of a circle as shown in Fig. 20.78. W_1 is the work done in taking a small positive charge + q_0 from A to B and W_2 is the work done in taking the same charge from A to C. Then $W_2 > W_1$

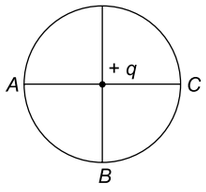


Fig. 20.78

Statement-2
Work done = Charge × potential difference.

3. Statement-1
A small test charge is initially at rest at a point in an electrostatic field of an electric dipole. When released, it will move along the line of force passing though that point.
Statement-2
The tangent at a point on a line of force gives the direction of the electric field at that point.

4. Statement-1
If electric field is zero at a point, the electric potential must also be zero at that point.
Statement-2
Electric field is equal to the negative gradient of potential.

5. Statement-1
If electric potential is constant in a certain region of space, the electric field in that region must be zero.
Statement-2
Electric field is equal to the negative gradient of potential.

6. Statement-1
If an electron is moved from P to Q, its potential energy increases (see Fig. 20.79)
Statement-2
Potential at Q is less than that at P.

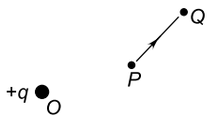


Fig. 20.79

7. Statement-1
In Q. 6 above, the work done to move an electron from P to Q and then back to P is zero.
Statement-2
Electric field is conservative.

8. Statement-1
The work done by the electric field of a nucleus in moving an electron around it in a complete orbit is greater if the orbit is elliptical than if the orbit is circular.
Statement-2
Electric field is conesvature.

9. Statement-1
Electrons move from a region of higher potential to a region of lower potential.
Statement-2 An electron has less potential energy at a point where the potential is higher and vice versa.

10. Statement-1
The equipotential surfaces corresponding to a constant electric field along the x-direction are equidistant planes parallel to the y-z plane.
Statement-2
Electric is normal to every point on an equipotential surface

11. Statement-1
The electric field in the region around a point charge is uniform.
Statement-2
The equipotential surface of the electric field of a point charge is a sphere with the charge at its center.

12. Statement-1
If a metallic sphere A of radius r carrying a charge Q is brought in contact with an uncharged metallic sphere B of radius $2r$, the charge on sphere A reduces to $Q/3$.
Statement-2
Charge flows from A to B until their potentials are equalised.

13. Statement-1
A metal ring of radius R carries a charge + Q distributed uniformly. A point charge – q is placed on the axis of the ring at a distance $x = 2R$ from the centre of the ring. If the charge is released from rest, it will execute a simple harmonic motion along the axis of the ring.
Statement-2
In simple harmonic motion, the restoring force acting on the oscillator is proportional to (– x), where x is the displacement from the mean position.

IIT, 1988

14. Statement-1
For practical purposes, the earth is used as a reference at zero potential in electrical circuits.
Statement-2
The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$.

IIT, 2008

SOLUTION

1. The correct choice is (a). Let E be the electric field between the plates; It is directed vertically downwords. If q is the charge of the particle, it will experience a force $F = qE$. Hence its acceleration (in the vertical direction) is

$$a = \frac{F}{m} = \frac{qE}{m}$$

where m is the mass of the particle. If t is time spent by the particle between the plates (i.e in the region of the electric field), the vertical distance travelled (i.e. deflection) of the partical is

$$y = \frac{1}{2}\ at^2 = \frac{qEt^2}{2m}$$

Thus $\frac{q}{m} \propto y$. Since particle B has a higher deflection, it has a higher charge to mass ratio.

2. The correct choice is (d). Points A, B and C are at the same distance from charge + q. Hence electric potential difference between points A, B and C is zero. Hence $W_1 = W_2 = 0$.

3. The correct choice is (d). The test charge will move along the line of force if the line of force is straight (as in the case of a single charge). If the lines of force is curved, the charge will not move along the line of force. The reason is that the line of force does not give the direction of velocity, it gives the direction of the force which is along the tangent to the curve at that point.

4. The correct choice is (d). Since $E = -dV/dr$, if $E = 0$, V = constant not necessarily equal to zero.

5. The correct choice is (a).

6. The correct choice is (c). Since charge of an electron is negative, P.E at P and Q is

$$U_P = -\frac{eq}{4\pi\varepsilon_0(OP)}$$

$$U_Q = -\frac{eq}{4\pi\varepsilon_0(OQ)}$$

Since $OQ > OP$, U_Q is less negative than U_P, i.e. $U_Q > U_P$. For the same reason, $V_Q > V_P$.

7. Since charge + q will attract the electron, work is done to move the electron from P to Q is negative because the work is done against the field. To move it from Q back to P an equal positive work is done by the field because electric field is conservative. So the correct choice is (a).

8. The correct choice is (d). The work done by the electric field in moving a charge around a closed path of any shape (circular or elliptical) is zero.

9. The correct choicce is (d). Since the electron has a negative charge, it has less energy at a point where the potenial is higher and vice versa. Hence in an electric field an electron moves from a region of lower potential to a region of higher potential.

10. The electric field is always normal to the equipotential surface. Therefore, for a constant electric field in the x-direction, the equipotential surfaces are planes parallel to the y-z plane. Since the field is constant, the equipotential surfaces are equidistant from each other. The correct choice is (a).

11. The correct choice is (d). The electric field in a region around a point charge varies with distance r:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

12. The correct choice is (a). Charge will flow from A to B until thier potentials become equal. If charge q flows from A to B, then

$$\frac{Q-q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0(2r)}$$

which gives $Q - q = \frac{q}{2} \Rightarrow q = \frac{2Q}{3}$. Hence charge left on $A = Q - \frac{2Q}{3} = \frac{Q}{3}$.

13. The electric field on the axis of the ring at a distance x from its centre is given by

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q\,x}{(R^2 + x^2)^{3/2}}$$

Force on charge $-q$ is

$$F = -q\ E = -\frac{1}{4\pi\varepsilon_0} \times \frac{q\,Q\,x}{(R^2 + x^2)^{3/2}}$$

The motion will be simple harmonic if $F \propto -x$ which is true only if $x << R$ and not when $x = 2R$.

Hence Statement-1 is false, but Statement-2 is true.

14. Both the statements are true but Statement-2 is not the correct explanation for Statement-1.

SECTION V

Integer Answer Type

1. Two identical charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density 800 kg m^{-3}, the angle remains the same. What is the dielectric constant of the liquid? The density of the material of the sphere is 1600 kg m^{-3}.

IIT, 1978

2. A circular ring of radius R with uniform positive charge density λ per unit length is located in the y-z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point P ($R\sqrt{3}$, 0, 0) on the positive x-axis directly towards O, with initial speed v. The smallest (non-zero) value of speed v such that the particle does not return to P is $v = \left(\frac{\lambda q}{n\varepsilon_0 m}\right)$, where n is an integer. Find the value of n.

IIT, 1993

3. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of a.

IIT, 2009

4. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where '$k$' is a constant. What is the value of N?

IIT, 2011

SOLUTIONS

1. It follows from Fig. 20.80 that

$$W = T\cos\theta \text{ and } F = T\sin\theta$$

$$\therefore \quad \frac{F}{W} = \tan\theta \qquad (1)$$

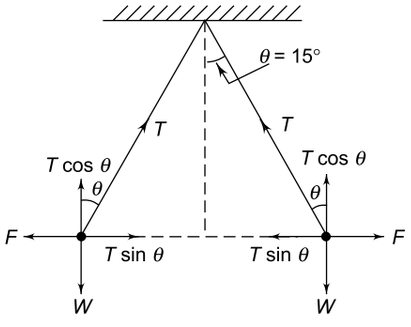


Fig. 20.80

If q is the charge on each sphere and r the separation between them, force F is given by

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2}$$

When the spheres are immersed in a liquid of dielectric constant K, the force of repulsion between them becomes

$$F' = \frac{F}{K}$$

Apparent weight is

$$W' = W - (\text{weight of liquid displaced})$$

or $$W' = W - \rho Vg$$

where ρ is the density of the liquid. If σ is the density of the material of the sphere, $W = \sigma Vg$. Thus

$$W' = W\left(1 - \frac{\rho V g}{W}\right)$$

$$\Rightarrow \qquad W' = W\left(1 - \frac{\rho}{\sigma}\right)$$

$$\frac{F'}{W'} = \frac{F}{W} \times \frac{1}{K\left(1 - \frac{\rho}{\sigma}\right)}$$

As the angle θ remains unchanged, it follow from Eq. (1) that

$$\tan\theta = \frac{F}{W} = \frac{F'}{W'}$$

Hence

$$K\left(1 - \frac{\rho}{\sigma}\right) = 1 \text{ or } K = \frac{1}{\left(1 - \frac{\rho}{\sigma}\right)}$$

Given ρ = 800 kg m^{-3} and σ = 1600 kg m^{-3}. Therefore,

$$K = \frac{1}{\left(1 - \frac{800}{1600}\right)} = \frac{1}{\left(1 - \frac{1}{2}\right)} = 2$$

2. The positive charge on the ring is

$$Q = 2\pi R\lambda$$

A particle with a positive charge q is projected towards the centre O of the ring with a certain velocity from a point P (lying on the x-axis) whose x-coordinate is $\sqrt{3}\ R$. Due to repulsion of charge Q, the particle will keep slowing down. Now, at the centre O of a ring, carrying charge, the electric field is zero. Therefore, the force on the particle will be zero when it reaches O. It will then stop there and will not return to P. Let v be the smallest velocity of the particle when it is at P which is just sufficient to take it to O where it comes to rest. [see Fig. 20.81]

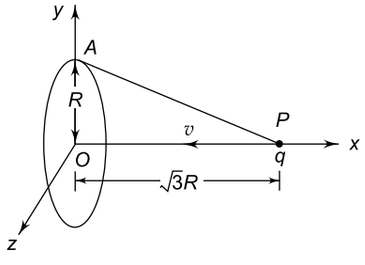


Fig. 20.81

The total energy of the particle at point P is E_P = KE due to its motion + PE due to charge Q on the ring

$$= \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{(AP)}$$

Now $(AP)^2 = (OA)^2 + (OP)^2 = R^2 + \left(\sqrt{3}\ R\right)^2 = 4R^2$

$\therefore \qquad AP = 2R.$

Therefore, $E_P = \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{2R}$

The total energy of the particle when it reaches O is (since its velocity at O is zero)

E_O = PE at O due to charge Q on the ring

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{R}$$

From the law of conservation of energy, $E_P = E_O$. Thus

$$\frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{2R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{R}$$

which gives $v = \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{mR}\right)^{1/2} = \left(\frac{\lambda q}{2\varepsilon_0 m}\right)^{1/2}$

$(\because\ Q = 2\pi R\lambda)$

Thus the value of $n = 2$.

3. According to Gauss's theorem

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0} = \frac{\int \rho dV}{\varepsilon_0} = \frac{\int 4\pi r^2 \rho dr}{\varepsilon_0}$$

where q is the charge enclosed inside a Gaussian sphere of radius r. If E_1 is the electric field at $r = R/2$, then [see Fig. 20.82]

$$E_1 \times 4\pi\left(\frac{R}{2}\right)^2 = \frac{1}{\varepsilon_0}\int_0^{R/2} 4\pi r^2 \rho dr$$

$$= \frac{4\pi}{\varepsilon_0}\int_0^{R/2} k\, r^a\, r^2 dr$$

$$= \frac{4\pi k}{\varepsilon_0}\int_0^{R/2} r^{(a+2)} dr$$

$$= \frac{4\pi k}{\varepsilon_0 (a+3)} \left| r^{(a+3)} \right|_0^{R/2}$$

$$= \frac{4\pi k}{\varepsilon_0 (a+3)} \times \left(\frac{R}{2}\right)^{a+3}$$

$$\Rightarrow \qquad E_1 = \frac{4\pi k}{\varepsilon_0 (a+3) R^2} \times \left(\frac{R}{2}\right)^{a+3}$$

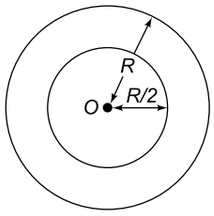


Fig. 20.82

If E_2 is the electric field at $r = R$, then

$$E_2 \times 4\pi R^2 = \frac{4\pi k}{\varepsilon_0}\int_0^R r^{(a+2)}\,dr$$

$$= \frac{4\pi k}{\varepsilon_0 (a+3)} \times (R)^{a+3}$$

$$\Rightarrow \qquad E_2 = \frac{k}{\varepsilon_0 (a+3)\,R^2} \times (R)^{a+3}$$

Given $\qquad \dfrac{E_1}{E_2} = \dfrac{1}{8}$

Hence

$$\frac{4k}{\varepsilon_0 (a+3) R^2} \times \left(\frac{R}{2}\right)^{a+3} = \frac{k}{8\varepsilon_0 (a+3) R^2} \times (R)^{a+3}$$

$$\Rightarrow \qquad 2^{(a+3)} = 32 \Rightarrow 2^{(a+3)} = (2)^5 \Rightarrow a = 2$$

4. Refer to Fig. 20.83.

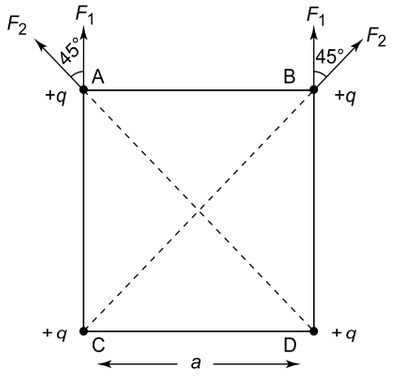


Fig. 20.83

$$F_1 = \frac{q^2}{4\pi\varepsilon_0 a^2}$$

$$F_2 = \frac{q^2}{4\pi\varepsilon_0 \left(\sqrt{2}a\right)^2} = \frac{q^2}{4\pi\varepsilon_0 \times 2a^2}$$

Net force on side AB of the film is

$$F = 2F_1 + 2F_2 \cos 45°$$

$$= \frac{2q^2}{4\pi\varepsilon_0 a^2} + \frac{2q^2}{4\pi\varepsilon_0 2\sqrt{2}a^2}$$

$$= \frac{q^2}{4\pi\varepsilon_0 a^2}\left(2 + \frac{1}{\sqrt{2}}\right)$$

Force on AB due to surface tension = $2\gamma a$. Hence

$$\frac{q^2}{4\pi\varepsilon_0 a^2}\left(2 + \frac{1}{\sqrt{2}}\right) = 2\gamma a$$

$$\Rightarrow \qquad a = k\left(\frac{q^2}{\gamma}\right)^{1/3}$$

where k is a constant. Hence $N = 3$

21 Chapter

Capacitance and Capacitors

REVIEW OF BASIC CONCEPTS

21.1 CAPACITANCE

When a conductor is given a charge Q, it acquires a potential V which is proportional to the charge given to it, i.e.

$$Q \propto V \quad \text{or} \quad Q = CV \quad \text{or} \quad C = \frac{Q}{V}$$

where C is a constant of proportionality and is called the capacitance which is defined as the amount of charge in coulomb necessary to increase the potential of a conductor by 1 volt. The SI unit of capacitance is farad (symbol F)

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

The farad is a large unit. More practical units are microfarad (μF) and picofarad (pF).

$$1\ \mu\text{F} = 10^{-6}\ \text{F} \text{ and } 1\ \text{pF} = 10^{-12}\ \text{F}$$

21.2 ENERGY OF A CHARGED CONDUCTOR

A charged conductor has electric field in the region around it. If additional similar charge is given to the conductor, work has to be done against the electrical repulsive force. This work is stored in the form of potential energy which resides in the electric field. If a charge Q is given to a conductor of capacitance C, the potential energy in its electric field is given by

$$U = \frac{Q^2}{2C} = \frac{1}{2} QV = \frac{1}{2} CV^2 \quad (\because Q = CV)$$

21.3 CAPACITANCE OF A SINGLE SPHERICAL CONDUCTOR

Consider a spherical conductor of radius r having a charge Q. Since the electric field is normal to the surface of the sphere, the lines of force appear to originate from its centre, i.e. the charge Q may be supposed to be concentrated at the centre. Therefore, the potential is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$$

Since $C = \frac{Q}{V}$, the capacitance of the sphere is given by

$$C = 4\pi\varepsilon_0 r$$

Thus, the greater the radius of the sphere, the higher is its capacitance.

21.4 CAPACITORS

Any isolated system of two conducting bodies, of any shape and size, separated by a distance is called a capacitor. If two conductors, carrying equal and opposite charge Q have a potential difference V between them, then

$$Q = CV$$

where C is the capacitance of the capacitor and its value depends on the size, the shape, the separation between the conductors and the nature of the medium between them. If C_0 is the capacitance of the capacitor when the medium is air (or vacuum) and C_m its capacitance when the medium is a dielectric other than air, then the dielectric constant of the medium is given by

$$K = \frac{C_m}{C_0}$$

21.5 EXPRESSIONS FOR CAPACITANCE

1. *Parallel Plate Capacitor*
   The capacitance of a parallel plate capacitor is given by
   $$C = \frac{K \varepsilon_0 A}{d}$$
   where A is the area of each plate and d is the distance between them. K is dielectric constant of the material between the plates. For air or vacuum, $K = 1$.
2. *Spherical Capacitor*
   A spherical capacitor consists of a solid charged sphere of radius a surrounded by a concentric hollow sphere of radius b. Its capacitance is given by
   $$C = 4\pi \varepsilon_0 K \left(\frac{ab}{b-a}\right)$$
3. *Cylindrical Capacitor*
   A cylindrical capacitor consists of two co-axial cylinders and its capacitance is given by
   $$C = \frac{2\pi \varepsilon_0 K l}{\log_e\left(\frac{b}{a}\right)}$$
   where l is the length of each cylinder and a and b are the radii of the inner and outer cylinders.
4. If the space between the plates of a parallel plate capacitor is filled with two media of thicknesses d_1 and d_2 having dielectric constants K_1 and K_2, then the capacitance of the capacitor is given by
   $$C = \frac{\varepsilon_0 A}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

21.6 CAPACITORS IN PARALLEL AND SERIES

In parallel arrangement of capacitors, the potential difference across individual capacitors is the same and the total charge is shared by them in the ratio of their capacitances.

$$Q = Q_1 + Q_2 + Q_3 + \ldots$$

and
$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3} = \ldots$$

$\therefore$
$$C = C_1 + C_2 + C_3 + \ldots$$

In series arrangement of capacitors, the charge on each capacitor is the same and the total potential difference is shared by them in the inverse ratio of their capacitances.

$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3 = \ldots$$
$$V = V_1 + V_2 + V_3 + \ldots$$

Therefore, the effective capacitance of the combination is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

21.7 ENERGY STORED IN A CAPACITOR

As in the case of a charged conductor, the energy stored in a capacitor is given by

$$U = \frac{Q^2}{2C} = \frac{1}{2} QV = \frac{1}{2} CV^2$$

where Q = charge on each plate of the capacitor, V = potential difference between plates and C = capacitance of the capacitor. This potential energy resides in the electric field in the medium between the plates.

21.8 LOSS OF ENERGY ON SHARING CHARGES

If two charged bodies carrying charges Q_1 and Q_2 and having capacitances C_1 and C_2 are connected with each other, then their common potential after the sharing of charges is given by

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

where V_1 and V_2 are the initial potentials of the charged bodies. The loss of energy is given by

$$\Delta E = \frac{1}{2} \frac{C_1 C_2}{(C_1 + C_2)} (V_1 - V_2)^2$$

21.9 FORCE BETWEEN PLATES OF A PARALLEL PLATE CAPACITOR

The plates of a capacitor carry equal and opposite charges. Therefore, they exert an attractive force on each other which is given by

$$F = \frac{Q^2}{2K\varepsilon_0 A}$$

The force per unit area of the plates is

$$f = \frac{F}{A} = \frac{Q^2}{2K\varepsilon_0 A^2} = \frac{\sigma^2}{2K\varepsilon_0}$$

where σ is the charge per unit area.

EXAMPLE 21.1

A parallel plate capacitor of capacitance 10 μF and plate separation 0.5 mm is connected to a 20 V battery.

(a) What is the charge on each plate?

(b) What is the energy stored in the capacitor?

(c) What is the electric field between the plates?

(d) If the battery is disconnected and then the plate

separation is doubled, what are the answers to parts (a), (b) and (c) above?

(e) If the battery is kept connected and the plate separation is doubled, then what are the answers to parts (a), (b) and (c) above?

SOLUTION

(a) $Q = CV = (10 \times 10^{-6}) \times 20 = 200 \times 10^{-6}\ \text{C} = 200\ \mu\text{C}$

(b) $U = \frac{1}{2}\ CV^2 = \frac{1}{2}\ (10 \times 10^{-6}) \times (20)^2 = 2 \times 10^{-3}\ \text{J}$

(c) $E = \frac{V}{d} = \frac{20}{\left(0.5 \times 10^{-3}\right)} = 4 \times 10^4\ \text{Vm}^{-1}$

NOTE

The engery is stored in the electric field between the plates

(d) If the battery is disconnected, the charge on the plates remains the same but the potential difference between the plates will charge. If the separation between the plates is doubled capacitance becomes

$$C' = \frac{\varepsilon_0 A}{2d} = \frac{C}{2} = 5\ \mu\text{F} = 5 \times 10^{-6}\ \text{F}$$

and potential difference between the plates becomes

$$V' = \frac{Q}{C'} = \frac{200\ \mu\text{C}}{5\ \mu\text{F}} = 40\text{V}$$

$$\therefore\ U' = \frac{1}{2}C'V'^2 = \frac{1}{2} \times 5 \times 10^{-6} \times (40)^2 = 4 \times 10^{-3}\text{J}$$

Alternatively, $U' = \frac{Q^2}{2C'} = \frac{(200 \times 10^{-6})^2}{2 \times (5 \times 10^{-6})} = 4 \times 10^{-3}\text{J}$

$$E' = \frac{V'}{d'} = \frac{40}{1 \times 10^{-3}} = 4 \times 10^4\ \text{Vm}^{-1}$$

(e) If the battery is kept connected, the potential difference between the plates always remains equal to the emf of the battery and hence is constant = 20 V. If d is doubled,

Capacitance becomes $C' = \frac{\varepsilon_0 A}{2d} = \frac{C}{2}$

$= 5\ \mu\text{F} = 5 \times 10^{-6}\ \text{F}$

Charge becomes $Q'' = C'V = 5 \times 10^{-6} \times 20$

$= 10^{-4}\ \text{C}$

Energy stored becomes $U'' = \frac{1}{2}\ C'V^2$

$= \frac{1}{2} \times 5 \times 10^{-6} \times (20)^2 = 10^{-3}\text{J}$

Electric field becomes $= \frac{V}{2d} = \frac{20}{20 \times (0.5 \times 10^{-3})}$

$= 2 \times 10^4\ \text{Vm}^{-1}$

EXAMPLE 21.2

Find the charge on each capacitor shown in Fig. 21.1. Given $C_1 = 2\ \mu\text{F}$, and $C_2 = 2\ \mu\text{F}$ and $C_3 = 1\ \mu\text{F}$.

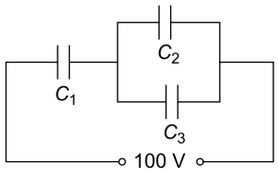


Fig. 21.1

SOLUTION

The capacitance of the parallel combination of C_2 and C_3 is $C' = C_2 + C_3 = 3\ \mu\text{F}$

The circuit can be redrawn as shown in Fig. 21.2.

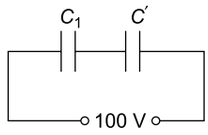


Fig. 21.2

The equivalent capacitance C of series combination of C_1 and C' is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C'}$$

$$\Rightarrow \qquad C = \frac{C_1 C'}{C_1 + C'} = \frac{2 \times 3}{2 + 3} = \frac{6}{5} = 1.2\ \mu\text{F}$$

Let Q_1, Q_2 and Q_3 be the charges on C_1, C_2, and C_3 respectively. Since C_1 and C' are in series, charge on C_1 = charge on $C' = Q_1$. Let V_1 and V' be the potential differences across C_1 and C' respectively. Then

$$Q_1 = C_1V_1 = C'V' \Rightarrow \frac{V'}{V_1} = \frac{C_1}{C'} = \frac{2}{3}$$

But $V_1 + V' = 100$. Therefore

$$V_1 + \frac{2}{3}\ V_1 = 100 \Rightarrow V_1 = 60\ \text{V}$$

Therefore $V' = 100 - 60 = 40$ V. Hence

$$Q_1 = C_1V_1 = 2\ \mu\text{F} \times 60\ \text{V} = 120\ \mu\text{C}$$

Since C_2 and C_3 are in parallel, potential difference V_2 across C_2 = potential difference V_3 across $C_3 = V'$ = 40 V. Therefore,

Charge on capacitor $C_2 = C_2V_2 = 2\ \mu\text{F} \times 40\ \text{V} = 80\ \mu\text{C}$ and Charge on capacitor $C_3 = C_3V_3 = 1\ \mu\text{F} \times 40\ \text{V} = 40\ \mu\text{C}$

Thus $Q_1 = 120\ \mu\text{C}$, $Q_2 = 80\ \mu\text{C}$ and $Q_3 = 40\ \mu\text{C}$

EXAMPLE 21.3

In the network shown in Fig 21.1 in Example 21.2, find the total energy stored in the network.

SOLUTION

Total (equivalent) capacitance of network is

$$C = 1.2\ \mu F = 1.2 \times 10^{-6}\ F.$$

Total potential difference across the network is $V = 100$ V.

Therefore, total energy stored is

$$U = \frac{1}{2}\ CV^2 = \frac{1}{2} \times (1.2 \times 10^{-6}) \times (100)^2 = 6 \times 10^{-3}\ J$$

Notice that

$$U = U_1 + U_2 + U_3 = \frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2V_2^2 + \frac{1}{2} C_3V_3^2.$$

EXAMPLE 21.4

Two capacitors of capacitances C_1 and C_2 have a total capacitance of 20 μF when connected in parallel and a total capacitance 4.8 μF when connected in series. Find C_1 and C_2.

SOLUTION

$$C_1 + C_2 = 20 \qquad (1)$$

$$\frac{C_1C_2}{C_2 + C_2} = 4.8 \qquad (2)$$

Using (1) in (2) we get

$$\frac{C_1\ C_2}{20} = 4.8 \Rightarrow C_1C_2 = 96\ \mu F$$

$$\therefore \qquad C_2 = \frac{96}{C_1}$$

Using this in (2) we get

$$C_1^2 - 20C_1 + 96 = 0$$

The two roots of this equation are $C_1 = 10$ μF and 12 μF. Hence the two capacitors have capacitances of 10 μF and 12 μF.

EXAMPLE 21.5

Four capacitors of capacitances $C_1 = 1$ μF, $C_2 = 2$ μF, $C_3 = 3$ μF and $C_4 = 4$ μF are connected as shown in Fig. 21.3.

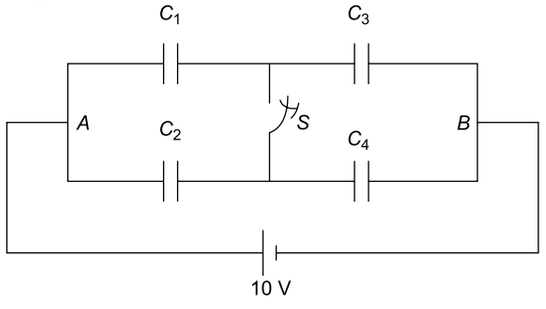


Fig. 21.3

Find the potential difference across C_3 when (a) switch S is open and (b) switch S is closed.

SOLUTION

(a) When switch S is open the circuit is [see Fig. 21.4]

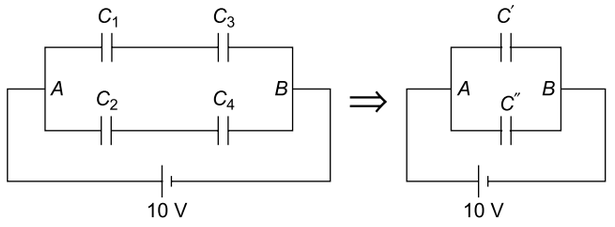


Fig. 21.4

C_1 and C_3 are in series, charge on C_1 = charge on C_3 = Q (say) and potential difference across the combination of C_1 and C_3 is 10 V.

$$\therefore \qquad Q = C_1V_1 = C_3V_3 \Rightarrow C_1V_1 = C_3V_3$$

$$\Rightarrow \qquad V_1 = 3V_3. \text{ Also } V_1 + V_3 = 10$$

Hence $\qquad V_3 = 2.5$ V.

(b) When switch S is closed, the circuit is as shown in Fig. 21.5.

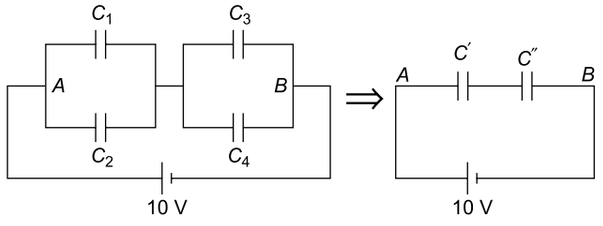


Fig. 21.5

$$C' = 1 + 2 = 3\ \mu F$$

$$C'' = 3 + 4 = 7\ \mu F$$

Since C' and C'' are in series, the charge on C' = charge on C''

or $\qquad C'V' = C''V'' \Rightarrow 3V' = 7V''$

$$\Rightarrow \qquad V' = \frac{7V''}{3}$$

Also $\qquad V' + V'' = 10 \Rightarrow \frac{7V''}{3} + V'' = 10$

$$\Rightarrow \qquad V'' = 3V$$

Since C_3 and C_4 are in parallel, potential difference across C_3 = potential difference across C_4 = 3V.

EXAMPLE 21.6

In Ex. 21.5 above, find the energy stored in the network of capacitors shown in Fig. 21.3 when (a) Switch S is open and (b) switch S is closed.

SOLUTION

(a) Refer to Fig. 21.4. The equivalent capacitance between point A and B is

$$C_a = C' + C''$$

$$= \frac{C_1 C_3}{C_1 + C_3} + \frac{C_2 C_4}{C_2 + C_4}$$

$$= \frac{1\times 3}{(1+3)} + \frac{2\times 4}{(2+4)} = \frac{25}{12}\mu\text{F} = \frac{25}{12}\times 10^{-6}\,\text{F}$$

Energy stored is

$$U_a = \frac{1}{2} C_a V^2 = \frac{1}{2}\times\frac{25}{12}\times 10^{-6}\times(10)^2 = 1.04\times 10^{-4}\,\text{J}$$

(b) Refer to Fig. 21.5. The equivalent capacitance between point A and B is

$$C_b = \frac{C' C''}{C' + C''} = \frac{3\times 7}{3+7} = \frac{21}{10}\mu\text{F} = \frac{21}{10}\times 10^{-6}\,\text{F}$$

Energy stored is

$$U_b = \frac{1}{2} C_b V^2 = \frac{1}{2}\times\frac{21}{10}\times 10^{-6}\times(10)^2 = 1.05\times 10^{-4}\,\text{J}$$

EXAMPLE 21.7

Find the equivalent capacitance between A and B in the circuit shown in Fig. 21.6. Given C_1 = 5 μF, C_2 = 20 μF, C_3 = 10 μF, C_4 = 40 μF and C_5 = 30 μF.

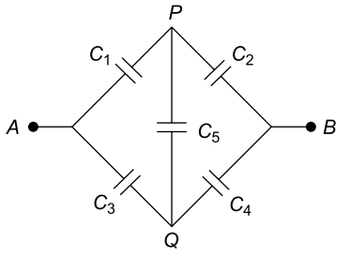


Fig. 21.6

SOLUTION

The network shown in the figure is a balanced wheatstone's bridge. If the values of C_1, C_2, C_3 and C_4 are such that the condition

$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

is satisfied, the bridge is said to be balanced. The potential at P = potential at Q. Since the potential difference between P and Q is zero (when a battery is connected across A and B), the capacitor C_5 is ineffective as no charge collects on its plates.

Hence the circuit reduces to [see Fig. 21.7]

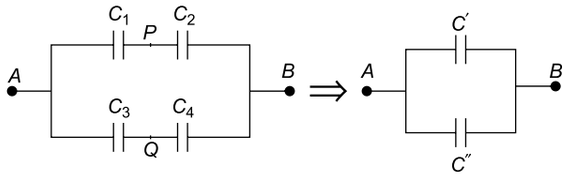


Fig. 21.7

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{5\times 20}{5+20} = 4\,\mu\text{F}$$

$$C'' = \frac{C_3 C_4}{C_3 + C_4} = \frac{10\times 40}{10+40} = 8\,\mu\text{F}$$

$$\therefore \quad C_{AB} = C' + C'' = 12\ \mu\text{F}$$

NOTE

For an unbalanced Wheelstone's bridge or for any other more complicated combinations of capacitors, it is not easy to find the equivalent capacitance using the formulae for series and parallel combinations.

For such cases, we should use the following procedure:

(1) Connect an imaginary battery between the points across which the equivalent capacitance is to be found.

(2) Send a positive charge + Q from the positive terminal of the battery and equal negative charge –Q from the negative terminal.

(3) Write the charges on each capacitor plate using the principle of charge conservation. Let Q_1,Q_2, ... etc. be the charges on the capacitors in the network and V_1, V_2,... etc. be the respective potential differences.

(4) Use $Q = CV$ for each capacitor. Eliminate Q_1,Q_2, ... etc. and V_1, V_2,... etc. to obtain the equivalent capacitance $C_{\text{eq}} = \frac{Q}{E}$, where E is the voltage of the battery

EXAMPLE 21.8

Find the equivalent capacitance between A and B in the circuit shown in Fig. 21.8. Given C = 5 μF.

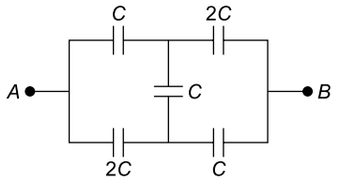


Fig. 21.8

SOLUTION

Refer to Fig. 21.9.

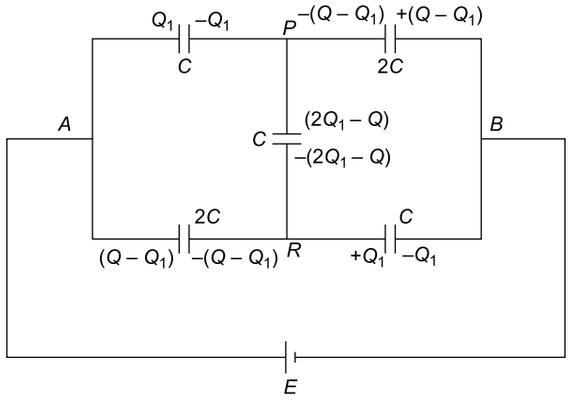


Fig. 21.9

Let us imagine a battery of emf E connected between A and B. Let the positive terminal send a charge $+Q$ and let negative terminal send a charge $-Q$. The distribution of charges on the capacitor plates is shown in the figure using the principle of charge conservation. Then

$$V_A - V_B = (V_A - V_P) + (V_P - V_B)$$

$$\Rightarrow \quad E = \frac{Q_1}{C} + \frac{(Q-Q_1)}{2C}$$

$$\Rightarrow \quad 2CE = 2Q_1 + (Q - Q_1) = Q_1 + Q \qquad (1)$$

Also $V_A - V_B = (V_A - V_P) + (V_P - V_R) + (V_R - V_B)$

$$\Rightarrow \quad E = \frac{Q_1}{C} + \frac{(2Q_1 - Q)}{C} + \frac{Q_1}{C}$$

$$\Rightarrow \quad CE = Q_1 + (2Q_1 - Q) + Q_1 = 4Q_1 - Q \quad (2)$$

Eliminating Q_1 from (1) and (2), we get

$$Q = \frac{7CE}{5}$$

$$\therefore \quad C_{eq} = \frac{Q}{E} = \frac{7C}{5} = \frac{7 \times 5\,\mu F}{5} = 7\,\mu F$$

$[\because\ C = 5\ \mu F$ (given)$]$

EXAMPLE 21.9

The circuit shown in Fig. 21.10 consists of two capacitors $C_1 = 2\ \mu F$ and $C_2 = 4\ \mu F$ and two batteries, each of emf $E = 3V$. Find the charge flowing through points A and B when switch S is closed

SOLUTION

When switch S is open, the equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{2 \times 4}{(2+4)} = \frac{4}{3}\,\mu F$$

Charge on the R.H.S. plate of C_1 and upper plate of C_2 is

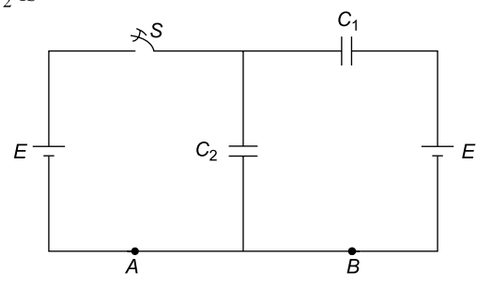


Fig. 21.10

$$Q = CE = \frac{4}{3}\,\mu F \times 3V = 4\ \mu C$$

When switch S is closed, the potential difference across C_1 is zero, because the batteries in opposition. Therefore, charge on the R. H. S. plate of $C_1 = 0$. If the charge flowing through B is q, then

$$Q + q = 0$$

$$\Rightarrow \quad q = -Q = -4\ \mu C$$

Now the charge on the upper plate of $C_2 = C_2E = 4\ \mu F \times 3V = 12\ \mu C$, which is the charge flowing through A.

EXAMPLE 21.10

The circuit shown in Fig. 21.11 consists of four capacitors C_1, C_2, C_3, and C_4 each of capacitance $4\ \mu F$ and a battery of emf $E = 5V$. Find the potential difference between A and B.

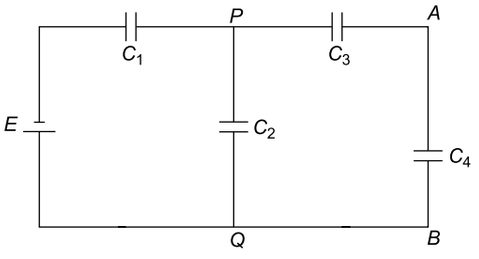


Fig. 21.11

SOLUTION

Capacitors C_3 and C_4 are in series, their equivalent capacitance is

$$C' = \frac{C_3 C_4}{(C_3 + C_4)} = \frac{4 \times 4}{(4+4)} = 2\mu F$$

C' is in parallel with C_2.

Their equivalent capacitance is

$C'' = C_2 + C' = 4 + 2 = 6\ \mu F$

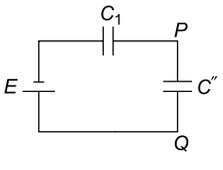


Fig. 21.12

Hence the circuit can be redrawn as shown in Fig. 21.12.

If V_1 is the p.d. across C_1 and V'' across C'', then (since they are in series)

$$C_1 V_1 = C'' V''$$

$$\Rightarrow \quad 4V_1 = 6V'' \Rightarrow V_1 = \frac{3V''}{2}$$

But $V_1 + V'' = E = 5$ Hence

$$\frac{3V''}{2} + V'' = 5 \Rightarrow V'' = 2\text{V}$$

Thus the p.d. between P and $Q = 2$V. By symmetry of C_3 and C_4, the p.d. between A and $B = 1$V.

EXAMPLE 21.11

Three capacitors $C_1 = 1$ μF, $C_2 = 2$ μF and $C_3 = 3$ μF are connected as shown in Fig. 21.13. Find the equivalent capacitance between points A and B.

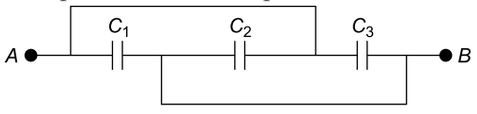


Fig. 21.13

SOLUTION

Refer to Fig. 21.14.

Plate 1 of C_1, plate 4 of C_2 and plate 5 of C_3 are connected to point A. Plate 2 of C_1, plate 3 of C_2 and plate 6 of C_3 are connected to point B. Thus C_1, C_2 and C_3 are connected in parallel. Therefore, the equivalent capacitance C_{eq} between point A and B is given by

$$C_{eq} = C_1 + C_2 + C_3$$
$$= 1 + 2 + 3 = 6\ \mu\text{F}$$

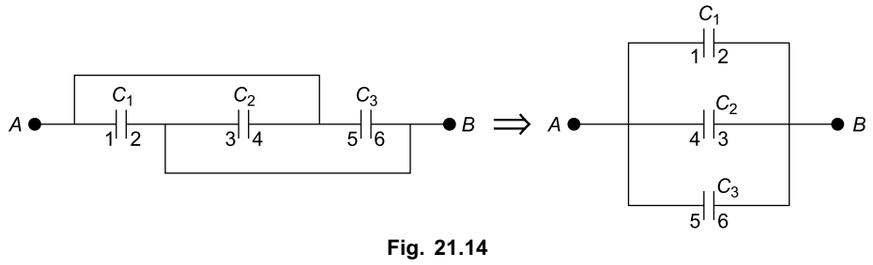


Fig. 21.14

21.10 CAPACITANCE OF DIELECTRIC FILLED CAPACITORS

(i) Capacitors as shown in Fig. 21.15

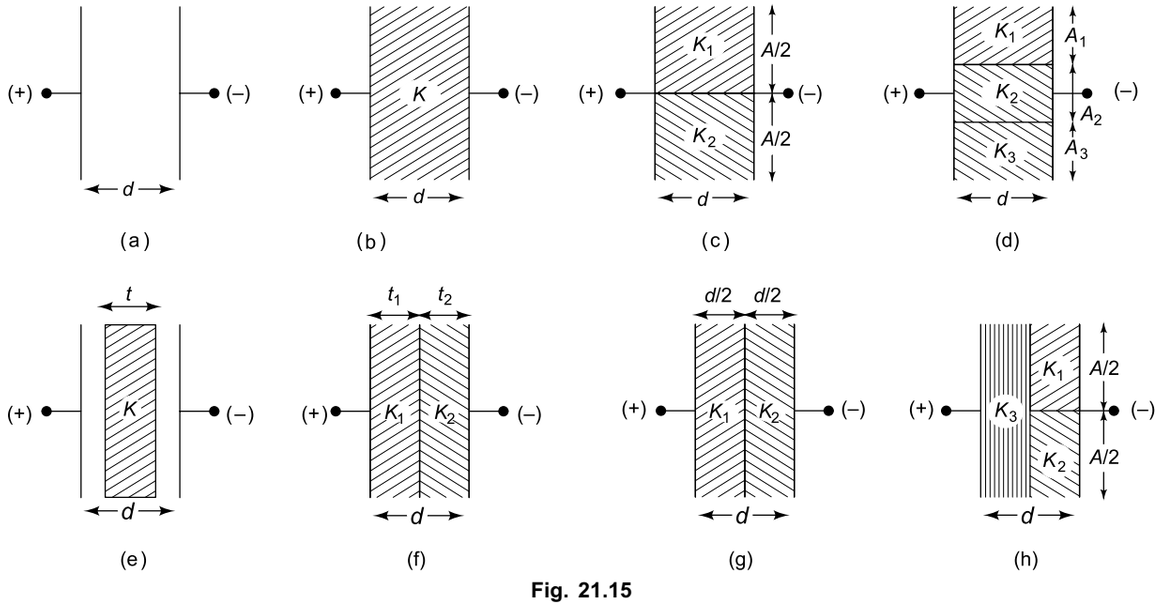


Fig. 21.15

(a) Capacitor with air as dielectric [Fig. 21.15(a)]

$$C_0 = \frac{\varepsilon_0 A}{d}$$

(b) Capacitor completely filled with a dielectric of dielectric constant K [Fig. 21.15(b)]

$$C = KC_0 = \frac{K\varepsilon_0 A}{d}$$

(c) Capacitor filled with two dielectrics as shown in Fig. 21.15 (c)
Capacitance of upper dielectric is

$$C_1 = \frac{K_1\varepsilon_0 A/2}{d} = \frac{K_1\varepsilon_0 A}{2d}$$

Capacitance of lower dielectric is $C_2 = \frac{K_2\varepsilon_0 A}{2d}$

Since positive plates of C_1 and C_2 are connected together, C_1 and C_2 are in parallel. Hence the capacitance of combination is

$$C = C_1 + C_2 = \left(\frac{K_1 + K_2}{2}\right)\frac{\varepsilon_0 A}{d} = \frac{K_{eq}\varepsilon_0 A}{d}$$

where $K_{eq} = \frac{1}{2}(K_1 + K_2)$ is the equivalent dielectric constant

(d) Capacitor filled with three dielectrics as shown in Fig. 21.15 (d)

$$C = C_1 + C_2 + C_3 = \frac{K_{eq}\varepsilon_0 A}{d}$$

$$\text{Where } K_{eq} = \frac{K_1A_1 + K_2A_2 + K_3A_3}{(A_1 + A_2 + A_3)}$$

(e) Capacitor partly filled with a dielectric as shown in Fig. 21.15(e)

$$C = \frac{\varepsilon_0 A}{\left[d - t + \frac{t}{K}\right]} = \frac{C_0}{\left[1 + \frac{t}{d}\left(\frac{1}{K} - 1\right)\right]}$$

Where $C_0 = \frac{\varepsilon_0 A}{d}$. If $t = d$, $C = KC_0$ as in Fig. 21.15 (b)

(f) Capacitor filled with two dielectrics as shown in Fig. 21.15 (f)

Capacitance of the left capacitor is $C_1 = \frac{K_1\varepsilon_0 A}{t_1}$

Capacitance of the right capacitor is $C_2 = \frac{K_2\varepsilon_0 A}{t_2}$

Since C_1 and C_2 are in series, the capacitance of the combination is

$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{K_1K_2\varepsilon_0{}^2A^2}{t_1t_2\,\varepsilon_0 A\left(\frac{K_1}{t_1} + \frac{K_2}{t_2}\right)}$$

$$\Rightarrow \quad C = \frac{K_1K_2\,\varepsilon_0 A}{(K_1t_2 + K_2t_1)} = \frac{K_{eq}\varepsilon_0 A}{d}$$

$$\text{Where} \quad \frac{K_{eq}}{d} = \frac{K_1K_2}{(K_1t_2 + K_2t_1)}$$

$$\Rightarrow \quad \frac{K_{eq}}{t_1 + t_2} = \frac{K_1K_2}{(K_1t_2 + K_2t_1)} \quad (\because\ d = t_1 + t_2)$$

$$\Rightarrow \quad K_{eq} = \frac{K_1K_2(t_1 + t_2)}{(K_1t_2 + K_2t_1)}$$

(g) Capacitor filled with two dielectrics as shown in Fig. 21.15(g)

$$C_1 = \frac{2K_1\varepsilon_0 A}{d},\ C_2 = \frac{2K_2\varepsilon_0 A}{d}$$

$$\therefore \quad C = \frac{C_1C_2}{(C_1 + C_2)} = \frac{K_{eq}\varepsilon_0 A}{d}$$

$$\text{Where} \quad K_{eq} = \frac{2K_1K_2}{(K_1 + K_2)}$$

(h) Capacitor filled with three dielectrics as shown in Fig. 21.13(h)
Capacitors with K_1 and K_2 are in parallel and this combination is in series with the capacitor with K_3

$$C = \frac{K_{eq}\varepsilon_0 A}{d}, \text{ Where } K_{eq} \text{ is given by}$$

$$\frac{1}{K_{eq}} = \frac{1}{(K_1 + K_2)} + \frac{1}{2K_3}$$

21.11 CHARGING AND DISCHARGING OF A CAPACITOR THROUGH A RESISTANCE

Consider a capacitor of capacitance C connected in series to a resistor R and a battery of emf E and negligible internal resistance through a two-way key as shown in Fig. 21.16.

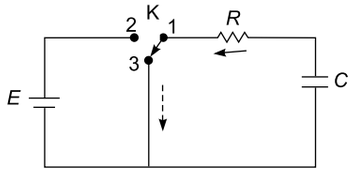


Fig. 21.16

Growth of Charge The battery is introduced in the circuit by connecting terminals 1 and 2 of the key. Initially, i.e. when $t = 0$, the charge on the capacitor plates is zero. As time passes, the charge flows into the capacitor plates and the potential difference q/C (q is the charge at any time t) between the plates rises. The charge on the plates

of the capacitor rises till the potential difference between the plates becomes E. The maximum charge collected is $q_0 = CE$. At this stage the current in the circuit becomes zero.

The growth of charge on the capacitor plates as a function of time is given by

$$q = q_0 (1 - e^{-t/RC}) \quad (1)$$

It is clear that the charge rises exponentially to a steady state maximum value q_0 as shown in Fig. 21.17 (a).

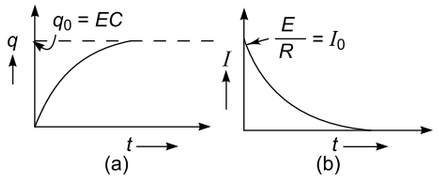


Fig. 21.17

Time Constant The product RC has the dimensions of time. If C is in farad and R in ohm, the product CR will be in seconds. Writing $RC = \tau$ in Eq. (1) we have

$$q = q_0 (1 - e^{-t/\tau})$$

At time $t = \tau$,

$$q = q_0 \left(1 - \frac{1}{e}\right) = q_0 \left(1 - \frac{1}{2.718}\right) = 0.63 q_0$$

Thus the time constant τ of a CR circuit may be defined as the time during which the charge on the capacitor grows from zero to 0.63 of its maximum value q_0. Whether the charge grows quickly or slowly depends on the value of the time constant, i.e. on the values of R and C. If the product CR (i.e. the time constant) is very small, the charge grows quickly.

The behaviour of current as a function of time t is given by

$$I = \frac{dq}{dt} = I_0 e^{-t/RC}$$

where $I_0 = E/R$ is the maximum current. Therefore, the current decreases exponentially from its maximum value I_0 to zero as shown in Fig. 21.17 (b).

Decay of Charge When the charge has attained a steady value EC, the battery is short-circuited by connecting the terminals 1 and 3 of the key K. In such a situation the capacitor starts discharging through the resistor, i.e. the charge on the capacitor starts flowing back through the resistor. The direction of the current is, therefore, reversed as shown by the broken arrow in Fig. 21.16.

The decay of charge with time is given by

$$q = q_0 e^{-t/RC} = q_0 e^{-t/\tau}$$

Figure 21.18 shows the decay of charge with time.

At time $t = \tau$, $q = q_0 e^{-1} = 0.368 q_0$. Thus in a time $t = \tau$, the time constant, the charge on the capacitor decays to 0.368 of its initial value q_0. So the charge decays exponentially with time t. Whether the charge decays slowly or quickly depends on the value of the time constant RC.

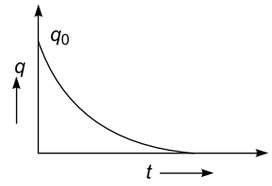


Fig. 21.18

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. A spherical capacitor consists of an inner sphere of radius $r_1 = r$ and the outer sphere of radius $r_2 = 2r$. The capacitance is C_1 when the inner sphere is charged and the outer sphere is earthed and C_2 when the inner sphere is earthed and the outer sphere is charged. The ratio C_1/C_2 is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) 2 (d) 3

2. In the circuit shown in Fig. 21.19, $C_1 = 3$ μF and $C_2 = 9$ μF. The charge on capacitor C_2 is

(a) 9 μC (b) 18 μC

(c) 27 μC (d) 81 μC

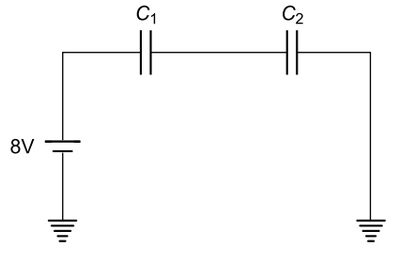


Fig. 21.19

3. Three capacitors, each of capacitance C are connected to a battery of voltage V. When key K is closed, the charge which will flow through the battery is (see Fig. 21.20)

(a) $2\,CV$ (b) CV

(c) $\frac{CV}{2}$ (d) $\frac{CV}{3}$

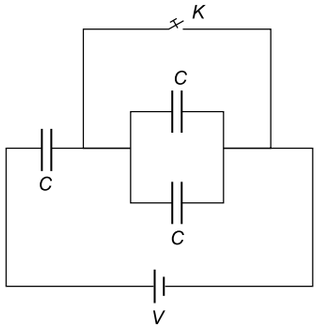


Fig. 21.20

4. In the circuit shown in Fig. 21.21, when the steady state is reached, the energy stored in C_1 is E_1 and that stored in C_2 is E_2. The ratio E_2/E_1 is

(a) 1.5 (b) 1.25

(c) 0.75 (d) 0.5

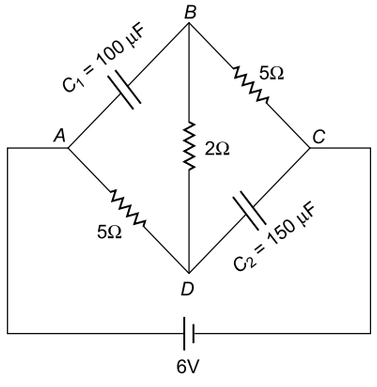


Fig. 21.21

5. A capacitor of capacitance 5 μF is fully charged by a 120 V battery. The battery is disconnected. If an additional charge of + 200 μC is given to the positive plate, the potential difference between the capacitor plates will be

(a) 100 V (b) 120 V

(c) 140 V (d) 160 V

6. A solid metallic sphere of radius r is enclosed by a thin metallic shell of radius $2\,r$. A charge q is given to the inner sphere. When the inner sphere is connected to the shell by a metal wire, the heat energy generated in it is given by

(a) $\frac{q^2}{\pi\varepsilon_0 r}$ (b) $\frac{q^2}{4\pi\varepsilon_0 r}$

(c) $\frac{q^2}{8\pi\varepsilon_0 r}$ (d) $\frac{q^2}{16\pi\varepsilon_0 r}$

7. A solid metallic sphere of radius r is charged to a voltage V. It is enclosed by a thin spherical shell of voltage $2\,V$. If q is the charge on the sphere and the shell, the potential difference between them will be

(a) $\frac{3V}{2}$ (b) V

(c) $\frac{V}{2}$ (d) $\frac{2V}{3}$

8. Two parallel plate capacitors of capacitances C and $2\,C$ are connected in parallel and charged to a potential difference V by a battery. The battery is then disconnected and the space between the plates of capacitor of capacitance C is completely filled with a material of dielectric constant K. The potential difference across the capacitors now becomes

(a) $\frac{V}{K+1}$ (b) $\frac{2V}{K+2}$

(c) $\frac{3V}{K+2}$ (d) $\frac{3V}{K+3}$

IIT, 1987

9. A parallel plate capacitor of plate area A has a charge Q. The force on each plate of the capacitor is

(a) $\frac{2Q^2}{\varepsilon_0 A}$ (b) $\frac{Q^2}{\varepsilon_0 A}$

(c) $\frac{Q^2}{2\varepsilon_0 A}$ (d) zero

10. If n drops, each of capacitance C, coalesce to form a single big drop, the capacitance of the big drop will be

(a) n^3C (b) nC

(c) $n^{1/2}C$ (d) $n^{1/3}C$

11. If n drops, each of capacitance C and charged to a potential V, coalesce to form a big drop, the ratio of the energy stored in the big drop to that in each small drop will be

(a) $n : 1$ (b) $n^{4/3} : 1$

(c) $n^{5/3} : 1$ (d) $n^2 : 1$

12. A parallel plate capacitor is made by stacking 10 identical metallic plates equally spaced from one another and having the same dielectric between plates. The alternate plates are then connected. If

the capacitor formed by two neighbouring plates has a capacitance C, the total capacitance of the combination will be

(a) $\frac{C}{10}$ (b) $\frac{C}{9}$

(c) $9\,C$ (d) $10\,C$

13. Figure 21.22 shows three capacitors connected to a 6 V power supply. What is the charge on the 2 μF capacitor?

(a) 1 μC
(b) 2 μC
(c) 3 μC
(d) 4 μC

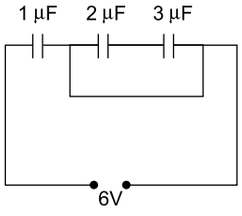


Fig. 21.22

14. Figure 21.23 shows five capacitors connected across a 12 V power supply. What is the charge the 2 μF capacitor?

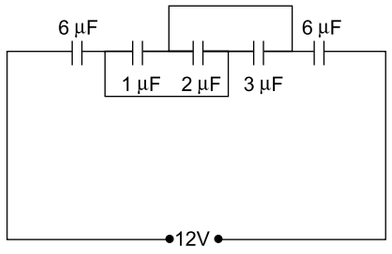


Fig. 21.23

(a) 6 μC (b) 8 μC

(c) 10 μC (d) 12 μC.

15. A capacitor of capacitance C is fully charged by a 200 V supply. It is then discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat 2.5×10^2 J kg^{-1} K^{-1} and of mass 0.1 kg. If the temperature of the block rises by 0.4 K, what is the value of C?

(a) 500 μF (b) 400 μF

(c) 300 μF (d) 200 μF

16. A capacitor is charged by using a battery which is then disconnected. A dielectric slab is then introduced between the plates which results in the

(a) reduction of charge on the plates and increase of potential difference across the plates
(b) increase is the potential difference across the plates and reduction in stored energy but no change in the charge on the plates
(c) decrease in the potential difference across the plates and reduction in the stored energy but no change in the charge on the plates
(d) none of these

IIT, 1987

17. The capacitance of a parallel plate capacitor is 5 μF. When a glass slab of thickness equal to the separation between the plates is introduced be-tween the plates, the potential difference reduces to 1/8 of the original value. The dielectric constant of glass is

(a) 1.6 (b) 5

(c) 8 (d) 40

18. The plates of a parallel plate capacitor are charged to 100 V. A 2 mm thick plate is inserted between the plates. Then to maintain the same potential difference, the distance between the capacitor plates is increased by 1.6 mm. The dielectric constant of the plate is

(a) 5 (b) 1.25

(c) 4 (d) 2.5

19. Four capacitors are connected as shown in Fig. 21.24. The effective capacitance between points A and B will be

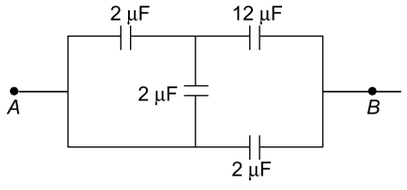


Fig. 21.24

(a) $\frac{28}{9}$ μF (b) 4 μF

(c) 5 μF (d) 18 μF

20. A parallel plate capacitor of value 1.77 μF is to be designed using a dielectric material (dielectric constant 200, breakdown strength of 3×10^6 Vm^{-1}). In order to make such a capacitor, which can withstand a potential difference of 20 V across the plates, the separation d between the plates and the area A of the plates can be ($\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2})

(a) $d = 10^{-6}$ m, $A = 10^{-3}$ m^2
(b) $d = 10^{-5}$ m, $A = 10^{-2}$ m^2
(c) $d = 10^{-4}$ m, $A = 10^{-4}$ m^2
(d) $d = 10^{-4}$ m, $A = 10^{-5}$ m^2

IIT, 1994

21. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is then disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) zero (b) $\frac{3}{2}CV^2$

(c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$

IIT, 1995

22. The magnitude of electric field E in the annular region of a charged cylindrical capacitor

(a) is the same throughout

(b) is higher near the outer cylinder than near the inner cylinder

(c) varies as $\frac{1}{r}$ where r is the distance from the axis

(d) varies as $\frac{1}{r^2}$ where r is the distance from the axis.

IIT, 1996

23. A parallel combination of 0.1 MΩ resistor and a 10 μF capacitor is connected across a 1.5 V source of negligible resistance. The time (in seconds) required for the capacitor to get charged up to 0.75 V is approximately

(a) ∞ (b) $\log_e 2$

(c) $\log_{10} 2$ (d) zero

IIT, 1997

24. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to $3d$,

(a) the magnitude of the electric field remains the same

(b) the direction of the electric field changes continuously

(c) the electric potential increases continuously

(d) the electric potential increases at first, then decreases and again increases.

IIT, 1998

25. Two identical metal plates are given positive charges Q_1 and Q_2 ($< Q_1$) respectively. If they are brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is

(a) $\frac{Q_1 + Q_2}{2C}$ (b) $\frac{Q_1 + Q_2}{C}$

(c) $\frac{Q_1 - Q_2}{C}$ (d) $\frac{Q_1 - Q_2}{2C}$

IIT, 1999

26. For the circuit shown in Fig. 21.25, which of the following statements is true?

(a) With S_1 closed, $V_1 = 15$V, $V_2 = 20$ V

(b) With S_3 closed, $V_1 = V_2 = 25$ V

(c) With S_1 and S_2 closed $V_1 = V_2 = 0$

(d) With S_1 and S_3 closed $V_1 = 30$ V and $V_2 = 20$ V

IIT, 1999

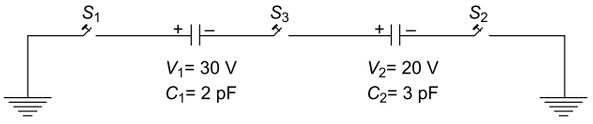


Fig. 21.25

27. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1, K_2 and K_3 as shown in Fig. 21.26. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant K is given by

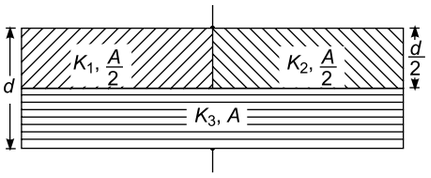


Fig. 21.26

(a) $\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{2K_3}$

(b) $\frac{1}{K} = \frac{1}{K_1 + K_2} + \frac{1}{2K_3}$

(c) $K = \frac{K_1 K_2}{K_1 + K_2} + 2K_3$

(d) $K = K_1 + K_2 + K_3$

IIT, 2000

28. The effective capacitance of two capacitors of capacitances C_1 and C_2 (with $C_2 > C_1$) connected in parallel is $\frac{25}{6}$ times the effective capacitance when they are connected in series. The ratio C_2/C_1 is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$

(c) $\frac{5}{3}$ (d) $\frac{25}{6}$

29. Consider the situation shown in Fig. 21.27. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is

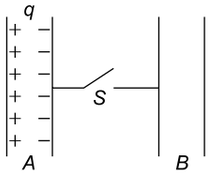


Fig. 21.27

(a) zero (b) $q/2$

(c) q (d) $2q$

IIT, 2001

30. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

(a) $\frac{1}{4}\ C(V_1^2 - V_2^2)$ (b) $\frac{1}{4}\ C(V_1^2 + V_2^2)$

(c) $\frac{1}{4}\ C(V_1 - V_2)^2$ (d) $\frac{1}{4}\ C(V_1 + V_2)^2$

IIT, 2002

31. If the charge on a capacitor is increased by 2 coulomb, the energy stored in it increases by 21%. The original charge on the capacitor (in coulomb) is

(a) 10 (b) 20

(c) 30 (d) 40

32. Figure 21.28 shows a network of capacitors where $C_1 = C_2 = C_3 = C_4 = 4\ \mu F$ and $C_5 = 5\ \mu F$. The equivalent capacitance between points A and B is

(a) 4 μF (b) 5 μF

(c) 16 μF (d) 20 μF

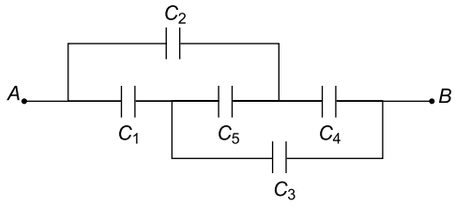


Fig. 21.28

33. The potential difference between points A and B in the circuit shown is Fig. 21.29 is

(a) 6 V (b) 2 V

(c) 10 V (d) 14 V

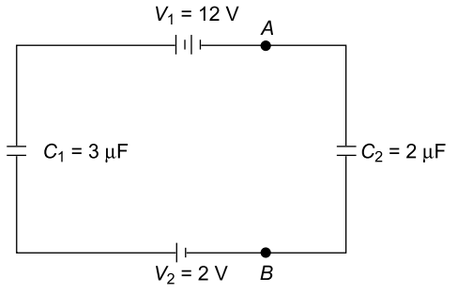


Fig. 21.29

34. An electric field of 200 Vm^{-1} exists in the region between the plates of a parallel plate capacitor of plate separation 5 cm. The potential difference between the plates when a slab of dielectric constant 4 and thickness 1 cm is inserted between the plates is

(a) 7.5 V (b) 8.5 V

(c) 9.0 V (d) 10 V

35. A parallel plate capacitor is maintained at a certain potential difference. When a dielectric slab of thickness 3 mm is introduced between the plates, the plate separation had to be increased by 2 mm in order to maintain the same potential difference between the plates. The dielectric constant of the slab is

(a) 2 (b) 3

(c) 4 (d) 5

36. A capacitor of capacitance C_1 is charged by connecting it to a battery. The battery is now removed and this capacitor is connected to a second uncharged capacitor of capacitance C_2. If the charge distributes equally on the two capacitors, the ratio of the total energy stored in the capacitors after connection to the total energy stored in them before connection is

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{3}$

37. Four metal plates numbered 1, 2, 3 and 4 are arranged as shown is Fig. 21.30. The area of each plate is A and the separation between adjacent plates is d. The capacitance of the arrangement is

(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$

(c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{d}$

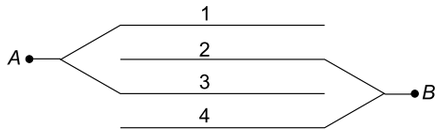


Fig. 21.30

38. Four metal plates numbered 1, 2, 3 and 4 are arranged as shown in Fig. 21.31. The area of each plate is A and the separation between the plates is d. The capacitance of the arrangement is

(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$

(c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{d}$

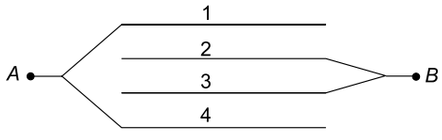


Fig. 21.31

39. A capacitor of capacitance $C_1 = 1.0$ μF can withstand a maximum voltage $V_1 = 6.0$ kV. Another capacitor of capacitance $C_2 = 2.0$ μF can withstand a maximum voltage $V_2 = 4.0$ kV. If the capacitors are connected in series, the combination can withstand a maximum voltage of

(a) 10 kV (b) 9 kV

(c) 8 kV (d) 6 kV

40. A leaky parallel plate capacitor is completely filled with a material having dielectric constant K and conductivity σ. The time constant of the capacitor is

(a) $\frac{K\varepsilon_0}{\sigma}$ (b) $\frac{\sigma}{K\varepsilon_0}$

(c) $\frac{K\sigma}{\varepsilon_0}$ (d) $\frac{\varepsilon_0}{K\sigma}$

41. The introduction of a metal plate between the plates of a parallel plate capacitor increases its capacitance by 4.5 times. If d is the separation of the two plates of the capacitor, the thickness of the metal plate introduced is

(a) $\frac{d}{3}$ (b) $\frac{5d}{9}$

(c) $\frac{7d}{9}$ (d) d

42. If the potential difference between the plates of a capacitor is increased by 20%, the energy stored in the capacitor increases by exactly

(a) 20% (b) 22%

(c) 40% (d) 44%

43. If the potential difference between the plates of a capacitor is increased by 0.1%, the energy stored in the capacitor increases by very nearly

(a) 0.1% (b) 0.11%

(c) 0.144% (d) 0.2%

44. Three capacitors connected in series have an effective resistance of 2 μF. If one of the capacitors is removed, the effective resistance becomes 3 μF. The capacitance of the capacitor that is removed is

(a) 1 μF (b) $\frac{3}{2}$ μF

(c) $\frac{2}{3}$ μF (d) 6 μF

45. The equivalent capacitance between points A and B in the network shown in Fig. 21.32 is ($C_1 = 2$ μF and $C_2 = 3$ μF)

(a) 1 μF (b) 2 μF

(c) 3 μF (d) 4 μF

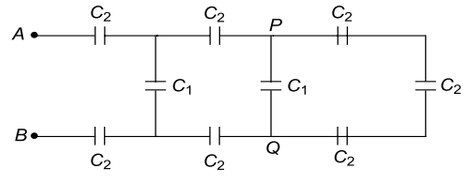


Fig. 21.32

46. The time constants in microsecond of circuits 1, 2 and 3 shown in Fig. 21.33 respectively are (here $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $C_1 = 2$ μF, $C_2 = 4$ μF)

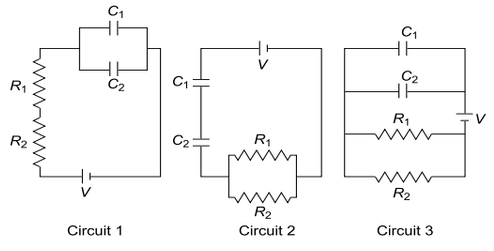


Fig. 21.33

(a) 4, 8/9, 18 (b) 4, 18, 8/9

(c) 18, 8/9, 4 (d) 18, 4, 8/9

IIT, 2006

47. A capacitior of capacitance $C_1 = C$ is charged to a voltage V. It is then connected in parallel with a series combination of two uncharged capacitors of capacitances $C_2 = C$ and $C_3 = C$. The charge that will flow through the connecting wires is

(a) $\frac{CV}{3}$ (b) $\frac{2CV}{3}$

(c) CV (d) zero

48. The capacitance of a sphere of radius R_1 is increased 3 times when it enclosed by an earthed sphere of radius R_2. The ratio R_2/R_1 is

(a) 2 (b) $\frac{3}{2}$

(c) $\frac{4}{3}$ (d) 3

49. A perallel plate rapacitor of plate area A and plate separation d is charged by a battery of voltage V. The battery is then disconnected. The work needed to pull the plates to a separation $2d$ is

(a) $\frac{AV^2\varepsilon_0}{d}$ (b) $\frac{2AV^2\varepsilon_0}{d}$

(c) $\frac{AV^2\varepsilon_0}{2d}$ (d) $\frac{3AV^2\varepsilon_0}{2d}$

50. One plate of a parallel plate capacitor of plate area A and plate separation d is connected to the positive terminal to a battery of the voltage V. The negative terminal of the battery and the other plate of the ceapacitor are earthed as shown in Fig. 21.34. The charge that flows from the battery to the capacitor plates is

(a) zero (b) $\frac{\varepsilon_0 AV}{d}$

(c) $\frac{Vd}{\varepsilon_0 A}$ (d) $\frac{\varepsilon_0 AV}{2d}$

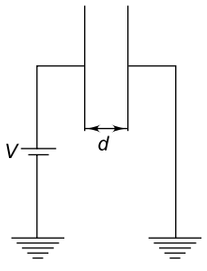


Fig. 21.34

51. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $\frac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is [see Fig. 21.35]

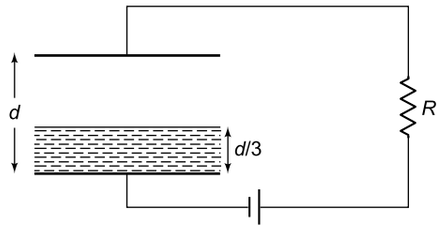


Fig. 21.35

(a) $\frac{6\varepsilon_0 R}{5d + 3Vt}$

(b) $\frac{(15d + 9Vt)\varepsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$

(c) $\frac{6\varepsilon_0 R}{5d - 3Vt}$

(d) $\frac{(15d - 9Vt)\varepsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

IIT, 2008

52. A 2 μF capacitor is charged as shown in Fig. 21.36. The percentage of its stored energy dissipated after the switch S is turned to position 2 is

(a) 0 % (b) 20 %

(c) 75 % (d) 80 %

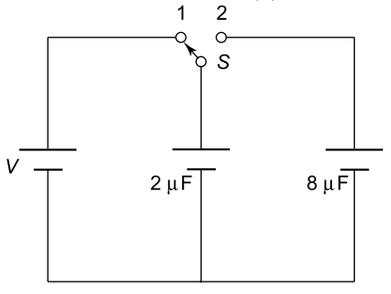


Fig. 21.36

IIT, 2011

ANSWERS

1. (b)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (d)
7. (c)	**8.** (c)	**9.** (c)	**10.** (d)	**11.** (c)	**12.** (c)
13. (b)	**14.** (b)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (a)
19. (c)	**20.** (b)	**21.** (b)	**22.** (c)	**23.** (d)	**24.** (c)
25. (d)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (c)
31. (b)	**32.** (a)	**33.** (a)	**34.** (b)	**35.** (b)	**36.** (b)
37. (c)	**38.** (b)	**39.** (b)	**40.** (a)	**41.** (c)	**42.** (d)
43. (d)	**44.** (d)	**45.** (a)	**46.** (c)	**47.** (c)	**48.** (b)
49. (c)	**50.** (b)	**51.** (a)	**52.** (d)		

SOLUTIONS

1. $C_1 = 4\pi\varepsilon_0\left(\dfrac{r_1 r_2}{r_2 - r_1}\right)$

$C_2 = 4\pi\varepsilon_0\left(\dfrac{r_2^2}{r_2 - r_1}\right)$

$\therefore \quad \dfrac{C_1}{C_2} = \dfrac{r_1}{r_2} = \dfrac{r}{2r} = \dfrac{1}{2}$

2. Since the capacitors are connected in series, the charge on each will be the same = Q (say). Equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{3\times 9}{3+9} = \frac{9}{4}\,\mu\text{F}$$

$$\therefore \quad Q = CV = \frac{9}{4}\times 8 = 18\ \mu\text{C}$$

3. The capacitance of the parallel combination is $C' = 2C$. The given circuit can be redrawn as shown in Fig. 21.37. When key K is open, the total capacitance $= \dfrac{CC'}{C+C'} = \dfrac{C\times 2C}{C+2C} = \dfrac{2C}{3}$

$\therefore$ Charge on capacitors is

$$Q_1 = \frac{2CV}{3}$$

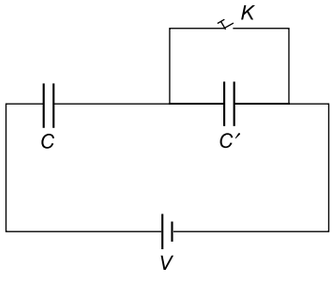


Fig. 21.37

When key K is closed, capacitor C' is short-circuited. The capacitance of the circuit now = C. Therefore, the charge on C is

$$Q_2 = CV$$

$\therefore$ Charge flowing through the battery is $(Q_2 - Q_1)$

$= CV - \dfrac{2CV}{3} = \dfrac{CV}{3}$, which is choice (d).

4. In the steady state, no current flows through the capacitors. Therefore, the total resistance = 5 + 2 + 5 = 12 Ω. Current in the circuit is

$$I = \frac{6\,\text{V}}{12\,\Omega} = 0.5\ \text{A}$$

$\therefore$ P.D. across $C_1 = 0.5\times(2+5) = 3.5$ V = P.D. across C_2. Therefore

$$E_1 = \frac{1}{2}C_1 V_1^2$$

$$E_2 = \frac{1}{2}C_2 V_2^2$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{C_2}{C_1} = \frac{150}{100} = 1.5 \qquad (\because V_1 = V_2)$$

5. Initial charge on the capacitor plates is

$$Q_0 = CV_0 = (5\ \mu\text{F})\times(120\ \text{V}) = 600\ \mu\text{C}$$

So, the charge of the positive plate is + 600 μC and of the negative plate is – 600 μC. If an additional charge of + 200 μC is given to the positive plate, its charge becomes = 600 + 200 = 800 μC. Let Q be the charge induced on the negative plate. The positive plate loses a charge Q and the negative plate gains a charge Q such that the total positive charge on the positive plate = total negative charge on the negative plate, i.e.

$$800 - Q = -600 + Q$$

$\Rightarrow Q = 700$ μC. Therefore, the potential difference between the plates now is

$$V = \frac{Q}{C} = \frac{700\,\mu\text{C}}{5\,\mu\text{F}} = 140\ \text{V}$$

So the correct choice is (c).

6. The capacitance of inner sphere is

$$C_1 = 4\pi\varepsilon_0 r$$

The capacitance of the outer shell is

$$C_2 = 4\pi\varepsilon_0(2r) = 8\pi\varepsilon_0 r$$

Before connection, the total energy is

$$U_1 = \frac{q^2}{2C_1} = \frac{q^2}{8\pi\varepsilon_0 r}$$

After connection, the entire charge q of the inner sphere is transferred to the outer shell. Hence, energy after connection is

$$U_2 = \frac{q^2}{2C_2} = \frac{q^2}{16\pi\varepsilon_0 r}$$

$\therefore$ Heat generated $= U_1 - U_2$

$$= \frac{q^2}{8\pi\varepsilon_0 r} - \frac{q^2}{16\pi\varepsilon_0 r} = \frac{q^2}{16\pi\varepsilon_0 r}$$

7. Charge on the inner sphere is

$$q = CV = 4\pi\varepsilon_0 rV$$

$\therefore$ Potential difference between the two spheres is

$$V' = V_1 - V_2$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{2r}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{2r} = \frac{V}{2}$$

Hence the correct choice is (c).

8. Original capacitance of the parallel combination of C and $2C = C + 2C = 3C$. Total charge $Q = 3CV$. When capacitor C is filled with the dielectric, its capacitance becomes KC. Therefore, the capacitance of the combination becomes $C' = KC + 2C = (K + 2)C$. Since charge remains the same = Q, the potential difference across the capacitors will be

$$\frac{Q}{C'} = \frac{3CV}{(K+2)C} = \frac{3V}{K+2}$$

9. To find the force between the charged capacitor plates we will use a method called the *method of virtual displacement.* We simply equate the work ΔW required to make a small change Δd in the plate separation d to the resulting change ΔU in the stored energy, i.e.

$$\Delta W = \Delta U \qquad \text{(i)}$$

If F is the magnitude of the force between the plates, then the work ΔW done to increase the plate separation by Δd is given by

$$\Delta W = F\,\Delta d \qquad \text{(ii)}$$

Now we know that the energy U of a parallel-plate capacitor of plate area A and capacitance C is

$$U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A}$$

where Q is the charge on the capacitor plates. The increase ΔU in U due to an increase Δd in d is, therefore, given by

$$\Delta U = \frac{Q^2 \Delta d}{2\varepsilon_0 A} \qquad \text{(iii)}$$

Equating Eqs. (ii) and (iii) we get

$$F = \frac{Q^2}{2\varepsilon_0 A}, \text{ which is choice (c).}$$

10. If ρ is the density of a small drop and r its radius, then the mass of each small drop is $m = \frac{4\pi}{3} r^3\rho$. If n such drops coalesce to form a big drop of radius R, then the mass of the big drop is $nm = \frac{4\pi}{3} R^3\rho$. Hence $R = n^{1/3} r$. Now, the capacitance of a sphere is proportional to its radius. Hence the capacitance of the big drop will be $C' = n^{1/3}C$. Hence the correct choice is (d).

11. $E = \frac{1}{2} CV^2$, $E' = \frac{1}{2} C'V'^2$. Therefore,

$$\frac{E'}{E} = \frac{C'}{C}\cdot\frac{V'^2}{V} = n^{1/3} \times (n^{2/3})^2 = n^{5/3}$$

$(\because\ Q = CV = C'V')$

Hence the correct choice is (c).

12. The combination is equivalent to (10 – 1) = 9 capacitors, each of capacitance C connected in parallel. Hence the correct choice is (c).

13. Capacitors of capacitances 2 μF and 3 μF are in parallel and this combination is in series with 1 μF capacitor. Thus we have 1 μF capacitor in series 5 μF capacitor and the potential difference across this series combination is 6V. Therefore, the potential difference across 5 μF capacitor (which consists of a parallel combination of 2 μF and 3 μF capacitors) is 1 V. Hence the charge on 2 μF capacitor = 2 μF × 1 V = 2 μC, which is choice (b).

14. Capacitors 1 μF, 2 μF and 3 μF are in parallel, their total capacitance is 6 μF. Thus, we have three capacitors in series each of capacitance 6 μF across the 12 V power supply. So the potential drop across each is 12/3 = 4 V. This is also the potential across 1 μF capacitor and 2 μF capacitor and 3 μF capacitor, because they are in parallel. Therefore, charge on 2 μF capacitor = 2 μF × 4V = 8 μC. Hence the correct choice is (b).

15. Energy stored in the capacitor is

$$\frac{1}{2} CV^2 = \frac{1}{2} \times C \times (200)^2 = 2 \times 10^4 \times C \text{ joule}$$

Energy appearing as heat in the block is

$$m\ c\ \theta = 0.1 \times 2.5 \times 10^2 \times 0.4 = 10 \text{ J}$$

Therefore,

$$2 \times 10^4 \times C = 10 \text{ or } C = 5 \times 10^{-4} \text{ F} = 500 \text{ μF}$$

16. When the battery is disconnected and a dielectric slab is introduced, the charge Q on the plates does not change but the capacitance C increases. Since $Q = CV$, the potential difference V decreases. Since stored energy = $\frac{1}{2} QV$, a decrease in V results in a decrease in stored energy. Hence the correct choice is (c).

17. The capacitance of a parallel plate capacitor with air (or vacuum) as dielectric is

$$C_0 = \frac{\varepsilon_0 A}{d}$$

When a dielectric of dielectric constant K is introduced, the capacitance becomes

$$C = \frac{K\varepsilon_0 A}{d}, \therefore \frac{C}{C_0} = K$$

Now $Q = C_0V_0$ and $Q = CV$. Therefore

$$\frac{V}{V_0} = \frac{C_0}{C} = \frac{1}{K}$$

But $V/V_0 = \frac{1}{8}$. Therefore $K = 8$.

18. When a dielectric slab of thickness t and dielectric constant K is introduced between the plates of a parallel plate capacitor, the potential difference between the plates is given by

$$V = E_0\left[d - t\left(1 - \frac{1}{K}\right)\right]$$

In order to maintain the same value of V, the separation between the plates should be increased by d' given by

$$d' = t\left(1 - \frac{1}{K}\right) \text{ or } K = \frac{t}{t - d'}$$

$$= \frac{2\,\text{mm}}{2\,\text{mm} - 1.6\,\text{mm}} = 5$$

19. The given network of capacitors can be redrawn as shown in Fig. 21.38.

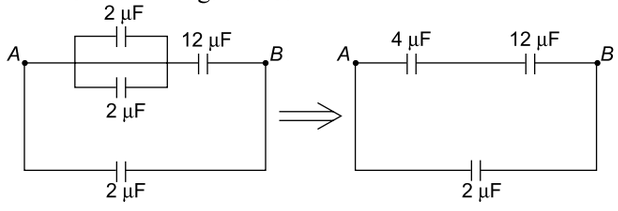


Fig. 21.38

The series combination of 4 μF and 12 μF gives 3 μF which is in parallel with 2 μF. Hence the effective capacitance between A and B = 3 + 2 = 5 μF, which is choice (c).

20. $C = \frac{K\varepsilon_0 A}{d}$. Therefore,

$$\frac{d}{A} = \frac{K\varepsilon_0}{C} = \frac{200 \times 8.85 \times 10^{-12}}{1.77 \times 10^{-6}} = 10^{-3}$$

Hence choices (c) and (d) are not possible. Now $E = \frac{V}{d}$. Therefore,

$$d_{\min} = \frac{V}{E_{\max}} = \frac{V}{\text{dielectric strength}} = \frac{20}{3 \times 10^6}$$

$$= 6.67 \times 10^{-6}\ \text{m}$$

Hence choice (a) is not correct. Thus the correct choice is (b).

21. $Q_1 = CV$ and $Q_2 = (2C) \times (2V) = 4CV$. Since the capacitors are connected in parallel such that the plates of opposite polarity are connected together, the common potential is

$$V' = \frac{Q_2 - Q_1}{C_1 + C_2} = \frac{4CV - CV}{C + 2C} = V$$

Equivalent capacitance $C' = C + 2C = 3C$. Therefore, the final energy of the configuration is

$$U' = \frac{1}{2}C'V'^2 = \frac{1}{2} \times 3C \times V^2 = \frac{3}{2}CV^2$$

22. The magnitude of the electric field in the annular region of a charged cylindrical capacitor is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\lambda}{r}$$

where λ is the charge per unit length and r is the distance from the axis of the cylinder. Hence $E \propto \frac{1}{r}$.

23. There is no resistance in the part of the circuit containing the battery and the capacitor, i.e. $R = 0$ in this circuit. Hence, the time constant $\tau = RC$ is zero. Thus the correct choice is (d).

24. The insertion of the dielectric slab decreases the electric field without changing its direction. The electric potential increases as we go from the negative to the positive plate. Hence the correct choice is (c).

25. Within the plates electric fields due to charges Q_1 and Q_2 are

$$E_1 = \frac{Q_1}{2\varepsilon_0 A} \text{ and } E_2 = \frac{Q_2}{2\varepsilon_0 A}$$

As these fields are in opposite directions and $Q_1 > Q_2$, the net electric field within the plates is

$$E = E_1 - E_2 = \frac{1}{2\varepsilon_0 A}(Q_1 - Q_2)$$

Hence $V = Ed = \frac{d}{2\varepsilon_0 A}(Q_1 - Q_2) = \frac{Q_1 - Q_2}{2C}$

$$\left(\because C = \frac{\varepsilon_0 A}{d}\right)$$

26. When switch S_3 is closed, the potential difference across C_1 and C_2 will become equal to the average of V_1 and V_2, i.e. (30 + 20)/2 = 25 V. Hence the correct choice is (b).

27. We have $C_1 = \frac{(A/2)K_1\varepsilon_0}{(d/2)} = \frac{AK_1\varepsilon_0}{d}$,

$$C_2 = \frac{(A/2)K_2\varepsilon_0}{(d/2)} = \frac{AK_2\varepsilon_0}{d}$$

and $C_3 = \dfrac{AK_3\varepsilon_0}{(d/2)} = \dfrac{2AK_3\varepsilon_0}{d}$

The capacitors C_1 and C_2 are in parallel and their equivalent capacitance is

$$C' = C_1 + C_2 = \frac{A\varepsilon_0}{d}(K_1 + K_2)$$

This combination is in series with C_3. Hence the net capacitance is

$$\frac{1}{C''} = \frac{1}{C'} + \frac{1}{C_3} = \frac{d}{\varepsilon_0 A(K_1 + K_2)} + \frac{d}{2AK_3\varepsilon_0}$$

$$= \frac{d}{\varepsilon_0 A}\left[\frac{1}{(K_1 + K_2)} + \frac{1}{2K_3}\right]$$

or $C'' = \dfrac{AK\varepsilon_0}{d}$ where $\dfrac{1}{K} = \dfrac{1}{(K_1 + K_2)} + \dfrac{1}{2K_3}$

Hence the correct choice is (b).

28. Given $C_1 + C_2 = \dfrac{C_1 C_2}{C_1 + C_2} \times \dfrac{25}{6}$

or $6(C_1 + C_2)^2 = 25\ C_1C_2$

or $6C_1^2 + 6C_2^2 + 12\ C_1C_2 = 25\ C_1C_2$

or $6C_1^2 + 6C_2^2 - 13\ C_1C_2 = 0$

Let $C_2 = x\ C_1$. Then, we have

$$6C_1^2 + 6x^2\ C_1^2 - 13\ x\ C_1^2 = 0$$

or $6x^2 - 13x + 6 = 0$

which gives $x = \dfrac{3}{2}$ or $\dfrac{2}{3}$. Since $C_2 > C_1$, $x = \dfrac{2}{3}$ is not possible. Hence the correct choice is (a).

29. Since the outer plate of B is free, charge cannot flow from A to B. Hence the correct choice is (a).

30. Initial Energy $U_i = U_1 + U_2 = \dfrac{1}{2}\ CV_1^2 + \dfrac{1}{2}\ CV_2^2$

When they are connected, the potential across each is $V = \dfrac{1}{2}(V_1 + V_2)$. Final energy is

$$U_f = \frac{1}{2}\ CV^2 + \frac{1}{2}\ CV^2 = CV^2 = C\left(\frac{V_1 + V_2}{2}\right)^2$$

$\therefore$ Decrease in energy $= U_i - U_f$

$$= \frac{1}{2}\ C(V_1^2 + V_2^2) - \frac{1}{4}\ C(V_1 + V_2)^2$$

$$= \frac{1}{4}\ C\ (V_1 - V_2)^2$$

Hence the correct choice is (c).

31. $U_1 = \dfrac{Q_1^2}{2C}$ and $U_2 = \dfrac{Q_2^2}{2C}$. Therefore,

$$\frac{U_2 - U_1}{U_1} = \frac{Q_2^2 - Q_1^2}{Q_1^2} \qquad \text{(i)}$$

Given $\dfrac{U_2 - U_1}{U_1} = 21\% = 0.21$. Using this in (i), we get $Q_2 = 1.1\ Q_1$. Therefore, $Q_2 - Q_1 = 0.1\ Q_1$.

Given $Q_2 - Q_1 = 2\text{C}$. Hence $0.1\ Q_1 = 2\text{C}$ or $Q_1 = 20$ C, which is choice (b).

32. The network can be redrawn as shown in Fig. 21.39.

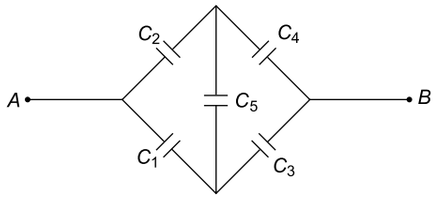


Fig. 21.39

Since $\dfrac{C_2}{C_1} = \dfrac{C_4}{C_3}$, the bridge is balanced. Hence no current flows through capacitor C_5, it is thus ineffective. Capacitors C_2 and C_4 are in series, hence their equivalent capacitance = 2 μF. Similarly, the equivalent capacitance of capacitors C_1 and C_3 = 2 μF. These two are in parallel with reference to points A and B. Hence the equivalent capacitance of the network = 2 + 2 = 4 μF.

33. The batteries are in opppsition as their positive terminals are connected together. Hence the effective voltage is

$$V = V_1 - V_2 = 12 - 2 = 10\text{ V}$$

As the capacitors C_1 and C_2 are in series, the effective capacitance of the circuit is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

or $C = \dfrac{6}{5} = 1.2\ \mu\text{F}.$

Therefore, charge on capacitors is

$$Q = CV = 1.2\ \mu\text{F} \times 10\text{V} = 12\ \mu\text{C}$$

$\therefore$ Potential difference across A and B = potential difference across capacitor C_2

$$\frac{Q}{C_2} = \frac{12\ \mu\text{C}}{2\ \mu\text{F}} = 6\text{ V}$$

34. Potential difference between the plates before the slab is introduced is

$$V = E \times d = 200 \times 0.05 = 10\text{ V}$$

The capacitance of the capacitor is given by

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{0.05} \text{ or } \varepsilon_0 A = 0.05\ C$$

When a slab of dielectirc constant K and thickness t is introduced, the capacitance becomes

$$C' = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)} = \frac{0.05C}{0.05 - 0.01\left(1 - \frac{1}{4}\right)} = \frac{20C}{17}$$

Now $Q = CV = C'V'$. Therefore,

$$V' = \frac{CV}{C'} = \frac{CV}{20C/17} = \frac{17V}{20} = \frac{17 \times 10}{20} = 8.5 \text{ V}$$

35. The capacitance before the introduction of the slab is $C = \dfrac{\varepsilon_0 A}{d}$

If Q is the charge on the plates, the potential difference is

$$V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} \qquad (1)$$

Let d' be the new separation between the plates. When a slab of thickness t and dielectric constant K is introduced, the new capacitance is

$$C' = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{K}\right)}$$

Since charge Q remains the same, the new potential difference is

$$V' = \frac{Q}{C'} = \frac{Q\left[d' - t\left(1 - \frac{1}{K}\right)\right]}{\varepsilon_0 A} \qquad (2)$$

Given $V' = V$. Equating Eqs. (1) and (2), we get

$$d = d' - t\left(1 - \frac{1}{K}\right) \text{ or } d' - d = t\left(1 - \frac{1}{K}\right)$$

Given $d' = d = 2$ mm and $t = 3$ mm. Thus

$$2 = 3\left(1 - \frac{1}{K}\right)$$

which gives $K = 3$. Hence the correct choice is (b).

36. If Q is the initial charge on capacitor C_1, the initial energy is given by

$$U_i = Q^2/2\ C_1$$

When the two capacitors are connected together, and as the charge is distributed equally, the charge on each capacitor is $Q/2$. Since the potential difference (in a parallel connection) across the two capacitors is also the same, it follows that their capacitances are equal (since $C = Q/V$). Thus $C_1 = C_2 = C$(say). Also $Q_1 = Q_2 = Q/2$.

Therefore, final energy stored in the two capacitors is

$$U_f = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \frac{(Q/2)^2}{2C} + \frac{(Q/2)^2}{2C} = \frac{Q^2}{4C}$$

But $\qquad U_i = \dfrac{Q^2}{2C}$

$\therefore \qquad \dfrac{U_f}{U_i} = \dfrac{1}{2}$, which is choice (b).

37. Plate 1 is connected to plate 3 and plate 2 is connected to plate 4. Thus, there are three capacitors in parallel, each of capacitance

$$C = \frac{\varepsilon_0 A}{d}$$

as shown in Fig. 21.40. Hence the equivalent capacitance is

$C' = 3C = \dfrac{3\varepsilon_0 A}{d}$, which is choice (c).

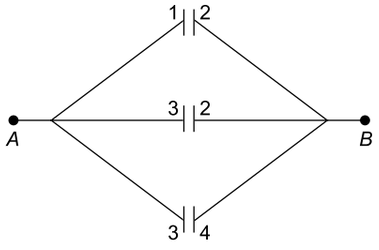


Fig. 21.40

38. The inner plates 2 and 3 are connected together. Hence they act as a single conductor. Since the outer plates 1 and 4 are connected together, there are effectively two capacitors (between plates 1 and 2 and plates 3 and 4) in parallel, each of capacitance $C = \varepsilon_0 A/d$ as shown in Fig. 21.41. Thus the equivalent capacitance is

$$C' = 2C = \frac{2\varepsilon_0 A}{d}, \text{ which is choice (b).}$$

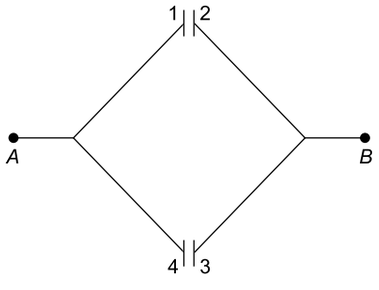


Fig. 21.41

39. The maximum charge the first capacitor can hold is

$$Q_1 = C_1\ V_1 = 1 \times 10^{-6} \times 6000 = 6 \times 10^{-3} \text{ C}$$

The maximum charge the second capacitor can hold is

$$Q_2 = C_2 V_2 = 2 \times 10^{-6} \times 4000 = 8 \times 10^{-3} \text{ C}$$

We know that in a series combination, the charge on each capacitor is the same. Now the first capacitor cannot hold a charge of 8×10^{-3} C; it can hold a maximum charge of 6×10^{-3} C. Therefore, the charge on the second capacitor must also be 6×10^{-3} C. Hence, the voltage across the second capacitor is

$$V_2 = \frac{6 \times 10^{-3}\,\text{C}}{2 \times 10^{-6}\,\text{F}} = 3000 \text{ volts} = 3 \text{ kilovolts}$$

Thus, the maximum voltage the system can withstand = $V_1 + V_2$ = 6 kilovolts + 3 kilovolts = 9 kilovolts. Hence the correct choice is (b).

40. Let A be the plate area and d the plate separation. The resistivity of the material is given by

$$\rho = \frac{RA}{d}$$

where R is the resistance of the material. The conductivity σ is

$$\sigma = \frac{1}{\rho} = \frac{d}{RA} \quad \text{or} \quad R = \frac{d}{\sigma A} \tag{1}$$

The capacitance of the capacitor is

$$C = \frac{K\varepsilon_0 A}{d} \tag{2}$$

Using Eqs. (1) and (2), the time constant of the capacitor is

$$\tau = RC = \frac{d}{\sigma A} \times \frac{K\varepsilon_0 A}{d} = \frac{K\varepsilon_0}{\sigma}$$

Hence the correct choice is (a).

41. Initial capacitance $C = \dfrac{\varepsilon_0 A}{d}$. When a metal plate of thickness t is introduced, the capacitance becomes $C' = \dfrac{\varepsilon_0 A}{(d - t)}$. Given $C' = 4.5\ C$. The correct choice is (c).

42. $U_1 = \dfrac{1}{2}\ CV^2$, $U_2 = \dfrac{1}{2}\ C(1.2\ V)^2 = \dfrac{1}{2}\ CV^2 \times 1.44$

$$\therefore \frac{U_2 - U_1}{U_1} \times 100 = (1.44 - 1) \times 100 = 44\%,$$

Thus the correct choice is (d).

43. $U = \dfrac{1}{2}\ CV^2$. Therefore, $\delta U = CV\ \delta V$. Therefore

$$\frac{dU}{U} \times 100 = \frac{CV\delta V}{\frac{1}{2}CV^2} \times 100 = \frac{2\delta V}{V} \times 100 = \frac{2 \times 0.1 \times 100}{100} = 0.2\%, \text{ which is choice (d)}$$

44. Given $\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{1}{2}$ and $\dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{3}$

Find C_3. The correct choice is (d).

45. The network reduces to that shown in Fig. 21.42. The correct choice is (a).

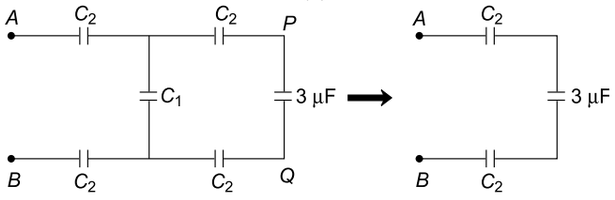


Fig. 21.42

46. Let R and C be the effective resistance and capacitance of the circuit.

For Circuit 1 : $R = R_1 + R_2 = 1 + 2 = 3\ \Omega$

$$C = C_1 + C_2 = 2 + 4 + 6 = 6\ \mu\text{F}$$

Time constant $\tau = RC = 3 \times 6 = 18\ \mu\text{s}$

For Circuit 2 : $R = \dfrac{R_1 \times R_2}{R_1 + R_2} = \dfrac{1 \times 2}{1 + 2} = \dfrac{2}{3}\ \Omega$

$$C = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{2 \times 4}{2 + 4} = \frac{4}{3}\ \mu\text{F}$$

$$\tau = RC = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}\ \mu\text{s}$$

For Circuit 3 : $R = \dfrac{R_1 \times R_2}{R_1 + R_2} = \dfrac{1 \times 2}{1 + 2} = \dfrac{2}{3}\ \Omega$

$$C = C_1 + C_2 = 2 + 4 = 6\ \mu\text{F}$$

$$\tau = RC = \frac{2}{3} \times 6 = 4\ \mu\text{s}$$

The correct choice (c).

47. Charge on C_1 is $Q_1 = C_1 V = CV$. This charge is shared by the three capacitors. The equivalent capacitance of the series combination of C_2 and C_3 is

$$C' = \frac{C_2 C_3}{C_2 + C_3} = \frac{C}{2} \quad (\therefore\ C_2 = C_3 = C)$$

The common potential of C_1 and C' is

$$V' = \frac{Q_1}{C_1 + C'} = \frac{C_1 V}{C_1 + C'} = \frac{CV}{C + C/2} = \frac{2V}{3}$$

$\therefore$ Final charge on C_1 is

$$Q_1' = C_1 V' = \frac{2CV}{3}$$

$\therefore$ Charge that will flow throught the connecting wires is

$Q'' = Q_1 = Q_1' = CV - \frac{2CV}{3} = \frac{CV}{3}$, which is choice (a).

48. $C_1 = 4\pi\varepsilon_0 R_1$

$$C_2 = \frac{4\pi\varepsilon_0(R_1R_2)}{(R_2 - R_1)}$$

Given $C_2 = 3C_1$ Hence

$$\frac{4\pi\varepsilon_0(R_1R_2)}{(R_2 - R_1)} = 3 \times 4\pi e_0 R_1$$

which gives $\frac{R_2}{R_1} = \frac{3}{2}$, which is choice (b).

49. Energy stored initially is $U_i = \frac{Q^2}{2C}$. if d is doubled, C becomes $C/2$. Hence, energy stored when d is doubled is $U_f = \frac{Q^2}{C}$

$\therefore$ Work needed is

$$W = U_f - U_i = \frac{Q^2}{C} - \frac{Q^2}{2C} = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \ (\therefore Q = CV)$$

Now $C = \frac{A\varepsilon_0}{d}$. Hence

$W = \frac{A\varepsilon_0 V^2}{2d}$, which is choice (c)

50. The circuit can be redrawn as shown in Fig. 21.43. The charge on the capacitor plates is

$$Q = CV = \frac{A\varepsilon_0 V}{d}$$

So the correct choice is (b).

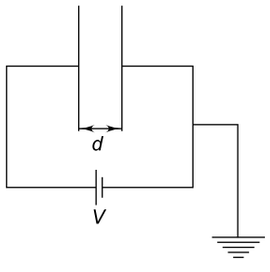


Fig. 21.43

51. Let the fall in the level in time t be x. Then $x = Vt$. At that time the thickness of air in the capacitor is $d_1 = \left(\frac{2d}{3} + x\right)$ and the thickness of water column is $d_2 = \left(\frac{d}{3} - x\right)$. At that time the equivalent capacitance is

$$C_{eq} = \frac{\varepsilon_0 A}{d_1 + \frac{d_2}{k}} \quad (1)$$

and time constant $\tau = RC_{eq}$ Putting $d_1 = \left(\frac{2d}{3} + x\right)$, $d_2 = \left(\frac{d}{3} - x\right)$, $K = 2$, $A = 1$ and $x = Vt$ in Eq. (1), we get $C_{eq} = \frac{6\varepsilon_0}{5d + 3Vt}$. Therefore $\tau = \frac{6\varepsilon_0 R}{5d + 3Vt}$

52. When switch S is connected to terminal 1, the potential difference across the 2 μF capacitor is V volt. Therefore, energy stored in the system is

$$U_1 = \frac{1}{2}C_1V^2 = \frac{1}{2} \times 2 \times V^2 = V^2 \ \mu J$$

When switch S is turned to terminal 2, the charge will flow from 2 μF capacitor to 8 μF capacitor until their potentials are equalized. The common potential is

$$V^2 = \frac{q}{C_1 + C_2} = \frac{C_1V}{C_1 + C_2} = \frac{2V}{(2+8)} = \frac{V}{5} \text{ volt}$$

$\therefore$ Energy stored in the system now will be

$$U_2 = \frac{1}{2}(C_1 + C_2)V_2^2 = \frac{1}{2}(2+8) \times \left(\frac{V}{5}\right)^2 = \frac{V^2}{5} \ \mu J$$

$\therefore$ Percentage loss of energy is

$$\frac{U_1 - U_2}{U_1} \times 100 = \frac{\left(V^2 - \frac{V^2}{5}\right)}{V^2} \times 100 = 80\ \%$$

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles

(a) the charge on the capacitor increases
(b) the voltage across the plates increases
(c) the capacitance increases
(d) the electrostatic energy stored in the capacitor increases

IIT, 1985

2. The plates of a parallel plate capacitor are 10 cm apart and have area equal to 2 m^2. If the charge on each plate is 8.85×10^{-10} C, the electric field at a point
(a) between the plates will be zero
(b) outside the plates will be zero
(c) between the plates will change from point to point
(d) between the plates will be 50 N C^{-1}

3. A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted), and the work done on the system in question, in the process of inserting the slab, then

(a) $Q = \varepsilon_0 \frac{AV}{d}$ (b) $Q = \varepsilon_0 KA \frac{V}{d}$

(c) $E = \frac{V}{Kd}$ (d) $W = \frac{\varepsilon_0 AV^2}{2d}\left(1 - \frac{1}{K}\right)$

IIT, 1991

4. A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous quantities as:

(a) $Q > Q_0$ (b) $V > V_0$
(c) $E > E_0$ (d) $U > U_0$

IIT, 1983

5. A parallel plate capacitor of plate area A has a charge Q. The electric field between the plates is E. The force on each plate of the capacitor is

(a) $\frac{Q^2}{2\varepsilon_0 A}$ (b) $\frac{Q^2}{\varepsilon_0 A}$
(c) QE (d) $\frac{1}{2}QE$

6. When a capacitor of capacitance C_1 is charge to a potential V_0, the energy stored in it is U_0. When this charged capacitor is connected to an uncharged capacitor of capacitance C_2, the common potential is V and the energy stored in the combination is U. Then

(a) $\frac{V}{V_0} = \frac{C_1}{C_1 + C_2}$ (b) $\frac{V}{V_0} = \frac{C_1}{C_2}$

(c) $\frac{U}{U_0} = \frac{C_1}{C_1 + C_2}$ (d) $\frac{U}{U_0} = \left(\frac{C_1}{C_2}\right)^2$

7. A parallel plate capacitor is charged by connecting it to a battery of voltage V. The battery is kept connected to the plates. If the space between the plates is filled with a dielectric,
(a) the potential difference between the plates remains unchanged
(b) the charge on the plates is increased
(c) the electric field between the plates is increased
(d) the energy stored in the capacitor increases.

8. A parallel plate capacitor is charged to a voltage V by connecting its plates to a battery. The battery is then disconnected. If the space between the plates is filled with a dielectric,
(a) the charge on the plates remains unchanged
(b) the potential difference between the plates decreases
(c) the electric field between the plates decreases
(d) the energy stored in the capacitor decreases.

9. A parallel plate capacitor is charged to a voltage V. The charging battery is then disconnected. If now the plate separation is doubled,
(a) the potential difference between the plates is doubled
(b) the charge on the plates is halved
(c) the electric field between the plates remains unchanged
(d) the energy stored in the capacitor is doubled.

10. A parallel plate capacitor is charged by connected it to a battery of voltage V. The battery is kept connected to the plates. If the plates separations is halved,

(a) the charge on the plates is doubled
(b) the potential difference between the plates is halved
(c) the electric field between the plates remains unchanged
(d) the energy stored in the capacitor is doubled.

11. Fig. 21.44 shows two identical parallel plate capacitors connected to a battery with switch S closed. The switch is now opened. If the space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3,

IIT, 1983

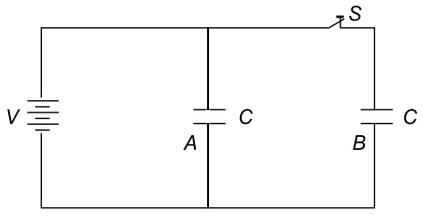


Fig. 21.44

(a) the charge on capacitor A remains equal to CV.
(b) the potential difference across capacitor B becomes $V/3$.
(c) the total energy stored increases by $\frac{2}{3}CV^2$.
(d) the total energy stored decreases by $2CV^2$.

12. An electric field of 200Vm^{-1} exists in the region between the plates of a parallel plate capacitor of plate separation 5 cm. When a slab of dielctric constant 4 is inserted parallel to the plates, the potential difference between the plates is found to be V'. If this slab is replaced by a metal plate of thickness x, the potenial difference between the plates remains equal to V'.

(a) $V' = 8.5$ V (b) $V' = 7.5$ V
(c) $x = 0.85$ cm (d) $x = 0.75$ cm

13. In the circuit shown in Fig. 21.45,

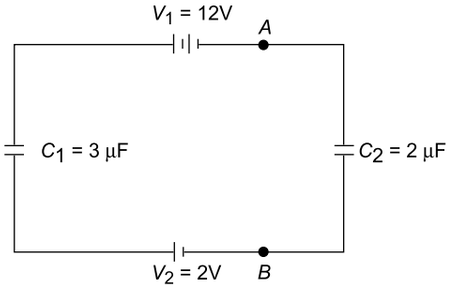


Fig. 21.45

(a) The effective capacitance of the circuit is 1.2 μF.
(b) The charge on capacitor C_1 is 12 μC.
(c) The charge on capacitor C_2 is 8 μC.
(d) The potential difference between A and B is 6 V.

14. Two capacitors, each of capacitance C, are connected to a battery of voltage V as shown in Fig. 21.46. One plate of a capacitor and the negative terminal of battery are earthed as shown.

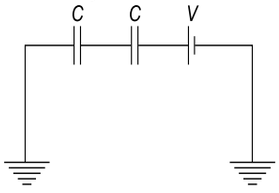


Fig. 21.46

If the combined capacitance of the arrangement is C' and the energy stored in the capacitors is U, then

(a) $C' = 2C$ (b) $C' = \frac{C}{2}$
(c) $U = \frac{1}{4}CV^2$ (d) $U = \frac{1}{2}CV^2$

15. Two capacitors A and B of capacitances C_1 and C_2 having dielectrics of dielectric constants 2 and 3 respectively are connected in series. When this combination is connected across a 220 V power supply, the potential difference across capacitor A is found to be 120 V. If the dielectric of capacitor A is replaced by a dielectric of dielectric constant 5, the new potential differences across A and B are V_1' and V_2' respectively. Then

(a) $\frac{C_1}{C_2} = \frac{2}{3}$
(b) $\frac{C_1}{C_2} = \frac{3}{2}$
(c) $V_1' = 75$ V, $V_2' = 125$ V
(d) $V_1' = 85$ V, $V_2' = 115$ V.

16. In the circuit shown in Fig. 21.47.

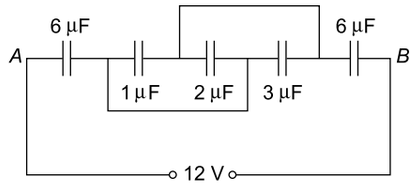


Fig. 21.47

(a) The equivalent capacitance between A and B is 18 μF.
(b) The charge on 2 μF capacitor is 8 μF.
(c) The potential drop across 3 μF capacitor is 4 V.
(d) The energy stored in the circuit is 1.44×10^{-4} J.

17. When the current in the circuit shown in Fig. 21.48 attains a steady value, then
(a) the current in branch fc is zero.
(b) the potential difference across the capacitor is $\frac{1}{3}$ V
(c) the charge on the capacitor is zero.
(d) the energy stored in the capacitor is nearly 5.5×10^{-8} J.

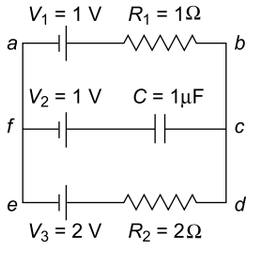


Fig. 21.48

18. When the current in the circuit shown in Fig. 21.49 attains a steady value, then
(a) the steady state current in the circuit is 8 A.
(b) the potential drop across each capacitor is 12 V.
(c) the charge on each capacitor is 24 μC.
(d) the energy stored in the circurit is 2.88×10^{-4} J.

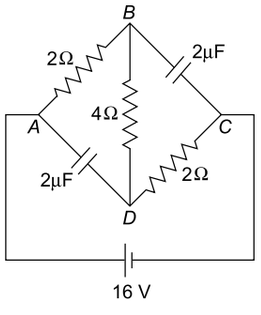


Fig. 21.49

19. In case (a) two identical parallel plate capacitors are connected in parallel and in case (b) they are connected in series. In each case, plate separation of one capacitor is decreased by x and of the other capacitor increased by the same amount x. Then the total capacitance
(a) of case (a) increases
(b) of case (a) decreases
(c) of case (b) increases
(d) of case (b) decreases

20. Five identical plates, each of area A are arranged such that adjacent plates are at a distance d apart. The plates are connected to a battery of voltage V as shown in Fig. 21.50. If Q_1 is the charge on plate 1 and Q_4 on plate 4 then

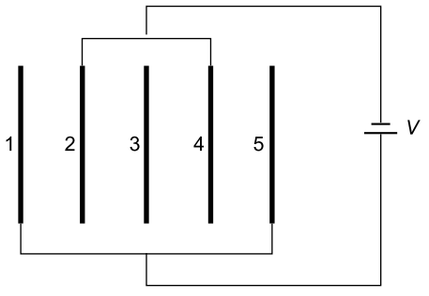


Fig. 21.50

(a) $Q_1 = \frac{\varepsilon_0 AV}{d}$ (b) $Q_1 = \frac{2\varepsilon_0 AV}{d}$

(c) $Q_4 = -\frac{2\varepsilon_0 AV}{d}$ (d) $Q_4 = \frac{4\varepsilon_0 AV}{d}$

IIT, 1983

21. Capacitor C_1 of capacitance 1 μF and capacitor C_2 of capacitance 2 μF are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time $t = 0$.
(a) The current in each of the two discharging circuits is zero at $t = 0$.
(b) The currents in the two discharging circuits at $t = 0$ are equal but not zero.
(c) The currents in two discharging circuits at $t = 0$ are unequal.
(d) Capacitor C_1 loses 50% of its initial charge sooner then capacitor C_2 loses 50% of its initial charge.

IIT, 1989

ANSWERS AND SOLUTIONS

1. If the plates of capacitor are moved further apart, its capacitance C decreases. Now $Q = CV$. Since charge Q remains unchanged, it follows that the voltage V across capacitor plates increases due to a decrease in capacitance. This is so because work has to be done in moving the plates further apart

against electrostatic field. Since V increases and Q remains the same, the energy stored $= \frac{1}{2} QV$ also increases. Hence the correct choices are (b) and (d).

2. The electric field outside the plates is zero and between the plates it is

$$E = \frac{Q}{\varepsilon_0 A} = \frac{8.85 \times 10^{-10}}{8.85 \times 10^{-12} \times 2} = 50 \text{ NC}^{-1}$$

Hence the correct choices are (b) and (d).

3. The charge on the capacitor plates remains unchanged and is given by

$$Q = CV = \frac{\varepsilon_0 AV}{d}$$

Hence choice (a) is correct.
The electric field which was V/d reduces by a factor $1/K$ and becomes

$$E = \frac{V}{Kd}$$

Hence choice (c) is also correct.
Energy stored in the capacitor before the dielectric slab is inserted is given by

$$U_1 = \frac{Q^2}{2C} = \frac{\varepsilon_0^2 A^2 V^2}{2d^2} \cdot \frac{d}{\varepsilon_0 A} = \frac{\varepsilon_0 AV^2}{2d}$$

After the dielectric slab is inserted, energy stored is

$$U_2 = \frac{Q^2}{2C'} \text{ where } C' = \frac{\varepsilon_0 KA}{d}$$

$$= \frac{\varepsilon_0 AV^2}{2Kd}$$

$$\therefore \text{Work done } W = U_1 - U_2 = \frac{\varepsilon_0 AV^2}{2d}\left(1 - \frac{1}{K}\right)$$

Hence the correct choices are (a), (c) and (d).

4. The potential difference between the plates remains unchanged (equal to the voltage of the battery) because the capacitor remains connected to the battery. Thus $V = V_0$. The introduction of the slab increases capacitance C. Hence Q $(= CV)$ increases. Thus $Q > Q_0$. Since the plate separation d remains unchanged, and V remains unchanged, the electric field $E = V/d$ remains unchanged, i.e. $E = E_0$. The energy stored $U = \frac{1}{2} CV^2$ increases because C increases. Thus $U > U_0$. Hence choices (a) and (d) are correct.

5. Refer to the solution of Q.13 of section I. The force on each plate of the capacitor is

$$F = \frac{Q^2}{2\varepsilon_0 A} \quad (1)$$

Electric field $E = \frac{V}{d}$

$$\text{Now } V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} \qquad \left(\because C = \frac{\varepsilon_0 A}{d}\right)$$

$$\therefore \quad E = \frac{Q}{\varepsilon_0 A} \quad (2)$$

From (1) and (2). we get $F = \frac{1}{2} QE$.

Hence the correct choice are (a) and (d).

6. Original charge on the first capacitor is $Q_0 = C_1 V_0$. The charge is shared by the two capacitors when they are connected. $Q_0 = Q_1 + Q_2$. Using $Q = CV$, we have $C_1 V_0 = C_1 V + C_2 V$, which gives

$$\frac{V}{V_0} = \frac{C_1}{C_1 + C_2} \quad (1)$$

Since the second capacitor is uncharged, if has no energy. Therefore, total energy before connection is

$$U_0 = \frac{1}{2} C_1 V_0^2 \quad (2)$$

The total energy after connection is

$$U = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 \quad (3)$$

Using (1) and (2) in (3), we get

$$\frac{U}{U_0} = \frac{C_1}{C_1 + C_2}$$

Hence the correct choices are (a) and (c).

7. Due to the introduction of dielectric the capacitance C of the capacitor increases. Since the battery is kept connected, the potential difference between the plates remains unchanged $= V$, the voltage of the battery. As $Q = CV$, the charge on the plated increases because C increases and V remains unchanged. Electric field $= V/d$ remains unchanged as both V and d remain the same. The energy stored in the capacitor is $U = \frac{1}{2} CV^2$. U increases because C increases and V remains unchanged. Hence the correct choices are (a), (b) and (d).

8. Due to the introduction of dielectric, the capacitance C of the capacitor increases. Since the battery is disconnected, the charge Q on the capacitor plates remains unchanged as there is no source (battery) to supply extra charge. The potential difference

between the plates is $V = Q/C$. Since Q remains unchanged but C increases, V will decrease. Hence electric field $E = V/d$ will also decrease. The energy stored in the capacitor is $U = \frac{1}{2} QV$. Since Q remains unchanged but V decreases, U will decrease. Hence all the four choices are correct.

9. Charge Q on the capacitor plates remains unchanged as there is no battery to supply extra change. The capacitance $C = \varepsilon_0 A/d$ becomes half is d is doubled. Therefore, $V = Q/C$ is doubled. The energy stored in the capacitor is $U = Q^2/2C$. U is doubled because C becomes half but Q remains unchanged. The extra energy is supplied by the external agent because work has to be done to pull the plates away from each other. Hence the correct choices are (a), (c) and (d).

10. Since the battery is kept connected to the plates, the potential difference between the plates remains unchanged equal to V, the voltage of the battery. Since the plate separation is reduced to half, the capacitance C is doubled. From $Q = CV$, we find that Q is doubled. From $E = V/d$, E is also doubled because V remains the same and d is halved. Energy stored $U = \frac{1}{2} CV^2$ is doubled because C is doubled and V remains the same. Hence the correct choices are (a) and (d).

11. When switch S closed, the potential differnce across capacitors A and B is the same = V volt. Therefore, the charges on capacitors A and B are $Q_1 = Q_2 = CV$. When the dielectric is introduced, the capacitance of each capacitor becomes.

$$C' = KC = 3C.$$

After the switch is opened, the potential difference across capacitor A remains V volt. Let V' be the potential difference across capaciteo B. When the dielectric is introduced (with switch S open), the charge on capacitor B remains unchange at Q_2. Thus

$$Q_2 = CV = C'V' \text{ or } V' = \frac{CV}{C'} = \frac{CV}{3C} = \frac{V}{3} \text{ volt}$$

Energy of both capacitors, before the dielectric is introduced, is

$$U = \frac{1}{2} CV^2 + \frac{1}{2} CV^2 = CV^2$$

Energy of both capacitors, after the dielectric is introduced, is

$$U' = \frac{1}{2} C'V^2 + \frac{1}{2} C'V'^2$$

$$= \frac{1}{2} \times 3C \times V^2 + \frac{1}{2} \times 3C \times \left(\frac{V}{3}\right)^2 = \frac{5}{3} CV^2$$

$$\therefore U' - U = \frac{5}{3} CV^2 - CV^2 = \frac{2}{3} CV^2.$$

So the correct choices are (a), (b) and (c).

12. Potential difference between the plates before the slab is introduced is

$$V = E \times d = 200 \times 0.05 = 10 \text{ V}$$

The capacitance of the capacitor is given by

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{0.05} \text{ or } \varepsilon_0 A = 0.05 \text{ C}$$

When a slab of dielectric constant K and thickness t is introduced, the capacitance becomes

$$C' = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

$$= \frac{0.05C}{0.05 - 0.01\left(1 - \frac{1}{4}\right)} = \frac{20C}{17}$$

Now $Q = CV = CV'$. Therefore

$$V' = \frac{CV}{C'} = \frac{CV}{20C/17} = \frac{17V}{20}$$

$$= \frac{17 \times 10}{20} = 8.5 \text{ V}$$

When a conducting (metal) plate of thickness x cm is introduced, the effective air gap between the plates = $(5 - x)$cm = $(5 - x) \times 10^{-2}$ m. The new capacitance is

$$C'' = \frac{\varepsilon_0 A}{(5-x) \times 10^{-2}} = \frac{0.05C}{(5-x) \times 10^{-2}}$$

Now $Q = C''V'' = C'V'$ = Given $V'' = V'$. Therefore, $C'' = C'$

$$\text{or } \frac{0.05C}{(5-x) \times 10^{-2}} = \frac{20C}{17} = \text{ or } \frac{5}{5-x} = \frac{20}{17}$$

which givens $x = 0.75$ cm.

Hence the correct chioce are (a) and (d).

13. As the batteries are acting in opposition (becasue their positive terminals are connected together), the effective voltage is

$$V = V_1 - V_2 = 12 - 2 = 10 \text{ V}$$

As the capacitors C_1 and C_2 are in series, the effective capacitance of the circuit is given by

$$\frac{1}{C} = \frac{1}{C_1} = \frac{1}{C_2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

or $C = \frac{6}{5} = 1.2\ \mu F$. Therefore, charge on capacitors is

$$Q = CV = 1.2\ \mu F \times 10\ V = 12\ \mu C$$

$\therefore$ Potential difference across A and B = potential difference across capacitor C_2

$$= \frac{Q}{C_2} = \frac{12\mu C}{2\mu F} = 6\ V$$

So the correct choices are (a), (b) and (d).

14. The capacitors are in series. So the combined capacitance is $C' = C/2$. Therefore, energy stored is

$$U = \frac{1}{2} C'V^2 = \frac{1}{4} CV^2$$

So the correct choices are (b) and (c).

15. Given $V_1 = 120$ V. Therefore, $V_2 = 200 - 120 = 80$ V. In a series arrangement, the charge is the same on each capacitor. If the charge is Q, then

$$Q = C_1V_1 = C_2V_2$$

$$\therefore \quad \frac{C_1}{C_2} = \frac{V_2}{V_1} = \frac{80}{120} = \frac{2}{3} \qquad (1)$$

When the dielectric of capacitor A is replaced by a dielectric of dielectric constant 5, its capacitance become

$$C_1' = \frac{5}{2} C_1$$

The capacitance of B remains unchanged i.e. $C_2' = C_2$. Therefore [using (1)], we have

$$\frac{C_1'}{C_2'} = \frac{5}{2} \times \frac{C_1}{C_2} = \frac{5}{2} \times \frac{2}{3} = \frac{5}{3}$$

If V_1' and V_2' are the new potential differences across A and B, then

$$\frac{V_2'}{V_1'} = \frac{C_1'}{C_2'} = \frac{5}{3} \qquad (2)$$

Also $$V_1' + V_2' = 200 \qquad (3)$$

Solving eqs. (2) and (3), we get $V_2' = 125$ V.

Thus the correct chocies are (a) and (c).

16. Capacitors 1 μF, 2 μF and 3 μF are in parallel, their combined capacitance is 6 μF. Thus, we have three capacitors each of capacitance 6 μF connected in series. The equivalent capacitance of the network is, therefore $C = 2$ μF. Potential drop across each 6μF capacitor = 12/3 = 4 V. Hence potential drop across 1 μF, 2 μF and 3 μF capacitor is 4V. So the charge on 2 μF capacitor = 2 μF × 4 V = 8 μC.

Energy stored in the circuit = $\frac{1}{2} CV^2$

$$= \frac{1}{2} \times (2 \times 10^{-6}) \times (12)^2 = 1.44 \times 10^{-4}\ J$$

Hence the correct choice are (b), (c) and (d).

17. In the steady state, no current flows through the branch cf containing the capacitor. So the circuit reduces to that shown in Fig. 21.51.

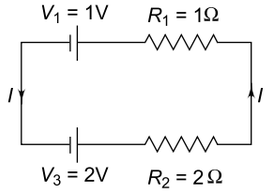


Fig. 21.51

Since the cells are in opposition, the current in the circuit is

$$I = \frac{V_3 - V_1}{R_1 + R_2} = \frac{2-1}{3} = \frac{1}{3}\ A$$

If V is the potential difference across the capacitor, then applying Kirchhoff's loop rule to loop $abcfa$ (Fig. 21.50)

we have

$$-V_1 - IR_1 + V + V_2 = 0$$

$$-1 - \frac{1}{3} + V + 1 = 0$$

$$\Rightarrow \qquad V = \frac{1}{3}\ \text{volt}$$

Charge on capacitor = $CV = (1 \times 10^{-6}) \times \frac{1}{3}$

$$= 0.33 \times 10^{-6}\ C.$$

Energy stored is $U = \frac{1}{2} CV^2$

$$= \frac{1}{2} \times (1 \times 10^{-6}) \times \left(\frac{1}{3}\right)^2 = 5.5 \times 10^{-8}\ J$$

So the correct choices are (b) and (d).

18. In the steady state, no current flows in branches BC and AD containing the capacitors. So the path of the current is $A \to B \to D \to C \to A$ through the battery. Hence the three resistors are in series and current in the circuit is

$$I = \frac{16}{2+4+2} = 2\ A$$

Now, p.d across A and $C = 16$ V and p.d across the 2 Ω resistor in arm $AB = 2\ \Omega \times 2\ A = 4$ V. Hence the p.d across the capacitor in arm $BC = 16 - 4 = 12$ V which is also the p.d across the capacitor in arm AD. Charge on each capacitor is $Q = CV = 2\ \mu F \times 12\ V = 24\ \mu C$.

Energy stored = $\frac{1}{2} CV^2 + \frac{1}{2} CV^2 = CV^2 = (2 \times 10^{-6})$ $\times (12)^2 = 2.88 \times 10^{-4}$ J

So the correct choices are (b) ,(c) and (d).

19. case (a): $C_1 = \dfrac{\varepsilon_0 A}{d}$, $C_2 = \dfrac{\varepsilon_0 A}{d}$

Total capacitance $C_a = C_1 + C_2 = \dfrac{2\varepsilon_0 A}{d}$

If the plate separation C_1 is decreased by x and of C_2 increased by x, then

$C_1' = \dfrac{\varepsilon_0 A}{(d-x)}$ and $C_2' = \dfrac{\varepsilon_0 A}{(d+x)}$

Total capacitance $C_a' = C_1' + C_2' = \dfrac{\varepsilon_0 A 2d}{(d^2 - x^2)}$

$= \dfrac{2\varepsilon_0 A}{d\left(1 - \dfrac{x^2}{d^2}\right)}$

Thus $C_a' > C_a$, which is choice (a), choice (b) is wrong.

Case (b) : $C_b = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{\varepsilon_0 A}{2d}$

$C_b' = \dfrac{C_1' C_2'}{C_1' + C_2'}$

Now $C_1' C_2' = \dfrac{\varepsilon_0^2 A^2}{(d-x)(d+x)}$

And $C_1' + C_2' = \dfrac{2\varepsilon_0 A d}{(d+x)(d-x)}$

$\therefore C_b' = \dfrac{\varepsilon_0 A}{2d} = C_b$

Hence choices (c) and (d) are wrong. Thus the only correct choice is (a).

20. Plate 1 is connected to the the positive terminal of the battery and plate 2 is connected to the negative terminal. Hence

$$Q_1 = CV = \frac{\varepsilon_0 AV}{d}$$

Plates 1, 2, 4 and 5 constitute two capacitors in parallel; their combined capacitance is $C' = 2C$. Hence

$$Q_4 = -C'V = -\frac{2\varepsilon_0 AV}{d}$$

The charge on plate 4 is negative because it is connected to the negative terminal of the battery. So the correct choices are (a) and (c).

21. At $t = 0$, current in each circuit $= \dfrac{V}{R}$; V = voltage of the battery. Current decays as $e^{-t/\tau}$ where $\tau = RC$, the time constant. Since the time constant of the second circuit is twice that of the first, it will take longer to lose 50% of the initial charge than the first circuit. Hence the correct choice are (b) and (d).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage

Passage I

A parallel plate capacitor consists of two metal plates, each of area A, separated by a distance d. A dielectric slab of the same surface area A and thickness t and dielectric constant K is introduced with its faces parallel to the plates as shown in Fig. 21.52.

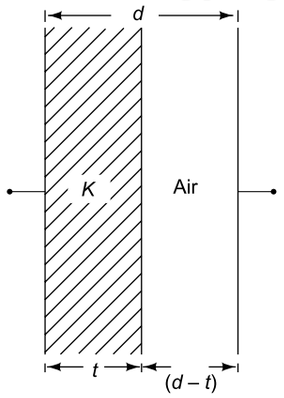


Fig. 21.52

1. The capacitance of the system is

(a) $C = \dfrac{\varepsilon_0 AK}{(d-t)}$

(b) $C = \dfrac{\varepsilon_0 A}{\left(d - \dfrac{t}{K}\right)}$

(c) $C = \dfrac{\varepsilon_0 A}{\left[d + t\left(\dfrac{1}{K} - 1\right)\right]}$

(d) $C = \dfrac{\varepsilon_0 A}{\left[d + t\left(1 - \dfrac{1}{K}\right)\right]}$

2. If $K = 3$, for what value of t/d will the capacitance of the system be twice that of the air capacitor alone ?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) $\frac{4}{5}$

3. If $K = 3$ and $t/d = 1/2$, the ratio of the energy stored in the system shown in the figure and the air capacitor alone if the charge is the same in both capacitors is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{4}$

4. In Q.3 above, the loss of energy is due to

(a) heating of the connecting wires which connect the capacitor with the battery.

(b) the flow of charge from the capacitor to the battery.

(c) leakage of the capacitor.

(d) polarization of the dielectric.

SOLUTION

1. The system is equivalent to a series combination of two capacitor one of thickness t filled with a dielectric of dielectric constant K and the other of thickness $(d - t)$ with air as dielectric. Their capacitances respectively are

$$C_1 = \frac{\varepsilon_0 KA}{t} \quad (1)$$

and $$C_2 = \frac{\varepsilon_0 A}{(d-t)} \quad (2)$$

The capacitance C of the system is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C = \frac{C_1 C_2}{C_1 + C_2} \quad (3)$$

Using Eqs. (1) and (2) in Eq. (3) and simplifying we get

$$C = \frac{\varepsilon_0 A}{\left[d + t\left(\frac{1}{K} - 1\right)\right]} \quad (4)$$

So the correct chioce is (c).

2. The capacitance of the air capacitor alone is

$$C_a = \frac{\varepsilon_0 A}{d} \quad (5)$$

Dividing Eq. (4) by Eq. (5), we get

$$\frac{C}{C_a} = \frac{1}{\left[1 + \frac{t}{d}\left(\frac{1}{K} - 1\right)\right]}$$

Given $K = 3$ and $C/C_a = 2$. Thus

$$2 = \frac{1}{\left[1 + \frac{t}{d}\left(\frac{1}{3} - 1\right)\right]}$$

which gives $\frac{t}{d} = \frac{3}{4}$, which is choice (c).

3. Putting $K = 3$ and $t/d = \frac{1}{2}$ in Eq (4), we get $C = \frac{3\varepsilon_0 A}{d}$.

Now $C_a = \frac{\varepsilon_0 A}{d}$. Hence $\frac{C_a}{C} = 3$. If Q is the same, $U_a = \frac{Q^2}{2C_a}$ and $U = \frac{Q^2}{2C}$. Thus $\frac{U}{U_a} = \frac{1}{3}$, which is choice (b).

4. The correct choice is (d).

Questions 5 to 7 are based on the following passage

Passage II

Fig. 21.53 shows a network of seven capacitors. The charge on the 5 μF capacitor is 10 μC.

5. The equivalent capacitance between points A and B is

(a) 2.5 μF (b) 5 μF

(c) 7.5 μF (d) 10 μF

6. The potential difference between points A and B is

(a) 2 V (b) 4 V

(c) 6 V (d) 8 V

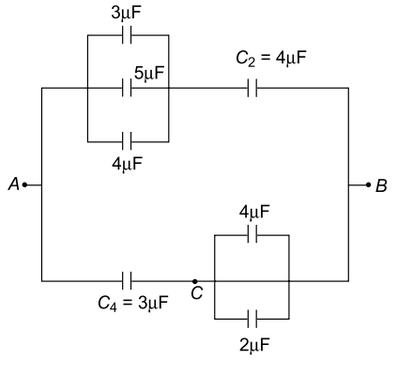


Fig. 21.53

7. The potential difference between points A and C is

(a) $\frac{16}{3}$ V (b) $\frac{12}{5}$ V (c) $\frac{8}{3}$ V (d) zero

SOLUTION

5. The equivalent capacitance C_1 of the parallel combination of 3 μF, 5 μF and 4 μF capacitors is

$$C_1 = 3 + 5 + 4 = 12\ \mu F$$

The equivalent capacitance C_3 of the parallel combination of 4 μF and 2 μF capacitors is

$$C_3 = 4 + 2 = 6\ \mu F$$

Now C_1 and C_2 are in series. Their equivalent capacitance C' is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{12} + \frac{1}{4} = \frac{1}{3}$$

or $C' = 3\ \mu F$. The equivalent capacitance C'' of the series combination of C_3 and C_4 is given by

$$\frac{1}{C''} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

or $C'' = 2\ \mu F$. The equivalent capacitance between A and B = capacitance of a parallel combination of C' and C'' = $C' + C'' = 3 + 2 = 5\ \mu F$.

Thus the correct choice is (b).

6. Given, charge on 5 μF capacitor = 10 μC = 10×10^{-6} C.

∴ Potential difference across 5 μF capacitor

$$= \frac{10 \times 10^{-6}}{5 \times 10^{-6}} = 2\ V$$

As the 3 μF, 4 μF and 5 μF capacitors are joined in parallel, the potential difference across each is the same = 2 V. Therefore,

Charge on 3 μF capacitor = $3 \times 10^{-6} \times 2$

$= 6 \times 10^{-6} = 6\ \mu C$

Charge on 4 μF capacitor = $4 \times 10^{-6} \times 2$

$= 8 \times 10^{-6}$ C = 8 μC

∴ Total charge flowing through C_1 and C_2 = 10 μC + 6 μC + 8 μC = 24 μC.

∴ Potential difference across $C_2 = \frac{24\ \mu C}{4\ \mu F} = 6$ V

∴ Total potential difference across AB = 2 V + 6 V

= 8 V

So the correct choice is (d).

7. The equivalent capacitance C_3 and C_4 is C'' = 2 μF. Therefore, charge flowing throught C_3 and $C_4 = 8\ V \times 2 \times 10^{-6}\ F = 16 \times 10^{-6}$ C = 16 μC. Hence the potential between A and C is $= \frac{16\ \mu C}{3\ \mu F} = \frac{16}{3}$ V, which is choice (a).

Questions 8 to 12 are based on the following passage

Passage III

Two capacitors A and B with capacitances 3 μF and 2 μF are charged to a potential difference of 100 V and 180 V respectively. They are connected to an uncharged 2 μF capacitor C throught a switch S as shown in Fig. 21.54.

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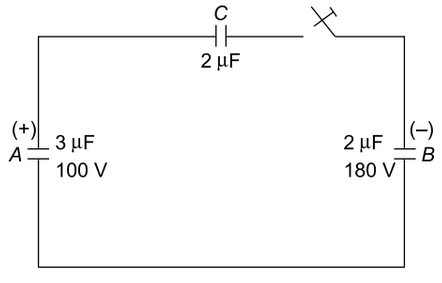


Fig. 21.54

8. When the switch is pressed, the charge flowing through the circuit is

(a) 180 μC (b) 190 μC
(c) 200 μC (d) 210 μC

9. When the switch is pressed, the final charge on capacitor A will be

(a) 60 μC (b) 90 μC
(c) 120 μC (d) 150 μC

10. When the switch is pressed, the final charge on capacitor B will be

(a) 150 μC (b) 160 μC
(c) 180 μC (d) 200 μC

11. When the switch is pressed, the final charge on capacitor C will be

(a) 90 μC (b) 150 μC
(c) 210 μC (d) 300 μC

SOLUTION

8. Refer to Fig. 21.55. Let Q be the charge flowing through the circuit. When the switch is pressed, the voltages developed on capacitors A, B and C are $V_A = \dfrac{Q}{3\times10^{-6}}$ volt, $V_B = \dfrac{Q}{2\times10^{-6}}$ volt and $V_C = \dfrac{Q}{2\times10^{-6}}$ volt.

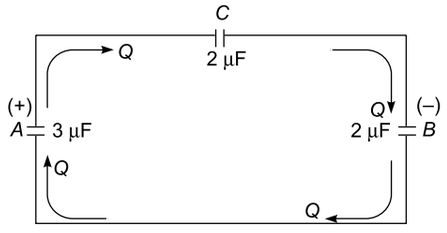


Fig. 21.55

Applying Kirchhoff's law to the loop, we have

$$\Delta V_A + \Delta V_B - \Delta V_C = 0 \quad \text{or}$$

$$\left(100 - \frac{Q}{3\times10^{-6}}\right) + \left(180 - \frac{Q}{2\times10^{-6}}\right) - \frac{Q}{2\times10^{-6}} = 0$$

which gives $Q = 210 \times 10^{-6}$ C = 210 μC. So the correct choice is (d).

9. The initial charge on capacitor A is $(Q_i)_A = (V_i\, C_i)_A$ = 100 V × 3 μF = 300 μC

∴ Final charge on A is $(Q_f)_A = (Q_i)_A - Q$ = 300 μC – 210 μC = 90 μC , which is choice (b).

10. Similarly, the final charge on B is

$(Q_f)_B = (Q_i)_B - Q = (180\text{ V}) \times (2\ \mu\text{F}) - 210\ \mu\text{C}$

= 360 μC – 210 μC = 150 μC

So the correct choice is (a).

11. From conservation of charge, we have

$(Q_f)_A + (Q_f)_C = 300\ \mu\text{C}$

∴ $(Q_f)_C = 300\ \mu\text{C} - (Q_f)_A$

= 300 μC – 90 μC = 210 μC,

which is choice (c).

Questions 12 to 15 are based on the following passage

Passage IV

In the circuit shown in Fig. 21.56 , emf E_1 = 14 V (internal resistance r_1 = 1 Ω), R_1 = 6 Ω, R_2 = 3.5 Ω, emf E_2 = 12 V (internal resistance r_2 = 0.5 Ω), C_1 = 4 μF and C_2 = 2 μF.

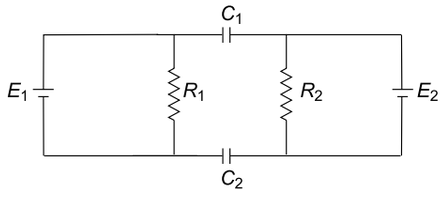


Fig. 21.56

12. The potential difference across R_1 is

(a) 2 V (b) 12 V

(c) 14 V (d) 26 V

13. The potential difference across R_2 is

(a) 10.5 V (b) 12.5 V

(c) 15.0 V (d) 26 V

14. The effective capacitance of the circuit is

(a) 2 μF (b) $\frac{2}{3}$ μF

(c) 3 μF (d) $\frac{4}{3}$ μF

15. The charge on capacitor C_2 is

(a) 20 μC (b) 30 μC

(c) 40 μC (d) 60 μC

SOLUTION

12. Current through R_1 is

$$I_1 = \frac{E_1}{R_1 + r_1} = \frac{14}{6+1} = 2\text{ A}$$

∴ Potential difference across R_1 is $V_1 = I_1 R_1 = 2 \times 6$ = 12 V, which is choice (b).

13. Current throught R_2 is

$$I_2 = \frac{E_2}{R_2 + r_2} = \frac{12}{3.5+0.5} = 3\text{ A}$$

∴ Potential difference across R_2 is $V_2 = I_2 R_2$

$= 3 \times 3.5 = 10.5$ V

So the correct choice is (a).

14. Total volage $V = V_1 + V_2 = 12 + 10.5 = 22.5$ V

Effective capacitance $C = \dfrac{C_1 C_2}{C_1 + C_2}$

$= \dfrac{4\times2}{4+2} = \dfrac{4}{3}$ μF.

The correct choice is (d).

15. Charge $Q = CV = \dfrac{4}{3} \times 10^{-6} \times 22.5 = 30 \times 10^{-6}$ C

= 30 μC

Thus the correct choice is (b).

Questions 16 to 18 are based on the following passage

Passage V

The inner and outer radii of a spherical capacitor are r_a and r_b respectively. The outer sphere is given a charge $+Q$ and the inner sphere is earthed.

16. Out of Q, a part Q' which will appear on the outer surface of the outer sphere is

(a) $\dfrac{Q(r_b - r_a)}{r_b}$ (b) $\dfrac{Q(r_b - r_a)}{r_a}$

(c) $\dfrac{Qr_b}{(r_b - r_a)}$ (c) $\dfrac{Qr_a}{(r_b - r_a)}$

17. The charge Q'' which appears on the inner surface of the outer sphere is

(a) $\dfrac{Qr_b}{r_a}$ (b) $\dfrac{Qr_a}{r_b}$

(c) $\dfrac{Q(r_b - r_a)}{r_b}$ (d) $\dfrac{Q(r_b - r_a)}{r_a}$

18. The capacitance of the spherical capacitor is

(a) $C = \dfrac{4\pi\varepsilon_0 r_b^2}{r_a}$ (b) $C = \dfrac{4\pi\varepsilon_0 r_a^2}{r_b}$

(c) $C = \dfrac{4\pi\varepsilon_0 r_a^2}{(r_b - r_a)}$ (d) $C = \dfrac{4\pi\varepsilon_0 r_b^2}{(r_b - r_a)}$

SOLUTION

16. Out of charge Q, a part Q' will be on the outer surface and another part Q'' will be on the inner surface of the outer shere such that

$$Q' + Q'' = Q \qquad (1)$$

The charge induced on the inner shhere $= -Q''$ (Fig. 21.57).

Electric potential at shere B is

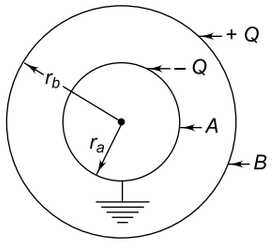


Fig. 21.57

$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q'}{r_b} + \frac{Q''}{r_b} - \frac{Q''}{r_b}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q'}{r_b} \qquad (2)$$

Electric potential at the inner sphere A is

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(-\frac{Q''}{r_a} + \frac{Q''}{r_b} + \frac{Q'}{r_b}\right)$$

Since sphere A is earthed, $V_A = 0$. Therefore

$$-\frac{Q''}{r_a} + \frac{Q''}{r_b} + \frac{Q'}{r_b} = 0 \text{ or } Q''\left(\frac{1}{r_b} - \frac{1}{r_a}\right) + \frac{Q'}{r_b} = 0$$

Now from Eq. (1), $Q'' = Q - Q'$. Therefore

$$(Q - Q')\left(\frac{r_a - r_b}{r_a r_b}\right) + \frac{Q'}{r_b} = 0$$

which given $Q' = \dfrac{Q(r_b - r_a)}{r_b}$ (3)

So the correct choice is (a).

17. $Q'' = Q - Q' = Q - \dfrac{Q(r_b - r_a)}{r_b} = \dfrac{Qr_a}{r_b}$. So the correct choice is (b).

18. The potential difference between A and B is

$$V = V_B - V_A = V_B - 0 = V_B$$

Using Eqs (2) and (3)we have

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q'}{r_b} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q(r_b - r_a)}{r_b^2}$$

The capacitance of the capacitor is

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0 r_b^2}{(r_b - r_a)}$$

The correct choice is (d).

SECTION IV

Assertion–Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. Statement-1

A parallel plate capacitor is charged by a d.c. source supplying a constant voltage V. If the plates are kept connected to the source and the space between the plates is filled with a dielectric, the charge on the plates will increase.

Statement-2

Additional charge will flow from the source to the plates.

2. Statement-1

A parallel plate capacitor is charged by a battery of voltage V. The battery is then disonnected. If the space between the plates is filled with a dielectric, the energy stored in the capacitor will decrease.

Statement-2

The capacitance of a capacitor increases due to the introduction of a dielectric between the plates.

3. Statement-1

A parallel plate capacitor is charged by a battery. The battery is then disconnected. If the distance between the plates is increased, the energy stored in the capacitor will decrease.

Statement-2

Work has to be done to increase the separation between the plates of a charged capacitor.

4. Statement-1

Two adjacent conductors when given the same charge will have a potential difference between them if they are of different shape and size.

Statement-2

The potential to which a conductor is raised depends not only on the amunt of charge but also on the shape and size of the conductor.

5. Statement-1

Two protons A and B are placed between the plates of a parallel plate capacitor charged to a potential difference V. The two protons will experience the same force. (Fig. 21.58)

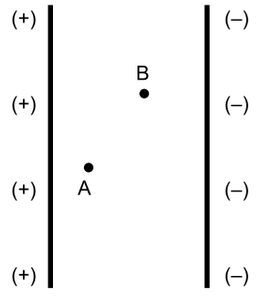


Fig. 21.58

Statement-2

The electric field is uniform between the plates.

IIT, 1985

SOLUTION

1. The correct choice is (a). Since the source supplies a constant voltage V, the potential difference between the plates remains equal to V because the source is not disconnected. The capacitance C increases due to the introduction of the dielectric. Since $Q = CV$, the charge Q on the capacitor plates will increase.

2. The correct choice is (b). The charge Q on the capacitor plates remains unchanged because there is no source to supply extra charge as the battery is disconnected. The capacitance C increases due to the introduction of the dielectric. Now, energy stored $U = Q^2/2C$. Since Q remains unchanged and C increases, U will decrease.

3. The correct choice is (d). The charge Q remains unchanged as the battery is disconnected. The capacitance C decreases if the separation between the plates is increased. Now, energy stored $U = Q^2/2C$. Since Q remains the same and C is decreased, U will increase.

4. The correct choice is (a).

5. The electric field E in the region between the plates in uniform except near the edges. Force experienced by charge q is $F = qE$. Since E is constant, force F is the same independent of the location of the charge beween the plates. Hence both the statements are correct and Statement-2 is the correct explanation for Statement-1

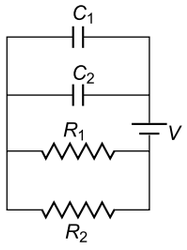
SECTION V

Integer Answer Type

1. A parallel plate capacitor is maintained at a certain potential difference. When a 3 mm thick dielectric slab is introduced between the plates, the plate separation had to be increased by 2 mm in order to maintain the same potential difference between the plates. Find the dielectric constant of the slab.

IIT, 1995

2. Find the time constant (in microsecond) of the circuit shown in the figure. Given $R_1 = 1\Omega$, $R_2 = 2\ \Omega$, $C_1 = 2\ \mu F$ and $C_2 = 4\ \mu F$. [2005]



SOLUTION

1. If Q is the charge on the plates, the potential difference is

$$V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} \qquad (1)$$

Let d' be the new separation between the plates. When a slab of thickness t and dielectric constant K is introduced, the new capacitance is

$$C' = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{K}\right)}$$

Since charge Q remains the same, the new potential difference is

$$V' = \frac{Q}{C'} = \frac{\left[d' - t\left(1 - \frac{1}{K}\right)\right]}{\varepsilon_0 A} \qquad (2)$$

Given $V' = V$. Equating Eqs. (1) and (2), we get

$$d = d' - t\left(1 - \frac{1}{K}\right) \text{ or}$$

$$d' - d = t\left(1 - \frac{1}{K}\right)$$

Given $d' - d = 2$ mm and $t = 3$ mm. Thus

$$2 = 3\left(1 - \frac{1}{K}\right)$$

Which gives $K = 3$.

2.
$$R = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{1 \times 2}{1 + 2} = \frac{2}{3}\Omega$$

$$C = C_1 + C_2 = 2 + 4 = 6\ \mu F$$

$$\therefore \quad \tau = RC = \frac{2}{3} \times 6 = 4\ \mu s$$

22 Chapter

Electric Current and D.C. Circuits

REVIEW OF BASIC CONCEPTS

22.1 ELECTRIC CURRENT

The rate of flow of charge is called electric current. If the rate of flow of charge does not change with time, the current is said to be *steady* or *constant* [Fig. 22.1(a)]. For steady current,

$$I = \frac{q}{t}$$

where q is the amount of charge flowing through any cross-section of the conductor in time t.

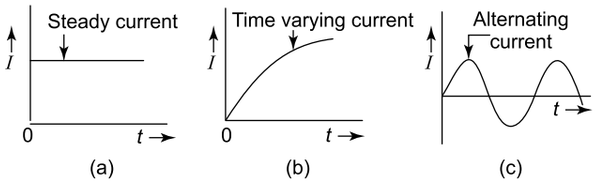


Fig. 22.1

In many situations, the current may vary with time [Fig. 22.1(b) and Fig. 22.1(c)]. In such situations, the current is given by

$$I = \frac{dq}{dt}$$

Convention regarding direction of current

In metallic conductors, the charge is carried by free electrons. In electrolytes, the charge is carried by positive and negative ions. The direction of current is taken to be the direction in which the positive charge moves. A positive charge moving in one direction is equivalent to negative charge moving in the opposite direction (Fig. 22.2)

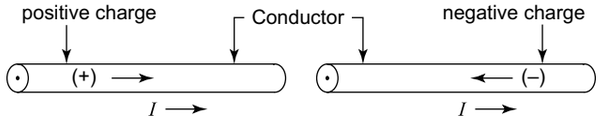


Fig. 22.2

22.2 DRIFT SPEED OF ELECTRONS IN A CONDUCTOR

A metallic conductor has a large number of free electrons. When a potential difference is applied across the ends of a conductor, an electric field is set up in the conductor which accelerates the electrons in the direction opposite to the direction of the electric field. Due to collisions with the atoms of the conductor, these electrons move with a velocity called the drift velocity $(\vec{v}_d)$ which is defined as the average velocity of the free electrons of the conductor under the influence of the electric field (E) and is given by

$$\vec{v}_d = -\frac{e\vec{E}\tau}{m}$$

where τ is the average time beween two successive collisions and is called the relaxation time. The negative sign indicates that the electrons drift in a direction opposite to that of the field.

Relations between drift speed and current

The drift speed is related to current I in a conductor as

$$v_d = \frac{I}{neA}$$

where n = number of free electrons per unit volume,
e = magnitude of charge of an electron
A = cross-sectional area of conductor

NOTE

For moderate electric field, the drift-speed of electrons in a conductor is of the of order of a few mms^{-1}.

22.3 OHM'S LAW

Ohm's Law states that *the current flowing through a conductor is directly proportional to the potential difference across its ends, provided the physical conditions of the conductor remain the same.*

Thus,

$$V \propto I \text{ or } V = RI$$

where R is the resistance of the given conductor.

22.4 ELECTRICAL RESISTIVITY

The resistance of a wire is directly proportional to its length (l) and inversely proportional to its area of cross-section (A), i.e.

$$R \propto \frac{l}{A} \text{ or } R = \rho\frac{l}{A}$$

or

$$\rho = \frac{RA}{l} \quad (1)$$

where ρ is a constant of proportionality and its value depends on the material of the wire. The constant ρ is called the *resistivity* of the material of the wire.

Unit of ρ: From Eq. (1) we have

$$\text{Unit of } \rho = \frac{\text{unit of } R \times \text{unit of } A}{\text{unit of } l} = \frac{\text{ohm} \times (\text{metre})^2}{\text{metre}}$$
$$= \text{ohm metre}$$

Hence ρ is measured in ohm metre (or Ω m).

Note that the value of ρ is independent of the length and the area of cross-section of the wire; it depends only on the material of the wire. Thus, resistivity is a characteristic of the material. The reciprocal of resistivity is called conductivity.

22.5 RESISTORS IN SERIES AND PARALLEL

1. *Resistors in series*
   When resistors are joined in series, the total resistance R of the combination is equal to the sum of the individual resistances, i.e.
   $$R = R_1 + R_2 + R_3 + \ldots$$
   The current is the same in all resistors. The total potential difference across the combination is equal to the sum of the potential differences across the individual resistors.
2. *Resistors in parallel*
   When resistors are connected in parallel, the effective resistance of the combination is given by
   $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$
   The potential difference is the same across each resistor. The total current is equal to the sum of the currents in the individual resistors.

22.6 EMF, TERMINAL VOLTAGE AND INTERNAL RESISTANCE OF A CELL

The potential difference between the terminals of a cell when it is on an open circuit, i.e. when no current is drawn from it is called its emf (E).

The potential difference between the terminals of a cell when it is in a closed circuit, i.e. when a current is drawn from it is called its terminal voltage (V).

Every cell has a resistance of its own called its internal resistance (r). The value of r depends upon the nature of electrodes, the nature of the electrolyte, size of electrodes and the distance between them.

Figure 22.3 shows a cell of emf E, internal resistance r connected to an external resistance R.

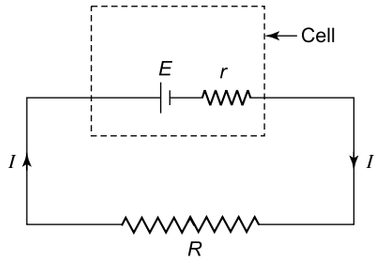


Fig. 22.3

Total resistance of the circuit = $R + r$. The current in the circuit is

$$I = \frac{E}{(R+r)}$$

Potential difference across r is $v = Ir$

Potential difference across R is $V = IR$. V is called the terminal voltage and v is the potential drop across the internal resistance.

Thus $E = V + v = I(R + r)$

Terminal voltage is $V = E - Ir$

22.7 GROUPING OF CELLS

(a) Cells connected in series

Consider two cells of emfs E_1 and E_2 and internal resistances r_1 and r_2 connected in series as shown in Fig. 22.4.

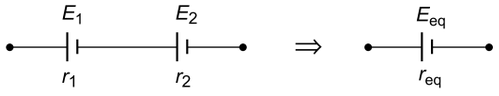


Fig. 22.4

The equivalent emf and equivalent internal resistance of the combination is given by

$$E_{eq} = E_1 + E_2$$

and $$r_{eq} = r_1 + r_2$$

For n cells in series

$$E_{eq} = E_1 + E_2 + \ldots + E_n$$

and $$r_n = r_1 + r_2 + \ldots + r_n$$

(b) Cells connected in parallel

If the cells are connected as shown in Fig. 22.5, then

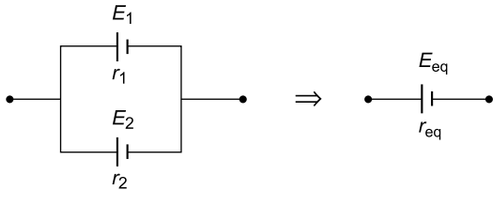


Fig. 22.5

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

where $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

For n cells in parallel

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \ldots + \frac{E_n}{r_n}$$

and $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots + \frac{1}{r_n}$$

1. If the individual cells have the same emf E, and the number of cells connected in series is n, the emf of the combination is nE and the total internal resistance is nr, where r is the internal resistance of each cell.
2. When identical cells are connected in parallel the total emf = E, the emf of any one of the cells. Here the sum of the reciprocals of the individual internal resistances is equal to the reciprocal of the total internal resistance.
3. Cells should be connected in series if the external resistance R is greater than internal resistance r.
4. Cells should be connceted in parallel if $R < r$.
5. Mixed grouping of cells is employed if R is comparable with r.

   The current in the circuit when n cells, each of emf E, are connected in series is given by.

$$I = \frac{nE}{R + nr}$$

The current in the circuit when n cells, each of emf E, are connected in parallel is given by

$$I = \frac{nE}{nR + r}$$

If the arrangement consists of n rows of cells in parallel each having m cells in series, then

$$I = \frac{mnE}{nR + mr}$$

EXAMPLE 22.1

A current of 1 A flows through a wire. Calculate the number of electrons passing through any cross-section of the wire in 8 seconds.

SOLUTION

If N is the number of electrons passing through any cross-section of the wire and e is the charge of an electrons, then

$q = Ne$ and $I = \frac{q}{t} = \frac{Ne}{t}$. Thus

$$N = \frac{It}{e} = \frac{1 \times 8}{1.6 \times 10^{-19}} = 5 \times 10^{19} \text{ electrons}$$

EXAMPLE 22.2

A copper wire of length 60 cm and cross-sectional area 10^{-7} m^2 carries a current of 1 A. How long will an electron take to drift from one end of the wire to the other. Assume that there are 10^{29} free electrons per m^3 in copper.

SOLUTION

$$v_d = \frac{I}{enA}$$

$$= \frac{1.6}{(1.6 \times 10^{-19}) \times (10^{29}) \times (10^{-7})}$$

$$= 10^{-3} \text{ ms}^{-1}$$

$$\therefore \quad t = \frac{L}{v_d} = \frac{0.6}{10^{-3}} = 600 \text{ s} = 10 \text{ min}$$

EXAMPLE 22.3

In a discharge tube, the number of hydrogen ions (protons) drifting through a cross-section is 1.0×10^{18} per second while the number of electrons drifting in the opposite direction is 3.0×10^{18} per second. If the

supply voltage is 240 V, what is the effective resistance of the tube.

SOLUTION

A proton has a positive charge and an electron has an equal negative charge. Since a positive charge and a negative charge drifting in opposite directions produce current in the same direction,

$$I = (N_p + N_e)e$$
$$= (1.0 \times 10^{18} + 3.0 \times 10^{18}) \times (1.6 \times 10^{-19})$$
$$= 0.64 \text{ A}$$
$$\therefore \quad R = \frac{V}{I} = \frac{240}{0.64} = 375 \ \Omega$$

EXAMPLE 22.4

A potential difference of 0.8 V is maintained between the ends of a metal wire of length 1.0 m. The number density of free electrons in the metal is 8.0×10^{28} per m^3 and the electrical conductivity of the metal is $6.4 \times 10^7 \ \Omega^{-1} \ m^{-1}$. Find the drift speed of electrons.

SOLUTION

Conductivity is $\sigma = \frac{1}{\rho} = \frac{L}{RA}$

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A} \text{ and } I = \frac{V}{R}. \text{ Therefore,}$$
$$I = \frac{V\sigma A}{L}$$

But $I = enAv_d$. Therefore,

$$enAv_d = \frac{V\sigma A}{L}$$
$$\Rightarrow \quad v_d = \frac{V\sigma}{Lne}$$
$$= \frac{0.8 \times (6.4 \times 10^7)}{1.0 \times (8.0 \times 10^{28}) \times (1.6 \times 10^{-19})}$$
$$= 4 \times 10^{-3} \text{ ms}^{-1} = 4 \text{ mms}^{-1}$$

EXAMPLE 22.5

The current I (in ampere) flowing in a wire varies with time t (in second) according to the equation $I = 5t + 3t^2$. Find the amount of charge which passes through a cross-section of the wire in the time interval $t = 2$ s to $t = 4$ s.

SOLUTION

$$I = \frac{dq}{dt} \quad \Rightarrow \quad dq = Idt = (5t + 3t^2)dt$$

Therefore, the amount of charge passing from $t = 2$ s to $t = 4$ s is

$$q = \int_2^4 (5t + 3t^2)\, dt$$
$$= \frac{5}{2}\left|t^2\right|_2^4 + \frac{3}{3}\left|t^3\right|_2^4$$
$$= \frac{5}{2} \times (4^2 - 2^2) + (4^3 - 2^3)$$
$$= \frac{5}{2} \times (16 - 4) + (64 - 8)$$
$$= 86 \text{ coulomb}$$

EXAMPLE 22.6

A wire of resistance 5 Ω is drawn out so that its length is increased to twice its original length. Find the new resistance of the wire.

SOLUTION

If a wire is stretched, its length L increases and its diameter and hence the cross-sectional area A decreases, but the volume of the wire remains the same. Hence AL = constant. Thus

$$A_1L_1 = A_2L_2$$

Given $L_2 = 2L_1$. Therefore $A_2 = \frac{A_1L_1}{L_2} = \frac{A_1}{2}$

Original resistance $R_1 = \frac{\rho L_1}{A_1}$

New resistance $R_2 = \frac{\rho L_2}{A_2}$

Resistivity ρ depends only on the material of the wire and hence it remains the same.

$$\frac{R_2}{R_1} = \frac{L_2}{L_1} \times \frac{A_1}{A_2} = 2 \times 2 = 4$$
$$\Rightarrow \quad R_2 = 4R_1 = 4 \times 5 = 20 \ \Omega$$

NOTE

If a wire of resistance R is stretched to n times its original length, the resistance of the stretched wire = n^2R.

EXAMPLE 22.7

A wire has a resistance of 9 Ω. It is cut into three equal pieces. Each piece is stretched uniformly to three times its original length. The three streched pieces are connected in parallel. Find the total resistance of the combination.

SOLUTION

Resistance each piece is $R = \frac{9}{3} = 3\ \Omega$. The new resistance of each stretched piece $= n^2R = (3)^2 \times 3 = 27\ \Omega$. The total resistance of the parallel combination is given by

$$\frac{1}{R_p} = \frac{1}{27} + \frac{1}{27} + \frac{1}{27} = \frac{1}{9}$$

$$\Rightarrow \qquad R_p = 9\ \Omega$$

EXAMPLE 22.8

A wire is stretched to make it 0.1% longer. What is the percentage change in the resistance?

SOLUTION

AL = constant $\Rightarrow$ $A_1L_1 = A_2L_2$

Given $L_2 = L_1 + \frac{0.1}{100} \times L_1 = L_1\,(1 + 0.001)$

$$\therefore \qquad A_2 = A_1 \times \frac{L_1}{L_2} = \frac{A_1}{(1 + 0.001)}$$

Now $R_1 = \frac{\rho L_1}{A_1}$ and $R_2 = \frac{\rho L_2}{A_2}$. Therefore

$$\frac{R_2}{R_1} = \frac{L_2}{L_1} \times \frac{A_1}{A_2}$$

$$= (1 + 0.001) \times (1 + 0.001)$$

$$= (1 + 0.001)^2 \simeq 1 + 0.002 = 1.002$$

$$\Rightarrow \qquad R_2 = 1.002\ R_1$$

Change in resistance $\Delta R = R_2 - R_1 = 0.002\ R_1$

Percentage increase in resistance $= \frac{\Delta R}{R_1} \times 100$

$= 0.002 \times 100 = 0.2\ \%$

Simple Method

If $\frac{\Delta L}{L} << 1$, the following simpler method may be used.

$$R = \frac{\rho L}{A}$$

$$\Rightarrow \qquad \log R = \log \rho + \log L - \log A$$

Differentiating ($\because$ ρ = constant)

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{\Delta A}{A} \qquad (1)$$

Also AL = constant

$$\therefore \qquad \frac{\Delta A}{A} + \frac{\Delta L}{L} = 0 \quad \Rightarrow \quad \frac{\Delta A}{A} = -\frac{\Delta L}{L} \qquad (2)$$

Using (2) in (1), we get

$$\frac{\Delta R}{R} = \frac{2\Delta L}{L} = 2 \times 0.1\% = 0.2\%$$

EXAMPLE 22.9

The resistance network shown in Fig. 22.6 is connected to a battery of emf 30 V and internal resistance 1 Ω. Find the current through the 6 Ω resistance and the terminal voltage of the battery.

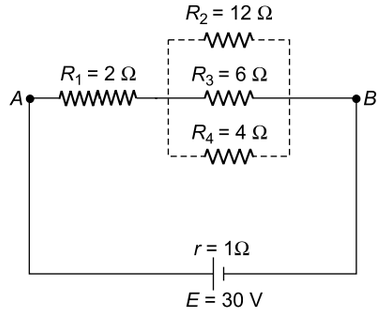


Fig. 22.6

SOLUTION

The equivalent resistance of the parallel combination of R_2, R_3 and R_4 is R', which is given by

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} \quad \Rightarrow \quad R' = 2\ \Omega$$

The circuit can be redrawn as shown in Fig. 22.7.

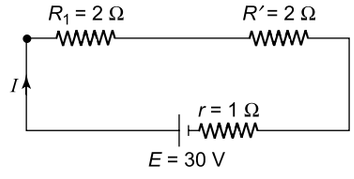


Fig. 22.7

Current in the circuit is

$$I = \frac{E}{R_1 + R' + r} = \frac{30}{2 + 2 + 1} = 6\ \text{A}$$

$\therefore$ Potential difference across R_1 is $V_1 = IR_1 = 6 \times 2 = 12$ V

Potential difference across R' is $V' = IR' = 6 \times 2 = 12$ V, which is same across R_2, R_3 and R_4 since these resistances are in parallel. Hence the current through $R_3 = 6\ \Omega$ is

$$I_3 = \frac{V'}{R_3} = \frac{12}{6} = 2 \text{ A}$$

The potential drop in the internal resistance is

$$v = Ir = 6 \times 1 = 6 \text{ V}$$

$\therefore$ Terminal voltage of the battery is

$$V = E - v = 30 - 6 = 24 \text{ V}$$

EXAMPLE 22.10

Calculate the steady state current in the 3 Ω resistor in the circuit shown in Fig. 22.8. The internal resistance of the cell is negligible.

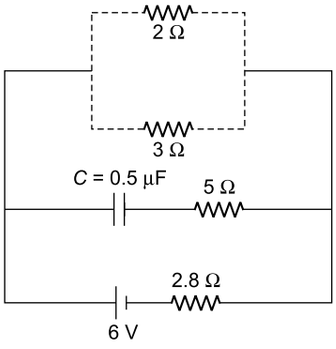


Fig. 22.8

SOLUTION

The resistance of the parallel combination of 2 Ω and 3 Ω is

$$R' = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

In the steady state, no current flows through the capacitor and hence through the 5 Ω resistor. Therefore, the ciruit reduces to that shown in Fig. 22.9.

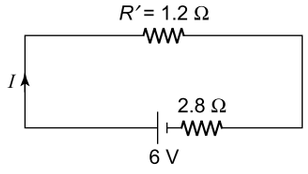


Fig. 22.9

$$\text{Current } I = \frac{6}{(1.2 + 2.8)} = 1.5 \text{ A}$$

$\therefore$ Potential difference across R' is

$$V' = IR' = 1.5 \times 1.2 = 1.8 \text{ V}$$

This is also the p.d. across the 3 Ω resistor. Hence the current through the 3 Ω resistor is $\frac{1.8}{3} = 0.6$ A.

EXAMPLE 22.11

A battery gives a current of 0.5 A when connected across an external resistor of resistance 12 Ω and a current of 0.25 A when connected across an external resistor of resistance 25 Ω. Find the emf and internal resistance of the battery.

SOLUTION

$$E = I(R + r)$$

$\therefore$ $$E = 0.5(12 + r) \quad (1)$$

and $$E = 0.25(25 + r) \quad (2)$$

Equating (1) and (2),

$$0.5(12 + r) = 0.25(25 + r) \quad \Rightarrow \quad r = 1\ \Omega$$

Using $r = 1\ \Omega$ in either (1) or (2) gives $E = 6.5$ V

EXAMPLE 22.12

Determine the current drawn from a 12 V supply of internal resistance 0.5 Ω by an infinite network shown in Fig. 22.10. Each resistor has a resistance of 1 Ω.

SOLUTION

Let R be the total resistance of the network. Since the circuit is infinitely long, its total resistance remains unaffected if one mesh is removed from it, i.e. the

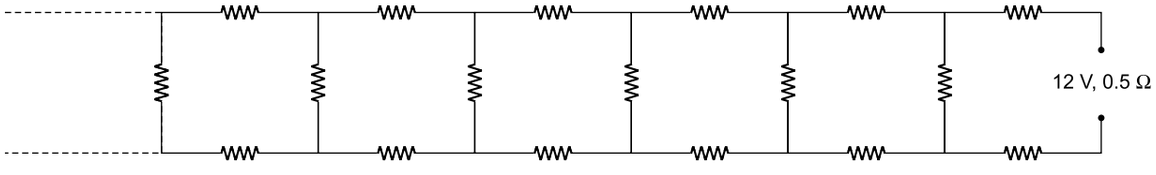


Fig. 22.10

resistance of the network to the left of PQ will also be R [see Fig. 22.11]

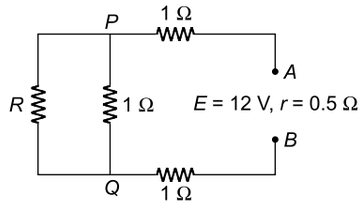


Fig. 22.11

$$R_{PQ} = \frac{R}{R+1}$$

The equivalent resistance between A and B is $R_{AB} = 2 + \dfrac{R}{R+1}$ which must be equal to R, i.e.

$$2 + \frac{R}{R+1} = R$$

$$\Rightarrow \quad R^2 - 2R - 2 = 0$$

The positive root of this quadratic equation is $R = 2.73\ \Omega$

$$\therefore \quad \text{Current } I = \frac{E}{R+r} = \frac{12}{(2.73+0.5)} = 3.7\ \text{A}$$

EXAMPLE 22.13

Find the equivalent resistance between points A and B in the network shown in Fig. 22.12. Each resistor has a resistance of 3 Ω.

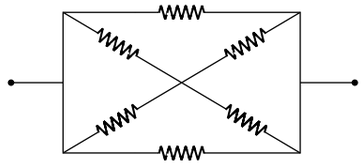


Fig. 22.12

SOLUTION

The given circuit can be redrawn as shown in Fig. 22.13.

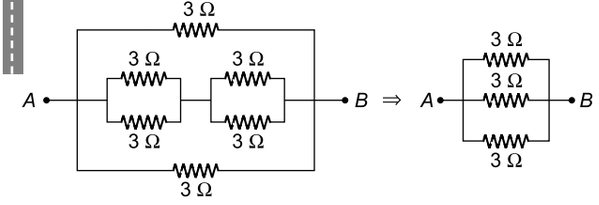


Fig. 22.13

It is clear that

$$\frac{1}{R_{AB}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \quad \Rightarrow \quad R_{AB} = 1\ \Omega$$

EXAMPLE 22.14

Find the equivalent resistance between points A and B in the network shown in Fig. 22.14, when

(a) switch S is open

(b) switch S is closed.

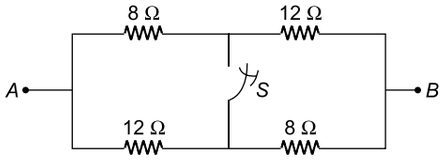


Fig. 22.14

SOLUTION

(a) When switch S is open, the circuit is as shown in Fig. 22.15.

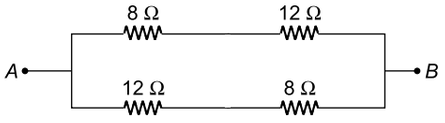


Fig. 22.15

$$R_{AB} = \frac{20 \times 20}{(20+20)} = 10\ \Omega$$

(b) When switch S is closed, the circuit can be redrawn as shown in Fig. 22.16

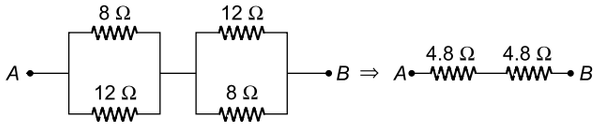


Fig. 22.16

It is clear that

$$R_{AB} = 4.8 + 4.8 = 9.6\ \Omega$$

EXAMPLE 22.15

Find the value of current I in the network shown in Fig. 22.17.

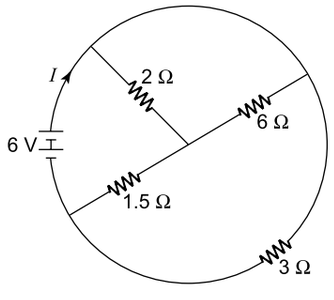


Fig. 22.17

SOLUTION

The circuit can be redrawn as shown in Fig. 22.18. The resistance between A and B is

$$R_1 = \frac{2 \times 6}{(2+6)} + 1.5 = 3\ \Omega$$

Resistance between C and D is

$$R = \frac{3 \times 3}{(3+3)} = 1.5\ \Omega$$

$$\therefore \quad \text{Current } I = \frac{V}{R} = \frac{6}{1.5} = 4\ \text{A}$$

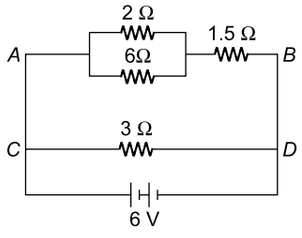


Fig. 22.18

22.8 KIRCHHOFF'S LAWS

First Law or Junction Rule

The algebraic sum of the currents at a junction in a circuit is zero. The currents entering the junction are taken as positive and those leaving the junction are taken as negative. Figure 22.19 shows a part of a circuit having a junction A.

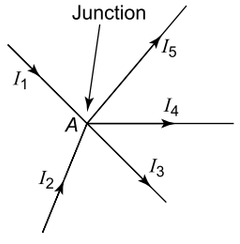


Fig. 22.19

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

$$\Rightarrow \quad I_1 + I_2 = I_3 + I_4 + I_5$$

i.e. sum of currents entering a junction = sum of currents leaving that junction.

Second Law or Loop Rule

In any closed circuit (or loop), the algebriac sum of the potential differernces across the sources of current and across the resistances in the circuit (or loop) is zero. The sources of current are the emfs of the cells and potential differences across the resistance are the voltage drops (IR).

Sign Convention for emfs and Voltage drops

Consider the circuit shown in Fig. 22.20. The circuit has three closed loops *abefa*, *bcdeb* and *acdfa*. To use the loop rule, follow the following steps.

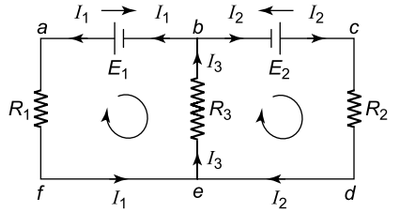


Fig. 22.20

(1) Draw a arrow on the top of each cell pointing from the positive to the negative terminal.

(2) Choose a closed loop and move in a clockwise direction in that loop.

(3) While crossing a cell, if you are moving in the direction of the arrow drawn on the cell, the emf of the cell is taken as positive but if you are moving opposite to the direction of the arrow, the emf is taken as negative. Do not worry about the direction of the current in the branch of the circuit containing that cell.

(4) While crossing a resistor, if you are moving in the direction of the current through that resistor, the potential drop (IR) across the resistor is taken as positive but if you are moving opposite to the direction of the current, the potential drop across the resistor is taken as negative.

In the circuit shown in Fig. 22.20, there are two junctions at b and e. Applying the junction rule at either b or e we have

$$I_1 + I_2 = I_3 \qquad (1)$$

Applying the loop rule to loops *abefa* and *bcdeb* we have

$$E_1 - I_3R_3 - I_1R_1 = 0 \qquad (2)$$

and $$-E_2 + I_2R_2 + I_3R_3 = 0 \qquad (3)$$

If the values of E_1, E_2, R_1, R_2 and R_3 are known, the values of I_1, I_2 and I_3 can be obtained by the simultaneous solutions of Eqs. (1), (2) and (3).

NOTE

1. Select as many loops as the number of unknowns. In the circuit shown in Fig. 22.20. There are two unknowns I_1 and I_2. The third unknown I_3 is determined by using the junction rule [Eq. (1)]. So we select two out of the three loops.

2. If the directions of currents are not given, choose any direction (clockwise or anticlockwise) in a loop and calculate the values of currents. If any current turns out to be negative, it indicates that our choice of the direction of that current is incorrect. So reverse the direction of that current. The magnitude of the current remains the same.

EXAMPLE 22.16

Figure 22.21 shows currents in a part of an electrical circuit. Find the value of current I.

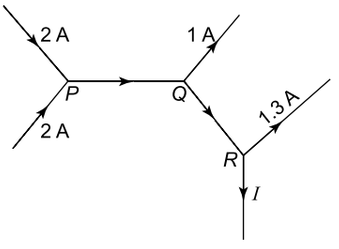


Fig. 22.21

SOLUTION

Applying the junction rule at P, the current in branch $PQ = 2 + 2 = 4$ A.

Using the junction rule at Q, the current in branch $QR = 4\text{ A} - 1\text{ A} = 3$ A.

Using the junction rule at R, we get

$$I + 1.3\text{ A} = 3\text{ A} \quad \Rightarrow \quad I = 1.7\text{ A}$$

EXAMPLE 22.17

In the circuit shown in Fig. 22.22, calculate

(a) the values of currents I_1, I_2 and I_3

(b) the potential difference between points B and E

Given $E_1 = 12$ V, $E_2 = 6$ V, $R_1 = 5\ \Omega$, $R_2 = 3\ \Omega$ and $R_3 = 2\ \Omega$

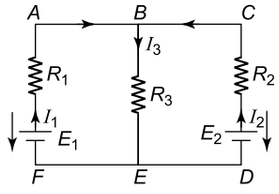


Fig. 22.22

SOLUTION

(a) Applying junction rule at B,

$$I_1 + I_2 = I_3 \qquad (1)$$

Applying loop rule to loop $ABEFA$

$$I_3R_3 - E_1 + I_1R_1 = 0$$

$$\Rightarrow \quad 2I_3 - 12 + 5I_1 = 0 \qquad (2)$$

Applying loop rule to loop $BCDEB$

$$-I_2R_2 + E_2 - I_3R_3 = 0$$

$$\Rightarrow \quad -3I_2 + 6 - 2I_3 = 0 \qquad (3)$$

Using (1) in (2) and (3), we get

$$2(I_1 + I_2) - 12 + 5I_1 = 0$$

$$\Rightarrow \quad 7I_1 + 2I_2 = 12 \qquad (4)$$

and $\quad -3I_2 + 6 - 2(I_1 + I_2) = 0$

$$\Rightarrow \quad 2I_1 + 5I_2 = 6 \qquad (5)$$

Solving (5) and (6) we get $I_1 = \dfrac{48}{31}$ A , $I_2 = \dfrac{18}{31}$ A and $I_3 = I_1 + I_2 = \dfrac{66}{31}$ A

(b) To find potential difference between B and E, we start from B and go to E.

$$V_B - V_E = I_3R_3 = \frac{66}{31} \times 2 = 4.26\text{ V}$$

EXAMPLE 22.18

Two cells of emfs $E_1 = 1.5$ V and $E_2 = 2.0$ V and internal resistances $r_1 = 1.0\ \Omega$ and $r_2 = 1.5\ \Omega$ respectively are connected to an external resistor $R = 2.5\ \Omega$ as shown in Fig. 22.23. Find the potential difference

(a) between points A and B

(b) between points A and C

(c) across R.

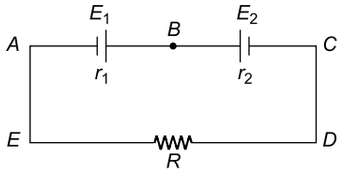


Fig. 22.23

SOLUTION

We choose any direction for the current I in the circuit. If the value of I turns out to be negative, then our choice is incorrect, the direction of the current has to be reversed. Let us choose the direction of I as shown in Fig. 22.24.

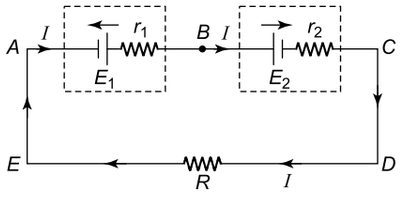


Fig. 22.24

Applying loop rule to loop *ABCDEA*,

$$-E_1 + Ir_1 + E_2 + Ir_2 + IR = 0$$

$$\Rightarrow \quad -1.5 + I \times 1.0 + 2.0 + I \times 1.5 + 2.5I = 0$$

$$\Rightarrow \quad I = -0.1 \text{ A}$$

Since I is negative, our choice for the direction of I is incorrect. So the direction of I will have to be reversed as shown in Fig. 22.25 where now $I = +0.1$ A.

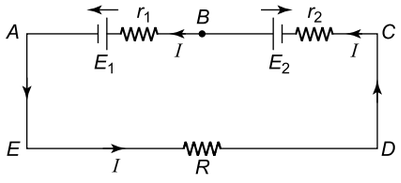


Fig. 22.25

(a) $V_A - V_B = -E_1 - Ir_1 = -1.5 - 0.1 \times 1.0 = -1.6$ V. The negative sign indicates that A is at a lower potential than B.

(b) $V_B - V_C = E_2 - Ir_2 = 2.0 - 0.1 \times 1.5 = 1.85$ V. Point B is at a higher potential that point C.

(c) $V_R = IR = 0.1 \times 2.5 = 0.25$ V.

EXAMPLE 22.19

Find the value of current I in the circuit shown in Fig. 22.26.

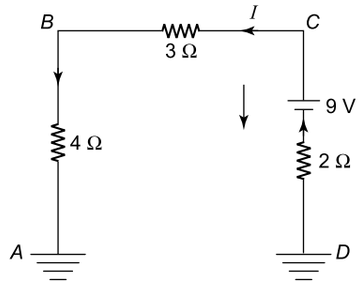


Fig. 22.26

SOLUTION

$$V_A - V_D = -4I - 3I + 9 - 2I$$

$$= -9I + 9$$

But $V_A = V_D$ because points A and D are earthed.

$\therefore \quad -9I + 9 = 0 \quad \Rightarrow \quad I = 1$ A

22.9 WHEATSTONE'S BRIDGE

The network of four resistances P, Q, R and S shown in Fig. 22.27 is called Wheatstone's Bridge. A battery is connected between A and C and a galvanometer is connected between B and D. If the values of P, Q, R and S are such that no current flows through the galvanometer, the bridge is said to be balanced. Then points B and D are at the same potential.

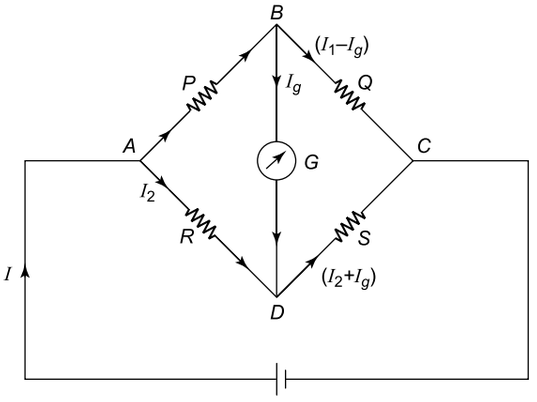


Fig. 22.27

Condition for balanced Wheatstone's Bridge

The currents in the branches of the bridge are shown in the figure. Applying the loop rule to loops *ABDA* and *BCDB*, we have

$$I_1P + I_gG - I_2R = 0 \qquad \text{(i)}$$

and $$(I_1 - I_g)Q - (I_2 + I_g)S - I_gG = 0 \qquad \text{(ii)}$$

where G is the galvanometer resistance. For a balanced bridge, $I_g = 0$. Putting $I_g = 0$ in (i) and (ii) we get

$$I_1P - I_2R = 0 \quad \Rightarrow \quad \frac{I_1}{I_2} = \frac{R}{P} \qquad \text{(iii)}$$

and $$I_1Q - I_2S = 0 \quad \Rightarrow \quad \frac{I_1}{I_2} = \frac{S}{Q} \qquad \text{(iv)}$$

From (iii) and (iv) we get

$$\frac{P}{Q} = \frac{R}{S} \quad \text{or} \quad \frac{P}{R} = \frac{Q}{S}$$

This is the condition for a balanced bridge.

Figure 22.28 shows a simple form of a wheatstone's bridge called the metre bridge.

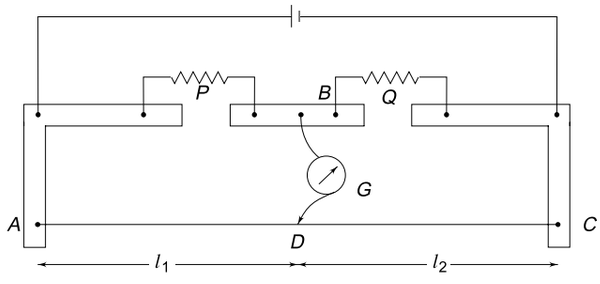


Fig. 22.28

The resistance of the wire of length $AD = l_1$ serves as the resistor R and the resistance of the wire of length $DC = l_2$ serves as the resistor S. If the wire AC is uniform,

the resistances of the parts AD and DC of the wire will be proportional to their lengths l_1 and l_2. For a balanced metre bridge, we have

$$\frac{P}{Q} = \frac{R}{S} = \frac{l_1}{l_2}$$

EXAMPLE 22.20

Find the equivalent resistance betwen points A and B in the circuits shown in Figs. 22.29(a), 22.29(b) and 22.29(c). All resistors have the same resistance R.

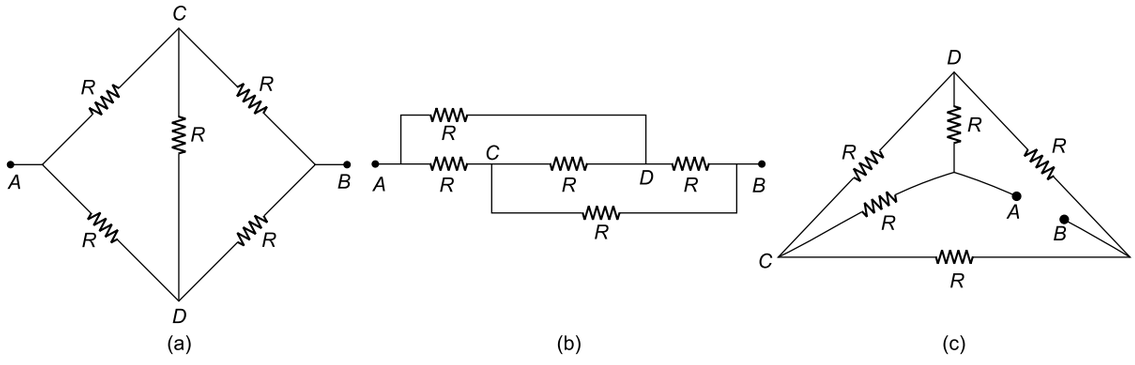


Fig. 22.29

SOLUTION

Circuits shown in Figs. 22.29(b) and (c) can be redrawn as the circuit shown in Fig. 22.29(a), which is a balanced Wheatstone's Bridge. Hence no current flows through the resistor between C and D because C and D are at the same potential. Hence this resistor is ineffective and circuits (a), (b) and (c) simplify as shown in Fig. 22.30. Hence the equivalent resistance between A and B is the resistance of the parallel combination of resistances $2R$ and $2R$.

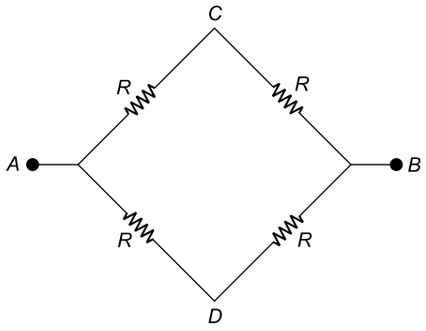


Fig. 22.30

$$\therefore \qquad R_{AB} = \frac{2R \times 2R}{(2R + 2R)} = R$$

EXAMPLE 22.21

Find the equivalent resistance between points A and B in the circuit shown in Fig. 22.31.

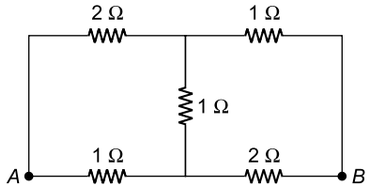


Fig. 22.31

SOLUTION

The network of resistances shown in Fig. 22.31 is not a balanced Wheatstone's bridge. In such a case, to find the resistance between A and B, we connect a battery of voltage V across A and B and find the current I drawn from the battery in terms of V and find the equivalent resistance from the relation $V = IR$. Refer to Fig. 22.32.

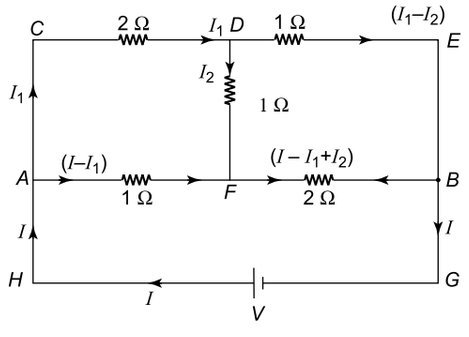


Fig. 22.32

The currents in various branches are shown in Fig. 22.32. Applying loops rule to loops $ABFGA$, $ACDFA$ and $DEBFD$ we have

$$V - 1(I - I_1) - 2(I - I_1 + I_2) = 0 \qquad \text{(i)}$$

$$2I_1 + I_2 - (I - I_1) = 0 \qquad \text{(ii)}$$

and $$(I_1 - I_2) + 2(I - I_1 + I_2) - I_2 = 0 \qquad \text{(iii)}$$

Simplyfing these equations we get

$$3I - 3I_1 + 2I_2 = V \qquad \text{(iv)}$$

$$I - 3I_1 - I_2 = 0 \qquad \text{(v)}$$

and $$2I - 3I_1 + 4I_2 = 0 \qquad \text{(vi)}$$

Eliminating I_1 from (v) and (vi), we have $I_2 = -\frac{I}{5}$.

Using this value of I_1 in (v) we get $I_1 = \frac{2}{5}I$.

Using these values of I_1 and I_2 in (iv) we get

$$7I = 5V \Rightarrow \frac{V}{I} = \frac{7}{5} = 1.4 \Rightarrow R_{AB} = 1.4\ \Omega$$

EXAMPLE 22.22

Figure 22.33 shows a metre bridge consisting of two resistances R_1 and R_2 connected to a wire AC of length 100 cm. The null point is found to be at a distance $l_1 = 40$ cm from end A. When a 12 Ω resistance is connected in parallel with R_2, the null point shifts to D_2 such that $l_2 = 60$ cm. Find R_1 and R_2.

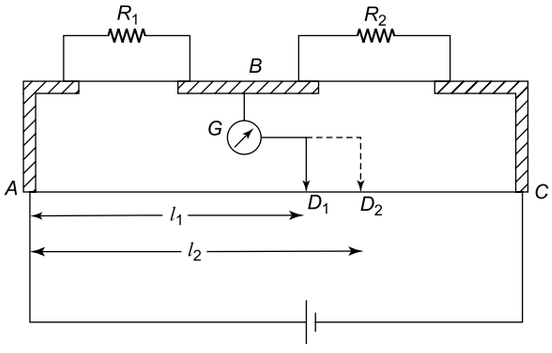


Fig. 22.33

SOLUTION

$$\frac{R_1}{R_2} = \frac{l_1}{100 - l_1} = \frac{40}{100 - 40} = \frac{40}{60} = \frac{2}{3} \qquad (1)$$

When a resistance of 12 Ω is connected in parallel with R_2, the resistance of the combination is

$$R_2' = \frac{12R_2}{12 + R_2}$$

$$\therefore \quad \frac{R_1}{R_2'} = \frac{60}{100 - 60} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{R_1 \times (12 + R_2)}{12R_2} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{R_1}{R_2} \times \frac{(12 + R_2)}{12R_2} = \frac{3}{2} \qquad (2)$$

Using (1) in (2),

$$\frac{2}{3} \times \frac{(12 + R_2)}{12R_2} = \frac{3}{2} \Rightarrow R_2 = 15\ \Omega$$

and $$R_1 = \frac{2}{3}R_2 = \frac{2}{3} \times 15 = 10\ \Omega$$

22.10 THE POTENTIOMETER

Figure 22.34 shows a potentiometer where AB is a wire of uniform cross-section, E is the driver battery and E_1 is a cell. The emf of the battery is greater than that of the cell.

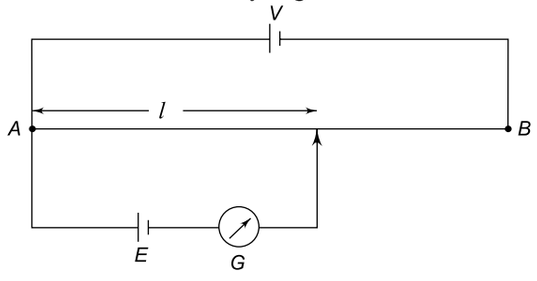


Fig. 22.34

The principle of a potentiometer is based on the fact that the potential difference across any length of the wire is proportional to that length of the wire. If R is the resistance of the potentiometer wire AB and L is its length, then $R = \rho L/A$ where ρ is the resistivity of the material of the wire and A its cross-sectional area. From Ohm's law, $V = IR$ or $V = I\rho L/A = KL$ where $K = I\rho/A$, which is constant for a given wire. Hence

$$V \propto L$$

or $\frac{V}{L} = K$ = potential gradient or fall of potential per unit length of the potentiometer wire. Hence potential difference v across any portion l of the wire is $v = Kl$. The galvanometer will show no deflection if v = emf E of the cell E or $E = v = Kl$. The length l is called the balancing length of the potentiometer wire. At balance point, potential difference across l due to the driver cell V = emf of cell E

Applications of potentiometer

1. *Comparison of emfs of two cells*
   If l_1 and l_2 are the balancing lengths will cells of emfs E_1 and E_2, then

   $$E_1 = Kl_1 \text{ and } E_2 = Kl_2$$

   $$\therefore \quad \frac{E_1}{E_2} = \frac{Kl_1}{Kl_2} = \frac{l_1}{l_2}$$

2. *Determination of internal resistance of a cell*
If l_1 is the balancing length will cell of emf E when switch S is open [Fig. 22.35], then

$$E = Kl_1$$

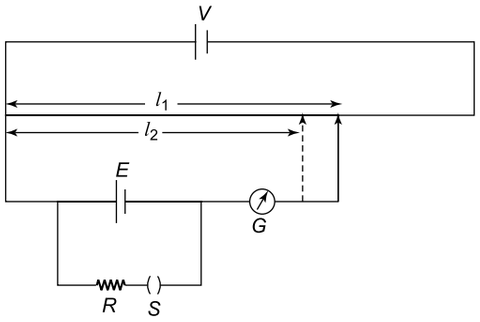


Fig. 22.35

A known resistance R is connected across the cell E, switch S is closed and the new balancing length l_2 is found, then (since a current is now drawn from cell E), the terminal voltage V of E is equal to Kl_2, i.e.

$$V = Kl_2$$

$$\therefore \quad \frac{E}{V} = \frac{Kl_1}{Kl_2} = \frac{l_1}{l_2}$$

Now $E = V + v = IR + Ir$ where I is the current drawn from E and r is its internal resistance. Thus

$$\frac{V+v}{V} = \frac{l_1}{l_2} \Rightarrow 1 + \frac{v}{V} = \frac{l_1}{l_2}$$

$$\Rightarrow \quad \frac{v}{V} = \frac{l_1}{l_2} - 1$$

$$\Rightarrow \quad \frac{Ir}{IR} = \frac{l_1}{l_2} - 1$$

$$\Rightarrow \quad r = R\left(\frac{l_1}{l_2} - 1\right)$$

Knowing the values of R, l_1 and R_2, the value of r is determined.

EXAMPLE 22.23

A uniform wire of length 400 cm and resistance 2 Ω is used in a potentiometer. The wire is connected in series with a battery of emf 5 V and an external resistance of 3 Ω. With a cell of unknown emf E, the balancing length is found to be 250 cm. Find (a) the potential gradient along the potentiometer wire and (b) the value of E.

SOLUTION

Refer to Fig. 22.36. $AB = L = 400$ cm $= 4$ m, $r = 2$ Ω, $R = 3$ Ω and $V = 5$ V, and $AC = l = 250$ cm $= 2.5$ m.

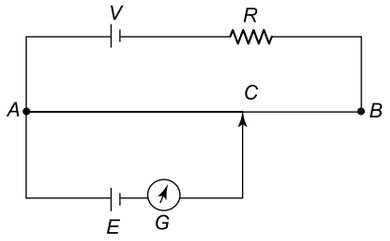


Fig. 22.36

(a) Current in AB due to driver cell is

$$I = \frac{V}{(R+r)} = \frac{5}{(3+2)} = 1 \text{ A}$$

Potential drop across $AB = I \times r$

$$= 1 \times 2 = 2 \text{ V}$$

$\therefore$ Potential gradient along AB is $K = \dfrac{2\text{V}}{4\text{m}}$

$$= 0.5 \text{ Vm}^{-1}$$

(b) $E = Kl = 0.5 \times 2.5 = 1.25$ V

EXAMPLE 22.24

In the potentiometer circuit shwon in Fig. 22.37, find the value of l when the galvanometer shows no deflection. The length of wire AB is 100 cm and its resistance is 3 Ω.

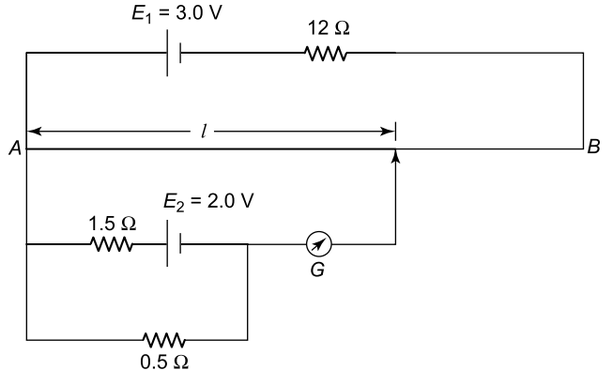


Fig. 22.37

SOLUTION

Current flowing in AB due to driver cell of em 3.0 V is

$$I_1 = \frac{3.0}{(12+3)} = 0.2 \text{ A}$$

$\therefore$ Potential difference across AB due to E_1 is

$$= 0.2 \times 3 = 0.6 \text{ V}$$

Potential difference across l due to E_1 is (l is in cm)

$$V_1 = \frac{0.6}{100} \times l \qquad (1)$$

Current through 0.5 Ω resistance due to E_2 is

$$I_2 = \frac{2.0}{(1.5 + 0.5)} = 1.0 \text{ A}$$

$\therefore$ Potential difference across 0.5 Ω resistance

$$= 0.5 \times 1.0 = 0.5 \text{ V}.$$

At balance length l, the potential difference across l due to E_2 is

$$V'_l = 0.5 \text{ V} \qquad (2)$$

At balance point $V_l = V'_l$. Equating (1) and (2), we have

$$\frac{0.6}{100} \times l = 0.5 \quad \Rightarrow \quad l = 83.3 \text{ cm}$$

EXAMPLE 22.25

The length of a potentiometer wire is 600 cm and it carries a current of 40 mA. For a cell of emf 2V and internal resistance 10 Ω, the balancing length is found to be 500 cm. If a voltmeter is connected across the cell, the balancing length is decreased by 10 cm. Find (a) the resistance of the potentiometer wire, (b) the resistance of voltmeter and (c) the reading of the voltmeter.

SOLUTION

Refer to Fig. 22.38. $AB = 600$ cm, $I = 40$ mA $= 0.04$ A, $AC = 500$ cm and $AC' = 490$ cm. Let r be the resistance of AB.

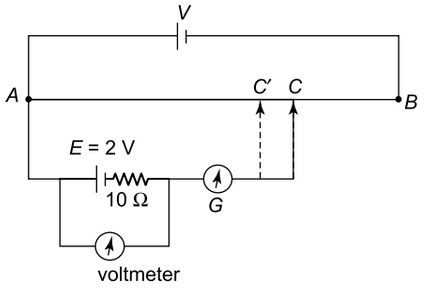


Fig. 22.38

(a) Resistance per cm of $AB = \frac{r}{600}$ Ω cm^{-1}. Therefore, the resistance of AC is $R_{AC} = \frac{r}{600} \times 500 = \frac{5r}{6}$ Ω

$\therefore$ Potential difference across AC due to driver cell is

$$V_{AC} = IR_{AC} = 0.04 \times \frac{5r}{6} \text{ volt} \qquad (1)$$

When the voltmeter is not connected, the potential difference across AC due to cell of emf E is 2 V (because no current is drawn from it at the balance point. Hence

$$V_{AC} = 2$$

$$\Rightarrow \quad \frac{0.04 \times 5r}{6} = 2 \quad \Rightarrow \quad r = 60 \text{ Ω}$$

(b) Let R be the resistance of voltmeter. When it is connected across E, the current drawn from the cell of emf 2 V is

$$I' = \frac{2}{R + 10}$$

$\therefore$ Potential difference across the voltmeter is

$$V_R = I'R = \frac{2R}{R + 10}$$

V_R must be equal to the potential difference across AC' due to the driver cell.

$\therefore$ V_R = resistance of $AC' \times$ current I

$$= \frac{60 \times 490}{600} \times 0.04 = 1.96$$

$$\Rightarrow \frac{2R}{(R + 10)} = 1.96 \quad \Rightarrow \quad R = 490 \text{ Ω}$$

(c) Voltmeter reading is

$$V_R = \frac{2R}{(R + 10)} = \frac{2 \times 490}{(490 + 10)} = 1.96 \text{ V}$$

22.11 AMMETER

Ammeter is used for measuring current in a circuit. It is a galvanometer having a very small resistance (called shunt) connected in parallel with it. The ammeter is always connected in series in the circuit the current through which is to be measured.

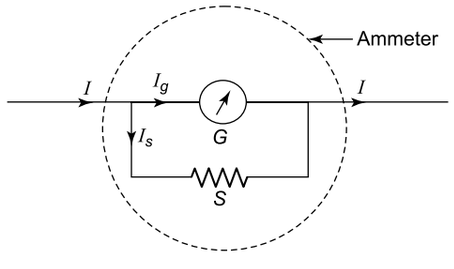


Fig. 22.39

The range of an ammeter is the maximum current it can measure. The value of the shunt resistance determines

the range of an ammeter. Figure 22.39 shows a part of a circuit, where

I = current to be measured,
G = galvanometer resistance,
I_g = current through the galvanometer for full scale deflection, and
S = shunt resistance

It is clear that (since p.d. across G = p.d. across S)

$$I_g G = I_s S. \quad \text{Also } I = I_g + I_s$$

$$\therefore \quad I_g G = (I - I_g) S$$

$$\Rightarrow \quad S = \left[\frac{I_g}{(I - I_g)}\right] G$$

EXAMPLE 22.26

A galvanometer of resistance 20 Ω gives full scale deflection when a current of 1 mA is passed through it

(a) How will you convert it into a ammeter that can reads currents upto 1.0 A?

(b) What is the resistance of the ammeter? When it is connected in a circuit, does it read slightly less or more than the actual current in the original circuit?

SOLUTION

(a) Given $I = 1.0$ A, $I_g = 1$ mA $= 00.1$ A and $G = 20$ Ω.

$$S = \left(\frac{I_g}{I - I_g}\right) G = \left(\frac{0.001}{1.0 - 0.001}\right) \times 20 \simeq 0.02 \ \Omega$$

The required shunt resistance is 0.02 Ω.

(b) Resistance of the ammeter is

$$R_A = \frac{GS}{G + S} = \frac{20 \times 0.02}{(20 + 0.02)} \simeq 0.02 \ \Omega$$

Since the ammeter is connected in series in the circuit, it will slightly increase the total resistance of the circuit and hence the current in the circuit will slightly decrease. Hence the ammeter will read slightly less than the actual current.

22.12 VOLTMETER

Voltmeter is used for measuring potential difference across a resistor in a circuit. It is a galvanometer having a very high resistance connected in series with it. The voltmeter is always connected in parallel with the resistor across which the potential difference is to be measured.

The range of a voltmeter is the maximum voltage it can measure. The range is determined by the value of high resistance connected in series with it. Figure 22.40 shows a voltmeter, where

V = voltage to be measured,
I_g = current for full scale deflection
R = required high resistance.

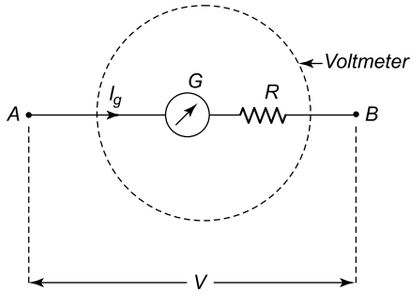


Fig. 22.40

It is clear that

$$V = V_G + V_R$$

$$= I_g G + I_g R$$

$$\Rightarrow \quad \frac{V}{I_g} = G + R \quad \Rightarrow \quad R = \frac{V}{I_g} - R$$

EXAMPLE 22.27

A galvanometer of resistance 20 Ω gives full scale deflection when a current of 1 mA is passed through it.

(a) How will you convert it into a voltmeter that can read voltages up to 5 V?

(b) What is the resistance of the voltmeter? When it is connected across a resistor in a circuit, will it read slightly less or more than the original voltage?

SOLUTION

(a) Given $G = 20$ Ω, $I_g = 1$ mA $= 0.001$ A and $V = 5$ V.

$$R = \frac{V}{I_g} - G = \frac{5}{0.001} - 20 = 4980 \ \Omega$$

The required high resistance to be connected in series with the galvanometer is 4980 Ω.

(b) Resistance of voltmeter is

$$R_V = R + G = 4980 + 20 = 5000 \ \Omega$$

When the voltmeter is connected across a resistor of resistance $R << R_V$, the current in the main

circuit divides between R and R_V. Since $R_V >> R$, a small current will flow through the voltmeter. Hence the current through R decreases slightly. From Ohm's law, the potential differernce across R will decrease slightly. Hence the voltmeter will read slightly less than the actual voltage across R.

EXAMPLE 22.28

Two resistors of 400 Ω and 600 Ω are connected in series with a 5.0 V battery of negligible internal resistance.

(a) An ammeter of resistance 20 Ω is used to measure the current in the circuit. Find the error in the measurement of current.

(b) A voltmeter of resistance 9600 Ω is used to measure the potential difference across the 400 Ω resistor. Find the error in the measurement of potential difference.

SOLUTION

(a) Actual current without ammeter in the circuit is (Fig. 22.41)

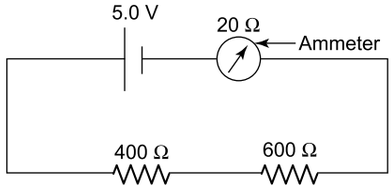


Fig. 22.41

$$I = \frac{5.0}{(400+600)} = 0.005 \text{ A} = 5 \text{ mA}$$

When the ammeter is connected, the total resistance = 400 + 600 + 20 = 1020 Ω. The current indicated by the ammeter will be

$$I' = \frac{5.0}{1020} = 0.0049 \text{ A} = 4.9 \text{ mA}$$

Hence the ammeter reads less than the actual current. Error in the measurement of current = 5.0 mA – 4.9 mA = 0.1 mA

(b) When a voltmeter of resistance 9600 Ω is connected across the 400 Ω resistor as shown in Fig. 22.42, the equivalent resistance of the combination is

$$R = \frac{400 \times 9600}{(400+9600)} = 384 \ \Omega$$

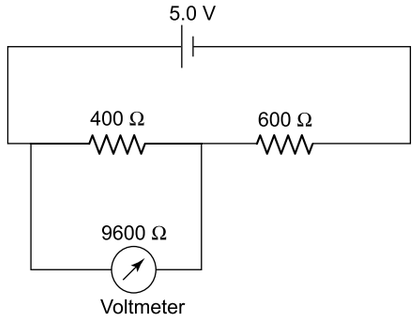


Fig. 22.42

∴ Total resistance of the circuit is

$$R' = 384 + 600 = 984 \ \Omega$$

∴ Current in the circuit is

$$I' = \frac{5.0}{984} \text{A}$$

The potential difference indicated by the voltmeter will be

$$V' = \frac{5.0}{984} \times 384 = 1.95 \text{ V}$$

Actual potential difference across the 400 Ω resistance (without voltmeter connected across it) is

$$V = I \times 400 = 0.005 \times 400 = 2.0 \text{ V}$$

Hence the voltmeter reads less than the actual voltage. Error in measurement = 2.0 – 1.95 = 0.05 V

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. Two wires of equal lengths, equal diameters and having resistivities ρ_1 and ρ_2 are connected in series. The equivalent resistivity of the combination is

(a) $(\rho_1 + \rho_2)$ (b) $\frac{1}{2}(\rho_1 + \rho_2)$ (c) $\frac{\rho_1\rho_2}{(\rho_1+\rho_2)}$ (d) $\sqrt{\rho_1\rho_2}$

2. A voltmeter of resistance 400 Ω is used to measure the emf of a cell of internal resistance 2 Ω. The

percentage error in reading of the voltmeter is very nearly equal to

(a) 0.5% (b) 0.8%

(c) 1% (d) 1.25%

3. In the circuit shown in Fig. 22.43, the current through the 10 Ω resistor is

(a) $\frac{1}{9}$ A (b) $\frac{4}{9}$ A

(c) $\frac{2}{3}$ A (d) $\frac{5}{6}$ A

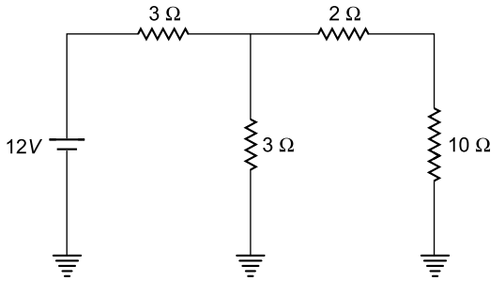


Fig. 22.43

4. In the circuit shown in Fig. 22.44, if a wire of negligible resistance is connected between P and Q, a current of

(a) $\frac{1}{3}$ A flows from P to Q

(b) $\frac{1}{3}$ A flows from Q to P

(c) $\frac{2}{3}$ A flows from P to Q

(d) $\frac{2}{3}$ A flows from Q to P

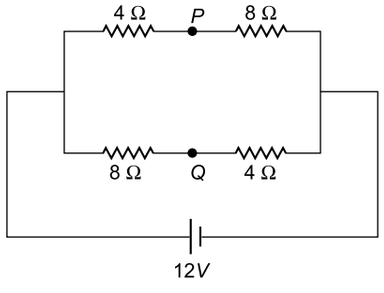


Fig. 22.44

5. In the circuit shown in Fig. 22.45, current $I_2 = 0$. The value of E is

(a) 3 V (b) 6 V

(c) 9 V (d) 12 V

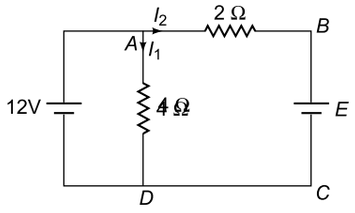


Fig. 22.45

6. In the circuit shown in Fig. 22.46, the reading of ammeter is

(a) 1 A (b) 2 A

(c) 3 A (d) 4 A

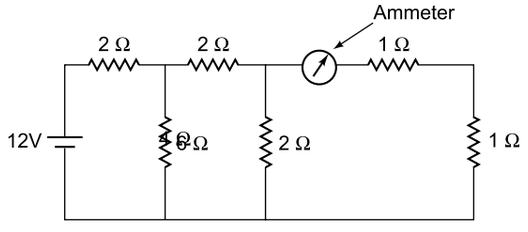


Fig. 22.46

7. In the circuit shown in Fig. 22.47, the equivalent resistance between points A and B is

(a) $\frac{3R}{4}$

(b) $\frac{5R}{6}$

(c) $\frac{7R}{10}$

(d) $\frac{11R}{9}$

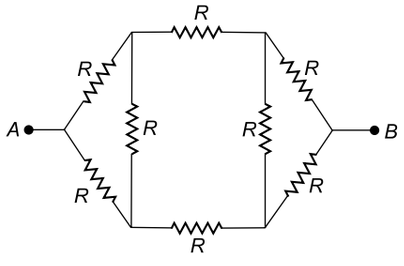


Fig. 22.47

8. In the circuit shown in Fig. 22.48, the equivalent resistance between points A and B is

(a) $\frac{3R}{4}$

(b) $\frac{R}{2}$

(c) $\frac{5R}{8}$

(d) $2R$

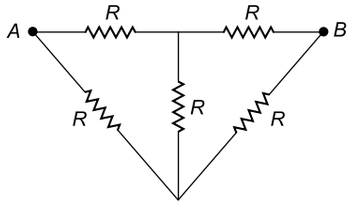


Fig. 22.48

9. A voltmeter having a resistance of 1800 Ω is used to measure the potential difference across a 200 Ω resistor which is connected to a power supply of emf 50 V and internal resistance of 20 Ω. The percentage decrease in the potential difference across the 200 Ω resistor when the voltmeter is connected across it is

(a) 1% (b) 5%
(c) 10% (d) 25%

10. The driver cell of a potentiometer has an emf of 2 V and negligible internal resistance. The potentiometer wire is 1 m long and has a resistance of 5 Ω. The resistance which must be connected in series with the wire so as to have a potential difference of 5 mV across the whole wire is
(a) 1985 Ω (b) 1990 Ω
(c) 1995 Ω (d) 2000 Ω

11. A potentiometer wire has length L and the emf of the driver cell is E_0 with negligible internal resistance. With a cell of emf E, the balance length is $L/4$. If the length of the potentiometer wire is doubled (without changing its diameter), the new balance length with the same cell will be
(a) $\frac{L}{4}$ (b) $\frac{L}{3}$
(c) $\frac{L}{2}$ (d) $\frac{2L}{3}$

12. In Fig. 22.49, the galvanometer shows no deflection. What is the resistance X?
(a) 7 Ω
(b) 14 Ω
(c) 21 Ω
(d) 28 Ω

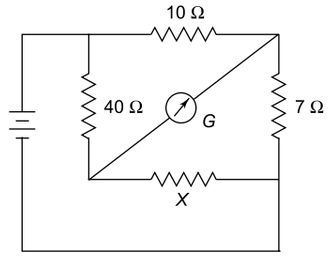


Fig. 22.49

13. Three resistances of 4 Ω each are connected as shown in Fig. 22.50. If the point D divides the resistance into two equal halves, the resistance between points A and D will be
(a) 12 Ω (b) 6 Ω
(c) 3 Ω (d) $\frac{1}{3}$ Ω

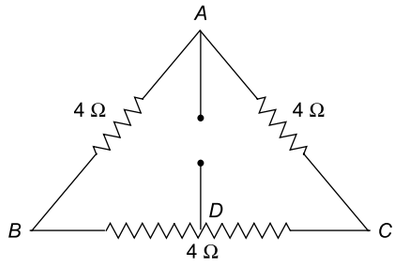


Fig. 22.50

14. The range of a voltmeter is 5 V and its resistance is 5000 Ω (4900 Ω connected in series with a coil of resistance 100 Ω). What additional resistance should be connected in series with it so that its range is doubled?
(a) 5000 Ω (b) 4500 Ω
(c) 4000 Ω (d) 3500 Ω

15. Three unequal resistances connected in parallel are equivalent to a single resistance of 1 Ω. If two resistances are in the ratio of 1:2 and if no resistance is fractional, the largest of three resistances is
(a) 4 Ω (b) 6 Ω
(c) 8 Ω (d) 12 Ω

16. The smallest resistance that can be obtained by combining n resistors, each of resistance R is
(a) n^2R (b) nR
(c) $\frac{R}{n}$ (d) $\frac{R}{n^2}$

17. A wire of resistance 4 Ω is bent into a circle. The resistance between the ends of a diameter of the circle is
(a) 4 Ω (b) 1 Ω
(c) $\frac{1}{4}$ Ω (d) $\frac{1}{16}$ Ω

18. Figure 22.51 shows a network of eight resistors numbered 1 to 8, each equal to 2 Ω, connected to a 3 V battery of negligible internal resistance. The current I in the circuit is
(a) 0.25 A (b) 0.5 A
(c) 0.75 A (d) 1.0 A

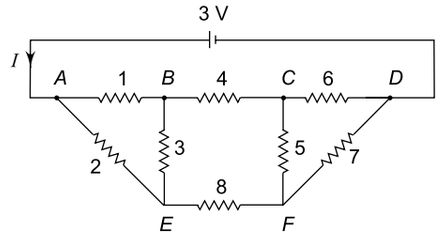


Fig. 22.51

19. A uniform wire of resistance 4 Ω is bent into the form of a circle of radius r. A specimen of the same wire is connected along the diameter of the circle. What is the equivalent resistance across the ends of this wire?
(a) $\frac{4}{(4+\pi)}$ Ω (b) $\frac{3}{(3+\pi)}$ Ω
(c) $\frac{2}{(2+\pi)}$ Ω (d) $\frac{1}{(1+\pi)}$ Ω

20. A battery of emf 10 V is connected to resistances as shown in Fig. 22.52. The potential difference between points A and B is
(a) − 2 V (b) 2 V
(c) 5 V (d) $\frac{20}{11}$ V

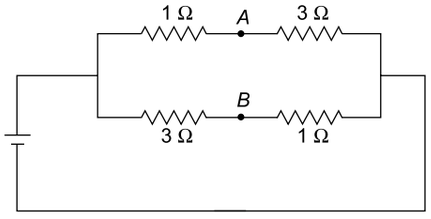


Fig. 22.52

21. In the circuit shown in Fig. 22.53, when the key K is pressed at time $t = 0$, which of the following statements about current I in the resistor AB is true?

(a) $I = 2$ mA at all t
(b) I oscillates between 1 mA and 2 mA
(c) $I = 1$ mA at all t
(d) At $t = 0$, $I = 2$ mA and with time it goes to 1 mA.

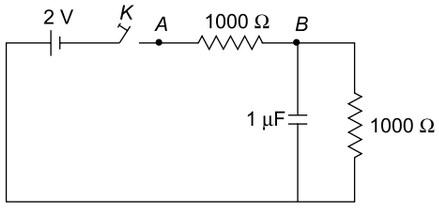


Fig. 22.53

22. You are given several identical resistances each of value $R = 10$ Ω and each capable of carrying a maximum current of 1 A. It is required to make a suitable combination of these resistances to produce a resistance of 5 Ω which can carry a current of 4 A. The minimum number of resistances required is

(a) 4 (b) 10
(c) 8 (d) 20

23. Two cells of the same emf E and different internal resistances r_1 and r_2 are connected in series to an external resistance R. The value of R for which the potential difference across the first cell is zero is given by (see Fig. 22.54)

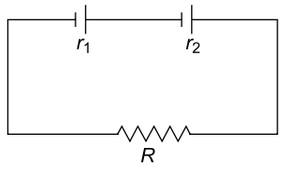


Fig. 22.54

(a) $R = \dfrac{r_1}{r_2}$ (b) $R = r_1 + r_2$
(c) $R = r_1 - r_2$ (d) $R = r_1 = r_2$

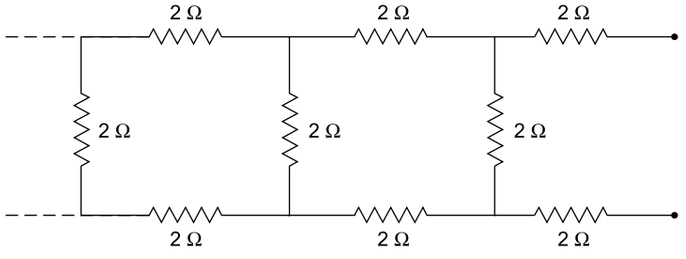


Fig. 22.55

24. The equivalent resistance of the following infinite network shown in Fig. 22.55 is

(a) less than 4 Ω
(b) 4 Ω
(c) more than 4 Ω but less than 12 Ω
(d) 12 Ω

25. A steady current flows in a metallic conductor of non-uniform cross-section. The quantity/quantities that remains/remain constant along the length of the conductor is/are

(a) current, electric field and drift speed
(b) drift speed only
(c) current and drift speed
(d) current only

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26. In the circuit shown in Fig. 22.56, the current through

(a) the 3 Ω resistor is 0.50 A
(b) the 3 Ω resistor is 0.25 A
(c) the 4 Ω resistor is 0.50 A
(d) the 4 Ω resistor is 0.25 A

IIT, 1998

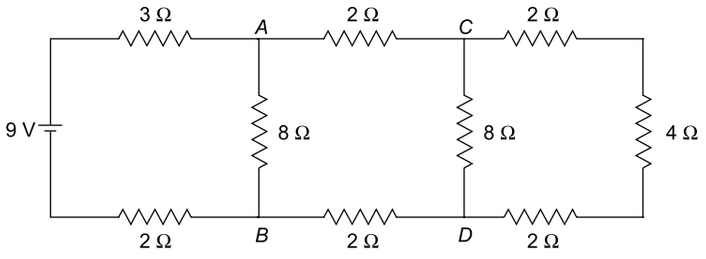


Fig. 22.56

27. A conductor of resistance 3 Ω is stretched uniformly till its length is doubled. The wire is now bent in the form of an equilateral triangle. The effective resistance between the ends of any side of the triangle in ohms is:

(a) $\frac{9}{2}$ (b) $\frac{8}{3}$

(c) 2 (d) 1

28. The resistance of the series combination of two resistances is S. When they are joined in parallel, the total resistance is P. If $S = nP$, the minimum possible value of n is

(a) 1 (b) 2

(c) 3 (d) 4

29. An electric current is passed through a circuit consisting of two wires of the same material, connected in parallel. If the lengths and radii of the wires are in the ratio of $\frac{4}{3}$ and $\frac{2}{3}$ respectively, then the ratio of the currents passing through the wires will be

(a) 3 (b) $\frac{1}{3}$

(c) $\frac{8}{9}$ (d) 2

30. In a metre bridge experiment null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y?

(a) 50 cm (b) 80 cm

(c) 40 cm (d) 70 cm

31. Seven resistors, each of resistance 5 Ω, are connected as shown in Fig. 22.57. The equivalent resistance between points A and B is

(a) 1 Ω (b) 7 Ω

(c) 35 Ω (d) 49 Ω

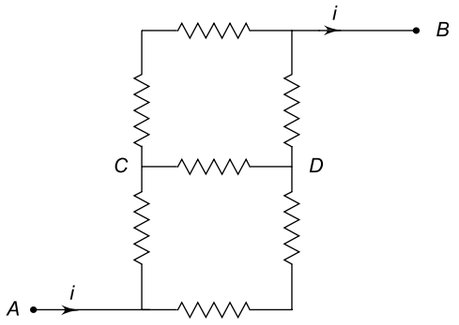


Fig. 22.57

32. Figure 22.58 shows a circuit with two cells in opposition to each other. One cell has an emf of 6 V and internal resistance of 2 Ω and the other cell has an emf of 4 V and internal resistance of 8 Ω. The potential difference across the terminals X and Y is

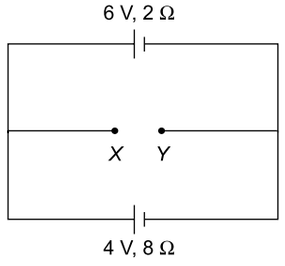


Fig. 22.58

(a) 5.4 V (b) 5.6 V

(c) 5.8 V (d) 6.0 V

33. A copper wire of length 50 cm and area of cross-section 10^{-6} m^2 carries a current of 1 A. If the resistivity of copper is 1.8×10^{-8} Ω m, the electric field across the wire is

(a) 9 Vm^{-1} (b) 0.9 Vm^{-1}

(c) 0.09 Vm^{-1} (d) 0.009 Vm^{-1}

34. Figure 22.59 shows a network of seven resistors numbered 1 to 7, each equal to 1 Ω, connected to a 4 V battery of negligible internal resistance. The current I in the circuit is

(a) 0.5 A (b) 1.5 A

(c) 2.0 A (d) 3.5 A

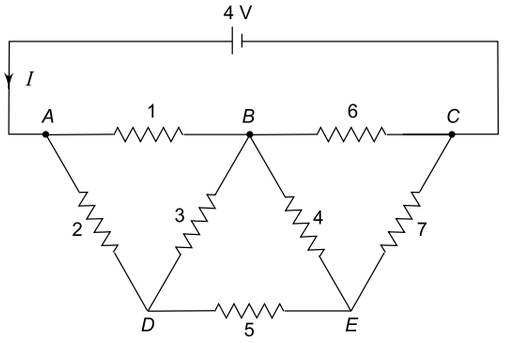


Fig. 22.59

35. In the circuit shown in Fig. 22.60, the ammeter A reads zero. If the batteries have negligible internal resistance, the value of R is

(a) 10 Ω (b) 20 Ω

(c) 30 Ω (d) 40 Ω

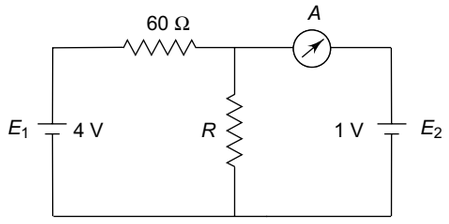


Fig. 22.60

36. What is the potential difference between points C and D in the circuit shown in Fig. 22.61?

(a) 3.6 V (b) 7.2 V

(c) 10.8 V (d) 12 V

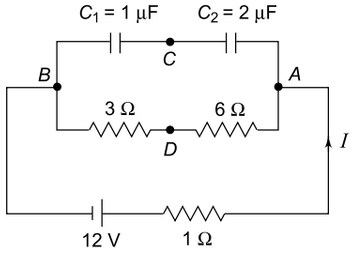


Fig. 22.61

37. In the circuit shown in Fig. 22.62, the effective resistance between A and B is

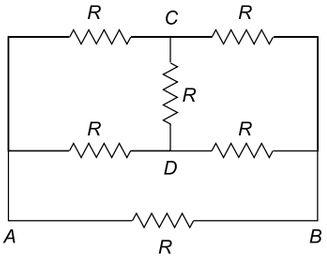


Fig. 22.62

(a) $\frac{R}{2}$ (b) R

(c) $2R$ (d) $4R$

38. The resistance of each arm of a Wheatstone's bridge is 10 Ω. A resistance of 10 Ω is connected in series with the galvanometer. The equivalent resistance across the battery will be

(a) 10 Ω (b) 15 Ω

(c) 20 Ω (d) 40 Ω

39. A set of n identical resistors, each of resistance R ohm when connected in series has an effective resistance of x ohm. When the resistors are connected in parallel, the effective resistance is y ohm. What is the relation between R, x and y?

(a) $R = \frac{xy}{(x+y)}$ (b) $R = (y - x)$

(c) $R = \sqrt{xy}$ (d) $R = (x + y)$

40. The effective resistance of a number of resistors connected in parallel in x ohm. When one of the resistors is removed, the effective resistance becomes y ohm. The resistance of the resistor that is removed is

(a) $\frac{xy}{(x+y)}$ (b) $\frac{xy}{(y-x)}$

(c) $(y - x)$ (d) $\sqrt{xy}$

41. You are given 48 cells each of emf 2 V and internal resistance 1 Ω. How will you connect them so that the current through an external resistance of 3 Ω is the maximum?

(a) 8 cells in series, 6 such groups in parallel

(b) 12 cells in series, 4 such groups in parallel

(c) 16 cells in series, 3 such groups in parallel

(d) 24 cells in series, 2 such groups in parallel

42. In the electrical circuit shown in Fig. 22.63, the current through the galvanometer will be zero if the value of resistance x is

(a) 4 Ω (b) 8 Ω

(c) 16 Ω (d) 24 Ω

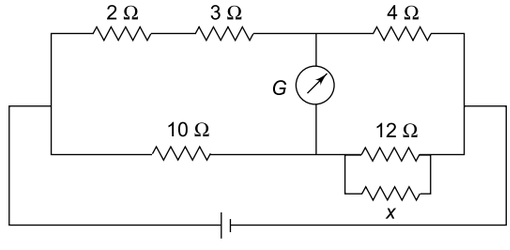


Fig. 22.63

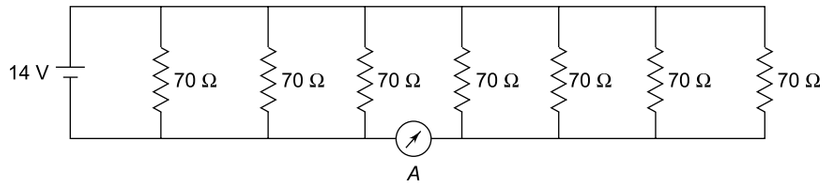


Fig. 22.64

43. The reading of the ammeter in the circuit in Fig. 22.64 is

(a) $\frac{3}{5}$ A (b) $\frac{4}{5}$ A

(c) $\frac{6}{5}$ A (d) A

44. Eight cells marked 1 to 8, each of emf 5 V and internal resistance 0.2 Ω are connected as shown in Fig. 22.65. What is the reading of the ideal voltmeter V?

(a) 40 V (b) 20 V

(c) 5 V (d) zero

IIT, 1987

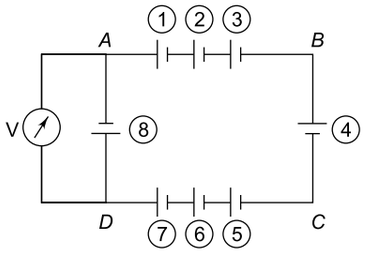


Fig. 22.65

45. In the given circuit, with steady current, the potential drop across the capacitor must be (see Fig. 22.66)

(a) V (b) $V/2$

(c) $V/3$ (d) $2V/3$

IIT, 2001

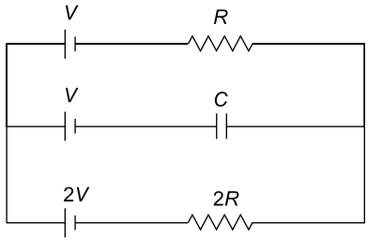


Fig. 22.66

46. In the given circuit it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy (see Fig. 22.67)

(a) $R_1\ R_2\ R_5 = R_3\ R_4\ R_6$

(b) $\frac{1}{R_5}+\frac{1}{R_6}=\frac{1}{R_1+R_2}+\frac{1}{R_3+R_4}$

(c) $R_1\ R_4 = R_2\ R_3$

(d) $R_1\ R_3 = R_2\ R_4 = R_5\ R_6$

IIT, 2001

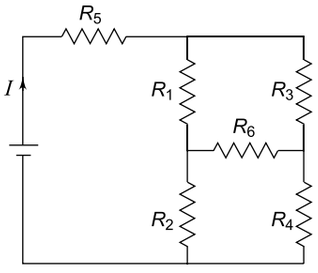


Fig. 22.67

47. The effective resistance between points P and Q of the electrical circuit shown in Fig. 22.68 is

(a) $2Rr/(R + r)$

(b) $8R(R + r)/(3R + r)$

(c) $2r + 4R$

(d) $5R/2 + 2r$

IIT, 2002

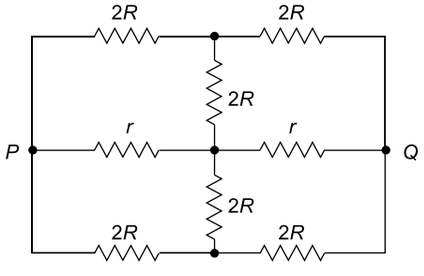


Fig. 22.68

48. In the meter bridge experiment shown in Fig. 22.69, the balance length AC corresponding to null deflection of the galvanometers is x. What would be the balance length if the radius of the wire AB is doubled?

(a) $\frac{x}{2}$ (b) x

(c) $2x$ (d) $4x$

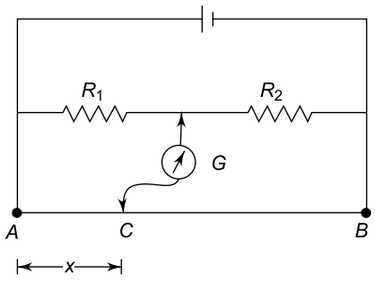


Fig. 22.69

IIT, 2003

49. Which of the circuits shown in Fig. 22.70 can be used to verify Ohm's law?

IIT, 2003

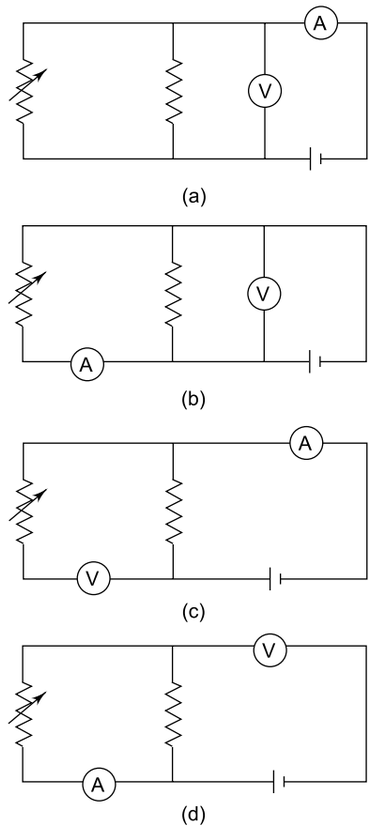


Fig. 22.70

50. Figure 22.71 shows a circuit in which the potential differences across the resistances are given in the diagram. If point Q is grounded, the potential at point S will be

(a) 10 V (b) – 10 V
(c) 40 V (d) – 40 V

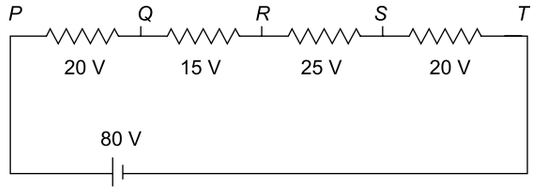


Fig. 22.71

51. The internal resistances of the cells in the circuit shown in Fig. 22.72 are negligible. The current in the circuit is

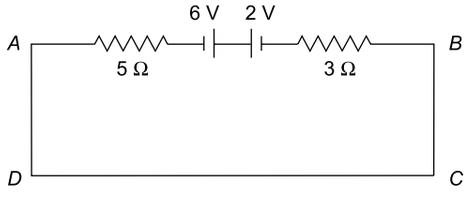


Fig. 22.72

(a) 0.5 A flowing from A to D
(b) 0.5 A flowing from B to C
(c) 1.0 A flowing from A to B
(d) 1.0 A flowing from B to A.

52. A wire of resistance 0.1 Ω/cm is bent to form a square $ABCD$ of side 10 cm. A similar wire is connected between corners B and D to form the diagonal BD. The effective resistance of this combination between corners A and C is

(a) 1 Ω (b) 2 Ω
(c) 4 Ω (d) 8 Ω

53. The voltmeter shown in Fig. 22.73 reads 2 V. What resistance should be connected in parallel with the 2 Ω resistor so that the voltmeter reads 3 V?

(a) 1 Ω (b) 2 Ω
(c) 4 Ω (d) 8 Ω

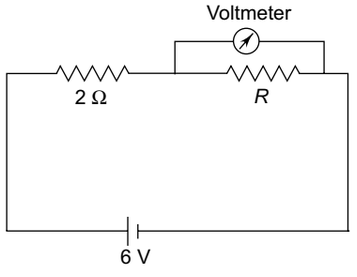


Fig. 22.73

54. Six equal resistances are connected between points A, B and C as shown in Fig. 22.74. If R_1, R_2 and R_3 are the net resistances between A and B, between B and C and between A and C respectively, then $R_1 : R_2 : R_3$ will be equal to

(a) 6 : 3 : 2 (b) 1 : 2 : 3
(c) 5 : 4 : 3 (d) 4 : 3 : 2

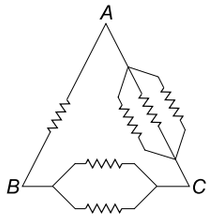


Fig. 22.74

IIT, 2004

55. An RC circuit consists of a resistance $R = 5$ MΩ and a capacitance $C = 1.0$ μF connected in series with a battery. In how much time will the potential difference across the capacitor become 8 times that across the resistor? (Given $\log_e (3) = 1.1$)

(a) 5.5 s (b) 11 s
(c) 44 s (d) 88 s

IIT, 2005

56. The potential difference through the 3 Ω resistor shown in Fig. 22.75 is

(a) zero (b) 1 V
(c) 3.5 V (d) 7 V

IIT, 2005

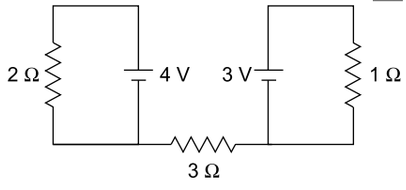


Fig. 22.75

57. A galvanometer of resistance 20 Ω gives full scale deflection when a current of 1 mA is passed through it. It is converted into a voltmeter by connecting a resistance of 4980 Ω in series with it. The maximum potential difference this voltmeter can measure is

(a) 5 mV (b) 0.05 V
(c) 5.0 V (d) 50 V

IIT, 2005

58. If a wire is stretched to make it 0.1% longer, its resistance will

(a) increase by 0.05% (b) increase by 0.2%
(c) decrease by 0.2% (d) remain unchanged.

59. The effective resistance between points A and B in the network shown in Fig. 22.76 is

(a) 1 Ω (b) 2 Ω
(c) 3 Ω (d) 4 Ω

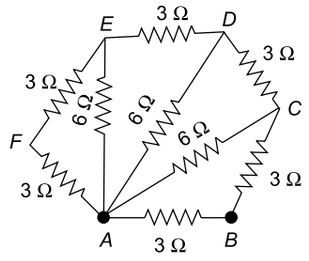


Fig. 22.76

60. A galvanometer together with an unknown resistance in series is connected across two identical batteries each of 1.5 V. When the batteries are connected in series, the galvanometer records a current of 1 A, and when the batteries are in parallel, the current is 0.6 A. What is the internal resistance of the battery?

(a) 1 Ω (b) $\frac{1}{2}$ Ω
(c) $\frac{2}{3}$ Ω (d) $\frac{1}{3}$ Ω

61. In the circuit shown in Fig. 22.77, A and B are two cells of the same emf E and of internal resistances r_A and r_B respectively. L is an ideal inductor and C is an ideal capacitor. The key K is closed. When the current in the circuit becomes steady, what should be the value of R so that the potential difference across the terminals of cell A is zero.

IIT, 2004

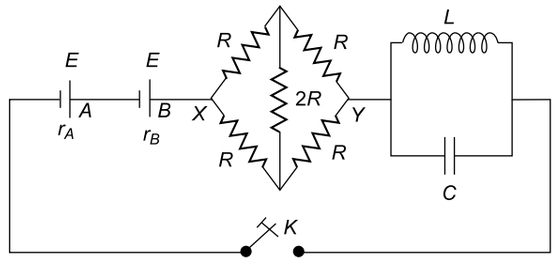


Fig. 22.77

(a) $R = r_A - r_B$ if $r_A > r_B$.

(b) $R = \sqrt{r_A r_B}$

(c) $R = \frac{1}{2}(r_A + r_B)$

(d) For no value of R will the potential difference between the terminals of cell A be equal to zero.

62. An RC circuit consists of a resistance $R = 5$ MΩ and a capacitance $C = 1.0$ μF connected in series

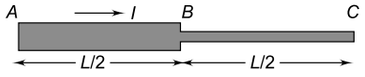
with a battery. In how much time will the potential difference across the capacitor become 8 times that across the resistor? (Given $\log_e(3) = 1.1$)

(a) 5.5 s (b) 11 s
(c) 44 s (d) 88 s

IIT, 2005

63. Two wires AB and BC, each of length $L/2$ are made of the same material. The radius of wire AB is $2r$ and of wire BC is r. The current I flows through the composite wire (see Fig.). Choose the correct statement from the following.

(a) Potential difference across BC is twice that across AB.
(b) Power dissipated in BC is four times the power dissipated in AB.
(c) Current densities in AB and BC are equal.
(d) Electric fields in AB and BC are equal.



IIT, 2006

64. A circuit is connected as shown in Fig. 22.78 with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is

(a) 0 (b) 54 μC
(c) 27 μC (d) 81 μC

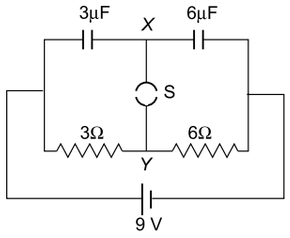


Fig. 22.78

IIT, 2007

65. A meter bridge is set-up as shown in Fig. 22.79 to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is

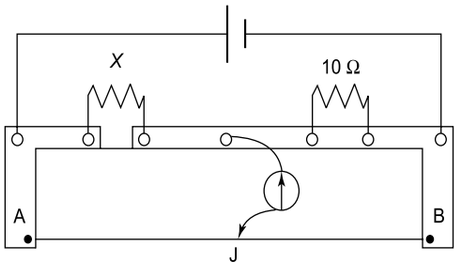


Fig. 22.79

(a) 10.2 ohm (b) 10.6 ohm
(c) 10.8 ohm (d) 11.1 ohm

IIT, 2011

ANSWERS

1. (b)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (a)
7. (b)	**8.** (c)	**9.** (a)	**10.** (c)	**11.** (c)	**12.** (d)
13. (c)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (d)
19. (a)	**20.** (c)	**21.** (d)	**22.** (c)	**23.** (c)	**24.** (b)
25. (d)	**26.** (d)	**27.** (b)	**28.** (d)	**29.** (b)	**30.** (a)
31. (b)	**32.** (b)	**33.** (d)	**34.** (d)	**35.** (b)	**36.** (a)
37. (a)	**38.** (a)	**39.** (c)	**40.** (b)	**41.** (b)	**42.** (d)
43. (b)	**44.** (d)	**45.** (c)	**46.** (c)	**47.** (a)	**48.** (b)
49. (a)	**50.** (d)	**51.** (b)	**52.** (a)	**53.** (b)	**54.** (c)
55. (b)	**56.** (a)	**57.** (c)	**58.** (b)	**59.** (b)	**60.** (d)
61. (a)	**62.** (b)	**63.** (b)	**64.** (c)	**65.** (b)	

SOLUTIONS

1. Equivalent resistance of the combination is

$$R = R_1 + R_2$$

$$= \frac{\rho_1 L}{A} + \frac{\rho_2 L}{A} = \frac{L}{A}(\rho_1 + \rho_2) \quad (1)$$

where L = length of each wire and A = cross-sectional area of each wire = $\pi d^2/4$ (d = diameter). If ρ is the equivalent resistivity,

$$R = \frac{\rho(2L)}{A} \quad (2)$$

Equations (1) and (2) give $\rho = \frac{1}{2}(\rho_1 + \rho_2)$

2. $E = V + Ir = I(R + r)$, where r is the internal resistance of the cell and the current in the circuit is

$$I = \frac{E}{R+r};\ R = \text{resistance of voltmeter.}$$

The voltmeter reads the terminal voltage $V = IR$ and not the emf. Therefore, error in the reading is

$$\Delta E = E - V$$

$$\therefore \quad \frac{\Delta E}{E} = 1 - \frac{V}{E} = 1 - \frac{IR}{I(R+r)}$$

$$= 1 - \frac{R}{(R+r)} = \frac{r}{(R+r)}$$

$$\therefore \text{ Percentage error} = \frac{r}{(R+r)} \times 100$$

$$= \frac{200}{(400+2)} \simeq 0.5\%$$

3. Refer to Fig. 22.80. Let V be the potential at point E. Points A, B and C are earthed. So the potential at A, B and $C = 0$. Potential at $D = 12$ V. Potential difference between D and E is $V_1 = 12 - V$, between E and B is $V_2 = V$ and between E and $C = V$. Therefore

$$I_1 = \frac{12 - V}{3}$$

$$I_2 = \frac{V}{3}$$

$$I_3 = \frac{V}{12}$$

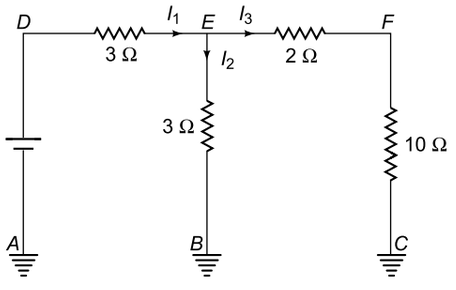


Fig. 22.80

From Kirchhoff's junction rule, $I_1 = I_2 + I_3$, i.e.

$$\frac{12 - V}{3} = \frac{V}{3} + \frac{V}{12} \quad \Rightarrow V = \frac{16}{3} \text{ V}$$

Hence $I_3 = \frac{16/3}{12} = \frac{4}{9}$ A

So the correct choice is (b).

4. Refer to Fig. 22.81. The equivalent resistance is

$$R = \frac{12 \times 12}{(12+12)} = 6\ \Omega$$

$$\therefore \quad I = \frac{12 \text{ V}}{6\Omega} = 2 \text{ A}$$

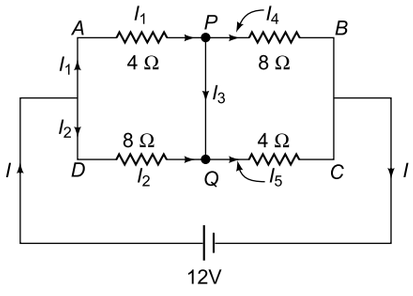


Fig. 22.81

From Kirchhoff's junction rule,

$$I_1 + I_2 = I \qquad (1)$$

Using Kirchhoff's loop rule to loop $APQDA$, we have

$$4I_1 - 8I_2 = 0 \quad \Rightarrow I_1 = 2I_2 \qquad (2)$$

Equations (1) and (2) give $I_1 = \frac{4}{3}$ A and $I_2 = \frac{2}{3}$ A.

Similarly $I_4 = \frac{2}{3}$ A and $I_5 = \frac{4}{3}$ A.

Applying junction rule at P,

$$I_1 = I_3 + I_4$$

$$\therefore \quad I_3 = I_1 - I_4 = \frac{4}{3} - \frac{2}{3} = \frac{2}{3} \text{ A}$$

The positive sign shows that current I_3 flows from P to Q. Hence the correct choice is (c).

5. $I_1 = \frac{12 \text{ V}}{4\Omega} = 3$ A

Applying Kirchhoff's loop rule to loop $ABCDE$,

$$2I_2 + E - 4I_1 = 0$$

Putting $I_1 = 3$ A and $I_2 = 0$, we get $E = 12$ V

6. Refer to Fig. 22.82.

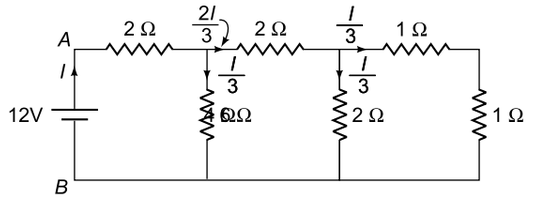


Fig. 22.82

The equivalent resistance between A and $B = 4\ \Omega$.

Current $I = \dfrac{12\text{ V}}{4\Omega} = 3\text{ A}$

The current divides into different branches as shown in the figure. It is clear that the ammeter will read $\dfrac{I}{3} = \dfrac{3\text{A}}{3} = 1\text{ A}$. So the correct choice is (a).

7. The circuit shown in Fig. 22.47 can be redrawn as shown in Fig. 22.83.

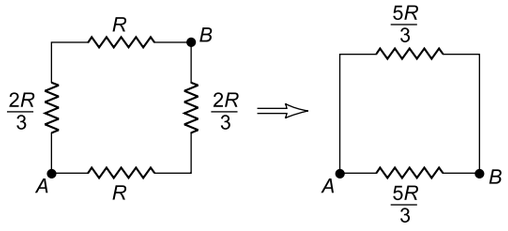


Fig. 22.83

$\therefore \quad R_{AB} = \dfrac{5R}{6}$, which is choice (b).

8. The circuit shown in Fig. 22.48 can be redrawn as shown in Fig. 22.84.

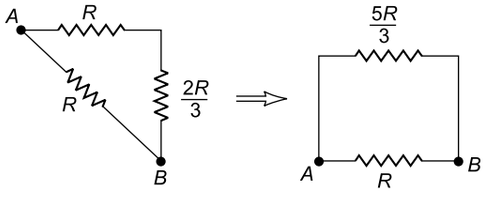


Fig. 22.84

$\therefore \quad R_{AB} = \dfrac{5R}{8}$, which is choice (c).

9. Current $I = \dfrac{50}{(200+20)} = \dfrac{5}{22}\text{ A}$

Potential difference across 200 Ω resistor is

$$V = \frac{5}{22} \times 200 = \frac{500}{11}\text{ volt}$$

When the voltmeter is connected across the 200 Ω resistor, the effective resistance becomes

$$R = \frac{1800 \times 200}{(1800+200)} = 180\ \Omega$$

The current in the circuit becomes

$$I' = \frac{50}{(180+20)} = \frac{1}{4}\text{ A}$$

The potential difference becomes

$$V' = \frac{1}{4} \times 180 = 45\text{ volt}$$

Decrease in p.d. $= V - V' = \dfrac{500}{11} - 45 = \dfrac{5}{11}$ volt

$\therefore$ Percentage decrease $= \dfrac{5}{11} \times \dfrac{11}{500} \times 100 = 1\%$

So the correct choice is (a).

10. In order to have a potential difference of 5 mV = 5×10^{-3} V across a wire of resistance 5 Ω, the current in the wire should be

$$I = \frac{5 \times 10^{-3}}{5} = 10^{-3}\text{ A}$$

If R is the resistance to be connected in series with the wire, then

$$\frac{2}{R+5} = 10^{-3} \Rightarrow R = 1995\ \Omega$$

11. In the first case, the drop of potential per unit length of the potential wire is E_0/L. Hence

$$E = \left(\frac{L}{4}\right) \times \frac{E_0}{L} = \frac{E_0}{4} \qquad (1)$$

In the second case, the potential drop per unit length $= E_0/2L$. If x is the balance length now, then

$$E = \frac{xE_0}{2L} \qquad (2)$$

Equating (1) and (2) we get $x = \dfrac{L}{2}$.

12. This is a balanced Wheatstone's bridge. Therefore

$$\frac{10}{40} = \frac{7}{X}$$

which gives $X = 28\ \Omega$. Hence the correct choice is (d).

13. The circuit can be rearranged as shown in Fig. 22.85.

The resistance between points A and D is given by

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

or $R = 3\ \Omega$. Hence the correct choice is (c)

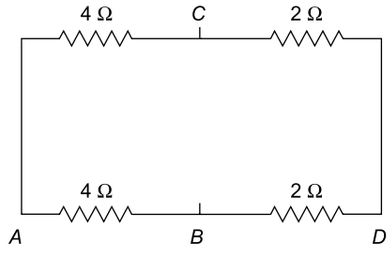


Fig. 22.85

14. If I is the maximum current that can flow through the coil,

$$100\ I + 4900\ I = 5\ \text{V} \quad \text{or} \quad I = \frac{1}{1000}\ \text{A}$$

If R is the required resistance,

$100\ I + R'\ I = 10$, which gives $R' = 9900\ \Omega$

Since $R' = R + 4900$, $R = 5000\ \Omega$. Hence the correct choice is (a).

15. Let the three resistances be X, $2X$ and Y. Then X and $2X$ connected in parallel are equivalent to a single resistance $R_1 = \frac{2}{3}X$.

Also, $$\frac{1}{R_1} + \frac{1}{Y} = 1 \quad \text{or} \quad \frac{3}{2X} + \frac{1}{Y} = 1$$

The minimum volume of X which satisfies the given condition is $X = 3$. Hence $Y = 2$. Three resistances are, therefore, 2, 3 and 6 Ω. Hence the correct choice is (b).

16. The smallest resistance is obtained by a parallel combination. The smallest resistance is given by

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} + \ldots + n \text{ terms} = \frac{n}{R}$$

or $R' = R/n$. Hence the correct choice is (c).

17. A diameter divides a circle into two equal halves. Therefore, the resistance between the ends of a diameter is the resistance of a parallel combination of two resistances, each equal to 2 Ω. Hence the correct choice is (b).

18. The potential at B = potential at E. Hence no current flows through the resistor numbered 3. Similarly, no current flows through the resistor numbered 5. These resistors are not effective. Therefore, the equivalent resistance between A and D is the resistance of the parallel combination of resistances of 1, 4, 6 (in series) and resistances of 2, 8, 7 (in series). Thus

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

or $R = 3\ \Omega$. Therefore, current $I = 3\text{V}/3\Omega = 1.0\ \text{A}$, which is choice (d).

19. Circumference of the circle = $2\pi r$. Therefore, the resistance per unit length of the wire = $R/2\pi r$, where $R = 4\ \Omega$ is the resistance of the wire. Now, the length of the specimen connected along the diameter = $2r$. Therefore, the resistance of this specimen is

$$R_1 = \frac{R}{2\pi r} \times 2r = \frac{R}{\pi}$$

Also, the resistance of each semicircle is $R_2 = \frac{R}{2}$

$\therefore$ Equivalent resistance R' across the specimen is given by

$$\frac{1}{R'} = \frac{2}{R} + \frac{2}{R} + \frac{\pi}{R} = \frac{4+\pi}{R}$$

or $$R' = \frac{R}{4+\pi} = \frac{4}{4+\pi}\ \Omega$$

20. The total resistance of the circuit is given by

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

which gives $R = 2\ \Omega$. Therefore, the current in the circuit is

$$\text{I} = \frac{V}{R} = \frac{10}{2} = 5\ \text{A}$$

Since the two arms have the same resistance = 4 Ω, the current divides equally, i.e. 2.5 A current flows in each arm. Therefore, the potential difference across the 1 Ω resistor in the upper arm = $1 \times 2.5 = 2.5$ V and the across the 3 Ω resistor in the lower arm = $3 \times 2.5 = 7.5$ V. Hence the potential difference between points A and $B = 7.5 - 2.5 = 5$ V, which is choice (c).

21. At time $t = 0$, when the key is pressed, the capacitor begins to charge and the current through the resistor AB is

$$I = \frac{2}{1000} = 2\ \text{mA}$$

The current in the capacitor varies with time t as

$$I = I_0\ e^{-t/\tau}$$

where τ is the time constant of circuit. Thus at $t = 0$, $I = I_0 = 2$ mA. At time $t >> \tau$, the current through the capacitor goes to zero, i.e. its effective resistance become infinite. Then the current will pass through the resistor connected in parallel with the capacitor. At that time, the current in the circuit goes to

$$I' = \frac{2}{1000} = 1\ \text{mA}$$

Hence the correct choice is (d).

22. Since the maximum current through a 10 Ω resistor is 1 A, a current of 4 A is to be divided into 4 equal parts, each going through a branch of a circuit. Thus we need 4 branches in parallel and having equal resistance. If r is the resistance of each branch, the resistance of a parallel combination of four branches will be $r/4$ which equals 5 Ω. Therefore, $r = 20\ \Omega$, i.e. each branch has two 10 Ω resistors connected in series. Hence the total number of resistors is 8.

23. Since the cells are connected in series, the total emf is 2 E. Therefore, the current in the circuit is

$$I = \frac{2E}{R + r_1 + r_2} \qquad \text{(i)}$$

The potential difference across the cell of internal resistance r_1 is $E - I r_1$, = 0, i.e. $E = I r_1$. Using this in (i) we have

$$I = \frac{2 I r_1}{R + r_1 + r_2}$$

which gives $R = r_1 - r_2$. Hence the correct choice is (c).

24. Since the circuit is infinitely long, its total resistance remains unaffected by remvoing one mesh from it. Let the effective resistance of the infinite network be R. The network can be broken as shown in Fig. 22.86. The effective resistance of the remaining network beyond AB will also be R. The resistances R and 4 Ω are in parallel. Their combined resistance is

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{4} \text{ or } R' = \frac{4R}{R+4}$$

Resistance R' is in series with the remaining 2 Ω resistance. The total combined resistance is

$$R' + 2 = \frac{4R}{R+4} + 2$$

which must be equal to the total resistance R of the infinite network. Thus

$$\frac{4R}{R+4} + 2 = R$$

which gives $R^2 - 2R + 8 = 0$

The roots are $R = 1 \pm \frac{1}{2}(36)^{1/2} = 1 \pm 3 = 4\ \Omega$ or $-2\ \Omega$.

The negative value is not permissible. Hence $R = 4\ \Omega$, which is choice (b).

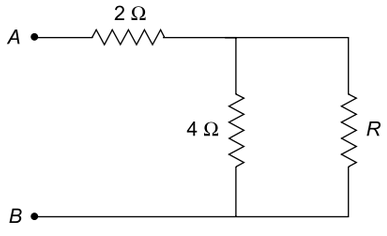


Fig. 22.86

25. The drift speed depends on A, the cross-sectional area of the conductor but the current is independent of A. Hence the correct choice is (d).

26. The equivalent resistance between A and B of the circuit to the right of AB is 4 Ω. Therefore, total resistance = 3 + 4 + 2 = 9 Ω. Current I = 9 V/9 Ω = 1 A. This current is equally divided in the 8 Ω resistor between A and B and the remainder 8 Ω. Hence current in AC = 0.5 A. This current is equally divided between CD and the circuit to the right of CD. Therefore, current in the 4 Ω resistor = 0.25 A which is choice (d).

27. The resistance of a conductor of length l, cross-sectional area A and made of a material of resistivity ρ is given by

$$\therefore \qquad R = \frac{\rho l}{A} = \frac{\rho l^2}{Al} = \left(\frac{\rho}{V}\right) l^2$$

where $V = Al$ is the volume of the conductor. Since ρ is a constant and volume V cannot change if the conductor is stretched, it follows that R is proportional to l^2. Thus if l is doubled, R becomes four times. Hence the new resistance is 3 × 4 = 12 Ω. Hence each side of the equilateral triangle has a resistance of 4 Ω as shown in Fig. 22.87. Therefore, the effective resistance between the ends of any side of the triangle (such as side AB) is equal to the resistance of a parallel combination of $R_1 = 4\ \Omega$ and $R_2 = 4 + 4 = 8\ \Omega$, which is given by

$$R_e = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{4 \times 8}{12} = \frac{8}{3}\ \Omega$$

Hence the correct choice is (b).

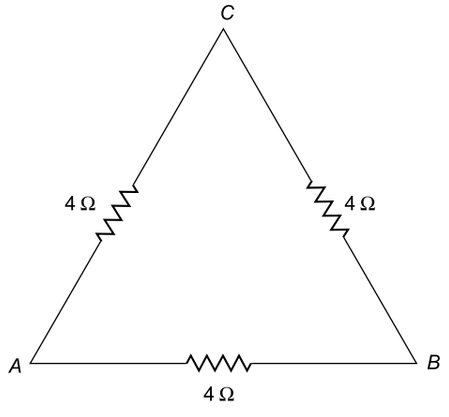


Fig. 22.87

28. Let x and y be the value of the two resistances. Then $S = x + y$ and $P = \frac{xy}{x+y}$. Given $S = nP$ or

$$x + y = \frac{n\,x\,y}{x+y}$$

which gives $x^2 - (n-2)\,xy + y^2 = 0$

For a given y, the two roots of x are

$$x = \frac{1}{2}(n-2)y \pm \frac{1}{2}\,[n-2)^2\,y^2 - 4y^2]^{1/2}$$

$$\text{or} \quad x = \frac{1}{2}\,(n-2)y\,[1 \pm \{(n-2)^2 - 4\}^{1/2}]$$

The value of x will be real if $(n-2)^2 \geq 4$ which gives $n \geq 4$. Hence the minimum possible value of $n = 4$, which is choice (d).

29. Since the wires are made of the same material, they have the same resistivity. Therefore,

$$\rho = \frac{R_1 A_1}{l_1} = \frac{R_2\,A_2}{l_2}$$

$$\text{or} \quad \frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \frac{l_1}{l_2} \times \frac{\pi r_2^2}{\pi r_1^2} = \frac{l_1}{l_2} \times \left(\frac{r_2}{r_1}\right)^2$$

Given $\frac{l_1}{l_2} = \frac{4}{3}$ and $\frac{r_1}{r_2} = \frac{2}{3}$. Hence

$$\frac{R_1}{R_2} = \frac{4}{3} \times \left(\frac{3}{2}\right)^2 = 3 \qquad (1)$$

Since the wires ae connected in parallel, the potential difference across them is the same, i.e., $I_1 R_1 = I_2 R_2$

$$\text{or} \quad \frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{1}{3}, \text{ which is choice (b).}$$

30. $\frac{X}{Y} = \frac{20}{(100-20)} = \frac{20}{80} = \frac{1}{4}$. If the new balance length is l cm, then

$$\frac{4X}{Y} = \frac{l}{(100-l)}$$

Now $\frac{X}{Y} = \frac{1}{4}$. Hence, we have

$$4 \times \frac{1}{4} = \frac{l}{(100-l)}$$

or $100 - l = l$ or $l = 50$ cm, which is choice (a).

31. The circuit can be redrawn as shown in Fig. 22.88 Let a battery of emf E and of negligible resistance be connected as shown. The currents in various branches are shown. Applying Kirchhoff'second law to loops $ACDA$, $CBDC$ and $ADBGFA$, we have

$$5i_2 + 5i_3 - 10i_1 = 0 \qquad \text{(i)}$$

$$10\,(i_2 - i_3) - 5\,(i_1 + i_3) - i_3 = 0 \qquad \text{(ii)}$$

$$10i_1 + 5\,(i_1 + i_3) - E = 0 \qquad \text{(iii)}$$

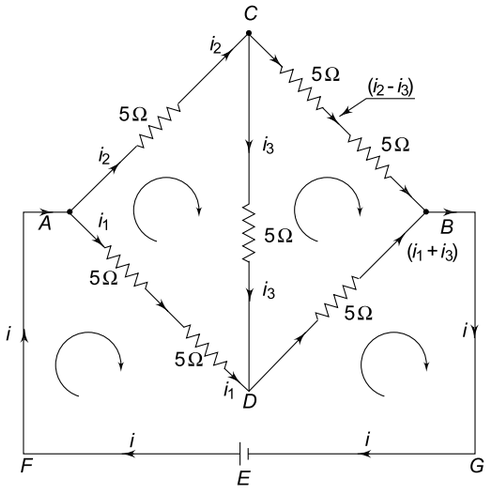


Fig. 22.88

Equations (i) and (ii) give $i_2 = \frac{3i_1}{2}$ and $i_3 = \frac{i_1}{2}$. If R is the effective resistance between A and B, then

$$R = \frac{E}{i_1 + i_2} = \frac{2E}{5i_1}$$

From Eq. (iii) we have

$$E = 15i_1 + 5i_3 = \frac{35i_1}{2}$$

Therefore $R = \frac{2E}{5i_1} = \frac{2 \times 35 i_1}{5i_1 \times 2} = 7\ \Omega$

Hence the correct choice is (b).

32. Since the two cells are in opposition, the effective voltage = 6 – 4 = 2 V. The current in the circuit is

$$I = \frac{2}{2+8} = 0.2 \text{ A}$$

$\therefore$ Terminal voltage of 6 V cell = $6 - 2 \times 0.2 = 5.6$ V
Terminal voltage of 4 V cell = $4 + 8 \times 0.2 = 5.6$ V.
Therefore, the potential difference across terminals X and Y is 5.6 V. Hence the correct choice is (b).

33. Potential difference across a wire of length l is

$$V = IR = \frac{I\rho l}{A}$$

$\therefore$ Electric field $E = \frac{V}{l} = \frac{I\rho}{A} = \frac{0.5 \times 1.8 \times 10^{-8}}{10^{-6}}$

$= 0.009\ \text{Vm}^{-1}$

Hence the correct choice is (d).

34. It follows from the symmetry of the network shown in Fig. 22.89 that no distribution of current takes place at point B. Therefore, the same current

flows through resistors numbered 3 and 4. Thus, it appears that there is no junction at point B. Therefore, the equivalent circuit is as shown in Fig. 22.55. Now, the resistance R_1 between D and E is given by

$$\frac{1}{R_1} = 1 + \frac{1}{2} = \frac{3}{2}$$

or $R_1 = \frac{2}{3}\ \Omega$. Therefore, the equivalent resistance R between A and C is given by

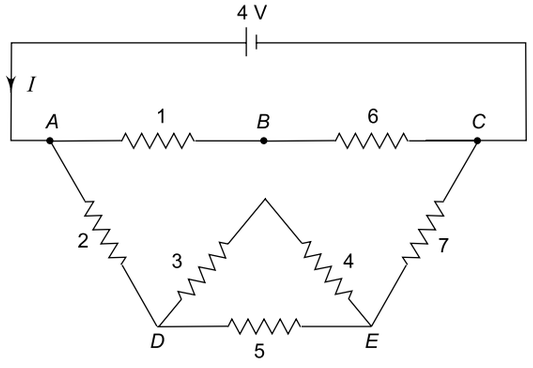


Fig. 22.89

$$\frac{1}{R} = \frac{1}{2} + \frac{3}{8} = \frac{7}{8}$$

or $R = \frac{8}{7}\ \Omega$. Therefore, $I = \frac{4V}{8/7\,\Omega} = 3.5$ A.

35. Current due to E_1 is

$$I = \frac{E_1}{R+60} = \frac{4}{R+60} \qquad \text{(i)}$$

$\therefore$ Potential drop across $R = IR$. Since there is no current due to E_2,

$$IR = E_2 = 1$$

$\therefore$ $I = 1/R$. Using this in Eq. (*i*) we get

$$\frac{1}{R} = \frac{4}{R+60}$$

which gives $R = 20\ \Omega$. Hence the correct choice is (b).

36. Current in the main circuit is $I = \frac{12\text{V}}{3\,\Omega + 6\,\Omega + 1\,\Omega}$

$= 1.2$ A

$\therefore$ Potential difference across AB is

$$V_{AB} = 12 - 1.2 \times 1 = 10.8 \text{ V}$$

Also, potential difference across AD is

$$V_{AD} = 6 \times 1.2 = 7.2 \text{ V}$$

Effective capacitance is

$$C_{\text{eff}} = \frac{C_1C_2}{C_1+C_2} = \frac{1\times 2}{1+2} = \frac{2}{3}\,\mu\text{F} = \frac{2}{3} \times 10^{-6}\ \text{F}$$

Since the capacitors are in series, charge Q is the same and is given by

$$Q = V_{AB} \times C_{\text{eff}} = 10.8 \times \frac{2}{3} \times 10^{-6}$$

$$= 7.2 \times 10^{-6}\ \text{C}$$

Therefore, potential difference across AC is

$$V_{AC} = \frac{Q}{C_2} = \frac{7.2\times 10^{-6}}{2\times 10^{-6}} = 3.6 \text{ V}$$

$\therefore$ Potential difference between points C and D is

$$V_{CD} = V_{AD} - V_{AC} = 7.2 - 3.6 = 3.6 \text{ V}$$

Hence the correct choice is (a).

37. The upper part of the circuit is a balanced Wheatstone's bridge. Hence resistance R between CD is ineffective as no current will flow in this branch. The circuit, therefore, reduces to three parallel branches having resistance R, $R + R$ and $R + R$, i.e. R, $2R$ and $2R$. The effective resistance R' is given by

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{2R}$$

which gives $R' = \frac{R}{2}$. Hence the correct choice is (a).

38. It is a balanced Wheatstone's bridge because

$$\frac{P}{Q} = \frac{R}{S}, \text{ i.e. } \frac{10}{10} = \frac{10}{10}$$

Therefore, no current flows through the galvanometer and the resistance in series with the galvanometer will be ineffective. The equivalent resistance across the battery is the resistance of the parallel combination of the resistances $10 + 10 = 20\ \Omega$ and $10 + 10 = 20\ \Omega$. Thus

$$\frac{1}{R'} = \frac{1}{20} + \frac{1}{20}$$

which gives $R' = 10\ \Omega$. Hence the correct choice is (a).

39. For series connection $x = nR$. For parallel connection $y = \frac{R}{n}$. Therefore $xy = nR \times \frac{R}{n} = R^2$. Hence the correct choice is (c).

40. Given $\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{n-1}} + \frac{1}{R_n} = \frac{1}{x}$ (1)

If the nth resistor is removed, then

$$\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{n-1}} = \frac{1}{y} \qquad (2)$$

Subtracting (2) from (1), we have

$$\frac{1}{R_n} = \frac{1}{x} - \frac{1}{y}$$

which gives $R_n = \dfrac{xy}{(y-x)}$ which is choice (b).

41. Let m cells be connected in series and n such groups are connected in parallel. If the emf of each cell is E and internal resistance r, then the total emf of m cells in series in mE and the total internal resistance is mr. When n such groups are in parallel, the effective internal resistance is mr/n. Then the current through an external resistance R is

$$I = \frac{mE}{R + \dfrac{mr}{n}} = \frac{mnE}{nR + mr} \qquad (1)$$

$$= \frac{mnE}{\left(\sqrt{nR} - \sqrt{mr}\right)^2 + 2\sqrt{mnRr}}$$

Now, I will be maximum if the denominator is the minimum, i.e. if $nR = mr$. Given $R = 3\ \Omega$ and $r = 1\ \Omega$. Using these values, we have $3n = m$. But $mn = 48$ (given). Therefore $\dfrac{m \times m}{3} = 48$, which gives $m = 12$. Thus $n = 4$. Hence the correct choice is (b).

42. The given circuit is a Wheatstone's bridge. The current through the galvanometer will be zero if the bridge is balanced, i.e. if

$\dfrac{P}{Q} + \dfrac{R}{S}$ where $P = 2 + 3 = 5\ \Omega$, $Q = 10\ \Omega$ and $R = 4\ \Omega$. The value of S is given by

$$\frac{5}{10} = \frac{4}{S}$$

or $S = 8\ \Omega$. Thus the effective resistance of the parallel combination of 12 Ω and x ohm must be 8 Ω. Therefore

$$\frac{1}{12} + \frac{1}{x} = \frac{1}{8}$$

which gives $x = 24\ \Omega$. Hence the correct choice is (d).

43. Since the seven resistances are in parallel, the effective resistance is $R = 70/7 = 10\ \Omega$. Therefore, the current in the circuit is $I = 14/10 = 7/5$A. The given circuit can be redrawn as shown in Fig. 22.90 where $R_1 = \dfrac{70}{3}$ and $R_2 = \dfrac{70}{4}\ \Omega$. The current I_2 is given by $I_2 = I \times \dfrac{R}{R_2} = \dfrac{7}{5} \times \dfrac{10}{70/4}$ $= \dfrac{4}{5}$ A, which is choice (b).

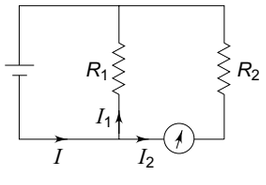


Fig. 22.90

44. The emfs of cells connected in reverse polarity cancel each other. Hence cells marked 2, 3 and 4 together cancel the effect of cells marked 5, 6 and 7 and the circuit reduces to that shown in Fig. 22.91. Now cells 1 and 8 are in reverse polarity. Hence the voltmeter reading = 5 – 5 = 0 V. Hence the correct choice is (d).

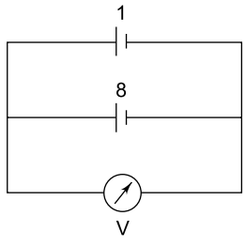


Fig. 22.91

45. In the steady state, no current flows in the branch containing the capacitor. Thus, the current, say I, flows in the branches containing R and $2R$. Applying Kirchhoff's second rule to the loop *abcdefa*, we have (see Fig. 22.92)

$$2V - I(2R) - IR - V = 0 \text{ or } I = \frac{V}{3R}$$

∴ Potential drop across capacitor

$$= 2V - V - I(2R) = V - \frac{V}{3R} \times 2R$$

$$= V - \frac{2V}{3} = \frac{V}{3}$$, which is choice (c).

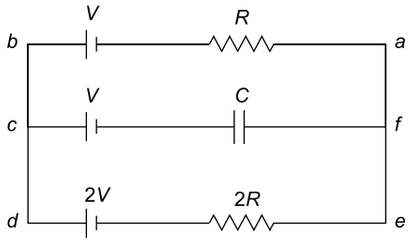


Fig. 22.92

46. Since no current flows through R_6, resistances R_1, R_2, R_3 and R_4 constitute the four arms of a balanced Wheatestone's bridge. Hence

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \text{ or } R_1 R_4 = R_2 R_3$$

Thus the correct choice is (c).

47. Refer to Fig. 22.93.

The branches $ABPQ$ and $PQCD$ are a balanced Wheatstone's bridge. Therefore, resistances (each equal to $2R$) between E and F and between F and G do not contribute and the circuit simplies to the one shown in the figure. The effective resistance R_e between P and Q is given by

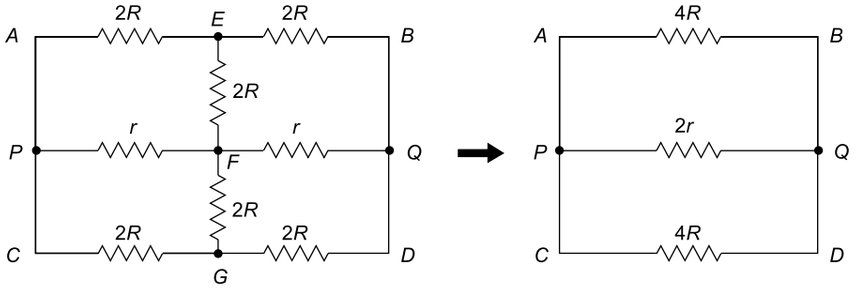


Fig. 22.93

$$\frac{1}{R_e} = \frac{1}{4R} + \frac{1}{2r} + \frac{1}{4R}$$

which gives $R_e = \dfrac{2Rr}{(R+r)}$. Hence the correct choice is (a)

48. The condition for no deflection of the galvanometer is

$$\frac{R_1}{R_2} = \frac{R_{AC}}{R_{CB}}$$

where R_{AC} and R_{CB} are the resistances of the bridge wire of length AC and CB respectively. If the radius of the wire AB is doubled, the ratio R_{AC}/R_{CB} will remain unchanged. Hence the balance length will remain the same. Thus, the correct choice is (b).

49. The voltmeter must be connected in parallel with the resistor and the ammeter must be connected in series with the resistor. Hence the correct circuit is (a).

50. The potential at Q with respect to R is 15 V and R is at 25 V higher potential than S. Thus Q is 40 V higher than S. When Q is grounded, its potential becomes zero. thus, $V_s = -40$ V. Hence the correct choice is (d).

51. Since the cells are in opposition, the effective emf = 6 – 2 = 4 V. Since the current is taken to flow from the positive to the negative terminal of the battery, a current of

$$\frac{4}{5+3} = 0.5 \text{ A}$$

flows from B to C. Hence the correct choice is (b).

52. Resistance of each side of the square = $10 \times 0.1 = 1\ \Omega$. As shown in Fig. 22.94, the square forms a Wheatstone's bridge which satisfies the balancing condition. Thus, no current flows along the diagonal BD. Hence the correct choice is (a).

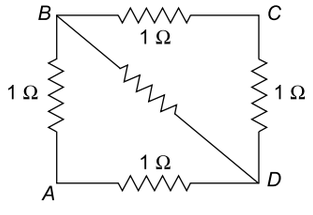


Fig. 22.94

53. The correct choice is (b).

54. Let the value of each resistance be r. The network can be redrawn as shown in the Fig. 22.95.

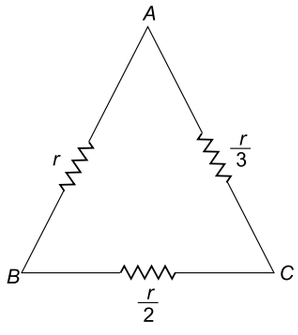


Fig. 22.95

(i) *Net resistance R_1 between points A and B*

The series combination of resistances $\frac{r}{3}$ and $\frac{r}{2}$, which has an equivalent resistance $r_1 = \frac{r}{3} + \frac{r}{2} = \frac{5r}{6}$, is in parallel with resistance r. Hence

$$R_1 = \frac{r \times r_1}{r + r_1} = \frac{r \times \frac{5r}{6}}{r + \frac{5r}{6}} = \frac{5r}{11}$$

(ii) *Net resistance R_2 between points B and C*

The series combination of resistances r and $\frac{r}{3}$, which has an equivalent resistance $r_2 = r + \frac{r}{3} = \frac{4r}{3}$, is in parallel with resistance $\frac{r}{2}$. Hence

$$R_2 = \frac{\frac{r}{2} \times r_2}{\frac{r}{2} + r_2} = \frac{\frac{r}{2} \times \frac{4r}{3}}{\frac{r}{2} + \frac{4r}{3}} = \frac{4r}{11}$$

(iii) *Net resistance R_3 between points A and C*

The series combination of resistances r and $\frac{r}{2}$,

which has an equivalent resistance $r_3 = r + \frac{r}{2} = \frac{3r}{2}$, is in parallel with resistance $\frac{r}{3}$. Hence

$$R_3 = \frac{\frac{r}{3}\times r_3}{\frac{r}{3}+r_3} = \frac{\frac{r}{3}\times\frac{3r}{2}}{\frac{r}{3}+\frac{3r}{2}} = \frac{3r}{11}$$

Hence $R_1 : R_2 : R_3 = \frac{5r}{11} : \frac{4r}{11} : \frac{3r}{11} = 5 : 4 : 3$. Thus the correct choice is (c).

55. At instant of time t, the charge on the capacitor is given by

$$q = q_0\,(1 - e^{-t/RC})$$

and the potential drop across the capacitor is given by ($\because V = q/C$)

$$V_C = V_0\,(1 - e^{-t/RC})$$

where V_0 is the voltage of the battery. The potential drop across the resistor is

$$V_R = V_0 - V_C = V_0 - V_0\,(1 - e^{-t/RC}) = V_0\,e^{-t/RC}$$

$$\therefore \frac{V_C}{V_R} = \frac{1-e^{-t/RC}}{e^{-t/RC}} = e^{t/RC} - 1$$

Given $\frac{V_C}{V_R} = 8$. Therefore,

$$8 = e^{t/RC} - 1$$

or $e^{t/RC} = 9 = (3)^2$

or $\frac{t}{RC} = 2\log_e(3)$

or $t = RC \times 2\log_e(3)$

$= (5\times10^6)\times(1\times10^{-6})\times 2\times 1.1$

$= 11$ s

Hence the correct choice is (b).

56. The two sub circuits are closed loops. They cannot send any current through the 3 Ω resistor. Hence the potential difference across the 3 Ω resistor is zero, which is choice (a).

57. Given $I = 1$ mA $= 10^{-3}$ A, $G = 20$ Ω and $R = 4980$ Ω.

Now $I = \frac{V}{R+G}$

or $V = I(R + G) = 10^{-3}\times(4980 + 20)$

$= 5.0$ V

Hence the correct choice is (c).

58. The mass of a wire of length l, cross sectional area A and density d is given by

$$m = Ald \text{ or } A = \frac{m}{ld}$$

$\therefore$ The resistance of wire of resistivity ρ is

$$R = \frac{\rho l}{A} = \frac{\rho d l^2}{m} = kl^2 \qquad (1)$$

where $k = \rho\, d/m$ is a constant of the wire. Taking logarithm of both sides of (1) we have

$$\log R = \log k + 2\log l$$

Differentiating

$$\frac{\delta R}{R} = 0 + \frac{2\delta l}{l} = \frac{2\delta l}{l}$$

Given $\frac{\delta l}{l} = 0.1\%$. Therefore, $\frac{\delta R}{R} = 2\times 0.1\% = 0.2\%$. Thus, the resistance of the increases by 0.2%.

which is choice (b).

59. The resistance between points A and E is given by

$$\frac{1}{R_{AE}} = \frac{1}{6}+\frac{1}{6}$$

by giving $R_{AE} = 3$ Ω. The network reduces to that shown in Fig. 22.96 (a). Similarly the resistance

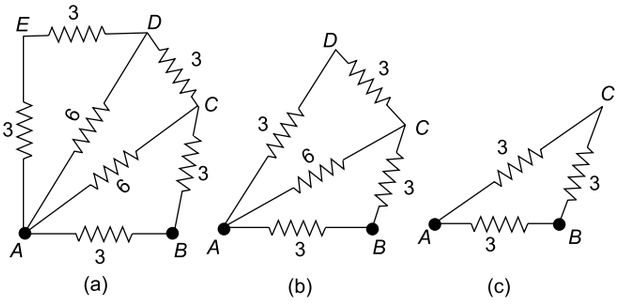


Fig. 22.96

between points A and D in Fig. 22.96 (a) is $R_{AD} = 3$ Ω. The network reduces to that shown in Fig. 22.96 (b). It follows from this figure that the resistance between points A and C is $R_{AC} = 3$ Ω. The network simplifies to that shown in Fig. 22.96 (c). Clearly the effective resistance between points A and B is 2 Ω

So the correct choice is (b).

60. Let the internal resistance of each battery be r. Let R be the unknown resistance and G be the resistance of the galvanometer. Let E be the emf of each battery. When the batteries are connected in series, the total emf = $2E = 2\times 1.5 = 3$ V and total internal resistance is $2r$. The current in the circuit will be

$$I = \frac{3}{R+G+2r}$$

Given $I = 1$ A. Therefore

$$1 = \frac{3}{R+G+2r} \quad \text{or} \quad R + G = (3 - 2r) \quad (1)$$

When the batteries are connected in parallel, the total emf = E = 1.5 V and the total internal resistance is $r/2$. Hence the current in the circuit will be

$$I' = \frac{1.5}{R+G+\frac{r}{2}}$$

Given I' = 0.6 A. Therefore,

$$0.6 = \frac{1.5}{R+G+\frac{r}{2}} \quad \text{or} \quad R + G = \left(2.5 - \frac{r}{2}\right) \quad (2)$$

From Eqs. (1) and (2), we have

$$3 - 2r = 2.5 - \frac{r}{2}$$

which gives $r = \frac{1}{3}$ ohm, which is choice (d)

61. When the key K is inserted, the current starts growing and after some time it acquires a steady value. At this stage, no current flows through the capacitor (because an ideal capacitor offers an infinite resistance to a steady current). All the current flows through the inductor (because an ideal inductor offers zero resistance to a steady current). Now, the network of resistors is a balanced Wheatstone's bridge. Hence, no current flows through the resistance 2 R. Therefore, this resistance can be ignored. The net resistance between points X and Y = resistance of the parallel combination of

$$2\ R \text{ and } 2\ R = \frac{2R \times 2R}{(2R+2R)} = R.$$

Hence the current in the circuit is

$$I = \frac{E+E}{R+r_A+r_B} = \frac{2E}{R+r_A+r_B}$$

Now, the terminal voltage of cell A is

$$V_A = E - Ir_A = E - \frac{2Er_A}{R+r_A+r_B}$$

$$V_A = 0, \text{ if } E - \frac{2Er_A}{R+r_A+r_B} = 0$$

which gives $2\ r_A = R + r_A + r_B$ or $R = r_A - r_B$

So the correct choice is (a).

62. At instant of time t, the charge on the capacitor is given by

$$q = q_0\ (1 - e^{-t/RC})$$

and the potential drop across the capacitor is given by ($\because$ $V = q/C$)

$$V_C = V_0\ (1 - e^{-t/RC})$$

where V_0 is the voltage of the battery. The potential drop across the resistor is

$$V_R = V_0 - V_C = V_0 - V_0\ (1 - e^{-t/RC}) = V_0\ e^{-t/RC}$$

$$\therefore \frac{V_C}{V_R} = \frac{1 - e^{-t/RC}}{e^{-t/RC}} = e^{t/RC} - 1$$

Given $\frac{V_C}{V_R} = 8$. Therefore,

$$8 = e^{t/RC} - 1$$

or $e^{t/RC} = 9 = (3)^2$

or $\frac{t}{RC} = 2 \log_e (3)$

or $t = RC \times 2 \log_e (3)$

$= (5 \times 10^6) \times (1 \times 10^{-6}) \times 2 \times 1.1$

$= 11$ s

63. $R = \frac{\rho l}{\pi r^2}$. Since the two wires are made of the same material, resistivity ρ is the same for wires AB and BC. Since the wires have equal lengths, it follows that $R \propto 1/r^2$. Hence

$$\frac{R_{AB}}{R_{BC}} = \frac{1}{4}, \text{ i.e } R_{BC} = 4R_{AB}$$

Since the current, is the same in the two wires, it follows from Ohm's law ($V = IR$) that $V_{BC} = 4\ V_{AB}$. Hence choice (a) is wrong. Now power dissipated is $P = I^2\ R$. Since I is the same, $P \propto R$. Hence

$$\frac{P_{BC}}{P_{AB}} = \frac{R_{AB}}{R_{BC}} = 4$$

Hence chioce (b) is correct. Choice (c) is wrong because current density (i.e. current per unit area) is different in wires AB and BC because their cross-sectional areas are different. The electric field in a wire is $E = V/l$. Since the two wires have the same length (l), E is proportional to potential difference (V). Since $V_{BC} = 4\ V_{AB}$, $E_{BC} = 4E_{AB}$. Hence choice (d) is also incorrect.

64. Refer to Fig. 22.97.

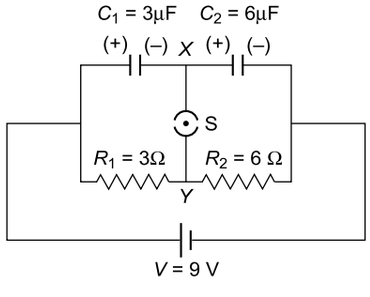


Fig. 22.97

When the switch S is open, capacitors C_1 and C_2 are in series and their combined capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2\ \mu F$$

$\therefore$ Charge on each capacitor = 2 μF × 9V = 18 μC

When the switch S is closed, in the steady state, no current flows through the capacitors. Therefore, resistors R_1 and R_2 will be in series and their combined resistance is $R = R_1 + R_2 = 3 + 6 = 9\ \Omega$. Therefore, the current in each resistor is

$$I = \frac{9V}{9\Omega} = 1\ A$$

$\therefore$ Potential difference across R_1 is $V_1 = IR_1 = 1 \times 3 = 3$ V

Potential difference across R_2 is $V_2 = IR_2 = 1 \times 6 = 6$ V

Since capacitor C_1 is connected across R_1, the potential difference across C_1 is V_1. Similarly, the potential difference across C_2 is V_2.

$\therefore$ Charge on capacitor C_1 in $Q_1 = C_1 V_1 = 3\ \mu F \times 3V = 9\ \mu C$

Charge on capacitor C_2 is $Q_2 = C_2 V_2 = 6\ \mu F \times 6V = 36\ \mu C$

Now, when the switch is open, the initial charge flowing from Y to X = 18 – 18 = 0 μC because the right plate of C_1 has a charge – 18 μC and left plate of C_2 has a charge + 18 μC. When the switch S is closed, the final charge flowing from Y to X = – 9 + 36 = + 27 μC. Therefore, the net charge flowing from Y to X when the switch is closed = 27 μC – 0 = 27 μC.

65. Corrected length L_1 (= AJ) = 52 + 1 = 53 cm

Corrected length L_2 (BJ) = (100 – 52) + 2 = 50 cm

For a balanced Wheatstone bridge,

$$\frac{X}{10} = \frac{L_1}{L_2} = \frac{53}{50}$$

$$\Rightarrow \quad X = 10.6\ \Omega$$

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. A galvanometer has a resistance of 100 Ω and full-scale range of 50 μA. It can be used as a voltmeter or an ammeter, provided a resistance is connected to it. Pick the correct range and resistance combination (s):

(a) 50 V range with 10 kΩ resistance in series

(b) 10 V range with 200 kΩ resistance in series

(c) 5 mA range with 1 Ω resistance in parallel

(d) 10 mA range with 2 Ω resistance in parallel

IIT, 1991

2. Choose the correct statements from the following.

(a) A low voltage supply of, say, 6 V must have a very low internal resistance.

(b) A high voltage supply of, say, 6000 V must have a very high internal resistance.

(c) A wire carrying current stays electrically neutral.

(d) A high resistance voltmeter is used to measure the emf of a cell.

3. The terminal voltage of a battery is

(a) always equal to its emf

(b) always less than its emf

(c) greater or less than its emf depending on the direction of the current through the battery

(d) greater or less than its emf depending on the magnitude of its internal resistance

4. The internal resistance of the cell shown in Fig. 22.98 is negligible. On closing the key K, the ammeter reading changes from 0.25 A to 5/12 A, then

(a) $R_1 = 10\ \Omega$

(b) $R_1 = 15\ \Omega$

(c) the power drawn from the cell increases

(d) the current through R decreases by 40%

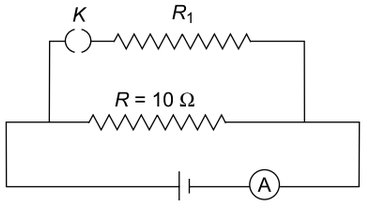


Fig. 22.98

5. A current I flows in the circuit shown in Fig. 22.99. Then

(a) If a resistance $R_2 = R$ is connected in parallel with $R_1 = R$, the current through R_1 will remain equal to I.

(b) If a resistance $R_2 = 2R$ is connected in parallel with $R_1 = R$, the current through R_1 will remain equal to I.

(c) If a resistance $R_2 = 2R$ is connected in parallel with $R_1 = R$, the current through R_1 will become $I/3$.

(d) If a resistance $R_2 = 2R$ is connected in parallel with $R_1 = R$, the current through R_2 will be $I/2$.

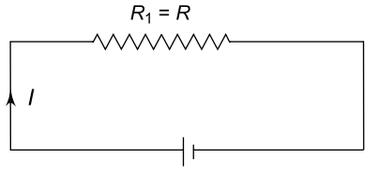


Fig. 22.99

6. Two equal resistances $R_1 = R_2 = R$ are connected with a 30 Ω resistor and a battery of terminal voltage E. The currents in the two branches are 2.25 A and 1.5 A as shown in Fig. 22.100. Then

(a) $R_2 = 15\ \Omega$ (b) $R_2 = 60\ \Omega$
(c) $E = 36$ V (d) $E = 180$ V

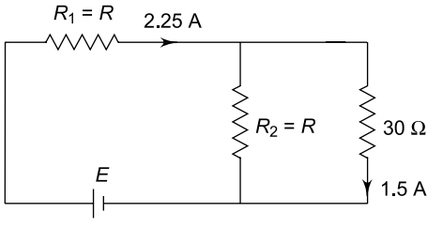


Fig. 22.100

7. Which of the following statements are correct about the circuit shown in Fig. 22.101 where 1 Ω and 0.5 Ω are the internal resistances of the 6 V and 12 V batteries respectively?

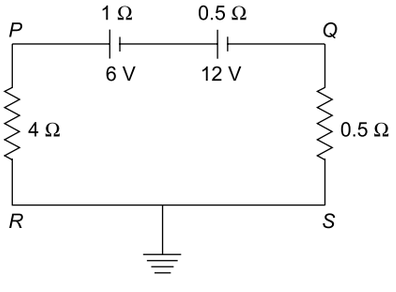


Fig. 22.101

(a) The potential at point P is 6 V.
(b) The potential at point Q is – 0.5 V
(c) If a voltmeter is connected across the 6 V battery, it will read 7 V.
(d) If a voltmeter is connected across the 6 V battery, it will read 5 V.

8. In the circuit shown in Fig. 22.102,

(a) the current through NP is 0.5 A
(b) the value of $R_1 = 40\ \Omega$
(c) the value of $R = 14\ \Omega$
(d) the potential difference across $R = 49$ V

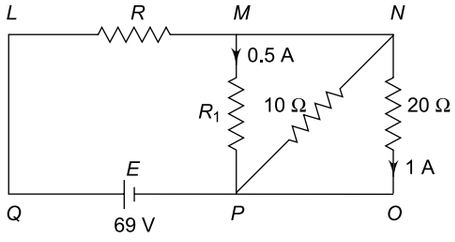


Fig. 22.102

9. Choose the correct statements from the following.

(a) If n identical cells, each of emf E and internal resistance r are connected in series, the emf of the combination is nE and the internal resistance of the combination is nr.

(b) If n identical cells, each of emf E and internal resistance r are connected in parallel, the emf of the combination is E/n and the internal resistance of the combination is r/n.

(c) Cells should be connected in series if the external resistance R is greater than internal resistance r.

(d) Cells should be connected in parallel of R is smaller than r.

10. The resistance network shown in Fig. 22.103 is connected to a battery of emf 30 V and internal resistance of 1 Ω. Then

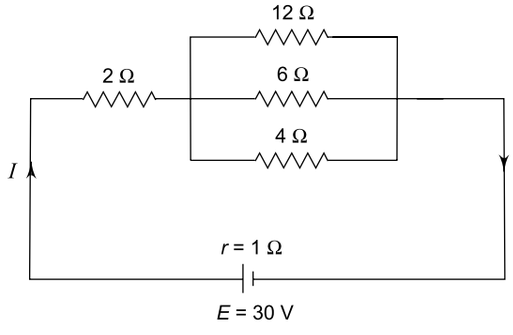


Fig. 22.103

(a) the voltage drop across the 2 Ω resistor is 12 V.
(b) the voltage drop across the 12 Ω resitor is 12 V.
(c) the terminal voltage of the battery is 24 V.
(d) the voltage drop across the internal resistance of the battery is 6 V.

11. A steady current flows in a matallic conductor of nonuniform cross-section. Which of the following quantities remain constant along the length of the conductor?

(a) Current (b) drift speed
(c) resistivity (d) electric field

12. A cell of emf E and internal resistance r supplies a current of 0.9 A through a 2 Ω resistor and a current of 0.3 A through a 7 Ω resistor. Then

(a) $r = 1.0\ \Omega$ (b) $r = 0.5\ \Omega$
(c) $E = 2.0$ V (d) $E = 2.25$ V

13. A voltmeter graded as 6000 Ω per volt reads 3 V at full-scale deflection. When a resistance R' is connected in series with it the reading of the new voltmeter is 12 V at full-scale deflection. The resistance of the new voltmeter is R''. Then

(a) $R' = 5.4 \times 10^4\ \Omega$
(b) $R' = 3.6 \times 10^4$ V
(c) $R'' = 5.4 \times 10^4\ \Omega$
(d) $R'' = 7.2 \times 10^4$ V

14. An infinite network of resistances is constructed with 1 Ω and 2 Ω resistances as shown in Fig. 22.104. A 6 V battery of negligible internal resistance is connected between A and B.

IIT, 1987

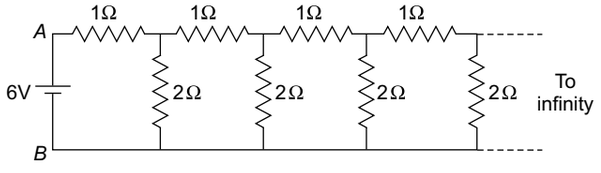


Fig. 22.104

(a) The effective resistance between A and B is 3 Ω.
(b) The effective resistance between A and B is 2 Ω.
(c) The current in the 1 Ω resistance closest to the battery is 3.0 A.
(d) The current in the 2 Ω resistance closest to the battery is 1.5 A.

15. A single battery of emf E and internal resistance r is equivalent to a parallel combination of two batteries of emfs $E_1 = 2$ V and $E_2 = 1.5$ V and internal resistances $r_1 = 1\ \Omega$ and $r_2 = 2\ \Omega$ respectively, with polarities as shown in Fig. 22.105.

IIT, 1997

(a) $E = \frac{5}{6}$ V (b) $E = 1.2$ V
(c) $r = 1.5\ \Omega$ (d) $r = \frac{2}{3}\ \Omega$

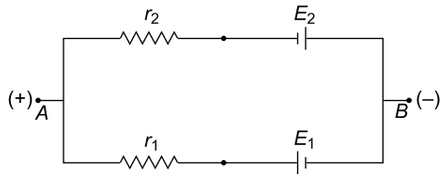


Fig. 22.105

16. In the circuit shown in Fig. 22.106, cells E_1 and E_2 have emfs 4 V and 8 V and internal resistances 0.5 Ω and 1 Ω respectively.

IIT, 1978

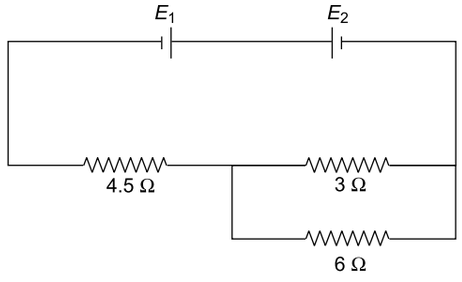


Fig. 22.106

(a) The potential difference across E_1 is 4.25 V
(b) The potential difference across E_1 is 3.75 V
(c) The potential difference across E_2 is 8.5 V
(d) The potential difference across E_2 is 7.5 V.

17. In the circuit shown in Fig. 22.107, if the galvanometer resistance is 6 Ω, then in the steady state

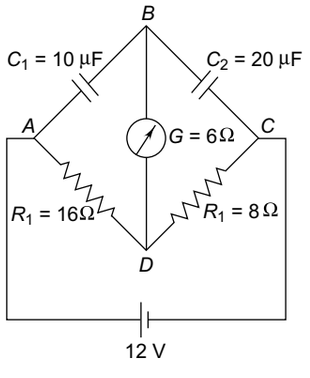


Fig. 22.107

(a) no current flows through the galvanometer.

(b) the current through R_2 is 4 A.
(c) the charge on C_1 is 80 μC.
(d) the charge on C_2 is 80 μC.

18. In the cirucit shown Fig. 22.108, the current through
(a) the 3 Ω resistor is 1.0 A
(b) the 3 Ω resistor is $\frac{9}{15}$ A
(c) the 4 Ω resistor is 0.50 A
(d) the 4 Ω resistor is 0.25 A

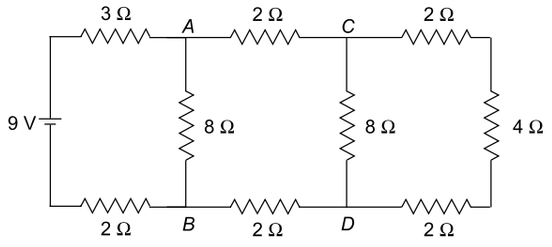


Fig. 22.108

IIT, 1998

ANSWERS AND SOLUTIONS

1. For voltmeter, the resistance R to be connected in series with the galvanometer is given by

$$R = \frac{V}{I_g} - G$$

For 50 V range, $R = \frac{50}{50 \times 10^{-6}} - 100$

$= 10^{-6} - 100 \simeq 10^6 \simeq 1000$ kΩ.

Hence choice (a) is incorrect.

For 10 V range, $R = \frac{10}{50 \times 100^{-6}} - 100$

$= 2 \times 10^5 - 100 \simeq 2 \times 10^5$

$\simeq 200$ kΩ. Hence choice (b) is correct.

For ammeter, the shunt resistance is given by

$$S = \frac{I_g}{I_s} \cdot G$$

For 5 mA range, $S = \frac{50 \times 10^{-6}}{5 \times 10^{-3}} \times 100 = 1\ \Omega$

Hence choice (c) is correct.

For 10 mA range, $S = \frac{50 \times 10^{-6}}{10 \times 10^{-3}} \times 100 = 0.5\ \Omega$

Hence choice (d) is incorrect. Thus the correct choices are (b) and (c).

2. Statement (a) is correct.
The current I drawn from a supply of voltage E and internal resistance r is given by $I = E/r$. So r must be very small so that a high current can be drawn.
Statement (b) is also correct. At such a high voltage, the current drawn from the supply will become dangerously large if its internal resistance is small. Hence a high voltage supply must have a very high internal resistance so that the current drawn from it does not exceed the safe limit.
Statement (c) is correct. A wire carrying current is not charged. It stays neutral because as many electrons enter one end of the wire as leave it from the other end.
Statement (d) is incorrect. A voltmeter does not measure the emf; it measures only the potential difference because it draws some current from the cell. A potentiometer is used for emf measurement because at balance point, no current is drawn from the cell. The potential difference measured with a voltameter is always less than the emf. Thus the correct choices are (a), (b) and (c).

3. Terminal voltage $V = E - Ir$, where E is the emf and r the internal resistance of the battery. If the current is flowing from the + ve to the negative terminal in the external circuit $V < E$. However, if the current is flowing in the opposite direction, a voltmeter across the battery would show $V > E$. Hence the correct choice is (c).

4. Before closing the key,

$$I_1 = \frac{E}{R} \Rightarrow 0.25 = \frac{E}{10} \Rightarrow E = 2.5\ \text{V}$$

After closing the key,

Effective resistance is $R_2 = \frac{RR_1}{(R + R_1)} = \frac{10R_1}{(10 + R_1)}$.

$$I_2 = \frac{E}{R_2}$$

or $$\frac{5}{12} = \frac{2.5 \times (10 + R_1)}{10R_1}$$

which gives $R_1 = 15\ \Omega$.
Before closing the key,

Power $P_1 = I_1^2 R_2 = (0.25)^2 \times 10 = 0.625$ W

After closing the key,

Power $P_2 = I_2^2 R_2 = \left(\frac{5}{12}\right)^2 \times \frac{10 \times 15}{(10 + 15)} = 1.04$ W

Hence power drawn from the cell increases.
Before closing the key, the current through R is $I_1 = 0.25$ A. After closing the key, the current through R is

$I_1' = \frac{5}{12} \times \frac{3}{5} = \frac{1}{4} = 0.25$ A, which is equal to I_1.

Hence the correct choices are (b) and (c).

5. When $R_2 = R$ is connected in parallel with $R_1 = R$, the resistance of the combination is $R/2$. Therefore, the current in the circuit is $V/R/2 = 2V/R = 2I$, where $I = V/R$ was the current in the circuit when R_2 was not connected. Current $2I$ divides equally among two equal parallel resistors. Hence the current through R_1 will still be I. Thus choice (a) is correct. When $R_2 = 2R$ is connected in parallel with $R_1 = R$, the total resistance in the circuit is $2R/3$ and the current in the circuit is

$$\frac{V}{2R/3} = \frac{3V}{2R} = \frac{3I}{2}$$

$\therefore$ Current through $R_1 = \frac{3I}{2} \times \frac{2}{3} = I$. Hence choice (b) is also correct. The remaining current $I/2$ flows through R_2. Hence the correct choices are (a), (b), and (d).

6. Using Kirchhoff's Ist law current through R_2 is 2.25 – 1.5 = 0.75 A. Also since R_2 is in parallel with the 30 Ω resistance, R_2 must be 60 Ω since only half the current flows through it compared to the current through 30 Ω resistor. Total resistance in the circuit becomes 60 + 20 = 80 Ω

Potential drop across the battery $E = IR = 2.25 \times 80 = 180$ V.

Hence the correct choices are (b) and (d).

7. Total resistance = 4 + 1 + 0.5 + 0.5 = 6 Ω. Net voltage in the circuit is 6 V. Current $I = \frac{6}{6} = 1$ A in the anticlockwise direction

$$V_{PR} = 1 \times 4 = 4 \text{ V}$$

Since R is connected to earth, $V_R = 0$. Hence $V_P = 4$ V

$$V_{SQ} = 0.5 \times 1 = 0.5 \text{ V}.$$ S is at a higher potential than Q

$$\therefore \quad V_Q = -0.5 \text{ V}$$

Current is being forced into the 6 V battery in the opposite direction. Hence $V_6 = E + Ir = 6 + 1 \times 1 = 7$ V. Hence the correct choices are (b) and (c).

8. Potential difference across MP = p.d. across NO = p.d. across NP (see Fig. 22.104)

Current across NP, $I_{NP} \times 10 = 20 \times 1$ or $I_{NP} = 2$ A

Across MP, $0.5\ R_1 = 20$ or $R_1 = 40$ Ω

Total current = 2 + 0.5 + 1.0 = 3.5 A

$3.5 = \frac{69}{R + 40/7}$ yields $R = 14$ Ω

Hence the correct choices are (b) and (c).

9. The correct choices are (a), (c) and (d). In choice (b) the emf of the combination E and not E/n.

10. Total resistance of parallel combination is given by

$$\frac{1}{R_1} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4}$$

or $R_1 = 2$ Ω. Total resistance in circuit is $R = R_1 + 2 = 4$ Ω. Therefore, current in the circuit is

$$I = \frac{E}{R + r} = \frac{30}{4 + 1} = 6 \text{ A}$$

$\therefore$ Potential drop across 2 Ω resistor = 2 × 6 = 12 V.

The voltage drop across the 2 Ω resistor is 12 V. Since the resistance of the parallel combination is also 2 Ω, the voltage drop across this combination is also 12 V. Therefore, the total voltage drop across the network = 12 + 12 = 24 V. Thus the terminal voltage of the battery is 24 V.

Voltage drop across the battery = 30 – 24 = 6 V. All the four choices are correct.

11. Current does not depend on the cross-section of the conductor. Drift speed is inversely proportional to the area of cross-section. Electric field depends upon the drift speed of free electrons. Resistivity depends only on the material of the conductor. Hence the correct choices are (a) and (c).

12. If E is the emf of the cell and r its internal resistance, then

$$\frac{E}{2 + r} = 0.9 \text{ and } \frac{E}{7 + r} = 0.3$$

Dividing the two equations, we get

$$\frac{7 + r}{2 + r} = \frac{9}{3}$$

or $r = 0.5$ Ω.

Using $r = 0.5$ Ω in $\frac{E}{(2 + r)} = 0.9$, we get $E = 2.25$ V.

Hence the correct choices are (b) and (d).

13. A voltmeter is graded according to its resistance and the voltage it reads at full scale deflection. If a voltmeter has a resistance R ohms and it reads V volts at full scale deflection, it is said to be graded as R/V ohm per volt. It is given that $V = 3.0$ V and the voltmeter is graded as 6000 Ω/V. Hence the resistance of the voltmeter is

$$R = 6000 \text{ Ω/V} \times 3.0 \text{ V} = 1.8 \times 10^4 \text{ Ω}$$

At full-scale deflection, the current through the voltmeter is

$$I = \frac{V}{R} = \frac{3.0}{1.8\times10^4} = 1.67 \times 10^{-4}\ \text{A}$$

In order to convert this instrument into a voltmeter that reads 12 V at full-scale deflection, the resistance R' that must be connected in series with it is given by (here $V' = 12$ V)

$$R' = \frac{V'}{I} - R = \frac{12}{1.67\times10^{-4}} - 1.8\times10^4$$

$$= 5.4 \times 10^4\ \Omega$$

The resistance of the new voltmeter $= R + R' = 1.8\times10^4 + 5.6\times10^4 = 7.2\times10^4\ \Omega$. Hence the correct choice are (a) and (d).

14. Let circuit be broken as shown in Fig. 22.109(a). Since the circuit is infinitely long, its total resistance remains unaffected by removing one mesh from it. Let the effective resistance of the infinite network be R. The effective resistance of the remaining part of the circuit beyond CD is also R. The circuit can be recombined as shown in Fig. 22.109 (b). The resistance R and 2 Ω are in parallel. Their combined resistance is

$$R' = \frac{2R}{R+2}$$

R' is the series with remaining 1 Ω resistance. The total combined resistance is

$$\frac{2R}{R+2} + 1$$

which must be equal to the total resistance of the infinite net work. Therefore

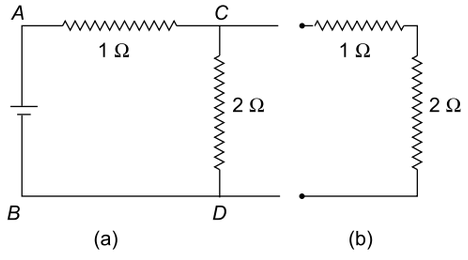


Fig. 22.109

$$R = \frac{2R}{R+2} + 1 = \frac{3R+2}{R+2}$$

or $R^2 + 2R = 3R + 2$ or $R^2 - R - 2 = 0$

or $R^2 - 2R + R - 2 = 0$

or $R(R-2) + (R-2) = 0$

or $(R+1)(R-2) = 0$

which gives $R = -1\ \Omega$ or $2\ \Omega$. Since negative value of R is not admissible, $R = 2\ \Omega$.

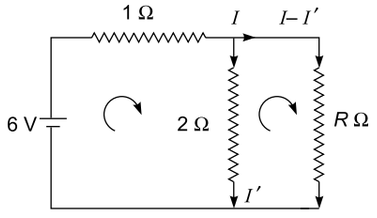


Fig. 22.110

Applying Kirchhoff's loop rule to the two meshes in Fig. 22.110, we have ($\because R = 2\ \Omega$)

$$1 \times I + 2I' = 6 \qquad (1)$$

and $2(I - I') - 2I' = 0$

which gives $I = 2I'$. Therefore, using Eq. (1), we have

$$2I' + 2I' = 6 \text{ or } I' = \frac{6}{4} = 1.5\ \text{A}.$$

So the correct choices are (b), (c) and (d).

15. Refer to Fig. 22.111. Applying Kirchhoff's loop rule to loop *abcda*, we have

$$-Ir_2 + E_2 + E_1 - Ir_1 = 0$$

which gives $I = \frac{E_1 + E_2}{r_1 + r_2} = \frac{2+1.5}{1+2} = \frac{7}{6}$ A

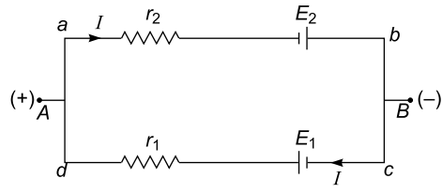


Fig. 22.111

The emf across A and B is

$$E = -E_2 + Ir_2$$

$$= -1.5 + \frac{7}{6} \times 2 = \frac{5}{6}\ \text{V}$$

This is the emf of the single battery. The internal resistance of the single battery is the resistance r of the parallel combination of internal resistances r_1 and r_2 which is

$$r = \frac{r_1 r_2}{(r_1 + r_2)} = \frac{1\times2}{1+2} = \frac{2}{3}\Omega$$

So the correct choices are (a) and (d).

16. Equivalent resistance of the parallel combination of 3 Ω and 6 Ω is $R = 3\times6/(3+6) = 2\ \Omega$. As the cells are in opposition, net emf $E = E_2 - E_1 = 8 - 4 = 4$ V.

Therefore, current is

$$I = \frac{E}{R + 4.5 + r_1 + r_2} = \frac{4}{2 + 4.5 + 0.5 + 1} = 0.5 \text{ A}$$

Potential difference across E_1 is $V_1 = E_1 + Ir_1 = 4 + 0.5 \times 0.5 = 4.25$ V

Potential difference across E_2 is $V_2 = E_2 - Ir_2 = 8 - 0.5 \times 1 = 7.5$ V

Hence the correct choices are (a) and (d).

17. In the steady state, no current flows through the branch ABC containing the capacitors. Thus all the current flows through ADC. So choice (a) is correct. Current through R_1 = current through R_2 = I = 12/24 = 0.5 A. Now p.d. across A and C = 12 V. Therefore $Q = V_1C_1 = V_2C_2$. Thus $\frac{V_1}{V_2} = \frac{C_2}{C_1}$ = 2. Also $V_1 + V_2 = 12$. Therefore, $V_1 = 8$ V and $V_2 = 4$V. Thus $Q = C_1V_1 = 10\ \mu\text{F} \times 8\ \text{V} = 80\ \mu\text{C}$ = charge on C_2. Thus the correct choices are (a), (c) and (d).

18. The equivalent resistance between A and B of the circuit to the right of AB is 4 Ω. Therefore, total resistance = 3 + 4 + 2 = 9 Ω. Current I = 9 V/9 Ω = 1 A. The current is equally divided in the 8 Ω resistor between A and B and the remainder 8 Ω. Hence current in AC = 0.5 A. This current is equally divided between CD and the circuit to the right of CD. Therefore, current in the 4 Ω resistor = 0.25 A. Hence the corrects choice are (a) and (d).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage

Passage I

The circuit shown in Fig. 22.112 consists of the following

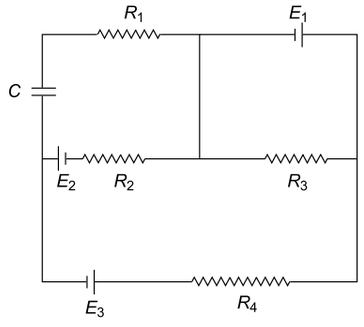


Fig. 22.112

$E_1 = 3E_2 = 2E_3 = 6$ volt

$R_1 = 2R_4 = 6$ ohm

$R_3 = 2R_2 = 4$ ohm

$C = 5\ \mu\text{F}$

IIT, 1988

1. The current in resistance R_1 is

(a) 0.5 A (b) 1.0 A

(c) 1.5 A (d) zero

2. The current through resistance R_3 is

(a) 1.5 A (b) 1.2 A

(c) 0.9 A (d) 0.6 A

3. The current through resistance R_4 is

(a) 0.3 A (b) 0.25 A

(c) 0.2 A (d) zero

4. The energy stored in the capacitor is

(a) 4.8×10^{-6} J (b) 9.6×10^{-6} J

(c) 1.44×10^{-5} J (d) 1.92×10^{-5} J

SOLUTION

1. Refer to Fig. 22.113 on page 22.43.

In the steady state, no current flows through the capacitor. Hence the current through R_1 is zero, which is choice (d).

2. Applying Kirchhoff's rule to loop $EBCDE$, we get

$$I_2 R_3 = E_1 \text{ or } I_2 = \frac{E_1}{R_3} = 1.5 \text{ A}$$

The correct choice is (a).

3. Applying Kirchhoff's second law to the loop $FDHGF$, we get

$$I_3 R_2 - I_2 R_3 + I_3 R_4 = -E_2 - E_3$$

or $I_3 (R_2 + R_4) = I_2 R_3 - E_2 - E_3$

or $I_3 (2 + 3) = 1.5 \times 4 - 2 - 3$

or $5I_3 = 1$ or $I_3 = 0.2$ A

So the correct choice is (c).

4. Potential difference between F and E is $V = E_2 + I_3 R_2 = 2 + 0.2 \times 2 = 2.4$V. The points E and A are at the same potential. Therefore potential difference between F and A is 2.4 volt. Energy stored in the capacitor $= \frac{1}{2} CV^2$

$= \frac{1}{2} \times 5 \times 10^{-6} \times (2.4)^2 = 14.4 \times 10^{-6}$ J

Hence the correct choice is (c).

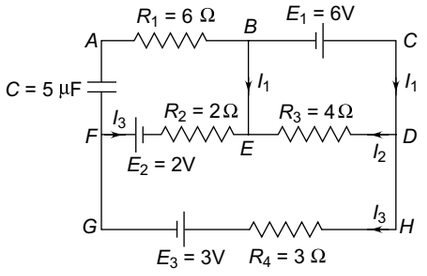


Fig. 22.113

Questions 5 to 8 are based on the following passage

Passage II

The circuit shown in Fig. 22.114, consists of the following $E_1 = 3$ V, $E_2 = 2$ V, $E_3 = 1$ V and $r_1 = r_2 = r_3 = R = 1\Omega$.

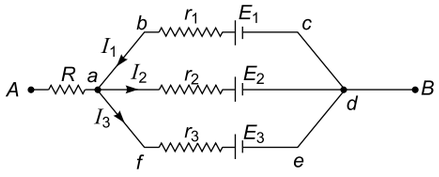


Fig. 22.114

IIT, 1981

5. Current I_1 through resistance r_1 is
(a) zero (b) 0.5 A
(c) 1.0 A (d) 1.5 A

6. Current I_2 through resistance r_2 is
(a) zero (b) 1.0 A
(c) 2.0 A (d) 3.0 A

7. Current I_3 through resistance r_3 is
(a) 1.0 A (b) 1.5 A
(c) 2.0 A (d) 2.5 A

8. If r_2 is short-circuited and point A is connected to point B, the current through resistor R would be
(a) 1.0 A (b) 2.0 A
(c) 3.0 A (d) zero

SOLUTION

5. Applying Kirchhoff's rule to loops *abcda* and *abcdefa*, we have

$$-I_1 r_1 + E_1 - E_2 - I_2 r_2 = 0$$

and $-I_1 r_1 + E_1 - E_3 - I_3 r_3 = 0$

which give $E_1 - I_1 r_1 = E_2 + I_2 r_2 = E_3 + I_3 r_3$ (i)

Applying Kirchhoff's first rule to junction a, we have

$$I_1 = I_2 + I_3 \quad \text{(ii)}$$

Using Eq. (ii) in Eq. (i), we get

$E_1 - (I_2 + I_3) r_1 = E_3 + I_3 r_3$ or $2I_3 + I_2 = 2$ (iii)

Also $E_2 + I_2 r_2 = E_3 + I_3 r_3$ or $I_3 - I_2 = 1$ (iv)

Equations (ii), (iii) and (iv) give $I_1 = 1$ A. So the correct choice is (c).

6. Equations (ii), (iii) and (iv) give $I_2 = 0$, which choice (a).

7. From Eq. (ii) we get $I_3 = I_1 - I_2 = 1 - 0 = 1$ A. So the correct choice is (a).

8. Since $I_2 = 0$, the potential difference between points a and d = emf $E_2 = 2$ V and remains equal to 2 V even when r_2 is short-circuited. Because the potential difference across a and d remains unchanged, the currents I_1 and I_3 through cells E_1 and E_2 do not change i.e. $I_1 = I_3 = 1$ A, even when point A is connected to point B. Hence the current through R will be zero, which is choice (d).

Questions 9 to 12 are based on the following passage

Passage III

Figure 22.115 shows a network of six resistors connected a battery of emf 8.5 V and of negligible internal resistance. The direction of one of the currents in the branches is wrongly marked.

IIT, 1991

9. In branch ab, the current i_1 is
(a) 0.1 A flowing from a to b

(b) 0.1 A flowing from b to a
(c) 0.2 A flowing from a to b
(d) 0.2 A flowing from b to a

10. In branch ad, the current i_2 is
(a) 0.2 A flowing from a to d
(b) 0.2 A flowing from d to a
(c) 0.3 A flowing from a to d
(d) 0.3 A flowing from d to a

11. In branch bd, the current I_3 is
(a) 0.3 A flowing from b to d
(b) 0.3 A flowing from d to b
(c) 0.1 A flowing from b to d
(d) 0.1 A flowing from d to b.

12. The total resistance of the network is
(a) 8.0 Ω (b) 8.5 Ω
(c) 17 Ω (d) 17.5 Ω

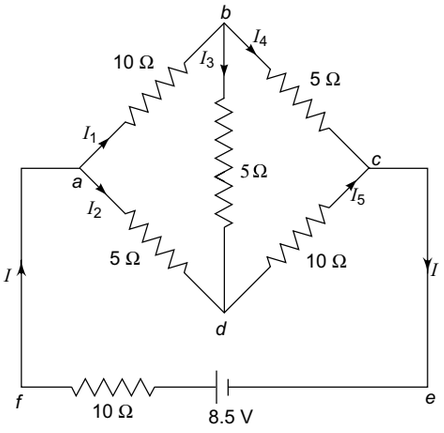


Fig. 22.115

SOLUTION

Applying Kirchhoff's junction rule to junctions a, b and d, we have

$$I = I_1 + I_2,\ I_4 = I_1 - I_3 \text{ and } I_5 = I_2 + I_3.$$

Applying Kirchhoff's loop rule to loops $abda$, $bcdb$ and $adcefa$, we get

$$10\ I_1 + 5\ I_3 - 5\ I_2 = 0$$

or $$I_2 = 2\ I_1 + I_3 \quad \text{(i)}$$

$$5(I_1 - I_3) - 10(I_2 + I_3) - 5\ I_3 = 0$$

or $$I_1 - 2\ I_2 = 4\ I_3 \quad \text{(ii)}$$

and $$5\ I_2 + 10(I_2 + I_3) - 8.5 + 10(I_1 + I_2)$$

or $$2\ I_1 + 5\ I_2 + 2\ I_3 = 1.7 \quad \text{(iii)}$$

Solving Eqs. (i), (ii) and (iii) we get $I_1 = 0.2$ A, $I_2 = 0.3$ A and $I_3 = -0.1$ A. Since I_1 and I_2 are positive, their directions are correct. But the sign of I_3 is negative which indicates that the direction I_3 should be from d to b and not from b to d.

9. The correct choice is (c).
10. The correct choice is (c).
11. The correct choice is (d).
12. Total current is $I = I_1 + I_2 = 0.2 + 0.3 = 0.5$ A. Hence the total resistance of the network is

$$R = \frac{V}{I} = \frac{8.5}{0.5} = 17\ \Omega$$

So the correct choice is (c).

Questions 13 to 16 are based on the following passage

Passage IV

Figure 22.116 shows four cells E, F, G and H of emfs 2 V, 1 V, 3 V and 1 V and internal resistances 2 Ω, 1 Ω, 3 Ω and 1 Ω respectively.

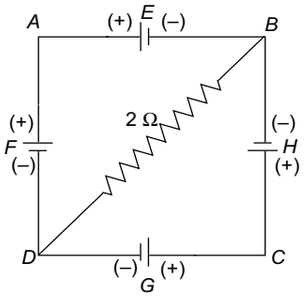


Fig. 22.116

IIT, 1984

13. The current flowing in the 2 Ω resistor is
(a) $\frac{1}{7}$ A (b) $\frac{1}{9}$ A
(c) $\frac{1}{11}$ A (d) $\frac{1}{13}$ A

14. The potential difference between points B and D is
(a) $\frac{2}{7}$ V (b) $\frac{2}{9}$ V
(c) $\frac{2}{11}$ V (d) $\frac{2}{13}$ V

15. The potential difference between the terminals of cell G is
(a) equal to 3 V
(b) more than 3 V
(c) between 2 V and 3 V
(d) between 1 V and 1.5 V.

16. The potential difference between the terminals of cell H is
(a) equal to 1 V
(b) more than 2 V
(c) between 1.5 V and 2 V
(d) between 1 V and 1.5 V

SOLUTION

13. Let I_1 and I_2 be the currents in branches BAD and DCB respectively as shown in Fig. 22.117. Let I_3 be the current along DB through the 2 Ω resistor.

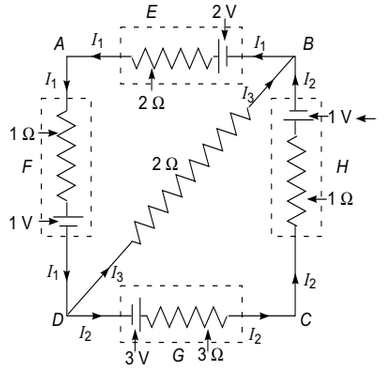


Fig. 22.117

Applying Kirchhoff's first law at function D, we have

$$I_1 = I_2 + I_3 \text{ or } I_3 = I_1 - I_2$$

Applying Kirchhoff's second law to loops $BADB$ and $DCBD$ we have

$$2I_1 + I_1 + 2(I_1 - I_2) = 2 - 1 = 1$$

or $$5I_1 - 2I_2 = 1 \quad \text{(i)}$$

and $$3I_2 + I_2 - 2(I_1 - I_2) = 3 - 1 = 2$$

or $$6I_2 - 2I_1 = 2 \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$I_1 = \frac{5}{13} \text{ A and } I_2 = \frac{6}{13} \text{ A}$$

$$\therefore \quad I_3 = I_1 - I_2 = -\frac{1}{13} \text{ A}$$

The negative sign shows that the current through the 2 Ω resistor flows along BD and not along DB as assumed. So the correct choice is (d).

14. Potential difference between B and $D = 2 \times (1/13) = 2/13$ V, which is choice (d).

15. Potential difference between the terminals of cell $G = 3 - (6/13) \times 3 = 21/13 \simeq 1.6$ V. So the correct choice is (d).

16. Potential difference between the terminals of cell $H = 1 + 6/13 = 19/13 \simeq 1.46$. Thus the correct choice is (d).

Questions 17 to 19 are based on the following passage

Passage V

Two resistances 300 Ω and 400 Ω are connected to a 60 V power supply as shown in Fig. 22.118. A voltmeter connected across the 400 Ω resistor reads 30 V.

IIT, 1980

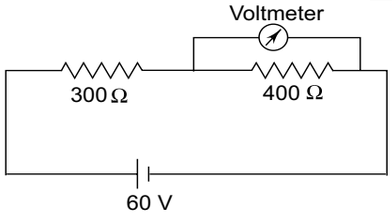


Fig. 22.118

17. The resistance of the voltmeter is
(a) 600 Ω (b) 800 Ω
(c) 1000 Ω (d) 1200 Ω

18. The current in the circuit is
(a) $\frac{3}{32}$ A (b) $\frac{3}{16}$ A
(c) $\frac{3}{8}$ A (d) $\frac{3}{4}$ A

19. When the same voltmeter is connected across the 300 Ω resistor, it will read
(a) 40 V (b) 22.5 V
(c) 37.5 V (d) 25 V

SOLUTION

17. Potential difference across the 400 Ω resistance = 30 V. Therefore, potential difference across the 300 Ω resistance = 60 – 30 V = 30 V. Let R be the resistance of the voltmeter. As the voltmeter is in parallel with the 400 Ω resistance, their combined resistance is

$$R' = \frac{400\,R}{(400 + R)}$$

As the potential difference of 60 V is equally, shared between the 300 Ω and 400 Ω resistances, R' should be equal to 300 Ω. Thus

$$300 = \frac{400\,R}{(400+R)}$$

which gives $R = 1200\ \Omega$, which is choice (d).

18. When the voltmeter is connected across the 300 Ω resistance, their combined resistance is

$$R'' = \frac{300\,R}{(300+R)} = \frac{300\times 1200}{(300+1200)} = 240\ \Omega$$

$\therefore$ Total resistance in the circuit = 400 + 240 =640 Ω

$\therefore$ Current in the circuit is

$$I = \frac{60\text{ V}}{640\ \Omega} = \frac{3}{32}\text{ A}$$

So the correct choice is (a).

19. Voltmeter reading = Potential difference across 240 Ω resistance

$$= \frac{3}{32} \times 240 = 22.5\text{ V}$$

Thus the correct choice is (b).

Questions 20 to 23 are based on the following passage

Passage VI

Figure 22.119 shows a part of the circuit in the steady state. The currents, the values of resistances and emfs of the cells are shown in the figure. The circuit also contains a capacitor of capacitance $C = 4\ \mu$F.

IIT, 1986

20. The value of current i_1 is

(a) 1 A (b) 2 A

(c) 3 A (d) 4 A

21. The value of current i_2 is

(a) 1 A (b) 2 A

(c) 3 A (d) 4 A

22. The value of current i_3 is

(a) 1 A (b) 2 A

(c) 3 A (d) 4 A

23. The energy stored in the capacitor is

(a) 2×10^{-4} J (b) 4×10^{-4} J

(c) 6×10^{-4} J (d) 8×10^{-4} J

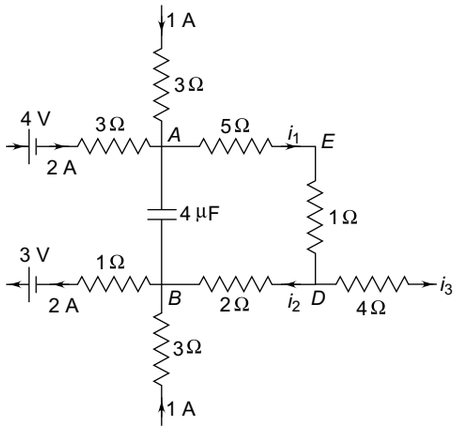


Fig. 22.119

SOLUTION

20. Applying Kirchhoff's junction rule to junction A, we have $i_1 = 2 + 1 = 3$ A, which is choice (c).

21. At junction B, $i_2 + 1 = 2$ or $i_2 = 1$ A, which is choice (a).

22. At junction D, $i_1 = i_2 + i_3 \Rightarrow i_3 = i_1 - i_2 = 3 - 1 = 2$ A.

So the correct choice is (b).

23. Potential drop along $AEDB$ is given by

$$V = 5\,i_1 + 1\,i_1 + 2\,i_2 = 15 + 3 + 2 = 20\text{ V}$$

$\therefore$ Potential difference across the capacitor $C = 20$ V

Energy stored in the capacitor $= \frac{1}{2}\,CV^2$

$$= \frac{1}{2}\,4\times 10^{-6}\times(20)^2 = 8\times 10^{-4}\text{ J}$$

Thus the correct choice is (d).

Questions 24 to 26 are based on the following passage

Passage VII

An electrical circuit is shown in Fig. 22.120. The values of resistances and the directions of the currents are shown. A voltmeter of resistance 400 Ω is connected across the 400 Ω resistor. The battery has negligible internal resistance.

IIT, 1996

24. The value of current i_1 is

(a) $\frac{1}{10}$ A (b) $\frac{1}{20}$ A

(c) $\frac{1}{30}$ A (d) $\frac{1}{40}$ A

25. The value of current i_2 is

(a) $\frac{1}{30}$ A (b) $\frac{1}{15}$ A

(c) $\frac{1}{10}$ A (d) $\frac{2}{15}$

26. The reading of the voltmeter is

(a) $\frac{10}{3}$ V (b) 5 V

(c) $\frac{20}{3}$ V (d) 4 V

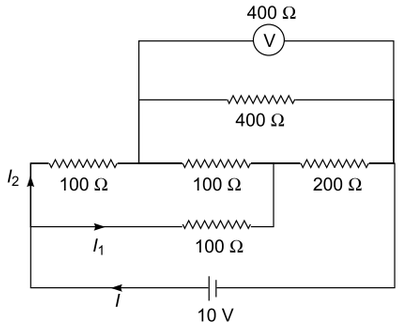


Fig. 22.120

SOLUTION

24. Refer to Fig. 22.121.

Resistance R = 200 Ω shown in the circuit is the equivalent resistance of the parallel combination of the 400 Ω resistor and the resistance of 400 Ω of the voltmeter.

Applying Kirchhoff's loop rule to mesh I, we have

$$-100\,I_2 - 100(I_2 - I_3) + 100\,I_1 = 0$$

or $$I_1 - 2\,I_2 + I_3 = 0 \qquad (1)$$

Applying Kirchhoff's loop rule to mesh II, we have

$$-100\,I_1 - 200(I_1 + I_2 + I_3) + 10 = 0$$

or $$3\,I_1 + 2\,I_2 - 2\,I_3 - 0.1 = 0 \qquad (2)$$

Applying Kirchhoff's loop rule to mesh III, we have

$$-200\,I_3 + 200(I_1 + I_2 - I_3) + 100(I_2 - I_3) = 0$$

or $$2\,I_1 + 3\,I_2 - 5\,I_3 = 0 \qquad (3)$$

Simultaneous solution of Eqs. (1), (2) and (3) yields

$$I_1 = I_2 = I_3 = \frac{1}{30}\ \text{A}$$

So the correct choice is (c).

25. The correct choice is (a).

26. Voltmeter reading = potential difference across R(= 200 Ω)

$$= I_3 R = \frac{1}{30} \times 200 = \frac{20}{3}\ \text{V}.$$

Hence the correct choice is (c).

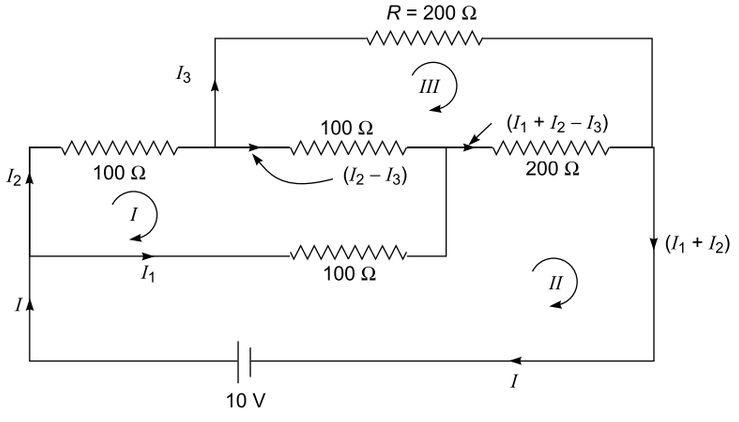


Fig. 22.121

SECTION IV

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. Statement-1

If an electric field is applied to a metallic conductor, the free electrons experience a force but do not accelerate; they only drift at a constant speed.

Statement-2

The force exerted by the electric field is completely balanced by the Coulomb force between electrons and protons.

2. Statement-1

The drift speed of electrons in metals is small (of the order of a few mm s^{-1}) and the charge of an electron is also very small (= 1.6×10^{-19} C), yet we can obtain a large current in a metal.

Statement-2

At room temperature, the thermal speed of electrons is very high (about 10^7 times the drift speed)

3. Statement-1

A wire carrying a current has no electric field around it.

Statement-2

A wire carrying current stays electrically neutral because rate of flow of electrons in one direction equals the rate of flow of protons in the opposite direction.

4. Statement-1

In the metre bridge experiment shown in Fig. 22.122, the balance length AC corresponding to null deflection of the galvanometer is x. If the radius of the wire AB is doubled, the balanced length becomes 4 x.

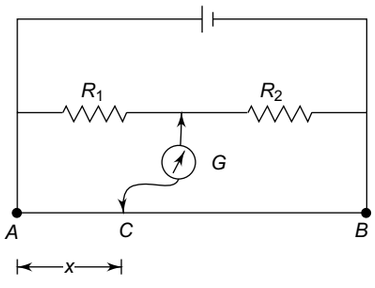


Fig. 22.122

Statement-2

The resistance of a wire is inversely proportional to the square of its radius.

5. Statement-1

In the potentiometer circuit shown in Fig. 22.123, E_1 and E_2 are the emfs of cells C_1 and C_2 respectively with $E_1 > E_2$. Cell C_1 has negligible internal resistance. For a given resistor R, the balance length is x. If the diameter of the potentiometer wire AB is increased, the balance length x will decrease.

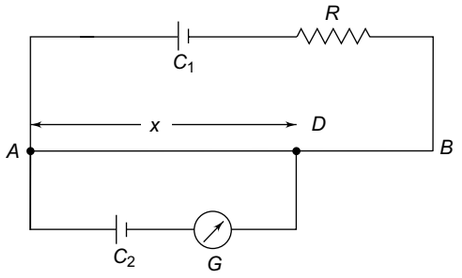


Fig. 22.123

Statement-2

At the balance point, the potential difference between AD due to cell $C_1 = E_2$, the emf of cell C_2.

6. Statement-1

In the potentiometer circuit shown in Q.5 above, the wire AB is not changed but the value of resistor R is decreased. Then the balance length x will decrease.

Statement-2

At the balance point, the potential difference between A and D due to cell C_1 = emf E_2 of cell C_2.

7. Statement-1

Electrons in a metallic conductor have no motion if no potential difference is applied across it.

Statement-2

The potential difference gives rise to electric field which exerts a force on the electrons.

IIT, 1982

8. Statement-1

The resistivity of a semiconductor decreases with increase of termperature.

Statement-2

In a conducting solid, the rate of collisions between free electrons and ions increases with increase of temperature.

IIT, 1993

9. Statement-1

In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

Statement-2

Resistance of a metal increases with increase in temperature.

IIT, 2008

SOLUTIONS

1. The correct choice is (c). The electrons suffer a large number of collisions with the positive ions of the conductor. Although the electric field accelerates an electron between two collisions, it is decelerated by collision. The net acceleration averages out to zero and the electron acquires a constant average speed. The gain in speed between collisions is lost in the next collision.
2. The correct choice is (b). The current in a metal depends not only on the charge of an electron and its drift speed but also on the number of free electrons per unit volume in a metal which is of the order of 10^{29} per m^3.
3. The correct choice is (c). A wire carrying a current stays neutral because as many electrons enter one end of the wire as leave it from the other end. Since there is no net charge on the wire, there is no electric field around it.
4. The correct choice is (d). The condition for no deflection of the galvanometer is

$$\frac{R_1}{R_2} = \frac{R_{AC}}{R_{CB}}$$

where R_{AC} and R_{CB} are the resistances of the bridge wire of length AC and CB respectively. If the radius of the wire AB is doubled, the ratio R_{AC}/R_{CB} will remain unchanged. Hence the balance length will remain the same.

5. The correct choice is (d). If the diameter of wire AB is increased, its resistance will decrease. Hence The potential difference between A and B due to cell C_1 will decrease. Therefore, the null point will be obtained at a higher value of x.
6. The correct choice is (a). If the value of R is decreased, the potential difference between A and B due to cell C_1 will increase. Hence the balance length will be smaller than x.
7. Electrons in a conductor have random thermal motion. Statement-1 is false and Statement-2 is true.
8. Both the statements are true and Statement-2 is the correct explanation for Statement-1.
9. Refer to Fig. 22.124 where R is the standard resistance and X is the unknown resistance.

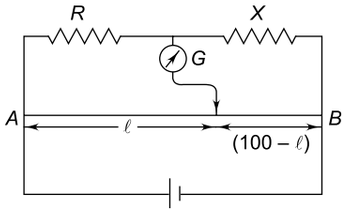


Fig. 22.124

$$\frac{X}{R} = \frac{(100 - l)}{l}$$

The value of X increases with increase in temperature. Hence to keep the value of l the same, the value of R must be increased. So Statement-1 is false, Statement-2 is true.

SECTION V

Integer Answer Type Questions

1. In the circuit shown in Fig. 22.125, each cell has an emf of 5 V and an internal resistance of 0.2 Ω. Find the reading of the ideal voltmeter V in volts.

IIT, 1997

2. A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to

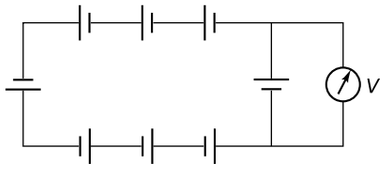


Fig. 22.125

the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross section but the length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. Find the value of N.

IIT, 2001

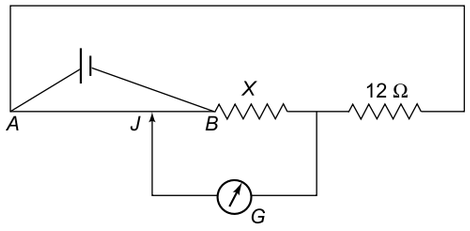


Fig. 22.126

3. A uniform wire AB of length 1 m, an unknown resistance X and a resistance of 12 Ω are connected to a battery and a galvanometer G as shown in Fig. 22.126. The galvanometer shows no deflection when $AJ = 60$ cm. Find the value of X (in ohm).

IIT, 2002

4. Two batteries of different emfs and different internal resistances are connected as shown in Fig. 22.127. The voltage across AB in volts is

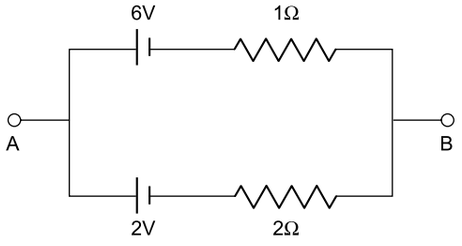


Fig. 22.127

IIT, 2011

SOLUTIONS

1. All the cells are in series. Therefore, total emf = 40 V and total internal resistance = 1.6 Ω. Therefore, current in the circuit is

$$I = \frac{40}{1.6} = 25 \text{ A}$$

Voltmeter reading = terminal voltage of the cell across which it is connected = $E - Ir = 5 - 25 \times 0.2 = 5 - 5 = 0$ volt.

2. The heat energy generated in time t is $Q = V^2t/R$, where V is the terminal voltage of a single cell. Here $Q = ms\Delta T$ where m is the mass of the wire, s its specific heat and R its resistance.
In the first case:

$$ms\Delta T = \frac{(3V)^2 t}{R} \qquad (1)$$

Now $m \propto$ length and $R \propto$ length. Hence, in the second case,

$$(2m)\, s\Delta T = \frac{(NV)^2 t}{(2R)} \qquad (2)$$

Dividing (2) by (1), we have

$$2 = \frac{N^2/2}{9} \text{ or } N^2 = 36 \text{ or } N = 6,$$

3. In the balanced condition

$$\frac{X}{12} = \frac{BJ}{AJ} = \frac{40}{60} \Rightarrow X = 8\ \Omega$$

4. Refer to Fig. 22.128.

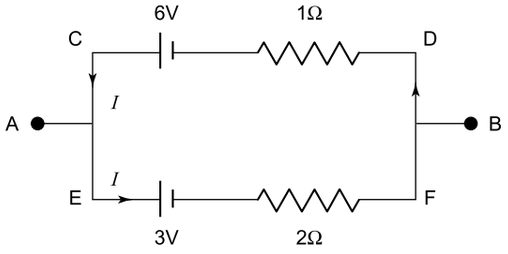


Fig. 22.128

Applying Kirchhoff's loop rule to $CDFEC$

$$6 - 1 \times I - 2 \times I - 3 = 0$$

$$\Rightarrow \qquad I = 1 \text{ A}$$

Since the current is anticlockwise,

$$V_A - 6 + 1 \times I - V_B = 0$$

$$\Rightarrow \qquad V_A - V_B = 6 - 1 \times 1 = 5 \text{ V}$$

23 Chapter

Heating Effect of Current

REVIEW OF BASIC CONCEPTS

23.1 HEATING EFFECT OF CURRENT

If a current I amperes flows for time t seconds through a resistor R ohms across which a potential difference V volts is maintained, the amount of heat energy H (in joules) delivered is given by

$$H = VIt = I^2 Rt = \frac{V^2 t}{R}$$

To obtain heat is calories, H is divided by $J = 4.2$ joules per calorie is called the mechanical equivalent of heat.

Electrical Power If E is the emf of a source of internal resistance r, the power delivered to an external circuit of resistance R (called the output power) is given by

$$P_{out} = VI = I^2 R = \frac{V^2}{R} = \frac{E^2 R}{(R+r)^2}$$

$[\because E = I(R + r)]$

Electrical Energy The SI unit of energy is joule. *A* practical unit for electrical energy is called kilowatt hour (kWh). Since watt = joule per second, watt second is the same as joule. Hence

$$1 \text{ watt hour} = 1 \text{ watt} \times (60 \times 60) \text{ seconds}$$
$$= 3600 \text{ watt seconds}$$
$$= 3600 \text{ joules}$$

$\therefore$ $1 \text{ kilowatt hour} = 1000 \text{ watt hours}$
$$= 1000 \times 3600 \text{ joules}$$
$$= 3.6 \times 10^6 \text{ joules}$$

Note that watt and kilowatt are units of electrical power and watt hour and kilowatt hour are units of electrical energy.

23.2 POWER-VOLTAGE RATING OF ELECTRICAL APPLIANCES

Every electrical appliance has a specified power-voltage ($P - V$) rating which determines the resistance of the appliance and the current it will draw. Since $P = VI$, the current the appliance will draw is given by

$$I = \frac{P}{V}$$

The electrical wiring should be able to withstand this current. The resistance of the appliance is given by

$$R = \frac{V}{I} = \frac{V}{P/V} = \frac{V^2}{P}$$

1. *Power of Electrical Appliances Connected in Parallel*

Let R_1, R_2, R_3, ... be the resistances of the electrical appliances meant to operate at the same voltage V and let P_1, P_2, P_3, ... be their respective electrical powers. Then

$$R_1 = \frac{V^2}{P_1}, R_2 = \frac{V^2}{P_2}, R_3 = \frac{V^2}{P_3}, \ldots$$

or
$$P_1 = \frac{V^2}{R_1}, P_2 = \frac{V^2}{R_2}, P_3 = \frac{V^2}{R_3}, \ldots$$

When the appliances are connected in parallel, their combined resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

$\therefore$ Total power consumed is

$$P = \frac{V^2}{R} = V^2 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \right)$$

$$= \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3} + \ldots$$

or
$$P = P_1 + P_2 + P_3 + \ldots$$

2. *Power of Electrical Appliances Connected in Series*

The total resistance R when the appliances are connected in series is given by

$$R = R_1 + R_2 + R_3 + ...$$

$\therefore$ Total power consumed is

$$P = I^2R = I^2 (R_1 + R_2 + R_3 + ...)$$

$$\Rightarrow \quad P = P_1 + P_2 + P_3 + ...$$

EXAMPLE 23.1

A 1 kW heater is designed to operate on a 200 volt supply. Find (a) the resistance of the heater and (b) by what percentage will its power drop if the supply voltage drops to 160 V?

SOLUTION

(a) P = 1 kW = 1000 W, V = 200 volt. The resistance of the heater is

$$R = \frac{V^2}{P} = \frac{200 \times 200}{1000} = 40\ \Omega$$

(b) Since R remains unchange, the power of the heater when the voltage drops to V' = 160 V will be

$$P' = \frac{V'^2}{R} = \frac{160 \times 160}{40} = 640\ \text{W}$$

$$\therefore \text{ Percentage drop in power} = \frac{P - P'}{P} \times 100$$

$$= \left(\frac{1000 - 640}{1000}\right) \times 100 = 36\%$$

EXAMPLE 23.2

An electric bulb has a rating of 40 W, 200 V. Can the bulb be safely used across of 300 V supply? If not, what will you do so that it can glow with normal brightness when it is connected across 300 V supply?

SOLUTION

Given P = 40 W, V = 200 volt. The resistance of the bulb is

$$R = \frac{V^2}{R} = \frac{200 \times 200}{40} = 1000\ \Omega$$

If this bulb is connected to a 300 V supply, the power dissipated is

$$P_1 = \frac{V_1^2}{R} = \frac{300 \times 300}{1000} = 90\ \text{W}$$

Since $P_1 >> P$, the bulb will get damaged because now it glows much more brightly and hence cannot be safely used across a 300 V supply. It will glow with normal brightness if it dissipated 40 W. This will happen if the voltage across it is 200 V. To make it glow with normal brightness when connected to 300 V supply, a resistance R_1 must be connected in series with it as shown in Fig. 23.1 so that the voltage drop across R is 200 V and across R_1 is 100 V. The current in the circuit is

$$I = \frac{V}{R + R_1} = \frac{300}{1000 + R_1}$$

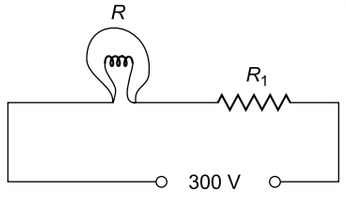


Fig. 23.1

$\therefore$ Voltage drop across $R_1 = IR_1 = \frac{300 R_1}{(1000 + R_1)}$ which must be equal to 100 V, i.e.

$$\frac{300 R_1}{(1000 + R_1)} = 100 \Rightarrow R_1 = 500\ \Omega$$

EXAMPLE 23.3

Two bulbs A and B are rated 40 W, 200 V and 60 W, 200 V respectively. Which bulb will glow more brightly if

(a) the two bulbs are connected in series and this combination is connected across a 200 V supply?

(b) the two bulbs are connected in parallel and this combination is connected across a 200 V supply?

SOLUTION

Resistance of bulb A is $R_A = \frac{V^2}{P_A}$

$$= \frac{200 \times 200}{40} = 1000\ \Omega$$

Resistance of bulb B is $R_B = \frac{V^2}{P_B}$

$$= \frac{200 \times 200}{60} = 666.7\ \Omega$$

(a) If the two bulbs are connected in series, the current flowing to each will be the same. Let the current be I.

Power dissipated in A is $P'_A = I^2 R_A$

Power dissipated in B is $P'_B = I^2 R_B$

$$\therefore \quad \frac{P'_A}{P'_B} = \frac{I^2 R_A}{I^2 R_B} = \frac{R_A}{R_B}$$

Since $R_A > R_B$; $P'_A > P'_B$. Hence bulb A will glow more brightly than bulb B.

(b) If the two bulbs are connected in parallel, the potential difference across each bulb will be the same = supply voltage V.

Power dissipated in A is $P_A'' = \dfrac{V^2}{R_A}$

Power dissipated in B is $P_B'' = \dfrac{V^2}{R_B}$

$$\therefore \quad \frac{P_A''}{P_B''} = \frac{R_B}{R_A}$$

Since $R_B < R_A$; $P_A'' < P_B''$. Hence bulb B will glow more brightly than bulb A.

EXAMPLE 23.4

An electric cable having a resistance of 0.2 Ω delivers 10 kW of power at 200 V to a factory. Find the efficiency of transmission.

SOLUTION

$R = 0.2\ \Omega$, $P = 10\ \text{kW} = 10 \times 10^3\ \text{W}$, $V = 200$ volt

Current in the cable is $I = \dfrac{P}{V} = \dfrac{10 \times 10^3}{200} = 50\ \text{A}$

Power loss in the cable $= I^2R = (50)^2 \times 0.2$

$= 500\ \text{W} = 0.5\ \text{kW}$

$\therefore$ Total power supplied by source = 10 kW + 0.5 kW = 10.5 kW

$\therefore$ Efficiency $= \dfrac{\text{Power delivered to factory}}{\text{Power supplied by source}}$

$= \dfrac{10\ \text{kW}}{10.5\ \text{kW}} = 0.95$ or 95%

EXAMPLE 23.5

100 MW of power from a power station is transmitted to a distant substation through a cable of resistance 10 Ω at (a) 20,000 V and (b) 200 V. Find the power loss in the cable in each case.

SOLUTION

$P = 100\ \text{MW} = 100 \times 10^6\ \text{W} = 10^8\ \text{W}$, $R = 10\ \Omega$

(a) $V_1 = 20{,}000$ volt. Current in the cable is

$$I_1 = \frac{P}{V_1} = \frac{10^8}{20{,}000} = 5000\ \text{A}$$

Power loss in cable is $P_1 = I_1^2R$

$= (5000)^2 \times 10 = 2.5 \times 10^8\ \text{W}$

(b) $V_2 = 200$ volt. Current in the cable is

$$I_2 = \frac{P}{V_2} = \frac{10^8}{200} = 5 \times 10^5\ \text{A}$$

Power loss in cable is $P_2 = I_2^2R$

$= (5 \times 10^5)^2 \times 10 = 2.5 \times 10^{12}\ \text{W}$

Notice that $P_1 << P_2$. It is for this reason that electrical power from a generating power station is transmitted at a very high voltage of about 30,000 V to a distant substation and not at a low voltage of 200 V. At low voltage the power loss is very high.

EXAMPLE 23.6

A battery of emf E and internal resistance r is connected to an external resistance R as shown in Fig. 23.2, show that the power drawn from the battery is maximum when $R = r$

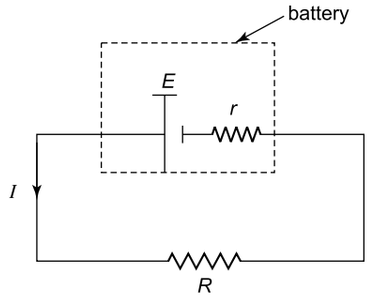


Fig. 23.2

SOLUTION

The current in the circuit is

$$I = \frac{E}{(R+r)}$$

The power dissipated in R is

$$P = I^2R = \left(\frac{E}{R+r}\right)^2 R = \frac{E^2R}{(R+r)^2} \qquad \text{(i)}$$

P is maximum if $\dfrac{dP}{dR} = 0$ and $\dfrac{d^2P}{dR^2} < 0$. Differentiating (i) with respect to R we have

$$\frac{dP}{dR} = E^2\left[\frac{1}{(R+r)^2} - 2R \times (R+r)^{-3}\right]$$

$$= \frac{E^2}{(R+r)^3}[(R+r) - 2R]$$

$$\frac{dP}{dR} = 0 \text{ if } (R+r) - 2R = 0$$

$\Rightarrow \qquad R = r.$

It is easy to check that $\dfrac{d^2P}{dR^2} < 0$ when $R = r$. Hence the power drawn form the battery is maximum when the external resistance is equal to the internal resistance of the battery.

23.3 VARIATION OF RESISTANCE AND RESISTIVITY WITH TEMPERATURE

The resistance of a conductor (and hence its resistivity) depends upon its temperature. For metallic conductors, the resistance increases with temperature. For small changes in temperature, we have

$$R_2 = R_1[1 + \alpha(t_2 - t_1)]$$

where R_1 is the resistance at a temperature t_1, R_2 is the resistance at temperature t_2 and α is the temperature coefficient of resistance. The variation of resistivity with temperature is given by

$$\rho_2 = \rho_1[1 + \alpha(t_2 - t_1)]$$

For metallic conductors, the temperature coefficient of resistance is positive. Some materials, such as carbon and semi-conductors have a negative temperature coefficient of resistance. The resistance of such materials decreases with increase in temperature.

EXAMPLE 23.7

The resistance of a heating element is 100 Ω at 27°C. What is the temperature of the element if its resistance is 117 Ω. Temperature coefficient of resistance $= 1.7 \times 10^{-4}\ \text{K}^{-1}$.

SOLUTION

$$R_2 = R_1[1 + \alpha(t_2 - t_1)]$$

$$\Rightarrow t_2 - t_1 = \frac{R_2 - R_1}{\alpha R_1} = \frac{117 - 100}{(1.7 \times 10^{-4}) \times 100} = 1000°\text{C}$$

$$\therefore \quad t_2 = 1000 + t_1 = 1000 + 27 = 1027°\text{C}$$

EXAMPLE 23.8

The resistance of a wire is 3.00 Ω at 0°C and 3.75 Ω at 100°C. Its resistance is measured to be 3.15 Ω at room temperature. Find the room temperature.

SOLUTION

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{3.75 - 3.00}{3.00(100 - 0)} = 0.0025\ \text{K}^{-1}$$

$$R_t = R_0(1 + \alpha t) \Rightarrow \frac{R_t - R_0}{R_0} = \alpha t$$

$$\Rightarrow \quad t = \frac{R_t - R_0}{\alpha R_0} = \frac{3.15 - 3.00}{0.0025 \times 3.00} = 20°\text{C}$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. When a cell is connected to a resistance R_1, the rate at which heat is generated in it is the same as when the cell is connected to a resistance $R_2 (< R_1)$. The internal resistance of the cell is

(a) $(R_1 - R_2)$ (b) $\frac{1}{2}(R_1 - R_2)$

(c) $\frac{R_1 R_2}{(R_1 + R_2)}$ (d) $\sqrt{R_1 R_2}$

2. In the circuit shown in Fig. 23.3 (a), the resistor which dissipates the maximum power has a resistance

(a) 1 Ω (b) 2 Ω

(c) 3 Ω (d) 4 Ω

3. In the circuit shown in Fig. 23.3 (b) the ratio of the power dissipated in 1 Ω resistor to that dissipated in 2 Ω resistor is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{8}{9}$

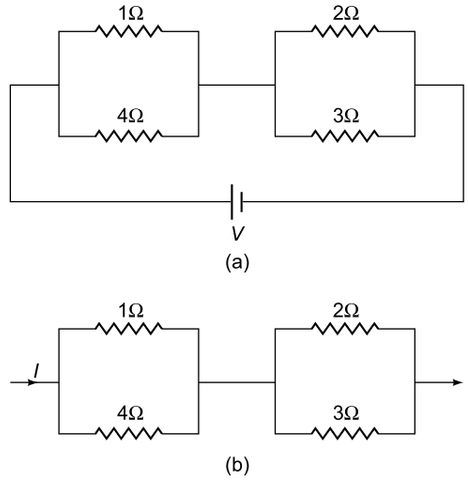


Fig. 23.3

4. A metre bridge wire is connected across a d.c. source of voltage V. When the steady state is reached, the temperature of wire is T and the rate of loss of heat from the wire is $P = k(T - T_0)$ where k is a constant and T_0 is the temperature of the room. If R is the resistance of the bridge wire at temperature T, α its coefficient of linear expansion and Y its Young's modulus, the stress developed in the wire is given by

(a) $\frac{\alpha k Y}{RV^2}$ (b) $\frac{kV^2Y}{\alpha R}$

(c) $\frac{\alpha V^2 Y}{k R}$ (d) $\frac{kYR}{\alpha V^2}$

5. A galvanometer of resistance G is converted into an ammeter by using a shunt of resistance S. When a current is passed through the ammeter for a certain time, the rate of heat dissipated through G and through S is 2/3. Then

(a) $S = \frac{1}{3}\ G$ (b) $S = \frac{3}{4}\ G$

(c) $S = \frac{3}{2}\ G$ (d) $S = \frac{2}{3}\ G$

6. A 12 V, 24 W filament bulb is supplied current from n cells connected in series. Each cell has an emf 1.5 V and internal resistance 0.25 Ω. What is the value of n so that the bulb runs at its rated power?

(a) 6 (b) 8

(c) 12 (d) 16

7. An electric bulb has a rating 500 W, 100 V. It is used in a circuit having at 200 V supply. What resistance must be connected in series with the bulb so that it delivers 500 W?

(a) 10 Ω (b) 20 Ω

(c) 30 Ω (d) 40 Ω

IIT, 1995

8. An electric motor working on a 100 V dc supply draws a current of 15 A. If the efficiency of the motor is 40%, the resistance of the windings of the motor is

(a) 4 Ω (b) 8 Ω

(c) 16 Ω (d) 32 Ω

9. In Q.8, the value of the back emf is

(a) 20 V (b) 40 V

(c) 60 V (d) 80 V

10. If electric power from 100 MW power station is transmitted to a substation at 20,000 V, the power loss during transmission is P_1. If the same power is transmitted at 200 V, the power loss is P_2. The ratio P_2/P_1 will be

(a) 10^4 (b) 10^3

(c) 10^2 (d) 10

11. The current through a bulb is increased by 1%. Assuming that the resistance of the filament remains unchanged, the power of the bulb (i.e. its wattage) will

(a) increase by 1% (b) decrease by 1%

(c) increase by 2% (d) decrease by 2%

12. The voltage across a bulb is decreased by 2%. Assuming that the resistance of the filament remains unchanged, the power of the bulb will

(a) decrease by 2% (b) increase by 2%

(c) decrease by 4% (d) increase by 4%

13. The current at which a fuse wire melts does not depend on

(a) cross-sectional area (b) length

(c) resistivity (d) density

14. A fuse wire should have

(a) high resistivity and high melting point

(b) low resistivity and high melting point

(c) high resistivity and low melting point

(d) low resistivity and low melting point.

15. Three equal resistors, connected in series with a battery, dissipate P watts of power. What will be the power dissipated if the same resistances are connected in parallel across the same battery?

(a) P (b) $3\ P$

(c) $9\ P$ (d) $27\ P$

16. Two electrical devices of power rating 1 kW and 2 kW are connected (a) in series and (b) in parallel. The ratio of the power ratings of the combination in case (a) to that in case (b) is

(a) 1 : 4 (b) 1 : 3

(c) 1 : 2 (d) 1 : 1

17. Two 500 W heaters when connected in series across a source of constant voltage V supply Q joules of heat in a certain time t. When the heaters are connected in parallel across the same source, the amount of heat energy supplied in the same time t will be

(a) $\frac{Q}{4}$ (b) $\frac{Q}{2}$

(c) $2\ Q$ (d) $4\ Q$

18. A battery of emf E and internal resistance r is connected to a resistor of resistance r_1 and Q joules of heat is produced in a certain time t. When the same battery is connected to another resistor of

resistance r_2, the same quantity of heat is produced in the same time t, the value of r is

(a) $\frac{r_1^2}{r_2}$ (b) $\frac{r_2^2}{r_1}$

(c) $\frac{1}{2}(r_1 + r_2)$ (d) $\sqrt{r_1 r_2}$

19. A heater boils a certain quantity of water in time t_1. Another heater boils the same quantity of water in time t_2. If both heaters are connected in series, the combination will boil the same quantity of water in time

(a) $\frac{1}{2}(t_1 + t_2)$ (b) $(t_1 + t_2)$

(c) $\frac{t_1 t_2}{(t_1 + t_2)}$ (d) $\sqrt{t_1 t_2}$

20. A heater boils a certain quantity of water in time t_1. Another heater boils the same quantity of water in time t_2. If both heaters are connected in parallel, the combination will boil the same quantity of water in time.

(a) $\frac{1}{2}(t_1 + t_2)$ (b) $(t_1 + t_2)$

(c) $\frac{t_1 t_2}{(t_1 + t_2)}$ (d) $\sqrt{t_1 t_2}$

21. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 300 K its resistance is 1 Ω. The resistance of the wire will be 2 Ω at

(a) 1154 K (b) 1100 K

(c) 1400 K (d) 1127 K

22. If a wire of resistance 20 Ω is covered with ice and a voltage of 210 V is applied across the wire, then the rate of melting of ice will be

(a) 0.85 g/s (b) 1.92 g/s

(c) 6.56 g/s (d) none of these

23. A heating coil is labelled 100 W, 200 V. The coil is cut in half and the two pieces are joined in parallel to the same source. The energy now liberated per second is

(a) 200 Js^{-1} (b) 400 Js^{-1}

(c) 25 Js^{-1} (d) 50 Js^{-1}

24. Two identical batteries, each of emf 2V and internal resistance 1 Ω are used to produce heat in a resistance R = 0.5 Ω by using the circuit shown in Fig. 23.4. The maximum power that can be developed across R is

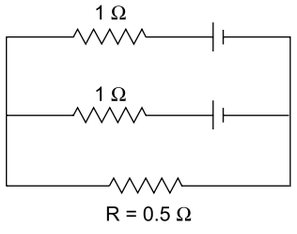


Fig. 23.4

(a) 1.28 W (b) 2.0 W

(c) $\frac{8}{9}$ W (d) 3.2 W

25. If a cell of a constant emf produces the same amount of heat during the same time in two independent resistances R_1 and R_2 when connected separately, one after the other, across the cell, then the internal resistance of the cell is

(a) $\frac{1}{2}(R_1 + R_2)$ (b) $\frac{1}{2}(R_1 - R_2)$

(c) $\frac{1}{2}(R_1 R_2)^{1/2}$ (d) $(R_1 R_2)^{1/2}$

26. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if

(a) both the length and radius of the wire are halved

(b) both the length and radius of the wire are doubled

(c) the radius of the wire is doubled

(d) the length of the wire is doubled

IIT, 1980

27. In the circuit shown in Fig. 23.5, the heat produced in the 5 Ω resistor due to a current flowing in it is 10 calories per second. The heat produced in the 4 Ω resistor is

(a) 1 cal s^{-1} (b) 2 cal s^{-1}

(c) 3 cal s^{-1} (d) 4 cal s^{-1}

IIT, 1985

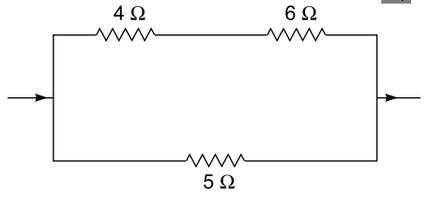


Fig. 23.5

28. A battery of internal resistance 4 Ω is connected to the network of resistances as shown in Fig. 23.6. In order that maximum power can be delivered to the network, the value of R in ohm should be

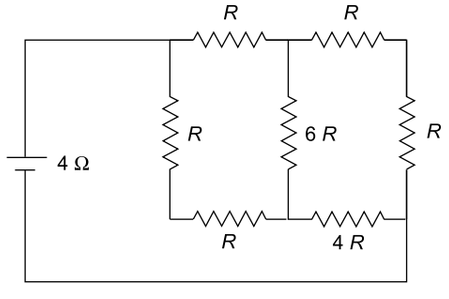


Fig. 23.6

(a) $\frac{4}{9}$ (b) 2

(c) $\frac{8}{3}$ (d) 18

IIT, 1995

29. The filament of an electric heater should have
(a) high resistivity and high melting point
(b) low resistivity and high melting point
(c) high resistivity and low melting point
(d) low resistivity and low melting point.

30. A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected, in combination with a resistance of 10 Ω and a resistance R to a 100 V line as shown in Fig. 23.7. What should be the value of R so that the heater operates with a power of 62.5 W?

(a) 10 Ω (b) 62.5 Ω

(c) $\frac{1}{5}$ Ω (d) 5 Ω

IIT, 1978

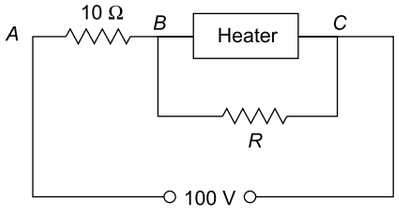


Fig. 23.7

31. A piece of copper and another of germanium are cooled from room temperature to 40 K. The resistance of
(a) each of them decreases
(b) each of them increases
(c) copper increases and of germanium decreases
(d) copper decreases and of germanium increases

IIT, 1990

32. A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross section but the length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is

(a) 4 (b) 6

(c) 8 (d) 9

IIT, 2001

33. A 100 W bulb B_1, and two 60 W bulbs B_2 and B_3, are connected to a 250 V source, as shown in Fig. 23.8. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1, B_2 and B_3, respectively. Then

(a) $W_1 > W_2 = W_3$ (b) $W_1 > W_2 > W_3$

(c) $W_1 < W_2 = W_3$ (d) $W_1 < W_2 < W_3$

IIT, 2002

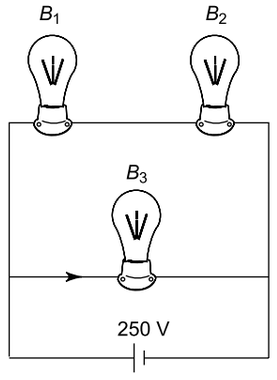


Fig. 23.8

34. Three resistances of equal value are connected in four different combinations as shown in Fig. 23.9. Arrange them in increasing order of power dissipation.

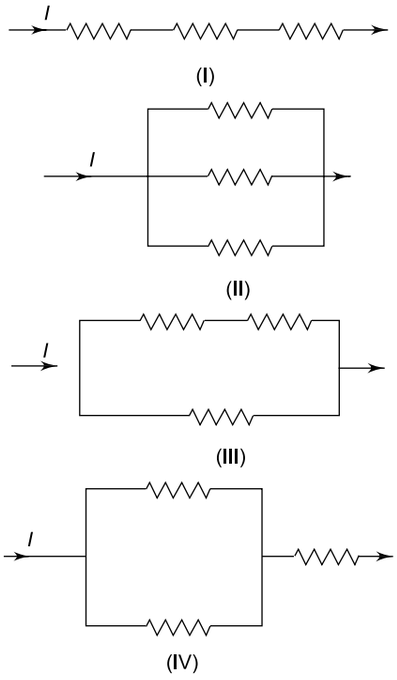


Fig. 23.9

(a) III < II < IV < I (b) II < III < IV < I

(c) I < IV < III < II (d) I < III < II < IV

IIT, 2003

35. A 220 V-100 W bulb is connected to a 110 V source. The power consumed by the bulb will be
(a) 200 W (b) 100 W
(c) 25 W (d) zero

36. The maximum power rating of a 20 Ω resistor is 1.0 kW. The resistor will melt if it is connected across a d.c. source of voltage.
(a) 160 V (b) 140 V
(c) 120 V (d) 100 V

37. The walls of a closed cubical box of edge 50 cm are made of a material of thickness 1 mm and thermal conductivity 1.68×10^{-1} W m^{-1} K^{-1}. The interior of the box is maintained at 100°C above the outside temperature by a heater placed inside the box and connected to a 420 V d.c. source. What must be the resistance of the heater element?
(a) 5 Ω (b) 6 Ω
(c) 7 Ω (d) 8 Ω

38. A constant voltage is applied across a uniform wire, which produces some heat in the wire. If a second wire of the same material but of length twice that of the first wire is used, the heat produced in the second wire will the same as in the first wire if the area of cross-section of the second wire is
(a) increased by a factor of four
(b) decreased by a factor of four
(c) increased by a factor of two
(d) decreased by a factor of two

39. Two heater coils *A* and *B* made of the same material are connected in parallel across the mains. The length and diameter of *A* are twice those of *B*. In a given time, the ratio of heat produced in *A* to that produced in *B* is
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 2 (d) 4

40. Two bulbs *A* and *B* of ratings 40 W - 200 V and 100 W - 200 V respectively are connected in series to a 400V d.c. supply. Then
(a) both bulbs will fuse.
(b) bulb A will fuse but bulb *B* will not
(c) bulb B will fuse but bulb *A* will not
(d) both bulbs will not fuse.

41. If the two bulbs of Q.40 above are connected in parallel to a 400 V d.c. supply, then
(a) both bulbs will fuse
(b) no bulb will fuse
(c) bulb *A* will fuse but bulb *B* will not
(d) bulb *B* will fuse but bulb *A* will not

42. Two electric bulbs, each designed to operate with a power of 500 W in a 200 V supply line, are connected in series to a 100 V supply line. The power generated in each bulb is
(a) 31.0 W (b) 31.25 W
(c) 31.5 W (d) 31.75 W

43. Two wires *AB* and *BC*, each of length *L*/2 are made of the same material. The radius of wire *AB* is 2*r* and of wire *BC* is *r*. The current *I* flows through the composite wire (see Fig. 23.10). Choose the correct statement from the following.
(a) Potential difference across *BC* is twice that across *AB*.
(b) Power dissipated in *BC* is four times the power dissipated in *AB*.
(c) Current densities in *AB* and *BC* are equal.
(d) Electric fields in *AB* and *BC* are equal.

IIT, 2006

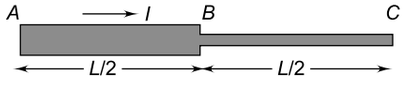


Fig. 23.10

44. In the circuit shown in Fig. 23.11, the resistors of 2 Ω and 3 Ω together dissipte 30 W of power. The current through the 4 Ω resistor is
(a) 0.5 A (b) 1.0 A
(c) 1.5 A (d) 2.0 A

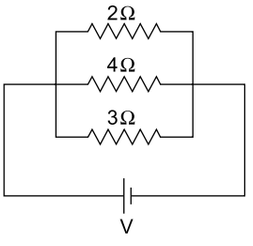


Fig. 23.11

45. In the circuit shown in Fig. 23.12, the current in the 1 Ω resistor is 2 A. The power developed in the 3 Ω resistor is
(a) 3 W (b) 9 W
(c) 27 W (d) 81 W

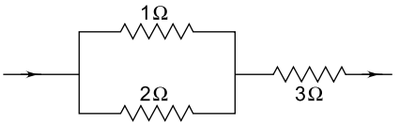


Fig. 23.12

46. Figure 23.13 shows three resistor configurations R_1, R_2 and R_3 connected to 3 V battery. If the power dissipated by the configuration R_1, R_2 and R_3 is P_1, P_2 and P_3, respectively, then

(a) $P_1 > P_2 > P_3$ (b) $P_3 > P_2 > P_1$
(c) $P_2 > P_1 > P_3$ (d) $P_1 = P_2 = P_3$

IIT, 2008

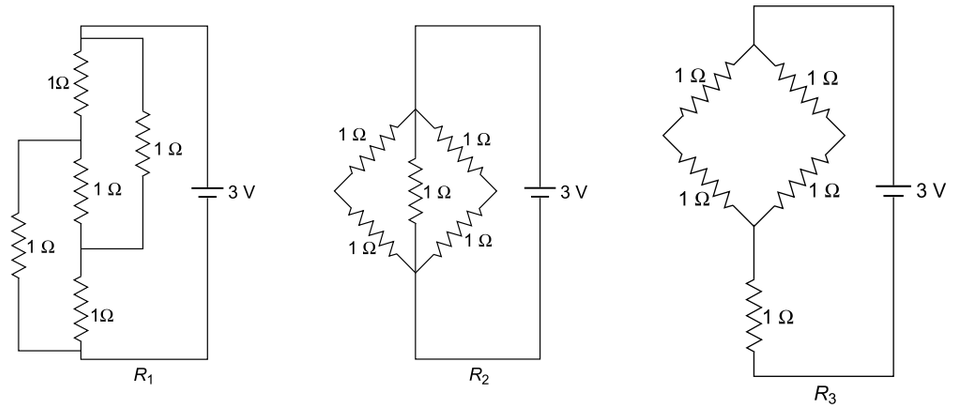


Fig. 23.13

ANSWERS

1. (d)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (c)
7. (b)	**8.** (a)	**9.** (b)	**10.** (a)	**11.** (c)	**12.** (c)
13. (b)	**14.** (c)	**15.** (c)	**16.** (d)	**17.** (d)	**18.** (d)
19. (b)	**20.** (c)	**21.** (d)	**22.** (c)	**23.** (b)	**24.** (b)
25. (d)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (d)
31. (d)	**32.** (b)	**33.** (d)	**34.** (b)	**35.** (c)	**36.** (a)
37. (c)	**38.** (c)	**39.** (b)	**40.** (b)	**41.** (a)	**42.** (b)
43. (b)	**44.** (c)	**45.** (c)	**46.** (c)		

SOLUTIONS

1. If E is the emf and r the internal resistance of the cell, the current through R_1 is

$$I = \frac{E}{(R_1 + r)}$$

The rate at which heat is generated in R_1 is

$$Q_1 = I^2 R_1 = \left[\frac{E}{(R_1 + r)}\right]^2 R_1$$

Similarly, $Q_2 = \left[\frac{E}{(R_2 + r)}\right]^2 R_2$

Equating Q_1 and Q_2 and simplifying, we get

$$r = \sqrt{R_1 R_2}\,.$$

2. Refer to Fig. 23.14 (a). The parallel combination of 1 Ω and 4 Ω has a resistance $\frac{4}{5}$ Ω and the parallel combination of 2 Ω and 3 Ω has a resistance $\frac{6}{5}$ Ω.

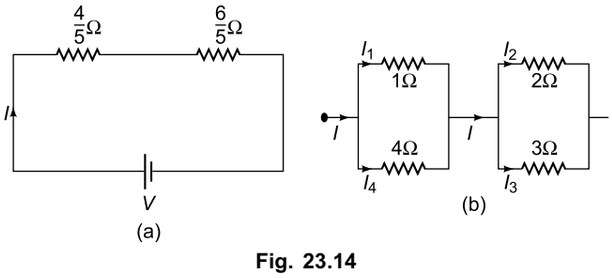


Fig. 23.14

Total resistance = $\frac{4}{5} + \frac{6}{5} = 2$ Ω. The current in the circuit is

$$I = \frac{V}{2}$$

$\therefore$ Potential drop across $\frac{4}{5}$ Ω $= \frac{V}{2} \times \frac{4}{5} = \frac{2V}{5}$. Hence p.d. across 1 Ω is $V_1 = \frac{2V}{5}$ and p.d. across 4 Ω is

$V_4 = \frac{2V}{5}$. Similarly p.d. across 2 Ω is $V_2 = \frac{3V}{5}$ and p.d. across 3 Ω is $V_3 = \frac{3V}{5}$.

The powers dissipated are

$$P_1 = \frac{V_1^2}{R_1} = \frac{4V^2}{25}$$

$$P_2 = \frac{V_2^2}{R_2} = \frac{9V^2}{25 \times 2} = \frac{4.5\,V^2}{25}$$

$$P_3 = \frac{V_3^2}{R_3} = \frac{9V^2}{25 \times 3} = \frac{3V^2}{25}$$

$$P_4 = \frac{V_4^2}{R_4} = \frac{4V^2}{25 \times 4} = \frac{V^2}{25}$$

Power P_2 is the maximum. Hence the correct choice is (b).

3. Refer to Fig. 23.14 (b). The currents is various branches are

$$I_1 = \frac{4I}{5}, \quad I_4 = \frac{I}{5}$$

$$I_2 = \frac{3I}{5}, \; I_3 = \frac{2I}{5}$$

$$\therefore \quad P_1 = I_1^2 R_1 = \frac{16I^2}{25} \times 1 = \frac{16I^2}{25}$$

$$P_2 = I_2^2 R_2 = \frac{9I^2}{25} \times 2 = \frac{18I^2}{25}$$

$$P_3 = I_3^2 R_3 = \frac{4I^2}{25} \times 3 = \frac{12I^2}{25}$$

$$P_4 = I_4^2 R_4 = \frac{I^2}{25} \times 4 = \frac{4I^2}{25}$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{8}{9}, \text{ which is choice (d).}$$

4. In the steady state, rate at which heat is generated in the wire = rate at which heat is lost to the surroundings. Hence

$$\frac{V^2}{R} = k(T - T_0) = k\Delta T$$

$$\Delta L = \alpha L \Delta T \Rightarrow \Delta T = \frac{\Delta L}{\alpha L}$$

$$\therefore \quad \frac{V^2}{R} = \frac{k\Delta L}{\alpha L} = \frac{k\varepsilon}{\alpha} \qquad (1)$$

where $\varepsilon = \frac{\Delta L}{L}$ is the strain. Now

$$Y = \frac{S}{\varepsilon}, \text{ where } S = \text{stress}$$

$$\Rightarrow \quad \varepsilon = \frac{S}{Y} \qquad (2)$$

Using (2) in (1) we get

$$S = \frac{\alpha V^2 Y}{k R}, \text{ which is choice (c).}$$

5. Since resistances G and S are connected in parallel, the potential difference across G = potential difference across $S = V$ (say). Heat dissipated in time t is

$$H = \frac{V^2 t}{R}. \text{ Hence } H \propto \frac{1}{R}$$

$$\therefore \quad \frac{H_G}{H_S} = \frac{S}{G}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{S}{G} \Rightarrow S = \frac{2}{3} G$$

6. $W = V \times I$. Therefore, current $I = W/V = 24/12 = 2$ A. Hence the terminal voltage of each cell $= 1.5 - 0.25 \times 2 = 1.0$ V. The resistance of the bulb is $R = 12/2 = 6$ Ω. If n cells are used, then

$$\frac{n \times 1.0}{6} = 2 \Rightarrow n = 12$$

7. The current flowing through the bulb when operated at 100 V is

$$I = \frac{500}{100} = 5 \text{ A}$$

Resistance of bulb is $R = \frac{100}{5} = 20\ \Omega$

To deliver 500 W at 200 V, the current must remain 5 A. Hence resistance R to be connected in series with the bulb is given by

$$\frac{200}{R + 20} = 5 \Rightarrow R = 20\ \Omega$$

8. A motor is not a passive resistor; it develops a back emf which is caused by an increase in current when the motor is suddenly started. The total power supplied to the motor is partly converted into mechanical energy and partly lost as heat in the windings of the motor. The total power supplied to the motor is $VI = 100 \times 15 = 1500$ W. Since the efficiency of the motor is 40%, it means that 60% of the total power is lost as heat. Thus the power lost as heat is 60% of 1500 W = 900 W. If R is the resistance of the windings, then $I^2R = 900$

or $$R = \frac{900}{15 \times 15} = 4.0\ \Omega$$

9. Now potential drop across winding is $V' = IR = 15 \times 4.0 = 60$ V. Therefore back emf is $V - V' = 100 - 60 = 40$ V.

10. $P = 100\ \text{MW} = 100 \times 10^6$ W. Let R be the resistance of the cable. At 20,000 V, the current in the cable is

$$I_1 = \frac{100 \times 10^6}{20{,}000} = 5000\ \text{A}$$

$\therefore$ Power dissipated is

$$P_1 = I_1^2 R = (5000)^2 R = 2.5 \times 10^7\ R$$

At 200 V, the current in the cable is

$$I_2 = \frac{100 \times 10^6}{200} = 5 \times 10^5\ \text{A}$$

$\therefore$ Power dissipated is

$$P_2 = I_2^2 R = 2.5 \times 10^{11}\ R$$

$$\therefore \quad \frac{P_2}{P_1} = \frac{2.5 \times 10^{11}}{2.5 \times 10^7} = 10^4$$

Hence the correct choice is (a).

11. Power $P = I^2 R$. Therefore, $\Delta P = 2I\,\Delta I R$. Hence

$$\frac{\Delta P}{P} = \frac{2\Delta I}{I}$$

Now $\frac{\Delta I}{I} = +1\%$. Therefore $\frac{\Delta P}{P} = +2\%$. Hence the correct choice is (c).

12. Power $P = \frac{V^2}{R}$. Therefore $\Delta P = \frac{2V\Delta V}{R}$. Hence

$$\frac{\Delta P}{P} = \frac{2\Delta V}{V}$$

Now $\frac{\Delta V}{V} = -2\%$. Therefore $\frac{\Delta P}{P} = -2 \times 2 = -4\%$, i.e. the power decreases by 4%. Hence the correct choice is (c).

13. The correct choice is (b).

14. The correct choice is (c).

15. Let R be the value of each resistance and V, the voltage of the battery. When the three resistances are connected in series, the power dissipated is

$$P = \frac{V^2}{3R}$$

When they are connected in parallel, the power dissipated is

$$P' = \frac{3V^2}{R}$$

$\therefore$ $P' = 9P$. Hence the correct choice is (c).

16. The power ratings are always added irrespective of whether the resistance are connected in series or in parallel. Hence the correct choice is (d).

17. Let R be the resistance of each heater. When they are connected in series, the resistance of the combination is $2R$. Therefore, heat supplied in time t is

$$Q = \frac{V^2 t}{2R}$$

When they are connected in parallel, the resistance of the combination is $R' = R/2$. Therefore, heat supplied in time t is

$$Q' = \frac{V^2 t}{R'} = \frac{2V^2 t}{R}$$

Thus $Q' = 4Q$. Hence the correct choice is (d).

18. In the first case, the current in the circuit is

$$I_1 = \frac{E}{r_1 + r}$$

$$\therefore \quad Q_1 = I^2 r_1 t = \left(\frac{E}{r_1 + r}\right)^2 \times r_1 t \qquad \text{(i)}$$

In the second case,

$$Q_2 = \left(\frac{E}{r_2 + r}\right)^2 \times r_2 t \qquad \text{(ii)}$$

Equating (i) and (ii) we get

$$\frac{r_1}{(r_1 + r)^2} = \frac{r_2}{(r_2 + r)^2}$$

or $$r_1 (r_2 + r)^2 = r_2 (r_1 + r)^2$$

or $$r_1 (r_2^2 + 2r r_2 + r^2) = r_2 (r_1^2 + 2r r_1 + r^2)$$

or $$r^2 (r_1 - r_2) = r_1 r_2 (r_1 - r_2)$$

or $$r = \sqrt{r_1 r_2}$$

Hence the correct choice is (d).

19. Let R_1 and R_2 be the resistances of the heaters. When they are connected in series, the resistance of the combination is $(R_1 + R_2)$. If V is the voltage of the supply, then, if Q is the heat energy needed to boil water,

$$Q = \frac{V^2 t}{(R_1 + R_2)} = \frac{V^2 t_1}{R_1} = \frac{V^2 t_2}{R_2}$$

$$\therefore \quad t = \frac{Q(R_1 + R_2)}{V^2} = \frac{QR_1}{V^2} + \frac{QR_2}{V^2}$$

$$= t_1 + t_2$$

Hence the correct choice is (b).

20. When the two heaters are connected in parallel, the resistance of the combination is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Now $$\frac{1}{t_1} = \frac{V^2}{QR_1} \text{ and } \frac{1}{t_2} = \frac{V^2}{QR_2}$$

Also $$\frac{1}{t} = \frac{V^2}{Q}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{1}{t_1} + \frac{1}{t_2}$$

or $$t = \frac{t_1 t_2}{(t_1 + t_2)}$$

Hence the correct choice is (c).

21. $R_1 = R_0 (1 + \alpha t_1)$ and $R_2 = R_0(1 + \alpha t_2)$ where t_1 and t_2 are in °C. Thus

$$\frac{R_2}{R_1} = \frac{1+\alpha t_2}{1+\alpha t_1}$$

Here $R_2/R_1 = 2$, $t_1 = 300 - 273 = 27°C$ and $\alpha = 0.00125$ per °C. Using these values, we get $t_2 = 854°C = 854 + 273 = 1127$ K. Hence the correct choice is (d).

22. The rate at which heat is produced in the wire is

$$\frac{Q}{t} = \frac{V^2}{R} = \frac{210 \times 210}{20} = 2205 \text{ Js}^{-1}$$

The latent heat of ice = 80 cal/g = 80 × 4.2 J/g. Hence the rate of melting of ice will be

$$\frac{2205 \text{ Js}^{-1}}{80 \times 4.2 \text{ J/g}} = 6.56 \text{ g/s}, \text{ which is choice (c).}$$

23. Let R be the resistance of the complete coil. When it is cut into two equal pieces, the resistance of each piece becomes $R/2$. If two pieces are joined in parallel, their combined resistance will be

$$R' = \frac{R}{4}$$

$\therefore$ Rate at which energy is liberated will be

$$\frac{V^2}{R'} = \frac{4V^2}{R} = 4 \text{ times the wattage of the complete coil}$$

$$= 4 \times 100 = 400 \text{ Js}^{-1}$$

Hence the correct choice is (b).

24. Applying Kirchhoff's loop rule to the loop consisting of the two batteries, we find the current in the loop is 1 A. Since the two parallel arms of the loop have identical batteries and identical resistances, the current through the $R = 0.05\ \Omega$ resistor is $I = 2$ A. Hence power developed is

$$P = I^2R = (2)^2 \times 0.5 = 2 \text{ W}$$

25. It is given that

$$I_1^2 R_1 t = I_2^2 R_2 t$$

But $$I_1 = \frac{E}{R_1 + r} \text{ and } I_2 = \frac{E}{R_2 + r}.$$ Therefore,

$$\frac{E^2}{(R_1 + r)^2} \times R_1 = \frac{E^2}{(R_2 + r)^2} \times R_2$$

or $$(R_1 + r)^2 \times R_2 = (R_2 + r)^2 \times R_1$$

which gives $r = \sqrt{R_1 \times R_2}$. Hence the correct choice is (d).

26. $H = \frac{V^2 t}{R}$, but $R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$. Hence

$$H = \frac{V^2 t \pi r^2}{\rho l}$$

H is doubled if r and l are both doubled. Hence the correct choice is (b).

27. Let I be the current in the 5 Ω resistor. Then the current in the 4 Ω resistor and 6 Ω resistor will be $I/2$. Therefore, the rates of production of heat in the 5 Ω and 4 Ω resistors respectively are

$$P_1 = I^2 \times 5 \text{ and } P_2 = \left(\frac{I}{2}\right)^2 \times 4 = I^2$$

$\therefore$ $$\frac{P_2}{P_1} = \frac{1}{5} \text{ or } P_2 = \frac{P_1}{5} = \frac{10}{5} = 2 \text{ cal s}^{-1}$$

28. The given network of resistances is a balanced Wheatstone's bridge. Therefore, no current flows through the 6 Ω resistor and it can be omitted. The equivalent resistance of the network is that of a parallel combination of $R_1 = R + R + R = 3R$ and $R_2 = R + R + 4R = 6R$ which is

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{3R \times 6R}{3R + 6R} = 2R$$

The power delivered will be maximum if the resistance of the emf source = external resistance, i.e. if

$$4\ \Omega = R' = 2R$$

or $R = 2\ \Omega$, which is choice (b).

29. The correct choice is (a).

30. Resistance of heater $(R_H) = \frac{V^2}{P} = \frac{100 \times 100}{1000} = 10\ \Omega$. Current in the circuit is

$$I = \frac{100}{10 + \frac{10R}{(10 + R)}}$$

$\therefore$ Current in heater is $I_H = I \times \frac{R}{(10 + R)}$

$$= \frac{100}{\left[10 + \frac{10R}{(10+R)}\right]} \times \frac{R}{(10+R)}$$

$$= \frac{10R}{\left[1 + \frac{R}{(10+R)}\right](10+R)} = \frac{5R}{(5+R)}$$

$$\therefore \qquad Q = I_H^2 \times 10$$

$$\text{or} \quad \frac{25R^2}{(5+R)^2} \times 10 = 62.5 \text{ (given)}$$

which gives $R = 5\ \Omega$. Hence the correct choice is (d).

31. The correct choice is (d).

32. The heat energy generated in time t is $Q = V^2t/R$, where V is the terminal voltage of a single cell. Here $Q = ms\Delta T$ where m is the mass of the wire, s its specific heat and R its resistance.

$$\text{In the first case: } ms\Delta T = \frac{(3V)^2 t}{R} \qquad (1)$$

Now $m \propto$ length and $R \propto$ length. Hence, in the second case,

$$(2m)\ s\Delta T = \frac{(NV)^2 t}{(2R)} \qquad (2)$$

Dividing (2) by (1), we have

$$2 = \frac{N^2/2}{9} \text{ or } N^2 = 36 \text{ or } N = 6,$$

which is choice (b).

33. The resistances of bulbs B_1, B_2 and B_3 respectively are

$$R_1 = \frac{V^2}{W_1} = \frac{(250)^2}{100} = 625\ \Omega$$

$$R_2 = \frac{(250)^2}{60} = 1042\ \Omega = R_3$$

Voltage across B_3 is $V_3 = 250$ V

$$\text{Voltage across } B_1 \text{ is } V_1 = \frac{VR_1}{(R_1 + R_2)}$$

$$= \frac{250 \times 625}{(625 + 1042)} = 93.7 \text{ V}$$

Voltage across B_2 is $V_2 = 250 - 93.7 = 156.3$ V

$$\text{Power output } W_1 = \frac{V_1^2}{R_1} = \frac{(93.7)^2}{625} = 14 \text{ W}$$

$$W_2 = \frac{(156.3)^2}{1042} = 23 \text{ W}$$

$$W_3 = \frac{(250)^2}{1042} = 60 \text{ W}$$

Hence $W_1 < W_2 < W_3$, which is choice (d).

34. Let R be the value of each resistance. The resistances of combinations I, II, III and IV are $3R$, $R/3$, $2R/3$ and $3R/2$ respectively. Now, power dissipation is inversely proportional to resistance. Hence the correct choice is (b).

35. Given $V = 220$ V, $P = 100$ W, Now $P = VI$ and the resistance of the bulb $R = \frac{V}{I}$. But $I = \frac{V}{P}$. Therefore

$$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$$

When the bulb is connected to $V' = 110$ V supply, the power consumed by the bulb will be

$$P' = \frac{(V')^2}{R} = \frac{(110)^2}{484} = 25 \text{ W}.$$

36. The power rating of a resistor is the maximum power the resistor can dissipate as heat without melting. Now $R = 20\ \Omega$ and $P_{max} = 1.0$ kW $= 1000$ W. Since $P = V^2/R$, we have $V^2 = RP$ or $V = \sqrt{RP}$

$$\therefore \quad V_{max} = \sqrt{R \times P_{max}} = \sqrt{20 \times 1000}$$

$$= 100\sqrt{2} = 141 \text{ V}$$

Thus the resistor will melt if V exceeds 141 V. Hence the correct choice is (a).

37. The heat transmitted per second through the walls of the closed box is given by

$$\frac{Q}{t} = \frac{K A(\theta_2 - \theta_1)}{d}$$

where A is the total surface area of the box.
Since a cube has 6 faces, the total surface area is

$$A = 6 \times \text{surface area of one face}$$

$$= 6 \times (0.5 \text{ m} \times 0.5 \text{ m}) = 1.5 \text{ m}^2$$

$$\therefore \quad \frac{Q}{t} = \frac{1.68 \times 10^{-1} \times 1.5 \times (100)}{1 \times 10^{-3}}$$

$$= 2.52 \times 10^4 \text{ J s}^{-1}$$

In order to maintain the temperature difference between the inside and the outside, the heat lost must be compensated by the production of heat through electric current in the coil. The heat produced per second is given by

$$H = \frac{V^2}{R} = 2.52 \times 10^4$$

or $$R = \frac{420 \times 420}{2.52 \times 10^4} = 7\ \Omega$$, which is choice (c).

38. Heat produced $= \frac{V^2}{R} = \frac{V^2 A}{\rho l}$, where ρ is the specific resistance (or resistivity) of the wire, which is the same for both wires. Therefore, the heat produced will be same in the second wire of length 2 l, if its area of cross-section A is doubled. Hence the correct choice is (c).

39. $R_A = \frac{\rho L}{\pi r^2}$ and $R_B = \frac{\rho (2L)}{\pi (2r)^2}$

$$\therefore \quad \frac{R_A}{R_B} = 2 \quad \text{or} \quad R_A = 2R_B$$

Now $Q \propto \frac{1}{R}$. Hence the correct choice is (b).

40. The resistances of bulbs A and B are

$$R_A = \frac{V^2}{P_A} = \frac{(200)^2}{40} = 1000\ \Omega$$

and $$R_B = \frac{V^2}{P_B} = \frac{(200)^2}{100} = 400\ \Omega$$

The current ratings (i.e. maximum current they can withstand) are

$$I_A = \frac{P_A}{V} = \frac{40}{200} = 0.2\ \text{A}$$

$$I_B = \frac{P_B}{V} = \frac{100}{200} = 0.5\ \text{A}$$

When the bulbs are connected in series, their combined resistance is $R = R_A + R_B = 1000 + 400 = 1400\ \Omega$. The current through each bulb is

$$I = \frac{V'}{R} = \frac{400}{1400} = 0.286\ \text{A}$$

Since I is greater than I_A but less than I_B; bulb A will fuse but bulb B will not. Hence the correct choice is (b).

41. When the bulbs are connected in parallel, the potential difference across each bulb is $V' = 400$ V. Therefore, the currents in A and B are

$$I'_A = \frac{V'}{R_A} = \frac{400}{1000} = 0.4\ \text{A}$$

and $$I'_B = \frac{V'}{R_B} = \frac{400}{400} = 1\ \text{A}$$

Since I'_A is greater than I_A and I'_B is greater than I_B, both the bulbs will fuse. Hence the correct choice is (a)

42. Resistance of each bulb is $R = \frac{(200)^2}{500} = 80\ \Omega$.

When the bulbs are connected in series to a 100 V source, the potential difference across each bulb is $V = 100/2 = 50$ V. Therefore, power generated in each bulb is

$$P = \frac{V^2}{R} = \frac{(50)^2}{80} = 31.25\ \text{W}$$

Hence the correct choice is (b).

43. $R = \frac{\rho l}{\pi r^2}$. Since the two wires are made of the same material, resistivity ρ is the same for wires AB and BC. Since the wires have equal lengths, it follows that $R \propto 1/r^2$. Hence

$$\frac{R_{AB}}{R_{BC}} = \frac{1}{4}, \text{ i.e } R_{BC} = 4R_{AB}$$

Since the current, is the same in the two wires, it follows from Ohms law ($V = IR$) that $V_{BC} = 4\ V_{AB}$. Hence choice (a) is wrong. Now power dissipated is $P = I^2 R$. Since I is the same, $P \propto R$. Hence

$$\frac{P_{BC}}{P_{AB}} = \frac{R_{AB}}{R_{BC}} = 4$$

Hence chioce (b) is correct. choice (c) is wrong because current density (i.e. current per unit area) is different in wires AB and BC because their cross-sectional areas are different. The electric field in a wire is $E = V/l$. Since the two wires have the same length (l), E is proportional to potential difference (V). Since $V_{BC} = 4\ V_{AB}$, $E_{BC} = 4E_{AB}$. Hence choice (d) is also incorrect.

44. Given $\frac{V^2}{2} + \frac{V^2}{3} = 30 \quad \Rightarrow \quad V^2 = 36$ or $V = 6$ volt.

So the potential difference across the 4 Ω resistor is 6 V. Therefore, the current through this resistor = 6/4 = 1.5 A, which is choice (c).

45. Since the current through the 1 Ω resistor is 2 A, the current in the 2 Ω resistor will be 1 A. So the current in the 3 Ω resistor = 1 + 2 = 3 A. Hence power

developed in the 3 Ω resistor = $(3)^2 \times 3 = 27$ W. So the corrent choice is (c).

46. The effective resistance across the battery in configuration R_1 is $R_1 = 1\ \Omega$. For configuration R_2, the effective resistance is $R_2 = 0.5\ \Omega$ and $R_3 = 2\ \Omega$.

Now power dissipated is $P = \frac{V^2}{R}$. Since V = constant, $P \propto \frac{1}{R}$. Hence $P_2 > P_1 > P_3$.

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Two wires of the same material and having same uniform area of cross-section are connected in an electrical circuit. The masses of the wires are m and 2 m respectively. When a current I flows through the circuit, the heat produced in them in a given time is in the ratio
- (a) 2 : 1 when they are connected in series
- (b) 2 : 1 when they are connected in parallel
- (c) 1 : 2 when they are connected in parallel
- (d) 1 : 2 when they are connected in series

2. If three bulbs of 40 W, 60 W and 100 W are connected in series with a 200 V power supply, then
- (a) the potential difference will be maximum across the 40 W bulb
- (b) the current will be maximum through the 100 W bulb
- (c) the 40 W bulb has the maximum resistance
- (d) the current in the circuit is 1.2 A

3. A constant voltage is applied between two ends of a uniform conducting wire. If both the length and radius of the wire are doubled
- (a) the heat produced in the wire will be doubled
- (b) the electric field across the wire will be doubled.
- (c) the heat produced will remain unchanged
- (d) the electric field across the wire will become one half.

4. Choose the correct statements from the following.
- (a) A 100 W filament bulb has a higher resistance that a 1 kW electric heater both marked for 200 V.
- (b) Three bulbs of 40 W, 60 W and 100 W are connected in series and this combination is connected across the mains. The potential difference across the 40 W bulb is the lowest.
- (c) A 60 W bulb is connected in series with a room heater and this combination is connected across the mains. If the 60 W bulb is replaced by a 100 W bulb, the heat produced by the heater will decrease.
- (d) A 60 W bulb is connected in parallel with a room heater and this combination is connected across the mains. If 60 W bulb is replaced by a 100 W bulb, the heat produced by the heater will remain unchanged.

5. A wire of length l, cross-sectional area A and resistivity ρ is connected to a source of a constant voltage V. The rate at which heat is produced in the wire is inversely proportional to
- (a) V (b) A
- (c) ρ (d) l

6. A battery consists of a series combination 6 lead accumulators, each of emf 2.0 V and internal resistance 0.5 Ω. The battery is charged by a 100 V d.c. supply, A series resistance $R = 8\ \Omega$ is used in the charging circuit. Then
- (a) The current in the circuit is 8 A.
- (b) The power supplied by the d.c. source is 800 W.
- (c) The power dissipated as heat is 704 W.
- (d) The power stored in the battery is 96 W.

7. A cell of emf 1.5 V and internal resistance 0.1 Ω is connected across a resistor R. The current in the circuit is 2 A. Then
- (a) the rate of chemical energy consumption in the cell is 3.0 W.
- (b) the rate of energy dissipation in the cell is 0.4 W.
- (c) the power output of the source is 2.6 W.
- (d) the value of $R = 0.5\ \Omega$.

8. A battery of emf E and internal resistance r is connected across a pure resistive device of resistance R. The power output of the device is P. Then
- (a) P is maximum if $R = r$.
- (b) P is maximum if $R = 2r$.
- (c) the maximum power output is $E^2/2r$.
- (d) the maximum power output is $E^2/4r$.

9. An eletric motor runs on a dc source of emf E & internal resistance r. The power output of the source is maximum = P_{max} when the current drawn by the motor is I. Then

(a) $I = \frac{E}{r}$ (b) $I = \frac{E}{2r}$

(c) $P_{max} = \frac{E^2}{r}$ (d) $P_{max} = \frac{E^2}{4r}$

10. A copper wire and a silicon crystal are heated from 50 K to room temperature. The resistivity of

(a) copper decreases (b) copper increases
(c) silicon decreases (d) silicon increases.

11. In the circuit shown in Fig. 23.15, the rate of heat produced in the 5 Ω resistor is 20 J s^{-1}.

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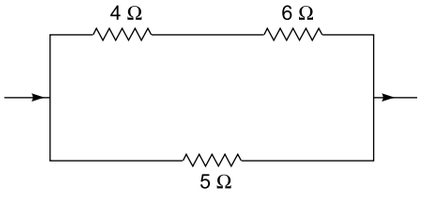


Fig. 23.15

(a) The current in the 6 Ω resistor is 1 A.
(b) The current in the 5 Ω resistor is 2 A.
(c) The rate of heat produced in the 4 Ω resistor is 4 J s^{-1}.
(d) The potential difference across 4 Ω resistor is 4 V.

12. The battery of emf E and internal resistance r supplies 1.50 W of power in a 6 Ω resistor and 1.96 W of power in a 4 Ω resistor. Then

(a) E = 3.5 V (b) E = 1.5 V
(c) r = 1 Ω (d) r = 2 Ω

13. An electric heater A operated at 100 V consumes electrical energy at the same rate as another electric heater B when operated at 200 V. If the heaters are connected in series across a 300 V power supply, the ratio of the

(a) potential difference across A to that across B is 1 : 4
(b) current in A to that in B is 1 : 1
(c) power consumed by A to that consumed by B is 2 : 3
(d) power consumed by A to that consumed by B is 1 : 4.

14. In the circuit shown in Fig. 23.16.

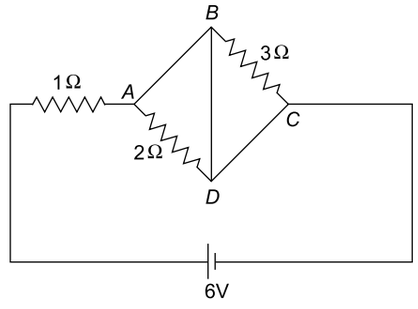


Fig. 23.16

(a) the current in the circuit is 6 A.
(b) the current flowing in branch BD is zero.
(c) the power dissipated in 1 W resistor is 36 W.
(d) the power dissipated in 3 W resistor is 12 W

15. Figure 23.17 shows a part of a circuit. The currents, resistances and emfs are indicated in the figure.

(a) The power dissipated in the 2 Ω resistor is 128 W.
(b) The power dissipated in the 3 Ω resistor is 108 W.
(c) The potential difference between A and D is − 15 V.
(d) The potential difference between A and D is + 15 V.

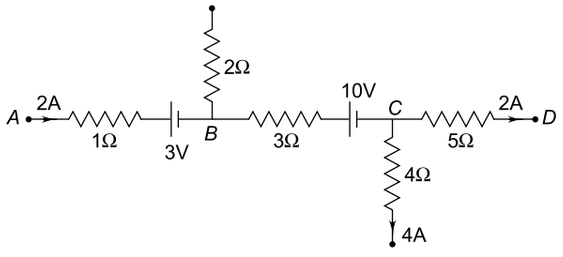


Fig. 23.17

ANSWERS AND SOLUTIONS

1. The mass of a wire of length l, cross-sectional area A and density d is given by

$$m = A l d$$

Since A and d are the same for the two wires and since their masses are in the ratio of 1 : 2, their lengths and hence their resistances are in the ratio of 1 : 2. When the wires are connected in series, the same current I flows in each. Since the heat H produced in time t is given by

$$H = I^2 R t \quad \text{or} \quad H \propto R,$$

the choice (d) is correct. On the other hand, if the wires are connected in parallel, the potential difference V is the same across the two wires. Since H is also given by

$$H = \frac{V^2 t}{R} \qquad \text{or} \qquad H \propto \frac{1}{R},$$

the choice (b) is also correct. Hence the correct choices are (b) and (d).

2. Resistance of 40 W bulb $= \frac{V^2}{W} = \frac{200 \times 200}{40}$

$= 1000\ \Omega$

Resistance of 60 W bulb $= \frac{200 \times 200}{60}$

$= 666.7\ \Omega$

Resistance of 100 W bulb $= \frac{200 \times 200}{100}$

$= 400\ \Omega$

Hence choice (c) is correct. Since the bulbs are connected in series, the current in each is the same. Hence choice (b) is incorrect. The total resistance = 1000 + 666.7 + 400 = 2066.7 Ω. Therefore, the current in the circuit is

$$I = \frac{200}{2066.7} = 0.097 \text{ A}$$

Hence the choice (d) is also incorrect. Since the 40 W bulb has the highest resistance and the current in each bulb is the same, the potential difference across the 40 W bulb is the highest. Hence the correct choices are (a) and (c).

3. $R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$

If l and r are doubled, R will become one half.

Thus heat produced per second will become twice because $H = V^2/R$. Also, since $E = \frac{V}{l}$, E will become one half. Hence the correct choices are (a) and (d).

4. Statement (a) is correct. The resistance of the bulb is

$$R_b = \frac{V^2}{P_b} = \frac{(200)^2}{100} = 400\ \Omega$$

The resistance of the heater is

$$R_h = \frac{(200)^2}{1000} = 40\ \Omega$$

Statement (b) is incorrect. Since the bulbs are connected in series, the current in each is the same. Therefore, the potential difference across a bulb will be proportional to its resistance. We have seen above that $R \propto 1/P$. Thus the 40 W bulb has the highest resistance and the 100 W bulb has the lowest resistance. Hence the potential difference across the 40 W bulb is the highest and that across the 100 W bulb is the lowest.

Statement (c) is also incorrect. Let R_1 be the resistance of the 60 W bulb and R_2 that of the 100 W bulb. Since $R \propto 1/P$, R_1 is greater than R_2. If R is the resistance of the heater and V the voltage of the mains, the current through heater and 60 W bulb is

$$I_1 = \frac{V}{R + R_1}$$

and the current through heater and 100 W bulb is

$$I_2 = \frac{V}{R + R_2}$$

Since $R_1 > R_2$, $I_2 > I_1$. We know that heat produced by the heater is proportional to the square of the current flowing through it. Hence heat produced by the heater will be more when 100 W bulb is in series with it than when a 60 W bulb is in series with it.

Statement (d) is correct. When a bulb and heater are connected in parallel and this combination is connected across the mains, the potential difference across each is equal to the voltage of the mains irrespective of the resistance of the bulb. Hence, on replacing the 60 W bulb by a 100 W bulb, the heat produced by the heater will remain unchanged which will be V^2/R in both the cases.

5. Heat produced per second $= \frac{V^2}{R} = \frac{V^2 A}{\rho l}$. Hence the correct choices are (c) and (d).

6. Emf of battery = 6 × 2 = 12V. Resistance of battery (r) = 6 × 0.5 = 3 Ω. Since the polarity of the charging d.c. source is opposite to that of the battery, the effective emf (E) = 100 – 12 = 88 V.

∴ Current in the circuit is

$$I = \frac{E}{R + r} = \frac{88}{8 + 3} = 8 \text{ A}$$

Power supplied by source = 100 V × 8 A = 800 W.

Power dissipated as heat = $I^2 (R + r) = (8)^2 \times (8 + 3)$ = 704 W.

Power stored in the battery = power supplied – power dissipated

= 800 – 704 = 96 W

Hence all the four choices are correct.

7. $V = 1.5$ V, $I = 2$ A and $r = 0.1$ Ω. The rate of chemical energy consumption in the cell is

$$P = V\,I = 1.5 \times 2 = 3.0 \text{ W}$$

The rate of energy dissipation inside the cell is

$$P_c = I^2 r = (2)^2 \times 0.1 = 0.4 \text{ W}$$

The rate of energy dissipation in resistor R is

$$P_R = P - P_c = 3.0 - 0.4 = 2.6 \text{ W}$$

Potential drop across R is

$$\frac{1.5 \times R}{R + 0.10}$$

Therefore rate of energy dissipation in R = potential drop across $R \times$ current in R

$$= \frac{1.5 \times R \times 2.0}{R + 0.10}$$

which is $P_R = 2.6$ W. Hence

$$\frac{1.5 \times R \times 2.0}{R + 0.10} = 2.6$$

which gives $R = 0.65\ \Omega$.

Power output of the source = rate of dissipation of energy in the resistor = 2.6 W.

Thus the correct choices are (a), (b) and (c).

8. The current in the circuit is

$$I = \frac{E}{(R + r)}$$

Therefore the power output of the device is given by

$$P = I^2 R = \frac{E^2 R}{(R + r)^2} \qquad \text{(i)}$$

For given values of E and r, power output P will be maximum if $dP/dR = 0$ and $d^2P/dR^2 < 0$. Differentiating (i) with respect to R we get (with E and r fixed)

$$\frac{dP}{dR} = \frac{E^2}{(R + r)^2}\left\{1 - \frac{2R}{(R + r)}\right\} \qquad \text{(ii)}$$

Now $dP/dR = 0$ if

$$1 - \frac{2R}{(R + r)} = 0$$

which gives $R = r$. Thus, P will be either maximum or minimum when $R = r$. To decide whether P is maximum at $R = r$, we find d^2P/dR^2 at $R = r$. If its value is negative, P will be maximum. Differentiating (ii) we have

$$\frac{d^2P}{dR^2} = \frac{2E^2}{(R + r)^3}\left\{\frac{3R}{(R + r)} - 2\right\}$$

$$\therefore \qquad \left(\frac{d^2P}{dR^2}\right)_{\text{at } R=r} = -\frac{E^2}{8r^3}$$

which is negative. Hence P is maximum when $R = r$. Putting $R = r$ in Eq. (i) above we get

$$P_{\max} = \frac{E^2}{4r}$$

So the correct choices are (a) and (d).

9. When the motor is running (it is an active device), a back emf, say E' is developed in the circuit. If I is the current in the circuit, the power output of the source is given by

$$P = (E - E')I$$

But $\quad E' = Ir.$

Hence $\quad P = EI - I^2 r \qquad \text{(i)}$

Differentiating (i) with respect to I, we have

$$\frac{dP}{dI} = E - 2\,Ir \qquad \text{(ii)}$$

P will be maximum or minimum if $\frac{dP}{dI} = 0$, i.e. if $I = \frac{E}{2r}$. Differentiating (ii), we have

$$\frac{d^2P}{dI^2} = -2\,r$$

which is negative. Hence P is maximum when $I = E/2r$.

Putting $I = E/2r$ in (i) above, we get

$$P_{\max} = \frac{E^2}{4r}$$

Thus the correct choices are (b) and (d).

10. The correct choices are (b) and (c). The resistance and hence resistivity of metallic conductors (such as copper) increases and of semi-conductors (such as silicon) decreases with increase in temperature.

11. The current I divides into I_1 and I_2 as shown in Fig. 23.18. The potential difference between A and B = potential difference between C and D

or $\quad I_1 \times (6 + 4) = I_2 \times 5$

or $\quad I_1 = \frac{I_2}{2}$.

Now $Q = I_2^2 \times 5 \Rightarrow 20 = I_2^2 \times 5 \Rightarrow I_2 = 2$ A. Therefore, $I_1 = 1$ A. The rate at which heat is produced in the 4 Ω resistor $= I_1^2 \times 4 = (1)^2 \times 4 = 4$ J s^{-1}. Potential difference across 4 Ω resistor $= I_1 \times 4 = 1 \times 4 = 4$ V. Hence all the four choices are correct.

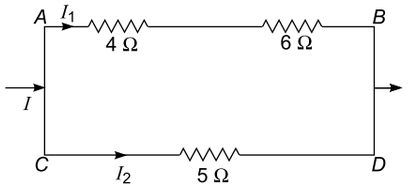


Fig. 23.18

12. $I = \dfrac{E}{(R+r)}$, power $P = I^2R = \dfrac{E^2R}{(R+r)^2}$

$$P_1 = \frac{E^2R_1}{(R_1+r)^2},\ P_2 = \frac{E^2R_2}{(R_2+r)^2}$$

$$\therefore \frac{P_1}{P_2} = \left(\frac{R_1}{R_2}\right)\times\left(\frac{R_2+r}{R_1+r}\right)^2$$

$$\Rightarrow \frac{1.5}{1.96} = \left(\frac{6}{4}\right)\times\left(\frac{4+r}{6+r}\right)^2 \Rightarrow r = 1\Omega$$

Also $1.5 = \dfrac{E^2\times 6}{(6+1)^2} \Rightarrow E = 3.5$ V.

Thus the correct choices are (a) and (c).

13. $P_1 = \dfrac{(100)^2}{R_1}$ and $P_2 = \dfrac{(200)^2}{R_2}$. Given $P_1 = P_2$.

Hence $\dfrac{R_1}{R_2} = \dfrac{1}{4}$. When the heaters are connected in series across 300 V supply, ($\therefore$ current I is the same in both)

$$\frac{V_1}{V_2} = \frac{IR_1}{IR_2} = \frac{R_1}{R_2} = \frac{1}{4}$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{I^2R_1}{I^2R_2} = \frac{R_1}{R_2} = \frac{1}{4}$$

Hence the correct choices are (a), (b) and (d).

14. Resistors 2 Ω and 3 Ω are short-circuited. So current flows from A to B, from B to D and from D to C and then through the battery and the 1 Ω resistor. The current in the circuit is

$$I = \frac{6}{1} = 6\text{ A}$$

which is the current flowing in 1 Ω resistor and in the branch BD. Power dissipated in 1 Ω resistor = $(6)^2 \times 1 = 36$ W. No current flows in the 2 Ω and 3 Ω resistors. So no power is dissipated in these resistors. Hence the correct choices are (a), (b) and (c).

15. Applying Kirchhoff's junction rule at junction C, we find that a current of (4 + 2) = 6 A flows in brance CB from C to B as shown in Fig. 23.19. At junction B, the current through the 2 Ω resistor = 6 + 2 = 8 A.

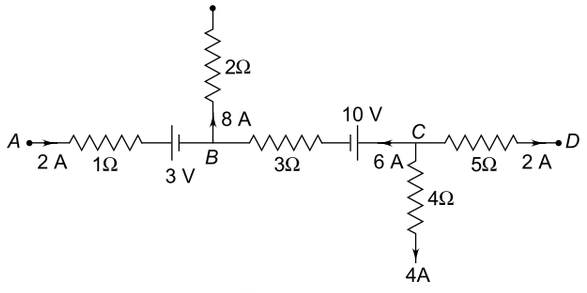


Fig. 23.19

$\therefore$ Power dissipated in 2 Ω resistor = $(8)^2 \times 2 = 128$ W.
Power dissipated in 3 Ω resistor = $(6)^2 \times 3 = 108$ W.
Potential difference between A and D = (2 A × 1 Ω + 3 V – 6 A × 3 Ω –10 V + 2 A × 5 Ω)
= 2 + 3 – 18 – 10 + 10 = – 15 V.
Hence the correct choices are (a), (b) and (c).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 5 are based on the following passage

passage I

Joule's Law

Consider a conductor having a finite resistance. A steady potential difference V is maintained between the ends A and B of the conductor. The end A of the conductor is connected to the positive terminal of the battery and is at a higher potential than end B which is connected to the negative terminal of the battery. A steady current I flows through the conductor. When a charge q moves from A to B in time t, the current in the conductor is

$$I = \frac{q}{t}$$

In moving from A to B, the charge has to do work which (from the definition of potential difference V) is given by

$$W = qV$$

This work represents the decrease in the electrical potential energy of the charge. The principle of conservation of energy tells us that this energy is transformed from electrical energy to some other form of energy. If the conductor is a metal (pure resistor), then the energy appears as heat in the metal, which is given by

$$H = qV = ItV = I^2Rt = \frac{V^2t}{R} \qquad (\therefore V = IR)$$

The power P associated with the transfer of heat is

$$P = \frac{H}{t} = IV = I^2R = \frac{V^2}{R}$$

1. The end A of a metallic conductor is connected to the positive terminal and end B of the conductor is connected to the negative terminal of a battery. Then
   (a) the free electrons move from A to B.
   (b) the free electrons move from B to A.
   (c) the electrons lose potential energy as they move in the conductor.
   (d) the electrons gain potential energy as they move in the conductor.

2. A constant potential difference is maintained between the ends of a metallic conductor. As the electrons move from one end of the conductor to the other,
   (a) their kinetic energy increases and potential energy decreases.
   (b) their kinetic energy decreases and potential energy increases.
   (c) their kinetic energy increases but potential energy ramains unchanged.
   (d) their potential energy decreases but kinetic energy remains unchanged.

3. A potential difference V applied across a resistance R causes a current I to flow through the resistance. In which of the following cases is the rate at which electrical energy is converted to thermal energy the maximum?
   (a) V is doubled with R unchanged
   (b) R is doubled with V unchanged
   (c) R is doubled with I unchanged
   (d) Both R and V are doubled

4. In Q.3 above, in which of the four cases (a), (b), (c) and (d) is the rate of conversion of electrical energy into thermal energy the least.

5. A uniform wire is connected across the mains. It dissipates power P. The wire is now cut into two equal pieces which are then joined in parallel. If this combination is connected across the mains, the power dissipated will be
   (a) $\frac{P}{4}$ (b) $\frac{P}{2}$
   (c) $2P$ (d) $4P$

ANSWERS AND EXPLANATIONS

1. Since the charge of an electron is negative, it will move towards the positive terminal. As it moves, it loses potential energy and this loss appears as heat in the conductor. Thus the correct choices are (b) and (c).

2. The electrons loose potential energy as they move from one end of the conductor to the other. The kinetic energy ramians unchanged because electrons drift at a constant speed called the drift speed. Hence the correct choice is (d).

3. In case (a), power $= \frac{(2V)^2}{R} = 4\,\frac{V^2}{R} = 4P.$

In case (b), power $= \frac{V^2}{2R} = \frac{P}{2}$

In case (c), power $= I^2\,(2R) = 2\,I^2\,R = 2P$

In case (d), power $= \frac{(2V)^2}{2R} = 2\,\frac{V^2}{R} = 2P$

Hence the correct choice is (a).

4. The correct choice is (b).

5. Let R be the resistance of the complete wire and V the voltage of the mains. Then $P = V^2/R$. If the wire is cut into two equal pieces and these pieces are connected in parallel, the resistance of the combination will be $R/4$. Hence power $= \frac{V^2}{R/4} = \frac{P}{2} = 4P.$

Thus the correct choice is (d).

Questions 6 to 9 are based on the following passage

Passage II

A 50 V dc power supply is used to charge a battery of eight lead accumulators, each of emf 2 V and internal resistance 1/8 Ω. The charging current also runs a motor connected in series with the battery. The resistance of the motor is 5 Ω and the steady current supply is 4 A.

6. The total power lost due to heat dissipation is
(a) 88 W (b) 92 W
(c) 96 W (d) 100 W

7. The useful power available is
(a) 112 W (b) 108 W
(c) 104 W (d) 100 W

8. The chemical power stored in the battery is
(a) 80 W (b) 76 W
(c) 64 W (d) 50 W

9. The mechanical power stored in the motor is
(a) 20 W (b) 30 W
(c) 40 W (d) 50 W

SOLUTION

6. Power supplied is $P = 50\text{ V} \times 4\text{ A} = 200$ W. A part of this power is lost due to heat in the battery and the windings of the motor. Heat lost in the battery is

$$P_1 = (\text{current})^2 \times (\text{resistance of battery})$$
$$= (4)^2 \times \left(8 \times \frac{1}{8}\right)$$
$$= 16 \text{ W}$$

Power lost as heat in the motor is

$$P_2 = (\text{current})^2 \times (\text{resistance of motor})$$
$$= (4)^2 \times 5 = 80 \text{ W}$$

$\therefore$ Total power lost due to heat dissipation $= P_1 + P_2 = 16 + 80 = 96\ \Omega$. Hence the correct choice is (c).

7. Useful power available = power supplied – power lost = 200 W – 96 W = 104 W. Hence the correct choice is (c).

8. Chemical power stored in the battery = (voltage of battery) × (current)
$= 16\text{ V} \times 4\text{ A} = 64$ W.
Hence the correct choice is (c).

9. Mechanical power supplied by the motor = total available power – power stored in the battery = 104 – 64 = 40 W. Hence the correct choice is (c).

Questions 10 to 12 are based on the following passage

Passage III

In the circuit shown in Fig. 23.20, the 5 Ω resistance develops 45 Js^{-1} of heat due to the flow of current through it.

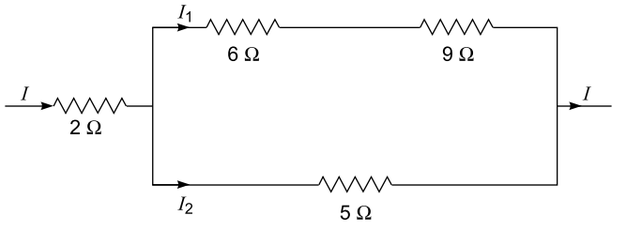


Fig. 23.20

10. The value of current I is
(a) 1 A (b) 2 A
(c) 3 A (d) 4 A

11. The power dissipated in the 2 Ω resistor is
(a) 96 W (b) 48 W
(c) 32 W (d) 16 W

12. The potential difference across the 6 Ω resistor is
(a) 2 V (b) 4 V
(c) 6 V (d) 9 V

SOLUTION

10. Current through the 5 Ω resistor is

$$I_2 = \frac{15I}{20} = \frac{3I}{4}$$

Current through 6 Ω and 9 Ω resistors is

$$I_1 = \frac{5I}{20} = \frac{I}{4}$$

Heat energy developed per second in the 5 Ω resistors is $I_2^2 \times 5 = 45$ or

$$\left(\frac{3I}{4}\right)^2 \times 5 = 45 \quad \Rightarrow \quad I = 4 \text{ A}.$$

So the correct choice is (d).

11. Power developed in the 2 Ω resistor $= I^2 \times 2 = (4)^2 \times 2 = 32$ W, which is choice (c).

12. Potential difference across the 6 Ω resistor $= I_1 \times 6 = \frac{I}{4} \times 6 = \frac{4}{4} \times 6 = 6$ V, which is choice (c).

Questions 13 to 16 are based on the following passage

Passage IV

Figure 23.21 shows four resistances $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 12\ \Omega$ and $R_4 = 10\ \Omega$ connected with a 120 V battery and a capacitor of capacitance $C = 500\ \mu F$, by means of two switches S_1 and S_2. Switch S_1 is closed so that the capacitor is fully charged. Switch S_1 is then opened and switch S_2 is closed.

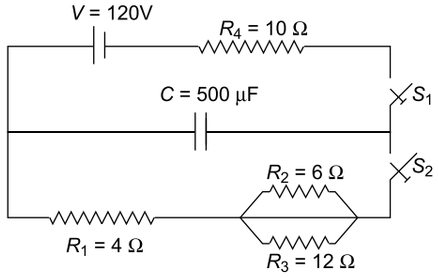


Fig. 23.21

13. The power developed in resistor R_1 is
(a) 1.8 W (b) 3.6 W
(c) 5.4 W (d) 7.2 W

14. The power developed in resistor R_2 is
(a) 0.3 W (b) 0.6 W
(c) 0.9 W (d) 1.2 W

15. The power developed in resistor R_3 is
(a) 0.2 W (b) 0.4 W
(c) 0.6 W (d) 0.8 W

16. The power developed in resistor R_4 is
(a) 3.6 W (b) 1.2 W
(c) 0.4 W (d) zero

SOLUTION

When switch S_1 is closed, the capacitor is charged to the voltage of the battery. Energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 500 \times 10^{-6} \times (120)^2 = 3.6 \text{ J}.$$

When S_1 is opened and switch S_2 is closed, no current flows through R_4. Hence power developed in $R_4 = 0$, i.e. $P_4 = 0$.

Let I_1 be the current through the 4 Ω resistor at an instant of time t. This current divides into I_2 flowing through R_2 and I_3 flowing through R_3 such that $I_1 = I_2 + I_3$. Since the potential difference across R_2 and R_3 is the same, we have, $I_2R_2 = I_3R_3$ or $6I_2 = 12\ I_3$ or $I_2 = 2I_3$. But $I_1 = I_2 + I_3$. Therefore,

$$I_1 = 2I_3 + I_3 \text{ or } I_3 = \frac{I_1}{3}. \text{ Similarly } I_2 = \frac{2I_1}{3}. \text{ Thus}$$

$$I_1 : I_2 : I_3 = I_1 : \frac{2I_1}{3} : \frac{I_1}{3} = 1 : \frac{2}{3} : \frac{1}{3} = 3 : 2 : 1$$

Now $P = I^2R$. Therefore,

$$P_1 : P_2 : P_3 = I_1^2R_1 : I_2^2R_2 : I_3^2R_3$$
$$= (3)^2 \times 4 : (2)^2 \times 6 : (1)^2 \times 12$$
$$= 36 : 24 : 12 = 3 : 2 : 1$$

Thus energy $U = 3.6$ J stored in the capacitor is distributed among resistors R_1, R_2 and R_3 in the ratio 3 : 2 : 1.

13. $P_1 = \dfrac{3.6 \times 3}{6} = 1.8$ W, which is choice (a).

14. $P_2 = \dfrac{3.6 \times 2}{6} = 1.2$ W, which is choice (d).

15. $P_3 = \dfrac{3.6 \times 1}{6} = 0.6$ W, which is choice (c).

16. Since no current flows through R_4, $P_4 = 0$, which is choice (d).

SECTION IV

Assertion–Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. Statement 1

The adjoining Fig. 23.22 shows the current-voltage ($I - V$) graphs for a given metallic wire at two different tempera-tures T_1 and T_2. It follows from the graphs that T_2 is greater than T_1.

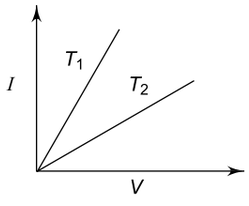


Fig. 23.22

Statement-2

The resistance of a metallic conductor increases with increase in temperature.

2. Statement 1

Two bulbs of 60 W and 100 W are connected in series and this combination is connected a d.c. power supply. The potential difference across the 60 W bulb is higher than that across the 100 W bulb.

Statement-2

In a series combination, the potential difference across a bulb a proportional to its resistance.

3. Statement 1

A 60 W bulb is connected in series with a room heater and this combination is connected across the mains. If the 60 W bulb is replaced by a 100 W bulb, the heat produced by the heater will be more.

Statement-2

The heat produced is inversely proportional to resistance when the resistances are connected in series across the mains.

4. Statement 1

A 60 W bulb is connected in parallel with a room heater and this combination is connected across the mains. If the 60 W bulb is replaced by a 100 W bulb, the heat produced by the heater will remain the same.

Statement-2

When resistance are connected in parallel across the mains, the heat produced is inversely proportional to the resistance.

SOLUTIONS

1. The correct choice is (a). From Ohm's law, the slope of the $I - V$ graph gives the reciprocal of the resistance of the wire. Since the slope of the graph is smaller at temperature T_2, the resistance of the wire is greater at temperature T_2 than at temperature T_1. Hence T_2 is greater than T_1.

2. The correct choice is (a). Since the bulbs are connected in series, the current in each is the same. Therefore, the potential difference across a bulb will be proportional to its resistance R. Now $R \propto 1/P$, where P is the power. Hence the 60 W bulb has a higher resistance than the 100 W bulb.

3. The correct choice is (c). The resistance (R_1) of the 60 W bulb is higher than resistance (R_2) of the 100 W bulb. If R is the resistance of the heater and V the voltage of the mains, the currents through the 60 W bulb and 100 W bulb respectively are

$$I_1 = \frac{V}{R + R_1} \quad \text{and} \quad I_2 = \frac{V}{R + R_2}$$

Since $R_1 > R_2 : I_2 > I_1$. Since the heater and the bulb are connected in series, the current through the bulb = current through the heater. Now, heat produced by the heater is proportional to the square of the current flowing through it.

4. The correct choice is (a). When a bulb and a heater a connected in parallel and this combination is connected across the main, potential difference across each is the same equal to the voltage V of the mains, irrespective of the resistance of the bulb. If R is the resistance of the heater, the rate at which heat is produced will be V^2/R in both cases.

24 Chapter
Magnetic Effect of Current and Magnetism

REVIEW OF BASIC CONCEPTS

24.1 BIOT-SAVART LAW

According Bipt-Savart law, the magnetic field $d\mathbf{B}$ at a point whose position vector with respect to a current element dl is $\mathbf{r}$ is given by

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{(d\mathbf{l}\times\mathbf{r})}{r^3} \qquad (1)$$

where $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}

24.2 AMPERE'S CIRCUITAL LAW

The loop or line circuit integral of the magnetic field along a closed curve is proportional to the current threading or passing through the closed circuit i.e.

$$\oint \mathbf{B}\cdot\mathbf{dI} = \mu_0 I$$

where μ_0 is the permeability of free space.
Biot-Savart law and Ampere's circuital law are used to find the magnetic field due to current carrying conductors.

24.3 MAGNETIC FIELD DUE TO CURRENT CARRYING CONDUCTORS

(i) *Magnetic field at point P due to an infinitely long wire carrying a current I* (Fig. 24.1)

$B = \frac{\mu_0 I}{2\pi r}$ directed into the page (away from the reader) if the I is upwards and towards the reader if I is downwards. At points Q or S, $B = 0$

(ii) *Magnetic field at the center of a circular loop of radius r* (Fig. 24.2)

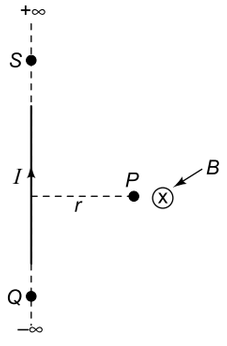


Fig. 24.1

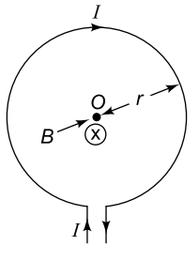


Fig. 24.2

$B = \frac{\mu_0 I}{2r}$ directed into the page if I is clockwise and outside the page if I is anticlockwise.

For a coil of N turns.

$$B = \frac{\mu_0 NI}{2r}$$

(iii) *Magnetic field at the centre of a curved element* (Fig. 24.3).

$$B = \frac{\mu_0 I}{2r}\times\frac{\theta}{2\pi}$$

directed into the page.
For a semi-circular element ($\theta = \pi$)

$$B = \frac{\mu_0 I}{4r}$$

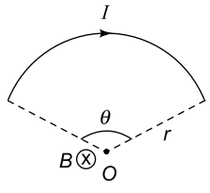


Fig. 24.3

(iv) *Magnetic field at point P due to a straight wire XY of finite length* (Fig. 24.4)

$$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

directed into the page

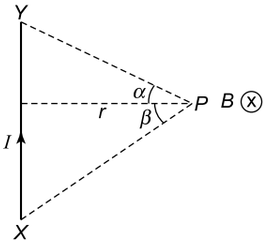


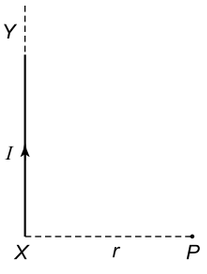
Fig. 24.4

Special Cases

(a) If the conductor *XY* is of infinite length and point *P* lies near the centre of the conductor (as in Fig. 24.1), $\alpha = \beta = 90°$ so

$$B = \frac{\mu_0 I}{4\pi r}(\sin 90° + \sin 90°)$$

$$= \frac{\mu_0 I}{2\pi r}$$



(b) If the conductor *XY* is of infinite length but point *P* lies near the end *X* or *Y* as shown in Fig. 24.5, then $\alpha = 90°$ and $\beta = 0°$, then

Fig. 24.5

$$B = \frac{\mu_0 I}{4\pi r}(\sin 90° + \sin 0°)$$

$$= \frac{\mu_0 I}{4\pi r}$$

Note: *For an infinitely long straight conductor carrying a current, the magnetic field near its centre is twice that near one of its ends.*

(c) If the conductor *XY* has a finite length *L* and point *P* lies on the right bisector of the conductor, as shown in Fig. 24.6, then

$\alpha = \beta$ and

$$\sin\alpha = \sin\beta = \frac{L/2}{x}$$

$$= \frac{L/2}{\sqrt{r^2 + \left(\frac{L}{2}\right)^2}}$$

$$= \frac{L}{\sqrt{4r^2 + L^2}}$$

so $B = \dfrac{\mu_0 I}{2\pi r} \cdot \dfrac{L}{\sqrt{4r^2 + L^2}}$

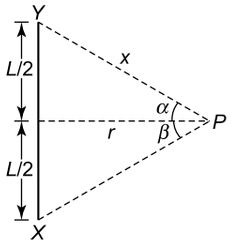


Fig. 24.6

(d) If the point *P* lies on the straight conductor or on its axis, then **dl** and **r** for each element of the straight conductor are parallel. Therefore, **dl** × **r** = 0. Hence **B** = 0 at point *P*.

(v) *Magnetic field at centre due to a wire PQRST* (Fig. 24.7)

Magnetic field at *O* due to straight portions *PQ* and *ST* is zero and due to semicircular part *QRS* is

$$B = \frac{\mu_0 I}{4r}$$ directed into the page.

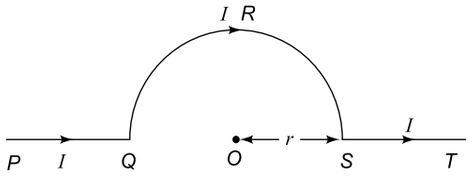


Fig. 24.7

If the current in anticlockwise, *B* is directed towards the reader.

(vi) *Magnetic field at Centre O of a rectangular coil (Fig. 24.8)*

$$B = \frac{2\mu_0 I}{\pi} \times \frac{\sqrt{a^2 + b^2}}{ab}$$ directed into the page

For a square coil ($b = a$)

$$B = \frac{2\sqrt{2}\,\mu_0 I}{\pi a}$$

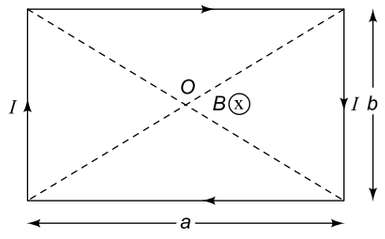


Fig. 24.8

(vii) *Magnetic field due to a hollow metal pipe of radius R carrying current in its walls* (Fig. 24.9)

At point *P*, $B = \dfrac{\mu_0 I}{2\pi r}$

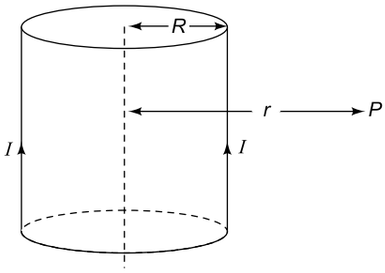


Fig. 24.9

For a solid pipe of radius R (Fig. 24.9)
Inside the pipe at a point at a distance r from the axis,

$$B = \frac{\mu_0 Ir}{2\pi R^2}$$

Outside the pipe at a point P at a distance r from the axis

$$B = \frac{\mu_0 I}{2\pi r}$$

(viii) *Magnetic field on the axis of a circular coil of radius R* (Fig. 24.10)

At point P, $B = \frac{\mu_0}{4\pi} \frac{2M}{(R^2 + r^2)^{3/2}}$

where $M = IA = I \times \pi R^2$ is the magnetic moment. If current I is anticlockwise **B** is directed from O to P. For clockwise current **B** is form P to O.

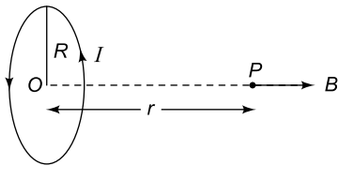


Fig. 24.10

(ix) *Magnetic field due to an electron (charge e) revolving in a circular orbit of radius r with speed r with speed v and frequency n* (Fig. 24.11)

Current along orbit $I = \frac{e}{T} = en = \frac{ev}{2\pi r}$

The direction of I is opposite to the direction of motion of the electron. Magnetic field at O is

$$B = \frac{\mu_0 I}{4r}$$

where $I = \frac{ev}{2\pi r}$ and is directed into the page.

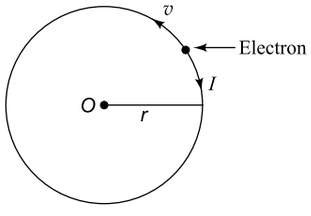


Fig. 24.11

Magnetic moment $M = IA = I \times \pi r^2 = \frac{evr}{2}$

(x) *Magnetic field due to a current carrying straight solenoid*
In the middle region $B = \mu_0 nI$; n = no. of turns per unit length.

At the ends of solenoid, $B = \frac{\mu_0 nI}{2}$

For a toroid of radius R, $B = \mu_0 nI$, where $n = \frac{N}{L} = \frac{N}{2\pi R}$; N = total no. of turns. Outside the solenoid, $B = 0$.

EXAMPLE 24.1

Figure 24.12 shows two stationary and infinitely long bent wires PQR and STU lying in the x-y plane and each carrying a current I as shown. Find the magnitude and direction of the magnetic field at origin O. Given $OQ = OT = a$

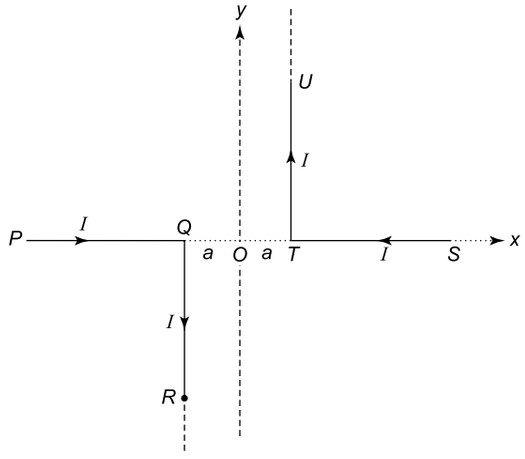


Fig. 24.12

SOLUTION

As point O is along the line segments PQ and ST, the magnetic field at O due to PQ and ST is zero. The magnetic field at O due to wires QR and TU respectively are

$$\mathbf{B}_1 = \frac{\mu_0 I\,(\hat{\mathbf{k}})}{4\pi(OQ)} \text{ and } \mathbf{B}_2 = \frac{\mu_0 I\,(\hat{\mathbf{k}})}{4\pi(OT)}$$

both directed along the positive z-axis. The resultant field at O is ($\because OQ = OT = a$)

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = 2 \times \frac{\mu_0 I}{4\pi a}(\hat{\mathbf{k}}) = \frac{\mu_0 I}{2\pi a}\hat{\mathbf{k}}$$

EXAMPLE 24.2

Two infinitely long wires carrying equal current I in the opposite direction are placed perpendicular to the x-y plane. One wire is located at point P (0, a, 0) and the other wire at Q (0, $-a$, 0). Find the magnitude and direction of at point $A(x, 0, 0)$.

SOLUTION

Refer to Fig. 24.13. Wire 1 carries a current I along the positive z-direction and wire 2 carries a current I along the negative z-direction.

$$OP = OQ = a,\ OA = x,\ PA = QA = r.$$

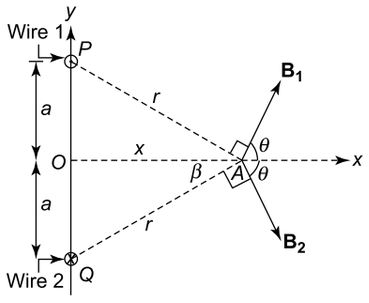


Fig. 24.13

Magnetic field at A due to wire 1 is

$$B_1 = \frac{\mu_0 I}{2\pi(PA)} = \frac{\mu_0 I}{2\pi\sqrt{a^2 + x^2}}$$

According to Biot-Savart law, $\mathbf{B}_1$ is perpendicular to both PA and wire 1 and therefore in the x-y plane. Similarly, magnetic field at A due to wire 2 is

$$B_2 = \frac{\mu_0 I}{2\pi\sqrt{a^2 + x^2}}$$

The y-components of $\mathbf{B}_1$ and $\mathbf{B}_2$ cancel each other but the x-components add up. These components are $B_1 \cos\theta$ and $B_2 \cos\theta$ both along the positive x-direction. Therefore, the resultant magnetic field at A is

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = (B_1 \cos\theta + B_2 \cos\theta)\,\hat{\mathbf{i}}$$

$$= \frac{\mu_0 I}{\pi\sqrt{(a^2 + x^2)}} \times \frac{a}{\sqrt{a^2 + x^2}} \times (\hat{\mathbf{i}})$$

$$\left(\because \cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2 + x^2}}\right)$$

$$= \frac{\mu_0 I\, a}{\pi(a^2 + x^2)}\,(\hat{\mathbf{i}})$$

EXAMPLE 24.3

Two wires A and B have the same length L and carry equal currents I. Wire A is bent into a circle and wire B is bent into a square. (a) Obtain expression for the magnitude of the magnetic field at (*i*) the centre of the circular loop and (*ii*) the centre of the square. Which wire produces a greater magnetic field at the centre?

SOLUTION

(a) (i) Radius r of wire A when it is bent into a circle is given by

$$2\pi r = L \Rightarrow r = \frac{L}{2\pi}$$

Magnetic field at the centre of the circular loop is

$$B_1 = \frac{\mu_0 I}{2r} = \frac{\mu_0 I \pi}{L} \qquad (1)$$

(ii) Refer to Fig. 24.14. The magnetic field at O due to wire PQ is ($OT = a$)

$$B_{PQ} = \frac{\mu_0 I}{4\pi a}(\sin 45° + \sin 45°)$$

$$= \frac{\mu_0 I}{2\sqrt{2}\,\pi a}$$

$$= \frac{4\,\mu_0 I}{\sqrt{2}\,\pi L} \qquad \left(\because a = \frac{L}{8}\right)$$

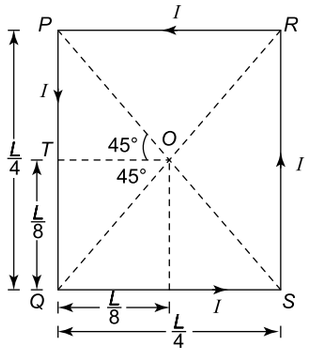


Fig. 24.14

Since centre O of the square is at the same distance from each side of the square and each arm carries the same current, the magnetic field due to each side of the square is of the same and in the same direction. Hence the total magnetic field at O is

$$B_2 = 4B_{PQ} = \frac{16\,\mu_0 I}{\sqrt{2}\,\pi L} = \frac{8\sqrt{2}\,\mu_0 I}{\pi L} \qquad (2)$$

(b) Dividing (2) by (1) we get

$$\frac{B_2}{B_1} = \frac{8\sqrt{2}}{\pi^2} = \frac{8 \times 1.41}{(3.14)^2} = 1.16$$

Hence $B_2 > B_1$. The magnetic field at the centre due to the square loop will be greater than that due to the circular loop.

EXAMPLE 24.4

Figure 24.15 shows a wire loop *ABCDEA* carrying a current *I* as shown.

Given $AE = ED = a$ and $AB = CD = a/2$.

Find the magnitude and direction of the magnetic field at point *F* where $BF = CF = a/2$.

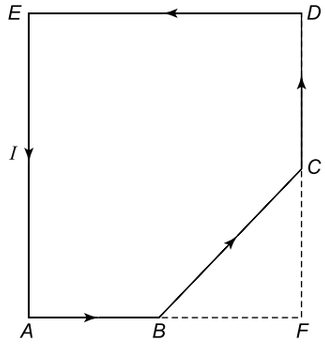


Fig. 24.15

SOLUTION

Magnetic field at *F* is

$$\mathbf{B} = \mathbf{B}_{AB} + \mathbf{B}_{BC} + \mathbf{B}_{CD} + \mathbf{B}_{DE} + \mathbf{B}_{EA}$$

Since point *F* lies in line with current elements *AB* and *CD*, $B_{AB} = B_{CD} = 0$

Also $B_{DE} = B_{EA} = \dfrac{\mu_0 I}{4\pi a}(\sin 0° + \sin 45°) = \dfrac{\mu_0 I}{4\sqrt{2}\,\pi a}$

directed out of the page and towards the reader.

$$B_{BC} = \frac{\mu_0 I}{4\pi\, BC/2}(\sin 45° + \sin 45°)$$

directed into the page and away from the reader. Now

$$BC = \sqrt{BF^2 + FC^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$$

$$\therefore \quad B_{BC} = \frac{\mu_0 I}{4\pi\left(\dfrac{a}{2\sqrt{2}}\right)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{\pi a}$$

directed into the page.

Now $B_{DE} + B_{EA} = \dfrac{\mu_0 I}{2\sqrt{2}\pi a}$ directed out of the page.

Since $B_{BC} > B_{DE} + B_{EA}$ the net field, **B** is directed into the page and has a magnitude

$$B = \frac{\mu_0 I}{\pi a} - \frac{\mu_0 I}{2\sqrt{2}\,\pi a} = \frac{\mu_0 I}{\pi a}\left(1 - \frac{1}{2\sqrt{2}}\right)$$

EXAMPLE 24.5

A wire *ABCDE* is bent as shown in Fig. 24.16. The wire carries a current *I* and the radius of the bent coil *BCD* is *r*. Find the magnitude and direction of the magnetic field at centre *O*.

SOLUTION

The straight line segments *AB* and *DE* are collinear with *O*. Hence the magnetic field due to *AB* and *DE* at *O* is zero. Angle subtended at *O* by arc $BCD = 2\pi - \theta$. The magnetic field due to *BCD* at *O* is

$$B = \frac{\mu_0 I}{2r}\left(\frac{2\pi - \theta}{2\pi}\right)$$

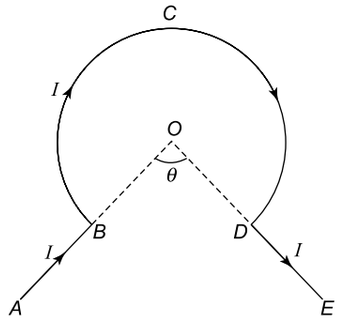


Fig. 24.16

The current through *BCD* is clockwise. Therefore, the direction of the magnetic field at *O* is into the page and away from the reader.

EXAMPLE 24.6

A wire *ABCEF* is bent as shown in Fig. 24.17 and caries a current *I*. The radius of the smaller arc *ABC* is $r_1 = r$ and that of the bigger arc is $r_2 = 2r$. Find the magnitude of the magnetic field at centre *O*.

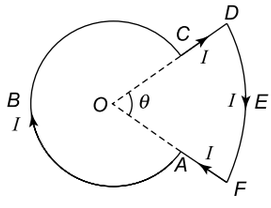


Fig. 24.17

SOLUTION

Magnetic field due to arc *ABC* at *O* is

$$B_1 = \frac{\mu_0 I}{2r_1}\left(\frac{2\pi - \theta}{2\pi}\right)$$

Magnetic field due to arc *DEF* at *O* is

$$B_2 = \frac{\mu_0 I}{2r_2}\cdot\frac{\theta}{2\pi}$$

since B_1 and B_2 are both directed into the page, the total magnetic field at O is

$$B = B_1 + B_2 = \frac{\mu_0 I}{2r_1}\left(\frac{2\pi - \theta}{2\pi}\right) + \frac{\mu_0 I}{2r_2}\frac{\theta}{2\pi}$$

Putting $r_1 = r$ and $r_2 = 2r$, we get

$$B = \frac{\mu_0 I}{2r}\left(1 - \frac{\theta}{4\pi}\right)$$

EXAMPLE 24.7

A long straight cylinder of radius R carries a current I which is uniformly distributed across its cross-section. Find the magnetic field at a point at a distance r from the axis of the cylinder in cases (a) $r > R$ and (b) $r < R$.

SOLUTION

Figure 24.18 shows the cross-sectional view of the cylinder.

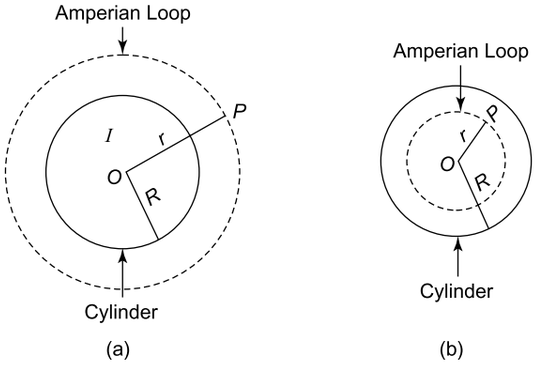


Fig. 24.18

Case (a) $r > R$. For this case, the Amperian loop is a circle of radius r concentric with the cross-section [Fig. 24.18 (a)]. For this loop, $L = 2\pi r$ and the current threading the loop is $i = I$. From Ampere's circuital law.

$$BL = \mu_0 i \Rightarrow B \times 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Case (b) $r < R$. For this case, $L = 2\pi r$ [Fig. 24.18 (b)] and the current threading the loop is

i = current per unit cross-sectional area of the cylinder × cross-sectional area of the Amperian loop

$$= \frac{I}{\pi R^2} \times \pi r^2 = \frac{Ir^2}{R^2}$$

From Ampere's law,

$$B \times 2\pi r = \mu_0 i = \frac{\mu_0 I r^2}{R^2}$$

$$\Rightarrow \qquad B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$$

NOTE

(1) For $r < R$; $B \propto r$ and for $r > R$; $B \propto \frac{1}{r}$. Figure 24.19 shows the variation of B with r.

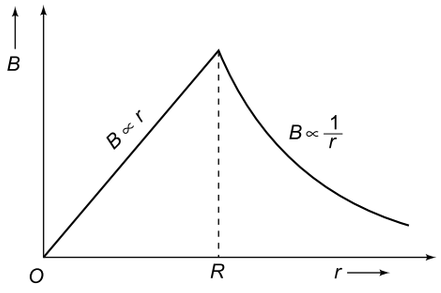


Fig. 24.19

(2) For a hollow cylinder, the current flows along its walls. Therefore, in the case $r < R$ [Fig. 24.18 (b)], no current threads the Amperian loop. Hence $B = 0$ for points inside a hollow cylinder.

EXAMPLE 24.8

The current density J (current per unit area) in a solid cylinder of radius R varies with distance r from its axis as $J = kr$ where k is a constant. Find the magnetic field at a point P where (a) $r > R$ and (b) $r < R$.

SOLUTION

Current $I = \int JdA = \int kr \times (2\pi r dr)$

Case (a) We take the Amperian loop of radius $r > R$. Since the loop is outside the cylinder, the current through the loop is

$$I = \int_0^R kr \times (2\pi r dr) = 2\pi k \int_0^R r^2 dr = \frac{2\pi\, kR^3}{3}$$

$$\therefore \qquad B \times 2\pi r = \mu_0 I = \frac{2\pi\, kR^3}{3} \Rightarrow B = \frac{\mu_0 k R^3}{r}$$

Case (b) For $r < R$, the current through the Amperian loop is

$$I = \int_0^r kr(2\pi r dr) = \frac{2\pi k r^3}{3}$$

$$\therefore \quad B \times 2\pi r = \mu_0 I = \frac{\mu_0 \times 2\pi k r^3}{3} \Rightarrow B = \frac{\mu_0 k r^2}{3}$$

24.4 FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

The force on a charge q moving with a velocity **v** in a magnetic field **B** is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

The direction of **F** is perpendicular to both **v** and **B**. The magnitude F of vector **F** is given by

$$\mathbf{F} = qvB \sin\theta$$

where θ is the angle between vectors **v** and **B**.

(1) **F** = 0 if **v** = 0, i.e. a charge at rest does not experience any magnetic force.

(2) **F** = 0 if θ = 0 or 180°, i.e. the magnetic force vanishes if **v** is either parallel or antiparallel to the direction of **B**.

(3) F is maximum = F_{max} if θ = 90°, i.e. if **v** is perpendicular to **B**, the magnetic force has a maximum value given by

$$F_{max} = qvB$$

The direction of the force when **v** ⊥ **B** is given by Fleming's left hand rule.

(4) If **v** is perpendicular to both **E** and **B** and **E** is perpendicular to **B**, then **F** = 0 if $v = \frac{E}{B}$.

24.5 MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

Case (a): If **v** is perpendicular to **B**, the particle describes a circle in the region of the magnetic field because **F** ⊥ **v**.

(i) The speed along the circular path is constant.

(ii) The kinetic energy is constant.

(iii) Velocity and momentum continually change.

(iv) The radius r of the circular path is given by $\frac{mv^2}{r}$

qvB

$$\Rightarrow r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where m = mass of particle and K = kinetic energy. If the particle is accelerated through a potential difference V, then $K = qV$.

(v) Time period of revolution is $T = \frac{2\pi m}{qB}$

(vi) Frequency of revolution is $v = \frac{qB}{2\pi m}$ which is independent of both v and r.

Case (b): If **v** is inclined to **B** at an angle θ, the particle moves in a helical path. The radius of helix is $r = \frac{mv \sin\theta}{qB}$, time period $T = \frac{2\pi m}{qB}$ and pitch of the helix = $v \cos\theta \times T$

Applications

(i) *Particle moving horizontally and entering a region of magnetic field B as shown in Fig. 24.20. Particle describes a semi-circle of radius.*

$$r = \frac{mv}{qB}$$

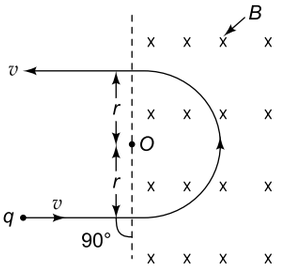


Fig. 24.20

Time spend in the region of magnetic field is

$$t = \frac{T}{2} = \frac{\pi m}{qB}$$

(ii) *If the particle enters the region of magnetic field as shown in Fig. 24.21, then*

$$r = \frac{mv}{qB}$$

and $$t = \frac{2\theta m}{qB}$$

where θ is in radian.

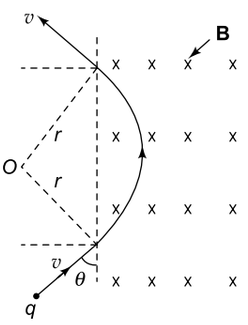


Fig. 24.21

(iii) *In Fig. 24.22, the particle wall not be able to hit the wall if d > r, i.e.*

$$d > \frac{mv}{qB} \Rightarrow B > \frac{mv}{qd}$$

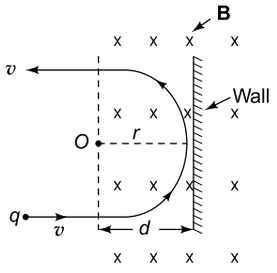


Fig. 24.22

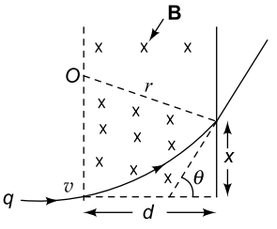


Fig. 24.23

(iv) *If $d < r$, as shown in Fig. 24.23, the deflection θ when the particle leaves the field is given by*

$$\sin\theta = \frac{d}{r} = \frac{qBd}{mv}$$

Linear defection $x = r(1 - \cos\theta)$

EXAMPLE 24.9

An electron emitted from a hot filament is accelerated through a potential difference of 18 kV and enters a region of a uniform magnetic field of 0.1 T with a certain initial velocity. What is the trajectory of the electron if the magnetic field (a) is transverse to the initial velocity and (b) makes an angle of 30° with the initial velocity? Mass of electron = 9×10^{-31} kg.

SOLUTION

$V = 18 \times 10^3$ V

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \left(\frac{2eV}{m}\right)^{1/2}$$

$$= \left[\frac{2 \times (1.6 \times 10^{-19}) \times (18 \times 10^3)}{9 \times 10^{-31}}\right]^{1/2} = 8 \times 10^7 \text{ ms}^{-1}$$

(a) Since v is $\perp$ to **B**, $\theta = 90°$, the trajectory of the electron is circular having a radius

$$r = \frac{mv}{eB} = \frac{(9 \times 10^{-31}) \times (8 \times 10^7)}{(1.6 \times 10^{-19}) \times 0.1}$$

$$= 4.5 \times 10^{-3} \text{ m} = 4.5 \text{ mm}$$

(b) The trajectory of the electron is helical. The radius of heix is

$$r = \frac{mv_\perp}{eB} = \frac{mv}{eB} \times \sin\theta$$

$$= 4.5 \text{ mm} \times \sin 30° = 2.25 \text{ mm}$$

EXAMPLE 24.10

A long straight wire lying along the y-axis carries a current of 10 A along the positive y-direction. A proton moving with a velocity of 10^7 ms^{-1} is at a distance 5 cm from the wire at a certain instant. Find the magnitude and direction of the force acting on the proton at that instant if its velocity is directed

(a) along the negative x-direction
(b) along the positive y-direction and
(c) along the positive z-direction

SOLUTION

Refer to Fig. 24.24.

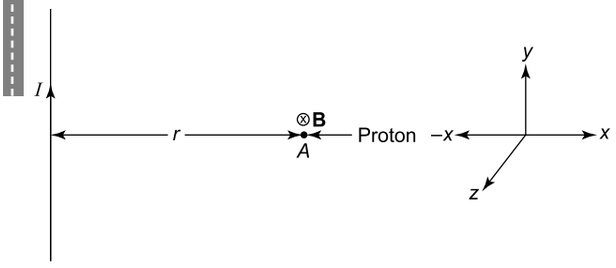


Fig. 24.24

Magnetic field at A is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.05} = 4 \times 10^{-5} \text{ T}$$

directed inwards along the negative z-direction

(a) $\theta = 90°$. Therefore, force on proton is
$F = qvB \sin\theta$
$= (1.6 \times 10^{-19}) \times 10^7 \times (4 \times 10^{-5}) \times \sin 90°$
$= 6.4 \times 10^{-17}$ N
According to Fleming's L.H. rule, the direction of the force is parallel to the wire and opposite to the direction of current I, i.e. **F** is along the negative y-direction

(b) $\theta = 90°$, $F = qvB = 6.4 \times 10^{-17}$ N. The force is directed towards the wire, i.e. along negative x-direction

(c) $\theta = 180°$. $F = qvB \sin 180° = 0$

EXAMPLE 24.11

A proton and an α-particle move perpendicular to a uniform magnetic field. The mass of an α-particle is four times that of a proton and its charge is twice that of a proton. Find the ratio of radii of the circular path followed by them if both

(a) have equal velocities,
(b) have equal linear momenta,
(c) have equal kinetic energies and
(d) are accelerated through the same potential difference.

SOLUTION

Given $\frac{m_\alpha}{m_p} = 4$ and $\frac{q_\alpha}{q_p} = 2$

(a) $r = \dfrac{mv}{qB} \Rightarrow r_p = \dfrac{m_p v}{q_p B}$ and $r_\alpha = \dfrac{m_\alpha v}{q_\alpha B}$

$\therefore \dfrac{r_p}{r_\alpha} = \dfrac{m_p}{m_\alpha} \times \dfrac{q_\alpha}{q_p} = \dfrac{1}{4} \times 2 = \dfrac{1}{2}$

(b) $r = \dfrac{mv}{qB}$

Kinetic energy $K = \dfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2K}{m}}$

$\therefore r = \dfrac{m}{qB} \times \sqrt{\dfrac{2K}{m}} = \dfrac{1}{qB}\sqrt{2\,mK}$

$\therefore \dfrac{r_p}{r_\alpha} = \dfrac{q_\alpha}{q_p} \times \sqrt{\dfrac{m_p}{m_\alpha}} = 2 \times \sqrt{\dfrac{1}{4}} = 1$

(c) $r = \dfrac{mv}{qB} = \dfrac{p}{qB}$

$\therefore \dfrac{r_p}{r_\alpha} = \dfrac{q_\alpha}{q_p} = 2$

(d) $K = qV$. Therefore

$r = \dfrac{1}{qB}\sqrt{2mqV} = \dfrac{1}{B}\sqrt{\dfrac{2mV}{q}}$

$\therefore \dfrac{r_p}{r_\alpha} = \sqrt{\dfrac{m_p}{m_\alpha} \times \dfrac{q_\alpha}{q_p}} = \sqrt{\dfrac{1}{4} \times 2} = \dfrac{1}{\sqrt{2}}$

24.6 FORCE ON A CURRENT CARRYING CONDUCTOR IN A MAGNETIC FIELD

(i) *Force on a straight conductor placed perpendicular to magnetic field (Fig. 24.25).*
$F = BIL$ upwards if current I is from left to right and downwards if I is from right to left (given by Fleming's left hand rule)

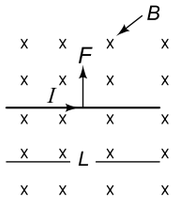


Fig. 24.25

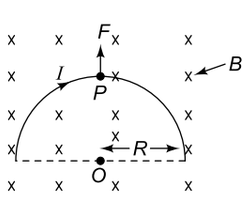


Fig. 24.26

(ii) *Force at point P on a semicircular wire of radius R (Fig. 24.26)*
$F = BI(2R) = 2BIR$ vertically upward for clockwise current and downward for anticlockwise current

(iii) *Force on a circular wire of radius R (Fig. 24.27)*
Net force $F = F_1 - F_2 = 0$

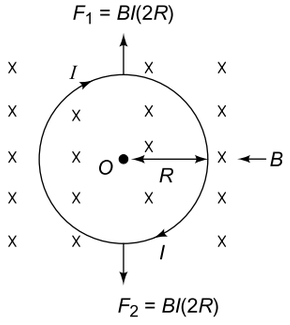


Fig. 24.27

(iv) *Force per unit length between two long straight and parallel wires carrying currents I_1 and I_2 and separated by distance r is*

$$f = \frac{\mu_0 I_1 I_2}{2\pi r}$$

attractive if I_1 and I_2 are in the same direction and repulsive if I_1 and I_2 are in opposite directions. Force on a segment of length l of either wire is $F = f \times l$.

(v) *Force on a rod carrying a current I_1 placed at a distance x from an infinitely long wire carrying a current I_2 as shown in Fig. 24.28.*

$F = \dfrac{\mu_0 I_1 I_2}{2\pi r} \log_e\left(1 + \dfrac{L}{x}\right)$ vertically upwards.

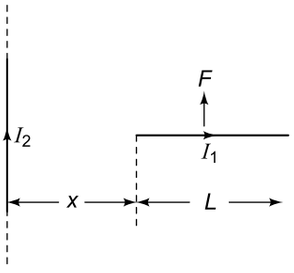


Fig. 24.28

(vi) *Force on a rectangular coil carrying a current I_1 placed at a distance x from an infinitely long wire carrying a current I_2 as shown in Fig. 24.29.*

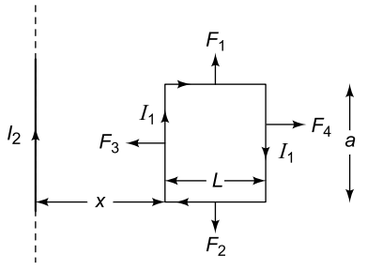


Fig. 24.29

Force F_1 and F_2 being equal and opposite cancel and F_3 and F_4 are given by expression above. Net force on coil is

$$F = F_3 - F_4 = \frac{\mu_0 I_1 I_2 a}{2\pi x} - \frac{\mu_0 I_1 I_2 a}{2\pi(x+L)}$$

$$\Rightarrow \qquad F = \frac{\mu_0 I_1 I_2 aL}{2\pi x(x+L)}$$

directed towards the wire (attractive)

EXAMPLE 24.12

The battery of a car is connected to the motor by 50 cm long wires which are 1.0 cm apart. If the current in the wires is 200 A, find the force between the wires. Is the force attractive or repulsive.

SOLUTION

Force per unit length is

$$f = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7}) \times 200 \times 200}{2\pi \times (1.0 \times 10^{-2})} = 0.8 \text{ Nm}^{-1}$$

$\therefore F = f \times l = 0.8 \times 0.5 = 0.4$ N

Since the currents in the wires are in opposite direction, the force is repulsive.

EXAMPLE 24.13

A small rectangular loop $ABCD$ of sides 5 cm and 3 cm carries a current of 5 A. It is placed with its longer side parallel to a long straight conductor PQ of length 5 m at a distance of 2 cm from it as shown in Fig. 24.30. If the current in PQ is 20 A, find the net force on the loop. Is the loop attracted towards PQ or repelled away from it?

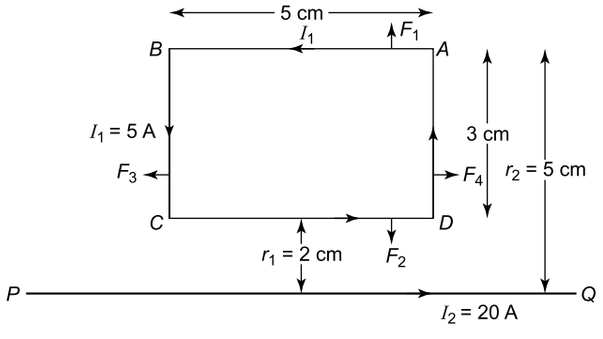


Fig. 24.30

SOLUTION

Force exerted by PQ on AB is

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi r_1} \times AB$$

$$= \frac{(4\pi \times 10^{-7}) \times 5 \times 20 \times 5 \times 10^{-2}}{2\pi \times 5 \times 10^{-2}}$$

$= 2 \times 10^{-5}$ N (repulsive since I_1 and I_2 are in opposite directions)

Force exerted by PQ on CD is

$$F_2 = \frac{\mu_0 I_1 I_2}{2\pi r_2} \times CD$$

$$= \frac{(4\pi \times 10^{-7}) \times 5 \times 20 \times 5 \times 10^{-2}}{2\pi \times 2 \times 10^{-2}}$$

$= 5 \times 10^{-5}$ N (attractive since I_1 and I_2 are in the same direction)

From Fleming's L.H. rule the magnetic field due to current in PQ is directed outwards (towards the reader) and perpendicular to the plane of the coil. Therefore, forces F_3 and F_4 on BC and AD are equal and opposite and hence cancel each other. Therefore, the net force on coil $ABCD$ is

$$F = F_2 - F_1 = 5 \times 10^{-5} - 2 \times 10^{-5} = 3 \times 10^{-5} \text{ N}$$

(attractive). Hence coil is attracted towards PQ.

EXAMPLE 24.14

A particle of charge q and mass m moving in region I with a velocity v enters normally a region II of width d where a uniform magnetic field B (directed inwards) exists as shown in Fig. 24.31. There is no magnetic field in regions I and III.

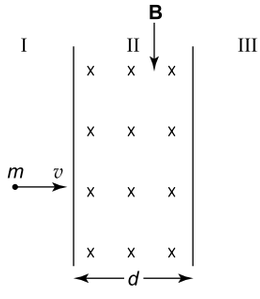


Fig. 24.31

(a) What is the maximum speed (v_{max}) of the particle so that it returns back in region I?

(b) What will happen if $v = \sqrt{2}\, v_{max}$?

SOLUTION

(a) Refer to Fig. 24.32(a). The particle describes a circular path of radius $r = mv/qB$ in region II. It will return to region I if it describes a semicircle in region II. This happens if

$$r < d \Rightarrow \frac{mv}{qB} < d \text{ or } v < \frac{qBd}{m}$$

$$\therefore v_{max} = \frac{qBd}{m}$$

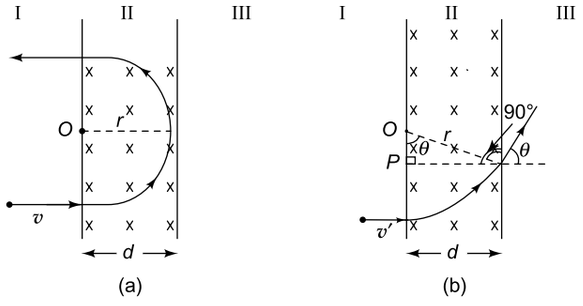


Fig. 24.32

(b) Refer to Fig. 24.32 (b). If $v > v_{max}$, the particle is cross over to region III after describing a circular trajectory in rejoin II with O as the centre. In region III, te particle is move along the tangent at Q. The particle will suffer a deviation θ.

In triangle OPQ

$$\sin\theta = \frac{PQ}{OQ} = \frac{d}{r}$$

$$\Rightarrow \sin\theta = \frac{qBd}{mv} = \frac{v_{max}}{v}$$

If $v = \sqrt{2}\, v_{max}$, then $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

EXAMPLE 24.15

A straight horizontal conducting rod of length 50 cm and mass 60 g is suspended by two vertical wires at its ends. A current of 5 A set up in the rod.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

(b) What will be the tension in the wires if the direction of the current is reversed, keeping the magnetic field the same?

Ignore the mass of the wires and take $g = 10$ m/s^{-2}.

SOLUTION

Refer to Fig. 24.33.

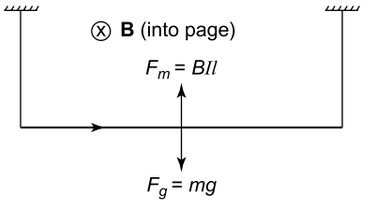


Fig. 24.33

(a) Tension in the wires will be zero if

$$F_m = F_g$$

$$\Rightarrow BIl = m_g$$

$$\Rightarrow B = \frac{mg}{Il} = \frac{(60\times10^{-3})\times10}{5\times(50\times10^{-2})} = 0.24 \text{ T}$$

(b) If current I is reversed, force F_m acts downwards. Hence

Tension $T = BIl + mg$

$= 0.24 \times 5 \times 0.5 + 60 \times 10^{-3} \times 10 = 1.2$ N

24.7 TORQUE ON A CURRENT CARRYING COIL IN A MAGNETIC FIELD

The torque on a coil of N turns, area A carrying a current I in a magnetic field B is given by

$$\vec{\tau} = \mathbf{M} \times \mathbf{B}$$

Magnitude of torque is $\tau = MB \sin\theta = NIAB \sin\theta$ where $M = NIA$ is the magnetic moment and θ is the angle between the normal to the plane of coil and magnetic field.

The magnitude of torque on a coil in radial magnetic field in moving coil galvanometer is

$$\tau = k\alpha$$

where k is the restoring couple per unit twist and α is the deflection of the coil. For radial magnetic field, $\alpha = 90°$. Then

$$NIAB = k\alpha \Rightarrow I = \frac{k\alpha}{NAB} \text{ or } I \propto \alpha$$

Current sensitivity of the galvanometer is

$$C_s = \frac{\alpha}{I} = \frac{NAB}{k}$$

24.8 TORQUE ON A BAR MAGNET IN A MAGNETIC FIELD

The magnetic dipole moment of a bar magnet of pole strength q and length $(2a)$ is defined as

$$\mathbf{M} = q(2\mathbf{a})$$

It is a vector pointing from the south to the north pole of a magnet.

Force on north pole N of magnet = $q\mathbf{B}$ (in the direction of **B**)

Force on south pole S of the magnet = $-q\mathbf{B}$ (opposite to **B**)

Thus the magnetic field exerts two equal, parallel and opposite forces on the magnet. The two forces, therefore, constitute a coupe which tends to rotate the magnet in the clockwise direction. The arm of the couple is 2 **a**. The torque is given by

$$\boldsymbol{\tau} = \text{arm of the coupe} \times \text{force}$$

$$= 2\mathbf{a} \times q\mathbf{B} = q(2\mathbf{a}) \times \mathbf{B}$$

or $$\boldsymbol{\tau} = \mathbf{M} \times \mathbf{B}$$

where $\mathbf{M} = q(2\mathbf{a})$ is called the magnetic moment of the bar magnet. The direction of $\boldsymbol{\tau}$ is perpendicular to both **M** and **B**. If **M** and **B** are both in the plane of the paper then the torque $\boldsymbol{\tau}$ will be perpendicular to the plane of the paper and directed into it away from the reader. The magnitude of the torque is

$$\tau = MB \sin \theta$$

where θ is the angle between **M** and **B**.
The SI unit of **M** is Nm T^{-1} or JT^{-1} (joule per tesla).

24.9 POTENTIAL ENERGY OF A MAGNETIC DIPOLE

The magnetic potential energy of a magnetic dipole in any orientation θ with an external uniform magnetic field **B** is defined as the work that an external agent must do to turn the dipole from its zero energy position ($\theta = 90°$) to the given position. θ.

$$U = -MB \cos \theta$$

In vector notation,

$$\mathbf{U} = -(\mathbf{M} \cdot \mathbf{B})$$

For stable equilibrium U is minimum. Hence $\theta = 0$ and $\tau = 0$. For unstable equilibrium, U is maximum i.e. $\theta = 180°$. Hence $\tau = 0$

24.10 SOME USEFUL TIPS

1. Magnetic dipole moment of a bar magnet is $M = m \times l$, where m is pole strength and l is the length of the magnet. The value of M depends on the volume of the magnet.
   (a) If a magnet is cut into two equal parts by cutting it by a plane along its length, its volume is halved, Hence the magnetic dipole moment of a piece is halved = $M/2$. The pole strength $m = \frac{M}{l}$ is also halved as length l remains the same.
   (b) If a magnet is cut into two equal parts by cutting it by a plane transverse to its length, the volume and length are both halved. Hence the magnetic moment becomes $M/2$ but pole strength m remains the same.
   (c) If a wire of magnetic dipole moment M and length l is bent as shown in Fig. 24.34, the distance between the pole becomes $\frac{1}{\sqrt{2}}$ and magnetic moment becomes

$$M' = m \times \frac{l}{\sqrt{2}} = \frac{M}{\sqrt{2}}$$

   If the wire is bent as shown in Fig. 24.35, the magnetic moment becomes

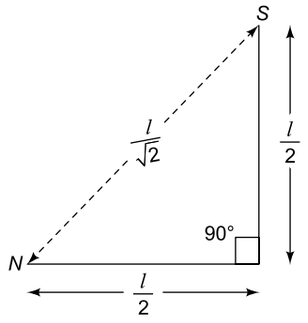


Fig. 24.34

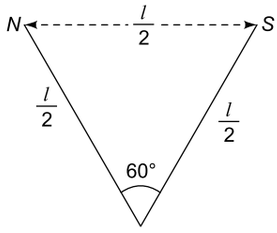


Fig. 24.35

$$M' = m \times \frac{l}{\sqrt{2}} = \frac{M}{\sqrt{2}}$$

NOTE

Magnetic dipole moment M is a vector quantity directed from south pole to north pole.

2. A bar magnet placed in a uniform magnetic field experiences no net force but experiences a torque $\vec{\tau} = \boldsymbol{M} \times \boldsymbol{B}$. The magnitude of $\vec{\tau}$ is $\tau = MB \sin \theta$ where θ is the angle between $\boldsymbol{M}$ and $\boldsymbol{B}$.
   (a) when $\theta = 90°$, $\tau_{max} = MB$
   (b) when $\theta = 0°$, $\tau_{min} = 0$ (stable equilibrium)
   (c) when $\theta = 180°$, $\tau_{min} = 0$ (unstable equilibrium)
   Potential energy is $U = -\boldsymbol{M}.\boldsymbol{B} = -MB \cos \theta$. When $\theta = 0°$, P.E is minimum $U_{min} = -\boldsymbol{MB}$. U is max = $U_{max} = MB$ when $\theta = 180°$
   (d) Work done in rotating the magnet from θ_1 to θ_2 is
   $W = MB (\cos \theta_1 - \cos \theta_2)$
   (e) In a non-uniform magnetic field, a bar magnet experiences a force as well as a torque.
3. The time period of a bar magnet oscillating in a uniform magnetic field is $T = 2\pi\sqrt{\frac{I}{MB}}$, where I is the moment of inertia of the bar magnet = $\frac{ml^2}{12}$, m = mass of magnet and l = length of magnet.
   (a) If a bar magnet is cut into two equal parts by cutting along its length, then each part has $M' = M/2$ and $I' = I/2$. Hence $T' = T$.
   (b) If a bar magnet is cut into two equal parts by cutting perpendicular to its length, then each part has $M' = M/2$ and $I' = I/8$. Hence $T' = T/2$.
   (c) If two bar magnets of magnetic moments M_1 and M_2 are placed one on top of the other as shown n Fig. 24.36, then time period is given by (since $I = I_1 + I_2$ and $M = M_1 + M_2$)

$$T_1 = 2\pi\sqrt{\frac{(I_1 + I_2)}{(M_1 + M_2)B}}$$

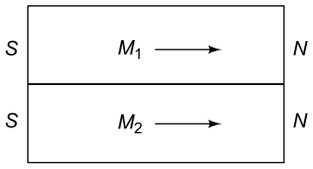


Fig. 24.36

If the magnets are placed as shown in Fig. 24.37, then $I = I_1 + I_2$ but $M = M_1 - M_2$ and $T_2 = 2\pi\sqrt{\frac{(I_1 + I_2)}{(M_1 + M_2)B}}$

T_1 and T_2 are related as $\frac{M_1}{M_2} = \frac{(T_2^2 + T_1^2)}{(T_2^2 - T_1^2)}$

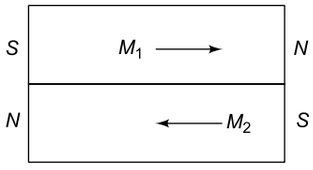


Fig. 24.37

4. Magnetic field due to a bar magnet

(a) At a point P on axial line (Fig. 24.38)

$B_a = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2}$ parallel to $\mathbf{M} = m \times 2\mathbf{l}$.

For a very short magnet ($l << r$), $B_a = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$

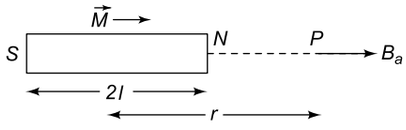


Fig. 24.38

(b) At a point Q on the equatorial line (Fig. 24.39)

$B_e = \frac{\mu_0}{4\pi}\frac{M}{(r^2 + l^2)^{3/2}}$

antiparallel to **M**

For $l << r$,

$B_e = \frac{\mu_0 M}{4\pi r^3}$

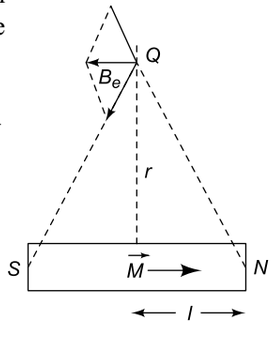


Fig. 24.39

EXAMPLE 24.16

A closely wound solenoid of 1000 turns and area of cross-section 5 cm^2 carries a current of 3 A. It is suspended through its centre (a) what is the magnetic moment? (b) What is the force and torque acting on the solenoid if a uniform magnetic field of 8×10^{-2} T is set up at an angle of 30° with the axis of the solenoid?

SOLUTION

(a) Magnetic moment $M = NIA = 1000 \times 3 \times (5 \times 10^{-4}) = 1.5$ JT^{-1} or Am^2

(b) Since the magnetic field is uniform, the force acting on the solenoid is zero

Torque $\tau = MB \sin\theta = 1.5 \times (8 \times 10^{-2}) \times \sin 30°$
$= 6 \times 10^{-2}$ J

EXAMPLE 24.17

In a hydrogen atom, the electron moves in a circular orbit of radius 0.5 Å making 10^{16} revolutions per second. Calculate the magnetic moment associated with the orbital motion of electron.

SOLUTION

$M = \pi evr^2 = 3.14 \times (1.6 \times 10^{-19}) \times 10^{16} \times (0.5 \times 10^{-10})^2 = 1.26 \times 10^{-23}$ Am^2

EXAMPLE 24.18

A bar magnet is suspended at a place where it is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude $\sqrt{2} \times 10^{-2}$ T. The magnet attains stable equilibrium at an angle of 30° with this field. Find the magnitude of the other field.

SOLUTION

Magnetic field $\mathbf{B}_1$ exerts anticlockwise torque τ_1 to orient **M** along itself and magnetic field $\mathbf{B}_2$ exerts a stockwise torque τ_2 to orient **M** along itself (Fig. 24.40). For equilibrium,

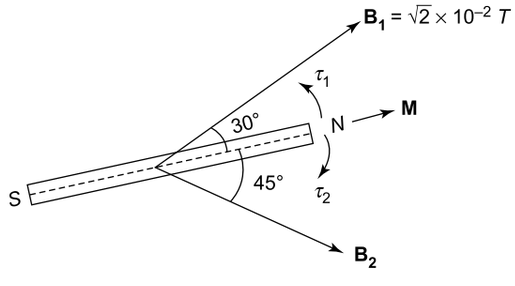


Fig. 24.40

$$\tau_1 = \tau_2$$

$$MB_1 \sin\theta_1 = MB_2 \sin\theta_2$$

$$\Rightarrow \quad B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2} = \frac{\sqrt{2}\times 10^{-2}\times \sin 30^\circ}{\sin 45^\circ} = 10^{-2}\ \text{T}$$

EXAMPLE 24.19

A uniform wire is bent into the shape of an equilateral triangle of side a. It is suspended from a vertex at place where a uniform magnetic field B exists parallel to its plane. Find the magnitude of the torque acting on the coil when a current I is passed through it.

SOLUTION

Area of the coil is ($AB = a$, $BD = a/2$)

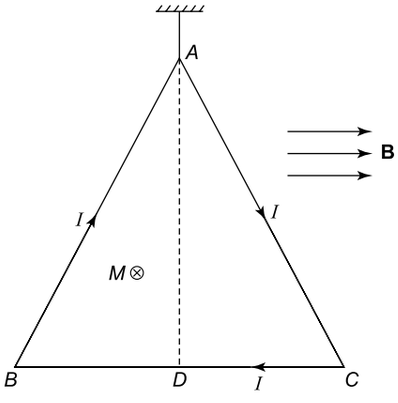


Fig. 24.41

$$A = 2 \times \text{area of triangle } ABD$$

$$= 2 \times \left(\frac{1}{2} \times AD \times BD\right)$$

$$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} a \times \frac{a}{2}$$

$$= \frac{\sqrt{3}}{4} a^2$$

Magnetic moment of the loop is

$$M = IA = I \times \frac{\sqrt{3}}{4} a^2$$

Since the current is clockwise the direction of vector **M** is perpendicular to the plane of the coil directed inwards as shown in Fig. 24.41. Hence $\theta = 90°$. The magnitude of the torque acting on the coil is

$$\tau = MB \sin\theta = \frac{\sqrt{3}}{4} Ia^2B \sin 90^\circ = \frac{\sqrt{3}}{4} Ia^2B$$

EXAMPLE 24.20

A wire loop $ABCD$ carrying a current I_2 is placed on a frictionless horizontal table as shown in Fig. 24.42. A long straight wire PQ carrying a current I_1 is placed at a distance a from side $AB = l$. Find the work done by the magnetic field in shifting the wire from position PQ to position $P'Q'$.

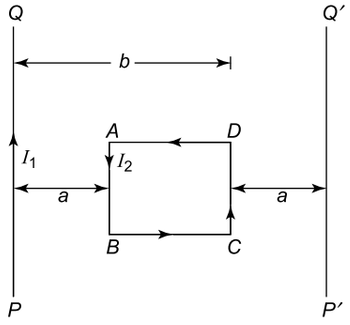


Fig. 24.42

SOLUTION

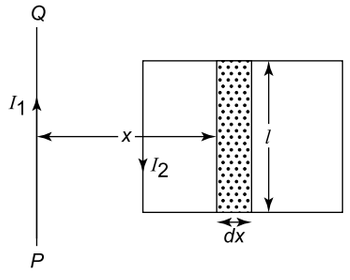


Fig. 24.43

Divide the loop into a large number of elements each of a very small width dx. Consider one such element at a distance x from PQ [see Fig. 24.43]. The magnetic field at any point on the element is

$$B = \frac{\mu_0 I_1}{2\pi x} \text{ directed inwards}$$

Magnetic moment of the element is

$$dM = I_2 \times \text{area of element}$$

$$= I_2 \times l dx$$

Since the current in the coil is anticlockwise, $d\mathbf{M}$ is directed outwards. Hence angle between vectors **B** and $d\mathbf{M}$ is $\theta = 180°$.

Potential energy when the wire is a distance x from the elements is

$$dU = -d\boldsymbol{M}\cdot\mathbf{B} = -dM \times B \cos\theta$$

$$= -I_2\ ldx \times \frac{\mu_0 I_1}{2\pi x}\ \cos\ 180°$$

$$= \frac{\mu_0 I_1 I_2 l}{2\pi}\frac{dx}{x}$$

$\therefore$ Potential energy of the system when the wire is at position PQ is

$$U_{PQ} = \frac{\mu_0 I_1 I_2 l}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 I_1 I_2 l}{2\pi}\ln\left(\frac{b}{a}\right)$$

By symmetry, the potential energy of the system when the wire is shifted to position $P'Q'$ is

$$U_{P'Q'} = -U_{PQ}$$

$\therefore$ Work done in shifting the wire from position PQ to $P'Q'$ is

$$W = -(U_{P'Q'} - U_{PQ})$$

$$= 2U_{PQ} = \frac{\mu_0 I_1 I_2 l}{\pi}\ln\left(\frac{b}{a}\right)$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. The dimensions of $\frac{B^2RC}{\mu_0}$ (where B is magnetic field, R is resistance, C is capacitance and μ_0 is permeability of free space) are the same as those of
(a) impulse
(b) angular momentum
(c) energy
(d) viscosity

2. A current carrying metal wire of diameter 2 mm produces a maximum magnetic field of magnitude 2×10^{-3} T. The current in the wire is
(a) 10 A (b) 20 A
(c) 40 A (d) $40\sqrt{2}$ A

3. A charged particle enters a uniform magnetic field with velocity vector making an angle of 30° with the magnetic field. The particle describes a helical trajectory of pitch x. The radius of the helix is
(a) $\frac{x}{2\pi}$ (b) $\frac{x}{2\sqrt{2}\pi}$
(c) $\frac{x}{2\sqrt{3}\pi}$ (d) $\frac{\sqrt{3}x}{2\pi}$

4. A current carrying wire AB is placed near another CD as shown in Fig. 24.44. Wire CD is fixed while wire AB is free to move. When a current is passed through wire AB, it will have
(a) only translational motion
(b) only rotational motion
(c) both translational as well as rotational motions
(d) neither translational nor rotational motion.

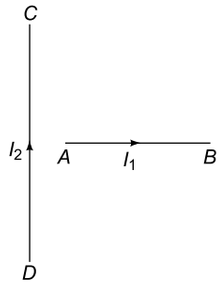


Fig. 24.44

5. A straight wire PQ of length L carries at current I. The magnitude of the magnetic field at point R is [see Fig. 24.45]
(a) zero
(b) $\frac{\mu_0 I}{2\sqrt{2}\pi L}$
(c) $\frac{\mu_0 I}{4\sqrt{2}\pi L}$
(d) $\frac{\mu_0 I}{2\pi L}$

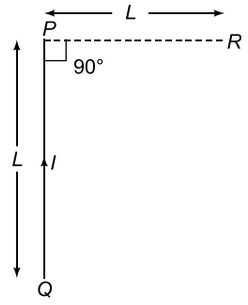


Fig. 24.45

6. A metal rod of length L and mass M is slipping down a smooth inclined plane of inclination 30° with a constant velocity v. The rod carries a current I directed inwards perpendicular to the plane of the paper. A magnetic field directed vertically upwards perpendicular to the length of the rod exists in space. The magnitude of the magnetic field is
(a) $\frac{Mg}{IL}$ (b) $\frac{Mg}{2IL}$

(c) $\frac{\sqrt{3}\,Mg}{2IL}$ (d) $\frac{Mg}{\sqrt{3}\,IL}$

7. Equal current I flows in circular wire segments ACB and ADB of equal radius r as shown in Fig. 24.46. If $\theta = 60°$, the magnetic field at centre O is

(a) $\frac{\mu_0 I}{r}$

(b) $\frac{\mu_0 I}{2r}$

(c) $\frac{\mu_0 I}{3r}$

(d) $\frac{\mu_0 I}{4r}$

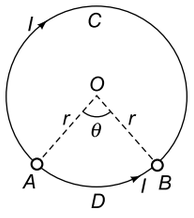


Fig. 24.46

8. The wire loop $PQRSP$ formed by joining two semi–circular wires to radii R_1 and R_2 carries a current I as shown in Fig. 24.47. The magnitude of the magnetic induction at centre C is

(a) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (b) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

(c) $\frac{\mu_0 I}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (d) $\frac{\mu_0 I}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

IIT, 1983

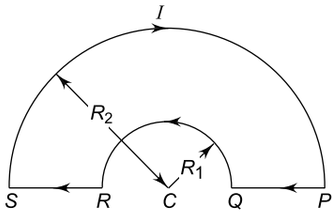


Fig. 24.47

9. The magnetic field at the point of intersection of diagonals of a square loop of side L carrying a current I is

(a) $\frac{\mu_0 I}{\pi L}$ (b) $\frac{2\mu_0 I}{\pi L}$

(c) $\frac{\sqrt{2}\,\mu_0 I}{\pi L}$ (d) $\frac{2\sqrt{2}\,\mu_0 I}{\pi L}$

10. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of mass of X to the mass of Y is

(a) $\left(\frac{R_2}{R_1}\right)^{1/2}$ (b) $\frac{R_2}{R_1}$

(c) $\left(\frac{R_1}{R_2}\right)^{2}$ (d) $\frac{R_1}{R_2}$

IIT, 1988

11. Two identical coils carry equal currents and have a common centre, but their planes are at right angles to each other. What is the magnitude of the resultant magnetic field at the centre if the field due to one coil alone is B?

(a) zero (b) $B/\sqrt{2}$

(c) $\sqrt{2}\,B$ (d) $2B$

12. When a charged particle moves perpendicular to a uniform magnetic field, its

(a) energy and momentum both change
(b) energy changes but momentum remains unchanged
(c) momentum changes but energy remains unchanged
(d) energy and momentum both do not change.

13. A proton with kinetic energy K describes a circle of radius r in a uniform magnetic field. An α–particle with kinetic energy K moving in the same magnetic field will describe a circle of radius

(a) $\frac{r}{2}$ (b) r

(c) $2\,r$ (d) $4\,r$

14. An α–particle moving with a velocity v in a uniform magnetic field is moving in a circular path at frequency ν called the cyclotron frequency. The cyclotron frequency of a proton moving with a speed $2v$ in the same magnetic field will be

(a) $\frac{\nu}{4}$ (b) $\frac{\nu}{2}$

(c) ν (d) $2\,\nu$

15. An electron emitted from a hot filament is accelerated through a potential difference of 2.88 kV and enters a region of a uniform magnetic field of 0.1 T at an angle of 30° with the field. Take the mass of an electron = 9×10^{-31} kg. The trajectory of the electron is a

(a) circle of radius 1.6 mm
(b) circle of radius 0.9 mm
(c) helix of radius 0.9 mm
(d) helix of radius 1.6 mm

16. In the region around a charge at rest, there is

(a) electric field only
(b) magnetic field only

(c) neither electric nor magnetic field
(d) electric as well as magnetic field.

17. In the region around a moving charge, there is
(a) electric field only
(b) magnetic field only
(c) neither electric nor magnetic field
(d) electric as well as magnetic field.

18. An electron is accelerated to a high speed down the axis of a cathode ray tube by the application of a potential difference of V volts between the cathode and the anode. The particle then passes through a uniform transverse magnetic field in which it experiences a force F. If the potential difference between the anode and the cathode is increased to 2 V, the electron will now experience a force
(a) $F/\sqrt{2}$ (b) $F/2$
(c) $\sqrt{2}\ F$ (d) $2\ F$

19. A magnetic needle is kept in a non–uniform magnetic field. It experiences
(a) a force as well as a torque
(b) a force but no torque
(c) a torque but no force
(d) neither a force nor a torque.

20. A conducting circular loop of radius r carries a constant i. It is placed in a uniform magnetic field B such that B is perpendicular to the plane of the loop. The magnetic force acting on the loop is
(a) $i\,rB$ (b) $2\,\pi\,i\,r\,B$
(c) zero (d) $\pi\,i\,r\,B$

IIT, 1983

21. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop. If a steady current I is established in the wire, as shown in Fig. 24.48, the loop will
(a) rotate about an axis parallel to the wire
(b) move away from the wire
(c) move towards the wire
(d) remain stationary.

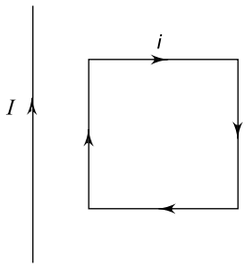


Fig. 24.48

22. A wire of length l metres carrying a current I amperes is bent in the form of a circle. The magnitude of the magnetic moment is
(a) $\dfrac{l\,I^2}{2\pi}$ (b) $\dfrac{l\,I^2}{4\pi}$
(c) $\dfrac{l^2\,I}{2\pi}$ (d) $\dfrac{l^2\,I}{4\pi}$

23. Two circular current carrying coils of radii 3 cm and 6 cm are each equivalent to a magnetic dipole having equal magnetic moments. The currents through the coils are in the ratio of
(a) $\sqrt{2}\ :\ 1$ (b) $2\ :\ 1$
(c) $2\sqrt{2}\ :\ 1$ (d) $4\ :\ 1$

24. An electron of charge e moves in a circular orbit of radius r around a nucleus. The magnetic field due to orbital motion of the electron at the site of the nucleus is B. The angular velocity ω of the electron is
(a) $\omega = \dfrac{2\mu_0\,e\,B}{4\pi r}$ (b) $\omega = \dfrac{\mu_0\,e\,B}{\pi r}$
(c) $\omega = \dfrac{4\pi r B}{\mu_0\,e}$ (d) $\omega = \dfrac{2\pi r B}{\mu_0\,e}$

25. Three long, straight and parallel wires C, D and G carrying currents are arranged as shown in Fig. 24.49. The force experienced by a 25 cm length of wire C is
(a) 0.4 N (b) 0.04 N
(c) 4×10^{-3} N (d) 4×10^{-4} N

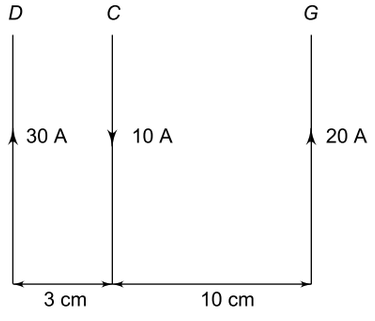


Fig. 24.49

26. A potential difference of 600 V is applied across the plates of a parallel plate capacitor. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of 2×10^6 ms^{-1} moves undeflected between the plates. What is the magnitude of the magnetic field between the capacitor plates?
(a) 0.1 T (b) 0.2 T
(c) 0.3 T (d) 0.4 T

27. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then
(a) the magnetic field at all points inside the pipe is the same but not zero.

(b) the magnetic field at any point inside the pipe is zero.
(c) the magnetic field is zero only on the axis of the pipe.
(d) the magnetic field is different a different points inside the pipe.

IIT, 1993

28. Two straight and long conductors *AOB* and *COD* are perpendicular to each other and carry currents of I_1 and I_2. The magnitude of the magnetic field at a point *P* at a distance *a* from point *O* in a direction perpendicular to the plane *ABCD* is

(a) $\frac{\mu_0}{2\pi a}(I_1 + I_2)$ (b) $\frac{\mu_0}{2\pi a}(I_1 - I_2)$

(c) $\frac{\mu_0}{2\pi a}(I^2_1 + I^2_2)^{1/2}$ (d) $\frac{\mu_0}{2\pi a}\frac{I_1 I_2}{(I_1 + I_2)}$

29. If a magnetic dipole of dipole moment *M* is rotated through an angle θ with respect to the direction of the field *H*, then the work done is
(a) $MH \sin\theta$ (b) $MH(1 - \sin\theta)$
(c) $MH \cos\theta$ (d) $MH(1 - \cos\theta)$

30. A 2 MeV proton is moving perpendicular to a uniform magnetic field of 2.5 T. The force on the proton is
(a) 2.5×10^{-10} N (b) 8×10^{-11} N
(c) 2.5×10^{-11} N (d) 8×10^{-12} N

31. A straight section *PQ* of a circuit lies along the *x*-axis from $x = -\frac{a}{2}$ to $x = \frac{a}{2}$ and carries a current *I*. The magnetic field due to the section *PQ* at point $x = +a$ will be
(a) proportional to *a* (b) proportional to a^2
(c) proportional to $\frac{1}{a}$ (d) equal to zero.

32. Two charged particles *M* and *N* enter a region of uniform magnetic field with velocities perpendicular to the field. The paths of particles are shown in Fig. 24.50. The possible reason is
(a) The charge of *M* is greater than that of *N*
(b) The momentum of *M* is greater than that of *N*
(c) The charge to mass ratio of *M* is greater than that of *N*
(d) The speed of *M* is greater than that of *N*.

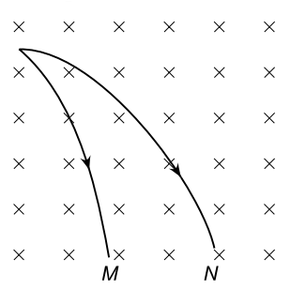


Fig. 24.50

33. The monoenergetic beam of electrons moving along $+y$ direction enters a region of uniform electric and magnetic fields. If the beam goes straight undeflected, then fields **B** and **E** are directed respectively along
(a) $-y$ axis and $-z$ axis
(b) $+z$ axis and $+x$ axis
(c) $+x$ axis and $+z$ axis
(d) $-x$ axis and $-y$ axis

34. A proton of mass 1.67×10^{-27} kg and charge 1.6×10^{-19} C is projected with a speed of 2×10^6 ms^{-1} at an angle of 60° to the *x*-axis. If a uniform magnetic field of 0.104 T is applied along the *y*-axis, the path of the proton is
(a) a circle of radius 0.1 m and time period $2\pi \times 10^{-7}$ s
(b) a circle of radius 0.2 m and time period $\pi \times 10^{-7}$ s
(c) a helix of radius 0.1 m and time period $2\pi \times 10^{-7}$ s
(d) a helix of radius 0.2 m and time period $4\pi \times 10^{-7}$ s

IIT, 1995

35. A proton, a deuteron and an alpha particle having the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote respectively the radii of trajectories of these particles, then
(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$
(c) $r_\alpha = r_d > r_p$ (d) $r_p = r_d = r_\alpha$

IIT, 1997

36. Two particles each of mass *m* and charge *q*, are attached to the two ends of a light rigid rod of length 2*l*. The rod is rotated at a constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is

(a) $\frac{q}{2m}$ (b) $\frac{q}{m}$

(c) $\frac{2q}{m}$ (d) $\frac{q}{\pi m}$

IIT, 1998

37. Two very long straight parallel wires carry steady currents *I* and $-I$. The distance between the wires is *d*. At a certain instant of time, a point charge *q* is at a point equidistant from the two wires, in the plane of the wires. Its instantaneous velocity *v* is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(a) $\frac{\mu_0 I q v}{2\pi d}$ (b) $\frac{\mu_0 I q v}{\pi d}$

(c) $\frac{2\mu_0 I q v}{\pi d}$ (d) zero

IIT, 1998

38. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a
(a) straight line (b) circle
(c) helix (d) cycloid

IIT, 1999

39. An ionized gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the + x direction and a magnetic field along the + z direction, then
(a) positive ions deflect towards + y direction and negative ions towards – y direction
(b) all ions deflect towards + y direction
(c) all ions deflect towards – y direction
(d) positive ions deflect towards – y direction and negative ions towards + y direction

IIT, 2000

40. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper, as shown in Fig. 24.51. The variation of the magnetic field B along the line XX' is given by

IIT, 2000

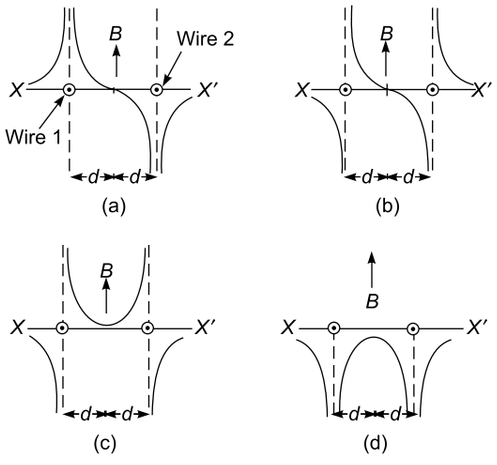


Fig. 24.51

41. An electron moves with a speed of 2×10^5 ms^{-1} along the positive x-direction in a magnetic field $\mathbf{B} = (\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$ tesla. The magnitude of the force (in newton) experienced by the electron is (the charge on electron = 1.6×10^{-19} C)
(a) 1.18×10^{-13} (b) 1.28×10^{-13}
(c) 1.6×10^{-13} (d) 1.72×10^{-13}

42. An electron revolves in a circle of radius 0.4 Å with a speed of 10^6 m/s in Hydrogen atom. The magnetic field produced at the centre of the orbit due to the motion of the electron in tesla is [$\mu_0 = 4\pi \times 10^{-7}$ H/m; Charge on the electron = 1.6×10^{-19} C]
(a) 0.1 (b) 1.0
(c) 10 (d) 100

43. A proton of velocity $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ ms^{-1} enters a field of magnetic induction $(2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ tesla. The acceleration produced in the proton is (charge to mass ratio of proton = 0.96×10^8 C kg^{-1})
(a) 2.8×10^8 $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$
(b) 2.88×10^8 $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
(c) 2.8×10^8 $(2\hat{\mathbf{i}} + 3\hat{\mathbf{k}})$
(d) 2.88×10^8 $(\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

44. A long wire carries a steady current. It is bent into a circular loop of one turn and the magnetic field at the centre of the loop is B. The wire is then bent into a circular coil of n turns and the same current is passed through it. The magnetic field at the centre of coil will be
(a) nB (b) n^2B
(c) $2nB$ (d) $2n^2B$

45. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on its axis at a distance of 4 cm from the centre is 54 μT. The magnetic field (in μT) at the centre of the loop will be
(a) 250 (b) 150
(c) 125 (d) 72

46. A wire $ABCDEF$ (with each side of length L) bent as shown in Fig. 24.52 and carrying a current I is placed in a uniform magnetic field B parallel to the positive y–direction. What is the magnitude and direction of the force experienced by the wire?

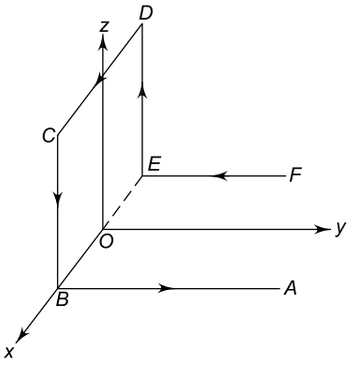


Fig. 24.52

(a) BIL along positive z-direction

(b) BI^2/L along positive z-direction
(c) BIL along negative z-direction
(d) BL/I along negative z-direction

47. A pair of stationary and infinitely long bent wires are placed in the x-y plane as shown in Fig. 24.53. The wires carry a current $I = 10$ A each as shown. The segments RL and SM are along the x-axis. The segments PR and QS are along the y-axis, such that $OS = OR = 0.02$ m. What is the magnitude and direction of the magnetic induction at the origin O?
(a) 100 Wb m^{-2} vertically upward
(b) 10^{-4} Wb m^{-2} vertically downward
(c) 10^{-4} Wb m^{-2} vertically upward
(d) 10^{-2} Wb m^{-2} vertically downward

IIT, 1989

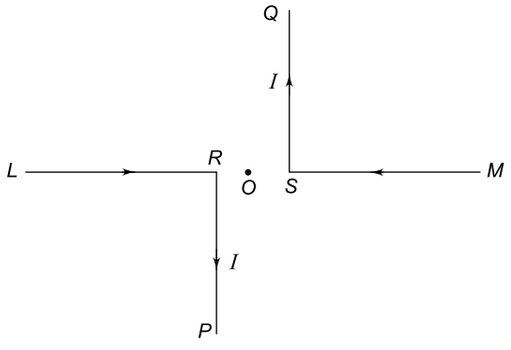


Fig. 24.53

48. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on
(a) ω and q (b) ω, q and m
(c) q and m (d) ω and m

IIT, 2000

49. Two long parallel wires P and Q are held perpendicular to the plane of the paper at a separation of 5 m. If P and Q carry currents of 2.5 A and 5 A respectively in the same direction, then the magnetic field at a point mid-way between P and Q is
(a) $\dfrac{\mu_0}{\pi}$ (b) $\dfrac{\sqrt{3}\,\mu_0}{\pi}$
(c) $\dfrac{\mu_0}{2\pi}$ (d) $\dfrac{3\mu_0}{2\pi}$

50. A proton of mass m and charge $+e$ is moving in a circular orbit in a magnetic field with energy 1 MeV. What should be the energy of an α-particle (mass 4 m and charge + 2e) so that it revolves in a circular orbit of the same radius in the same magnetic field?
(a) 1 MeV (b) 2 MeV
(c) 4 MeV (d) 0.5 MeV

51. A loosely wound helix made of stiff wire is mounted vertically with the lower end just touching a dish of mercury. When a current from the battery is started in the coil through the mercury
(a) the wire oscillates
(b) the wire continues making contact
(c) the wire breaks contact just when the current is passed
(d) the mercury will expand by heating due to passage of current.

IIT, 1981

52. A non-planar loop of conducting wire carrying a current is placed as shown in Fig. 24.54. Each of the straight sections of the loop is of length 2a. The magnetic field due to this loop at the point $P(a, 0, a)$ points in the direction
(a) $\dfrac{1}{\sqrt{2}}\left(-\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$
(b) $\dfrac{1}{\sqrt{3}}\left(-\hat{\mathbf{j}}+\hat{\mathbf{k}}+\hat{\mathbf{i}}\right)$
(c) $\dfrac{1}{\sqrt{3}}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$
(d) $\dfrac{1}{\sqrt{2}}\left(\hat{\mathbf{i}}+\hat{\mathbf{k}}\right)$

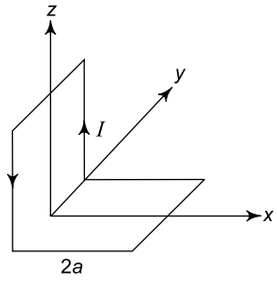


Fig. 24.54

IIT, 2001

53. Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B respectively and the trajectories are as shown in Fig. 24.55. Then
(a) $v_A < m_B\, v_B$
(b) $m_A\, v_A > m_B\, v_B$
(c) $m_A < m_B$ and $v_A < v_B$
(d) $m_A = m_B$ and $v_A = v_B$

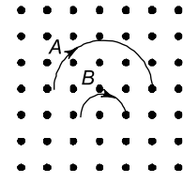


Fig. 23.55

IIT, 2003

54. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the centre is
(a) $\dfrac{\mu_0 NI}{b}$ (b) $\dfrac{2\mu_0 NI}{a}$

(c) $\frac{\mu_0 NI}{2(b-a)} \ln \frac{b}{a}$ (d) $\frac{\mu_0 NI}{2(b-a)} \ln \frac{a}{b}$

IIT, 2001

55. A particle of mass m and charge q moves with a constant velocity v along the positive x direction. It enters a region containing a uniform magnetic field B directed along the negative z direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is

(a) qbB/m (b) $q(b - a)B/m$
(c) qaB/m (d) $q(b + a)B/2m$

IIT, 2002

56. Which pattern shown in Fig. 24.56 correctly represents the magnetic field lines due to bar magnet?

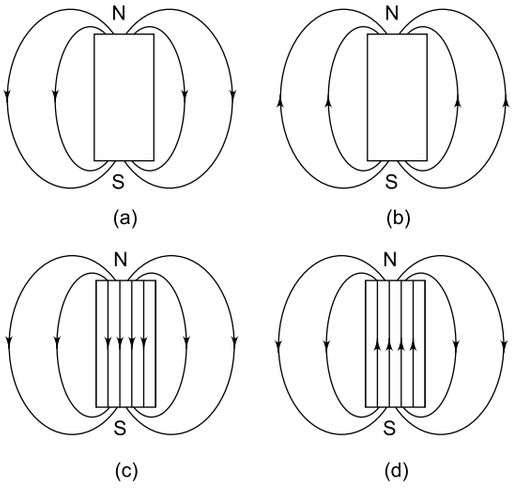


Fig. 24.56

57. An electron is moving in the x-y plane along the positive x-axis. There is a sudden change in its path due to the presence of electric and/or magnetic fields beyond P as shown in Fig. 24.57. The curved path lies in the x-y plane and is found to be non-circular. Which of the following combinations is possible?

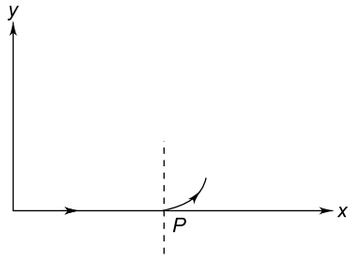


Fig. 24.57

(a) $\mathbf{E} = 0;\ \mathbf{B} = b\hat{\mathbf{i}} + c\hat{\mathbf{k}}$

(b) $\mathbf{E} = a\hat{\mathbf{i}};\ \mathbf{B} = c\hat{\mathbf{k}} + b\hat{\mathbf{i}}$

(c) $\mathbf{E} = 0;\ \mathbf{B} = c\hat{\mathbf{j}} + b\hat{\mathbf{k}}$

(d) $\mathbf{E} = a\hat{\mathbf{i}};\ \mathbf{B} = c\hat{\mathbf{k}} + b\hat{\mathbf{j}}$

IIT, 2003

58. A magnetized wire of magnetic moment M is bent into an arc of a circle that subtends an angle of 60° at the centre. The equivalent magnetic moment is

(a) $\frac{M}{\pi}$ (b) $\frac{2M}{\pi}$
(c) $\frac{3M}{\pi}$ (d) $\frac{4M}{\pi}$

59. Two straight infinitely long and thin parallel wires are held 0.1 m apart and carry a current of 5 A each in the same direction. The magnitude of the magnetic field at a point distant 0.1 m from both wires is

(a) 10^{-5} T (b) $\sqrt{2} \times 10^{-5}$ T
(c) $\sqrt{3} \times 10^{-5}$ T (d) 2×10^{-5} T

60. A proton moving with a speed u along the positive x-axis enters at $y = 0$ a region of uniform magnetic field $\mathbf{B} = \mathbf{B}_0\hat{\mathbf{k}}$ which exists to the right of y-axis as shown in Fig. 24.58. The proton leaves the region after some time with a speed v at co-ordinate y. Then

(a) $v > u,\ y < 0$ (b) $v = u,\ y > 0$
(c) $v > u,\ y > 0$ (d) $v = u,\ y < 0$

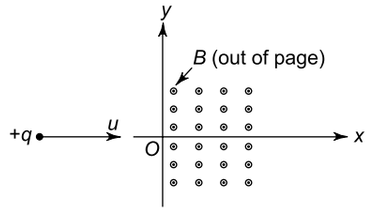


Fig. 24.58

IIT, 2004

61. A particle of charge q moves with a velocity $\mathbf{v} = a\hat{\mathbf{i}}$ in a magnetic field $\mathbf{B} = b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ where a, b and c are constants. The magnitude of the force experienced by the particle is

(a) zero (b) $qa(b + c)$
(c) $qa(b^2 - c^2)^{1/2}$ (d) $qa(b^2 + c^2)^{1/2}$

62. A current I is flowing through the sides of an equilateral triangle of side a. The magnitude of the magnetic field at the centroid of the triangle is

(a) $\frac{3\mu_0 I}{2\pi a}$ (b) $\frac{9\mu_0 I}{2\pi a}$

(c) $\frac{3\sqrt{3}\mu_0 I}{2\pi a}$ (d) zero

63. A particle of mass m and charge q, accelerated by a potential difference V enters a region of a uniform transverse magnetic field B. If d is the thickness of the region of B, the angle θ through which the particle deviates from the initial direction on leaving the region is given by

(a) $\sin\theta = Bd\left(\frac{q}{2mV}\right)^{1/2}$

(b) $\cos\theta = Bd\left(\frac{q}{2mV}\right)^{1/2}$

(c) $\tan\theta = Bd\left(\frac{q}{2mV}\right)^{1/2}$

(d) $\cot\theta = Bd\left(\frac{q}{2mV}\right)^{1/2}$

64. A metal wire of mass m slides without friction on two rails spaced at a distance d apart. The track lies in a vertical uniform magnetic field B. A constant current I flows along one rail, across the wire and back down the other rail. If the wire is initially at rest, the time taken by it to move through a distance x along the track is

(a) $t = \sqrt{\frac{BId}{2xm}}$ (b) $t = \sqrt{\frac{2xm}{BId}}$

(c) $t = \sqrt{\frac{BIdm}{2x}}$ (d) $t = \sqrt{\frac{2dm}{BIx}}$

65. A particle of charge q and mass m is released from the origin with a velocity $\mathbf{v} = a\hat{\mathbf{i}}$ in a uniform magnetic field $\mathbf{B} = b\hat{\mathbf{k}}$. The particle will cross the y-axis at a point whose y-coordinate is

(a) $y = \frac{ma}{qb}$ (b) $y = \frac{2ma}{qb}$

(c) $y = -\frac{ma}{qb}$ (d) $y = -\frac{2ma}{qb}$

66. A thin wire loop carrying a current I is placed in a uniform magnetic field B pointing out of the plane of the coil as shown in Fig. 24.59. The loop will tend to

(a) move towards positive x-direction
(b) move towards negative y-direction
(c) contract
(d) expand

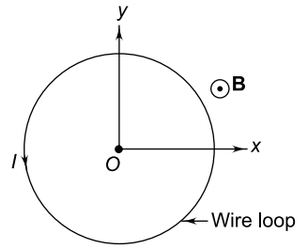


Fig. 24.59

67. Which of the field patterns given in Fig. 24.60 is valid for electric field as well as for magnetic field?

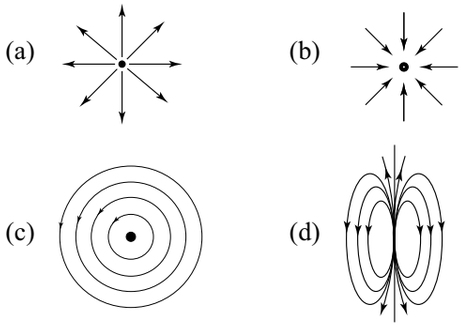


Fig. 24.60

IIT, 2011

68. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the XY plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the centre of the spiral is [see Fig. 24.61]

(a) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$ (b) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b+a}{b-a}\right)$

(c) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b}{a}\right)$ (d) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b+a}{b-a}\right)$

IIT, 2011

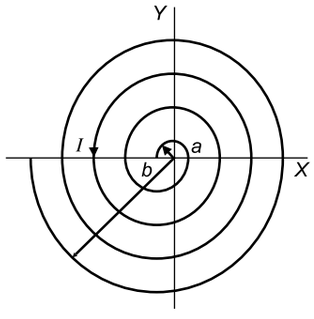


Fig. 24.61

ANSWERS

1. (d)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (d)
7. (c)	**8.** (a)	**9.** (d)	**10.** (c)	**11.** (c)	**12.** (c)
13. (b)	**14.** (d)	**15.** (c)	**16.** (a)	**17.** (d)	**18.** (c)
19. (a)	**20.** (c)	**21.** (c)	**22.** (d)	**23.** (d)	**24.** (c)
25. (d)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (d)	**30.** (d)
31. (d)	**32.** (c)	**33.** (c)	**34.** (c)	**35.** (a)	**36.** (a)
37. (d)	**38.** (a)	**39.** (c)	**40.** (b)	**41.** (c)	**42.** (c)
43. (b)	**44.** (b)	**45.** (a)	**46.** (a)	**47.** (c)	**48.** (c)
49. (c)	**50.** (a)	**51.** (a)	**52.** (d)	**53.** (b)	**54.** (c)
55. (b)	**56.** (d)	**57.** (b)	**58.** (c)	**59.** (c)	**60.** (d)
61. (d)	**62.** (b)	**63.** (a)	**64.** (b)	**65.** (d)	**66.** (c)
67. (c)	**68.** (a)				

SOLUTIONS

1. $\frac{B^2}{2\mu_0}$ = magnetic energy density. Therefore

$$\left[\frac{B^2}{\mu_0}\right] = \frac{\text{energy}}{\text{volume}} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}\ T^{-2}]$$

RC = time constant = [T]

$$\therefore \left[\frac{B^2RC}{\mu_0}\right] = [ML^{-1}\,T^{-2}] \times [T] = [ML^{-1}\,T^{-1}],$$ which are the dimensions of viscosity.

2. The magnitude the magnetic field is at the surface of the wire and is given by

$$B = \frac{\mu_0 I}{2\pi R} \qquad (1)$$

$R = 1$ mm $= 1 \times 10^{-3}$ m, $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1} and $B = 2 \times 10^{-3}$ T. Using these values in Eq. (1), we get $I = 10$ A.

3. Pitch $x = \frac{2\pi mv \cos 30°}{qB} = \frac{2\pi mv}{qB} \cdot \frac{\sqrt{3}}{2}$ (1)

Radius $r = \frac{mv \sin 60°}{qB} = \frac{mv}{2qB}$ (2)

From Eqs. (1) and (2), we get $r = \frac{x}{2\sqrt{3}\,\pi}$, which is choice (c).

4. Since the magnetic field due to wire CD is non-uniform, wire AB will experience as force as well as a torque. Hence it will have both translational as well as rotational motions.

5. Refer to the Fig. 24.62.

Magnetic field at point R in Fig. (a) is given by

$$B = \frac{\mu_0 I}{4\pi r}\ (\sin\ \alpha + \sin\ \alpha) \qquad (1)$$

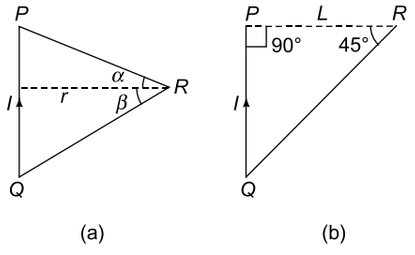


Fig. 24.62

In Fig. (b), $r = L$, $\alpha = 0°$ and $\beta = 45°$. Using these in Eq. (1), we find that the correct choice is (c).

6. Figure 24.63 shows the cross-sectional view of the rod. From Fleming left-hand rule, the magnetic force F acting on the rod is directed to the right and is given by $F = B\,I\,L$. Since the rod moves with a constant velocity, no net force acts on it. Hence the components of Mg and F parallel to the inclined plane must balance, i.e.

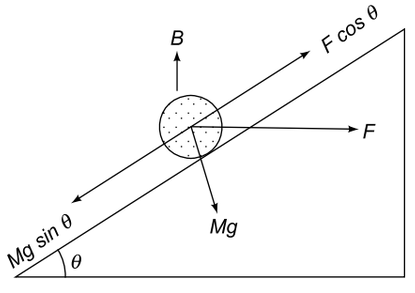


Fig. 24.63

$$F\ \cos\ \theta = Mg\ \sin\ \theta$$

or $$B\ I\ L\ \cos\ \theta = Mg\ \sin\ \theta$$

or $$B = \frac{Mg\ \tan\theta}{IL}$$

Putting $\theta = 30°$, we get $B = \dfrac{Mg}{\sqrt{3}\,IL}$, which is choice (d).

7. Magnetic field due to a complete circular loop at its centre is

$$B = \frac{\mu_0 I}{2r}$$

$\therefore$ Magnetic field due to segment ADB at centre O is

$$B_1 = \frac{\mu_0 I}{2r} \times \frac{\theta}{2\pi} \text{ directed out of the page.}$$

Magnetic field due to segment ACB at centre O is

$$B_2 = \frac{\mu_0 I}{2r} \times \left(\frac{2\pi - \theta}{2\pi}\right) \text{ directed into the page}$$

$\therefore$ Net magnetic field at O is

$$B = B_2 - B_1$$

$$= \frac{\mu_0 I}{2\pi r} (\pi - \theta) \text{ directed into the page.}$$

Putting $\theta = 60° = \dfrac{\pi}{3}$, we get $B = \dfrac{\mu_0 I}{3r}$.

8. The magnitude of magnetic induction due to a circular loop of radius R carrying a current I is given by $B = \dfrac{\mu_0 I}{2R}$

The field due to semi-circular loop is given by

$$B = \frac{\mu_0 I}{4R}$$

The direction of B is normal to the plane of the loop. Since the current in the bigger loop is clockwise and that in the smaller loop is anticlockwise (see Fig. 24.46), the fields produced by them at centre C are in opposite directions. Therefore, the magnetic induction at the centre is given by

$$B = B_1 - B_2 = \frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The wires PQ and SR do not produce any field at the centre C. Hence the correct choice is (a).

9. Refer to Fig. 24.64. Here $r = OE = \dfrac{L}{2}$. Referring to Fig. 24.64, the magnetic field at the centre O due to the current element AE is given by

$$B_{AE} = -\frac{\mu_0 I}{4\pi r} \int_{90°}^{45°} \sin\theta\, d\theta = \frac{\mu_0 I}{4\pi r} \left|\cos\theta\right|_{90°}^{45°}$$

$$= \frac{\mu_0 I}{4\pi r}(\cos 45° - \cos 90°)$$

$$= \frac{\mu_0 I}{4\pi r}(\cos 45° - 0) = \frac{\mu_0 I}{4\sqrt{2}\,\pi r}$$

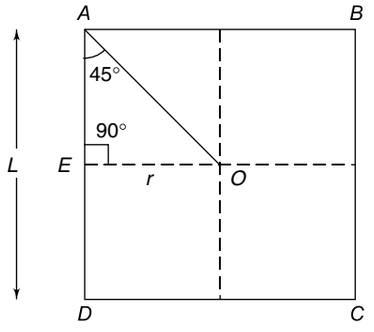


Fig. 24.64

It is clear that the magnetic field at O due to current element DE is the same as that due to AE. Hence, the magnetic field at O due to one side AD is

$$B_{AD} = \frac{2\mu_0 I}{4\sqrt{2}\,\pi r} = \frac{\sqrt{2}\,\mu_0 I}{4\pi r}$$

Since the centre of the square is equidistant from the ends A, B, C and D of each side of the square and each side produces at the centre O the same magnetic field, the field due to the square is 4 times that due to one side. Hence (because $r = L/2$)

$$B = 4B_{AD} = \frac{\sqrt{2}\,\mu_0 I}{\pi r} = \frac{2\sqrt{2}\,\mu_0 I}{\pi L}.$$

Hence the correct choice is (d).

10. Let the masses of X and Y be m_1 and m_2 and let their velocities after being accelerated be v_1 and v_2 respectively. Since the particles have equal charges and have been accelerated through the same potential difference, their kinetic energies are equal, i.e.

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$$

In a uniform magnetic field B, the radii of the circular paths are given by

$$\frac{m_1 v_1^2}{R_1} = qBv_1 \text{ or } qB = \frac{m_1 v_1}{R_1}$$

and $\dfrac{m_2 v_2^2}{R_2} = qBv_2$ or $qB = \dfrac{m_2 v_2}{R_2}$

Therefore $\dfrac{m_1 v_1}{R_1} = \dfrac{m_2 v_2}{R_2}$ or $\dfrac{m_1^2 v_1^2}{R_1^2} = \dfrac{m_2^2 v_2^2}{R_2^2}$

But $m_1 v_1^2 = m_2 v_2^2$. Therefore, we have

$$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$$

Hence the correct choice is (c).

11. The magnitude of the magnetic field at the centre due to each coil is B. Since the planes of the coils are at right angles to each other, the directions of the fields will be at right angles to each other. Therefore, the resultant field is $B_r = \sqrt{B^2 + B^2} = \sqrt{2}\ B$

Hence the correct choice is (c).

12. Since the force exerted by the magnetic field is perpendicular to the direction of motion of the particle, the speed of the particle cannot change but its velocity changes. Hence the correct choice is (c).

13. The radius of the circular orbit is given by

$$r = \frac{\sqrt{2mK}}{qB}$$

The charge of an α–particle is twice that of a proton and its mass is four times the mass of a proton. Therefore $\sqrt{m}/q$ is the same for both. Hence r will the same for both particles. Thus the correct choice is (b).

14. The cyclotron frequency is given by

$$\nu = \frac{qB}{2\pi m}$$

It is independent of the speed of the particle and the radius of its circular path. Now $\nu \propto q/m$. The charge of a proton is half that of an α-particle and the mass of a proton is one-fourth. Therefore, ν will be doubled. Hence the correct choice is (d).

15. $V = 2.88 \times 10^3$ V. The velocity of the electron is given by

$$\frac{1}{2} mv^2 = eV$$

or
$$v = \left(\frac{2eV}{m}\right)^{1/2}$$

$$= \left(\frac{2 \times 1.6 \times 10^{-19} \times 2.88 \times 10^3}{9 \times 10^{-31}}\right)^{1/2}$$

$$= \left(\frac{2 \times 1.6 \times 10^{-19} \times 2.88 \times 10^3}{9 \times 10^{-31}}\right)^{1/2}$$

$$= 3.2 \times 10^7 \text{ ms}^{-1}$$

If the field makes an angle θ with the velocity **v**, then $v_\perp = v \sin\theta$ and $v_{11} = v \cos\theta$. The electron has two motions: a linear motion parallel to magnetic field and a circular motion in a plane perpendicular to the field. Hence the trajectory of the electron is a helix whose radius is

$$r = \frac{mv \sin\theta}{eB} = \frac{9 \times 10^{-31} \times 3.2 \times 10^7 \times \sin 30°}{1.6 \times 10^{-19} \times 0.1}$$

$$= 9 \times 10^{-4} \text{ m} = 0.9 \text{ mm}$$

Hence the correct choice is (c).

16. The correct choice is (a).

17. The correct choice is (d).

18. The velocity when the potential difference is V is

$$v = \sqrt{\frac{2eV}{m}}$$

and force $F = e\ v\ B$

When the potential difference is doubled, i.e. $V' = 2V$, the velocity is

$$v' = \sqrt{\frac{2eV'}{m}} = \sqrt{\frac{2e \times 2V}{m}} = \sqrt{2}\ v$$

$\therefore$ Force $F' = ev'B = \sqrt{2}\ evB = \sqrt{2}\ F$. Hence the correct choice is (c).

19. The correct choice is (a).

20. The correct choice is (c) because the magnetic field produced by the current in the loop and the external magnetic field are along the same direction.

21. Referring to Fig. 24.65, the forces acting on arms BC and AD are equal and opposite. The force on arm AB is given by

$$F_1 = \frac{\mu_0 I i}{2\pi a}$$

which is directed towards the wire. The force on arm CD is given by

$$F_2 = \frac{\mu_0 I i}{2\pi (a+b)}$$

which is directed away from the wire. Since $F_1 > F_2$, the loop will move towards the wire. Hence the correct choice is (c).

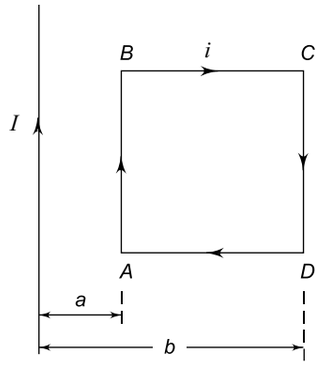


Fig. 24.65

22. Magnetic moment $m = AI = \pi r^2 I$, where r is the radius of the circular loop. Now, the circumference of the circle = length of the wire, i.e.

$$2\pi r = l \text{ or } r^2 = \frac{l^2}{4\pi^2}$$

Therefore, $m = \pi r^2 I = \frac{\pi l^2 I}{4\pi^2} = \frac{l^2 I}{4\pi}$,

which is choice (d).

23. Magnetic moment $m = IA = I\pi r^2$.

$$m = I_1 \pi r_1^2 = I_2 \pi r_2^2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} = \left(\frac{6}{3}\right)^2 = 4$$

Hence the correct choice is (d).

24. An electron moving in a circular orbit is equivalent to a current carrying loop. As explained above, the current is

$$I = \nu e = \frac{e}{T}$$

where T is the time period of the motion of the electron around the nucleus. If v is the speed of the electron,

$$T = \frac{2\pi r}{v}$$

$$\therefore \quad I = \frac{ev}{2\pi r} = \frac{e\omega}{2\pi} \qquad (\because v = r\omega)$$

Now, the magnetic field at the centre of the loop is

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 e\omega}{4\pi r} \text{ or } \omega = \frac{4\pi r B}{\mu_0 e}$$

Hence the correct choice is (c).

25. The magnetic field due to wire D at wire C is

$$B_D = \left(\frac{\mu_0}{4\pi}\right) \frac{2I}{r} = \frac{10^{-7} \times 2 \times 30}{0.03} = 2 \times 10^{-4} \text{ T}$$

which is directed into the page.

Similarly, the field due to wire G at C is

$$B_G = \frac{10^{-7} \times 2 \times 20}{0.1} = 0.4 \times 10^{-4} \text{ T}$$

which is directed out of the page.

Therefore, the field at the position of the wire C is

$$B = B_D - B_G = 2 \times 10^{-4} - 0.4 \times 10^{-4}$$
$$= 1.6 \times 10^{-4} \text{ T}$$

and is directed into the page.

The force on 25 cm of wire C is

$$F = BIl \sin 90° = 1.6 \times 10^{-4} \times 10 \times 0.25$$
$$= 4 \times 10^{-4} \text{ N}$$

26. Electric intensity $E = \frac{V}{d}$

where V is the potential difference between the plates and d, the separation between them.

$$d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

$$E = \frac{V}{d} = \frac{600}{3 \times 10^{-3}} = 2 \times 10^5 \text{ V m}^{-1}$$

Since the electron moves undeflected between the plates, the force due to magnetic field must balance the force due to electric field. Thus

$$B e v = e E \text{ or } B = \frac{E}{v} = \frac{2 \times 10^5}{2 \times 10^6} = 0.1 \text{ T}$$

27. From Ampere's law, we have

$$\oint \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_0 I$$

Since no current exists in the medium (air) inside the pipe $I = 0$. Hence $\mathbf{B} = 0$. Hence the correct choice is (b).

28. The magnetic field at point P due to current I_1 in conductor AOB is

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

and the magnetic field at point P due to current I_2 in conductor COD is

$$B_2 = \frac{\mu_0 I_2}{2\pi a}$$

Since the two conductors are perpendicular to each other, fields B_1 and B_2 will be perpendicular to each other. Therefore, the resultant field at P is

$$B = (B^2_1 + B^2_2)^{1/2} = \frac{\mu_0}{2\pi a} (I^2_1 + I^2_2)^{1/2}$$

Hence the correct choice is (c).

29. The work done is given by

$$W = \int_0^\theta M H \sin\theta \, d\theta = MH \,|-\cos\theta|_0^\theta$$
$$= MH(1 - \cos\theta)$$

Hence the correct choice is (d).

30. The kinetic energy of proton is

$$K = 2 \text{ MeV} = 2 \times 10^6 \text{ eV}$$
$$= 2 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} = 3.2 \times 10^{-13} \text{ J}$$

$$\therefore \quad \frac{1}{2} mv^2 = 3.2 \times 10^{-13}$$

Now, mass of proton is $m = 1.67 \times 10^{-27}$ kg. Therefore,

$$v^2 = \frac{2 \times 3.2 \times 10^{-13}}{1.67 \times 10^{-27}} = 3.83 \times 10^{14}$$

or $v = 1.96 \times 10^7$ ms^{-1}.

Now force on proton is

$$F = evB$$
$$= 1.6 \times 10^{-19} \times 1.96 \times 10^7 \times 2.5$$
$$= 7.84 \times 10^{-12} \text{ N}$$

Hence the closest choice is (d).

31. The point $x = + a$ lies along the line of the straight section PQ of the circuit. Hence the magnetic field at point $x = a$ is zero.

32. The radius of the circular path of a particle of mass m, charge e moving with a speed v perpendicular to a magnetic field B is given by

$$\frac{mv^2}{r} = evB \text{ or } r = \left(\frac{m}{e}\right)\frac{v}{B}$$

Thus, r is inversely proportional to $\left(\frac{e}{m}\right)$, the charge to mass ratio. Hence the correct choice is (c).

33. The total Lorentz force on the electron is

$$\mathbf{F} = -e\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

The electron will be undeflected if $\mathbf{v} \perp \mathbf{B}$. If $\mathbf{E}$ is along + z-direction, the force $-e\,\mathbf{E}$ will be along $-z$-direction. If $\mathbf{B}$ is along + x direction, force $-e\,(\mathbf{v} \times \mathbf{B})$ will be along + z direction. When $eE = evB$, the electron moves along + y-direction undeflected. Hence the correct choice is (c). Thus, for an electron moving along + y direction, the electric field should be along + z direction and magnetic field along + x direction, then the electron will be undeflected.

34. Refer to the solution of Q. 15. The correct choice is (c).

35. The radius of the circular path is given by

$$r = \frac{mv^2}{qB} = \frac{\sqrt{2mK}}{qB}, \text{ where, } K = \frac{1}{2}mv^2.$$

Thus $r \propto \frac{\sqrt{m}}{q}$ since K and B are the same for the three particles. If m_p is the mass of a proton and q_p its charge, then $m_d = 2m_p$ and $q_d = q_p$ and $m_\alpha = 4\,m_p$ and $q_\alpha = 2q_p$. From these it follows that $r_\alpha = r_p < r_d$.

36. According to Ampere's Law, the magnetic moment of a current I flowing in a circular path of area of cross-section A is given by

$$\mu_m = IA$$

It is given that the charge q is moving in a circular path of radius $2l$. Therefore, the time period $= 2\pi(2l)/v$. Hence

$$\mu_m = \frac{qv}{2\pi(2l)} \times \pi(2l)^2 = qvl$$

The angular momentum $L = mv(2l)$. Therefore,

$$\frac{\mu_m}{L} = \frac{qvl}{mv(2l)} = \frac{q}{2m}$$

37. Since currents in the wires are flowing in opposite directions, the magnetic fields due to the wires at a point equidistant from the two wires will be equal and opposite. The net magnetic field at this point is zero. Hence the force on a charge at this point is zero.

38. Due to electric field $\mathbf{E}$, the force on a particle of charge q is $\mathbf{F} = q\mathbf{E}$ in the direction of the electric field. Since $\mathbf{E}$ is parallel to $\mathbf{B}$, the velocity $\mathbf{v}$ of the particle is parallel to $\mathbf{B}$. Hence $\mathbf{B}$ will not affect the motion of the particle since $\mathbf{v} \times \mathbf{B} = 0$. Thus the correct choice is (a).

39. The effect of the electric field $\mathbf{E}$ on a particle of charge q is to impart to it a velocity $\mathbf{v}$ which is proportional to $q\mathbf{E}$, i.e.

$$\mathbf{v} \propto q\mathbf{E} \propto qE\,\hat{\mathbf{i}}$$

The effect of the magnetic field $\mathbf{B}$ on a charge moving with a velocity $\mathbf{v}$ is to exert on it a force

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \propto q\,(qE\,\hat{\mathbf{i}}) \times B\,\hat{\mathbf{k}}$$

Thus $\mathbf{F} \propto q^2EB\,\hat{\mathbf{i}} \times \hat{\mathbf{k}} \propto q^2EB\,(-\hat{\mathbf{j}})$

$$\left(\because \hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}\right)$$

Since $F \propto q^2$, both positive and negative ions will be deflected towards the $-y$ direction.

40. The magnetic fields due to the currents in wires 1 and 2 at a point between them act in opposite directions. But at a point to the left of wire 1 and to the right of wire 2, the magnetic fields act in the same direction. Hence the variation of magnetic field B along the line XX' is as shown in choice (b).

41. Given $v = (2 \times 10^5\ \hat{\mathbf{i}})$ ms^{-1}. The force vector is given by

$$\mathbf{F} = q(v \times \mathbf{B})$$
$$= q\ \{2 \times 10^5\ \mathbf{i} \times (\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}})\}$$
$$= 2 \times 10^5 \times q\ (-4\hat{\mathbf{k}} + 3\hat{\mathbf{j}})$$

Therefore, the y and z components of the force are

$$F_y = 6 \times 10^5 \times q$$

and $F_z = -8 \times 10^5 \times q$

$\therefore$ Magnitude of force $= \sqrt{F_y^2 + F_z^2}$

$$= q\ \sqrt{(6\times10^5)^2 + (-8\times10^5)^2}$$

$$= q \times 10 \times 10^5$$

$$= 1.6 \times 10^{-19} \times 10 \times 10^5$$

$$= 1.6 \times 10^{-13}\ \text{N, which is choice (c).}$$

42. Given $r = 0.4$ Å $= 0.4 \times 10^{-10}$ m, $v = 10^6$ ms^{-1}

Speed of electron in the orbit is

$v = \dfrac{2\pi r}{t}$; here t is time taken by the electron to complete one revolution. Thus

$$t = \frac{2\pi r}{v} = \frac{2\pi\times0.4\times10^{-10}}{10^6} = 8\pi \times 10^{-17}\ \text{s}$$

Current $I = \dfrac{e}{t} = \dfrac{1.6\times10^{-19}}{8\pi\times10^{-17}} = \dfrac{2}{\pi} \times 10^{-3}$ A

Magnetic field at the centre of the orbit is

$$B = \frac{\mu_0 I}{2r} = \frac{(4\pi\times10^{-7})\times\frac{2}{\pi}\times10^{-3}}{2\times0.4\times10^{-10}} = 10 \text{ tesla}$$

Hence the correct choice is (c).

43. Given $\mathbf{v} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ ms^{-1} and $B = (2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ tesla.

Force experienced by the proton is

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = q\ (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \times (2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$= q\ (6\hat{\mathbf{i}} \times \hat{\mathbf{j}} + 9\hat{\mathbf{i}} \times \hat{\mathbf{k}} + 4\hat{\mathbf{j}} \times \hat{\mathbf{j}} + 6\hat{\mathbf{j}} \times \hat{\mathbf{k}})$$

$$= q\ (6\hat{\mathbf{k}} - 9\hat{\mathbf{j}} + 0 + 6\hat{\mathbf{i}})$$

$$= 3q\ (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \text{ newton}$$

$\therefore$ Acceleration $= \dfrac{\mathbf{F}}{m} = \dfrac{3q}{m}(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$$= 3 \times (0.96 \times 10^8)(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

$$= 2.88 \times 10^8\ (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})\ \text{ms}^{-2}$$

Hence the correct choice is (b).

44. Let L be the length of the wire and let R be the radius of the circle when the wire is bent into one circular turn and r be the radius of the coil of n turns. Then

$$L = 2\pi R = 2\pi nr \text{ or } R = nr \text{ or } r = \frac{R}{n}$$

Now $B = \dfrac{\mu_0 I}{2R}$. Magnetic field at the centre of the coil of n turns and radius r is

$$B' = \frac{\mu_0 nI}{2r} = \frac{\mu_0 nI}{2R/n} = \frac{\mu_0 In^2}{2R} = n^2 B$$

Hence the correct choice is (b).

45. Given $x = 4$ cm $= 0.04$ m and $r = 3$ cm $= 0.03$ m. The magnetic field at a point on the axis of the loop is given by

$$B = \frac{\mu_o I r^2}{2(r^2 + x^2)^{3/2}} \qquad (1)$$

Magnetic field at the centre of the coil is given by

$$B_0 = \frac{\mu_o I}{2r} \qquad (2)$$

Dividing (1) by (2), we get

$$\frac{B_0}{B} = \frac{r^3}{(r^2 + x^2)^{3/2}}$$

Substituting the values of r and x, we get

$$\frac{B_0}{B} = \frac{125}{27} \text{ or}$$

$$B_0 = \frac{125}{27}\ B = \frac{125}{27} \times 54\ \mu\text{T} = 250\ \mu\text{T}$$

Hence the correct choice is (a).

46. Wires AB and EF experience no forces since currents in them are parallel to the magnetic field. The forces on BC and DE are equal in magnitude but are directed in opposite directions. Hence their resultant is zero. Only force acting is on CD. Hence the correct choice is (a).

47. Since point O lies along the segments LR and SM, the magnetic field due to these segments is zero at point O. As point O is close to R and S, the net magnetic field at O due to segments PR and QS is

$$B = B_P + B_Q = \frac{\mu_0 I}{4\pi RO} + \frac{\mu_0 I}{4\pi SO} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{d} + \frac{1}{d}\right)$$

$$= \frac{\mu_0}{4\pi}\left(\frac{2I}{d}\right) = 10^{-7} \times \frac{2\times10}{0.02} = 10^{-4}\ \text{Wb m}^{-2}$$

The direction of this field is vertically upward, i.e. outside the plane of the paper. Hence the correct choice is (c).

48. Magnetic moment M = current × area

$$= \frac{\text{charge} \times \text{area}}{\text{time period}}$$

$$= \frac{q \times \pi r^2}{T} = \frac{1}{2}\, q\omega r^2 \quad \left(\because \omega = \frac{2\pi}{T}\right)$$

Angular momentum $L = m\omega r^2$

$\therefore \quad \dfrac{M}{L} = \dfrac{\frac{1}{2} q\omega r^2}{m\omega r^2} = \dfrac{q}{2m}$. Hence the correct choice is (c).

49. Magnetic field $B = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2I_1}{r_1} - \dfrac{\mu_0}{4\pi} \cdot \dfrac{2I_2}{r_2}$

Given $I_1 = 2.5$ A, $I_2 = 5$ A and $r_1 = r_2 = 2.5$ m. Using these values, we get $B = -\dfrac{\mu_0}{2\pi}$. The magnitude of B is $\mu_0/2\pi$. Hence the correct choice is (c).

50. For proton: $r = \dfrac{mv}{eB}$

For α-particle $r' = \dfrac{m'v'}{e'B} = \dfrac{4mv'}{2eB} = \dfrac{2mv'}{eB}$

Given $r = r'$. Hence $v' = \dfrac{v}{2}$.

Energy of proton $E = \dfrac{1}{2} mv^2$. Energy of α-particle is

$$E' = \frac{1}{2}\, m'v'^2 = \frac{1}{2} \times 4m \times \left(\frac{v}{2}\right)^2 = \frac{1}{2}\, mv^2 = E$$

Hence $E' = 1$ MeV which is choice (a).

51. When a current is passed through the helix, the neighbouring coils of the helix attract each other due to which it contracts. As a result the contact is broken and the coils will recover their original state under the influence of a restoring force. The contact is made again and the process continues. Thus the wire oscillates. Hence the correct choice is (a).

52. The magnetic field due to the complete loop shown in the figure is the vector sum (i.e. resultant) of two magnetic fields: (i) one due to the planar loop in the y-z plane, which is along the x-direction and (ii) the other due to the planar loop in the x-y plane, which is along the z-direction. The direction of the resultant of these two fields is given by the vector $(\hat{\mathbf{i}} + \hat{\mathbf{k}})$. Hence the correct choice is (d).

53. The radius r of the circular path of a particle of mass m and charge q moving with velocity v perpendicular to a magnetic field B is given by

$$\frac{mv^2}{r} = qvB$$

or $mv = qrB$. Hence $m_A\, v_B = qr_A\, B$ and $m_B\, v_B = q\, r_B\, B$.

$$\therefore \quad \frac{m_A v_A}{m_B v_B} = \frac{r_A}{r_B}$$

It follows from the figure that $r_A > r_B$. Hence $m_A\, v_A > m_B\, v_B$. Thus the correct choice is (b).

54. The correct choice is (c). For derivation of the expression, refer to a Textbook of Physics.

55. The radius r of the circular path is given by (see Fig. 24.66)

$$\frac{mv^2}{r} = qvB$$

or $\quad v = \dfrac{qB}{m}(r)$

$$\therefore\ v_{\min} = \frac{qB}{m}(r_{\min}) = \frac{qB}{m}(b - a),$$

which is choice (b).

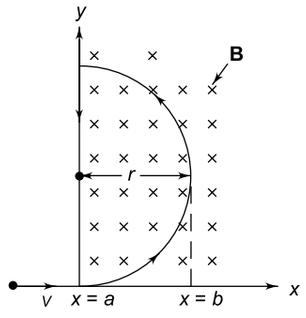


Fig. 24.66

56. The correct choice is (d) because the lines of force are continuous inside the magnet.

57. Since the path of the particle beyond P is non-circular, both **E** and **B** fields must be present beyond P. Hence choices (a) and (c) are incorrect. Since the curved path lies in the x-y plane, the magnetic field must be in the x-z plane. Hence choice (d) is also incorrect. Thus, the correct choice is (b).

58. Let r be the radius of the circle. The length of the arc $= (2\pi r) \times \dfrac{60^\circ}{360^\circ} = \dfrac{\pi r}{3}$. Therefore, the length $2l$ of the magnet is

$$2l = \frac{\pi r}{3} \quad \text{or} \quad r = \frac{6l}{\pi}$$

If m is the pole strength of each pole of the magnet, the magnetic moment of the arc $= m \times r = m \times \dfrac{6l}{\pi}$

$$= \frac{3 \times (2ml)}{\pi} = \frac{3M}{\pi}$$

Hence the correct choice is (c).

59. Wires A and B carry current $I = 5$ A each coming out of the plane of the page as shown in Fig. 24.66. The magnitude of magnetic field at point P due wire A is equal to that due to wire B, i.e.

$$B_A = B_B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}$$

$$= \frac{10^{-7} \times 2 \times 5}{0.1}$$

$$= 10^{-5} \text{ T}$$

The direction of field B_A is perpendicular to PA and that of field B_B is perpendicular to PB. Therefore, the angle between the two fields is $\theta = 60°$. The magnitude of the resultant field at P is given by

$$B_R^2 = B_A^2 + B_B^2 + 2B_A B_B \cos\theta$$

which gives $B_R = 2B_A \cos\left(\frac{\theta}{2}\right)$

$$= 2 \times 10^{-5} \times \frac{\sqrt{3}}{2} = \sqrt{3} \times 10^{-5} \text{ T}$$

Hence the correct choice is (c).

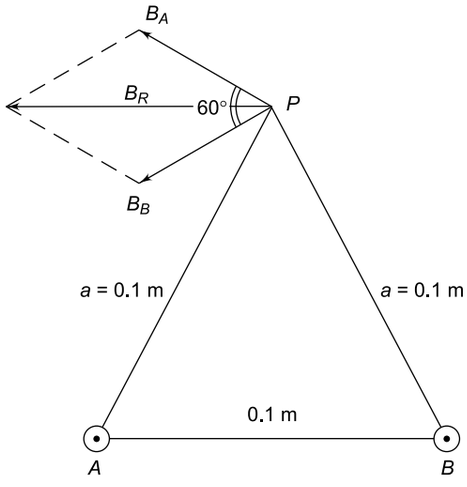


Fig. 24.67

60. When the proton enters the region of the magnetic field, it will experience a force **F** given by

$$\mathbf{F} = q\ (\mathbf{u} \times \mathbf{B})$$

where q is the charge of the proton. The force **F** is perpendicular to both **u** and **B**. Since the force is perpendicular to the velocity of the particle, it does not do any work. Hence the magnitude of the velocity of the particle will remain unchanged; only the direction of the velocity changes. Hence $v = u$. Since **u** is perpendicular to **B**, the proton moves in a circular path. Since the charge of proton is positive, **u** is along posotive x-axis and **B** is directed out of the page, the proton will move in a circle in the x-y plane in the clockwise direction. Hence its y coordinate will be negative, when it leaves the region. Thus the correct choice is (d).

61.
$$\mathbf{F} = q(\boldsymbol{v} \times \boldsymbol{B}) = q\{a\hat{\mathbf{i}} \times (b\hat{\mathbf{j}} + c\hat{\mathbf{k}})\}$$
$$= q(ab\hat{\mathbf{i}} \times \hat{\mathbf{j}} + ac\hat{\mathbf{i}} \times \hat{\mathbf{k}})$$
$$= q(ab\hat{\mathbf{k}} - ac\hat{\mathbf{j}}) = qa(b\hat{\mathbf{k}} - c\hat{\mathbf{j}})$$

Magnitude of $F = [(q\ ab)^2 + (q\ ac)^2]^{1/2}$

$$= qa(b^2 + c^2)^{1/2}$$

The correct choice is (d).

62. Refer to Fig. 24.68. Let $AB = BC = AC = a$. Let $OD = r$.

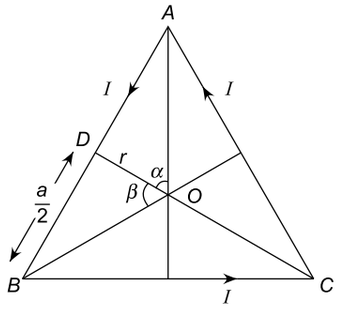


Fig. 24.68

The magnetic field at centroid O due to current I flowing in side AB of the triangle is given by

$$B_{AB} = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

It is clear that $\alpha = \beta = 60°$ and

$$OD = r = \frac{AD}{\tan\alpha} = \frac{a/2}{\tan 60°} = \frac{a/2}{\sqrt{3}} = \frac{a}{2\sqrt{3}}$$

$$\therefore\ B_{AB} = \frac{\mu_0 I}{4\pi r} \times \frac{2\sqrt{3}}{a} \times (\sin 60° + \sin 60°)$$

$$= \frac{3\mu_0 I}{2\pi a}$$

By symmetry, the magnetic fields due to current in sides BC and AC = that due to side AB. Hence, the magnetic field at O due to the current in the three sides of triangle ABC is

$$B = B_{AB} + B_{BC} + B_{CA} = 3B_{AB}$$

Hence the correct choice is (b).

63. Refer to Fig. 24.69. Let v be the velocity of the particle. Its kinetic energy is

$$\frac{1}{2}mv^2 = qV \text{ or } v = \left(\frac{2qV}{m}\right)^{1/2} \quad (1)$$

The particle follows a circular path from A to B of radius r which is given by

$$\frac{mv^2}{r} = qvB \text{ or } r = \frac{mv}{qB} \quad (2)$$

Using (1) and (2), we have

$$r = \frac{m}{qB}\left(\frac{2qV}{m}\right)^{1/2} = \frac{1}{B}\left(\frac{2mV}{q}\right)^{1/2}$$

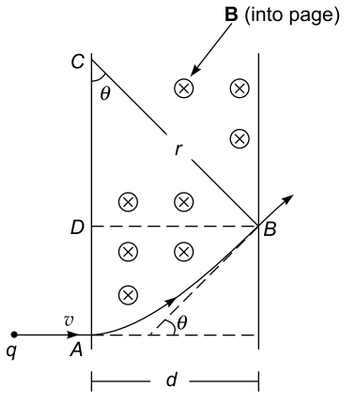


Fig. 24.69

In triangle BCD, $\sin\theta = \dfrac{BD}{BC} = \dfrac{d}{r}$. Therefore,

$\sin\theta = Bd\left(\dfrac{q}{2mV}\right)^{1/2}$, which is choice (a).

64. Refer to Fig. 24.70. Wire PQ of length d, the spacing between rails carries a current I vertically downwards in a magnetic field pointing towards the reader and perpendicular to the length PQ of the wire. Thus angle θ between I and B is 90°. The force exerted on the wire of length d by the magnetic field is

$$F = BId \sin 90° = BId$$

Using Fleming's left hand rule, the direction of the force is to the left. The acceleration of the wire is

$$a = \frac{\text{force}}{\text{mass}} = \frac{F}{m} = \frac{BId}{m}$$

Now $x = \dfrac{1}{2}at^2 \Rightarrow t = \sqrt{\dfrac{2x}{a}}$. Hence the correct choice is (b).

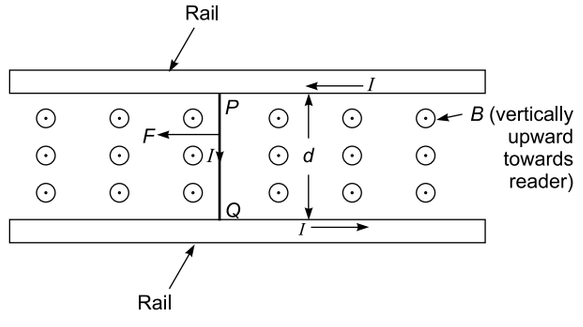


Fig. 24.70

65. Refer to Fig. 24.71. Since the velocity of the particle is $v = a$ along the positive x-axis and the direction of the magnetic field $B = b$ in the positive z-direction, and the charge of the particle is positive, the path of the particle is a circle as shown in the figure. The radius of the circular path is

$$r = \frac{mv}{qB} = \frac{ma}{qb}$$

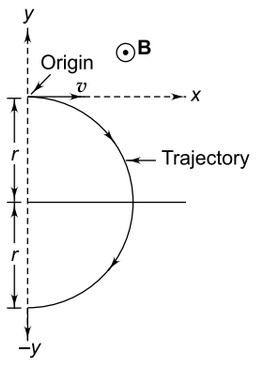


Fig. 24.71

Thus $y = -2$

$$r = -\frac{2ma}{qb}$$

So the correct choice is (d).

66. A circular metal loop carries a current. Hence charge, say, q moves along the circle with a velocity, say v which is tangential to the circle at every point (Fig. 24.59 on page 24.22). The force experienced by the charge is $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. Since $\mathbf{v}$ is along the tangent and $\mathbf{B}$ is directed out of the $x-y$ plane, the direction of the force is towards the centre O of the loop. Hence the force tends to contract the loop. Further, since $\mathbf{F}$ is perpendicular to $\mathbf{v}$, no work is done on the loop. Hence it cannot have any translational motion. Thus the correct choice is (c).

67. The field lines of a magnetic field form closed loops. From Maxwell's equations, we find that a time-varying magnetic field produces an electric field. Hence the field lines of an induced electric field also form closed loops like a magnetic field. Hence the correct choice is (c).

68. Magnetic field at the centre of a circular loop of radius r and carrying a current $I = \dfrac{\mu_0 I}{2r}$. The direction of the field is along z-direction if the current is anticlockwise.

Consider a small element of width dr. The current through the element is

$$dI = \frac{\text{total current in spiral}}{\text{total width of spiral}} \times \text{width of element}$$

$$= \frac{Idr}{(b-a)}$$

$$\therefore \quad B = \int_a^b \frac{\mu_0 N dI}{2r} = \int_a^b \frac{\mu_0 NI}{2(b-a)}\frac{dr}{r}$$

$$= \frac{\mu_0 NI}{2(b-a)}\int_a^b \frac{dr}{r} = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

SECTION II

Multiple Choice Questions with One or More Choice Correct

1. A straight wire carrying current is parallel to the y–axis as shown in Fig. 24.72. The
(a) magnetic field at the point P is parallel to the x–axis
(b) magnetic field is parallel to the z–axis
(c) magnetic lines are concentric circles with the wire passing through their common centre
(d) magnetic fields to the left and right of the wire are oppositely directed.

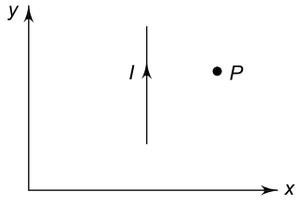


Fig. 24.72

2. The magnetic field due to a current carrying toroidal solenoid
(a) is independent of the radius of the solenoid
(b) depends on the number of turns and the current in the solenoid
(c) is constant in magnitude inside the solenoid
(d) is always radial inside the solenoid.

3. The magnetic field inside a straight solenoid
(a) is independent of the radius of the solenoid
(b) depends on the number of turns and current in the solenoid
(c) is uniform throughout the solenoid
(d) is perpendicular to the axis of the solenoid.

4. A charged particle of mass m and charge q enters a magnetic field $\mathbf{B}$ with a velocity $\mathbf{v}$ at an angle θ with $\mathbf{B}$.
(a) The kinetic energy of the particle will not change if $\theta \neq 0$.
(b) The momentum of the particle will not change if $\theta = 0$.
(c) The particle moves in a circle of radius mv/qB if $\theta = 90°$.
(d) The frequency of circular motion is independent of the speed of the particle if $\theta = 90°$.

5. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If E and B represent the electric and magnetic fields respectively, this region of space may have
(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$
(c) $E \neq 0, B = 0$ (d) $E \neq 0, B \neq 0$

IIT, 1985

6. The force $\mathbf{F}$ experienced by a particle of charge q moving with a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is given by $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. Which pairs of vectors are at right angles to each other?
(a) $\mathbf{F}$ and $\mathbf{v}$ (b) $\mathbf{F}$ and $\mathbf{B}$
(c) $\mathbf{B}$ and $\mathbf{v}$ (d) $\mathbf{F}$ and $(\mathbf{v} \times \mathbf{B})$

7. Choose the correct statements from the following.
(a) A non-uniform magnetic field that varies in magnitude from point to point but has a constant direction, is set up in a region of space. A charged particle enters that region and travels undeflected in a straight line with a constant speed. The initial velocity of the particle is either along the direction of the field or opposite to it.
(b) A non-uniform magnetic field that varies from point to point in magnitude and direction is set up in a certain region of space. A charged particle enters the region with a certain initial velocity. The direction of the final velocity will be different from that of the initial velocity.
(c) A electron travelling in the positive x–direction enters a region of space having an electrostatic field in the negative y–direction. If a magnetic field in the region is along the positive z–direction, the electron will travel undeflected.
(d) A charged particle moves in a uniform magnetic field for some time. The kinetic energy of the particle will change during this time.

8. Which of the following statements are correct?
(a) A current carrying coil, free to rotate, when placed in a uniform magnetic field will orient itself such that its plane becomes perpendicular to the magnetic field.
(b) The trajectory of a charged particle moving in a uniform magnetic field with its velocity parallel to the field is a circle in a plane perpendicular to the field.
(c) The magnetic field at a point midway between two long parallel wires carrying equal currents in the same direction is zero.

(d) An electron is moving in the anticlockwise sense in a horizontal circular orbit. The direction of the magnetic field at the centre of the orbit will be vertically upward.

9. A small circular flexible loop of wire of radius r carries a current I. It is placed in a uniform magnetic field B. The tension in the loop will be doubled if
(a) I is doubled
(b) B is doubled
(c) r is doubled
(d) Both B and I are doubled.

10. A particle having a mass of 0.5 g carries a charge of 2.5×10^{-8} C. The particle is given an initial horizontal velocity of 6×10^4 ms^{-1}. To keep the particle moving in a horizontal direction
(a) the magnetic field should be perpendicular to the direction of the velocity
(b) the magnetic field should be along the direction of the velocity
(c) magnetic field should have a minimum value of 3.27 T
(d) no magnetic field is required.

11. A wire is bent into a circular loop of radius R and carries a current I. The magnetic field at the centre of the loop is B. The same wire is bent into a double loop. If both loops carry the same current in the same direction, the magnetic field at the centre of the double loop is B_1. If they carry the same current I in opposite directions, the magnetic field at their centre is B_2. Then
(a) $B_1 = 0$ (b) $B_1 = 4B$
(c) $B_2 = 0$ (d) $B_2 = \frac{B}{4}$

12. A straight horizontal conducting rod of mass m and lenght l is suspended by two vertical wires (of negligible mass) at its ends. A current I is set up in the rod. A magnetic field B normal to the conductor is required to keep the tension in the wires equal to zero. If the direction of the current is reversed then, for the same magnatic field B, a tension T is developed in the wires. Then
(a) $B = \frac{mg}{Il}$ (b) $B = \frac{2mg}{Il}$
(c) $T = BIl + mg$ (d) $T = BIl - mg$

13. In the hydrogen atom the electron (charge e) moves around the proton with a speed v in a circular orbit of radius r. The magnetic dipole moment of the circulating electron is M and the magnetic field at the site of the proton (i.e. at the centre of the orbit) is B. Then
(a) $M = \frac{evr}{4}$ (b) $M = \frac{evr}{2}$
(c) $B = \frac{\mu_0 ev}{4\pi r^2}$ (d) $B = \frac{\mu_0 ev}{2\pi r^2}$

14. The magnitude of the magnetic field at the centre of a circular coil of radius r, having n turns and carrying a current I can be doubled by
(a) changing I to $2I$, keeping n unchanged
(b) changing n to $2n$, keeping I unchanged
(c) changing I to $2I$ and n to $2n$
(d) changing n to $\frac{n}{2}$, keeping I unchaged.

15. The force experienced by a charged particle moving in a magnetic field **B** with a velocity **v** is zero if
(a) **v** = 0
(b) **v** is parallel to **B**
(c) **v** is perpendicular to **B**
(d) **v** is antiparallel to **B**

16. The magnetic field due to a toroidal solenoid depends upon
(a) the number of turns in the solenoid
(b) the current in the solenoid
(c) the radius of the toroid
(d) the permeability of the core of the solenoid.

17. The plates of a parallel plate capacitor are in the $y - z$ plane. The separation between the plates is 3 mm and a potential difference of 600 V is applied across the plates. An electron is projected between the plates with a velocity of 2×10^6 ms^{-1} along the positive y-direction. The electron moves undeflected between the plates.
(a) The electric field between the plates is 2×10^5 V m^{-1}.
(b) The magnitude of the magnetic field in the region between the plates is 0.1 T.
(c) The direction of the magnetic field is along the positive z-direction.
(d) The direction of the magnetic field is along the negative z

IIT, 1981

18. An alpha particle and a deuteron, after being accelerated through the same potential difference, enter a uniform magnetic field whose direction is perpendicular to their velocities. If r_α and r_d are the radii of the circular paths of the alpha particle and the deuteron respectively and v_α and v_d their respective frequency of revolution, then
(a) $r_\alpha = 2\, r_d$ (b) $r_\alpha = r_d$
(c) $v_\alpha = \frac{v_d}{2}$ (d) $v_\alpha = v_d$

IIT, 2004

19. Three infinitely long thin wires, each carrying current I in the same direction are in the x-y plane in a gravity free space (see Fig. 24.73). The central wire is at $x = 0$ while the other two wires are at $x = \pm d$.

IIT, 1997

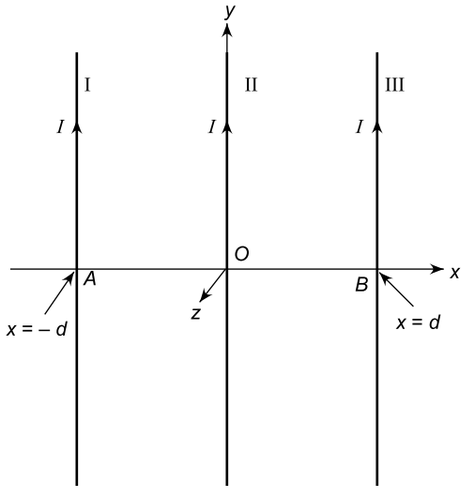


Fig. 24.73

The net magnetic field
(a) cannot be zero beyond B (i.e. $x > d$)
(b) cannot be zero beyond A (i.e. $-x > -d$)
(c) will be zero between $x = 0$ and $x = d$.
(d) will be zero between $x = 0$ and $x = -d$.

20. Two charged particles 1 and 2 of masses m_1 and m_2, charges q_1 and q_2 enter a uniform magnetic field with velocities v_1 and v_2 normal to the field as shown in Fig. 24.74.

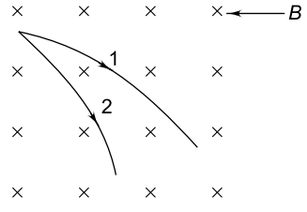


Fig. 24.74

(a) If $q_1 = q_2$, $m_1v_1 > m_2v_2$
(b) If $q_1 = q_2$, $m_1v_1 < m_2v_2$
(c) If $v_1 = v_2$, $\frac{m_1}{q_1} > \frac{m_2}{q_2}$
(d) If $m_1 = m_2$, $v_1q_1 < v_2q_2$.

21. A charged particle is accelerated through a potential difference V. It then enters a region of uniform magnetic field. It moves in a circle of radius r and its frequency of revolution is ν. If V is doubled.
(a) the kinetic energy is doubled
(b) the magnitude of the angular momentum about the centre is doubled.
(c) the radius r becomes half
(d) the frequency ν remains unchanged.

22. A long, thin and hollow cylindrical metal pipe of radius R carries a current I along its length. For such a pipe,
(a) the magnetic field is zero at all points inside the pipe
(b) the magnetic field is zero on the axis of the pipe and increases as we go towards the wall.
(c) On the surface of the pipe, $B = \frac{\mu_0 I}{2\pi R}$.
(d) at a point outside the pipe at distance r from its axis, the magnitude field varies as $1/r_2$.

23. A particle of charge q moving with a velocity $v = v_0\hat{\mathbf{i}}$ enters a magnetic field $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$.
(a) The particle describes a circle in the magnetic field.
(b) The magnitude of the force acting on the particle is qv_0B_0
(c) The magnitude of the force on the particle is $\sqrt{2}\, qv_0B_0$.
(d) The force vector lies in the y-z plane.

24. A thin rod AB of length l carries a current I_1. It is placed in the magnetic field of a long wire PQ carrying a current I_2 as shown in Fig. 24.75.

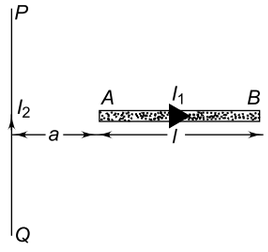


Fig. 24.75

(a) The force experienced by the rod is $F = \frac{\mu_0 I_1 I_2}{4\pi}\log_e\left(1 + \frac{l}{2a}\right)$
(b) The force experienced by the rod is $F = \frac{\mu_0 I_1 I_2}{4\pi}\log_e\left(1 + \frac{l}{a}\right)$
(c) The rod experiences no torque
(d) The rod experiences a force as well as a torque.

25. Choose the correct statements from the following.
(a) The dimensional formula of magnetic field B is $[ML^0\ T^{-2}\ A^{-1}]$
(b) μ_0 is dimensionless.

(c) The dimensions of $\sqrt{\mu_0 \varepsilon_0}$ are the same as those of speed.
(d) The dimensions of $\frac{E}{B}$ are the same as those of speed.

26. An annular wire loop $ABCD$ carries a current I_1 as shown in Fig. 24.76. O is the common centre of the curved parts AB and CD of the loop. A straight wire passing through O and perpendicular to the plane of the loop carries a current I_2 directed towards the reader. Then

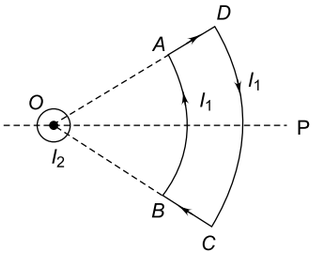


Fig. 24.76

(a) the net force on the loop is zero.
(b) the net torque on the loop is zero.
(c) As seen from O the loop will rotate in clockwise sense about axis OP.
(d) As seen from O the loop will rotate in anticlockwise sense about axis OP.

IIT, 2006

27. A neutron, a proton, an electron and an alpha particle enter a region of uniform magneti field with equal velocities. The magnetic field is along the inward normal to the plane of the paper. The tracks of the particles are labeled as shown in Fig. 24.77.
(a) The electron follows track D and alpha particle follows track B
(b) The proton follows track A and alpha particle follows track B

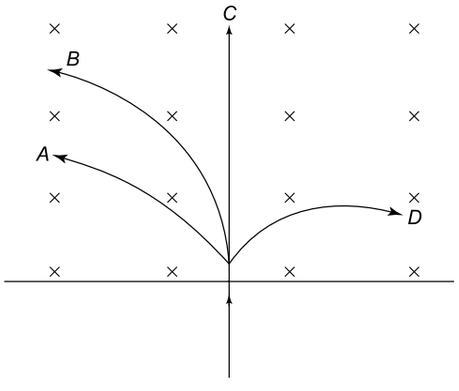


Fig. 24.77

(c) The electron follows track A and neutron follows tracks C
(d) The proton follows track D and electron follows track A.

IIT, 1984

28. A particle of charge + q and mass m moving under the influence of a uniform electric field $E\hat{\mathbf{i}}$ and a uniform magnetic field $B\hat{\mathbf{k}}$ follows a trajectory from P to Q as shown in Fig. 24.78. The velocity at P is $v\ \hat{\mathbf{i}}$ and at Q is $-2v\ \hat{\mathbf{j}}$. Which of the following statements is/are correct ?

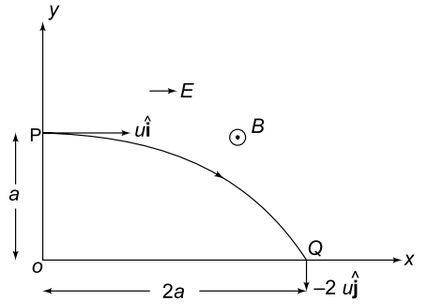


Fig. 24.78

(a) $E = \frac{3mv^2}{4qa}$
(b) The rate of work done by electric field at P is $\frac{3mv^3}{4a}$.
(c) The rate of work done by electric field at P is zero.
(d) The rate of work done by both the fields at Q is zero.

IIT, 1991

29. A particle of mass m and charge q moving with velocity v enters Region II normal to the boundary as shown in Fig. 24.79. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is l. Choose the correct choice(s)

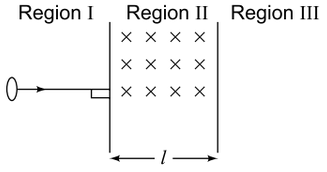


Fig. 24.79

(a) The particle entres Region III only if its velocity $v > \frac{qlB}{m}$
(b) The particle enters Region III only if its velocity $v < \frac{qlB}{m}$

(c) Path length of the particle in Region II is maximum when velocity $v = \frac{qlB}{m}$

(d) Time spent is Region II is the same for any velocity v as long as the particle returns to Region I

IIT, 2008

30. H^+, He^+, and O^{++} all having the same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{++} are lu, 4u and 16u respectively. Then

(a) H^+ will be deflected the most

(b) O^{++} will be deflected the most

(c) He^+ and O^{++} will be deflected equally

(d) All will be deflected equally.

IIT, 1994

31. A proton and an electron moving with the same velocity v enter a region of uniform magnetic field **B** which is perpendicular to their velocity. If r_p and r_e are the radii of their circular trajectories and t_p and t_e the time after which each particle comes out of the region of magnetic field, then

(a) $r_q < r_e$ (b) $r_p > r_e$

(c) $t_p < t_e$ (d) $t_p > t_e$

IIT, 2011

ANSWERS AND SOLUTIONS

1. The direction of the magnetic field is perpendicular to the plane containing the point P and the current element I, which is the z-direction. Hence choice (a) is wrong and choice (b) is correct. Also the fields to the left and to the right of the current element are oppositely directed. Hence choice (d) is correct. Further, the lines of force of the magnetic field are concentric circles with their common centre at the wire. Hence choice (c) is also correct.

2. The correct choices are (a), (b) and (c). The magnetic field inside the solenoid is always tangential.

3. The correct choices are (a) and (b). The field is not uniform at the edges of the solenoid and is parallel to the axis.

4. Since the force exerted by the magnetic field is perpendicular to the velocity of the particle; the speed of the particle cannot change; only the direction of motion, i.e. velocity (and hence momentum) will change. The correct choices are (a), (c) and (d).

5. The force on a charge q moving with a velocity **v** is given by

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

There will be no change in velocity if $\mathbf{F} = 0$. This can happen in three cases. (i) Both **E** and **B** are zero which is choice (a). (ii) $\mathbf{E} = 0$ and **v** and **B** are parallel so that $\mathbf{v} \times \mathbf{B} = 0$ in which case $\mathbf{B} \neq 0$ which is choice (b). (iii) The electric force $q\mathbf{E}$ is equal and opposite to magnetic force $q\,(\mathbf{v} \times \mathbf{B})$ in which case the net force is zero which is choice (d). Hence the correct choices are (a), (b) and (d).

6. The pairs **F** and **v** and **F** and **B** are always at right angles to each other, because **F** is always perpendicular to the plane containing **B** and **v**. Vectors **B** and **v** may have any angle between them.

7. Statement (a) is correct. Since the direction of the velocity of the particle remains unchanged, no magnetic force acts on the particle. The force experienced by a particle of charge q moving with a velocity **v** in a magnetic field **B** is

$$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

Since $\mathbf{F} = 0$, $(\mathbf{v} \times \mathbf{B}) = 0$, i.e. **v** and **B** are parallel to each other. Thus, the initial velocity of the particle is either along the direction of the field or opposite to it.

Statement (b) is also correct. Since the magnetic force is always perpendicular to particle velocity, it cannot change the magnitude of the velocity (i.e. speed); it can only change the direction of the velocity. Hence, the final speed of the particle is equal to its initial speed. The direction of the final velocity is, however, different from that of the initial velocity.

Statement (c) is wrong. Under the influence of an electric field in the negative y-direction, the electron will be deflected in the positive y-direction. It will travel undeflected, if the magnetic field imparts an equal deflection in the negative y-direction. Since the magnetic force is perpendicular to the magnetic field and the charge on the electron is negative, the direction of the magnetic field should be along the negative z-direction (use Fleming's left-hand rule).

Statement (d) is also incorrect. No work is done by a uniform magnetic field on a charged particle. Hence its kinetic energy remains constant.

8. Statement (a) is correct. The loop will rotate until equilibrium state is attained. It will then come to rest because the torque acting on it becomes zero. We know that the torque is given by

$$\tau = BIA \sin \theta$$

where θ is the angle between the direction of magnetic field **B** and the normal to the plane of the loop. It is clear that $\tau = 0$ when $\theta = 0$. Thus, in the equilibrium state, the orientation of the loop is such that the magnetic field is perpendicular to the plane of the loop.

Statement (b) is incorrect. If **v** is parallel to **B**, the particle does not experience any force. Hence its trajectory will be a straight line. Statement (c) is correct. Since currents are equal and in the same direction, the magnetic fields due to them at a point midway between the wires will be equal and opposite and hence they will cancel each other.

Statement (d) is incorrect. The direction of the magnetic field, which is given by Fleming's left hand rule, will be vertically downward. Note that the charge of an electron is negative.

9. The force acting on the loop is given by

$$F = m\,B \sin\theta$$

where $m = \pi r^2 I$. Force will be doubled if I or B are doubled. Hence the correct choices are (a) and (b).

10. In the absence of a magnetic field, the particle will experience gravitational force mg. As a result the particle will not continue moving in the horizontal direction but will describe a parabolic path. So a magnetic field must be present and its direction must be perpendicular to the direction of the velocity. The magnetic force experienced by the particle is given by

$$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

The magnitude of the force is $F = q\,v\,B \sin\theta$. If the particles is to move in the horizontal direction, this force must balance the force of gravity, i.e.

$$mg = q\,v\,b \sin\theta$$

The minimum value of B corresponds to $\sin\theta = 1$ or $\theta = 90°$. Thus

$$mg = \theta\,v\,B$$

$$\text{or } B = \frac{mg}{qv} = \frac{0.5\times10^{-3}\times9.8}{2.5\times10^{-8}\times6\times10^{4}} = 3.27 \text{ T}$$

Hence the correct choices are (a) and (c).

11. The radius of the double loop $r = R/2$. Now

$$B = \frac{\mu_0 I}{2R}$$

Magnetic field due to a loop of radius r at the centre of the loop is

$$B_1 = \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{2R} = 2\,B \qquad (\because r = 1/2)$$

Similarly for the second loop of the double loop,

$$B_2 = 2\,B$$

Since the currents in the two loops are in the same direction, the net magnetic field at the centre $= B_1 + B_2 = 4B$.

Since the currents in the two loops are in opposite directions, fields B_1 and B_2 are equal and opposite. Therefore, the net magnetic field at the centre of the double loop $= B_1 - B_2 = 0$. Hence the correct choices are (b) and (c).

12. In order that the tension in the supporting wires is zero the downward gravitational force mg on the rod must be balanced by an upward force BIl due to magnetic field, i.e. $BIl = mg$

$$\text{or} \qquad B = \frac{mg}{Il}$$

If the current is reversed, the direction of the force due to **B** becomes downwards, in the direction of the gravitational force. Hence the tension in the string is

$$T = BIl + mg$$

The correct choices are (a) and (c).

13. Time period $T = \dfrac{2\pi r}{v}$. Current $I = \dfrac{e}{T} = \dfrac{ev}{2\pi r}$.

Magnetic moment M = current × area of orbit

$$= I \times \pi r^2 = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

Magnetic field at the centre of the orbit is

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 ev}{4\pi r^2}$$

So the correct choices are (b) and (c).

14. The magnitude of the magnetic field at the centre of the coil is given by

$$B = \frac{\mu_0 nI}{2r}$$

Hence the correct choices are (a) and (b).

15. $\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$. The magnitude of the force is

$$F = q\,v\,B \sin\theta$$

Thus $F = 0$ if $v = 0$ or $\theta = 0°$ or $180°$. Hence the correct choices are (a), (b) and (d).

16. The correct statements are (a), (b) and (d). The magnetic field is given by

$$B = \mu nI$$

where μ is the permeability of the core of the solenoid. B is independent of the radius of the toroid.

17. Refer to Fig. 24.80. The electric field between the plates is $E = \dfrac{V}{d} = \dfrac{600}{3\times10^{-3}} = 2\times10^5 \text{ V m}^{-1}$.

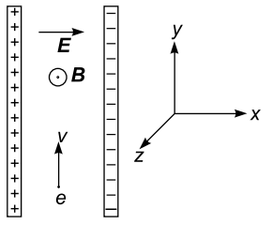


Fig. 24.80

The electron will experience a force qE in the negative x-direction. If the electron is to move undeflected in the region between the plates, the magnetic force experienced by it must be along the positive x-direction and its magnitude must be equal to qE. Since magnetic force = $q(\boldsymbol{v} \times \mathbf{B})$, it follows that the field must be perpendicular to $\boldsymbol{v}$ (as well as **E**) and directed along the negative z-direction. The magnitude of the magnetic force =qvB sin 90° = qvB.
Thus

$$qvB = qE \text{ or } B = \frac{E}{v} = \frac{2\times10^5}{2\times10^6} = 0.1 \text{ T}$$

So the correct choices are (a), (b) and (d).

18. If a particle of mass m and charge q is accelerated through a potential difference V, it acquires kinetic energy = qV. Hence

$$\frac{1}{2}mv^2 = qV$$

or $$v = \sqrt{\frac{2qV}{m}}$$

where v is the velocity acquired by the particle. The radius of the circular path is given by

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{\sqrt{2V}}{B}\times\sqrt{\frac{q}{m}}$$

For given V and B, $r \propto \sqrt{\frac{q}{m}}$. Hence

$$\frac{r_\alpha}{r_d} = \sqrt{\frac{q_\alpha}{q_d}\times\frac{m_d}{m_\alpha}} \quad (1)$$

Now, alpha particle has 2 protons and 2 neutrons and a deuteron has 1 proton and 1 neutron. Therefore, $m_\alpha = 4\ m_p$, $m_d = 2\ m_p$, $q_\alpha = 2\ q_d$ and $q_d = q_p$. Using these in (1), we get

$$\frac{r_\alpha}{r_d} = \sqrt{\frac{2q_p}{q_p}\times\frac{2m_p}{4m_p}} = 1$$

The frequency of revolution is given by

$$\nu = \frac{qB}{2\pi m} \propto \frac{q}{m}$$

Hence $$\frac{\nu_\alpha}{\nu_d} = 1$$

Thus the correct choices are (b) and (d).

19. By using the right hand rule, the following conclusions can be drawn:
For Points on the x-axis beyond B: The directions of the magnetic fields due to currents in wires I, II and III are along the negative z-axis. Hence the net magnetic field B cannot be zero beyond B (i.e. $x > d$).
For Points on the x-axis beyond A: The directions of the magnetic fields due to currents in wires I, II and III are along the positive z-axis. Hence the net magnetic field B cannot be zero beyond A, (i.e. $-x > -d$).
For Points between O and B: The directions of the magnetic fields due to currents in wires I and II are along the negative z-axis but the direction of the magnetic field due to current in wire III is along the positive z-axis. Hence the net magnetic field B will be zero for some value of x lying between $x = 0$ and $x = d$.
For Points between O and A: The directions of the magnetic fields due to currents in wires I and II are along the positive z-axis but that due to current in wire III is along the negative z-axis. Hence the net magnetic field B will be zero for some value of x lying between $x = 0$ and x = $-d$.
So all the four choices are correct.

20. The radius r of the circular path of a particle of mass m and charge q moving with a velocity v perpendicular to a uniform magnetic field B is given by

$$\frac{mv^2}{r} = qvB \Rightarrow mv = qrB \quad (1)$$

If $q_1 = q_2$, then $\frac{m_1v_1}{m_2v_2} = \frac{r_1}{r_2}$

It follows from Fig. 24.66 that $r_1 > r_2$. Hence $m_1v_1 > m_2v_2$.
If $v_1 = v_2$, then from Eq. (1), we have

$$\frac{m_1}{m_2} = \frac{q_1r_1}{q_2r_2}$$

Since $\frac{r_1}{r_2} > 1, m_1q_2 > m_2q_1$ or $\frac{m_1}{q_1} > \frac{m_2}{q_2}$. If $m_1 = m_2$, then from Eq. (1) we find that $v_1q_2 > v_2q_1$. So the correct choices are (a) and (c).

21. Kinetic energy = qV. Hence if V is doubled, K.E. is also doubled. The magnitude of the angular momentum about the centre is $L = mvr$. Now $r = mv/qB$.
Therefore,

$$L = \frac{m^2v^2}{qB} = \frac{2m}{qB}\left(\frac{1}{2}mv^2\right) = \frac{2m}{qB}\times(\text{K.E.})$$

Thus if V is doubled, K.E. is doubled and L is also doubled. The radius of the circular path is given by

$$r^2 = \frac{m^2v^2}{q^2B^2} = \frac{2m}{q^2B^2}\times\left(\frac{1}{2}mv^2\right) = \frac{2m}{q^2B^2}\times qV$$

$$= \frac{2mV}{qB^2}$$

Thus $r \propto \sqrt{V}$. The frequency of revolution is

$$\nu = \frac{qB}{2\pi m}$$

Hence the correct choices are (a), (b) and (d).

22. Consider a point P at a distance r from the axis of the pipe. For all points inside the pipe, $r < R$. To find the magnetic field at P, we choose an Amperean loop to be a cylindrical loop of radius R as shown in Fig. 24.81. Since the current enclosed in the Amperean surface is zero, it follows from Ampere's circuital law, that the magnetic field is zero at all points inside the pipe. For points outside the pipe, $r > R$, we have from Ampere's law,

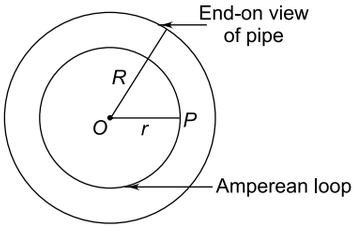


Fig. 24.81

$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 I$$

$$\Rightarrow B \times \int dl = \mu_0 I \qquad (\mathbf{B} \text{ is } \| \ \mathbf{dl})$$

$$\Rightarrow B \times 2\pi r = \mu_0 I$$

$$\Rightarrow \qquad B = \frac{\mu_0 I}{2\pi r} \text{ , i.e. } B \propto \frac{1}{r}$$

At the surface of the pipe ($r = R$)

$$B = \frac{\mu_0 I}{2\pi R}$$

Hence the correct choices are (a) and (c).

23. Since v is not perpendicular to **B**, the particle will not describe a circle. The force acting on the particle is

$$\mathbf{F} = q(v \times \mathbf{B})$$

$$= q[v_0 \hat{\mathbf{i}} \times B_0(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})]$$

$$= qv_0B_0(\hat{\mathbf{i}} \times \hat{\mathbf{i}} + \hat{\mathbf{i}} \times \hat{\mathbf{j}} + \hat{\mathbf{i}} \times \hat{\mathbf{k}})$$

$$= qv_0B_0(\hat{\mathbf{k}} - \hat{\mathbf{j}})$$

Thus the magnitude of the force is

$F = qv_0B_0 = qv_0B_0(1^2 + 1^2)^{1/2} = \sqrt{2}\, qv_0B_0$

and it lies in the y-z plane. Hence the correct choices are (c) and (d).

24. Consider a small element of length dx of the rod at a distance x from wire PQ as shown in Fig. 24.82. The magnetic field due to wire PQ at the element is

$$B = \frac{\mu_0 I_2}{2\pi x}$$

Therefore, force exerted on the element is

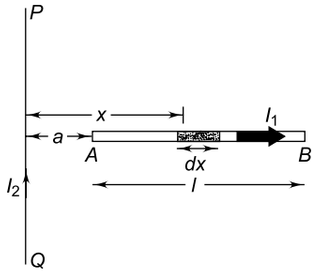


Fig. 24.82

$$dF = BI_1\, dx = \frac{\mu_0 I_1 I_2 dx}{2\pi x}$$

$\therefore$ Total force experienced by the rod is

$$F = \int dF$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \int_{x=a}^{x=(a+l)} \frac{dx}{x}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \log_e \left(\frac{a+l}{a}\right)$$

Now, the magnetic field B is not uniform; it decreases as we go from and A to end B of the rod. Hence the rod will also experience a torque. Thus the correct choices are (b) and (d).

25. From $F = qvB$, it is easy to see that the dimensional formula of B is $[ML^0\, T^{-2}\, A^{-1}]$, which is choice (a). Choice (b) is wrong. From $= \mu_0/(2\pi)$, the dimensions of μ_0 are

$$[\mu_0] = \frac{[B] \times [r]}{[I]}$$

$$= \frac{ML^0T^{\pm 2}A^{-1} \times L}{A} = [MLT^{-2}\, A^{-2}]$$

From $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$, the dimensions of ε_0 are

$$[\varepsilon_0] = \frac{[q_1 q_2]}{[F][r^2]} = \frac{A^2T^2}{MLT^{-2} \times L^2} = M^{-1}\, L^{-3}\, T^4\, A^2$$

$\therefore$ $[\mu_0 \varepsilon_0] = [MLT^{-2}\, A^{-2}] \times [M^{-1}\, L^{-3}\, T^4\, A^2] = [L^{-2}\, T^2]$

$\therefore$ $\sqrt{\mu_0 \varepsilon_0} = [L^{-1}\, T]$, which is reciprocal to speed.

Hence choice (c) is incorrect.

From $qE = qvB$, we find that $\frac{E}{B} = v$. Hence choice (d) is correct.

26. The magnetic field due to current I_2 is tangential to the curved parts AB and CD of the loop. Hence every current element $d\mathbf{l}$ of parts AB and CD is parallel or antiparallel to $\mathbf{B}$. The magnetic force on AB or CD is zero since $\theta = 0°$ or $180°$ in the expression $dF = Bldl \sin\theta$. The magnetic force on straight parts AD and BC is not zero. The magnetic force on AD is directed towards the reader which is equal and opposite to the force on BC which is directed away from the reader. These equal and opposite forces cancel each other. Therefore, the net force on the loop $ABCD$ is zero. Since these equal and opposite forces do not act at the same point, they will exert a net torque on the loop which will rotate it in the clockwise sense when viewed from O. Hence the correct choices are (a) and (c).

27. The radius of the circular track is given by

$$r = \frac{mv}{qB}$$

For a neutron $q = 0$. For an electron $q = -e$ and for a proton $q = +e$. For an alpha particle $q = 2e$ and m = mass of four protons. Hence the correct choices are (a) and (b).

28. In going from P to Q, the change in the kinetic energy of the particle $= \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{3}{2}mv^2$. The work done by the electric field in moving the particle from P to $Q = \mathbf{F}\cdot\mathbf{d}$ where $\mathbf{d}$ is the displacement in the direction of $\mathbf{E}$. So work done $W = q\mathbf{E}\hat{\mathbf{i}}\cdot 2a\hat{\mathbf{i}} = 2qaE\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = 2qaE$. Now work done = change in K.E. Hence

$$2qaE = \frac{3}{2}mv^2 \quad \Rightarrow \quad E = \frac{3mv^2}{4qa}$$

Thus choice (a) is correct.

Rate of work done by electric field at P is

$$\mathbf{F}\cdot v_p = qE\hat{\mathbf{i}}\cdot v\hat{\mathbf{i}} = qEv = q\times\frac{3mv^2}{4qa}\times v$$

$$= \frac{3mv^3}{4a}$$

So choice (b) is also correct.

Rate of work done by both the field at Q is zero because $\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = 0$ and $\hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = 0$. Hence the correct choices are (a), (b) and (d).

29. In region II, the particle follows a circular path of radius

$$r = \frac{mv}{qB}$$

Therefore, the particle can enter region III if $r > l$, i.e. if $v > \frac{qBl}{m}$.

In region II, the maximum path length is $r = l$, which gives $v = \frac{qBl}{m}$

The time period of the circular motion is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\times\frac{mv}{qB} = \frac{2\pi m}{qB}$$

The particle will return to region I if the time spent by it in region II is $\frac{T}{2} = \frac{\pi m}{qB}$, which is independent of the velocity. Hence the correct choices are (a), (c) and (d).

30. $\frac{1}{2}m_H v_H^2 = \frac{1}{2}m_{He}v_{He}^2 = \frac{1}{2}m_O v_O^2$. Given m_H = 1u, m_{He} = 4u and m_O = 16u. Hence $v_{He} = \frac{1}{2}v_H$ and $v_O = \frac{1}{4}v_H$.

The radius of the circular path is given by

$$R = \frac{mv}{qB};\ q = \text{charge}$$

Now charge of $H^+ = e$, charge of $He^+ = e$ and charge of $O^{++} = 2e$. It is easy to check that $r_{He} = r_O$ and $r_H = \frac{1}{2}r_{He}$

Smaller the value of r, greater is the deflection. Thus the correct choices are (a) and (c).

31. The following figure shows the trajectories of the electron and proton in the region of magnetic field.

Since $r = \frac{mv}{qB}$, $r_p > r_e$

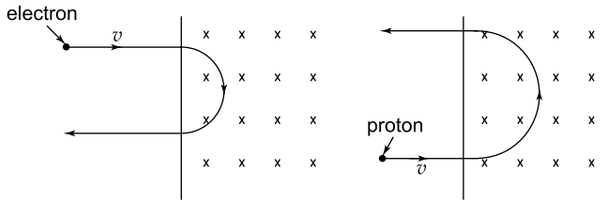


Fig. 24.83

Since $v \perp \mathbf{B}$, the charged particle describes a semicircular trajectory. Hence the time after which the particle comes out of the field = half the time period, i.e.

$$t = \frac{T}{2} = \frac{\pi m}{qB}$$

Since $m_p > m_e$ and q and B are the same, $t_p > t_e$. Hence the correct choices are (b) and (d).

SECTION III

Multiple Choice Questions based on Passage

Questions 1 to 3 are based on the following passage

Passage I

Two long parallel wires carrying currents 2.5 amperes and I ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of the paper. The points P and Q are located at a distance of 5 m and 2 m, respectively, from a collinear point R (see Fig. 24.84).

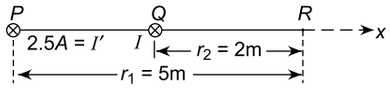


Fig. 24.84

An electron moving with a velocity of 4×10^5 m/s along the positive x-direction experiences a force of magnitude 3.2×10^{-20} N at the point R.

IIT, 1990

1. The magnitude of magnetic field at point R is
(a) 2.5×10^{-7} T (b) 5.0×10^{-7} T
(c) 5.0×10^{-6} T (d) 2.5×10^{-6} T

2. The magnitude of magnetic field at point R due to current $I' = 2.5$ A in wire P is
(a) 1×10^{-7} T (b) 2×10^{-7} T
(b) 3×10^{-7} T (d) 4×10^{-7} T

3. The current I in wire Q is
(a) 1 A (b) 2 A
(c) 3 A (d) 4 A

SOLUTION

1. The magnitude of the force experienced by a particle of charge q moving with a velocity v in a magnetic field is given by

$$F = qvB \sin\theta$$

where θ is the angle between v and **B**. Given $F = 3.2 \times 10^{-20}$ N, $v = 4 \times 10^5$ ms^{-1} and $\theta = 90°$. For electron $q = 1.6 \times 10^{-19}$ C. Using these value we get $B = 5 \times 10^{-7}$ T, which is choice (b)

2. The magnetic field at point R due to currect I' in wire P is

$$B_1 = \frac{\mu_0 I'}{2\pi r_1} = \frac{4\pi \times 10^{-7} \times 2.5}{2\pi \times 5} = 1 \times 10^{-7} \text{ T}$$

The correct choice is (a).

3. The magnetic field at point R due to currect I in wire Q is

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times I}{2\pi \times 2} = I \times 10^{-7} \text{ T}$$

Both fields B_1 and B_2 will be in the downwar direction, parallel and colliner. Hence the resultant magnetic field at point R is

$B = B_1 + B_2 = (1 + I) \times 10^{-7}$ T

Now $B = 5 \times 10^{-7}$ T. Therefore

$(1 + I) \times 10^{-7} = 5 \times 10^{-7}$

or $1 + I = 5$ or $I = 4$ A.

So the correct choice is (d).

Questions 4 to 7 are based on the following passage

Passage II

The region between $x = 0$ and $x = L$ is filled with a uniform, steady magnetic field $B_0 \hat{\mathbf{k}}$. A particle of mass m, positive charge and velocity $v_0 \hat{\mathbf{i}}$ travels along x-axis and enters the region of the magnetic field. Neglect gravity.

IIT, 1999

4. The force experienced by the charged particle in the magnetic field is
(a) along the positive y-direction
(b) along the negative y-direction
(c) in the x-y plane
(d) in the y-z plane.

5. If the particle emerges from the region of magnetic field with its final velocity at an angle of 30° to its initial velocity, the value of L is

(a) $\frac{2mv_0}{qB_0}$ (b) $\frac{mv_0}{qB_0}$

(c) $\frac{mv_0}{2qB_0}$ (d) $\frac{\sqrt{3}mv_0}{2qB_0}$

6. If the magnetic field now extends up to $x = 2.1\ L$, the final velocity of the particle when it emerges out of the region of magnetic field will be

(a) $v_0\,\hat{\mathbf{i}}$ (b) $-v_0\,\hat{\mathbf{i}}$

(c) $v_0\,\hat{\mathbf{j}}$ (d) $-v_0\,\hat{\mathbf{j}}$

7. In Q. 6, the time spent by the particle in the magnetic field is

(a) $t = \frac{2\pi m}{qB_0}$ (b) $t = \frac{\sqrt{2}\pi m}{qB_0}$

(c) $t = \frac{\sqrt{3}\pi m}{2qB_0}$ (d) $t = \frac{\pi m}{qB_0}$

SOLUTION

4. The force experienced by the charged particle is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = (v_0\,\hat{\mathbf{i}}) \times (B_0\,\hat{\mathbf{k}})$$

$$= qv_0B_0(\hat{\mathbf{i}} \times \hat{\mathbf{k}})$$

$$= qv_0B_0(-\hat{\mathbf{j}}) \qquad (1)$$

The force is along the negative y-direction, which is choice (b).

5. Refer to Fig. 24.85.

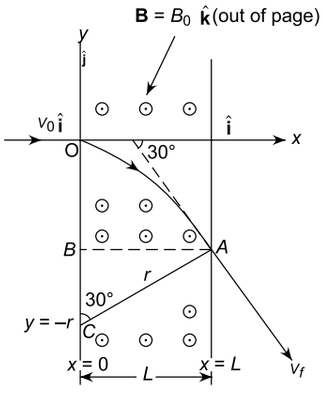


Fig. 24.85

The particle describes a circle of radius

$$r = \frac{mv_0}{qB_0} \qquad (2)$$

Since the particle emerges from the region of the magnetic field with the velocity vector making an angle of 30° with the initial vector, it follows from triangle ABC that

$$AB = AC \sin 30°$$

or $$L = r \sin 30°$$

$$= \frac{mv_0 \sin 30°}{qB_0} = \frac{mv_0}{2qB_0} \qquad (3)$$

Thus the correct choice is (c).

6. Comparing (2) and (3) we find that = 2. Since the magnetic field now extends up to = 2. 1 , the particle will continue to move in a circular path till it completes half the circular path and emerges out of the region of the magnetic field with a velocity $-v_0\,\hat{\mathbf{i}}$ moving along the negative x-axis as shown in Fig. 24.39.

7. Distance travelled by the particle in the magnetic field = half the circumference = π. Therefore, time spent in the magnetic field is

$$t = \frac{\pi r}{v_0} = \frac{\pi m}{qB_0} \qquad \text{[Use Eq. (2)]}$$

So the correct choice is (d).

Questions 8 to 11 are based on the following passage

Passage III

A wire loop consists of a straight segment AB and a circular arc ACB of radius r. The segment AB subtends an angle of 60° at the centre O of the circular arc. The wire loop carries a current I in the clockwise direction (Fig. 24.86).

8. The magnetic field B_1 at O due to the straight segment AB is

(a) $\frac{\mu_0 I}{2\pi r}$ (b) $\frac{\mu_0 I}{2\sqrt{2}\pi r}$

(c) $\frac{\mu_0 I}{2\sqrt{3}\pi r}$ (d) $\frac{\mu_0 I}{4\pi r}$

9. The magnetic field B_2 at O due to the circular arc ACB is

(a) $\frac{5\mu_0 I}{12r}$

(b) $\frac{\mu_0 I}{2r}$

(c) $\frac{3\mu_0 I}{8r}$

(d) $\frac{7\mu_0 I}{18r}$

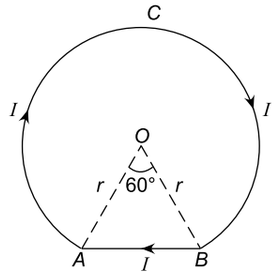


Fig. 24.86

10. The net magnetic field B at O due to the whole wire loop is

(a) $B = B_1 + B_2$ (b) $B = B_2 - B_1$

(c) $B = \sqrt{B_1^2 + B_2^2}$ (d) $B = \sqrt{B_2^2 - B_1^2}$

11. The direction of the magnetic field B is

(a) parallel to the plane of the coil

(b) perpendicular to the plane of the coil and directed out of the page

(c) perpendicular to the plane of the coil and directed into the page.

(d) inclined at an angle of 60° with the plane of the coil.

SOLUTION

8. As shown in Fig. 24.87, the magnetic field at O due to current I in AB is given by (use Biot-Savart law)

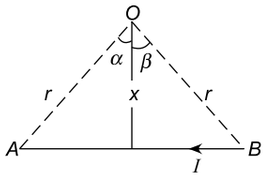


Fig. 24.87

$$B_{AB} = \frac{\mu_0 I}{4\pi x}(\sin\alpha + \sin\beta)$$

Here $\alpha = \beta = 30°$. Also $x = r\cos\alpha = r\cos 30° = \frac{\sqrt{3}r}{2}$. Therefore,

$$B_1 = \frac{\mu_0 I}{4\pi \times \sqrt{3}r/2} \times (\sin 30° + \sin 30°)$$

$$= \frac{\mu_0 I}{2\sqrt{3}\pi r} \times (0.5 + 0.5) = \frac{\mu_0 I}{2\sqrt{3}\pi r},$$

which is choice (c).

The direction of the field is perpendicular to the plane of the paper directed into the page.

9. The magnetic field at the centre of a complete (n = 1 turn) circular loop of radius r and carrying a current I is

$$B = \frac{\mu_0 nI}{2r}$$

Here loop ACB is a fraction of a circle i.e. $n < 1$. Since ACB subtends an angle (360° – 60°) = 300° at O, hence the fraction n is

$$n = \frac{300°}{360°} = \frac{5}{6}$$

Therefore, magnetic field due to arc ACB is

$$B_2 = \frac{\mu_0 \times \frac{5}{6} \times I}{2r} = \frac{5\mu_0 I}{12r}$$

As the current in ACB is clockwise, the direction of the magnetic field is perpendicular to the plane of the paper and directed into the page.

The correct choice is (a).

10. Since B_1 and B_2 are in the same direction, the net field is $B = (B_1 + B_2)$, which is choice (a).

11. The correct choice is (c).

Questions 12 to 15 are based on the following passage

Passage IV

A moving coil galvanometer consists of a coil of N turns and area suspended by a thin phosphor bronze strip in radial magnetic field B. The moment of inertia of the coil about the axis of rotation is I and C is the torsional constant of the phosphor bronze strip. When a current is passed through the coil, it deflects through an angle θ (in radian).

IIT, 2004

12. Choose the correct statement from the following. The magnitude of the torque experienced by the coil is independent of

(a) N (b) B

(c) i (d) I

13. The current sensitivity of the galvanometer is increased if

(a) N, A and B are increased and C is decreased.

(b) N and A are increased and B and C are decreased

(c) N, B and C are increased and A is decreased

(d) N, A, B and C are all increased.

14. When a charge is passed almost instantly through the coil, the angular speed ω acquired by the coil is

(a) $\frac{NAB}{QI}$ (b) $\frac{BAQ}{NI}$

(c) $\frac{NABQ}{I}$ (d) $\frac{NAQI}{B}$

15. In Q. 14, the maximum angular deflection (in radian) of the coil is

(a) $\theta_{max} = \omega\sqrt{\frac{I}{C}}$ (b) $\theta_{max} = \frac{1}{C}\sqrt{I\omega}$

(c) $\theta_{max} = I\sqrt{\frac{\omega}{C}}$ (d) $\theta_{max} = \omega\sqrt{IC}$

SOLUTION

12. The magnitude of torque experienced by the coil is given by

$$\tau = iNAB \sin \alpha$$

where α is the angle which the normal to the plane of the coil makes with the direction of the magnetic field. If the magnetic field is radial, the plane of the coil is always parallel to the direction of the magnetic field, i.e. $\alpha = 90°$. Hence $\tau = iNAB = Ki =$ where

$$K = NAB$$

So the correct choice is (d).

13. Let θ be the angular deflection (in radian) when a current i is passed through the coil. Then, restoring torque = $C\theta$. When the coil is in equilibrium, deflecting torque = restoring torque, i.e.

$$iNAB = \theta$$

$\therefore$ Current sensitivity is $\frac{\theta}{i} = \frac{NAB}{C}$

Hence the correct choice is (a).

14. If ω is the angular speed acquired by the coil when a charge is passed through it for very short time Δt, then

$$\tau = \frac{\text{angular momentum}}{\text{time interval}} = \frac{I\omega}{\Delta t}$$

or $\omega = \tau \Delta t = \Delta = KQ \qquad \left(\therefore i = \frac{Q}{\Delta t}\right)$

or $I\omega = NABQ$ or $\omega = \frac{NABQ}{I}$, which is choice (c).

15. From the principle of conservation of energy, we have

$$\frac{1}{2}\omega^2 = \frac{1}{2}\theta_{\max}^2$$

which gives $\theta_{\max} = \omega\sqrt{\frac{I}{C}}$, which is choice (a)

Questions 16 to 18 are based on the following passage

Passage V

A particle of mass 1.6×10^{-27} kg and charge 1.6×10^{-19} C enters a region of uniform magnetic field of 1 T at E along the direction shown in Fig. 24.88. The speed of the particle is 10^7 ms^{-1}. The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at F.

IIT, 1984

16. The value of angle θ is

(a) 30° (b) 45°
(c) 60° (d) 75°

17. The radius of the circular path of the particle in the magnetic field is

(a) 0.1 m (b) 0.2 m
(c) 0.3 m (d) 0.4 m

18. The distance EF is

(a) $\sqrt{2}$ m (b) $\frac{\sqrt{2}}{5}$ m
(c) $\frac{\sqrt{2}}{10}$ m (d) $\frac{1}{\sqrt{2}}$ m

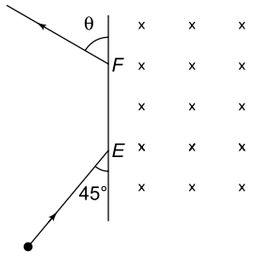


Fig. 24.88

SOLUTION

16. Let O be the centre of the circular path. It is obvious that the particle will leave the field at F such that BF is tangent to the circle. OE and OF are normals, which meet at O (see Fig. 24.89). Therefore, angle OFE = angle OEF. Hence $\theta = 45°$.

17. $r = \frac{mv}{qB}$. Substituting the given values and solving we get $r = 0.1$ m

18. $EF = 2r \cos 45° = \sqrt{2}\, r = \sqrt{2} \times 0.1 = \frac{\sqrt{2}}{10}$ m

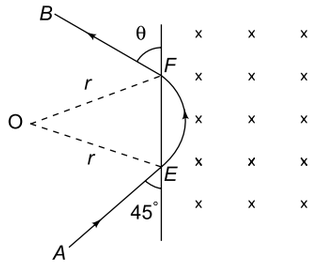


Fig. 24.89

SECTION IV

Matrix Matching Type

1. In Column I are listed some charged bodies and current carrying conductors. Match them with the effects they produce listed in column II

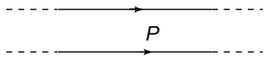
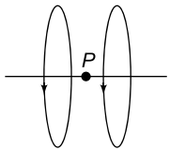
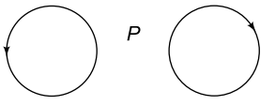
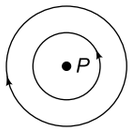
Column I	***Column II***
(a) A uniformly charged stationary ring	(p) Electric field
(b) A uniformly charged ring rotating with a constant angular velocity	(q) Magnetic field
(c) A coil carrying a current $I = I_0 \sin \omega t$	(r) Magnetic moment
(d) A wire carrying a constant current	(s) Issnduced electric field

IIT, 2006

ANSWER

(a) → (p)
(b) → (p), (q), (r)
(c) → (p), (r), (s)
(d) → (q)

2. Two wires each carrying a steady current I are shown in four configurations in **Column I.** Some of the resulting effects are described in **Column II.** Match the statemens in **Column I** with those in **Column II**.

Column I	***Column II***
(a) Point P midway between wires 	(p) The magnetic fields (B) at P due to the currents in the wires are in the same direction.
(b) Point P is situated at the mid-point of the line joining the centers of the circular wires which have same radii. 	(q) The magnetic fields (B) at P due to the current in the wires are in opposite directions.
(c) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii 	(r) There is no magnetic field at P.
(d) Point P is situated at the common centre of the wires. 	(s) The wires repel each other.

IIT, 2007

SOLUTION

(a) At point P midway between two wires carrying equal currents in the same direction, the magnetic fields due to the currents are equal but in opposite directions. Hence the correct choices are (q) and (r).

(b) The currents in the two coils are in the same sense (anticlockwise). Hence, at point, P, the magnetic fields due to the current are the same direction, which is choice (p).

(c) Since the currents in the circular coils are in the opposite sense, the magnetic fields at point P due to the currents are in the opposite directions. Since point P is the same distance from the centres of the coils, and their radii are equal the magnitudes of the magnetic fields due to the currents are equal. Therefore, the net magnetic fields at point P is zero. Hence the correct choices are (q) and (r).

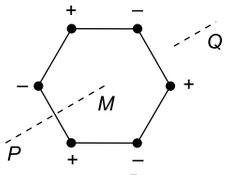
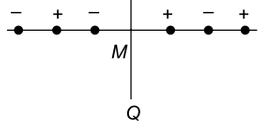
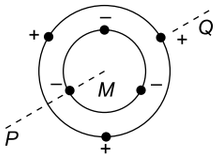
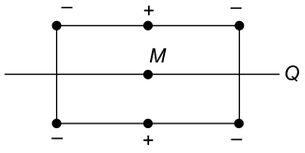
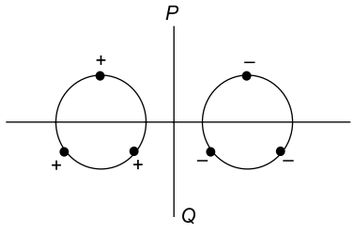
(d) The magnetic fields at the common centre P of the coils are not equal (because their radii are different). Further, since the currents in the coils are in the opposite sense, the magnetic fields due to the currents are in opposite direction. If the currents are in opposite directions in coils (or wires), they repel each other. Hence the correct choices are (q) and (s).

(a) $\rightarrow$ (q), (r) (b) $\rightarrow$ (p)

(c) $\rightarrow$ (q), (r) (d) $\rightarrow$ (q), (s)

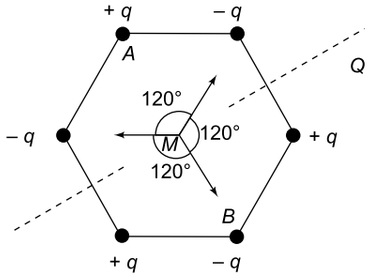
3. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column II**. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

IIT, 2007

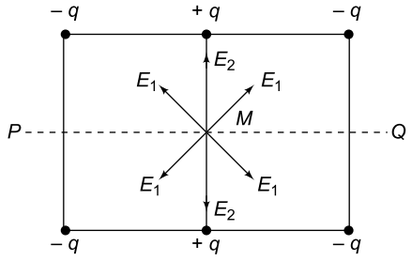
Column I	***Column II***
(a) $E = 0$ (p) 	(p) Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon.
(b) $V \neq 0$ (q) 	(q) Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.
(c) $B = 0$ (r) 	(r) Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.
(d) $\mu \neq 0$ (s) 	(s) Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid point of the longer sides. M is at the centre of the rectangle. PQ is parralled to the longer sides.
(t) 	(t) Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres coplanar to

SOLUTION

(p) Electric field at M due to charge at A and $B = \dfrac{2q}{4\pi\varepsilon_0 r^2}$ where $r = AM = BM$ directed from M to B. The electric fields at M due to other charges are shown. It follows that the net electric field at I due to all charges = 0. Also net $V = 0$. Since the total charge of the system is zero, on rotation net current = 0. Hence $B = 0$ and $\mu = 0$

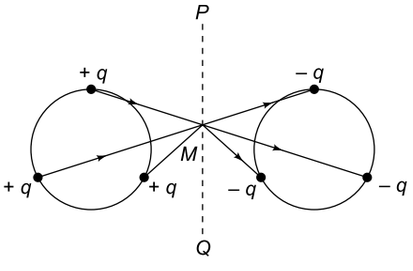


(q) The net charge on the right of M is $2q - q = q$ and on the left of M is $-q$. Hence the net electric field at M is finite and directed to the left of M. But net $V = 0$. Net charge = 0. On rotation, current = 0. Hence $B = 0$ and $\mu = 0$.



(r) Net electric field at $M = 0$ as the angles between individual fields is 120°. Net $V \neq 0$. On rotating, net current is not zero. Hence $B \neq 0$ and $\mu \neq 0$.

(s) It follows from the figure that net $E = 0$. But net $V \neq 0$. On rotating, net current $\neq 0$. Hence $B \neq 0$ and $\mu \neq 0$.
It follows from the figure that net electric field $E \neq 0$ and is directed to the right but net $V = 0$. On rotation, net current = 0. Hence $B = 0$ and $\mu = 0$.



ANSWERS

(a) → (p), (r), (s)
(b) → (r), (s)
(c) → (p), (q), (t)
(d) → (r), (s)

SECTION V

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by statement-2 (Reason). Each question has the following four options out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True and State-ment-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; but Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. Statement 1
A non-uniform magnetic field that varies in magnitude from point to point but has a constant direction, is set up in a region of space. If a charged particle enters the region in the direction of the magnetic field, it will be accelerated at non-uniform rate in the region.
Statement 2
The force $\vec{F}$ experienced by a particle of charge moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = (\vec{v} \times \vec{B})$.

2. Statement 1
A charged particle moves in a uniform magnetic field for some time. During this time, the kinetic energy of the particle cannot change but its momentum can change.
Statement 2
The magnetic force is always perpendicular to the velocity of the particle.
IIT, 1993

3. Statement 2
A current carrying loop is free to rotate. It is placed in a uniform magnetic field. It will attain equilibrium when its plane is perpendicular to the magnetic field.
Statement 2
The torque on the coil is zero when its plane is perpendicular to the magnetic field.

4. Statement 1
An electron moving in the positive -direction enters a region where uniform electric and magnetic fields exist perpendicular to each other. The electric field is in the negative -direction. If the electron travels undeflected in this region, the direction of the magnetic field is along the negative -axis.
Statement 2
If a charged particle moves in a direction perpendicular to a magnetic field, the direction of the force acting on it is given by Fleming's left-hand rule.

5. Statement 1
If a charged particle is released from rest in a region of uniform electric and magnetic fields parallel to each other, it will move in a straight line.
Statement 2
The electric field exerts no force on the particle but the magnetic field does.

6. Statement 1
A proton and an alpha particle having the same kinetic energy are moving in circular paths in a uniform magnetic field. The radii of their circular paths will be equal.
Statement 2
Any two charged particles having equal kinetic energies and entering a region of uniform magnetic field $\vec{B}$ in a direction perpendicular to $\vec{B}$, will describe circular trajectories of equal radii.

7. Statement 1
Two particles having equal charges and masses m_1 and m_2, after being accelerated by the same potential difference (V), enter a region of uniform magnetic field and describe circular paths of radii r_1 and r_2 respectively. Then

$$\frac{m_1}{m_2} = \sqrt{\frac{r_1}{r_2}}$$

Statement 2
Gain in kinetic energy = work done to accelerate the charged particle through potential difference .

8. Statement-1
No net force acts on a rectangular coil carrying a steady current when a suspended freely in a uniform magnetic field.
Statement-2
The magnitude of force experienced by a straight conductor of length L carrying a current I and placed perpendicular to a uniform magnetic field B is BIL.
IIT, 1981

9. Statement-I
There is no change in the energy of a charged particle moving in a magnetic field although a magnetic force is acting on it.
Statement-2
The magnetic force acting on a moving charged particle is always perpendicular to its velocity.

IIT, 1983

10. Statement-I
A charged particle enters a region of uniform magnetic field at an angle of 85° to the field lines. The path of the particle is a circle.
Statement-2
The field lines of a uniform magnetic field are parallel and equidistant.

IIT, 1983

11. Statement-I
An electron and a proton are moving with the same kinetic energy along the same direction. When they enter a uniform magnetic field perpendicular to their direction of motion, they describe circular path of the same radius.
Statement-2
The radius of the circular pathof a particle of charge q, mass m and moving with velocity v perpendicular to a magnetic field B is given by

$$r = \frac{mv}{qB}$$

IIT, 1985

12. Statement-1
A non-uniform magnetic field that varies in magnitude from point to point but has a constant direction, is set up in a region of space. If a charged particle enters the region in the direction of the magnetic field, it will be accelerated at non-uniform rate in the region
Statement-2
The force $\vec{F}$ experienced by a particle of charged q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = q(\vec{v} \times \vec{B})$

IIT, 1989

13. Statement-I
The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.
Statement-2
Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

IIT, 2008

SOLUTIONS

1. The correct choice is (d). If $\vec{v}$ is parallel to $\vec{B}$, $\vec{F} = 0$. Hence the particle does not experience any force and is, therefore, not accelerated in the region. It will travel undeflected with a constant speed.

2. The correct choice is (a). Since the magnetic force is always perpendicular to the velocity, no work is done by a uniform magnetic field on a charged particle. Hence magnetic force cannot change the magnitude of velocity (i.e. speed); it can only change the direction of velocity. Hence kinetic energy $\left(=\frac{1}{2}mv^2\right)$ remains unchanged but momentum $\vec{p} = \vec{v}$ will change.

3. The correct choice is (a). The loop will rotate and come to rest when the torque acting on it becomes zero. The magnitude to torque acting on a loop of area A and carrying a current I in a magnetic field B is given by

$$\tau = BIA \sin \theta$$

where θ is the angle between the direction of the magnetic field and the normal to the plane of the coil. It is clear that $\tau = 0$ when $\theta = 0$, i.e. when the plane of the coil is perpendicular to the magnetic field.

4. The correct choice is (a). Because electron has a negative charge, an electric field in the negative -direction will deflect it in the positive -direction. It will travel undeflected if the magnetic field imparts an equal deflection in the negative -direction. Since the magnetic force is perpendicular to the magnetic field and the charge of electron is negative, the direction of the magnetic field (according to Fleming's left-hand rule) should be along the negative -direction.

5. The correct choice is (c). Due to electric field, the force is $\vec{F} = \vec{E}$ in the direction of $\vec{E}$. Since $\vec{E}$ is parallel to $\vec{B}$, the particle velocity $\vec{v}$ (acquired due to force $\vec{F}$) is parallel to $\vec{B}$. Hence $\vec{B}$ will not exert any force since $\vec{v} \times \vec{B} = 0$ and the motion of the particle is not affected by $\vec{B}$.

6. The correct choice is (c). The radius of the circular path is given by

$$= \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB} \text{ ; where } = \frac{1}{2}v^2$$

Since and are the same for the two particles, $r \propto \frac{\sqrt{m}}{q}$.

Now, the charge of an alpha particle is twice that of a proton and its mass is four times the mass of a proton, $\sqrt{m}/q$ will be the same for both particles. Hence will be the same for both particles.

7. The correct choice is (d). Kinetic energy $K = qV$.

Therefore $r_1 = \frac{\sqrt{2m_1qV}}{qB}$ and $r_2 = \frac{\sqrt{2m_2qV}}{qB}$

Hence $\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^2$.

8. A coil *ABCD* is suspended in a magnetic field ***B***. Its planes makes an angle θ with ***B*** and it carries a current *I* as shown in the Fig. 24.90. The forces $F_1 = B\,Il$ on *AB* and *CD* are equal and opposite but constitute a couple which tends to rotate the coil.

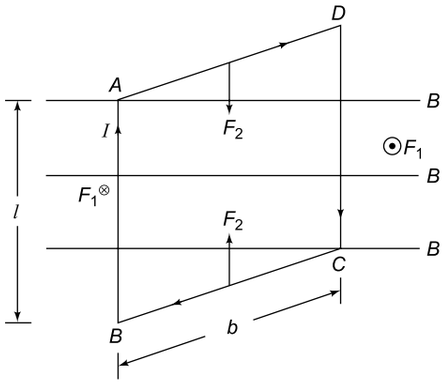


Fig. 24.90

These forces do not exert any net force on the coil. The force on arms *AD* and *BC* are $F_2 = B\,I\,b \sin\theta$. These forces merely compress the coil and are resisted by its rigidity. These forces also exert no net force on the coil. Thus, both the statements are true but Statement-2 is not the correct explanation for statement-1.

9. $\mathbf{F} = q(v \times \mathbf{B})$. Since **F** is perpendicular to v, power $p = \mathbf{F} \cdot v = 0$. Hence both the statements are true and statement-2 is the correct explanation for Statement-1.

10. The path of the Particle is a helix; it is a circle if the particle enters the field at angle of 90° with the field line. So, Statement-1 is false but Statement-2 is true.

11. $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$, where $K = \frac{1}{2}mv^2$

Since K and q are the same, $r \propto \sqrt{m}$. Hence the electron will describe a circle of a circle of a smaller radius. So. Statement-1 is false but Statement-2 is true.

12. If $\vec{v}$ is parallel to $\vec{B}, \vec{F} = 0$. Hence the particle does not experience any force and is, therefore, not accelerated in the region. It will travel undeflected with a constant speed. Statement-1 is false and Statement-2 is true.

13. Placing a soft iron core inside the coil makes the magnetic field radial which increases the torque acting on the coil due to a given current flowing in the coil. The sensitivity increases because, for the same current, the deflection of the coil is more when a core is inserted the coil. Statement-1 is true and Statement-2 is false.

SECTION VI

Integer Answer Type

1. A potential difference of 600 V is applied across the plates of a parallel plate capacitor. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates with a velocity of 2×10^5 ms^{-1} moves undeflected between the plates. Find the magnitude of the magnetic field (in tesla) in the region between the capacitor plates. Neglect edge effects.

IIT, 1981

2. Two long straight parallel wires are 2 m apart, perpendicular to the plane of the paper (Fig. 24.91) Wire *A* carries a current of 9.6 A directed into the plane of the paper. Wire *B* carries a current such that the magnetic field at a point *P* at a distance of 10/11 m from the wire is zero. Find the magnitude of the current (in ampere) in wire *B*.

3. A steady current current *I* goes through a wire loop *PQR* having shape of a right angle triangle with

$PQ = 3x$. $PR = 4x$ and $QR = 5x$. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k.

IIT, 1990

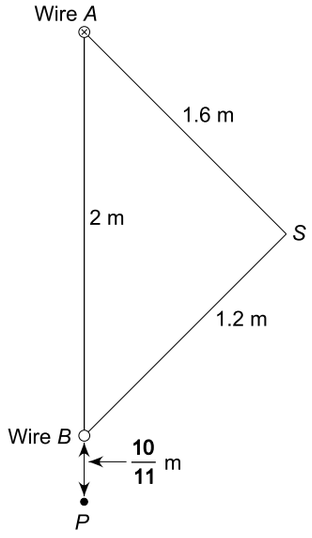


Fig. 24.91

4. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0\cos(300\,t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300\,t)$, then 'N' is

IIT, 2011

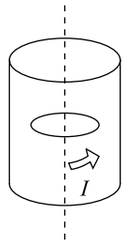


Fig. 24.92

SOLUTION

1. $E = \frac{V}{d} = \frac{600}{3\times10^{-3}} = 2\times10^5\ \text{Vm}^{-1}$

The electron will move undeflected if [see Fig. 24.93]

$e\,v\,\boldsymbol{B} = eE \Rightarrow B = \frac{E}{v} = \frac{2\times10^5}{2\times10^5} = 1\text{T}$

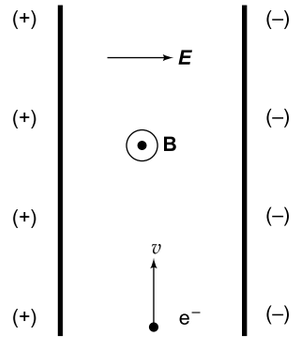


Fig. 24.93

2. Let the currents in A and B be I_1 and I_2 respectively. Let $AP = R_1$, $BP = R_2$. According to Ampere's right hand rule, the magnetic field at P due to I_1 is directed to the right in the plane of the paper. Its magnitude is given by (see Fig. 24.94 on page 24.52)

$$B_1 = \frac{\mu_0 I_1}{2\pi R_1}$$

The net field at P will be zero, if the field B_2 due to I_2 is equal and opposite to B_1. Therefore, the current in wire B should be normal to it and directed out the plane of the paper. Therefore

$$B_1 = B_2$$

or $$\frac{\mu_0 I_1}{2\pi R_1} = \frac{\mu_0 I_2}{2\pi R_2}$$

or $$I_2 = \frac{I_1 R_2}{2R_1}$$

Given $I_1 = 9.6$ A, $R_2 = BP = \frac{10}{11}$ m and

$$R_1 = AB + BP = 2 + \frac{10}{11} = \frac{32}{11}\ \text{m}.$$

Thus

$$I_2 = \frac{9.6\times10/11}{32/11} = \frac{9.6\times10}{32} = 3.0\ \text{A}$$

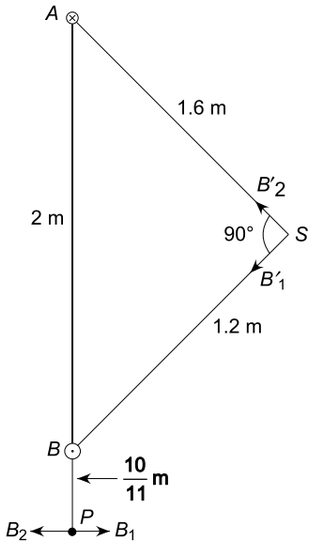


Fig. 24.94

3. The magnetic field at P is (Fig. 24.95)

$$B \text{ (at } P) = \frac{\mu_0 I}{4\pi r}[\sin\theta + \sin 90° - \theta]$$

$$= \frac{\mu_0 I}{4\pi r}(\sin\theta + \cos\theta)$$

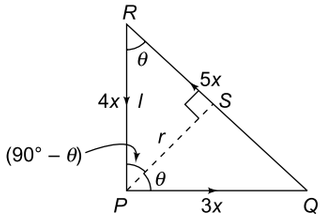


Fig. 24.95

Now $\sin\theta = \dfrac{r}{4x}$

and $\cos\theta = \dfrac{r}{3x}$

These equations give $\sin^2\theta + \cos^2\theta = \dfrac{r^2}{16x^2} + \dfrac{r^2}{9x^2}$

$$\Rightarrow \quad 1 = \frac{25r^2}{144x^2} \quad \Rightarrow \quad r = \frac{12x}{5}$$

$$\therefore \quad \sin\theta = \frac{3}{5} \text{ and } \cos\theta = \frac{4}{5}$$

$$\therefore \quad B(\text{at } P) = \frac{\mu_0 I}{4\pi r}\left(\frac{3}{5} + \frac{4}{5}\right) = \frac{7\mu_0 I}{20\pi r} = \frac{7\mu_0 I}{48\pi x}$$

$$\therefore \quad k = 7$$

4. From Ampere's circuital law, the magnetic field inside the tube is

$$B = \frac{\mu_0 I}{L};\ L = \text{length of tube}$$

$$= \frac{\mu_0 I_0 \cos(300\,t)}{L}$$

If r is the radius of the loop, the magnetic flux through it is

$$\phi = BA = \frac{\mu_0 I_0 \cos(300t)\pi r^2}{L}$$

Magnitude of induced emf is

$$|e| = \frac{d\phi}{dt} = \frac{\mu_0 I_0 (300)\pi r^2}{L} \times \sin(300\,t)$$

The induced current through the loop is

$$i = \frac{e}{R} = \frac{\mu_0 I_0 (300)\pi r^2}{LR} \sin(300t)$$

$\therefore$ Magnetic moment of the loop is

$$M = i \times \pi r^2 = \frac{300\pi^2 r^4 \mu_0 I_0}{RL} \sin(300t)$$

Given $M = N\mu_0 I_0 \sin(300t)$ (i)

Comparing (i) and (ii) we get (ii)

$$N = \frac{300\pi^2 r^4}{RL} = \frac{300 \times (3.14)^2 \times (0.1)^4}{0.005 \times 10}$$

$$= 5.92 \simeq 6$$

25 Chapter
Electromagnetic Induction and A.C. Circuits

REVIEW OF BASIC CONCEPTS

25.1 MAGNETIC FLUX

The magnetic flux through any surface placed in a magnetic field is determined by the number of the lines of force that cut through that surface. The magnetic flux through a coil of area A in a uniform magnetic field B is defined as

$$\phi = \mathbf{B} \cdot \mathbf{A} = B\,A\,\cos\,\theta$$

where θ is the angle between the normal to the plane of the coil and the magnetic field. If the coil has N turns, the magnetic flux through the coil is given by

$$\phi = N\,B\,A\,\cos\,\theta$$

The SI unit of flux is called weber (Wb).

For a curved surface,

$$\phi = \int \mathbf{B}\cdot\mathbf{dA}$$

25.2 FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

The magnitude and direction of induced emf can be determined by the application of two laws of electromagnetic induction: (i) Faraday's law, and (ii) Lenz's law.

Faraday's Law of Electromagnetic Induction

On the basis of various experiments, Faraday found that

1. whenever magnetic flux linked with a circuit changes, an induced emf is produced in the ciruit,
2. the induced emf lasts as long as the change in the magnetic flux is taking place, and
3. the magnitude of the induced emf is directly proportional to the rate of change of magnetic flux, i.e.

$$e \propto \frac{d\phi}{dt}$$

Lenz's Law

According to Lenz's law, the direction of the induced emf is such that it always opposes the cause that has produced it.

Thus

$$e = -\,k\,\frac{d\phi}{dt}$$

where k is a positive constant whose value depends on the system of units. In SI system of units, $k = 1$ and one can write

$$e = -\,\frac{d\phi}{dt}$$

Magnitude of induced emf is $|e| = \left|\frac{d\phi}{dt}\right|$

If ϕ is the flux through one turn of a coil, then for a coil of N turns

$$|\,e\,| = N\left|\frac{d\phi}{dt}\right|$$

The magnitude of the induced current is given by

$$i = \frac{\text{induced emf}}{\text{total resistance of circuit}} = \frac{1}{R}\left|\frac{d\phi}{dt}\right|$$

The direction of induced current is obtained by Lenz's law.

Flow of Induced Charge

When a current is induced in a circuit due to change in magnetic flux, induced charge q flows through the circuit.

$$q = \int i dt = \int \frac{1}{R}\left|\frac{d\phi}{dt}\right|\,dt = \frac{1}{R}\int |d\phi| = \frac{\text{change in flux}}{\text{Resistance}}$$

Heat Dissipation

Heat dissipated due to induced current is

$$H = \int eidt = \int \left|\frac{d\phi}{dt}\right| idt = i\int |d\phi|$$

= induced current × change in flux

Fleming's Right Hand Rule

This rule gives the direction of the induced emf when a conductor moves at right angles to a magnetic field. Hold the thumb and the first two fingers of your right hand mutually perpendicular to each other. Then, if the first finger points in the direction of the magnetic field and the thumb points in the direction of the motion of the conductor, then the second finger gives the direction of the induced emf (and hence of the induced current).

Applications of Lenz's Law

(i) If the magnet is moved towards the coil or coil is moved towards the magnet, the induced current i is anticlockwise Fig. 25.1. The current is clockwise if the magnet is moved away from the coil.

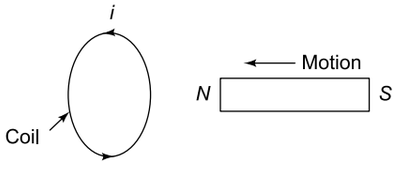


Fig. 25.1

(ii) The induced current i in the coil is anticlockwise if (Fig. 25.2)

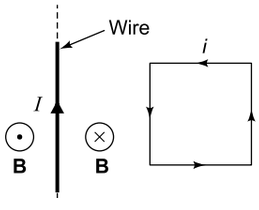


Fig. 25.2

(a) the coil is moved towards the long wire carrying current I

or

(b) the current I increases with time.

The current I is clockwise if

(a) the coil is moved away from wire

or

(b) the current I decreases with time.

(iii) Two coils carrying currents I_1 and I_2 placed with their planes parallel approach each other (Fig. 25.3).

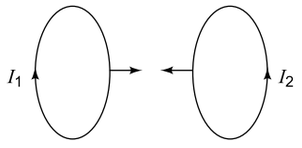


Fig. 25.3

(a) If I_1 and I_2 are both clockwise (or anticlockwise), then both I_1 and I_2 will decrease.

(b) If the currents I_1 and I_2 are in opposite sense, both the currents will increase.

25.3 EXPRESSION FOR INDUCED EMF

(i) **Change in flux due to change in magnetic field (B).**

If **B** increases with time, the induced current i is anticlockwise so that it produces a magnetic field pointing outwards (opposite to **B**). The induced emf is (Fig. 25.4)

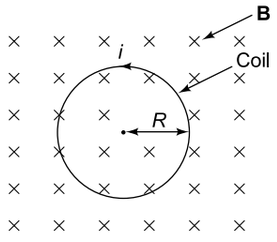


Fig. 25.4

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = \pi R^2 \frac{dB}{dt}$$

If **B** decreases with time, I will be clockwise.

If **B** remains constant but the radius of the coil increases at a rate $\frac{dR}{dt}$, then

$$|e| = B\frac{d}{dt}(\pi R^2) = B \times 2\pi R\frac{dR}{dt}.$$

(ii) **Change in flux due to change in area (A)**

If a rectangular coil $PQRS$ is moved out of a region of uniform magnetic field **B** with a velocity v, the emf induced is (Fig. 25.5)

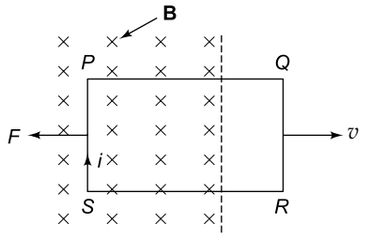


Fig. 25.5

$$|e| = Blv \text{ where } l = PS = QR$$

Induced current i is clockwise. If R is the resistance of the coil,

$$i = \frac{e}{R} = \frac{Blv}{R}$$

Force F required to pull the coil out with constant velocity v is

$$F = Bil = \frac{B^2l^2v}{R}$$

Power needed is $P = Fv = \dfrac{B^2l^2v^2}{R}$

= heat dissipated

The current will be anticlockwise, if the coil is pushed into the region of magnetic field.

NOTE

(a) If the coil is moved within the region of uniform magnetic field, no change in flux takes place and hence no emf is induced.

(b) If the magnetic field is non-uniform and the coil kept stationary in it, no change in flux occurs and hence no emf is induced.

The above results also hold in the case of rod XY sliding on metallic rails $PQRS$ to the right as shown in Fig. 25.6.

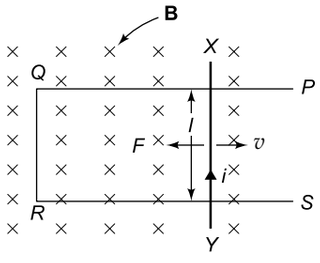


Fig. 25.6

(iii) **Change in flux due to change in orientation (θ) (A.C. generator)**

If a coil of area A, consisting of N turns is rotated in a magnetic field B with angular velocity ω, the emf induced in it is given by

$$e = e_0 \sin\theta = e_0 \sin\omega t$$

where $e_0 = NBA\omega$ is the amplitude (peak value). Thus an alternating emf is produced.

25.4 MOTIONAL EMF

(i) When a rod (or wire) of length l is moved with a velocity v in a magnetic field **B** as shown in Fig. 25.7(a), the emf induced between the ends P and Q of the rod is given by

$$e = Blv$$

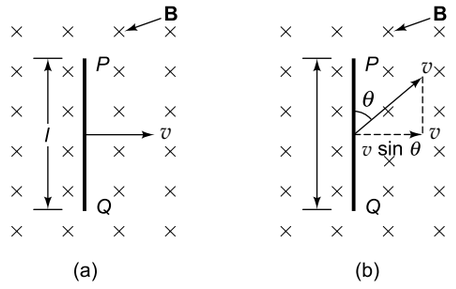


Fig. 25.7

If the rod is moved as shown in Fig. 25.7(b), then

$$e = Blv \sin\theta$$

(ii) When a semicircular rod (or wire) of radius R is moved with a velocity v in a magnetic field **B** as shown in Fig. 25.8, the emf induced between the ends P and Q of the rod is given by $e = Bv\,(2R) = 2BvR$

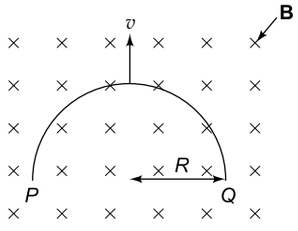


Fig. 25.8

(iii) When a rod PQ of length l pivoted at one end P is rotated with angular velocity ω in a magnetic field **B** as shown in Fig. 25.9, the emf induced between its ends is given by

$$e = \frac{1}{2} B\omega l^2$$

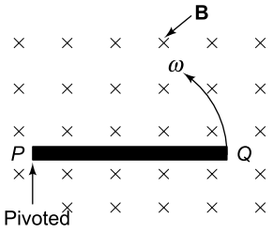


Fig. 25.9

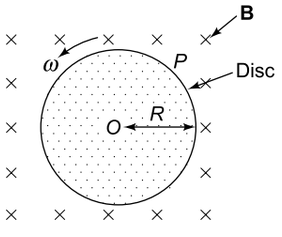


Fig. 25.10

(iv) When a disc of radius R is rotated about its centre with angular velocity ω in a magnetic field **B** as shown in Fig. 25.10, the emf induced between its centre O and a point P on its rim is given by

$$e = \frac{1}{2} B\omega R^2$$

25.5 ELECTRIC MOTOR

When a current is passed through a coil placed in a magnetic field by connecting its end to a source of voltage V, it experiences a torque which rotates it. As a result, an emf e is induced in it. This emf is called back emf as it opposes the applied voltage V (from Lenz's law). If R is the resistance of the coil, then current in it is

$$i = \frac{V - e}{R}$$

Input power = Vi and heat loss = i^2R. Hence output power = $Vi - i^2R = ei$.

Efficiency of motor $\eta = \frac{ei}{Vi} = \frac{e}{V}$

Some important points about a d.c. motor

(i) Back emf e and hence current i vary sinusoidally even if the source of voltage V is a d.c. (battery).

(ii) When output power is maximum, $e = \frac{V}{2}$ and $\eta = 50\%$.

(iii) Initially, i.e. when the motor is switched on, $e = 0$ and initial current = V/R which is very large. So, for safety, a starter is used.

(iv) At full speed, back emf is maximum and current i is minimum.

EXAMPLE 25.1

A square coil of resistance 2 Ω, 100 turns and side 10 cm is placed with its plane making an angle of 30° with a uniform magnetic field of 0.1 T. In 0.05 s the coil rotates until its plane becomes parallel to the magnetic field. Calculate the current induced in the coil.

SOLUTION

$R = 2\ \Omega$, $N = 100$, $A = 0.1 \times 0.1 = 10^{-2}\ \text{m}^2$,
$\theta_1 = 90° - 30° = 60°$,
$\theta_2 = 90° - 0 = 90°$, $B = 0.1$ T and $t = 0.05$ s

Change in flux = $NBA\ (\cos\theta_2 - \cos\theta_1)$

$$= 100 \times 0.1 \times 10^{-2} \times (\cos 90° - \cos 60°)$$

$$= 0.1 \times \left(0 - \frac{1}{2}\right) = 0.05\ \text{Wb}$$

$$\text{Induced emf } e = \frac{\text{change in flux}}{\text{time}} = \frac{0.05}{0.05} = 1\ \text{V}$$

$$\text{Induced Current } i = \frac{e}{R} = \frac{1}{2} = 0.5\ \text{A}$$

EXAMPLE 25.2

A solenoid of diameter 0.2 m has 500 turns per metre. At the centre of this solenoid, a coil of 100 turns is wrapped closely around it. If the current in the solenoid changes from zero to 2 A in 1 millisecond, calculate the induced emf developed in the coil.

SOLUTION

The magnetic field due to current I in a solenoid having n turns per unit length is

$$B = \mu_0 nI$$

This is the magnetic field threading the coil. The direction of the field is parallel to the axis of the solenoid. Hence angle between the normal to the plane of the coil and the magnetic field is $\theta = 0°$. If A is the cross-sectional area of the coil and N the number of turns in it, then the magnetic flux threading the coil is

$$\phi = NAB\cos\theta = NAB\cos 0° = NAB = N\pi r^2\mu_0 nI$$

Since the coil is wrapped closely around the solenoid, the radius of the coil (r) = radius of solenoid = 0.1 m.

Change in flux if the current change from $I_1 = 0$ to $I_2 = 2$ A is

$$\Delta\phi = N\pi r^2\,\mu_0 n\,(I_2 - I_1)$$

$$= 100 \times (\pi \times 0.1^2) \times (4\pi \times 10^{-7}) \times 500 \times (2 - 0)$$

$$= 4\pi^2 \times 10^{-4}\ \text{Wb}$$

$$\text{Induced emf } e = \left|\frac{\Delta\phi}{\Delta t}\right| = \frac{4\pi^2 \times 10^{-4}}{10^{-3}} = 0.4\pi^2 = 3.95\ \text{V}$$

EXAMPLE 25.3

The magnetic flux through a coil of resistance 6.5 Ω placed with its plane perpendicular to a uniform magnetic field varies with time t (in second) as

$$\phi = (3t^2 + 5t + 2)\ \text{milliweber}$$

Find the induced current in the coil at $t = 10$ s.

SOLUTION

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(3t^2 + 5t + 2) = (6t + 5)\ \text{mV}.$$

At $t = 10$ s, $e = (6 \times 10 + 5)\ \text{mV} = 65 \times 10^{-3}\ \text{V}$

Induced current at $t = 10$ s is

$$I = \frac{e}{R} = \frac{65 \times 10^{-3}}{6.5} = 10^{-2}\text{A}$$

EXAMPLE 25.4

A metal wheel with 8 metallic spokes, each 60 cm long is rotated at a speed of 100 rev./min in a plane perpendicular to earth magnetic field of 0.3×10^{-4} T. Find the magnitude of the induced emf between the axle and the rim of the wheel.

SOLUTION

$\nu = 100$ rev./min. $= \dfrac{100}{60} = \dfrac{5}{3}$ rev./s

$l = 0.6$ m. The emf developed between the ends of a spoke is (as $\omega = 2\pi\nu$)

$$e = \frac{1}{2} Bl^2 \omega$$

$$= \frac{1}{2} \times (0.3 \times 10^{-4}) \times (0.6)^2 \times (2\pi \times \frac{5}{3})$$

$$= 1.8\pi \times 10^{-5} \text{ V} = 5.65 \times 10^{-5} \text{ V}$$

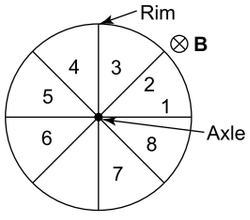


Fig. 25.11

The same emf is induced between the ends of each spoke. It is clear from Fig. 25.11 that the spokes are joined in parallel. Hence the emf between rim and axle = emf across each spoke = 5.65×10^{-5} V.

EXAMPLE 25.5

An aircraft with a wing span of 50 m is flying with a speed of 1080 kmh^{-1} in the eastward direction at a constant altitude at a place where the vertical component of earth's magnetic field is 2×10^{-5} T. Find the emf developed between the tips of the wing.

SOLUTION

$v = 1080 \text{ kmh}^{-1} = 300 \text{ ms}^{-1}$

$e = Blv = (2 \times 10^{-5}) \times 50 \times 300 = 0.3$ V

EXAMPLE 25.6

A circular coil of mean radius r and having N turns is kept in a horizontal plane. A magnetic field B exists in the vertical direction as shown in Fig. 25.12. Find the emf induced in the loop

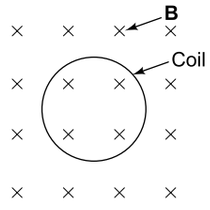


Fig. 25.12

(a) if it is held stationary and the magnetic field is uniform,
(b) if it is held stationary and the magnetic field is non-uniform,
(c) if it is held stationary and the magnetic field is varying with time,
(d) it is rotated with an angular velocity ω about an axis passing through its centre and perpendicular to its plane, and
(e) it is rotated with an angular velocity ω about its diameter. Assume that the normal to the plane of the coil makes an angle $\theta = 0$ with the magnetic field at time $t = 0$.

SOLUTION

The emf is induced if the magnetic flux through the coil changes with time.

(a) In this case there is no change in magnetic flux with time, hence no emf induced.

(b) In this case also the magnetic flux through the coil does not change with time, hence no emf induced.

(c) In this case, the number of field lines through the coil does not change with time, hence the magnetic flux does not change with time. So no emf is induced in the coil (see Fig. 25.13)

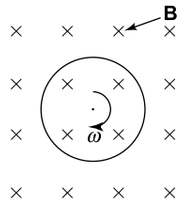


Fig. 25.13

(d) If the coil is rotated about a diameter, as shown in Fig. 25.14, there is a change in magnetic flux with time. Hence emf will be induced in the coil. Area of the coil is $A = \pi r^2$. If the normal to the plane of the coil makes an angle $\theta = 0$ with the magnetic field, then at time t, $\theta = \omega t$. The magnetic flux at this time is

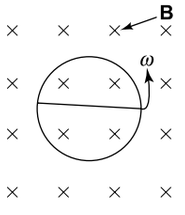


Fig. 25.14

$$\phi = NBA \cos\theta = NBA \cos \omega t$$

$\therefore$ Induced emf is

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\omega t)$$

$$= \omega NBA\sin\omega t$$

$$= \omega NB \times \pi r^2 \sin\omega t$$

$$= \pi Nr^2 B\omega \sin\omega t$$

EXAMPLE 25.7

A metal rod PQ moves with a velocity v parallal to a very long straight wire CD carrying a current I as shown in Fig. 25.15. The ends P and Q of the rod are at distances a and b from the wire as shown. Obtain the expression for the emf induced between the ends of the rod.

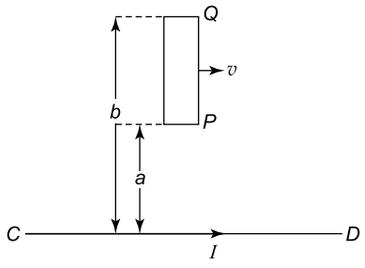


Fig. 25.15

SOLUTION

Divide the rod into a large number of very small elements, each of length dx. Consider one such element at distance x from wire CD as shown in Fig. 25.16.

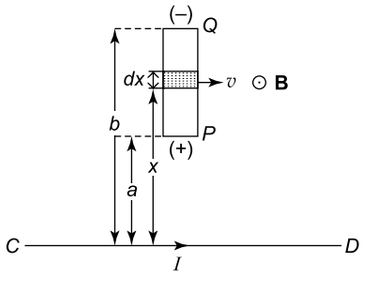


Fig. 25.16

The magnetic field at the element due to current I in wire CD is

$$B = \frac{\mu_0 I}{2\pi x}$$

The direction of the field is upwards perpendicular to velocity v. The magnitude of magnetic field is different at different points on the rod PQ. From Fleming's L.H. rule, the free electrons in the rod will experience force in the direction P to Q. So free electrons move from P to Q. Hence end P acquires a positive charge (due to loss of electrons) and end Q acquires a negative charge (due to gain of electrons).

Force on the element is

$$dF = qvB = qv \times \frac{\mu_0 I}{2\pi x}$$

Therefore, electric field set up in the element is

$$dE = \frac{dF}{q} = \frac{qv\mu_0 I}{2\pi xq} = \frac{\mu_0 Iv}{2\pi x}$$

Now $$dE = -\frac{dV}{dx}$$

$\Rightarrow$ $$dV = -dE \times dx = -\frac{\mu_0 Iv}{2\pi x}dx$$

Where dV is the voltage induced in the element. The voltage induced in the rod PQ is

$$|V| = \frac{\mu_0 Iv}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)$$

EXAMPLE 25.8

A metal rod PQ of length l slides with a velocity v on two parallel rails AB and CD parallel to a long straight wire XY carrying a current I as shown in Fig. 25.17. A resistance R is connected between the rails as shown. The velocity of rod PQ is kept constant by applying force.

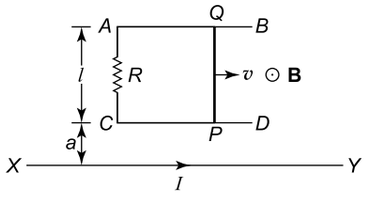


Fig. 25.17

(a) Obtain the expression for the current induced in resistance R.

(b) Obtain the expression for the force to be applied on rod PQ to keep its velocity constant at v.

SOLUTION

In this case the induced emf is due to change in magnetic flux which is due to the change in the area of $ACPQ$ with time.

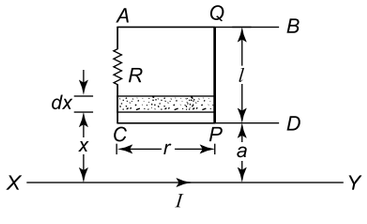


Fig. 25.18

Magnetic flux through an infinitesimal area element of with dx at a distance x from PQ is

$$d\phi = BdA = Brdx = \frac{\mu_0 Ir}{2\pi x}dx$$

where r is the position of PQ at an instant of time t. Magnetic flux through $ACPQ$ is

$$\phi = \int d\phi = \int_a^{(a+l)} \frac{\mu_0 Ir}{2\pi x}dx$$

$$= \frac{\mu_0 Ir}{2\pi}\int_a^{(a+l)} \frac{dx}{x} = \frac{\mu_0 Ir}{2\pi}\ln\left(\frac{a+l}{a}\right)$$

$\therefore$ Induecd emf $e = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0 I}{2\pi}\ln\left(\frac{a+l}{a}\right)\frac{dr}{dt}$

$$= \frac{\mu_0 Iv}{2\pi}\ln\left(\frac{a+l}{a}\right) \qquad \left(\because v = \frac{dr}{dt}\right)$$

(a) Current induced in R is

$$i = \frac{e}{R} = \frac{\mu_0 Iv}{2\pi R}\ln\left(\frac{a+l}{a}\right)$$

(b) This current will exert a force on the element of width dx which is given by

$$dF = Bidx = \frac{\mu_0 I}{2\pi x}idx$$

$\therefore$ Force to be applied on PQ is

$$F = \int dF = \frac{\mu_0 Ii}{2\pi}\int_a^{(a+l)} \frac{dx}{x}$$

$$= \frac{\mu_0 I}{2\pi} \times \frac{\mu_0 Iv}{2\pi R}\ln\left(\frac{a+l}{a}\right)\int_a^{(a+l)} \frac{dx}{x}$$

$$= \frac{v}{R}\left[\frac{\mu_0 I}{2\pi}\ln\left(\frac{a+l}{a}\right)\right]^2$$

EXAMPLE 25.9

A metal rod PQ of length l slides on two parallel rails AB and CD, each rail having a resistance k per unit length. The rod and the rails are in a region of a uniform magnetic field B directed into the plane of the paper as shown in Fig. 25.19. A resistance R is connected between the rails. A variable force F is applied to PQ so that it is accelerated to the right. Obtain the expression for the velocity v of rod PQ when it is at a distance x from R.

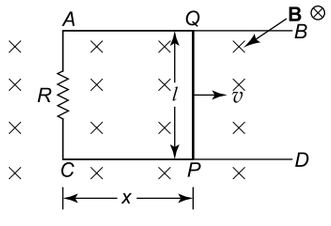


Fig. 25.19

SOLUTION

Magnetic flux through $ACPQ$ when the rod is at a distance x from R is

$$\phi = BA = Blx$$

Induced emf at that instant is

$$e = \left|\frac{d\phi}{dt}\right| = Bl\frac{dx}{dt} = Blv$$

Resistance of $ACPQ = R + 2kx$. Therefore, current induced in the circuit is

$$i = \frac{e}{(R+2kx)} = \frac{Blv}{(R+2kx)}$$

$$\Rightarrow \qquad v = \frac{i(R+2kx)}{Bl}$$

EXAMPLE 25.10

A metal rod PQ of mass m and of negligible resistance slides on two parallel metal rails AB and CD separated by a distance l. The rails have negligible resistance and have a resistance R connected between them as shown in Fig. 25.20. The rod and the rails are located in a region of uniform magnetic field direction into the plane of the loop $ACPQ$. The rod is given an initial velocity u. Obtain the expression for the distance x covered by the rod before it comes to rest. Neglect friction between the rod and the rails.

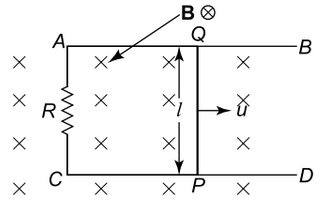


Fig. 25.20

SOLUTION

Induced currnet $i = \frac{Bvl}{R}$

Force $F = Bil = -ma = -m\frac{dv}{dt}$

The negative sign shows that force opposes the acceleration (Lenz's law)

$$\therefore \qquad m\frac{dv}{dt} = -B \times \frac{Bvl}{R} \times l$$

$$= -\frac{B^2vl^2}{R}$$

$$\Rightarrow \qquad \frac{dv}{v} = -\frac{B^2l^2}{mR}dt = -kdt$$

Where $k = \dfrac{B^2l^2}{mR}$

Intergrating $\displaystyle\int_u^v \frac{dv}{v} = -k\int_0^t dt$

$$\Rightarrow \qquad \ln\left(\frac{v}{u}\right) = -kt$$

$$\frac{v}{u} = e^{-kt}$$

$$\Rightarrow \qquad v = ue^{-kt}$$

$$\therefore \qquad \frac{dx}{dt} = ue^{-kt}$$

The rod comes to rest when $t = \infty$
Integrating

$$\int_0^x dx = u\int_0^\infty e^{-kt}\,dt$$

$$x = -\frac{u}{k}|e^{-kt}|_0^\infty = -\frac{u}{k}(0-1) = \frac{u}{k}$$

$$\therefore \qquad x = \frac{umR}{B^2l^2}$$

EXAMPLE 25.11

A circular coil of radius r has N turns and a resistance R. It is placed with its plane at right angles to a uniform magnetic field B. Find the expression for the amount of charge Q which passes through the coil when it is rotated through an angle of 180° in its plane.

SOLUTION

Area of the coil $(A) = \pi r^2$

Since the plane of the coil is normal to the magnetic field, the magnetic flux through the coil = NBA cos 0° = NBA. When the coil is rotated through 180°, the magnetic flux through it will be = NBAcos 180° = $-NBA$. Therefore, change in flux is

$$\phi = NBA - (-NBA) = 2NBA$$

Magnitude of induced emf is

$$|e| = \frac{d\phi}{dt}$$

$\therefore$ Induced current is $i = \dfrac{|e|}{R} = \dfrac{d\phi}{dt}\times\dfrac{1}{R}$

$$\Rightarrow \qquad iR = \frac{d\phi}{dt}$$

$$\frac{dq}{dt}R = \frac{d\phi}{dt}$$

$$\Rightarrow \qquad dqR = d\phi$$

$$\Rightarrow \qquad dq = \frac{d\phi}{R}$$

$$\Rightarrow \qquad Q = \frac{\phi}{R} = \frac{2NBA}{R} = \frac{2NB\times\pi r^2}{R}$$

EXAMPLE 25.12

Two circular coils A and B of radii a and b respectively (with $b > a$) have their plane perpendicular to the plane of the page. They are separated co-axially by a distance $x = \sqrt{3}\,b$ as shown in Fig. 25.21. A transient current I flows through coil B for a very short time interval. If the resistance of coil A is R obtain the expression for the charge that flows through coil A during the short time interval.

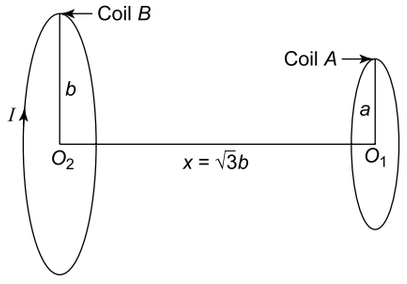


Fig. 25.21

SOLUTION

Magnetic field at the centre of coil A due to current I in coil B is

$$B_{AB} = \frac{\mu_0 Ib^2}{2(b^2+x^2)^{3/2}} = \frac{\mu_0 I}{16b} \quad (\because\ x = \sqrt{3}\,b)$$

Since the magnetic field is along the axis of coil A, it is perpendicular to the plane of A, hence $\theta = 0°$. Therefore, magnetic flux through A is

$$\phi = B_{AB}\times\text{area of coil } A\times\cos 0° = \frac{\mu_0 I\times\pi a^2}{16b}$$

$\therefore$ Induced emf is $|e| = \left|\dfrac{d\phi}{dt}\right|$

$$\Rightarrow \qquad IR = \frac{d\phi}{dt}$$

$$\Rightarrow \qquad IRdt = d\phi$$

$$\therefore \qquad \int Idt = \frac{1}{R}\int d\phi = \frac{\phi}{R}$$

or $$Q = \frac{\phi}{R} = \frac{\mu_0 I \times \pi a^2}{16bR}$$

EXAMPLE 25.13

A thin non-conducting disc of radius R and mass M is held horizontally and is capable of rotation about an axis passing through its centre and perpendicular to its plane. A charge Q is distributed uniformly over the surface of the disc.

A time-varying magnetic field $B = kt$ (where k is a constant and t is the time) directed perpendicular to the plane of the disc is applied to it. If the disc is stationary initially (i.e. at $t = 0$). Find

(a) The torque acting on the disc.

(b) The angular velocity acquired by the disc as a function of t.

SOLUTION

(a) Area of disc = πR^2

Charge per unit area = $\frac{Q}{\pi R^2}$

Area of a small element of width dx at a distance x from the centre of the disc = $2\pi x dx$. Therefore, charge of the element is

$$dq = \frac{Q}{\pi R^2} \times 2\pi x dx$$

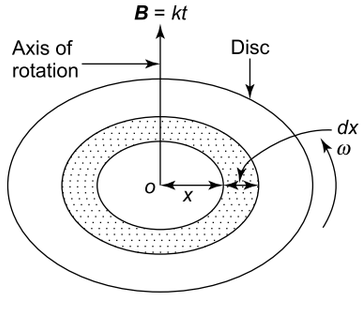


Fig. 25.22

A time-varying magnetic field gives rise to an electric field E. Since

$$E = -\frac{dV}{dl}$$

$$\int dV = -\int Edl = -E \times 2\pi x$$

$$\Rightarrow \qquad V = -E \times 2\pi x \qquad (1)$$

where V is emf induced in the element, which is given by

$$V = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA)$$

$$= -\frac{d}{dt}(kt \times \pi x^2) = -\pi k x^2 \qquad (2)$$

From (1) and (2) we get

$$-E \times 2\pi x = -\pi k x^2$$

$$\Rightarrow \qquad E = \frac{kx}{2} \qquad (3)$$

Force acting on the element is

$$dF = dq \times E = \frac{Q}{\pi R^2} \times 2\pi x dx \times \frac{kx}{2}$$

$$= \frac{kQ}{R^2} x^2 dx$$

Torque acting on the disc is

$$\tau = \int_0^R x dF = \frac{kQ}{R^2}\int_0^R x^3 dx = \frac{kQR^2}{4}$$

(b) $\tau = I\alpha$, where I is the moment of inertia of the disc about the axis of rotation and α is the angular acceleration

$I = \frac{1}{2}MR^2$ and $\alpha = \frac{d\omega}{dt}$. Hence

$$\frac{kQR^2}{4} = \frac{1}{2}MR^2 \times \frac{d\omega}{dt}$$

$$\Rightarrow \qquad d\omega = \frac{kQ}{2M}dt$$

$$\Rightarrow \qquad \int_0^\omega d\omega = \frac{kQ}{2M}\int_0^t dt$$

$$\Rightarrow \qquad \omega = \frac{kQt}{2M}$$

25.6 MUTUAL INDUCTANCE

If the current in a coil is i then the flux linked with a neighbouring coil is $\phi = Mi$ where M is the coefficient of mutual inductance. If current i is changing with time, the emf induced in the neighbouring coil is

$$e = -M\frac{di}{dt}$$

Expressions of M in some situations

(i) A small coil of length l, number of turns N_1 wound closely on a long coil of N_2 turns.

$M = \dfrac{\mu_0 N_1 N_2 A}{l}$; A = common cross-sectional area

(ii) Two coplanar and concentric coils of radii R and r ($R >> r$) Fig. (25.23)

$$M = \frac{\mu_0 \pi r^2}{2R}$$

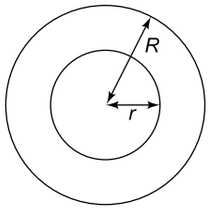


Fig. 25.23

(iii) A small circular coil of radius r at the centre of a large rectangular coil of sides a and b with $a, b >> r$ (Fig. 25.24)

$$M = \frac{2\mu_0 r^2 \sqrt{a^2 + b^2}}{ab}$$

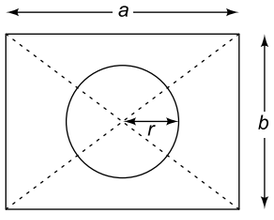


Fig. 25.24

(iv) a rectangular loop of sides a and b placed at a distance x from a long straight wire (Fig. 25.25)

$$M = \frac{\mu_0 a}{2\pi} \log_e \left(\frac{x+b}{x} \right)$$

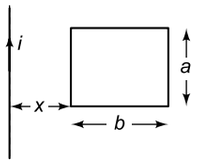


Fig. 25.25

NOTE

If the medium is different from air, μ_0 in above expressions is replaced by $\mu = \mu_0 \mu_r$, where μ_r is the relative permeability of the medium.

25.7 SELF INDUCTANCE

If i is the instantaneous current in a coil, flux $\phi = Li$, where L is the self inductance of the coil. Induced emf $e = -L\dfrac{di}{dt}$.

(i) The self inductance of a coil of N turns, cross-sectional area A and length l is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

(ii) Direction of induced emf is such that it opposes the change in current (Fig. 25.26(a) and (b))

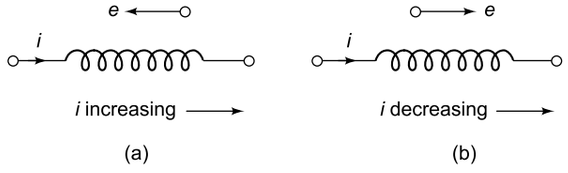


Fig. 25.26

(iii) Energy stored in the inductor $U = \dfrac{1}{2} Li^2$.

(iv) Inductors in series (Fig. 25.27)

Equivalent inductance is

(a) $L = L_1 + L_2$ (when the flux linked with one coil is not linked with the other, i.e. $M = 0$)

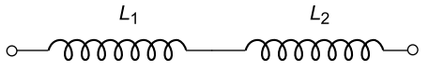


Fig. 25.27

(b) $L = L_1 + L_2 + 2M$ (when flux of one coil is in the same direction as that of the other coil)
$L = L_1 + L_2 - 2M$ (when the fluxes oppose each other)

(v) Inductors in parallel

$$\frac{1}{L} = \frac{1}{L} + \frac{1}{L_2} \quad (\text{when } M = 0)$$

(vi) $M = k\sqrt{L_1 L_2}$; k = coefficient of coupling.

25.8 GROWTH AND DECAY OF CURRENT IN A D.C L—R CIRCUIT (FIG. 25.28)

If switch S_1 is closed at $t = 0$, with switch S_2 open, no current flows in the beginning (as the inductor behaves as open switch). The current starts increasing and at time t, $i = i_o (1 - e^{-t/\tau})$, where $\tau = \dfrac{L}{R}$ is the time constant.

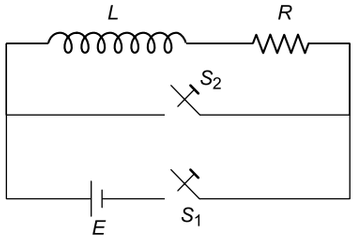


Fig. 25.28

After a long time ($t = \infty$), the current attains a steady value $i_o = E/R$ (now the ideal inductor behaves as a closed switch).

At $$t = \tau,\ i = i_o\left(1 - \frac{1}{e}\right) = 0.632\ i_0.$$

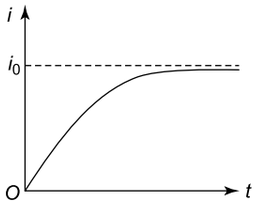


Fig. 25.29

Decay of current: At time $t = 0$, let $i_0 = E/R$ be the current in the circuit. If S_2 is closed (with S_1 open), the current decays as

$$i = i_0\ e^{-t/\tau}$$

At $$t = \tau,\ i = \frac{i_0}{e} = 0.368 i_0.$$

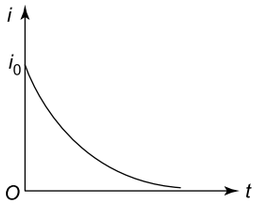


Fig. 25.30

25.9 ENERGY STORED IN AN INDUCTOR

If the current in a coil of self inductance L is increased from zero to a steady value I, the energy stored in the magnetic field of the coil is

$$U = \frac{1}{2}\ LI^2$$

EXAMPLE 25.14

A magnetic flux of 5 μWb is linked with a coil when a current of 1 mA flows through it. Find the self inductance of the coil.

SOLUTION

$$\phi = LI \Rightarrow L = \frac{\phi}{I} = \frac{5\times10^{-6}}{1\times10^{-3}}$$

$$= 5 \times 10^{-3}\ \text{H} = 5\ \text{mH}$$

EXAMPLE 25.15

An emf of 1 mV is induced in a coil when the current in it changes steadily from 2 A to 4 A in 0.1 s. Find the self inductance of the coil.

SOLUTION

$$\frac{dI}{dt} = \frac{4-2}{0.1} = 20\ \text{As}^{-1}$$

$$|e| = L\ \frac{dI}{dt}$$

$$1 \times 10^{-3} = L \times 20 \Rightarrow L = 5 \times 10^{-5}\ \text{H} = 50\ \mu\text{H}$$

EXAMPLE 25.16

A solenoid 1.0 m long and 0.05 m diameter has 700 turns. Another solenoid of 50 turns is tightly wound over the first solenoid. Find the emf induced in the second solenoid when the current in the first solenoid changes from 0 to 5 A in 0.01 s.

SOLUTION

Mutual inductance $M = \dfrac{\mu_0 A N_1 N_2}{l}$

$$= \frac{4\pi\times10^{-7}\times\pi(0.025)^2\times700\times50}{1.0}$$

$$= 8.6 \times 10^{-5}\ \text{H}$$

$$|e| = M\ \frac{dI}{dt} = 8.6 \times 10^{-5} \times \frac{5-0}{0.01} = 4.3 \times 10^{-2}\ \text{V}$$

EXAMPLE 25.17

An ideal inductor of inductance 5 H and a pure resistor of resistance 100 Ω are connected in series to a battery of emf 6 V of negligible internal resistance through a switch. The switch is closed at time $t = 0$

(a) Find the maximum (or steady) value of the current.
(b) What is the time constant τ of the circuit?
(c) How long does it take for the current to rise to 50% of the maximum value?
(d) Find the potential difference across the inductor at $t = 0.1$ s. Given $e^{-2} = 0.135$.

SOLUTION

$i = i_0 (1 - e^{t/\tau})$; i_0 = maximum value of i

(a) When $t \to \infty$, $i = i_0 = \dfrac{E}{R} = \dfrac{6}{100} = 0.06$ A

(b) Time constant $\tau = \dfrac{L}{R} = \dfrac{5}{100} = 0.05$ s

(c) $0.5\, i_0 = i_0 (1 - e^{-t/\tau})$

$$\Rightarrow \quad \frac{1}{2} = 1 - e^{-t/\tau}$$

$$\Rightarrow \quad e^{-t/\tau} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \quad e^{t/\tau} = 2 \Rightarrow \frac{t}{\tau} = \ln(2) = 0.693$$

$$\therefore \quad t = 0.693\ \tau = 0.693 \times 0.05 = 0.0346 \text{ s}$$

(d) $V_L = -L \dfrac{di}{dt} = -L \dfrac{d}{dt} [i_0(1 - e^{-t/\tau})]$

$$= -Li_0 \left(-\frac{1}{\tau}\right) e^{-t/\tau}$$

$$= \frac{Li_0}{\tau} e^{-t/\tau}$$

$$= \frac{5 \times 0.06}{0.05} \times e^{-0.1/.05} = 6 \times e^{-2} = 6 \times 0.135 = 0.8 \text{ V}$$

EXAMPLE 25.18

An inductor of inductance 10 H and a resistor of resistance 16 Ω are connected to a 12 V dc source.

(a) Find the final steady current.

(b) How much energy is consumed to attains this steady current?

(c) What is the power dissipated in the resistor at this current?

SOLUTION

(a) $i_0 = \dfrac{E}{R} = \dfrac{12}{16} = \dfrac{3}{4}$ A $= 0.75$ A

(b) $U = \dfrac{1}{2} L i_0^2 = \dfrac{1}{2} \times 10 \times \left(\dfrac{3}{4}\right)^2 = 2.8$ J

(c) $P = i_0 E = \dfrac{3}{4} \times 12 = 9$ W

or $\quad P = i_0^2 R = \left(\dfrac{3}{4}\right)^2 \times 16 = 9$ W

EXAMPLE 25.19

An inductor of inductance 100 mH and a resistor of resistance 10 Ω are connected in series to a 2 V battery. After some time the current attains a steady value. The battery is now short circuited. Calculate the time required for the current to fall to half the steady value.

SOLUTION

$L = 100$ mH $= 0.1$ H, $R = 50$ Ω, $E = 2$ V.

$$i_0 = \frac{E}{R} = \frac{2}{50} = 0.04 \text{ A}$$

$$\tau = \frac{L}{R} = \frac{0.1}{50} = 0.002 \text{ s}$$

$$\frac{i}{i_0} = e^{-t/\tau}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-t/\tau} \Rightarrow 2 = e^{-t/\tau} \Rightarrow \ln 2 = \frac{t}{\tau}$$

$$\therefore \quad t = \tau \ln 2 = 0.002 \times 0.693 = 1.386 \times 10^{-3} \text{ s}$$

25.10 TRANSFORMER

The transformer is a device used for converting a low ac voltage into a high ac voltage and vice versa. The former is called the step-up transformer and the latter the step-down transformer.

A transformer consists of two coils each of which is wound on an iron core. One of the coils is connected to a source of alternating emf. This coil is called the *primary* of the transformer while the other is called the *secondary* of the transformer. Any of the two coils can act as primary while the other as secondary. The alternating emf in one coil induces an alternating emf in the second coil. The presence of an iron core in the primary and secondary makes the flux linkage between the two coils very large. The alternating emf in the coil makes the magnetic flux in the iron also vary periodically. This varying magnetic flux in iron induces an alternating emf in the secondary.

If the magnetic field lines remain confined to the core, then all the field lines threading the primary also go across the secondary. Then the magnetic fluxes across the secondary and primary will be simply proportional to the number of turns in them, i.e.

$$\frac{\phi_s}{\phi_p} = \frac{N_s}{N_p} \quad \text{or} \quad \phi_s = \left(\frac{N_s}{N_p}\right) \phi_p$$

where N_s is the number of turns in the secondary and N_p is the number of turns in the primary. Now from Faraday's law the emf induced across the secondary is $e_s = -(d\phi_s/dt)$ and that across the primary is $e_p = -(d\phi_p/dt)$.

Thus

$$e_s = -\frac{d}{dt}\left(\frac{N_s}{N_p}\cdot\phi_p\right) = -\frac{N_s}{N_p}\frac{d\phi_p}{dt}$$

or $$e_s = \frac{N_s}{N_p}\ e_p$$

From this equation, it follows that if $N_s > N_p$, then $e_s > e_p$, i.e. the voltage across the secondary is greater than the input primary voltage. Such a transformer in which the number of turns in the secondary is more than in the primary is called a *step-up* transformer. But if $N_s < N_p$, then $e_s < e_p$. Such a transformer is called a *step-down* transformer. The former are used in TV, high-voltage power supplies and the latter in radio transmitter sets, battery eliminators, etc.

Usually, there are a number of energy losses in actual transformers. These are: (i) Joule heating (I^2R) losses in the primary and secondary coils due to their resistance (generally, these losses are minimized by using wires of large diameters so that resistance is low); and (ii) the losses in the iron core which include the heating of the core due to eddy currents and power loss due to hysteresis. The eddy currents can be minimized by using laminated iron.

In an ideal transformer, the entire power in the primary is transferred to the secondary. For an ideal transformer,

input power = output power

or $$e_s\ I_s = e_p\ I_p$$

Therefore, $$\frac{e_s}{e_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

where I_p and I_s are the currents in the primary and the secondary of the transformer.

EXAMPLE 25.20

A step down transformer is used to reduce the main supply of 220 V to 10 V. If the primary draws 5 A and secondary 88 A current, calculate the efficiency of the transformer.

SOLUTION

$e_p = 220$ V, $e_s = 10$ V , $I_p = 5$ A and $I_s = 88$ A

Input power $(P_i) = e_p \times I_p = 220 \times 5 = 1100$ W

Output power $(P_o) = e_s \times I_s = 10 \times 88 = 880$ W

Efficiency $\eta = \frac{P_o}{P_i} = \frac{880}{1100} = 0.8$ or 80%

EXAMPLE 25.21

A transformer has an efficiency of 75%. The power input is 4 kW at 100 V. If the secondary voltage is 200 V, calculate the currents in the primary and secondary.

SOLUTION

$P_i = e_p I_p = 4$ kW $= 4000$ W, $e_p = 100$ V and $e_s = 200$ V

$$I_p = \frac{P_i}{e_p} = \frac{4000}{100} = 40 \text{ A}$$

$$\eta = \frac{P_o}{P_i} \Rightarrow P_o = \eta\, P_i = 0.75 \times 4000 = 3000 \text{ W}$$

$$\therefore I_s = \frac{P_o}{e_s} = \frac{3000}{200} = 15 \text{ A}$$

EXAMPLE 25.22

The primary of a transformer has 400 turns while the secondary has 2000 turns. The power output from the secondary at 1000 V is 12 kW.

(a) Calculate the primary voltage.

(b) If the resistance of the primary is 0.9 Ω and that of the secondary is 5 Ω and the efficiency of the transformer is 90%, calculate the power loss in the primary coil and in the secondary coil.

SOLUTION

$N_p = 400$, $N_s = 2000$, $P_o = 12000$ W, $e_s = 1000$ V

(a) $$\frac{e_p}{e_s} = \frac{N_p}{N_s} \Rightarrow e_p = \frac{N_p}{N_s} \times e_s = \frac{400}{2000} \times 1000$$

$$= 200 \text{ V}$$

(b) $$\eta = \frac{P_o}{P_i} = \frac{P_o}{e_p I_p}$$

$$\Rightarrow I_p = \frac{P_o}{\eta e_p} = \frac{12000}{0.9 \times 200} = \frac{200}{3} \text{ A}$$

$$I_s = \frac{P_o}{e_s} = \frac{12000}{1000} = 12 \text{ A}$$

Power loss in primary $= I_p^2 \times R_p = \left(\frac{200}{3}\right)^2 \times 0.9$

$= 4000$ W

Power loss is secondary $= I_s^2 \times R_s = (12)^2 \times 5$

$= 720$ W

EXAMPLE 25.23

A power transformer is used to step up an emf of 220 V to 4.4 kV to transmit 6.6 kW of power. If the primary has 1000 turns, find (a) number of turns in the secondary and (b) the current rating of the secondary. Assume that the efficiency of the transformer is 80%.

SOLUTION

(a) $N_s = \dfrac{e_s}{e_p} \times N_p = \dfrac{4400}{220} \times 1000 = 20{,}000$

(b) $I_s = \dfrac{P_o}{e_s} = \dfrac{\eta P_i}{e_s} = \dfrac{0.8 \times 6600}{4400} = 1.2 \text{ A}$

I_s is called the current rating of the secondary.

25.11 ALTERNATING CURRENT

If an alternating voltage $V = V_0 \sin \omega t$ is applied across a resistance R, the current I in the circuit is

$$I = \frac{V}{R} = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t \qquad (1)$$

where $I_0 = V_0/R$, is the *maximum* or *peak value* of the current. It is clear from Eq. (1) that the current I varies sinusoidally with time; its magnitude changes continuously with time and its direction is reversed periodically. A *sinusoidally varying current whose magnitude changes continuously with time and whose direction reverses periodically is called an alternating current* (or simply ac).

The angular frequency ω of an alternating current is related to its time period T and frequency ν as

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

where ω is expressed in radians per second (rad s^{-1}), T in seconds (s) and ν in hertz. (Hz). In terms of T, Eq. (1) reads

$$I = I_0 \sin\left(\frac{2\pi t}{T}\right)$$

Root Mean Square Voltage and Current

The mean value of a periodic function $X(t)$ of time period T over one time period is defined as

$$\overline{X} = \frac{\int_0^T X(t)dt}{\int_0^T dt} = \frac{1}{T}\int_0^T X(t)dt$$

(i) Mean or average value of alternating voltage $V = V_0 \sin(\omega t) = V_0 \sin\left(\dfrac{2\pi t}{T}\right)$

$$\overline{V} = \frac{1}{T}\int_0^T V_0 \sin(\omega t)dt$$

$$= \frac{V_0}{T}\int_0^T \sin(\omega t)dt = -\frac{V_0}{T\omega}\left|\cos(\omega t)\right|_0^T$$

$$= -\frac{V_0}{T\omega}\left|\cos\left(\frac{2\pi t}{T}\right)\right|_0^T$$

$$= \frac{V_0}{2\pi}(\cos 2\pi - \cos 0) = 0$$

Similary mean value of alternating current $I = I_0 \sin(\omega t)$ over one time period is $\overline{I} = 0$

(ii) Mean square value of alternating voltage $V = V_0 \sin(\omega t)$ is

$$\overline{V^2} = \frac{1}{T}\int V_0^2 \sin(\omega t)dt$$

$$= \frac{V_o^2}{T}\int_0^T \frac{1}{2}(1 - \cos 2\omega t)dt$$

$$= \frac{V_0^2}{T}\left[\frac{1}{2}\int_0^T dt - \frac{1}{2}\int_0^T \cos(2\omega t)dt\right]$$

$$= \frac{V_0^2}{T}\left[\frac{T}{2} - \frac{1}{2}\left|\frac{\sin(2\omega t)}{2\omega}\right|_0^T\right]$$

$$= \frac{V_0^2}{T}\left[\frac{T}{2} - 0\right] = \frac{V_0^2}{2}$$

Root mean square (rms) value of the alternating voltage is

$$V_{\text{rms}} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \text{peak value of } V$$

Similarly, root mean square (rms) value of alternating current $I = I_0 \sin(\omega t)$ is

$$I_{\text{rms}} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \text{peak value of } I$$

EXAMPLE 25.24

An alternating voltage V (in volt) varies with time t (in second) as

$$V = 100 \sin(50\ \pi t)$$

Find the peak value, rms value and frequency of the alternating voltage.

SOLUTION

Comparing the given equation with

$$V = V_0 \sin(\omega t)$$

We get $V_0 = 100$ V,

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7 \text{ V}$$

and $\omega = 50\pi$

or $2\pi\nu = 50\pi \Rightarrow \nu = 25$ Hz

EXAMPLE 25.25

A 100 Ω electric iron is connected to a 200 V, 50 Hz a.c. supply. Find (a) rms value of voltage, (b) peak value of voltage, (c) rms value of current and (d) peak value of current.

SOLUTION

If an alternating supply is given to be 200 V, 50 Hz, it implies that the rms value of voltage is 200 V and the frequency is 50 Hz.

(a) $V_{rms} = 200$ V

(b) $V_0 = \sqrt{2}\, V_{rms} = \sqrt{2} \times 200 = 282.8$ V.

(c) $I_{rms} = \dfrac{V_{rms}}{R} = \dfrac{200}{100} = 2$ A

(d) $I_0 = \dfrac{V_0}{R} = \dfrac{282.8}{100} = 2.828 \simeq 2.83$ A

25.12 SERIES LCR CIRCUIT

The applied voltage V divides into three parts, V_L (across L), V_C (across C) and V_R (across R) such that (Fig. 25.31).

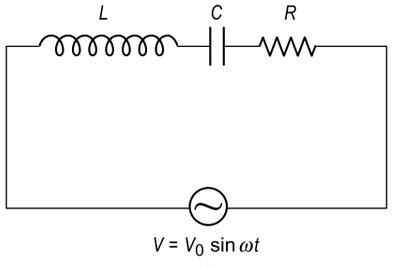


Fig. 25.31

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The current in the circuit is

$$I = I_0 \sin(\omega t - \phi)$$

$$\tan\phi = \left(\omega L - \frac{1}{\omega C}\right)$$

(i) If $\omega L > \dfrac{1}{\omega C}$, i.e. $\omega > \dfrac{1}{\sqrt{LC}}$, then tan ϕ is positive and voltage leads the current.

(ii) If $\omega L < \dfrac{1}{\omega C}$, i.e. $\omega < \dfrac{1}{\sqrt{LC}}$, then voltage lags behind current.

(iii) If $\omega L = \dfrac{1}{\omega C}$, i.e. $\omega = \dfrac{1}{\sqrt{LC}}$, then $\phi = 0$

This is the case of resonance. Voltage and current are in phase. $Z = R$ (minimum) and current is maximum.

Special Cases

(a) A.C. circuit containing only a pure resistor (Fig. 25.32)

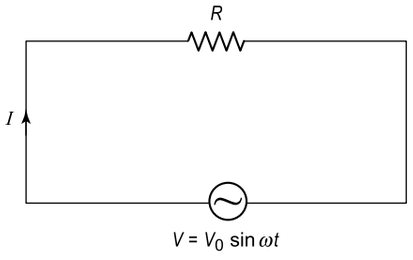


Fig. 25.32

$$V_R = V_0 \sin \omega t$$

$$I = I_0 \sin \omega t$$

where $I_0 = \dfrac{V_0}{R}$

The voltage across R is always in phase with the current in the circuit.

(b) A.C. circuit containing only an ideal inductor (Fig. 25.33)

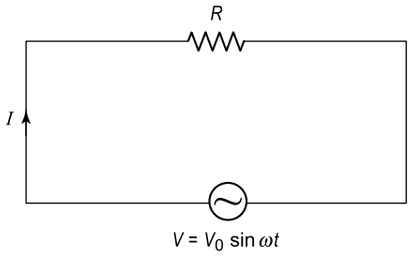


Fig. 25.33

$$V_L = V_0 \sin \omega t$$

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $I_0 = \dfrac{V_0}{X_L} = \dfrac{V_0}{\omega L}$

$X_L = \omega L$ is called inductive reactance.The voltage across the inductance leads the current in the circuit by a phase angle of $\pi/2$.

(c) A.C. circuit containing only an ideal capacitor (Fig. 25. 34)

$$V_C = V_0 \sin \omega t$$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $I_0 = \dfrac{V_0}{X_C} = \omega C V_0$

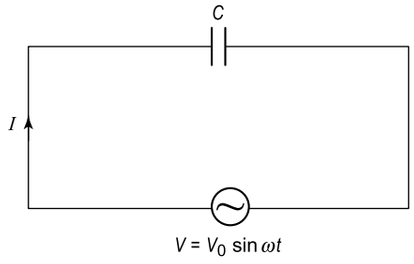


Fig. 25.34

$X_C = \dfrac{1}{\omega C}$ is called capacitative reactance. The voltage across the capacitor lags behind the current in the circuit by a phase angle of $\pi/2$.

(d) A.C. circuit containing an ideal inductor and a pure resistor (Fig. 25.35)

$$V_0 = I_0 Z$$

Where $V_0 = \sqrt{V_R^2 + V_L^2}$

$$V_R = IR,\ V_L = IX_L$$

and $Z = \dfrac{V_0}{I_0} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$ is called impedance.

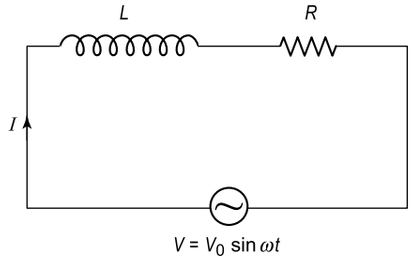


Fig. 25.35

$$I = I_0 \sin(\omega t - \phi)$$

where $\phi = \tan^{-1}\left(\dfrac{\omega L}{R}\right)$ is the phase angle between the voltage and current in the circuit.

(e) A.C. circuit containing an ideal capacitor and a pure resistor (Fig. 25.36)

$$V_0 = I_0 Z$$

where $V_0 = \sqrt{V_R^2 + V_C^2}$

$$V_R = IR,\ V_C = IX_C$$

and $Z = \dfrac{V_0}{I_0} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$

is called impedance.

$$I = I_0 \sin(\omega t + \phi)$$

where $\phi = \tan^{-1}\left(\dfrac{1}{R\omega C}\right)$ is the phase angle between the voltage and current in the circuit.

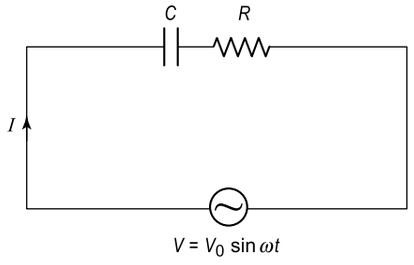


Fig. 25.36

25.13 POWER IN LCR CIRCUIT

In a series *LCR* circuit driven by an alternating voltage $V = V_0 \sin \omega t$, the current in the circuit is

$$I = I_0 \sin(\omega t \pm \phi)$$

depending upon whether $X_L < X_C$ or $X_L > X_C$.

$$I_0 = \frac{V_0}{Z};\ Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

and $\phi = \tan^{-1}\left(\dfrac{\omega L - \dfrac{1}{\omega C}}{R}\right)$

Instantaneous power supplied to the circuit by the A.C. source is

$$P(t) = VI = V_0 \sin \omega t \times I_0 \sin(\omega t \pm \phi)$$
$$= V_0 I_0 \sin \omega t \times \sin(\omega t + \phi)$$

$\therefore$ Average power supplied by the source in one complete cycle is

$$\overline{P} = \frac{1}{T}\int_0^T P(t)\,dt$$

$$= \frac{1}{T} \times V_0 I_0 \int_0^T \sin \omega t\,(\sin \omega t \cos\phi \pm \cos \omega t \sin \phi)\,dt$$

$$= \frac{V_0 I_0}{T}\left[\cos\phi \int_0^T \sin^2(\omega t)\,dt \pm \sin f \int_0^T \sin(\omega t) \times \cos(\omega t)\,dt\right]$$

$$= \frac{V_0 I_0}{T}\left[\cos\phi \times \frac{T}{2} \pm 0\right]$$

$$= \frac{V_0 I_0}{2}\cos\phi = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}}\cos\phi$$

or $\overline{P} = V_{\text{rms}}\, I_{\text{rms}} \cos \phi$

Power Factor of an A.C. Circuit

The power supplied by the source depends not only on V_{rms} and I_{rms} but also on cos ϕ. The quantity cos ϕ is called the power factor of the A.C. circuit. Now

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

and

$$\cos\phi = \frac{1}{(1+\tan^2\phi)^{1/2}}$$

$$= \frac{1}{1+\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)^2}$$

$$= \frac{R}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

$$= \frac{R}{Z}$$

$$\therefore \text{ Power factor} = \frac{\text{resistance}}{\text{impedance}}$$

Special Cases

(a) *For an A.C. circuit containing only a resistor,*
$Z = R$ and $\cos\phi = \frac{R}{R} = 1 \Rightarrow \phi = 0$ and $\overline{P} = V_{rms}\, I_{rms}$

(b) *For an A.C. circuit containing only an inductor or a capacitor,*
$\phi = 90°$. Hence $\overline{P} = 0$

(c) At resonance, $\phi = 0$ for an *LCR* circuit. Hence $\overline{P}$ = maximum, i.e. maximum power is delivered to the circuit form A.C. source.

Wattless Current

For an A.C. circuit containing only a pure inductor or an ideal capacitor $\phi = 90°$. Hence

$$\overline{P} = V_{rms}\, I_{rms} \cos 90° = 0$$

Such an A.C. circuit consumes no power. The current flowing through the inductor or capacitor consumes no power and is called wattless current.

Bandwidth and Quality Factor of LCR Circuit

For an *LCR* circuit driven by an alternating voltage $V = V_0 \sin \omega t$, the peak (amplitude) value of the current is given by

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

$$\therefore \quad (I_0)_{max} = \frac{V_0}{R} \quad \Rightarrow \quad V_0 = (I_0)_{max}\, R$$

In terms of $(I_0)_{max}$, I_0 is given by

$$I_0 = \frac{(I_0)_{max}\, R}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

Figure 25.37 shows the variation of I_0 versus ω.

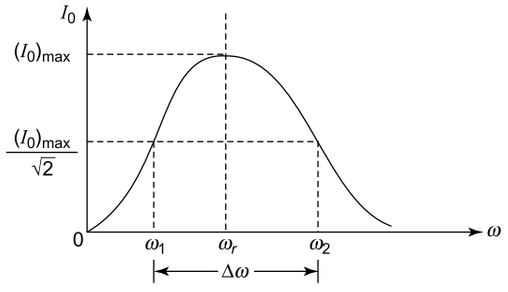


Fig. 25.37

Let ω_1 and ω_2 be the values of ω when $I_0 = \frac{(I_0)_{max}}{\sqrt{2}}$, i.e. when

$$\frac{(I_0)_{max}\, R}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}} = \frac{(I_0)_{max}\, R}{\sqrt{2}}$$

$$\Rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\Rightarrow \quad \omega L - \frac{1}{\omega C} = \pm R$$

Case 1: $\omega L - \frac{1}{\omega C} = +R \Rightarrow \omega^2 - \frac{1}{LC} = \frac{R\omega}{L}$

$$\Rightarrow \quad \omega^2 - \frac{R\omega}{L} - \omega_r^2 = 0, \text{ where } \omega_r = \frac{1}{\sqrt{LC}}$$

The positive root of this quadratic equation is

$$\omega_2 = \frac{R}{2L} + \left(1 + \frac{4\omega_r^2 L^2}{R^2}\right)^{1/2}$$

Case 2: $\omega L - \frac{1}{\omega C} = -R$

$$\Rightarrow \quad \omega^2 + \frac{R\omega}{L} - \omega_r^2 = 0$$

The positive root of this quadratic equation is

$$\omega_1 = -\frac{R}{2L} + \left(1 + \frac{4\omega_r^2 L^2}{R^2}\right)^{1/2}$$

Bandwidth $\Delta\omega = \omega_2 - \omega_1 = \dfrac{R}{L}$

Quality factor (or Q factor) of LCR circuit is defined as

$$Q = \frac{\text{resonant frequency}}{\text{bandwidth}} = \frac{\omega_r}{\Delta\omega}$$

$$= \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Q is a dimensionless number. Figure 25.38 shows the graph of $\overline{P}$ versus ω for some values of Q.

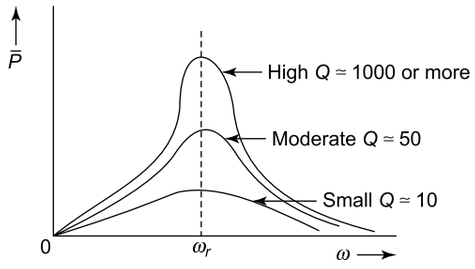


Fig. 25.38

The power peak is sharp for high Q. The resonance is then said to be sharp. Higher the value of Q, the sharper is the resonance and greater is the power absorbed from the source.

EXAMPLE 25.26

A coil of inductance 0.5 H and a resistor of resistance 100 Ω are connected in series to a 240 V, 50 Hz supply.

(a) Find the maximum current in the circuit.

(b) What is the time lag between voltage maximum and current maximum?

SOLUTION

Given $V_{rms} = 240$ V, $\omega = 2\pi\nu = 2\pi \times 50 = 100\pi$ rad s^{-1}, $L = 0.5$ H and $R = 100$ Ω.

$$V = V_0 \sin \omega t$$

$$I = I_0 \sin (\omega t - \phi);$$

$$I_0 = \frac{V_0}{(R^2 + \omega^2 L^2)^{1/2}},$$

$$\tan \phi = \frac{\omega L}{R}$$

(a) $V_0 = \sqrt{2}\, V_{rms} = \sqrt{2} \times 240$ V

$$\therefore I_0 = \frac{\sqrt{2} \times 240}{[(100)^2 + (100\pi \times 0.5)^2]^{1/2}} = 1.82 \text{ A}$$

(b) V is maximum when $\sin \omega t = +1 \Rightarrow \omega t_1 = \dfrac{\pi}{2}$

$\Rightarrow t_1 = \dfrac{\pi}{2\omega}$

I is maximum when $\sin (\omega t - \phi) = +1 \Rightarrow \omega t_2 - \phi = \dfrac{\pi}{2} \Rightarrow t_2 = \dfrac{\pi}{2\omega} - \dfrac{\phi}{\omega}$

$\therefore$ Time lag between voltage maximum and current maximum is

$$\Delta t = t_1 - t_2 = \frac{\pi}{2\omega} - \left(\frac{\pi}{2\omega} - \frac{\phi}{\omega}\right) = \frac{\phi}{\omega}$$

Now $\tan \phi = \dfrac{\omega L}{R} = \dfrac{100\pi \times 0.5}{100} = 1.57$

$$\Rightarrow \quad \phi = 57.5° = \frac{57.5 \times \pi}{180} \text{ rad}$$

$$\therefore \quad \Delta t = \frac{\phi}{\omega} = \frac{57.5 \times \pi}{180 \times 100\pi} = 3.2 \times 10^{-3} \text{ s}$$

EXAMPLE 25.27

A capacitor of capacitance 100 μF and a resistor of resistance 40 Ω are connected in series to a 110 V, 60 Hz supply.

(a) Find the maximum current in the circuit.

(b) What is the time lag between current maximum and voltage maximum?

SOLUTION

Given $C = 100$ μF $= 100 \times 10^{-6}$ F, $R = 40$ Ω, $V_{rms} = 110$ V, $\omega = 2\pi\nu = 2\pi \times 60 = 120$ rad s^{-1}

$$V = V_0 \sin \omega t$$

$$I = I_0 \sin (\omega t + \phi),$$

$$I_0 = \frac{V_0}{\left(R^2 + \dfrac{1}{\omega^2 C^2}\right)^{1/2}}, \quad \tan \phi = \frac{1}{\omega CR}$$

(a) $I_0 = \dfrac{\sqrt{2} \times 110}{\left[\left((40)^2 + \dfrac{1}{(120 \times 10^{-4})^2}\right)\right]^{1/2}} = 3.24$ A

(b) V is maximum when $\sin \omega t = +1 \Rightarrow \omega t_1 = \dfrac{\pi}{2}$

$\Rightarrow t_1 = \dfrac{\pi}{2\omega}$

I is maximum when $\sin (\omega t + \phi) = +1$

$\Rightarrow \omega t_2 + \phi = \dfrac{\pi}{2} \Rightarrow t_2 = \dfrac{\pi}{2\omega} - \dfrac{\phi}{\omega}$

$\therefore$ Time lag between current maximum and voltage maximum is

$$\Delta t = t_1 - t_2 = \frac{\phi}{\omega}$$

Now $\tan\phi = \dfrac{1}{\omega CR} = \dfrac{1}{120\pi \times 10^{-4} \times 40} = 0.663$

$$\Rightarrow \qquad \phi = 33.5° = \frac{33.5 \times \pi}{180} \text{ rad}$$

$$\therefore \qquad \Delta t = \frac{\phi}{\omega} = \frac{33.5 \times \pi}{180 \times 120\pi} = 1.55 \times 10^{-3} \text{ s}$$

EXAMPLE 25.28

A series LCR circuit with $L = 5$ H, $C = 80$ μF and $R = 40\ \Omega$ is connected to a variable frequency 230 V a.c. source.

(a) What is the source frequency which drives the circuit at resonance?

(b) What is the impedance of the circuit at resonance?

(c) Find peak value of the current at resonance.

(d) Find the rms potential differences across L, C and R at resonance.

(e) What is the total potential difference across the combination of L and C at resonance.

(f) Find the maximum power transferred to the circuit from the source in one complete cycle.

SOLUTION

(a) $\omega_r = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50 \text{ rad s}^{-1}$

$$\therefore \quad \nu_r = \frac{\omega_r}{2\pi} = \frac{50}{2\pi} = 7.96 \text{ Hz}$$

(b) $Z = \left[R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right]^{1/2}$

$= R \qquad \left(\because\ \omega L = \dfrac{1}{\omega C} \text{ at resonance}\right)$

$= 40\ \Omega$

(c) $I_0 = \dfrac{V_0}{Z} = \dfrac{\sqrt{2} \times 230}{40} = 8.1$ A

(d) $(V_L)_{\text{rms}} = I_{\text{rms}} \times X_L = I_{\text{rms}} \times \omega_r L$

$$= \frac{230}{40} \times 50 \times 5 = 1437.5 \text{ V}$$

$$(V_C)_{\text{rms}} = I_{\text{rms}} \times X_C = I_{\text{rms}} \times \frac{1}{\omega_r C}$$

$$= \frac{230}{40} \times \frac{1}{80 \times 10^{-6} \times 50} = 1437.5 \text{ V}$$

$$(V_R)_{\text{rms}} = I_{\text{rms}} \times R = \frac{230}{40} \times 40 = 230 \text{ V}$$

(e) $(V_{L,C})_{\text{rms}} = I_{\text{rms}} \times \left(\omega_r L - \dfrac{1}{\omega_r C}\right)$

$= 1437.5 - 1437.5 = 0$

(f) $P_{\max} = V_{\text{rms}} \times I_{\text{rms}} \times \cos\phi$

$= V_{\text{rms}} \times I_{\text{rms}} \qquad (\because\ \phi = 0 \text{ at resonance})$

$$= \frac{V_{\text{rms}}^2}{R} = \frac{230 \times 230}{40} = 1322.5 \text{ W}$$

EXAMPLE 25.29

When an alternating voltage of 220 V is applied across a device A, a current of 0.5 A flows through the circuit and it is in phase with the applied voltage. When the same voltage is applied across a device B, again the same current flows in the circuit but it leads the voltage by $\pi/2$. (a) Name devices A and B. (b) Compute the current when the same voltage is applied across a series combination of A and B.

SOLUTION

(a) Device A is a resistor and B is a capacitor.

(b) Given $V_{\text{rms}} = 220$ V, $I_{\text{rms}} = 0.5$ A

Resistance of A is $R = \dfrac{V_{\text{rms}}}{I_{\text{rms}}} = \dfrac{220}{0.5} = 440\ \Omega$

Reactance of B is $X_C = \dfrac{V_{\text{rms}}}{I_{\text{rms}}} = \dfrac{220}{0.5} = 440\ \Omega$

Impedance when A and B are connected in series is

$$Z = \sqrt{R^2 \times X_C^2} = \sqrt{(440)^2 + (440)^2} = 622.3\ \Omega$$

$$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220}{622.3} = 0.35 \text{ A}$$

25.14 LC OSCILLATIONS

In an electrical circuit consisting of an inductance L and a capacitance C, the charge (and hence current) oscillates harmonically with an angular frequency

$$\omega = \frac{1}{\sqrt{LC}}$$

and time period $T = 2\pi\sqrt{LC}$

The charge and current in the circuit vary with time as

$$q = q_0 \sin(\omega t + \phi) \text{ and } I = I_0 \cos(\omega t + \phi)$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. A square loop is placed in a uniform magnetic field **B** as shown in Fig. 25.39. The power needed to pull it out of the field with a velocity v is proportional to

(a) $v^{1/2}$ (b) v

(c) v^2 (d) $v^{3/2}$

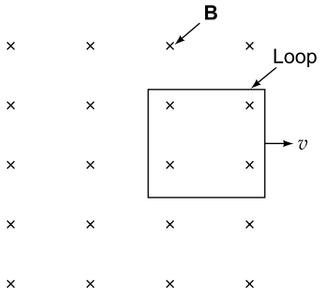


Fig. 25.39

2. Two circular co-axial coils of equal radii carry equal currents circulating in the same direction. If the coils are moved towards each other,

(a) the current in each coil will increase

(b) the current in each coil will decrease

(c) the current in each coil will remain the same

(d) the current in one coil will increase and in the other coil the current will decrease.

3. A circular coil with its plane vertical is released from rest. It enters a region of a uniform magnetic field B at time $t = t_1$ and leaves the region at time $t = t_2$ (Fig. 25.40). The acceleration of the coil is

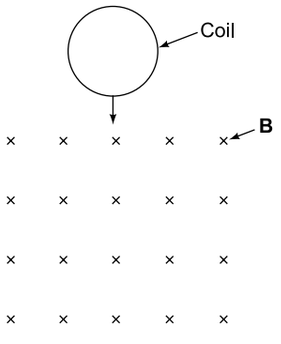


Fig. 25.40

(a) less than g for all values of t

(b) equal to g for all values of t

(c) equal to g before $t = t_1$ and after $t = t_2$ but less than g between t_1 and t_2.

(d) less than g when it is entering the field and when it is leaving the field.

4. An equilateral triangular loop PQR of side a is at the edge of a uniform magnetic field **B** at $t = 0$ as shown in Fig. 25.41 (a). It is pulled to the right with a constant velocity v and its edge R leaves the region of magnetic field at $t = t_0$. Which of the graphs shown in Fig. 25.41 (b) represents the variation of induced current i with time t?

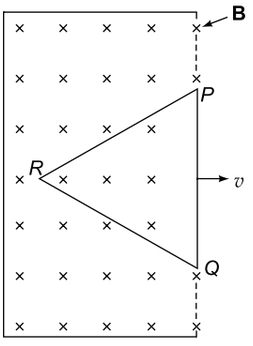


Fig. 25.41 (a)

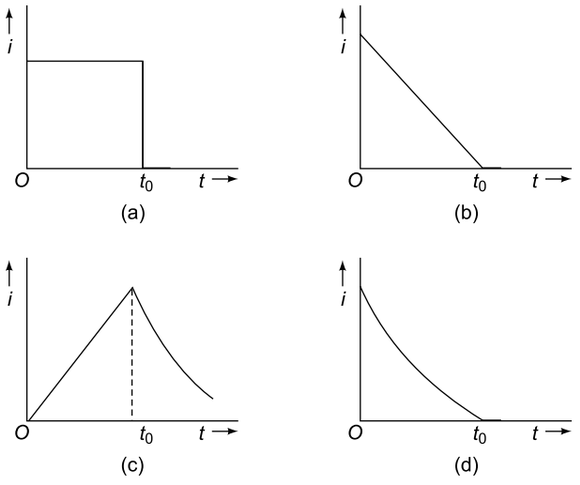


Fig. 25.41 (b)

5. Two parallel wires PQ and RS are connected by a capacitor and a metal rod CD and placed in a magnetic field directed into the page as shown in Fig. 25.42. If rod CD is moved with a velocity v as shown in the figure,

(a) plate 1 of the capacitor acquires a positive charge and plate 2 an equal negative charge
(b) plate 1 of the capacitor acquires a negative charge and plate 2 an equal positive charge

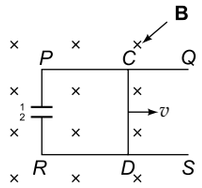


Fig. 25.42

(c) plates 1 and 2 do not acquire any charge
(d) the energy stored in the capacitor increases linearly with time.

6. A square loop of side a is placed such that its plane is the same as that of a very long straight wire carrying a current I. The centre O of the loop is at a distance x from the wire (Fig. 25.43). The loop is given a velocity v as shown. If $x >> a$, the magnitude of the emf induced in the loop is proportional to

(a) x
(b) x^2
(c) $\frac{1}{x}$
(d) $\frac{1}{x^2}$

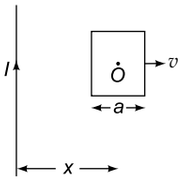


Fig. 25.43

7. Two ideal inductors are connected in parallel as shown in Fig. 25.44. A time-varying current flows as shown. The ratio I_1/I_2 at any time t is

(a) $\frac{L_1}{L_2}$ (b) $\frac{L_2}{L_1}$
(c) $\sqrt{\frac{L_1}{L_2}}$ (d) $\sqrt{\frac{L_2}{L_1}}$

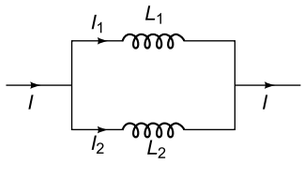


Fig. 25.44

8. In the circuit shown in Fig. 25.45, I_2 = 3A in the steady state. The potential difference across the 4 Ω resistor is

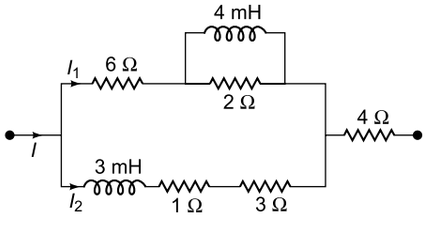


Fig. 25.45

(a) 12 V (b) 18 V
(c) 20 V (d) 24 V

9. An alternating current (in ampere) varies with time t as

$$I = 3 \sin \omega t + 4 \cos \omega t$$

The rms value of the current is

(a) $\frac{3}{\sqrt{2}}$ A (b) $\frac{4}{\sqrt{2}}$ A
(c) $\frac{5}{\sqrt{2}}$ A (d) $\frac{7}{\sqrt{2}}$ A

10. A series LCR circuit consists of a variable inductance L, variable capacitance C and variable resistance R. For a given set of values of L, C and R, the voltage lags the current. The power factor of the circuit can be increased by

(a) decreasing L
(b) decreasing C
(c) decreasing R
(d) increasing R or L or C.

11. A coil and a capacitor are connected in series with a 12 V, variable frequency a.c. source. By varying the frequency of the source, a maximum rms current of 6 A is observed. If the same coil is connected to a battery of emf 6 V, a current of 2 A flows through it. The internal resistance of the battery is

(a) 0.5 Ω (b) 1.0 Ω
(c) 1.5 Ω (d) 2.0 Ω

12. In a series LCR circuit, the voltage across resistance, capacitance and inductance is the same, each equal to 80 V. If the capacitor is short circuited, the voltage across the inductor becomes

(a) zero (b) 40 V
(c) 80 V (d) $40\sqrt{2}$ V

13. A circuit consists of a 2 μF capacitor connected through an a.c. ammeter to an a.c. source of voltage given by

$$V = 200\sqrt{2} \sin (100t + \pi/3)$$

where V is in volts and t in seconds. The reading of the ammeter is

(a) 10 mA (b) 20 mA
(c) 30 mA (d) 40 mA

14. A circuit has a self inductance of 1 henry and carries a current of 2 A. To prevent sparking when the circuit is broken, a capacitor which can withstand 400 volts is used. The least capacitance of the capacitor connected across the switch must be equal to

(a) 12.5 μF (b) 25 μF
(c) 50 μF (d) 100 μF

15. A 25 kW dc generator produces a potential difference of 250 V. If the resistance of the transmission line is 1 Ω, what percentage of the original power is lost during transmission?
(a) 40% (b) 50%
(c) 60% (d) 75%

16. Which one of the graphs shown in Fig. 25.46 represents the variation of current I in the circuit shown in Fig. 25.47 with time t, the key K being a plugged at $t = 0$

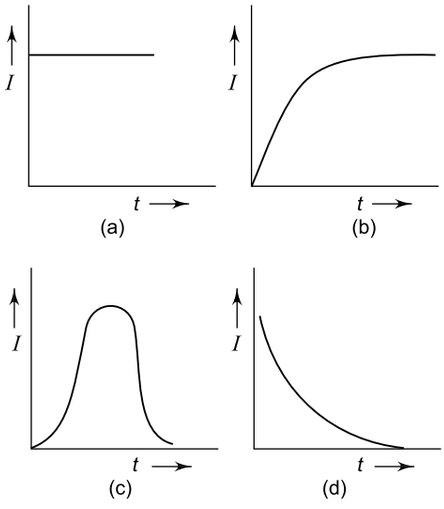


Fig. 25.46

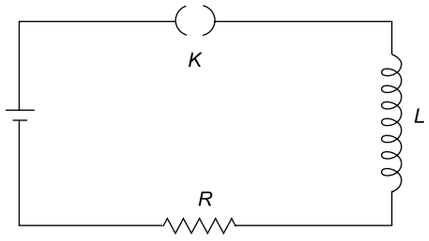
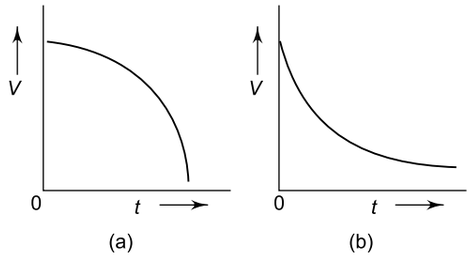


Fig. 25.47

17. Which one of the graphs shown in Fig. 25.48 represents the variation of potential difference V across the inductor L with time t, the key K being plugged at $t = 0$, in the circuit shown in Fig. 25.47?



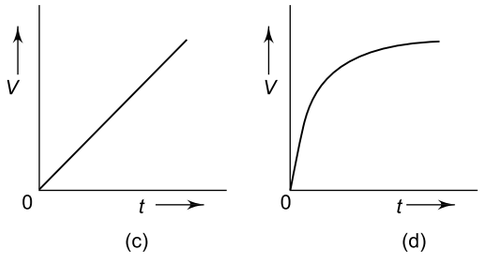


Fig. 25.48

18. An inductive coil has a resistance of 100 Ω. When an ac signal of frequency 1000 Hz is fed to the coil, the applied voltage leads the current by 45°. What is the inductance of the coil?
(a) 10 mH (b) 12 mH
(c) 16 mH (d) 20 mH

19. An ac source of variable frequency f is connected to an LCR series circuit. Which one of the graphs in Fig. 25.49 represents the variation of current I in the circuit with frequency f?

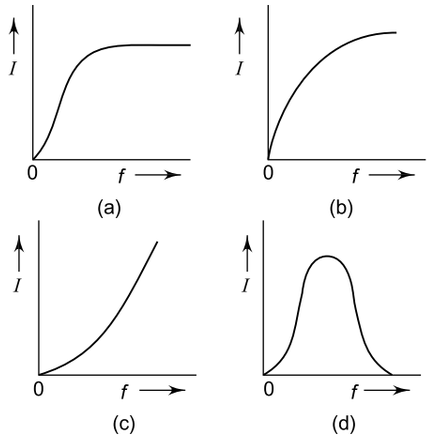


Fig. 25.49

20. An inductor of self inductance 100 mH and a resistor of resistance 50 Ω are connected to a 2 V battery. The time required for the current to fall to half its steady value is
(a) 2 millisecond
(b) 2 ln (0.5) millisecond
(c) 2 ln (1) millisecond
(d) 2 ln (2) millisecond

21. A radio tuner has a frequency range from 500 kHz to 5 MHz. If its LC circuit has an effective inductance of 400 μH, what is the range of its variable capacitor? Take $\pi^2 = 10$.
(a) 2.5 pF to 250 pF (b) 5.0 pF to 500 pF
(c) 7.5 pF to 750 pF (d) 10 pF to 1000 pF

22. L, C and R, respectively represent inductance, capacitance and resistance. Which one of the

following combinations has the dimensions of frequency?

(a) $1/RC$ (b) $1/LC$

(c) L/R (d) C/L

23. An alternating voltage (in volts) given by

$$V = 200\sqrt{2}\sin(100\,t)$$

is connected to 1 μF capacitor through an ac ammeter. The reading of the ammeter will be

(a) 10 mA (b) 20 mA

(c) 40 mA (d) 80 mA

24. Three pure inductances each of 3H are connected as shown in Fig. 25.50. The equivalent inductance between points A and B is

(a) 1 H (b) 2 H

(c) 3 H (d) 9 H

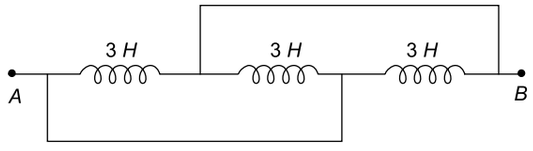


Fig. 25.50

25. In an ac circuit the potential difference V and current I are given respectively by

$$V = 100\sin(100\,t) \text{ volt}$$

and $$I = 100\sin\left(100t + \frac{\pi}{3}\right) \text{ mA}$$

The power dissipated in the circuit will be

(a) 10^4 W (b) 10 W

(c) 2.5 W (d) 5 W

26. A coil of metal wire is kept stationary in a non-uniform magnetic field,

(a) an emf and current are both induced in the coil

(b) a current but no emf is induced in the coil

(c) an emf but no current is induced in the coil

(d) neither emf nor current is induced in the coil.

IIT, 1987

27. A loosely wound helix made of stiff wire is mounted vertically with the lower end just touching a dish of mercury. When a current from the battery is started in the coil through the mercury

(a) the wire oscillates

(b) the wire continues making contact

(c) the wire breaks contact just when the current is passed

(d) the mercury will expand by heating due to passage of current.

IIT, 1981

28. In an ac circuit the potential differences across an inductance and a resistance connected in series are respectively 16 V and 20 V. The total potential difference across the circuit is

(a) 20.0 V (b) 25.6 V

(c) 31.9 V (d) 53.5 V

29. An alternating voltage $V = V_0\sin\omega t$ is applied across a circuit. As a result a current $I = I_0\sin(\omega t - \pi/2)$ flows in it. The power consumed per cycle is

(a) zero (b) 0.5 V_0I_0

(c) 0.707 V_0I_0 (d) 1.414 V_0I_0

30. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic field **B**, constant in space and time, pointing perpendicular and into the plane of the loop exists everywhere as shown in Fig. 25.51. The current induced in the loop is

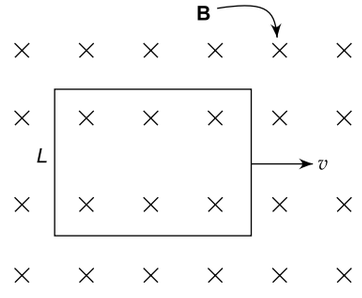


Fig. 25.51

(a) BLv/R clockwise

(b) BLv/R anticlockwise

(c) $2BLv/R$ anticlockwise

(d) zero

IIT, 1989

31. A thin circular ring of area A is held perpendicular to a uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is

(a) $\frac{BR}{A}$ (b) $\frac{AB}{R}$

(c) ABR (d) $\frac{B^2A}{R^2}$

IIT, 1995

32. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic field B (Fig. 25.52). At the position MNQ the

speed of the ring is v and the potential difference across the ring is

(a) zero

(b) $\frac{1}{2}$ $Bv\pi R^2$ and M is at higher potential

(c) πRBv and Q is at higher potential

(d) 2 RBv and Q is at higher potential.

IIT, 1996

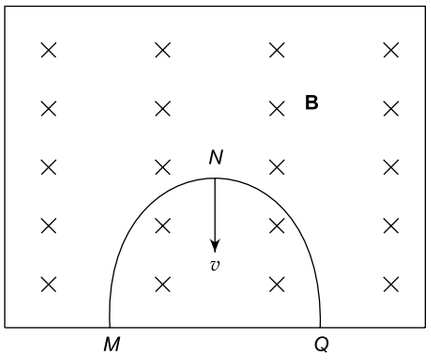


Fig. 25.52

33. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($L >> l$). The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to

(a) $\frac{l}{L}$ (b) $\frac{l^2}{L}$

(c) $\frac{L}{l}$ (d) $\frac{L^2}{l}$

IIT, 1998

34. A circular loop of radius R, carrying current I, lies in the x-y plane with its centre at the origin. The total magnetic flux through the x-y plane is

(a) directly proportional to I

(b) directly proportional to R

(c) inversely proportional to R

(d) zero

IIT, 1999

35. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time

(a) 500 s (b) 20 s

(c) 35 ms (d) 1 ms

IIT, 1999

36. A uniform but time varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown in Fig. 25.53. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region

(a) is zero (b) decreases as $\frac{1}{r}$

(c) increases as r (d) decreases as $\frac{1}{r^2}$.

IIT, 2000

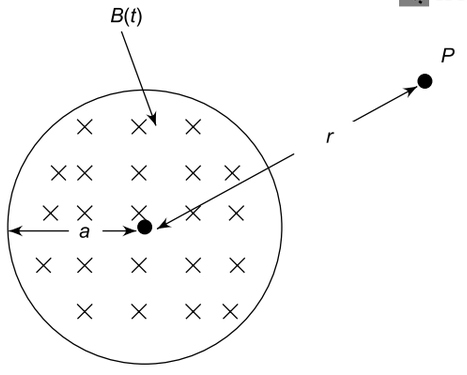


Fig. 25.53

37. In a uniform magnetic field B, a wire in the form of a semicircle of radius r rotates about the diameter of the circle with an angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R, the mean power generated per period of rotation is

(a) $\frac{\pi r^2 \omega B}{2R}$ (b) $\frac{(\pi r^2 \omega B)^2}{8R}$

(c) $\frac{(\pi r \omega B)^2}{2R}$ (d) $\frac{(\pi r \omega^2 B)^2}{8R}$

38. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 rad s^{-1}. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, the emf developed between the ends of the conductor is

(a) 5 μV (b) 50 μV

(c) 5 mV (d) 50 mV

39. A capacitor of capacitance 2 μF is charged to a potential difference of 12 V. The charging battery is then removed and the capacitor is connected across an inductor of self inductance 0.6 mH. The current in the circuit at a time when the potential difference across the capacitor is 6 V is

(a) 0.3 A (b) 0.6 A

(c) 0.9 A (d) 1.2 A

IIT, 1992

40. A wire in the form of a circular loop of radius r lies with its plane normal to a magnetic field B. If the wire is pulled to take a square shape in the

same plane in time t, the emf induced in the loop is given by

(a) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{10}\right)$ (b) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{8}\right)$

(c) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{6}\right)$ (d) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{4}\right)$

41. A square loop of side l, mass m and resistance R falls vertically into a uniform magnetic field directed perpendicular to the plane of the coil. The height h through which the loop falls so that it attains terminal velocity on entering the region of magnetic field is given by

(a) $\frac{mgR}{2Bl}$ (b) $\frac{m^2gR^2}{2B^2l^2}$

(c) $\frac{mgR^2}{4B^3l^3}$ (d) $\frac{m^2gR^2}{2B^4l^4}$

42. The mutual inductance between two planar concentric rings of radii r_1 and r_2 (with $r_1 > r_2$) placed in air is given by

(a) $\frac{\mu_0\pi r_2^2}{2r_1}$ (b) $\frac{\mu_0\pi r_1^2}{2r_2}$

(c) $\frac{\mu_0\pi (r_1+r_2)^2}{2r_1}$ (d) $\frac{\mu_0\pi (r_1+r_2)^2}{2r_2}$

43. A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v in a uniform magnetic field B = 2T as shown in Fig. 25.54. The magnetic field is perpendicular to the plane of the loop and directed into the paper. The loop is connected to a network of resistors, each equal to 3 Ω. What should be the speed of the loop so as to have a steady current of 1 mA in the loop?

(a) 1 cm s^{-1} (b) 2 cm s^{-1}

(c) 3 cm s^{-1} (d) 4 cm s^{-1}

IIT, 1983

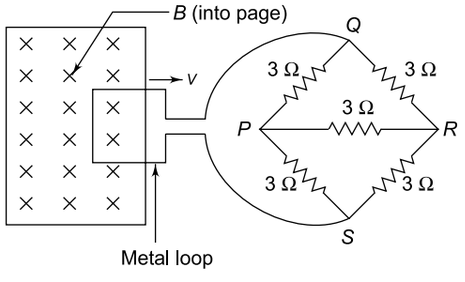


Fig. 25.54

44. Two circular coils can be arranged in any of the three situations shown in Fig. 25.55. Their mutual inductance will be

(a) maximum in situation (A)
(b) maximum in situation (B)
(c) maximum in situation (C)
(d) the same in all situations.

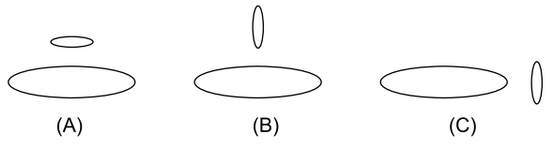


Fig. 25.55

IIT, 2000

45. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in Fig. 25.56. An electric field is induced

(a) in AD, but not in BC
(b) in BC, but not in AD
(c) neither in AD nor in BC
(d) in both AD and BC

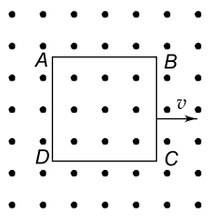


Fig. 25.56

IIT, 2001

46. As shown in Fig. 25.57, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_p flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the directions of I_{Q1} and I_{Q2} (as seen by E) are:

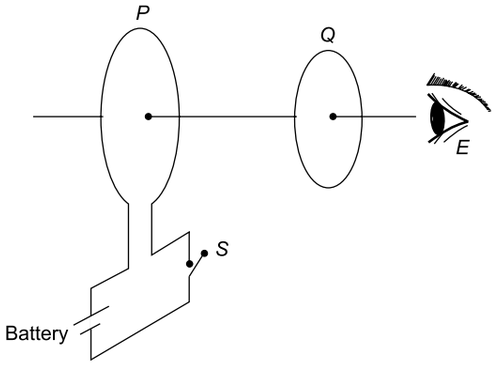


Fig. 25.57

(a) respectively clockwise and anti-clockwise
(b) both clockwise
(c) both anti-clockwise
(d) respectively anti-clockwise and clockwise

IIT, 2002

47. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be

(a) halved (b) the same

(c) doubled (d) quadrupled

IIT, 2002

48. When an AC source of e.m.f. $E = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the e.m.f. E and the current I in the circuit is observed to be $\pi/4$, as shown in the Fig. 25.58. If the circuit consists possibly only of R-C or R-L or L-C in series, what will be the relation between the two elements of the circuit?

(a) $R = 1\ \text{k}\Omega$, $C = 10\ \mu\text{F}$

(b) $R = 1\ \text{k}\Omega$, $C = 1\ \mu\text{F}$

(c) $R = 1\ \text{k}\Omega$, $L = 10\ \text{H}$

(d) $R = 1\ \text{k}\Omega$, $L = 1\ \text{H}$

IIT, 2003

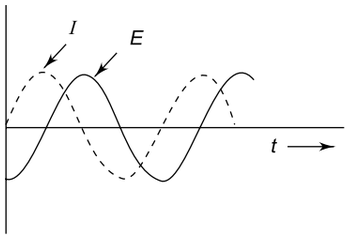


Fig. 25.58

49. An air plane, with 20 m wingspread is flying at 250 ms^{-1} parallel to the earth's surface at a place where the horizontal component of earth's magnetic field is 2×10^{-5} T and angle of dip is 60°. The magnitude of the induced emf between the tips of the wings is

(a) $\frac{1}{10}$ V (b) $\frac{\sqrt{2}}{10}$ V

(c) $\frac{\sqrt{3}}{10}$ V (d) $\frac{1}{5}$ V

50. A solenoid of inductance L and resistance R is connected to a battary. The time taken for the magnetic energy to reach $\frac{1}{4}$ of its maximum value is

(a) $\frac{L}{R}\log_e(1)$ (b) $\frac{L}{R}\log_e(2)$

(c) $\frac{L}{R}\log_e(3)$ (d) $\frac{L}{R}\log_e(4)$

IIT, 1996

51. An LCR series circuit with $R = 100\ \Omega$ is connected to a 200 V, 50 Hz a.c. source. When only the capacitance is removed, the voltage leads the current by 60°. When only the inductance is removed, the current leads the voltage by 60°. The current in the circuit is

(a) $\frac{2}{\sqrt{3}}$ A (b) $\frac{\sqrt{3}}{2}$ A

(c) 1 A (d) 2 A

52. A uniform metal rod is moving with a uniform velocity v parallel to a long straight wire carrying a current I. The rod is perpendicular to the wire with its ends at distances r_1 and r_2 (with $r_2 > r_1$) from it. The emf induced in the rod is

(a) zero (b) $\frac{\mu_0 I v}{2\pi}\log_e\left(\frac{r_2}{r_1}\right)$

(c) $\frac{\mu_0 I v}{2\pi}\log_e\left(\frac{r_1}{r_2}\right)$ (d) $\frac{\mu_0 I v}{4\pi}\left(1-\frac{r_1}{r_2}\right)$

53. The current in a coil of self inductance 2.0 H is increasing according to the equation $I = 2\sin(t^2)$ ampere. The amount of energy spent during the period when the current changes from zero to 2 A is

(a) 2 J (b) 4 J

(c) 8 J (d) 16 J

54. In a car spark coil, an emf of 40,000 volts is induced in its secondary when the current in its primary changes from 4 A to zero is 10 μs. The mutual inductance between the primary and the secondary windings of the spark coil is

(a) 0.1 H (b) 0.2 H

(c) 0.3 H (d) 0.4 H

55. A rectangular wire loop of sides a and b is placed in a non-uniform magnetic which varies with x as $B = kx$ where k is a constant. The magnetic field is directed perpendicular to the plane of the coil as shown in Fig. 25.59. The magnetic flux through the coil is

(a) zero

(b) kab^2

(c) $\frac{1}{2}kab^2$

(d) $\sqrt{2}\,kab^2$

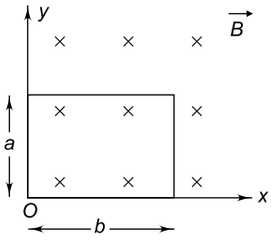


Fig. 25.59

56. A capacitor of capacitance 2 μF is charged to 50 V. The charging battery is then disconnected and a coil of inductance 5 mH is connected across it. Assuming that the coil has negligible resistance, the peak value of the current in the circuit will be

(a) 1 A (b) 2 A

(c) 3 A (d) 4 A

57. The variation of induced emf with time in a coil if a short bar magnet is moved along its axis, (shown in Fig. 25.60), with a constant velocity is best represented as

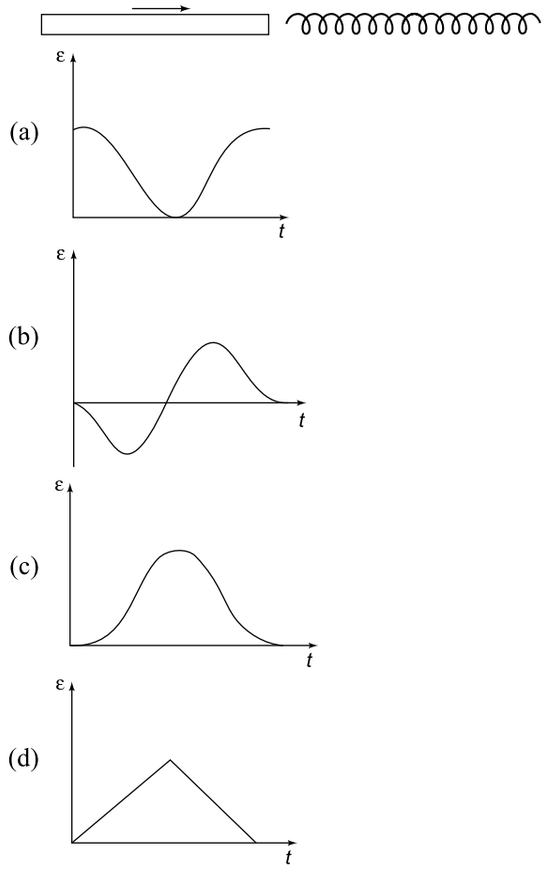


Fig. 25.60

IIT, 2004

58. A cylindrical conducting rod is kept with its axis along the x-axis. Also there exists a uniform magnetic field parallel to the x-axis. The current induced in the cylinder is

(a) clockwise as seen from the + x axis
(b) anticlockwise as seen from the + x axis
(c) Along the axis towards – x direction
(d) zero

IIT, 2005

59. Figure 25.61 shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments AB and CD. Then,

(a) $I_1 > I_2$
(b) $I_1 < I_2$
(c) I_1 is in the direction BA and I_2 is in the direction CD
(d) I_1 is in the direction AB and I_2 is in the direction DC

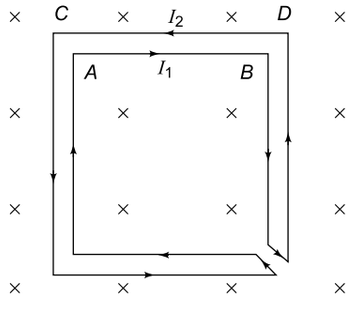


Fig 25.61

IIT, 2009

ANSWERS

1. (c)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (d)
7. (b)	**8.** (c)	**9.** (c)	**10.** (d)	**11.** (b)	**12.** (d)
13. (d)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (c)
19. (d)	**20.** (d)	**21.** (a)	**22.** (a)	**23.** (b)	**24.** (a)
25. (c)	**26.** (d)	**27.** (a)	**28.** (b)	**29.** (a)	**30.** (d)
31. (b)	**32.** (d)	**33.** (b)	**34.** (d)	**35.** (d)	**36.** (b)
37. (b)	**38.** (b)	**39.** (b)	**40.** (d)	**41.** (d)	**42.** (a)
43. (b)	**44.** (a)	**45.** (d)	**46.** (d)	**47.** (b)	**48.** (a)
49. (c)	**50.** (b)	**51.** (d)	**52.** (b)	**53.** (b)	**54.** (c)
55. (c)	**56.** (a)	**57.** (b)	**58.** (d)	**59.** (d)	

SOLUTIONS

1. If l is the side of the square loop, the magnitude of emf induced is $|e| = Blv$. If R is the resistance of the loop, the induced current is

$$i = \frac{Blv}{R}$$

The force required to pull the loop is $F = Bil = \dfrac{B^2 l^2 v}{R}$

$\therefore$ Power needed is $P = Fv = \dfrac{B^2 l^2 v^2}{R}$. So the correct choice is (c).

2. According to Lenz's law, the direction of the induced current in each coil will be opposite to the direction of the original current. Hence the current in each coil will decrease.

3. When the coil is within the region of the magnetic field, there is no change in the magnetic flux linked with it. Hence between t_1 and t_2 no current is induced in it and its acceleration will be g. But when it is entering or leaving the field, then according of Lenz's law, the induced current will oppose its motion. Therefore, its acceleration will be less than g. Hence the correct choice is (d).

4. Refer to Fig. 25.62. In time t, the loop moves to the right a distance $CD = vt$. Let $RD = b$.

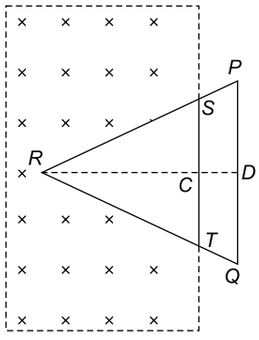


Fig. 25.62

Now

$$\frac{RC}{RD} = \frac{CT}{QD}$$

$$\Rightarrow \quad \frac{RD - CD}{RD} = \frac{CT}{QD}$$

$$\Rightarrow \quad \frac{b - vt}{b} = \frac{CT}{a/2}$$

$$\Rightarrow \quad CT = \frac{a\,(b - vt)}{2b}$$

$$\therefore \quad ST = 2CT = \frac{a}{b}\,(b - vt)$$

Induced emf $|e| = Bv \times ST = Bv\left(a - \dfrac{a}{b}vt\right)$

If R is the resistance of the loop, the induced current is

$$i = \frac{|e|}{R} = \frac{Bv}{R}\left(a - \frac{a}{b}vt\right)$$

$$\Rightarrow \quad i = -\frac{B v^2 a}{Rb}t + \frac{B v a}{R}$$

Thus the graph of i against t has a positive intercept $\left(= \dfrac{B v a}{R}\right)$ and a negative slope $\left(= -\dfrac{B v^2 a}{Rb}\right)$. Hence the correct choice is (b).

5. As the rod CD moves to the right, from Fleming's left hand rule, the free electrons in CD experience a force from C to D. Hence, the electrons move from C to D and, as a result, end C acquire a positive charge (due to loss of electrons) and end D acquires an equal negative charge (due to gain of electrons). Therefore, plate 1 acquires a positive charge and plate 2 acquires an equal negative charge.

6. Magnetic flux linked with the loop is

$$\phi = BA = B(a)^2 \text{ (Area of loop } A = a^2)$$

Now $B = \dfrac{\mu_0 I}{2\pi x}$

$$\therefore \quad \phi = \frac{\mu_0 I a^2}{2\pi x}$$

$$\therefore \text{ Induced emf is } e = -\frac{d\phi}{dt} = \frac{\mu_0 I a^2}{2\pi x^2}\cdot\frac{dx}{dt}$$

$$= \frac{\mu_0 I a^2 v}{2\pi x^2} \qquad \left(\because\ v = \frac{dx}{dt}\right)$$

Hence $e \propto \dfrac{1}{x^2}$, which is choice (d).

7. Since the inductors are connected in parallel, the potential difference across L_1 = potential difference across L_2 at any time t. Hence.

$$L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt}$$

$$\Rightarrow \quad L_1\, dI_1 = L_2\, dI_2$$

Integrating, we get $L_1 I_1 = L_2 I_2$

which gives $\dfrac{I_1}{I_2} = \dfrac{L_2}{L_1}$.

8. In the steady state, the resistance of the inductor is zero, i.e. it behaves as a short circuit. Hence the circuit can be drawn as follows. (Fig. 25.63)

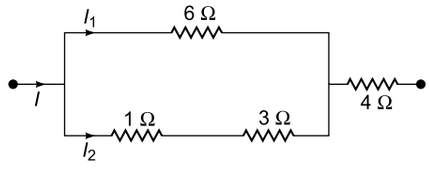


Fig. 25.63

$I_2 = 3$ A. Therefore, p.d. across $1 + 3 = 4\ \Omega$ resistance $= 3 \times 4 = 12$ V. Hence $I_1 = \dfrac{12}{6} = 2$ A. Therefore $I = I_1 + I_2 = 2 + 3 = 5$ A, which is the current flowing through 4 Ω resistance. So p.d. across 4 Ω resistance $= 5 \times 4 = 20$ V.

9. Peak value of current is

$$I_0 = \sqrt{(3)^2 + (4)^2} = 5 \text{ A}$$

$$\therefore \quad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ A}$$

10. Since the voltage lags current, $X_C > X_L$ or $\dfrac{1}{\omega C} > \omega L$

$$\text{Power factor } \cos\phi = \frac{R}{Z} = \left[\frac{R^2}{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}\right]^{1/2}$$

$$= \left[\frac{1}{1 + \left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)^2}\right]^{1/2}$$

It is clear that the power factor can be increased if the denominator is decreased, i.e. if R, C and L are increased. Hence the correct choice is (d).

11. With an a.c. source, the current is maximum at resonance, i.e. when $Z = R$, where R is the resistance of the coil. Given $V_{\text{rms}} = 12$ V. Hence

$$R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{12}{6} = 2\ \Omega$$

If the coil is connected to a d.c. source

$$I = \frac{E}{R + r}; \ r = \text{internal resistance of battery}$$

$$\Rightarrow \quad 2 = \frac{6}{2 + r} \quad \Rightarrow \quad r = 1.0\ \Omega$$

12. Given $V_R = V_L = V_C$. Therefore, $R = X_L = X_C$. When the capacitance is short circuited, $X_C = 0$ and the impedance is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + R^2} = \sqrt{2}\,R \quad (\because\ R = X_L)$$

The voltage of the a.c. source is given by ($\because\ V_L = V_C$)

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = V_R = 80 \text{ V}$$

$\therefore$ Current in the circuit is

$$I = \frac{V}{Z} = \frac{80}{\sqrt{2}\,R}$$

Hence $\quad V_L = IX_L = \dfrac{80}{\sqrt{2}\,R} \times R \qquad (\because\ X_L = R)$

$$= 40\sqrt{2} \text{ volt}$$

13. An a.c. ammeter reads the rms value of current. Given $V_0 = 200\sqrt{2}$ volts and $\omega = 100$ rad s^{-1}.

$$I_{\text{rms}} = V_{\text{rms}} \times \omega C$$

$$= \frac{200\sqrt{2}}{\sqrt{2}} \times 100 \times (2 \times 10^{-6}) = 40 \text{ mA}$$

14. The least capacitance is such that the energy stored in the capacitor is equal to that stored in the inductor, i.e.

$$\frac{1}{2} CV^2 = \frac{1}{2} LI^2 \text{ or } C = \frac{LI^2}{V^2}$$

$$= \frac{1 \times (2)^2}{(400)^2} = 25 \times 10^{-6} \text{ F} = 25\ \mu\text{F}$$

Hence the correct choice is (b).

15. Current in the transmission line is

$$I = \frac{\text{power}}{\text{voltage}} = \frac{25000}{250} = 100 \text{ A}$$

$\therefore$ Power loss $= I^2R = (100)^2 \times 1 = 10000$ W.

Therefore, the percentage of original power lost is

$$\frac{10000}{25000} \times 100 = 40\%$$

Hence the correct choice is (a).

16. The growth of current in LR circuit is given by

$$I = I_0\,(1 - e^{-Rt/L})$$

or $\quad I = I_0\,(1 - e^{-t/\tau})$

where $\quad \tau = L/R$ is the time constant. At $t = 0$, $I = 0$. At $t >> t$, $I = I_0$.

Hence the correct graph is (b).

17. The potential difference across the inductor is

$$V = -L\frac{dI}{dt} = -LI_0\frac{d}{dt}(1 - e^{-Rt/L})$$

$$= -LI_0\frac{R}{L}e^{-Rt/L}$$

or $|V| = V_0 e^{-t/\tau}$, where $V_0 = I_0 R$ is the initial voltage. Thus $V = V_0$ at $t = 0$ and then falls exponentially with time becoming zero at $t >> \tau$. Hence the correct graph is (b).

18. We know that $\tan\phi = \dfrac{\omega L}{R}$. Therefore

$$L = \frac{R\tan\phi}{\omega} = \frac{100 \times \tan 45°}{2\pi \times 1000}$$

$$\simeq 15.9 \times 10^{-3} \simeq 16 \text{ mH}$$

Hence the correct choice is (c).

19. The current in an LCR circuit is given by

$$I = \frac{V}{\left[R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right]^{1/2}}$$

where $\omega = 2\pi\nu$. Thus I increases with increase in ω upto a value of $\omega = \omega_c$ given by

$$\omega_c L = \frac{1}{\omega_c C} \text{ or } \omega_c = \frac{1}{\sqrt{LC}}$$

when I becomes maximum. At $\omega > \omega_c$, I decreases with increase in ω. Hence the correct graph is (d).

20. The time constant of the circuit is

$$\tau = \frac{L}{R} = \frac{100 \times 10^{-3}}{50} = 2 \times 10^{-3} \text{ s}$$

$$= 2 \text{ millisecond.}$$

Current at time t is given by

$$I = I_0 e^{-t/\tau}$$

where I_0 is the steady current. Therefore, time for I to fall to $I_0/2$ is

$$e^{-t/\tau} = \frac{1}{2} \text{ or } e^{t/\tau} = 2 \text{ or } t = \tau \ln(2).$$

Hence the correct choice is (d).

21. Frequency of oscillation of an LC circuit is

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

or $$C = \frac{1}{4\pi^2 \nu^2 L}$$

where $L = 400\ \mu\text{H} = 400 \times 10^{-6}$ H.

For $\nu = 500$ kHz $= 500 \times 10^3$ Hz, we have

$$C = \frac{1}{4\pi^2 \times \left(500 \times 10^3\right)^2 \times 400 \times 10^6}$$

$$= 250 \times 10^{-12} \text{ F} = 250 \text{ pF}$$

Similarly for $\nu = 1$ MHz $= 10^6$ Hz, we get $C = 2.5$pF.

Hence the range of the capacitor is 2.5 pF to 250 pF, which is choice (a).

22. The dimensions of RC are those of ohm × charge / voltage, i.e.

$$\frac{\text{voltage}}{\text{current}} \times \frac{\text{charge}}{\text{voltage}} = \frac{\text{charge}}{\text{charge/time}} = \text{time}$$

Hence the dimensions of $1/RC$ are those of frequency.

23. Given $V = 200\ \sqrt{2}\ \sin(100\ t)$. Comparing this equation with $V = V_0 \sin\omega t$, we have

$$V_0 = 200\ \sqrt{2}\ \text{V and } \omega = 100 \text{ rad s}^{-1}$$

The current in the capacitor is

$$I = \frac{V_{\text{rms}}}{Z_c} = V_{\text{rms}} \times \omega C \qquad \left(\because Z_c = \frac{1}{\omega C}\right)$$

$$= \frac{V_0}{\sqrt{2}} \times \omega C = \frac{200\sqrt{2}}{\sqrt{2}} \times 100 \times 1 \times 10^{-6}$$

$$= 20 \times 10^{-3} \text{ A} = 20 \text{ mA}$$

24. The circuit is equivalent to three inductances, each of value 3H, connected in parallel. The equivalent inductance L' is given by

$$\frac{1}{L'} = \frac{1}{L} + \frac{1}{L} + \frac{1}{L} = \frac{3}{L} = \frac{3}{3} = 1$$

or $L' = 1$ H. Hence the correct choice is (a).

25. Voltage amplitude $V_0 = 100$ V, current amplitude $I_0 = 100$ mA $= 100 \times 10^{-3}$ A and phase difference between I and V is $\phi = \dfrac{\pi}{3} = 60°$. Now power dissipated is given by

$$P = \frac{V_0 I_0}{2} \cos\phi$$

$$= \frac{100 \times 100 \times 10^{-3}}{2} \times \cos 60° = 2.5 \text{ W}$$

Hence the correct choice is (c).

26. If a coil is not moved in a magnetic field, the magnetic flux does not change. Hence no emf or current is induced in the coil.

27. When a current is passed through the helix, the neighbouring coils of the helix attract each other due to which it contracts. As a result the contact is broken and the coils will recover their original state under the influence of a restoring force. The contact is made again and the process continues. Thus the wire oscillates.

28. Since the voltage leads the current by a phase angle of 90°, the total potential difference across the circuit is

$$V = (V_R^2 + V_L^2)^{1/2}$$
$$= (20 \times 20 + 16 \times 16)^{1/2} = 25.6 \text{ V}$$

29. The phase angle between voltage V and current I is $\pi/2$. Therefore, power factor $\cos\phi = \cos(\pi/2) = 0$. Hence the power consumed is zero.

30. Since the magnetic field is constant in time and space and exists everywhere, there is no change in magnetic flux when the loop is moved in it. Hence no current is induced.

31. Induced charge $q = \dfrac{\text{change of flux}}{\text{resistance}} = \dfrac{\phi_f - \phi_i}{R}$.
But final area = 0, therefore, $\phi_f = 0$. Numerically, $\phi_i = BA$. Therefore, $q = BA/R$.

32. As the ring falls with a velocity v the decrease in area with time is

$$\frac{dA}{dt} = -(2R)v$$

$\therefore$ Induced emf,

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -B\frac{dA}{dt} = 2RBv.$$

From Lenz's law, the induced current in the ring must produce magnetic field in the upward direction. Hence Q is at higher potential.

33. Refer to Fig. 25.64. The magnetic field due a current I in the large loop at its centre is

B = 4 times that due to one side

$$= 4 \times \frac{\mu_0}{4\pi}\frac{I}{(L/2)}(\cos\alpha + \cos\beta)$$

$$= \frac{2\mu_0 I}{\pi L}(\cos 45° + \cos 45°)$$

$$= \frac{2\sqrt{2}\,\mu_0 I}{\pi L} \quad (\because \alpha = \beta = 45°)$$

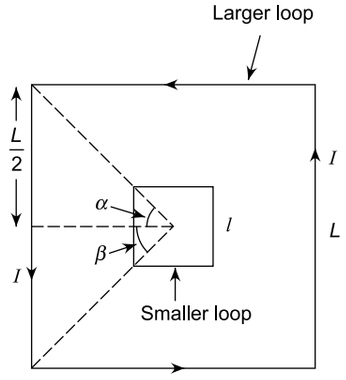


Fig. 25.64

The magnetic flux that links the larger loop with the smaller loop of side l ($l << L$) is

$$\phi_{12} = Bl^2 = \frac{2\sqrt{2}\,\mu_0 I l^2}{\pi L}$$

$\therefore$ Mutual inductance

$$M_{12} = \frac{\phi_{12}}{I} = \frac{2\sqrt{2}\,\mu_0}{\pi}\left(\frac{l^2}{L}\right)$$

i.e. $M_{12} \propto \dfrac{l^2}{L}$

34. Figure 25.65 shows the field lines (shown as broken curves) of the magnetic field due to the current flowing in the loop. It is clear from the figure that the magnetic flux in the x-y plane will be zero.

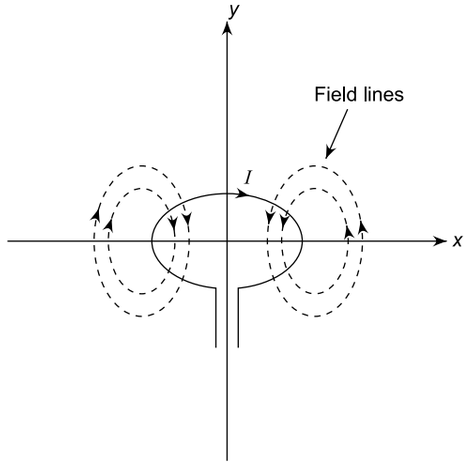


Fig. 25.65

35. The current in the inductor is given by

$$I = \frac{V}{R}(1 - e^{-t/\tau}), \text{ where } \tau = L/R.$$

Given, $\tau = \dfrac{L}{R} = \dfrac{8.4\,\text{mH}}{6\,\Omega}$

$= 1.4$ ms (millisecond)

$$\therefore \quad 1.0 = \frac{12}{6}(1 - e^{-t/1.4\text{ ms}})$$

or $e^{-t/1.4\text{ms}} = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

or $-\dfrac{t}{1.4\,\text{ms}} = \log_e\left(\dfrac{1}{2}\right) = -0.693$

or $t = 0.693 \times 1.4$ ms $= 0.97$ ms

Hence the correct choice is (d).

36. A time varying magnetic field produces an electric field. The magnitude of the electric field at a

distance r from the centre of a circular region of radius a where a time varying field B exists is given by

$$E = \frac{a^2}{2r}\ \frac{dB}{dt}$$

At $r = a$, $E = (a/2)\ dB/dt$, which the value of E at the edge of the circular region. For $r > a$, E decreases as $1/r$. Hence the correct choice is (b).

37. The induced voltage across the ends of the wire at an instant of time t is given by

$$V = V_0 \sin\ \omega t$$

per period of rotation. Here

$V_0 = AB\ \omega$ and

$A = \frac{1}{2}\ (\pi r^2)$ is the area of the semi-circular coil. Hence

$$V_0 = \frac{1}{2}(\pi\ r^2\ B\omega)$$

Power generated per period of rotation if

$$P = \frac{V^2}{R} = \frac{V_0^2 \sin^2 \omega t}{R}$$

$\therefore$ Mean power per period of rotation is

$$\langle P\rangle = \frac{V_0^2}{R}\ \langle \sin^2\ \omega t\rangle$$

$$= \frac{1}{2}\frac{V_0^2}{R} = \frac{1}{2R}\left(\frac{\pi\ r^2 B\omega}{2}\right)^2$$

$$= \frac{\left(\pi r^2 B\omega\right)^2}{8R}$$

Hence the correct choice is (b).

38. If L is the length of the conductor, it sweeps an area $A = \pi L^2$ in one rotation. Therefore, the change in magnetic flux $= BA = B(\pi L^2)$. Now, the time taken = time period $= \frac{2\pi}{\omega}$. Hence emf induced is

$$e = \frac{\text{change in flux}}{\text{time taken}}$$

$$= \frac{B\,(\pi L^2)}{2\pi/\omega} = \frac{1}{2}B\omega L^2$$

$$= \frac{1}{2} \times (0.2 \times 10^{-4}) \times 5 \times (1)^2$$

$$= 5 \times 10^{-5}\ \text{V} = 50 \times 10^{-6}\ \text{V} = 50\ \mu\text{V}$$

Hence the correct choice is (b).

39. We know that $Q = CV$ and $Q = Q_0 \cos\ \omega t$. Also $Q_0 = CV_0$.

$\therefore\quad \cos\ \omega t = \frac{Q}{Q_0} = \frac{V}{V_0} = \frac{6}{12} = \frac{1}{2}$ or $\omega t = \frac{\pi}{3}$.

Now ω is given by

$$\omega = \frac{1}{\sqrt{LC}} \qquad (1)$$

Given $L = 0.6 \times 10^{-3}$ H and $C = 2 \times 10^{-6}$ F. Using these values in Eq. (1) we get $\omega = \frac{10^5}{2\sqrt{3}}$ rad s^{-1}.

Now $\quad I = \frac{dQ}{dt} = \frac{d}{dt}\ (Q_0 \cos\ \omega t)$

$$= -\ Q_0\ \omega \sin\ \omega t$$

$\therefore\quad |I| = Q_0\ \omega \sin\ \omega t = CV_0\ \omega \sin\ \omega t$

$$= (2 \times 10^{-6}) \times 12 \times \frac{10^5}{2\sqrt{3}} \sin\left(\frac{\pi}{3}\right)$$

$$= 0.6\ \text{A}$$

Hence the correct choice is (b).

40. Induced emf $(e) = \frac{\text{magnetic field} \times \text{change in area}}{\text{time}}$

$$= \frac{B\Delta A}{t}$$

Since the circumference of the circular loop $= 2\pi r$, the side of the square loop $= \frac{2\pi r}{4} = \frac{\pi r}{2}$. Therefore,

$$\Delta A = \pi r^2 - \left(\frac{\pi r}{2}\right)^2 = \pi r^2\ \left(1 - \frac{\pi}{4}\right)$$

$\therefore\quad e = \frac{B\left(\pi r^2\right)}{t}\left(1 - \frac{\pi}{4}\right)$

Hence the correct choice is (d).

41. Velocity $v = \sqrt{2gh}$. Induced emf $e = Blv = Bl\sqrt{2gh}$. Therefore, the induced current in the loop is

$$I = \frac{Bl\sqrt{2gh}}{R}$$

$\therefore$ Force $\quad F = BIl = \frac{B^2 l^2 \sqrt{2gh}}{R}$

The loop will attain terminal velocity if this force equals mg, i.e. if

$$\frac{B^2 l^2 \sqrt{2gh}}{R} = mg$$

which gives $h = \dfrac{m^2 g R^2}{2B^4 l^4}$

Hence the correct choice is (d).

42. Magnetic field due to the larger coil at its centre is

$$B = \frac{\mu_0 I}{2r_1}$$

where I is the current in the larger coil. Flux through the inner coil is

$$\phi = B \times \pi r_2^2 = \frac{\mu_0 I}{2r_1} \times \pi r_2^2$$

But $\phi = MI$. Therefore

$$M = \frac{\mu_0 \pi r_2^2}{2r_1}$$

Hence the correct choice is (a).

43. The network $PQRS$ is a balanced Wheatstone's bridge. Hence the resistance of 3 Ω between P and R is ineffective. The net resistance of the network, therefore, is 3 Ω. Total resistance $R = 3\ \Omega + 1\ \Omega = 4\ \Omega$. Now, induced emf is $e = Blv = 2 \times 0.1 \times v = 0.2\ v$.

$\therefore$ Induced current $I = \dfrac{e}{R} = \dfrac{0.2v}{4}$.

Given $I = 1 \times 10^{-3}$ A

Hence $1 \times 10^{-3} = \dfrac{0.2v}{4}$

which gives $v = 2 \times 10^{-2}$ ms^{-1} = 2 cm s^{-1}, which is choice (b).

44. The mutual inductance between the two coils in orientation (A) is the maximum since the flux linkage in (A) is the maximum as shown in Fig. 25.66.

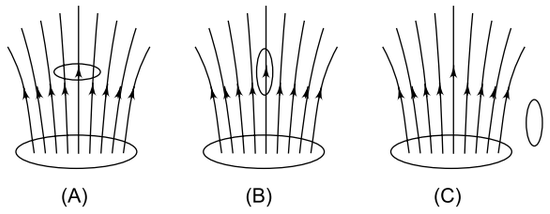


Fig. 25.66

45. Electric field will be induced in both AD and BC, since both are moving perpendicular to the direction of the magnetic field and the flux linked with them is changing with time. Hence the correct choice is (d).

46. Let the switch be closed at time $t = 0$. The current I_p flowing in P grows for a time, say t_0, after which it becomes steady. During this time the magnetic field (due to I_p) (from left to right) increases at the location of loop Q. According to Lenz's law, the induced current $(I_Q)_1$ should be such that it tries to decrease the magnetic field. Therefore, the magnetic field due to this current must be from right to left. Hence this induced current $(I_Q)_1$ should be anticlockwise (opposite to the direction of I_P). After the switch S is opened, the current I_P takes a finite time to decay to zero and the reverse of the above phenomenon is observed. Hence the induced current $(I_Q)_2$ should be clockwise. Thus the correct choice is (d).

47. The magnitude of the induced voltage is proportional to the rate of change of magnetic flux which, in turn, depends on the number of turns in the coil, i.e. $V \propto n$. The resistance of a wire is given by $R = \dfrac{\rho l}{\pi r^2}$ or $R \propto \dfrac{l}{r^2}$. Here ρ is the resistivity of the material of the wire.

$\therefore$ Power $P = \dfrac{V^2}{R} \propto \dfrac{n^2}{l/r^2}$ or $P \propto \dfrac{(nr)^2}{l}$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2 \times \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{l_1}{l_2}\right) \qquad (1)$$

Now, if a wire of length l_1 and radius r_1 is stretched to a length l_2 such that its radius reduced to r_2, then (since the mass of the wire remains constant)

$m = \pi r_1^2 l_1 d = \pi r_2^2 l_2 d$ (d is the density)

or $\dfrac{l_1}{l_2} = \left(\dfrac{r_2}{r_1}\right)^2$. Using this in Eq. (1), we get

$$\frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2 \times \left(\frac{r_2}{r_1}\right)^4$$

Given $\dfrac{n_2}{n_1} = 4$ and $\dfrac{r_2}{r_1} = \dfrac{1}{2}$. Using these values, we get

$$\frac{P_2}{P_1} = (4)^2 \times \left(\frac{1}{2}\right)^4 = 1,$$ which is choice (b).

48. Given $E = E_0 \sin(100\ t)$. Comparing this with $E = E_0 \sin \omega t$, we have $\omega = 100$ rad s^{-1}. It follows from the figure that the current leads the e.m.f. which is true only for R-C circuit, and not for R-L circuit. Hence the circuit does not contain an

inductor. Thus choices (c) and (d) are not possible. For *R*-*C* circuit, the phase difference between *E* and *I* is given by

$$\tan\,\phi = \frac{1}{\omega RC} \qquad \text{(i)}$$

Given $\phi = \pi/4$. Also $\omega = 100$ rad s^{-1}. Using these values in (i), we get

$$\tan\left(\frac{\pi}{4}\right) = \frac{1}{100\,RC} \quad \text{or} \quad RC = \frac{1}{100}$$

This relation between *R* and *C* is satisfied by choice (a) and not choice (b). Hence the correct choice is (a).

49. As the air plane is flying horizontally parallel to the earth's surface, the flux linked with it will be due to the vertical component B_V of the earth's field.
Now

$$B_V = B_H \tan\theta = 2\times10^{-5}\times\tan 60°$$

$$= 2\sqrt{3}\times10^{-5}\ \text{Wbm}^{-2}$$

$\therefore$ Induced emf is $|e| = B_V\,lv = 2\sqrt{3}\times10^{-5}\times 20\times 250 = \dfrac{\sqrt{3}}{10}$ V, which is choice (c).

50. The growth of current in an *LR* circuit is given by

$$I = I_0\,(I - e^{-Rt/L}) \qquad (1)$$

where I_0 is the maximum current. The energy stored at time *t* is

$$U = \frac{1}{2}LI^2$$

We are required to find the time at which the energy stored is one-forth the maximum value, i.e. when

$$U = \frac{U_0}{4} \text{ where}$$

$$U_0 = \frac{1}{2}LI_0^2$$

i.e. $\dfrac{1}{2}LI^2 = \dfrac{1}{4}\left(\dfrac{1}{2}LI_0^2\right)$ or $I = \dfrac{I_0}{2}$

Using this in Eq. (1), we get

$$\frac{I_0}{2} = I_0\,(1 - e^{-Rt/L}) \quad \text{or} \quad \frac{1}{2} = 1 - e^{-Rt/L}$$

or $e^{-Rt/L} = \dfrac{1}{2}$ or $-\dfrac{Rt}{L} = \log_e\left(\dfrac{1}{2}\right)$

or $t = \dfrac{L}{R}\log_e(2)$, which is choice (b).

51. When capacitance is removed, the circuit contains only inductance and resistance. Phase difference θ between the current and voltage is then given by

$$\tan\theta = \frac{\omega L}{R} \quad \text{or} \quad \omega L = R\tan\theta$$

$$= 100\tan 60°$$

When the circuit contains only capacitance and resistance, the phase difference between the voltage and current is given by

$$\tan\phi = \frac{1}{RC\omega}$$

$$\therefore \quad \frac{1}{C\omega} = R\tan\phi = 100\tan 60°$$

The impedance of the *LCR* circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{C\omega}\right)^2}$$

$$= \sqrt{R^2 + (100\tan 60° - 100\tan 60°)^2}$$

$$= R = 100\ \Omega$$

The current is given by

$I = \dfrac{V}{R} = \dfrac{200}{100} = 2$ A. Hence the correct choice is (d).

52. An emf is induced in the rod because it cuts through the lines of force of the magnetic field of the current carrying wire. Consider a small element of length *dr* of the rod at a distance *r* from the wire. The magnetic field due to a current *I* in the wire at a distance *r* from it is (Fig. 25.67)

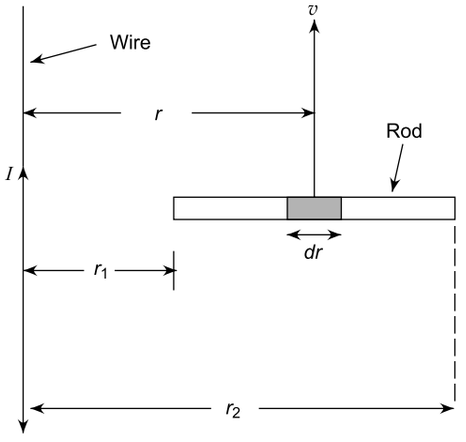


Fig. 25.67

$$B = \frac{\mu_0 I}{2\pi r}$$

The emf induced in the element of length dr is

$$de = Bvdr = \frac{\mu_0 I v}{2\pi}\ \frac{dr}{r}$$

$\therefore$ The emf induced in the whole rod is

$$e = \frac{\mu_0 I v}{2\pi}\ \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I v}{2\pi} \mid \log_e r \mid_{r_1}^{r_2}$$

or $$e = \frac{\mu_0 I v}{2\pi}\ \log_e \left(\frac{r_2}{r_1}\right)$$, which is choice (b)

53. $W = \frac{1}{2} L I_0^2$, where I_0 = peak value of I = 2 A. Thus

$$W = \frac{1}{2} \times 2.0 \times (2)^2 = 4 \text{ J}$$

Hence the current choice is (b).

54. $e = -M\frac{\Delta I}{\Delta t}$

or $$M = -\frac{e\Delta I}{\Delta I} = -\frac{40.000\,\text{V} \times (10 \times 10^{-6}\,\text{s})}{(-4-0)}$$

$$= 0.1 \text{ H}$$

The correct choice is (a).

55. Since the magnetic field varies with x, we find the flux by considering a small element of the loop of width dx and length a at a distance x from O, as shown in Fig. 25.68.

The total magnetic flux is

$$\phi = \int BdA = \int_0^b kx(adx)$$

$$= ka \int_0^b xdx = \frac{1}{2} kab^2$$

so the correct choice is (c).

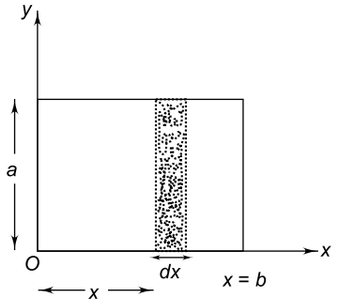


Fig. 25.68

56. $\frac{1}{2}LI_0^2 = \frac{1}{2}CV^2$ ($\because$ there is no loss of energy due to joule heating as $R = 0$). Hence

$$I_0 = V\sqrt{\frac{C}{L}} = 50 \times \sqrt{\frac{2\times10^{-6}}{5\times10^{-3}}} = 1 \text{ A}$$

which is choice (a).

57. As the bar magnet moves towards the coil, the induced emf $e = -d\phi/dt$ is negative and as it moves away, the induced emf is positive. Hence the correct choice is (b).

58. Since the magnetic field is constant, the rate of change of magnetic flux is zero. Hence the induced emf and current are zero. So the correct choice is (d).

59. From Lenz's law, the directions of currents I_1 and I_2 will be as shown in the figure of Question 59. So the correct choice is (d).

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. Choose the correct statements from the following.

(a) Induced current does not develop in a conductor when it is moved in a direction parallel to a magnetic field.

(b) Induced current does not develop in a conducting loop when it is moved in a direction perpendicular to an electric field.

(c) A constant induced emf is developed in a rectangular loop when it is moved with a constant velocity from a region of magnetic field out into a field-free region.

(d) A constant induced emf is developed in a circular loop when it moved with a constant velocity from a region of magnetic field out into a field-free region.

2. Choose the correct statements from the following.
- (a) A circuit consisting of an inductance is carrying a current. When the circuit is suddenly switched off, the direction of the induced current will be the same as that of the main current.
- (b) Sparking is observed in the switch when an electrical appliance is suddenly switched off because the induced emf suddenly falls to zero.
- (c) A coil is connected in series with a bulb and this combination is connected to a dc source. When an iron core is inserted in the coil, there will be no change in the brightness of the bulb.
- (d) A coil is connected in series with a bulb and this combination is connected to an ac source. When an iron core is inserted in the coil, the brightness of the bulb will increase.

3. Which of the following statements are correct?
- (a) A capacitor is connected in series with a bulb and this combination is connected to a variable voltage dc source. The brightness of the bulb will increase with increase in the voltage.
- (b) A capacitor is connected in series with a bulb. When this combination is connected to an ac source, the bulb does not glow.
- (c) a variable capacitor is connected in series with a bulb and this combination is connected to an ac source. The brightness of the bulb is reduced if the capacitance of the variable capacitor is decreased.
- (d) In a series ac circuit, the applied voltage is not equal to the algebraic sum of voltages across the different elements of the circuit.

4. Two circuits A and B are connected to identical dc sources each of emf 12 V. Circuit A has a self inductance $L_1 = 10$ H and circuit B has a self inductance $L_2 = 10$ mH. The total resistance of each circuit is 48 Ω. The ratio of
- (a) steady currents in circuits A and B is 1.
- (b) energy consumed in circuits A and B to build up the current to the steady value is 1.
- (c) energy consumed in circuits A and B to build up the current to the steady value is 1000.
- (d) power dissipated by circuits A and B after the steady state is reached is 1.

5. An alternating voltage (in volts) varies with time t (in seconds) as

$$V = 200 \sin (100\ \pi t)$$

- (a) The peak value of the voltage is 200 V.
- (b) The rms value of the voltage is 220 V.
- (c) The rms value of the voltage is $100\sqrt{2}$ V
- (d) The frequency of the voltage is 50 Hz.

6. A 50 Ω electric heater is connected to 100 V, 60 Hz ac supply.
- (a) The peak value of the voltage is 100 V.
- (b) The peak value of the current in the circuit is $2\sqrt{2}$ A.
- (c) The rms value of the voltage is 100 V.
- (d) The rms value of the current is 2 A.

7. Figure 25.69 shows a series LCR circuit connected to a variable frequency 200 V source. $L = 5$ H, $C = 80$ μF and $R = 40$ Ω.
- (a) The impedance of the circuit at resonance is 40 Ω.
- (b) The current amplitude at resonance is 5 A.
- (c) The rms potential drop across the inductor at resonance is 1250 V.
- (d) The rms potential drop across the resistor at resonance is 200 V.

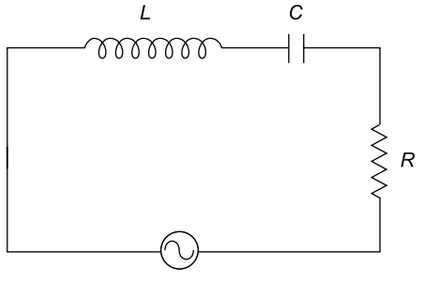


Fig. 25.69

8. L, C and R respectively represent inductance, capacitance and resistance. Which of the following combinations have the dimensions of frequency?

(a) $\frac{R}{L}$ (b) $\frac{1}{RC}$

(c) $\frac{R}{\sqrt{LC}}$ (d) $\frac{1}{\sqrt{LC}}$

IIT, 1984

9. The network shown in Fig. 25.70 is part of a circuit. The battery has negligible internal resistance.

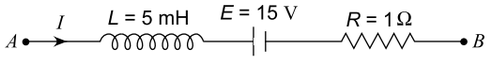


Fig. 25.70

At a certain instant the current $I = 5$A and is decreasing at a rate of 10^3 As^{-1}. At that instant, the potential difference

(a) across L is 5 mV.
(b) across L is 5 V.
(c) between points A and B is 15 V.
(d) between points A and B is 25 V.

IIT, 1997

10. In a coil of self inductance 10 mH, the current I (in ampere) varies with time t (in second) as

$$I = 2 + 8t$$

(a) The induced emf increases with time.
(b) The induced emf has a constant value of 80 mV.
(c) The power supplied to the inductor at $t = 1$ s is 0.8 W.
(d) The power supplied to the inductor at $t = 2$ s is 1.6 W.

11. A uniformly wound solenoid coil of self inductance L and resistance R is connected a battery of voltage V having negligible internal resistance. The time constant for the current in the circuit is τ and the steady current through the battery is I. The solenoid coil is now broken up into two indentical coils which are connected in parallel across the same battery. The time constant for the current in the circuit is now τ' and the steady current through the battery is I'. Then

(a) $\tau' = 2\tau$ (b) $\tau' = \tau$
(c) $I' = 2I$ (d) $I' = 4I$

IIT, 1989

12. A reatangular coil 20 cm × 10 cm having 500 turns rotates in a magnetic field of 5×10^{-3} T with a frequency of 1200 rev/min about an axis perpendicular to the field.

(a) The maximum value of the induced emf is $\frac{2\pi}{5}$ volt.
(b) The instantaneous emf when the plane of the coil is perpendicular to the field is zero.
(c) The instantaneous emf when the plane of the coil makes an angle of 60° with the field is $\frac{\pi}{5}$ volt.
(d) The instantaneous emf when the plane of the coil makes an angle of 30° with the field is $\frac{\pi}{10}$ volt.

13. In a transformer, the number of turns in the primary and secondary are 400 and 200 respectively. The power input to the primary is 10 kW at 200 V. The efficiency of the transformer is 90%.

(a) The output voltage is 100 V.
(b) The output power is 9 kW.
(c) The current in the primary is 50 A.
(d) The current in the secondary is 10 A.

14. Which of the following fields cannot possibly have a field line as shown in Fig. 25.71

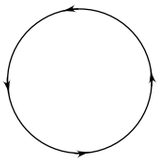


Fig. 25.71

(a) Electrostatic field
(b) Magnetostatic field
(c) Gravitational field
(d) Induced electric field

15. The resistance of the armature of a generator is 0.2 Ω. It yields an emf of 220 V in an open circuit and a potential difference of 210 V at full load. The current at full load is I and the power delivered to an external circuit is P. Then

(a) I = 50 A (b) I = 100 A
(c) P = 10 kW (d) P = 10.5 kW

16. Two parallel wires A_1L and B_1M placed at a distance w are connected by a resistor R and placed in a magnetic field B which is perpendicular to the plane containing the wires (Fig. 25.72). Another wire CD now connects the two wires perpendicularly and made to slide with velocity v. Neglect the resistance of all wires.

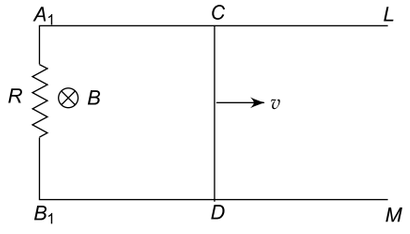


Fig. 25.72

(a) The magnitude of induced emf is $\frac{Bv^2}{R}$.
(b) The induced current is $\frac{Bwv}{R}$.
(c) The magnetic force on wire CD tends to move it away from resistor R.
(d) The work done per second needed to slide the wire CD is $(Bwv)^2/R$.

IIT, 2001

17. An LR series circuit consists of inductance $L = \frac{100}{\pi}$ mH and a resistance $R = 10$ Ω. A sinusoidal voltage $V = V_0 \sin(2\pi\nu t)$ is applied. It is given that $V_{rms} = 200$ V and $\nu = 50$ Hz.

IIT, 2004

(a) The peak value of the current in the steady state is 20 A.

(b) The phase difference between the current and the voltage is $\pi/2$.

(c) At time $t = 0$, the current in the circuit is $-10\sqrt{2}$ A.

(d) The current in the circuit is zero at $t = \frac{T}{8}$, $\frac{5T}{8}$, ..., where $T = 0.02$ s.

18. A metal rod PQ moves at a constant velocity v in a direction perpendicular to its length. A constant uniform magnetic field $\vec{B}$ exists in a direction perpendicular to the rod as well as its velocity as shown in Fig. 25.73. Select the correct statement(s) from the following.

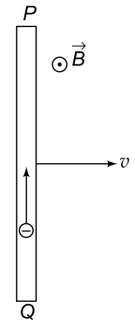


Fig. 25.73

(a) There is an electric field in the rod

(b) The electric potential is the same at every point on the rod

(c) There is no induced current in the rod

(d) The induced current flows from P to Q.

IIT, 1998

19. The SI unit of inductance (which is henry) can also be written as

(a) weber/ampere (b) volt second/ampere

(c) joule/(ampere)^2 (d) ohm second

IIT, 1998

20. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in Fig. 25.74

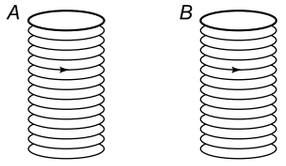


Fig. 25.74

When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are)

(a) $\rho_A > \rho_B$ and $m_A = m_B$

(b) $\rho_A < \rho_B$ and $m_A = m_B$

(c) $\rho_A > \rho_B$ and $m_A > m_B$

(d) $\rho_A < \rho_B$ and $m_A < m_B$

IIT, 2009

21. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$

(c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

IIT, 2011

ANSWERS AND SOLUTIONS

1. Statement (a) is correct. Let **v** be the velocity of the conductor in a magnetic field **B.** Since the free electrons in the conductor are moving with it, the magnetic force on these electrons is $\mathbf{F} = e(\mathbf{v} \times \mathbf{B})$. Since the conductor is moved parallel to the field, **v** is parallel to **B** and $\mathbf{v} \times \mathbf{B}$ will be zero. Thus no force will be acting on the free electrons of the conductor. Consequently no induced emf is developed across the ends of the conductor. Therefore, no current will be developed in a conductor if it is moved parallel to a magnetic field.

Statement (b) is also correct. No current is induced in a loop if it is moved in any direction in an electric field. A current is induced only if magnetic flux linked with the coil changes.

Statement (c) is correct but statement (d) is incorrect. The induced emf will remain constant in the case of the rectangular loop. The reason is that the rate of change of area is constant. But, in the case of the circular loop, the rate of change of area keeps varying as the loop is moving towards the field-free region.

2. Statement (a) is correct. This follows from Lenz's law. Statement (b) is incorrect. The sparking is caused because a large induced emf is developed at the breaking of the circuit. Statement (c) is correct. A coil offers no reactance to direct currents, i.e. inductance plays no role in dc circuits. Hence there will be no change in the brightness of the bulb when an iron core is inserted in the coil.

Statement (d) is incorrect. The coil will offer reactance ωL to alternating current. When an iron core is inserted in the coil the inductance L increases due to increase in flux, hence the reactance of the coil increases, causing a decrease in current in the circuit. As a result the brightness of the bulb will be reduced.

3. Statement (a) is incorrect. Since the capacitor offers infinite resistance to direct current, the bulb will not glow at all. Statement (b) is also incorrect. The capacitor will offer finite reactance to alternat-

ing current. Hence a current will flow in the circuit and the bulb will glow. Statement (c) is correct. The reactance of a capacitor is $1/\omega C$. Hence if C is decreases, the reactance will increase and as a result the current is decreased. Hence the brightness of the bulb is reduced.

Statement (d) is also correct, because the voltages across the different elements are not in phase. Therefore, they cannot be added algebraically; they are added vectorially.

4. $E = 12$ V, $L_1 = 10$ H, $L_2 = 10 \times 10^{-3}$ H and $R = 48\ \Omega$. Steady state current is $I_0 = E/R$ and is independent of the inductance. Hence, the value of the steady state current is the same for both circuits.

$$I_0 = \frac{E}{R} = \frac{12}{48} = 0.25 \text{ A}$$

The energy consumed by the circuit to build up the current I_0 is

$$E = \frac{1}{2} LI_0^2$$

For circuit 1,

$$E_1 = \frac{1}{2} L_1 I_0^2 = \frac{1}{2} \times 10 \times (0.25)^2 = 3.125 \times 10^{-1} \text{ J}$$

For circuit 2,

$$E_2 = \frac{1}{2} L_2 I_0^2 = \frac{1}{2} \times 10 \times 10^{-3} \times (0.25)^2 = 3.125 \times 10^{-4} \text{ J}$$

$$\therefore \quad \frac{E_1}{E_2} = 1000$$

Power dissipated at current I_0 is $I_0^2 R$. Since I_0 and R are the same for both the circuits, they dissipate the same power which is

$$P = I_0^2 R = (0.25)^2 \times 48 = 3.0 \text{ W}$$

So the correct choices are (a), (c) and (d).

5. Comparing the equation $V = 200 \sin (100\ \pi t)$ with $V = V_0 \sin \omega t$, we find that $V_0 = 200$ V and $\omega = 100\ \pi$ rad s^{-1} or $2\ \pi \nu = 100\ \pi$ or $\nu = 50$ Hz.

Also $V_{\text{rms}} = \dfrac{V_0}{\sqrt{2}} = \dfrac{200}{\sqrt{2}} = 100\sqrt{2}$ V.

Hence the correct choices are (a), (c) and (d).

6. $V_{\text{rms}} = 100$ V. Peak value of voltage $= 100\sqrt{2}$ V. Peak value of current $= \dfrac{100\sqrt{2}}{50} = \sqrt{2}$ A.

$$I_{\text{rms}} = \frac{2\sqrt{2}}{\sqrt{2}} = 2 \text{ A}.$$

Hence the correct choices are (b), (c) and (d).

7. The resonant angular frequency is

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{(5.0 \times 80 \times 10^{-6})^{1/2}} = 50 \text{ rad s}^{-1}$$

Therefore, the resonant frequency is

$$\nu_r = \frac{\omega_r}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi} \text{ Hz}$$

The impedance is given by

$$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right]^{1/2}$$

When $\omega = \omega_r = 1/\sqrt{LC}$ (i.e. at resonance), $\omega L = 1/\omega C$, and therefore

$$Z = R = 40\ \Omega$$

Current amplitude at resonance is

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{R} = \frac{\sqrt{2} V_{\text{rms}}}{R} = \frac{\sqrt{2} \times 200}{40} = 5\sqrt{2} \text{ A}.$$

The rms current in the circuit is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{200}{40} = 5 \text{ A}$$

$\therefore$ The rms potential drop across L is

$$= I_{\text{rms}} \times \omega_r \times L = 5 \times 50 \times 5$$

$$= 1250 \text{ V} = 1.25 \text{ kV}$$

The rms potential drop across C is

$$= I_{\text{rms}} \times \frac{1}{\omega_r C}$$

$$= 5 \times \frac{1}{50 \times 80 \times 10^{-6}} = 1.25 \text{ kV}$$

The rms potential drop across R is

$$= I_{\text{rms}} \times R = 5 \times 40 = 200 \text{ V}$$

Hence the correct choices are (a), (c) and (d).

8. The correct choices are (a), (b) and (d). The dimensions of ωL and $\dfrac{1}{\omega C}$ are the same as those of resistance, where $\omega = 2\pi\nu$.

9. Since inside the cell, the current is taken to flow from the negative to the positive terminal, we have

$$V_A - IR + E - L \frac{dI}{dt} = V_B$$

or $V_B - V_A = -IR + E - L\dfrac{dI}{dt}$

Since I is decreasing with t, $\dfrac{dI}{dt}$ is negative. Hence

$$V_B - V_A = -5 \times 1 + 15 - (5 \times 10^{-3}) \times (-10^3)$$
$$= -5 + 15 + 5 = 15 \text{ V}$$

So the correct choices are (b) and (c).

10. $|e| = L\dfrac{dI}{dt}$

$$= (10 \times 10^{-3}) \times \frac{d}{dt}(2 + 8t) = 80 \times 10^{-3} = 80 \text{ mV}$$

Power $P = eI = (80 \times 10^{-3}) \times (2 + 8t)$

Power at $t = 1$ s is $(80 \times 10^{-3}) \times (2 + 8) = 0.8$ W

Power at $t = 2$ s is $(80 \times 10^{-3}) \times (2 + 16) = 1.44$ W.

So the correct choices are (b) and (c).

11. Time constant $\tau = \dfrac{L}{R}$. Steady current $I = \dfrac{V}{R}$. If the solenoid is broken up into two identical parts, the self inductance of each part = $L/2$ and resistance of each part = $R/2$. When the two parts are connected in parallel, the self inductance and resistance of the combination become

$$L' = \frac{L}{4} \quad \text{and} \quad R' = \frac{R}{4}$$

The new time constant $\tau' = \dfrac{L'}{R'} = \dfrac{L/4}{R/4} = \dfrac{L}{R} = \tau.$

The steady current $I' = \dfrac{V}{R'} = \dfrac{4V}{R} = 4I.$

So the correct choices are (b) and (d).

12. $A = 20 \text{ cm} \times 10 \text{ cm} = 200 \text{ cm}^2 = 200 \times 10^{-4} \text{ m}^2$. $N = 100$, $B = 5 \times 10^{-3}$ T and frequency $\nu = \dfrac{1200}{60}$ = 20 rev/sec. Hence $\omega = 2\pi \times 20 = 40\,\pi$ rad s^{-1}. The induced emf is given by

$$e = e_0 \sin\theta$$

where $E_0 = NA\omega B$ and θ = angle which the normal to the plane of the coil subtends with the direction of the magnetic field. The correct choices are (a), (b) and (c).

13. Output voltage $e_s = \dfrac{N_s}{N_p} \times e_p = \dfrac{2000}{400} \times 200$

$$= 1000 \text{ V}$$

Output power = 90% of 10 kW = 9 kW

Current in primary $I_p = \dfrac{\text{Input power}}{e_p} = \dfrac{10{,}000 \text{ W}}{200 \text{ V}}$

$$= 50 \text{ A}$$

Current in secondary $I_s = \dfrac{\text{Output power}}{e_s}$

$$= \frac{9000 \text{ W}}{1000 \text{ V}} = 9 \text{ A}$$

Hence the correct choices are (a), (b) and (c).

14. The field line of an electrostatic field and of a gravitational field cannot originate and terminate at the same point. It is not a closed loop. The line of force of a magnetic field is a closed loop. The induced electric field is due to a magnetic field. Hence the correct choices are (a) and (c).

15. Resistance of armature = 0.2 Ω. Potential difference in open circuit = 220 V. Potential difference at full load = 210 V. Current in the circuit is

$$I = \frac{220 - 210}{0.2} = 50 \text{ A}$$

Power delivered = 210 × 50 = 10.5 kW. Hence the correct choices are (a) and (d).

16. When wire CD is made to slide on wires A_1L and B_1M, the flux linked with the circuit changes with time and hence an emf is induced in the circuit, which is given by

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = B\frac{dA}{dt}$$

If wire CD moves a distance dx is time dt, then $A = wdx$ (here $w = CD$) and

$$|e| = B\frac{d}{dt}(wdx) = Bw\frac{dx}{dt} = Bwv$$

The induced current is

$$I = \frac{e}{R} = \frac{Bwv}{R}$$

This current is caused by the motion of wire CD. From Lenz's law, the current I opposes the motion of wire CD. Therefore, work has to be done to slide the wire CD. Now, the magnetic force on wire CD (of length w) is

$$F = BIw = B\left(\frac{Bwv}{R}\right)w = \frac{B^2w^2v}{R} \qquad (1)$$

Work done is sliding wire CD through a small distance dx in time dt is

$$dW = Fdx$$

Therefore, the work done per second is

$$P = \frac{dW}{dt} = F\,\frac{dx}{dt} = Fv$$

Using (1), we get

$$P = \frac{B^2 w^2 v^2}{R}$$

So the correct choices are (b) and (d).

17. Inductive reactance $X_L = \omega L = 2\pi\nu L$

$= 2\pi \times 50 \times \frac{100}{\pi} \times 10^{-3} = 10\ \Omega$. Therefore, impedance is

$$Z = (X_L^2 + R^2)^{1/2} = (10^2 + 10^2)^{1/2} = 10\sqrt{2}\ \ \Omega$$

The amplitude of the current in the steady state is

$$I_0 = \frac{V_0}{Z} = \frac{V_{\text{rms}} \times \sqrt{2}}{Z} = \frac{200 \times \sqrt{2}}{10\sqrt{2}} = 20\ \text{A}$$

The phase difference between the current and the voltage is

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{10}{10}\right) = \tan^{-1} \qquad (1)$$

which gives $\phi = \frac{\pi}{4}$.

In an LR circuit, the current lags behind the voltage by a phase angle ϕ. In the given circuit $V = V_0 \sin(2\pi\nu t)$, $I_0 = 20$ A and $\phi = \pi/4$. Hence the current I varies with time t as

$$I = I_0 \sin\left(2\pi\nu t - \frac{\pi}{4}\right)$$

or $$I = 20 \sin\left(\frac{2\pi t}{T} - \frac{\pi}{4}\right) \text{ ampere} \qquad (1)$$

where $T = \frac{1}{\nu} = \frac{1}{50} = 0.02$ s.

At $t = 0$, $I = 20 \sin\left(-\frac{\pi}{4}\right) = -10\sqrt{2}$ A.

From Eq. (1) it follows that $I = 0$ at $t = \frac{T}{8}, \frac{5T}{8}, \cdots$ etc. Hence the correct choices are (a), (c) and (d).

18. From Fleming's left hand rule, the free electrons experience a force from Q to P. As a result a current flows from P to Q. Also end Q acquires a positive charge (due to deficiency of electrons) and end P acquires a negative charge (due to gain of electrons). Hence an electric field exists in the rod in the direction Q to P. Thus the correct choices are (a) and (d).

19. From $\phi = LI$, the SI unit of L is weber/ampere.

From $|e| = L\frac{dI}{dt}$, the SI unit of L is volt second/ampere.

From $U = \frac{1}{2} LI^2$, the SI unit of L is joule/(ampere)^2

Since ohm = volt/ampere, volt second/ampere = ohm second. Hence all the four choices are correct.

20. Since the rings are identical and the solenoids are identical, magnetic field $B = \mu_0 nI$ and hence magnetic flux $\phi = B \times A$ (here A = area of the ring) is the same for both the rings. Hence the magnitude of induced emf (e) is the same. Since $h_A > h_B$, the force exerted on ring A is greater than that on B. Hence, current induced in A is greater than that in B, i.e.

$$I_A > I_B \Rightarrow \frac{e}{R_A} > \frac{e}{R_B} \Rightarrow R_B > R_A$$

Now $R = \frac{\rho A}{l}$. Hence $\rho_A < \rho_B$. Mass m_A can be less than, equal to or greater than m_B. Hence the correct choices are (b) and (d).

21. *Case A:* $$I_{\text{R}}^{\text{A}} = \frac{V}{Z_A} = \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \qquad \text{(i)}$$

$$V_{\text{C}}^{\text{A}} = \frac{I_{\text{R}}^{\text{A}}}{\omega \text{C}} = \frac{V}{\sqrt{(R\omega C)^2} + \text{I}} \qquad \text{(ii)}$$

Case B: $C_B = KC = 4C$. Hence

$$I_{\text{R}}^{\text{B}} = \frac{V}{\sqrt{R^2 + \left(\frac{1}{4\omega C}\right)^2}} \qquad \text{(iii)}$$

$$V_{\text{C}}^{\text{B}} = \frac{I_{\text{R}}^{\text{B}}}{4\omega C} = \frac{V}{\sqrt{(4R\omega C)^2} + 1} \qquad \text{(iv)}$$

From Eqs. (i) to (iv) it follows that

$$I_{\text{R}}^{\text{A}} < I_{\text{R}}^{\text{B}} \text{ and } V_{\text{C}}^{\text{A}} > V_{\text{C}}^{\text{B}}$$

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 5 are based on the following passage

Passage I

The Alternating Current Genetator

The a.c. generator which is one of the most important applications of the phenomenon of electromagnetic induction converts mechanical energy into electrical energy. A rectangular coil consisting of a large number of turns of copper wire wound over a soft iron core is rotated between the pole pieces of a permanent strong magnet. The magnetic flux through the coil changes continuously with time, thus producing induced emf given by

$$E = E_0 \sin \omega t$$

When a load resistor R is connected across the terminals, a current I flows through the circuit.

$$I = \frac{E}{R} = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t$$

where $I_0 = E_0/R$. Such a current is called a.c. or alternating current.

1. In an a.c. generator, a coil of area A and having N turns rotates in a magnetic field B. The magnetic flux through the coil is

(a) maximum equal to NAB when the plane of the coil is perpendicular to the magnetic field.

(b) zero when the plane of the coil is parallel to the field.

(c) $\frac{1}{2}$ NAB when the plane of the coil makes an angle of 60° with the field.

(d) $\frac{1}{4}$ NAB when the plane of the coil makes an angle of 30° with the field.

2. In an a.c. generator, initially (i.e. at $t = 0$) the plane of the coil is normal to the magnetic field. Which graph shown in Fig. 25.75 represents the variation of induced emf E with time t?

3. In an a.c. generator, the peak value of the induced emf depends upon the

(a) frequency of rotation of the coil

(b) area of the coil

(c) number of turns in the coil

(d) strength of the magnetic field.

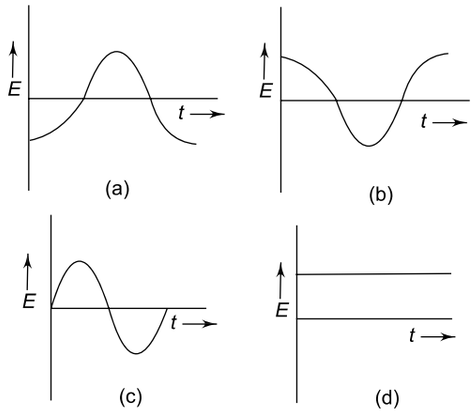


Fig. 25.75

4. In an a.c. generator

(a) the coil is wound over a soft iron core in order to increase the flux.

(b) an electromagnet fed with an alternating current is used.

(c) the output is always taken across a load resistor.

(d) the mechanical energy of the rotating coil is converted into electrical energy.

5. The emf of an a.c. generator is given by

$$E = 100 \sin \left(100\pi t + \frac{\pi}{3}\right)$$

where E is in volts and t in seconds.

(a) The peak value of the emf is $100\sqrt{2}$ volts.

(b) The frequecy of rotation of the armature is 50 Hz.

(c) At start (i.e. at $t = 0$), the plane of the armature makes an angle of 60° with the magnetic field.

(d) At start, the plane of the coil is perpendicular to the field.

SOLUTION

1. The correct choices are (a), (b) and (c). The magnetic flux is given by

$$\Phi = NAB \cos \theta$$

where θ is the angle which the normal to the plane of the coil subtends with the magnetic field.

2. When $\theta = 0$, i.e. when the plane of the coil is perpendicular to the magnetic field, the flux is maximum but induced emf $E = 0$. Since $\theta = \omega t$, $E = 0$ at $t = 0$. Hence the correct graph is (c).

3. All four choices are correct.

4. The correct choices are (a), (c) and (d). A permanent magnet is used.

5. Comparing the given expression with

$$E = E_0 \sin(\omega t + \theta)$$

We find that, peak value of $E = E_0 = 100$ volts. The angular frequency $\omega = 100\ \pi$ or $2\pi\nu = 100\pi$ or $\nu = 50$ Hz. At $t = 0$, $E = E_0 \sin\left(\frac{\pi}{3}\right)$, i.e. $\theta = \frac{\pi}{3} = 60°$, i.e. the normal to the plane of the armature makes an angle of 60° with the magnetic field. Hence the correct choice is (b).

Questions 6 to 8 are based on the following passage

Passage II

Two long parallel horizontal rails, distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see Fig. 25.76). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that as the rod moves, constant current flows through R.

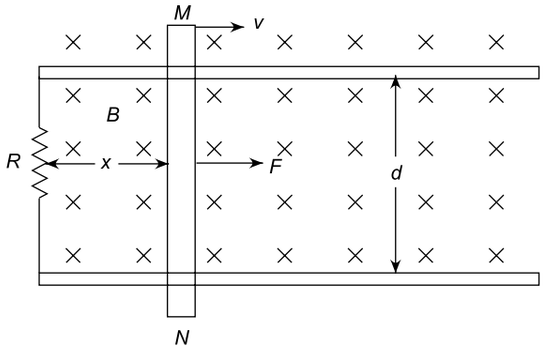


Fig. 25.76

IIT, 1988

6. The magnitude of the emf induced in the loop is

(a) $Bvd\left(\frac{2\lambda x}{R}\right)$ (b) $Bvd\left(\frac{R}{2\lambda x}\right)$

(c) Bvd (d) $\frac{1}{2}Bvd$

7. The current in the loop is

(a) $\frac{Bvd}{R}$ (b) $\frac{Bvd}{2\lambda x}$

(c) $\frac{2Bvd}{(R+2\lambda x)}$ (d) $\frac{Bvd}{(R+2\lambda x)}$

8. The velocity of the rod is

(a) $\frac{B^2d^2}{2\lambda m}\left(1+\frac{2\lambda x}{R}\right)$

(b) $\frac{B^2d^2}{R}\left(1-\frac{R}{2\lambda x}\right)$

(c) $\frac{B^2d^2}{2\lambda m}\log_e\left(1-\frac{R}{2\lambda x}\right)$

(d) $\frac{B^2d^2}{2\lambda m}\log_e\left(1+\frac{2\lambda x}{R}\right)$

SOLUTION

6. Let the distance from R to MN be x. Then the area of the loop between MN and R is xd and the magnetic flux linked with the loop is Bxd. As the rod moves, the emf induced in the loop is given by

$$|e| = \frac{d}{dt}(Bxd) = Bd\,\frac{dx}{dt} = Bvd$$

where v = velocity of MN.
So the correct choice is (c).

7. The total resistance of the loop between R and MN is $R + 2\ \lambda x$. The current in the loop is given by

$$I = \frac{|e|}{R+2\lambda x} = \frac{Bvd}{R+2\lambda x}$$

The correct choice is (d).

8. Force acting on the rod,

$$F = IBd = \frac{B^2d^2v}{R+2\lambda x}$$

$$\therefore \quad m\frac{dv}{dt} = \frac{B^2d^2}{R+2\lambda x}\cdot\frac{dx}{dt}$$

or $$dv = \frac{B^2d^2}{m} \times \frac{dx}{(R+2\lambda x)}$$

Integrating, we have

$$\int_0^v dv = \frac{B^2d^2}{m}\int_0^x \frac{dx}{(R+2\lambda x)}$$

or $$v = \frac{B^2d^2}{2\lambda m}\log_e\left(\frac{R+2\lambda x}{R}\right)$$

Hence the correct choice is (d).

Questions 9 to 11 are based on the following passage

Passage III

A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails without friction. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string, hangs vertically. A constant magnetic field B exists perpendicular to the table. The system is released from rest. (Fig. 25.77)

IIT, 1997

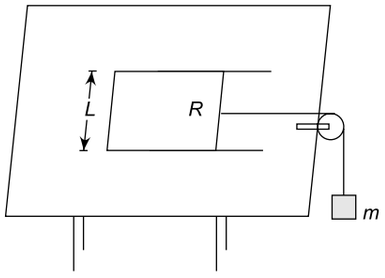


Fig. 25.77

9. The acceleration of the mass m moving in the downward direction is

(a) g (b) $\dfrac{B^2L^2v}{mR}$

(c) $\left(g-\dfrac{B^2L^2v}{mR}\right)$ (d) $\left(g+\dfrac{B^2L^2v}{mR}\right)$

10. The terminal velocity acquired by the rod is

(a) g (b) $\sqrt{gR}$

(c) $\dfrac{\sqrt{mgR}}{BL}$ (d) $\dfrac{mgR}{B^2L^2}$

11. The acceleration of mass m when the velocity of the rod is half the terminal velocity is

(a) g (b) $\dfrac{g}{2}$

(c) $\dfrac{g}{3}$ (d) $\dfrac{g}{4}$

SOLUTION

9. Refer to Fig. 25.78. Let v be the velocity of the rod along the positive x-direction at an instant of time and let the magnetic field B act perpendicular to the table along the positive y-direction. The emf induced in the rod is $e = BLv$. Therefore, the induced current is

$$I = \frac{e}{R} = \frac{BLv}{R} \tag{1}$$

The rod of length L carrying a current I in magnetic field will experience a force

$$F = BIL \tag{2}$$

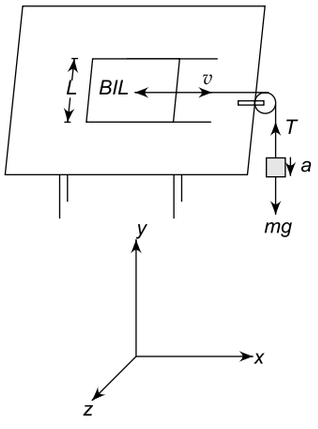


Fig. 25.78

along the negative x-direction. Since the rod is massless, this force will also be equal to the tension T in the string acting along the positive x-direction, i.e. $T = F = BIL$.

Let a be the acceleration of mass m moving in the downward direction, then

ma = net force acting on $m = mg - T = mg - F$

or $$a = g - \frac{F}{m} \tag{3}$$

Using (1) and (2) in (3), we have

$$a = g - \frac{BIL}{m} = g - \frac{B \times BL^2v}{mR}$$

$$= g - \frac{B^2L^2v}{mR} \tag{4}$$

So the correct choice is (c).

10. The rod will acquire terminal velocity v_t when $a = 0$. Putting $a = 0$ and $v = v_t$ in Eq. (4) we have

$$0 = g - \frac{B^2L^2v_t}{mR} \text{ or } v_t = \frac{mgR}{B^2L^2}$$

The correct choice is (d).

11. When the velocity of the rod is half the terminal velocity, i.e. when

$$v = \frac{v_t}{2} = \frac{mgR}{2B^2L^2},$$

then from Eq. (4), we have

$$a = g - \frac{B^2L^2v_t/2}{mR}$$

$$= g - \frac{B^2L^2}{2mR} \times \frac{mgR}{B^2L^2} = g - \frac{g}{2} = \frac{g}{2}$$

Thus the correct choice is (b).

Questions 12 to 15 are based on the following passage

Passage IV

An infinitesimally small bar magnet of dipole moment **M** is pointing and moving with a speed v in the x-direction. A small closed circular conducting loop of radius a and negligible self inductance lies in the y-z plane with its centre at $x = 0$, and its axis coinciding with the x-axis.

IIT, 1998

12. The magnitude of magnetic field at a distance x on the axis of the short bar manget is

(a) $\frac{\mu_0 M}{2\pi x}$ (b) $\frac{\mu_0 M}{2\pi x^2}$

(c) $\frac{\mu_0 M}{2\pi x^3}$ (d) $\frac{\mu_0 M}{2\pi x^4}$

13. If $x = 2\,a$, the magnetic flux through the loop is

(a) $\mu_0 M$ (b) $\frac{\mu_0 M}{2}$

(c) $\frac{\mu_0 M}{4a}$ (d) $\frac{\mu_0 M}{16a}$

14. If $x = 2\,a$, the emf induced in the loop is

(a) $\frac{3}{16}\frac{\mu_0 Mv}{a^2}$ (b) $\frac{3}{32}\frac{\mu_0 Mv}{a^2}$

(c) $\frac{1}{8}\frac{\mu_0 Mv}{a^2}$ (d) $\frac{1}{16}\frac{\mu_0 Mv}{a^2}$

15. If $x = 2\,a$, the magnetic moment of the loop is

(a) $\frac{3\pi\mu_0 Mv}{32R}$ (b) $\frac{3\pi\mu_0 Mv}{8R}$

(c) $\frac{\pi\mu_0 Mv}{2R}$ (d) $\frac{3\pi\mu_0 Mv}{4R}$

SOLUTION

12. Refer to Fig. 25.79. The magnetic field at a distance x on the axis of a magnet of length $2l$ and dipole moment M is given by

$$B = \frac{\mu_0}{2\pi} \cdot \frac{Mx}{\left(x^2 - l^2\right)^2}$$

Since $x >> l$, we have

$$B = \frac{\mu_0 M}{2\pi x^3}$$

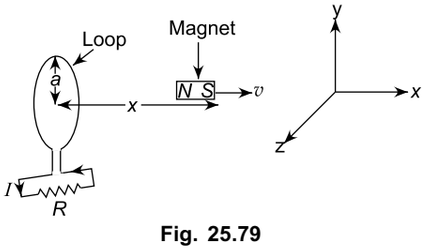


Fig. 25.79

So the correct choice is (c).

13. Due to B, the flux through the loop is

$$\phi = BA = B(\pi a^2) = \frac{\mu_0 M}{2\pi x^3} \times \pi a^2 = \frac{\mu_0 Ma^2}{2x^3}$$

If $x = 2\,a$, we find that the correct choice is (d).

14. Induced emf in the loop is

$$e = -\frac{d\phi}{dt} = -\frac{dx}{dt}\frac{d\phi}{dx} = -v\frac{d\phi}{dx}$$

$$= -\frac{\mu_0 Ma^2 v}{2}\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{3}{2}\frac{\mu_0 Ma^2 v}{x^4}$$

Putting $x = 2\,a$, we get $e = \frac{3\mu_0}{32}\frac{Mv}{a^2}$, which is choice (b).

15. Induced current in the loop is

$$I = \frac{e}{R} = \frac{3}{2}\frac{\mu_0 Ma^2 v}{x^4 R}$$

Magnetic moment of the loop is

$M_0 = I \times$ area enclosed by the loop $= I(\pi a^2)$

$$= \frac{3\pi}{2}\frac{\mu_0 Ma^4 v}{x^4 R}$$

Putting $x = 2\,a$, we find that the correct choice is (a).

Questions 16 to 19 are based on the following passage

Passage V

Two co-axial circular coils of radii R and $r = R/100$ are separated by a distance $x = \sqrt{3}\ R$ and carry currents $I_1 = 2I$ and $I_2 = I$ respectively.

16. The magnetic field at the centre of the smaller loop due to current $I_1 = 2\ I$ in the bigger loop is

(a) $\dfrac{\mu_0 I}{R}$ (b) $\dfrac{\mu_0 I}{4R}$

(c) $\dfrac{\mu_0 I}{8R}$ (d) $\dfrac{\mu_0 I}{3R}$

17. The magnetic flux (ϕ) linked with the smaller loop is

(a) $\dfrac{5\pi\mu_0 IR}{4}\times 10^{-5}$ (b) $\pi\ \mu_0\ IR \times 10^{-5}$

(c) $\dfrac{3\pi\mu_0 IR}{4}\times 10^{-5}$ (d) $\dfrac{\pi\mu_0 IR}{2}\times 10^{-5}$

18. The mutual inductance of the pair of coils is

(a) $\dfrac{\phi}{I}$ (b) $\dfrac{2\phi}{I}$

(c) $\dfrac{\phi}{2I}$ (d) zero

19. If M and m are the magnetic moments of the bigger and smaller loops respectively, then the ratio M/m is

(a) 10^4 (b) 2×10^4

(c) 10^2 (d) 2×10^2

SOLUTION

16. The magnetic field at the centre of the smaller loop due to current I_1 in the bigger loop is given by (see Fig. 25.80).

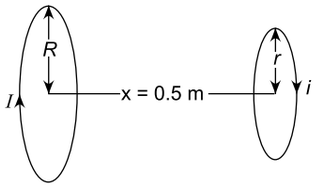


Fig. 25.80

$$B = \frac{\mu_0 I_1 R^2}{2(R^2 + x^2)^{3/2}}$$

Putting $I_1 = 2\ I$ and $x = \sqrt{3}\ R$, we find that

$$B = \frac{\mu_0 I}{8R}, \text{ which is choice (c).}$$

17. Since r is very small compared to x, the magnetic field over the entire area enclosed by the smaller loop may be treated to be uniform and equal to B. Therefore, the magnetic flux linked with the smaller loop is

$\phi = B \times$ area enclosed by the smaller loop

$= B \times \pi r^2$

$$= B \times \pi \left(\frac{R}{100}\right)^2$$

$$= \frac{\mu_0 I}{8R} \times \pi \times \frac{R^2}{10^4} = \frac{5\pi\mu_0 IR}{4} \times 10^{-5}$$

So the correct choice is (a).

18. The mutual inductance of the pair of loops is

$M = \dfrac{\phi}{I_2} = \dfrac{\phi}{I}$, which is choice (a).

19. A current carrying loop is a magnetic dipole whose magnetic moment = current × area of the loop. Therefore,

$$M = I_1\ \pi\ R^2 = 2\ \pi\ I\ R^2 \qquad (\because\ I_1 = 2I)$$

and $$m = I_2\ \pi\ r^2 = I\ \pi \left(\frac{R}{100}\right)^2 = \frac{\pi IR^2}{10^4}$$

Hence the correct choice is (b).

Questions 20 to 22 are based on the following passage

Passage VI

An LCR series circuit with 100 Ω resistance is connected to an a.c. source of 200 V and angular frequency 300 rad/sec. When only the capacitance is removed, the current leads the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°.

IIT, 1990

20. The impedance of the LCR circuit is

(a) 100 Ω (b) $100\sqrt{2}$ Ω

(c) 200 Ω (b) $200\sqrt{2}$ Ω

21. The current in the circuit is

(a) $\sqrt{2}$ A (b) 2 A

(c) $2\sqrt{2}$ A (d) 1 A

22. The power dissipated in the circuit is

(a) 200 W (b) 400 W

(c) 800 W (d) 100 W

SOLUTION

20. When capacitance is removed, the circuit contains only inductance and resistance. Phase difference θ between the current and voltage is then given by

$$\tan\theta = \frac{\omega L}{R} \text{ or } \omega L = R\tan\theta = 100\tan 60°$$

When the circuit contains only capacitance and resistance, the phase difference between the voltage and current is given by

$$\tan\phi = \frac{1}{RC\omega}$$

$$\therefore \quad \frac{1}{C\omega} = R\tan\phi = 100\tan 60°$$

The impedance of the *LCR* circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{R^2 + (100\tan 60° - 100\tan 60°)^2}$$

$= R = 100\ \Omega$, which is choice (a).

21. The current is given by

$$I = \frac{V}{R} = \frac{200}{100} = 2\text{ A}$$

The correct choice is (b).

22. The power dissipated in the circuit is

$P = I^2R = 4 \times 100 = 400$ W

So the correct choice is (b).

Questions 23 to 26 are based on the following passage.

Passage VII

A light horizontal rod is held horizontally by two light inextensible strings as shown in Fig. 25.81. A wheel of radius R having charge Q distributed uniformly on its rim is free to rotate about the rod. A uniform magnetic field B is applied as shown in Fig. 25.81. The angular frequency of the wheel is ω.

IIT, 2003

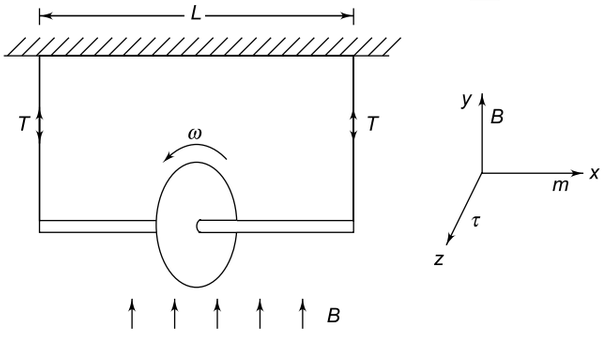


Fig. 25.81

23. The magnitude of the magnetic moment of the wheel is

(a) $QR^2\omega$ (b) $\frac{QR^2\omega}{\sqrt{2}}$

(c) $\frac{QR^2\omega}{2}$ (d) $2QR^2\omega$

24. The direction of the magnetic moment is along

(a) the positive x-axis
(b) the negative x-axis
(c) the positive y-axis
(d) the negative y-axis.

25. The magnitude of the torque produced is

(a) $\frac{1}{2}(QR^2\omega B)$ (b) $\frac{1}{2}(QRL\omega B)$

(c) $QR^2\omega B$ (d) $BRL\omega B$

26. The direction of the torque is along

(a) the positive x-axis
(b) the positive y-axis
(c) the positive z-axis
(d) the negative z-axis.

SOLUTION

23. If T is the time period of rotation of the wheel, the current due to the rotation of the charge is

$$I = \frac{Q}{T} = \frac{Q\omega}{2\pi}$$

The magnitude of the magnetic moment of the wheel is

$$m = I \times \pi R^2 = \frac{Q\omega}{2\pi}\ \pi R^2 = \frac{1}{2}\ (QR^2\omega)$$

So the correct choice is (c).

24. The direction of the magnitude moment vector **m** is perpendicular to the plane of the wheel and since the current is anticlockwise, from the screw rule, the direction of **m** is along the positive x-axis, which is choice (a)

25. Torque $\vec{\tau} = \mathbf{m} \times \mathbf{B} = m\hat{\mathbf{i}} \times B\hat{\mathbf{j}}$

$$= mB\hat{\mathbf{k}}$$

$$= \frac{1}{2}\ (B\,Q\,R^2\,\omega)\ \hat{\mathbf{k}}$$

So the correct choice is (a).

26. The correct choice is (c).

Questions 27 to 29 are based on the following passage

Passage VIII

An *LCR* circuit consists of an inductor, a capacitor and a resistor driven by a battery and connected by two switches S_1 and S_2 as shown in Fig. 25.82.

IIT, 2006

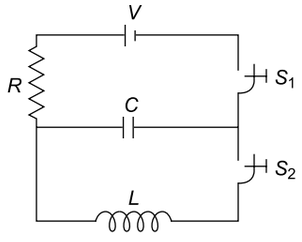


Fig. 25.82

27. At time $t = 0$ switch S_1 is closed and S_2 is left open. The maximum charge the capacitor plate can hold is q_0 and τ is the time constant of the *RC* circuit. Then
(a) at time $t = \tau$, the charge on the capacitor plates is $q = q_0/2$.
(b) at $t = 2\tau$, $q = q_0 (1 - e^{-2})$
(c) at $t = 2\tau$, $q = q_0 (1 - e^{-1})$
(d) work done by the battery is half the energy dissipated in the resistor.

28. At time $t = 0$ when the charge on the capacitor plates is q, switch S_1 is opened and S_2 is closed. The maximum charge the capacitor can hold is q_0. Choose the correct statement from the following.

(a) $q = q_0 \cos\left(\frac{t}{\sqrt{LC}} + \frac{\pi}{2}\right)$

(b) $q = q_0 \cos\left(\frac{t}{\sqrt{LC}} - \frac{\pi}{2}\right)$

(c) $q = -LC\frac{d^2q}{dt^2}$

(d) $q = -\frac{1}{\sqrt{LC}}\frac{d^2q}{dt^2}$

29. At an instant of time $t = 0$ when the capacitor has been charged to a voltage V, switch S_1 is opened and S_2 is closed. Then
(a) at $t = 0$, the energy is stored in the magnetic field of the inductor.
(b) at $t > 0$, there is no exchange of energy between the capacitor and the inductor.
(c) at $t > 0$, the current in the circuit flows only in one direction.
(d) the maximum value of the current in the circuit is $\sqrt{\frac{C}{L}}V$.

SOLUTION

27. In an *RC* circuit, the charge on the capacitor plates at a time t is given by

$$q = q_0 (1 - e^{-t/\tau})$$

where $\tau = RC$ is the time constant. At $t = 2\tau$, we have

$$q = q_0(1 - e^{-2t/\tau}) = q_0(1 - e^{-2})$$

Hence the correct choice is (b).

28. When S_2 is closed and S_1 is open, the charge oscillates in the *LC* circuit at an angular frequency given by

$$\omega = \frac{1}{\sqrt{LC}} \qquad (1)$$

Now $q \neq 0$ at $t = 0$. Hence choices (a) and (b) are wrong. The charge q varies with time t as

$$q = q_0\cos(\omega t + \phi) \qquad (2)$$

where ϕ is not equal to $\pi/2$. Differentiating Eq. (2) twice with respect to t, we get

$$\frac{d^2q}{dt^2} = -\omega^2 q_0 \cos(\omega t + \phi) = -\omega^2 q$$

$$q = -\frac{1}{\omega^2}\frac{d^2q}{dt^2} = -LC\frac{d^2q}{dt^2} \quad \text{[use Eq. (1)]}$$

Hence the correct choice is (c).

29. At $t = 0$, the energy is stored in the electric field in the space between the capacitor plates. As time passes (i.e. at $t > 0$), there is an exchange of energy between the capacitor and the inductor. The charge q varies with time t as

$$q = q_0 \cos\omega t, \quad \text{where} \quad \omega = \frac{1}{\sqrt{LC}}$$

The current in the circuit is given by

$$I = \frac{dq}{dt} = \frac{d}{dt}(q_0 \cos\omega t) = -\omega q_0 \sin\omega t$$

which is alternating and not unidirectional. The maximum value of current is

$$I_{max} = \omega q_0 = \frac{1}{\sqrt{LC}} \times CV \qquad (\because q_0 = CV)$$

$$= \sqrt{\frac{C}{L}}V$$

Hence the correct choice is (d).

Questions 30 to 32 are based on the following passage

Passage IX

MAGLEV Train

The MAGLEV train was developed by Japanese engineers. It is based on the phenomenon of electromagnetic induction. The word MAGLEV stands for Magnetic Lavitation. The train moves without any contact with the track; it remains suspended in air slightly above the track. Consequently, the power loss due to friction is considerably reduced. In a MAGLEV train, a coil is attached to the railway track and an electromagnet is fixed on the base of the train. When the train moves, the magnetic flux in the coil changes. From Lenz's law, the train is repelled by the track, thus lifting it above the track. Hence there is no power loss due to friction with the track. A disadvantage is that the induced circulating currents in the coil exert on opposing force on the train. As a result the train slows down and significant power is needed to propel the train against this force.

IIT, 2006

30. The MAGLEV train is lifted above the track due to
(a) electrostatic repulsion
(b) magnetic repulsion
(c) induced electric field
(d) time varying electric field

31. The advantage of MAGLEV train is that
(a) no power is required to propel the train
(b) no power is required to overcome gravity
(c) no power is dissipated due to friction between the train and the track
(d) the electrostatic forces moves the train.

32. The disadvantage of MAGLEV train is that
(a) the train cannot overcome gravity
(b) the train is repelled by the track and hence becomes unstable
(c) the magnetic force tends to bring down the train
(d) the induced currents in the coil exert a backward force on the train due to Lenz's law.

SOLUTION

30. The correct choice is (b).
31. The correct choice is (c).
32. The correct choice is (d).

SECTION IV

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by statement-2 (Reason). Each question has the following four options out of which only *one* option is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. Statement-1
No induced emf is developed across the ends of a conductor if it is moved parallel to a magnetic field.

Statement-2
No force acts on the free electrons of the conductor.

2. Statement-1
No current is induced in a metal loop if it is rotated in an electric field.

Statement-2
The electric flux through the loop does not change with time.

3. Statement-1
A rectangular loop and a circular loop are moved with a constant velocity from a region of magnetic field out into a field-free region. The field is normal to the loops. Then a constant emf will be induced in the circular loop and a time-varying emf will be induced in the rectangular loop.

Statement-2
The induced emf is constant if the magnetic flux changes at a constant rate.

4. Statement-1
A magnetised iron bar is dropped vertically through a hollow region of a thick cylindrical shell made of copper. The bar will fall with an acceleration less than g, the acceleration due to gravity.

Statement-2
The emf induced in the bar causes a retarding force to act on the falling bar.

5. Statement-1
A coil is connected in series with a bulb and this combination is connected to a d.c. source. If an iron core is inserted in the coil, the brightness of the bulb will increase.

Statement-2
The reactance offered by the coil to d.c. current is zero.

6. Statement-1
A coil is connected in series with a bulb and this combination is connected to an a.c. source. If an iron core is inserted in the coil, the brightness of the bulb will be reduced.

Statement-2
When an iron core is inserted in the coil, its inductance decreases.

IIT, 1997

7. Statement-1
A variable capacitor is connected in series with a bulb and this combination is connected to an a.c. source. If the capacitance of the variable capacitor is decreased, the brightness of the bulb is reduced.

Statement-2
The reactance of the capacitor increases if the capacitance is reduced.

8. Statement-1
If the current in a straight conductor increases from A to B, the direction of the current induced in the coil will be anticlockwise. (Fig. 25.83)

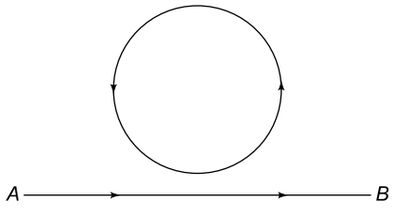


Fig. 25.83

Statement-2
According to Lenz's law, the direction of the induced current is such that it opposes the change which produces it.

IIT, 1979

9. Statement-1
An emf can be induced between the two ends of a straight copper wire when it moved through a uniform magnetic field.

Statement-2
As the straight wire moves through the magnetic field, the magnetic flux through the wire changes.

IIT, 1980

10. Statement-1
Three identical coils A, B and C are placed with their planes parallel to one another. Coils A and C carry equal currents as shown in Fig. 25.84 If coil A is moved towards B, with coils B and C fixed in position, the induced current in B will be in the anticlockwise direction.

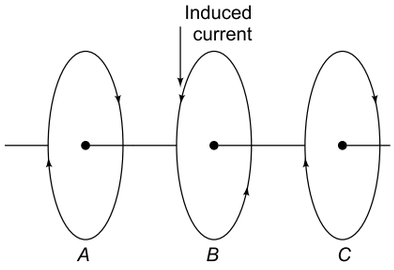


Fig. 25.84

Statement-2
The direction of the induced current is given by Lenz's law.

IIT, 1982

11. Statement-1
A coil of metal wire is kept stationary in a non-uniform magnetic field. An emf is induced in the coil.

Statement-2
Whenever the magnetic flux through a metal coil changes, an emf is induced in it.

IIT, 1986

12. Statement-1
If a conducting rod AB moves parallel to the x-axis in a uniform magnetic field $\vec{B}$ pointing in the

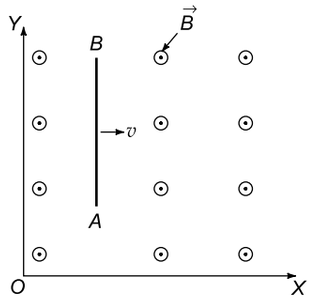


Fig. 25.85

positive z-direction as shown in Fig. 25.85, the end A of the rod gets positively charged.

Statement-2
The free electrons in the rod experience a force in positive y-direction and move from A to B.

IIT, 1987

13. Statement-1
A magnetized iron bar is dropped vertically through a hollow region of a thick cylindrical shell made of copper. The bar will fall with an acceleration less than g, the acceleration due to gravity.
Statement-2
The emf induced in the bar causes a retarding force to act on the falling bar.

IIT, 1991

14. Statement-1
A vertical iron rod has a coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in Fig. 25.86. The ring can float at a certain height above the coil.

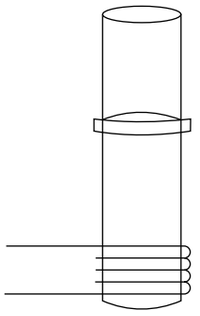


Fig. 25.86

Statement-2
In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction.

IIT, 2007

SOLUTIONS

1. The correct choice is (a). Let $\vec{v}$ be the velocity of the conductor in a magnetic field $\vec{B}$. Since the free electrons in the conductor are moving with it, force $\vec{F} = e(\vec{v} \times \vec{B})$ is zero because $\vec{v}$ is parallel to $\vec{B}$. Consequently, no induced emf is developed between the ends of the conductor.

2. The correct choice is (b). A current is induced in a loop only if magnetic flux linked with the coil changes.

3. The correct choice is (d). The induced emf is constant in the case of rectangular coil because the rate of change of area is constant. But in the case of the circular coil, the rate of change of area (and hence the rate of change of magnetic flux) keeps varying as the loop is moving towards the field-free region.

4. The correct choice is (a). The retarding force is caused by the eddy currents and according to Lenz's law, the induced emf must oppose the cause. The cause is the falling bar.

5. The correct choice is (d). A coil offers no reactance to d.c. currents. Hence there will be no change in the brightness of the bulb when an iron core is inserted in the coil.

6. The correct choice is (c). If an iron core is inserted in the coil, its inductance L increases. Hence its reactance ωL increases, causing a decrease in the current in the circuit. As a result, the brightness of the bulb will reduce.

7. The correct choice is (a). The reactance of a capacitor is $1/\omega C$. Hence if C is decreased, the reactance will increase and as a result the current in the circuit is decreased causing a decrease in the brightness of the bulb.

8. Statement-2 is correct. According to Lenz's law, the induced current in the coil should be clockwise. So Statement-1 is false and Statement-2 is true.

9. Statement-1 is true but Statement-2 is false. When the wire is moved through a magnetic field, the free electrons experience a force which causes them to move from one end of wire to the other. The end where the electrons pile up acquires a negative charge and the other end acquires an equal positive charge. This causes an induced emf between the ends of the wire. If the wire is connected to a closed circuit, an induced current will flow in the circuit.

10. Since coil C is not moved relative to coil B, the current in C does not produced any induced current in B. When coil A is moved towards B, then according to Lenz's law, the current in B will be anticlockwise. Hence both the statements are true and Statement B is the correct explanation for Statement-1.

11. There is no change in magnetic flux if the coil is kept stationary even if the magnetic field is non-uniform. Hence no emf is induced in the coil. Statement-1 is false and Statement-2 is true.

12. Both statements are true and Statement-2 is the correct explanation for Statement-1.

13. Both statements are true and Statement-2 is the correct explanation for Statement-1. The retarding force is caused by eddy currents and according to Lenz's law, the induced emf must oppose the cause. The cause is the falling bar.

14. According to Lenz's law, the coil will repel the ring. At a height where this upward force balances the weight of the ring, it will float at that height above the coil. Hence both statements are true and Statement-2 is the correct explanation for Statement-1.

SECTION V

Integer Answer Type

1. The two rails of a railway track, insulated from each other, and the ground, are connected to a millivoltmeter. What is the reading of the voltmeter when a train travels at a speed of 180 km h^{-1} along the track? Given vertical component of earth's magnetic field = 0.2×10^{-4} T and the separation between the rails = 1 m.

IIT, 1981

2. Space is divided by the line AD into two regions. Region I is field free and region II has a uniform magnetic field $B = 0.5$ T directed into the plane of the paper as shown in the Fig. 25.87. ACD is a semicircular conducting loop of radius $r = 10$ cm with centre at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity $\omega = 400$ rad s^{-1} about an axis passing through O and perpendicular to the plane of the paper. The effective resistance of the loop is $R = 0.5\ \Omega$. Find the induced current (in A) in the loop.

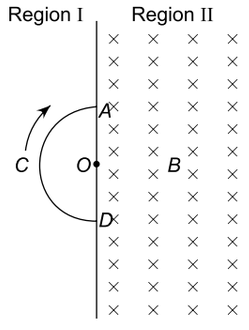


Fig. 25.87

IIT, 1985

3. An LCR series circuit with 100 Ω resistance is connected to an a.c. source of 200 V and angular frequency 300 rad/sec. When only the capacitance is removed, the current lags the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current (in ampere) in the LCR circuit.

IIT, 1990

4. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistances R_1 and R_2 as shown Fig. 25.88. A horizontal metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. Find the terminal velocity of bar L (in ms^{-1}).

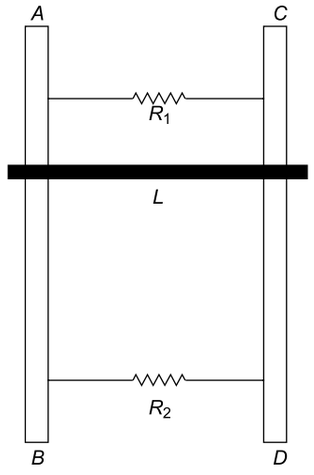


Fig. 25.88

IIT, 1994

5. An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of charge Q is 200 μC. Find the maximum value of I (in ampere).

IIT, 1998

6. A metal coil of area 5×10^{-3} m^2, number of turns 100 and resistance 0.5 Ω is lying horizontally at the bottom of a vessel made of an insulating material. A uniform magnetic field passing vertically through the coil changes from zero to 0.8 T in 0.2 s. Calculate the induced current (in ampere) flowing through the coil.

IIT, 2000

7. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

SOLUTIONS

1. Velocity $v = 180$ km $h^{-1} = 50$ m s^{-1}. The induced voltage between the rails is

$e = Blv = 0.2 \times 10^{-4} \times 1.0 \times 50 = 1 \times 10^{-3}$ V = 1 mV

2. Induced emf is $|e| = \frac{1}{2} Br^2\omega$. Therefore, the current in the loop is

$$I = \frac{|e|}{R} = \frac{Br^2\omega}{2R}$$

$$= \frac{0.5 \times (0.1)^2 \times 400}{2 \times 0.5} = 2 \text{ A}$$

3. When capacitance is removed, the circuit contains only inductance and resistance. Phase difference θ between the current and voltage is then given by

$$\tan\theta = \frac{\omega L}{R} \text{ or } \omega L = R \tan\theta$$

$$= 100 \tan 60°$$

When the circuit contains only capacitance and resistance, the phase difference between the voltage and current is given by

$$\tan\phi = \frac{1}{RC\omega}$$

$$\therefore \quad \frac{1}{C\omega} = R \tan\phi = 100 \tan 60°$$

The impedance of the *LCR* circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{R^2 + (100\tan 60° - 100\tan 60°)^2}$$

$$= R = 100 \ \Omega$$

The current is given by

$$I = \frac{V}{R} = \frac{200}{100} = 2 \text{ A}$$

4. The rod will acquire terminal velocity when [see Fig. 25.89]

$$BId = mg$$

or $$I = \frac{mg}{Bd} = \frac{0.2 \times 9.8}{0.6 \times 1} = \frac{49}{15} \text{ A}$$

Current I divides between parallel resistors R_1 and R_2. Let I_1 and I_2 be the currents in R_1 and R_2, then

$$I = I_1 + I_2$$

If e is the emf induced in the rod, we have (as R_1 and R_2 are in parallel)

$$e = I_1R_1 = I_2R_2 \qquad (1)$$

Powers dissipated in R_1 and R_2 are

$$P_1 = eI_1 \qquad (2)$$

and $$P_2 = eI_2$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{I_1}{I_2}$$

Given $P_1 = 0.76$ W and $P_2 = 1.2$ W. Therefore,

$$\frac{I_1}{I_2} = \frac{0.76}{1.2} = \frac{19}{30}$$

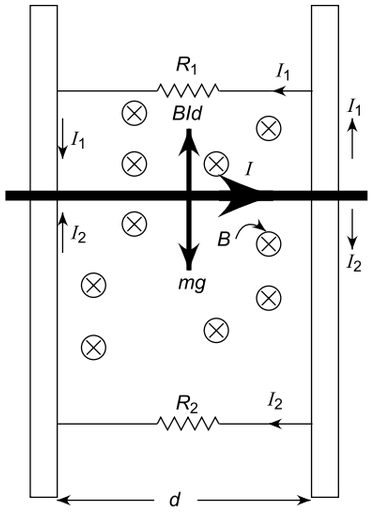


Fig. 25.89

But $I = I_1 + I_2$. Thus $\left(\because I = \frac{49}{15}\text{A}\right)$

$$\frac{49}{15} = I_1 + \frac{30I_1}{19} = \frac{49I_1}{19}$$

which gives $I_1 = \frac{19}{15}$ A. Hence

$$I_2 = I - I_1 = \frac{49}{15} - \frac{19}{15} = \frac{30}{15} = 2 \text{ A}.$$

From (1) we have $e = \frac{P_1}{I_1} = \frac{0.76}{19/15} = 0.6$ V

If v_t is the terminal velocity of the rod, the emf induced in it is

$$e = Bv_t d \text{ or } v_t = \frac{e}{Bd} = \frac{0.6}{0.6 \times 1} = 1 \text{ ms}^{-1}$$

5. The charge Q on the capacitor plates (and hence the current I in the circuit) oscillates an angular frequency ω given by

$$\omega = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{[(2.0 \times 10^{-3}) \times (5.0 \times 10^{-6})]^{1/2}}$$

$$= 10^4 \text{ rad s}^{-1}$$

Charge Q varies harmonically with time t at angular frequency ω. Since at $t = 0$, $Q = Q_0$, the maximum charge, we have

$$Q = Q_0 \cos \omega t \quad (1)$$

$$\text{Current } I = \frac{dQ}{dt} = Q_0 \frac{d}{dt}(\cos \omega t)$$

$$= -Q_0 \omega \sin \omega t \quad (2)$$

$$\therefore \quad I_{max} = \omega Q_0 = 10^{-4} \times (200 \times 10^{-6}) = 2 \text{ A}$$

6. $\frac{dB}{dt} = \frac{0.8}{0.2} = 4 \text{ T s}^{-1}$

Magnetic flux $\phi = NAB$. The induced emf is

$$e = \left| -\frac{d\phi}{dt} \right| = NA \frac{dB}{dt}$$

$$= 100 \times (5 \times 10^{-3}) \times 4 = 2 \text{ V}$$

$\therefore$ Induced current $I = \frac{e}{R} = \frac{2}{0.5} = 4 \text{ A}$

7. $$Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$$

$$\Rightarrow \quad Z^2 = R^2 + \frac{1}{\omega^2 C^2}$$

$$\left(\sqrt{1.25}\, R\right)^2 = R^2 + \frac{1}{\omega^2 C^2}$$

$$\Rightarrow \quad R = \frac{2}{\omega C}$$

Time constant $\tau = RC = \frac{2}{\omega} = \frac{2}{500} = 4 \times 10^{-3}$s = 4 millisecond

26 Chapter

Ray Optics and Optical Instruments

REVIEW OF BASIC CONCEPTS

26.1 REFLECTION OF LIGHT

(i) *Law of Reflection*
Angle of reflection = angle of incidence
A ray incident along the normal to a reflecting surface retraces its path after reflection.

(ii) *Reflection at a plane surface (mirror)*
(a) Distance of image from mirror = distance of object from mirror.
(b) Size of image = size of object.
(c) If the object moves with a certain velocity, the image moves with the same velocity but in the opposite direction.
(d) Keeping the incident ray fixed, if the mirror is rotated through an angle θ, the reflected ray rotates through an angle 2θ.
(e) If two mirrors are inclined at an angle θ (in degrees), the number of images formed by the mirrors of an object is
- $\left(\frac{360}{\theta}-1\right)$ if $\frac{360}{\theta}$ is an even number,
- $\left(\frac{360}{\theta}-1\right)$ if $\frac{360}{\theta}$ is an odd number and the object is placed at the same distance from the mirrors, and
- $\left(\frac{360}{\theta}\right)$ if $\frac{360}{\theta}$ is an odd number and the object is not placed at the same distance from the mirrors.

(f) If three mirrors are placed mutually perpendicular and adjacent to each other, the number images of an object placed in front of them is 7.

26.2 REFLECTION AT A SPHERICAL SURFACE (CONCAVE MIRROR OR CONVEX MIRROR)

(a) Sign conventions
(i) All distances are measured from the centre (pole) of the mirror.
(ii) Distances measured in the direction of incident rays are takes as positive while those measured opposite to the direction of incident rays are taken as negative.
(iii) Distances above the principal axis are taken as positive while those below the principal axis are taken as negative.

(b) The spherical mirror formula (for both concave and convex mirrors) is

$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f}; f = \frac{R}{2}, \text{ where}$$

u = object distance, v = image distance, f = focal length and R = radius of curvature. For convex mirror f is positive and for an concave mirror f is negative.

(c) Linear magnification $m = \frac{h_i}{h_0} = -\frac{v}{u} = \frac{f}{f-u}$
$= \frac{f-v}{f}$

For an erect image m is positive and for an inverted image m is negative.

(d) A concave mirror forms a real and inverted image of an object placed beyond its focus and a virtual and erect image if the object is placed between the pole and focus. A convex mirror forms a virtual and erect image for all positions of the object.

26.3 SNELL'S LAW OF REFRACTION

If a ray of light travelling in a medium of refractive index μ_1 is incident at angle i on the boundary of a medium of refractive index μ_2, then

$$\mu_1 \sin i = \mu_2 \sin r$$

where r is the angle of refraction in medium μ_2.

26.4 REFRACTIVE INDEX AND SPEED OF LIGHT

$$\mu = \frac{\text{speed of light in air or vacuum}}{\text{speed of light in the medium}} = \frac{c}{v}$$

The value of μ depends upon (i) nature of the medium and (ii) wavelength (colour) of light.

$$\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$$

26.5 REFRACTION OF LIGHT THROUGH A PLANE SLAB

When a ray of light passes through a glass slab of thickness t (Fig. 26.1),

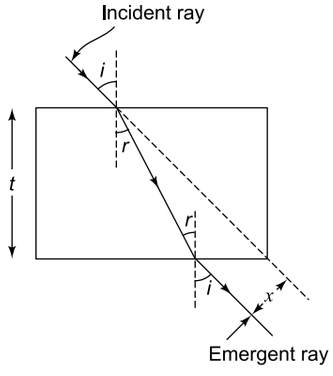


Fig. 26.1

(a) the emergent ray is parallel to the incident ray and

(b) lateral displacement $x = \dfrac{t\sin(i-r)}{\cos r}$

26.6 REAL AND APPARENT DEPTH (OR HEIGHT)

(a) An object in a denser medium (water) viewed by an observer in a rarer medium (air) from above [Fig. 26.2(a)]
Real depth $OA = d$, Apparent depth $AI = d'$ and $\mu = \dfrac{d}{d'}$
Apparent shift $OI = d - d' = d - \dfrac{d}{\mu} = d\left(1 - \dfrac{1}{\mu}\right)$; μ = refractive index of water.

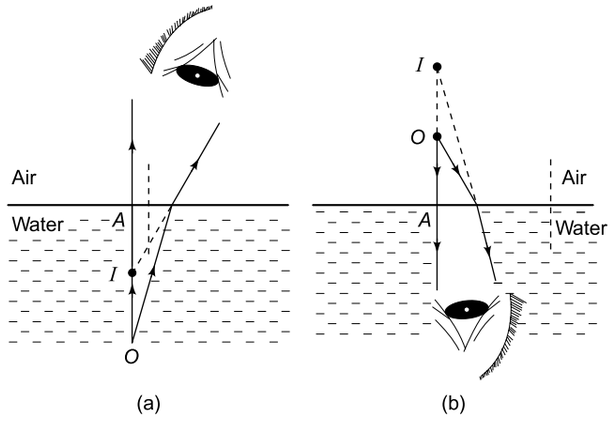


Fig. 26.2

(b) Object in air viewed by observer under water [Fig. 26.2(b)]
Real height $OA = h$, Apparent height $AI = h' = \mu h$.
Apparent shift $OI = h' - h = \mu h - h = (\mu - 1)h$.

26.7 TOTAL INTERNAL REFLECTION

For total internal reflection,

(a) the incident ray must travel in a denser medium (μ_2) to the boundary of a rarer medium μ_1 ($< \mu_2$) and

(b) the angle of incidence must be greater than critical angle i_c given by (Fig. 26.3) $\mu_2 \sin i_c = \mu_1 \sin 90°$

$\Rightarrow \sin i_c = \dfrac{\mu_1}{\mu_2}$. If rarer medium is air ($\mu_1 = 1$ and $\mu_2 = \mu$) then

$$\sin i_c = \frac{1}{\mu}$$

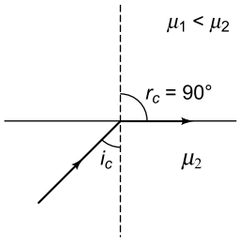


Fig. 26.3

The value of i_c depends upon μ_1, μ_2 and wavelength of light.

26.8 REFRACTION THROUGH A PRISM

When a ray of monochromatic light is refracted by a prism, the deviation δ produced by the prism is (Fig. 26.4)

$$\delta = i_1 + i_2 - A$$

where i_1 = angle of incidence, i_2 = angle of emergence and A = refracting angle of the prism. Also

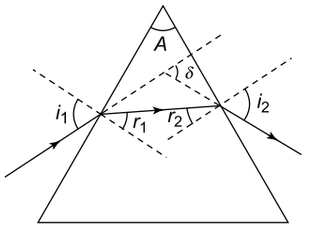


Fig. 26.4

$$A = r_1 + r_2$$

The deviation is minimum = δ_m, if $i_1 = i_2$. Then $r_1 = r_2$. Then

$$\frac{\mu_2}{\mu_1} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where μ_2 = refractive index of the material of the prism and μ_1 = refractive index of the medium surrounding the prism. If $\mu_1 = 1$ (air) and $\mu_2 = \mu$, then

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

For a thin prism, angles i_1, i_2, r_1, r_2 and A are small and

$$\delta = (\mu - 1)A$$

26.9 REFRACTION AT A SPHERICAL SURFACE

If the rays from an object travelling in a medium of refractive index μ_1 are refracted at the spherical surface (convex or concave) of a medium of refractive index μ_2 forming an image, then the object and image distances u and v are related as

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{(with sign convention)}$$

where R = radius of curvature of the spherical surface.

Linear magnification $m = \frac{h_1}{h_0} = \frac{v}{u}$.

If the first medium is air, $\mu_1 = 1$ and $\mu_2 = \mu$, then

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$$

If the incident rays from the object are in medium μ_2 and are refracted at the spherical surface of medium μ_1, then in the above formula μ_1 and μ_2 are interchanged, i.e.

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

Application: Image formed by a transparent sphere

Case (a): When the object O is in air outside the sphere of radius R. (Fig. 26.5)

I' is the angle of O due to refraction at P. For this refraction,

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{R}$$

(u is negative, and R is positive).

This image I' serves as the virtual object for refraction at Q forming the final image I. For refraction at Q (since the incident rays are in medium μ), we have

$$\frac{1}{v} - \frac{\mu}{u'} = \frac{1 - \mu}{R} \quad \text{where } u' = v' - R$$

(u' is positive and R is negative).

Case (b): When the object O is on the surface of the sphere (Fig. 26.6).

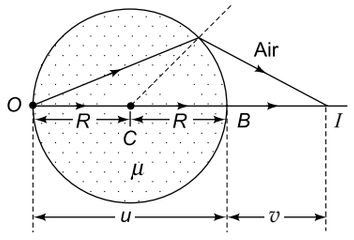


Fig. 26.6

$$\frac{1}{v} - \frac{\mu}{u} = \frac{1 - \mu}{R}$$

where $u = 2R$.

(u is negative and R is also negative).

Case (c): Object O inside the sphere (Fig. 26.7)

$$\frac{1}{v} - \frac{\mu}{u} = \frac{1 - \mu}{R}$$

(u is negative and R is also negative)

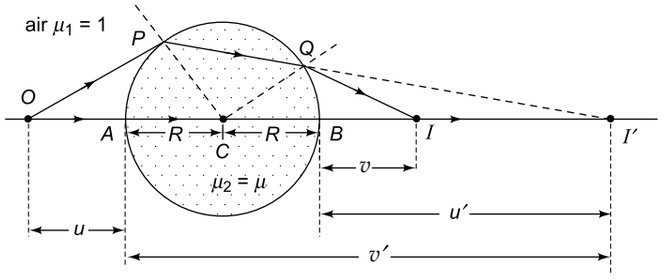


Fig. 26.5

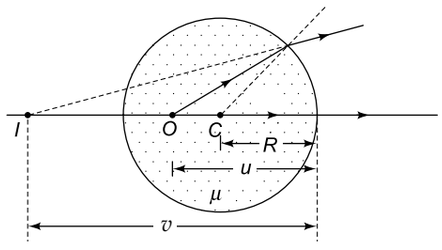


Fig. 26.7

26.10 REFRACTION THROUGH A LENS

(i) Relation between u, v and f for a lens (convex or concave) is

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\ \text{(with sign convention)}$$

Linear magnification $m=\frac{h_i}{h_i}=\frac{v}{u}=\frac{f-v}{f}=\frac{f}{f+u}$

The focal length f is positive for converging (convex) lens and negative for diverging (concave) lens.

(ii) Focal length f of a lens of refractive index μ_2 surrounded by a medium of refractive index μ_1 is given by

$$\frac{1}{f}=\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\ \text{(with sign convention)}$$

where R_1 = radius of curvature of the surface of the lens on which the rays are incident

R_2 = radius of curvature of the surface of the lens from which the rays emerge after refraction through the lens.

If the media on the two sides of the lens are different (e.g. a lens floating on water) as shown in Fig. 26.8, then

$$\frac{1}{f}=\frac{\mu_2-\mu_1}{\mu_3 R_1}+\frac{\mu_3-\mu_2}{\mu_3 R_2}$$

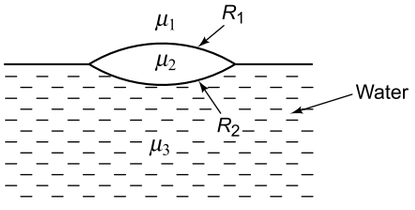


Fig. 26.8

(iii) Power of lens $P=\frac{1}{f\,(\text{in metre})}$

(iv) Lens maker's formula

If a lens is surrounded by air, $\mu_1=1$ and $\mu_2=\mu$, then

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

This is the lens maker's formula.

Applications of lens maker's formula

(a) Equi-convex lens (Fig. 26.9)

$R_1=+R,\ R_2=-R$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{-R}\right)$$

$$=(\mu-1)\frac{2}{R}$$

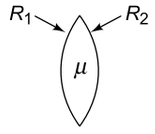


Fig. 26.9

f is positive.

(b) Equi-concave lens (Fig. 26.10)

$R_1=-R,\ R_2=+R$

$$\frac{1}{f}=-(\mu-1)\frac{2}{R}$$

f is negative.

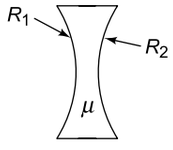


Fig. 26.10

(c) Double convex lens ($R_1\neq R_2$)

$R_1=+R_1,\ R_2=-R_2$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right);\ f\ \text{is positive}$$

(d) Double concave lens ($R_1\neq R_2$)

$R_1=-R_1,\ R_2=-R_2$

$$\frac{1}{f}=-(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right);\ f\ \text{is negative.}$$

(e) Plano-convex lens (Fig. 26.11)

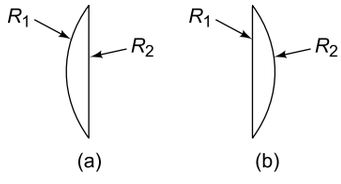


Fig. 26.11

Fig. (a): $R_1=+R,\ R_2=\infty$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\frac{(\mu-1)}{R}$$

Fig. (b): $R_1=\infty,\ R_2=-R$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)=\frac{(\mu-1)}{R}$$

f is positive.

(f) Plano-concave lens (Fig. 26.12)

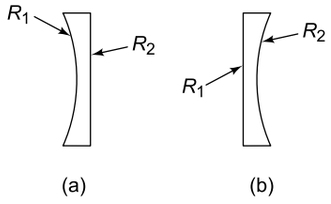


Fig. 26.12

Fig. (a): $R_1 = -R, R_2 = \infty$

$$\frac{1}{f} = (\mu - 1)\left(-\frac{1}{R} - \frac{1}{\infty}\right) = -\frac{(\mu - 1)}{R}.$$

Fig. (b): $R_1 = \infty, R_2 = +R$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{R}\right) = -\frac{(\mu - 1)}{R}$$

f is negative.

(g) First surface convex, second surface concave (Fig. 26.13)

$$R_1 = +R_1,\ R_2 = +R_2$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

since $R_2 > R_1$, f is positive

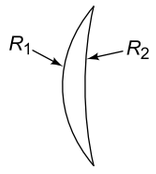


Fig. 26.13

(h) First surface concave, second surface convex (Fig. 26.14)

$$R_1 = -R_1,\ R_2 = -R_2$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

Since $R_2 > R_1$, f is negative.

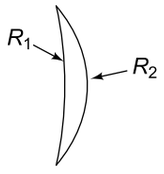


Fig. 26.14

(i) If a lens of focal length f is cut along AB into two equal pieces as shown in Fig. 26.15(a), the focal length of each piece is $2f$.

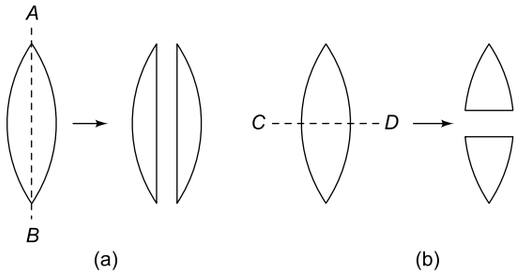


Fig. 26.15

But if the lens is cut along CD as shown in Fig. 26.15(b), the focal length remains the same equal to f.

26.11 POWER OF A LENS

The reciprocal of the focal length (expressed in metres) is known as the power of a lens.

$$\text{Power } P = \frac{1}{f(\text{in metre})}$$

The SI unit of P is called diopre (D)

If the lens is placed in medium of refractive index μ_m, then

$$P = \frac{\mu_m}{f_m(\text{in metre})}$$

where f_m = focal length in medium.

26.12 CO-AXIAL COMBINATION OF LENSES

If two thin lenses of focal lengths f_1 and f_2 are placed in contact co-axially (i.e. with their principal axes coinciding), the equivalent focal length F of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Power of combination $P = P_1 + P_2$

If the lenses are separated by a distance d,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

and $$P = P_1 + P_2 - P_1\, P_2 d$$

26.13 EFFECT OF SILVERING ONE OF THE REFRACTING FACES OF A LENS

If one surface of a convex lens is silvered, it behaves like a concave mirror. If one surface of a concave lens is silvered, it behaves like a convex mirror. In Fig. 26.16, the rays are refracted at surface 1, reflected at surface 2 and again refracted at surface 1. The effective focal length F is given by

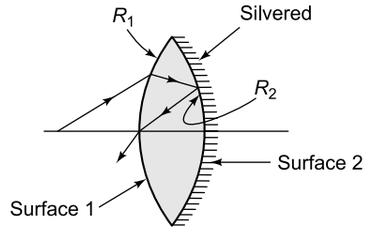


Fig. 26.16

$$\frac{1}{F} = \sum_{n=1,2} \frac{1}{f_n}$$

where f_n = focal length of lens or mirror repeated as many times as there are refractions and reflections. In the case shown in the figure, there are two refractions and one reflection. Hence

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f_m} = \frac{2}{f} + \frac{1}{f_m}$$

where f = focal length of the lens and f_m = focal length of the spherical mirror of radius of curvature R_2.

(a) If the face of radius of curvature R_2 of a double convex lens is silvered (Fig. 26.16),

$$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m}$$

where $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

and $f_m = \frac{R_2}{2}$

$$\therefore \quad \frac{1}{F} = 2(\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{2}{R_2}$$

(b) If the lens is equi-convex, then $R_1 = R_2 = R$

$$\frac{1}{F} = 2(\mu - 1)\frac{2}{R} + \frac{2}{R} = \frac{2(2\mu - 1)}{R}$$

$$\Rightarrow \quad F = \frac{R}{2(2\mu - 1)}$$

(c) If the plane face of a plano-convex lens is silvered (Fig. 26.17), then

($R_1 = R$, $R_2 = \infty$)

$$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m}$$

where $f = \frac{(\mu - 1)}{R}$

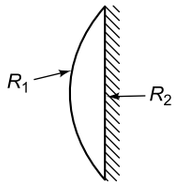


Fig. 26.17

and $f_m = \infty$

Thus $F = \frac{f}{2} = \frac{R}{2(\mu - 1)}$

(d) If the curved face of a plano-convex lens is silvered (Fig. 26.18), then $R_1 = \infty$ and $R_2 = R$.

$$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m}$$

where $\frac{1}{f} = \frac{(\mu - 1)}{R}$

and $f_m = \frac{R}{2}$

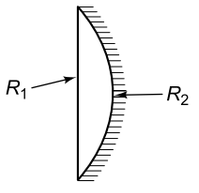


Fig. 26.18

Thus $\frac{1}{F} = \frac{2(\mu - 1)}{R} + \frac{2}{R} = \frac{2\mu}{R}$

$$\Rightarrow \quad F = \frac{R}{2\mu}$$

EXAMPLE 26.1

A vessel of depth t is half filled with oil of refractive index μ_1 and the other half is filled with water of refractive index μ_2. Find the apparent depth of the vessel when viewed from above.

SOLUTION

Refer to Fig. 26.19

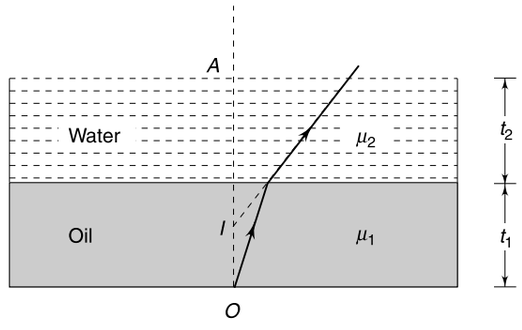


Fig. 26.19

The apparent depth is given by

$$AI = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} = \frac{t}{2}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$$

$$= \frac{t(\mu_1 + \mu_2)}{2\,\mu_1\,\mu_2}$$

EXAMPLE 26.2

A ray of light incident normally on face AB of an isosceles prism travels as shown in Fig. 26.20. What is the least value of the refractive index of the prism?

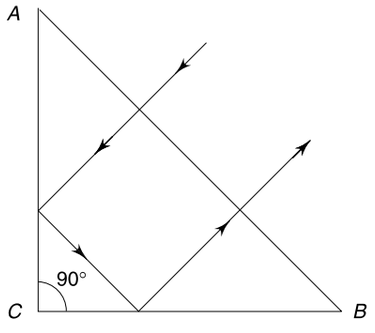


Fig. 26.20

SOLUTION

The ray is totally reflected at faces AC and CB. The angle of incident at each face is at least $i_c = 45°$.

Therefore

$$\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45^\circ} = \sqrt{2}$$

EXAMPLE 26.3

The linear magnification of an object placed on the principal axis of a convex lens of focal length 30 cm is found to be + 2. In order to obtain a magnification of –2, by how much distance should the object be moved?

SOLUTION

In the first case, the magnification is positive which implies that the image is erect, virtual and on the same side of the lens as the object. If a is the object distance then $u = -a$ and $v = -2a$. From the lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or $$\frac{1}{-2a} - \frac{1}{-a} = \frac{1}{30} \Rightarrow a = 15 \text{ cm}$$

So the object is at a distance of 15 cm from the lens. In the second case, the magnification is negative, the image is real, inverted and on the other side of the lens as the object. If b is the object distance, the $u = -b$ and $v = +2b$. Hence

$$\frac{1}{2b} - \frac{1}{-b} = \frac{1}{30} \Rightarrow b = 45 \text{ cm}$$

Thus the object has to be moved through a distance of (45 – 15) = 30 cm away from the lens.

EXAMPLE 26.4

A parallel beam of light falls on a convex lens. The path of the rays is shown in Fig. 26.21. It follows that

(a) $\mu_1 > \mu > \mu_2$ (b) $\mu_1 < \mu < \mu_2$

(c) $\mu_1 = \mu < \mu_2$ (d) $\mu_1 = \mu > \mu_2$

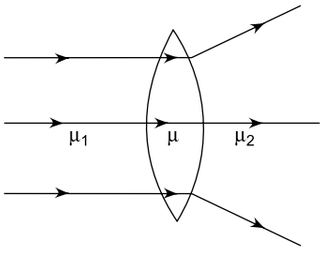


Fig. 26.21

SOLUTION

Since the ray does not bend at the first face of the lens, i.e. it is not refracted, $\mu_1 = \mu$. When this ray meets the second face, it bends towards the normal to that face. Hence $\mu_2 > \mu$. Hence the correct choice is (c).

EXAMPLE 26.5

A lens of power + 2.0 D is placed in contact with another lens of power – 1.0 D. The combination will behave like

(a) A converging lens of focal length 100 cm

(b) a diverging lens of focal length 100 cm

(c) a converging lens of focal length 50 cm

(d) a diverging lens of focal length 50 cm

SOLUTION

Given $P_1 = +2\text{D}$, $P_2 = -1\text{D}$. The power of the combination is

$$P = P_1 + P_2 = 2\text{D} - 1\text{D} = +1\text{D}$$

$\therefore$ Focal length of combination $F = +1\text{m} = +100$ cm. Since F is positive, the combination will behave like a converging lens of focal length 100 cm.

EXAMPLE 26.6

The distance between the object and the real image formed by a convex lens is d. If the linear magnification is m, find the focal length of the lens in terms of d and m.

SOLUTION

The convex lens formula for a real image is

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad \text{(i)}$$

where no sign conventions are to be used. Multiplying by u we get

$$\frac{u}{v} + 1 = \frac{u}{f} \text{ or } \frac{1}{m} = \frac{u}{f} - 1 = \frac{u-f}{f}$$

or $$m(u-f) = f$$

or $$u = \frac{(1+m)f}{m} \qquad \text{(ii)}$$

Multiplying (i) by v we get

$$1 + \frac{v}{u} = \frac{v}{f} \text{ or } 1 + m = \frac{v}{f}$$

or $$v = f(1+m) \qquad \text{(iii)}$$

Now $u + v = d$. Using (ii) and (iii) we have

$$d = \frac{(1+m)f}{m} + f\,(1 + m)$$

which gives $f = \dfrac{md}{(1+m)^2}$.

EXAMPLE 26.7

A concave lens of focal length f forms an image which is n times the size of the object. What the distance of the object from the lens in terms of f and n?

SOLUTION

The concave lens formula is $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

where no sign conventions are to be used. Thus

$$\frac{u}{v} - 1 = \frac{u}{f} \text{ or } \frac{1}{n} - 1 = \frac{u}{f} \quad \left(\because \frac{v}{u} = n\right)$$

or $$u = \left(\frac{1-n}{n}\right) f\,.$$

EXAMPLE 26.8

A prism PQR of refractive index $\sqrt{2}$ has the refracting angle of 30°. Face PR of the prism is silvered. Find the angle of incidence i on face PQ so that the ray retraces its path after reflection from face PR (Fig. 26.22)

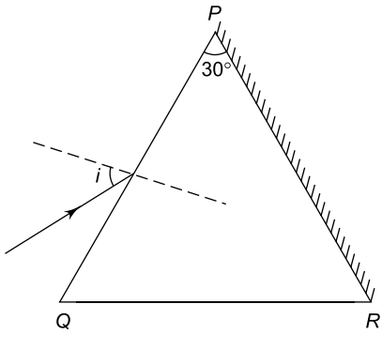


Fig. 26.22

SOLUTION

Refer to Fig. 26.23. The ray AB is refracted along BC in the prism. This ray will retrace its path if it falls normally on the silvered face PR. Therefore, $\angle PBC = 60°$ and angle $r = 90° - 60° = 30°$.

From Snell's law

$$\mu = \frac{\sin i}{\sin r} \Rightarrow \sin i = \mu \sin r = \sqrt{2} \times \sin 30° = \frac{1}{\sqrt{2}}$$

which gives $i = 45°$.

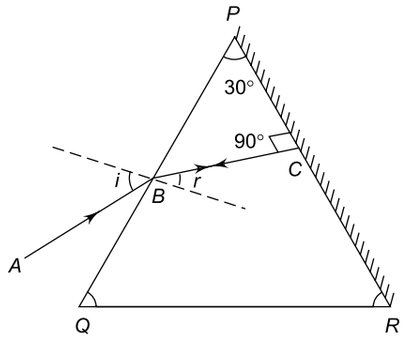


Fig. 26.23

EXAMPLE 26.9

A prism is made of glass of refractive index $\sqrt{3}$. The refracting angle of the prism is A. If the angle of minimum deviation is equal to A, find the value of A.

SOLUTION

Given $\mu = \sqrt{3}$ and $\delta_m = A$. Now

$$\mu = \frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \frac{\sin\left(\dfrac{A+A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \frac{\sin A}{\sin\left(\dfrac{A}{2}\right)}$$

$$= \frac{2\sin\left(\dfrac{A}{2}\right)\cos\left(\dfrac{A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = 2\cos\left(\frac{A}{2}\right)$$

$$\therefore \quad 2\cos\left(\frac{A}{2}\right) = \mu = \sqrt{3} \text{ or } \cos\left(\frac{A}{2}\right) = \frac{\sqrt{3}}{2}$$

or $$\frac{A}{2} = 30° \text{ or } A = 60°$$

EXAMPLE 26.10

The refractive index of the material of an equilateral prism is $\sqrt{3}$. What is the angle of minimum deviation?

SOLUTION

Refractive index $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$. Now $A = 60°$ and $\mu = \sqrt{3}$. Therefore

$$\sqrt{3} = \frac{\sin\left(30° + \dfrac{\delta_m}{2}\right)}{\sin 30°}$$

$$\text{or} \quad \sin\left(30° + \frac{\delta_m}{2}\right) = \sqrt{3} \sin 30° = \frac{\sqrt{3}}{2}$$

$$\text{or} \quad 30° + \frac{\delta_m}{2} = 60° \text{ which gives } \delta_m = 60°$$

26.14 COMPOUND MICROSCOPE

In its simplest form, a compound microscope consists of two convergent (convex) lenses, of a very short focal length called the *objective* and the other of a longer focal length called the *eye-piece*. The lenses are mounted co-axially and the separation between them can be varied.

The magnifying power of a microscope is

M = magnification by objective × magnification by eye-piece

$= m_0 \times m_e$

(a) If the final image is formed at the least distance of distinct vision (strained eye)

$$|M| = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

where u_0 = distance of object from objective, v_0 = distance (from the objective) of the image formed by objective, f_e = focal length of eye-piece and D = least distance of distinct vision.

(b) If the final image is formed at infinity (relaxed eye)

$$|M| = \frac{v_0}{u_0}\frac{D}{f_e}$$

26.15 TELESCOPE

A telescope consists of two convergent lenses called the objective and the eye-piece. The focal length of the objective is much larger than that of the eye-piece.

(a) The magnifying power of a telescope in normal adjustment (i.e. when the image of a distant object is formed at infinity), i.e. for relaxed eye

$$|M| = \frac{f_0}{f_e}$$

f_0 = focal length of objective, f_e = focal length of eye-piece

L = length of tube = $f_0 + f_e$.

(b) When the final image is formed at the least distance of distinct vision (i.e. for strained eye)

$$|M| = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

EXAMPLE 26.11

A wire mesh consists of squares each of side 1 mm. It is placed at a distance of 9.0 cm from a magnifying glass of focal length 10 cm. It is viewed by placing the eye close to the lens. (a) What is the magnification produced by the lens? (b) What is the area of each square in the virtual image of the mesh? (c) What is the magnifying power of the lens? (d) Is the magnification (a) equal to the magnifying power in (c)? If not, when can they be made equal to each other? Explain.

SOLUTION

A magnifying glass is just a convex lens. Therefore f = +10 cm. Also u = – 9.0 cm.

(a) The lens formula is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Putting f = +10 cm and u = – 9.0 cm, we have

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{9.0}$$

Which gives v = – 90 cm. Therefore the magnification is

$$m = \frac{v}{u} = \frac{-90}{-9.0} = +10$$

(b) Area of each square in the mesh is 1 mm × 1 mm, 1 mm^2. Since each side is magnified 10 times, the area of each square in the image is 10 mm × 10 mm = 100 mm^2

(c) The magnifying power of a magnifying glass is

$$M = \frac{D}{x}$$

Where x is the numerical value of the distance of the object from the lens. Here x = 9.0 cm. Also D, the least diatance of distinct vision is 25 cm. Therefore.

$$M = \frac{25}{9.0} = 2.8$$

(d) Magnification found in (a) is different from magnifying power obtained in (c). They are different quantities. However, they become equal to each other in the special situation when the distance of the object is such that its image is formed at the least distance of distinct vision (D). Putting $v = -D = -25$ cm and f = + 10 cm in the lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we have $-\frac{1}{u}=\frac{1}{10}+\frac{1}{25}=\frac{7}{50}$ or $u=-\frac{50}{7}$ cm

$\Rightarrow \quad x=\frac{50}{7}$ cm

Therefore the magnification is

$$m=\frac{v}{u}=\frac{-25}{-50/7}=+3.5$$

and the magnifying power is

$$M=\frac{D}{x}=1+\frac{D}{f}=1+\frac{25}{10}=3.5$$

EXAMPLE 26.12

Find the magnifying power of a compound miscroscope whose objective and eyepiece are of focal lengths 4.0 cm and 6.0 cm respectively and the object is placed 5.0 cm beyond the objective. Assume that the final image is formed at the least distance of distinct vision (25 cm).

SOLUTION

$u=-5.0$ cm, $f_o=+4.0$ cm and $f_e=+6.0$ cm. For the objective

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_o}$$

Putting $u=-5.0$ cm and $f_o=+4.0$ cm we have

$$\frac{1}{v}=\frac{1}{4}-\frac{1}{5}$$

which gives $v=20$ cm. Therefore, magnifying power is

$$M=\frac{v}{u}\left(1+\frac{D}{f_e}\right)=\frac{20}{-5}\left(1+\frac{25}{6}\right)=-20.7$$

The negative sign indicates that final image is virtual.

EXAMPLE 26.13

A compound microscope has a magnification of 30. Assuming that the final image is formed at the least distance of distinct vision (25 cm), find the magnification produced by the objective. Given, the focal length of the eyepiece is 5.0 cm.

SOLUTION

$M=30$, $f_e=5.0$ cm and $D=25$ cm. We know that

$$M=m_o\times m_e=m_o\left(1+\frac{D}{f_e}\right) \text{ or } 30=m_o\left(1+\frac{25}{5.0}\right)$$

which gives $m_o=5.0$

EXAMPLE 26.14

A magnifying power of 30 is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope? The final image is formed at $D=25$ cm.

SOLUTION

Required magnifying power $M=30$, $f_o=1.25$ cm and $f_e=5$ cm. We know that

$$M=m_o\times m_e=m_o\left(1+\frac{D}{f_e}\right) \text{ or } 30=m_o\left(1+\frac{25}{5}\right)$$

which gives $m_o=5$, i.e. the objective should produce a magnification $m_o=5$ and the eyepiece should produce a magnification $m_e=6$.

Position of object Let us first find where the object AB should be placed from the objective so that a real image $A'B'$ magnified five times is obtained. Since $A'B'$ is inverted we have [refer to rough ray diagram Fig. 26.24]

$$m_o=\frac{v}{u}=-5 \text{ or } v=-5u$$

Using the lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_0}$$

we have $\frac{1}{-5u}-\frac{1}{u}=\frac{1}{1.25}$

which gives $u=-1.5$ cm. Thus the object AB should be placed 1.5 cm to the left of the objective. The distance of $A'B'$ from the objective will then be

$$v=-5u=-5\times(-1.5)=7.5 \text{ cm}$$

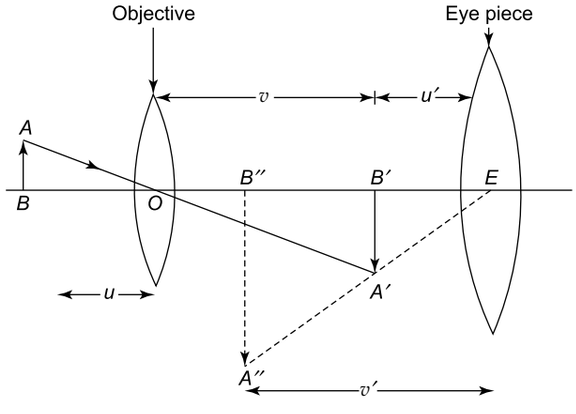


Fig. 26.24

Positioning of eyepiece We will now find where the eyepiece should be positioned so that it produces a magnification $m_e = 6$, i.e. the final image $A''B''$ is six times $A'B'$. It is given that $v' = -D = -25$ cm. Since the final image is virtual,

$$m_o = \frac{v'}{u'} = +6 \text{ or } v' = 6u'$$

Applying the lens formula to the eyepiece we have

$$\frac{1}{v'} - \frac{1}{u'} = \frac{1}{f_e} \text{ or } \frac{1}{6u'} - \frac{1}{u'} = \frac{1}{5}$$

which gives $$u' = -\frac{25}{6} = -4.17 \text{ cm}$$

Thus, the eyepiece should be at a distance of 4.17 cm to the right of $A'B'$. Hence the separation between the objective and the eyepiece should be

$$OE = OB' + B'E = v + |u'| = 7.5 + 4.17 = 11.67 \text{ cm}$$

Thus we find that the microscope will give a magnifying power of 30 if it is set up as follows: the distance between the objective and the eyepiece should be 11.67 cm and the object should be placed at a distance of 1.5 cm in front of the objective.

EXAMPLE 26.15

A refracting (astronomical) telescope, when in normal adjustment, has a magnifying power of 6 and the objective and the eyepiece are 14 cm apart. Find the focal lengths of the lenses of the telescope.

SOLUTION

In normal adjustment, the object and the final image are both at infinity. The separation between the two lenses is $(f_o + f_e)$. Therefore

$$f_o + f_e = 14 \quad \text{(i)}$$

Also $$M = \frac{f_o}{f_e} = 6 \text{ or } f_o = 6 f_e \quad \text{(ii)}$$

Using (ii) in (i) we get $f_e = 2$ cm and $f_o = 12$ cm.

EXAMPLE 26.16

A telescope has an objective of focal length 120 cm and an eyepiece of focal length 5.0 cm. Find the magnifying power of the telescope when it is used to view distant objects when (a) the telescope is in normal adjustment, and (b) the final image is formed at the least distance of distinct vision (25 cm).

SOLUTION

$f_o = 120$ cm and $f_e = 5.0$ cm.

(a) In normal adjustment, the magnifying power is

$$M = \frac{f_o}{f_e} = \frac{120}{5.0} = 24$$

(b) When the object is at infinity (distant object) and the final image formed at $D = 25$ cm, the magnifying power is

$$M = \frac{f_o}{f_e} \times \frac{D + f_e}{D} = \frac{120}{5.0} \times \frac{25 + 5.0}{25} = 28.8$$

26.16 DISPERSION OF LIGHT

Refraction of light occurs because the velocity of light changes as it travels from one medium into another. The velocity of light in a given medium depends upon its wavelength (or frequency). Light of a single wavelength or frequency is called monochromatic. White light (sunlight) is not monochromatic, it consists of many wavelengths ranging from violet (~ 300 nm) to red (~ 700 nm).

When white light (or any composite light) enters a refracting medium, the different constituent colours are refracted unequally. The red is refracted the least and the violet the most. From Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

it follows that the refractive index for red is less than for violet, i.e., the speed of red light is greater than that of violet light. Thus, a medium does not have one definite refractive index; it has a range of refractive indices corresponding to the range of colours or wavelengths of the composite light. Since each colour has its own characteristic wavelength (or frequency), the refractive index of a medium will be different for different wavelengths (or frequencies). The variation of refractive index of a medium (and hence of the velocity of light in the medium) with the wavelength or frequency is referred to as *dispersion.* The prism disperses the colours of white light and produces its spectrum.

Angular Dispersion and Dispersion Power

If δ_V and δ_R are the deviations of violet and red lights produced by a prism of refracting angle A, the angular dispersion θ is given by

$$\theta = \delta_V - \delta_R$$

Now $\delta_V = (\mu_V - 1)A$ and $\delta_R = (\mu_R - 1)A$, where μ_V and μ_R are the refractive indices of the material of the prism for violet and red lights respectively. Hence

$$\theta = (\mu_V - \mu_R)A$$

Mean deviation $\delta = (\mu - 1)A$, where $\mu = \frac{1}{2}(\mu_V + \mu_R)$.

The dispersive power of a prism is its ability to deviate the different colours of a composite light along different directions and is defined as

$$\omega = \frac{\theta}{\delta} = \frac{(\mu_V - \mu_R)A}{(\mu - 1)A} = \frac{\mu_V - \mu_R}{\mu - 1}$$

Dispersion without Deviation and Deviation without Dispersion

If two prisms are arranged as shown in Fig. 26.25, then net angular dispersion $\Delta\theta = \theta - \theta'$

$$= (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A'$$

Net deviation $\Delta\delta = \delta - \delta' = (\mu - 1)A + (\mu' - 1)A'$

(a) For dispersion without deviation

$$\Delta\delta = 0 \quad \Rightarrow \quad (\mu - 1)A + (\mu' - 1)A' = 0$$

(b) For deviation without dispersion

$$\Delta\theta = 0 \quad \Rightarrow \quad (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A' = 0$$

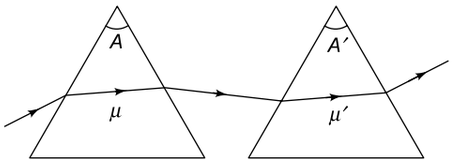


Fig. 26.25

26.17 ABERRATIONS

(i) Spherical aberration

Spherical aberration occurs in a lens or a spherical mirror due to spherical nature of the surface. This defect arises because the paraxial and marginal rays do not focus at a single point (Fig. 26.26).

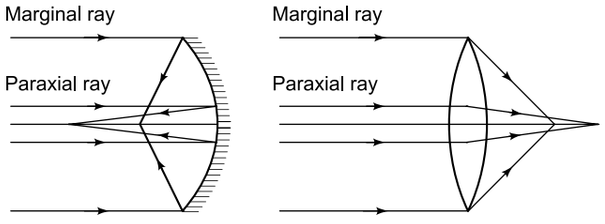


Fig. 26.26

Spherical aberration is minimized by

(a) using a plano-convex lens with the incident rays falling on the curved face

(b) using two plano-convex lens of focal lengths f_1 and f_2 separated by a distance $d = f_1 - f_2$.

(c) using a parabolic mirror.

(ii) Chromatic aberration

Chromatic aberration occurs only in lenses and not in mirrors. This defect arises because the refractive index and hence the focal length) of a lens is different for the different colours of light. In fact $f_R > f_Y > f_V$. Hence rays of white light do not focus at a single point (Fig. 26.27)

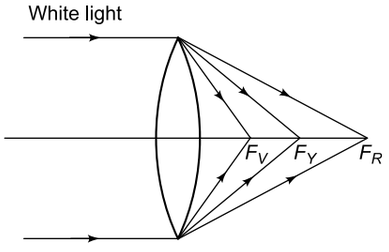


Fig. 26.27

Chromatic aberration is removed by

(a) using a convex lens is contact with a concave lens such that (see Fig. 26.28)

$$\frac{f}{f'} = -\frac{\omega}{\omega'}$$

where ω and ω' are dispersive powers of the material of the lenses.

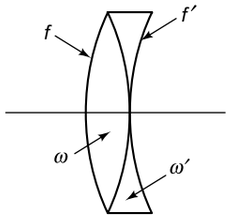


Fig. 26.28

(b) using two lenses made of the same material separated by a distance $d = \frac{1}{2}(f + f')$.

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. A plane mirror is placed at $x = 0$ with its plane parallel to the y-axis. An object starts from $x = 3$ m and moves with a velocity of $(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ ms^{-1} towards the mirror. The relative velocity of the image with respect to the object is

(a) $2\sqrt{2}$ ms^{-1} making an angle of 45° with the + x axis

(b) $2\sqrt{2}$ ms^{-1} making an angle of 135° with the + x axis

(c) 4 ms^{-1} along the $-x$ axis
(d) 4 ms^{-1} along the $+x$ axis

2. A plane mirror is made of glass of thickness 3 cm and refractive index 1.5 by silvering one of its faces. A point object is placed a distance of 6 cm in front of the unsilvered face. The distance of the image from the silvered face is
(a) 6 cm (b) 7 cm
(c) 12 cm (d) 15 cm

3. The image distance (v) is plotted against the object distance (u) for a concave mirror of focal length f. Which of the graphs shown in Fig. 26.29 represents the variation of v versus u as u is varied from zero to infinity?

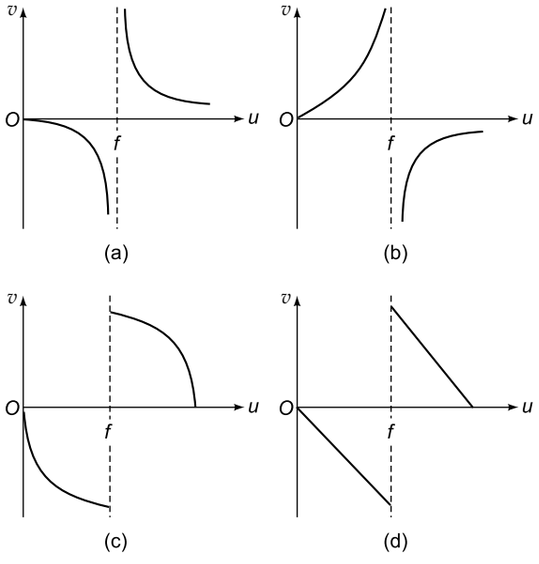


Fig. 26.29

4. A hollow thin convex lens made of glass is placed in air. It will behave like a
(a) convex lens (b) concave lens
(c) prism (d) glass plate

5. Two thin equi-convex lenses each of focal length f and made of glass $\left(\mu_g = \frac{3}{2}\right)$ are placed in contact. The space between them is filled with water $\left(\mu_\omega = \frac{4}{3}\right)$. The focal length of the combination is
(a) $\frac{f}{2}$ (b) $\frac{2f}{3}$
(c) $\frac{3f}{5}$ (d) $\frac{4f}{7}$

6. Two lenses A and B of focal lengths 30 cm and 20 cm are placed co-axially a distance d apart. A ray of light parallel to the common principal axis is incident on lens A as shown in Fig. 26.30. What should be the value of d so that the ray emerges from lens B without suffering any deviation?
(a) 50 cm (b) 40 cm
(c) 25 cm (d) 10 cm

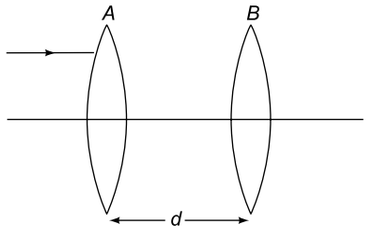


Fig. 26.30

7. A convex lens has a diameter d and focal length f. A point object is placed on its principal axis at a distance $3f$ from it. The eye of an observer is placed at a distance of $3f$ on the other side of the lens and at a distance h below the principal axis. The maximum value of h so that the observer can see the image of the object is
(a) d (b) $\frac{2d}{3}$
(c) $\frac{d}{2}$ (d) $\frac{d}{3}$

8. A glass prism ABC of refractive index 1.5 is immersed in water of refractive index 4/3 as shown in Fig. 26.31. A ray of light incident normally on face AB is totally reflected at face AC if
(a) $\sin\theta \geq 8/9$
(b) $\sin\theta \geq 2/3$
(c) $\sin\theta = \sqrt{3}/2$
(d) $2/3 < \sin\theta < 8/9$

IIT, 1981

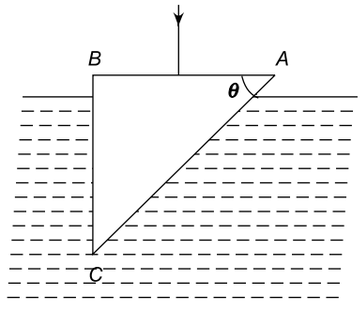


Fig. 26.31

9. What is the relation between refractive indices μ, μ_1 and μ_2 if the behaviour of light rays is as shown in Fig. 26.32.

(a) $\mu > \mu_2 > \mu_1$ (b) $\mu < \mu_2 < \mu_1$

(c) $\mu < \mu_2$; $\mu = \mu_1$ (d) $\mu_2 < \mu_1$; $\mu = \mu_2$

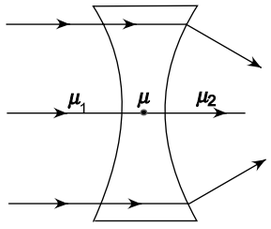


Fig. 26.32

10. A lens forms a sharp image on a screen. On inserting a parallel sided glass slab between the lens and the screen, it is found necessary to move the screen a distance d away from the lens in order to focus the image sharply. If the refractive index of glass relative to air is μ, the thickness of the glass slab is given by

(a) d/μ (b) $\mu\, d$

(c) $\dfrac{d}{\left(1-\dfrac{1}{\mu}\right)}$ (d) $\left(1-\dfrac{1}{\mu}\right) d$

11. A convex lens is placed between an object and a screen which are a fixed distance apart. For one position of the lens the magnification of the image obtained on the screen is m_1. When the lens is moved by a distance d, the magnification of the image obtained on the same screen is m_2. The focal length of the lens is $(m_1 > m_2)$.

(a) $\dfrac{d}{(m_1 - m_2)}$ (b) $\dfrac{d}{(m_1 + m_2)}$

(c) $d\,\dfrac{m_1}{m_2}$ (d) $d\,\dfrac{m_2}{m_1}$

12. A ray of light, travelling in a medium of refractive index μ, is incident at an angle i on a composite transparent plate consisting of three plates of refractive indices μ_1, μ_2 and μ_3. The ray emerges from the composite plate into a medium of refractive index μ', at angle x. Then

(a) $\sin x = \sin i$

(b) $\sin x = \dfrac{\mu}{\mu'} \sin i$

(c) $\sin x = \dfrac{\mu'}{\mu} \sin i$

(d) $\sin x = \dfrac{\mu_1}{\mu_2} \cdot \dfrac{\mu_3}{\mu_2} \cdot \dfrac{\mu}{\mu'} \sin i$

13. The principal section of a glass prism is an isosceles triangle ABC with $AB = AC$. The face AC is silvered. A ray incident normally on face AB, after two reflections, emerges from the base BC in a direction perpendicular to it. What is the $\angle BAC$ of the prism?

(a) 30° (b) 36°

(c) 60° (d) 72°

14. An object is placed at a distance of 10 cm from a co-axial combination of two lenses A and B in contact. The combination forms a real image three times the size of the object. If lens B is concave with a focal length of 30 cm, what is the nature and focal length of lens A?

(a) Convex, 12 cm (b) Concave, 12 cm

(c) Convex, 6 cm (d) Convex, 18 cm

15. A plano-convex lens acts like a concave mirror of 28 cm focal length when its plane surface is silvered and like a concave mirror of 10 cm focal length when its curved surface is silvered. What is the refractive index of the material of the lens?

(a) 1.50 (b) 1.55

(c) 1.60 (d) 1.65

16. A pin is placed 10 cm in front of a convex lens of focal length 20 cm and refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature of 22 cm. How far from the lens is the final image formed?

(a) 10 cm (b) 11 cm

(c) 12 cm (d) 13 cm

17. Two glasses have dispersive powers in the ratio of 2 : 3. These glasses are used in the manufacture of an achromatic objective of focal length 20 cm. What are the focal lengths of the two lenses of the objective?

(a) 6.67 cm, – 10 cm (b) 7.5 cm, – 12.5 cm

(c) 9.67 cm, – 15 cm (d) 12.5 cm, – 20 cm

18. A giant telescope in an observatory has an objective of focal length 19 m and an eye-piece of focal length 1.0 cm. In normal adjustment, the telescope is used to view the moon. What is the diameter of the image of the moon formed by the objective? The diameter of the moon is 3.5×10^6 m and the radius of the lunar orbit round the earth is 3.8×10^8 m.

(a) 10 cm (b) 12.5 cm

(c) 15 cm (d) 17.5 cm

19. Two convex lenses of focal lengths f_1 and f_2 are separated co-axially by a distance d. The power of the combination will be zero if

(a) $d = \dfrac{f_1 + f_2}{2}$ (b) $d = \dfrac{f_1 - f_2}{2}$

(c) $d = f_1 + f_2$ (d) $d = \sqrt{f_1 f_2}$

20. A thin lens of focal length f has a aperture of diameter d. It forms an image of intensity I. Now, the central part of the aperture upto diameter $d/2$ is blocked by opaque paper. The focal length and the image intensity will change to

(a) $\frac{f}{2}, \frac{I}{2}$ (b) $f, \frac{I}{4}$

(c) $\frac{3f}{4}, \frac{I}{2}$ (d) $f, \frac{3I}{4}$

21. The diameter of a plano-convex lens is 6 cm and the thickness at the centre is 3 mm. If the speed of light in the material of the lens is 2×10^8 ms^{-1}, the focal length of the lens is

(a) 15 cm (b) 20 cm

(c) 30 cm (d) 10 cm

22. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of prism P_2 is

(a) 5.33° (b) 4°

(c) 3° (d) 2.6°

IIT, 1990

23. A ray is incident at an angle of incidence i on one face of a prism of a small angle A and emerges normally from the opposite face. If the refractive index of the prism is μ, the angle of incidence i is nearly equal to

(a) $\frac{A}{\mu}$ (b) $\frac{A}{2\mu}$

(c) μA (d) $\frac{\mu A}{2}$

24. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen,

(a) half the image will disappear
(b) complete image will be formed
(c) intensity of the image will increase
(d) none of these

IIT, 1986

25. The angle of a prism is A and one of its refracting surfaces is silvered. Light rays falling at an angle of incidence 2 A on the first surface return back through the same path after suffering reflection at the second (silvered) surface. The refractive of the material of the prism is

(a) 2 sin A (b) 2 cos A

(c) $\frac{1}{2}$ cos A (d) tan A

26. When a ray of light enters a glass slab from air,

(a) its wavelength decreases
(b) its wavelength increases
(c) its frequency increases
(d) neither wavelength nor frequency changes

IIT, 1980

27. A beam of light consisting or red, green and blue colours is incident on a right-angled prism ABC as shown in Fig. 26.33. The refractive indices of the material of the prism for red, green and blue wavelengths, respectively are 1.39, 1.44 and 1.47. The prism will

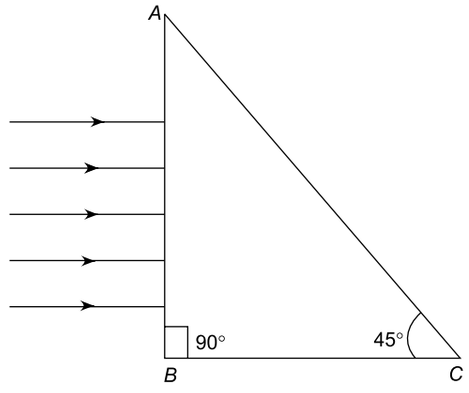


Fig. 26.33

(a) separate part of the red colour from green and blue colours
(b) separate part of the blue colour from red and green colours
(c) separate part of three colours from one another
(d) not separate even partially any colour from the other two colours.

IIT, 1989

28. Spherical aberration in a thin lens can be reduced by

(a) using a monochromasstic light
(b) using a doublet combination
(c) using a circular annular mask over the lens
(d) increasing the size of the lens.

IIT, 1994

29. A real image of a distant object is formed by a plano-convex lens on its principal axis. The spherical aberration

(a) is absent
(b) is smaller if the curved surfaces of the lens faces the object
(c) is smaller if the plane surface of the lens faces the object

(d) is the same whichever side of the lens faces the object

IIT, 1998

30. A concave mirror is placed on a horizontal table, with its axis directed vertically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be

(a) real and will remain at C
(b) real and located at a point between C and infinity
(c) virtual and located at a point between C and O
(d) real and located at a point between C and O.

IIT, 1998

31. A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at point O and $PO = OQ$. The distance PO is equal to

(a) $5R$ (b) $3R$
(c) $2R$ (d) $1.5R$

IIT, 1998

32. An eye specialist prescribes spectacles having a combination of a convex lens of focal length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination is

(a) + 1.5 D (b) – 1.5 D
(c) + 6.67 D (d) – 6.67 D

IIT, 1997

33. A concave lens of glass, refractive index 1.5, has both surfaces of the same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a

(a) convergent lens of focal length 3.5 R
(b) convergent lens of focal length 3.0 R
(c) divergent lens of focal length 3.5 R
(d) divergent lens of focal length 3.0 R

IIT, 1999

34. In a compound microscope, the intermediate image is

(a) real, inverted and magnified
(b) real, erect and magnified
(c) virtual, erect and magnified
(d) virtual, erect and reduced.

IIT, 2000

35. A hollow double concave lens is made of a very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices n_1 and n_2 respectively ($n_2 > n_1 > 1$). The lens will diverge a parallel beam of light if it is filled with

(a) air and placed in air
(b) air and immersed in L_1
(c) L_1 and immersed in L_2
(d) L_2 and immersed in L_1.

IIT, 2000

36. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown in Fig. 26.34. The angles of incidence for the two extreme rays are equal. If the thickness of the slab is t and refractive index is n, then the divergence angle of the emergent beam is

(a) zero (b) α
(c) $\sin^{-1}\left(\frac{1}{n}\right)$ (d) $2\sin^{-1}\left(\frac{1}{n}\right)$

IIT, 2000

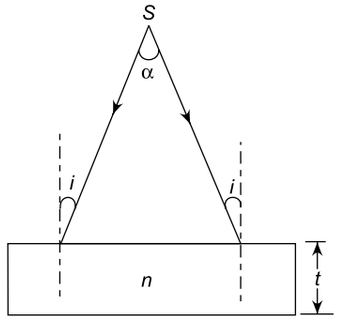


Fig. 26.34

37. A rectangular glass slab $ABCD$ of refractive index n_1 is immersed in water of refractive index n_2 ($n_1 > n_2$). A ray of light is incident at the face AB of the slab as shown in Fig. 26.35. The maximum value of the angle of incidence α_{max}, such that the ray comes out only from the other side CD is given by

(a) $\sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\frac{n_2}{n_1}\right)\right]$

(b) $\sin^{-1}\left[n_1\cos\left(\sin^{-1}\frac{1}{n_2}\right)\right]$

(c) $\sin^{-1}\left(\frac{n_1}{n_2}\right)$

(d) $\sin^{-1}\left(\frac{n_2}{n_1}\right)$

IIT, 2000

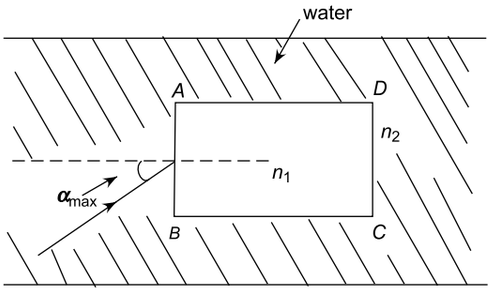


Fig. 26.35

38. When a glass prism of refracting angle 60° is immersed in a liquid, its angle of minimum deviation is 30°. The critical angle of glass with respect to the liquid medium is

(a) 42° (b) 45°

(c) 50° (d) 52°

39. In the visible region, the dispersive powers and the mean angular deviations for crown and flint glass prism are ω and ω' and d and d' respectively. When the two prisms are combined, the condition of zero dispersion by the combination is

(a) $\sqrt{\omega d} + \sqrt{\omega' d'} = 0$

(b) $\omega' d + \omega d' = 0$

(c) $\omega d + \omega' d' = 0$

(d) $(\omega d)^2 + (\omega' d')^2 = 0$

40. A ray of light is incident normally on one face of a 90° prism and is totally internally reflected at the glass-air interface as shown in Fig. 26.36. If the angle of reflection is 45° and the refractive index of the prism is n, then

(a) $n < \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$

(c) $n > \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$

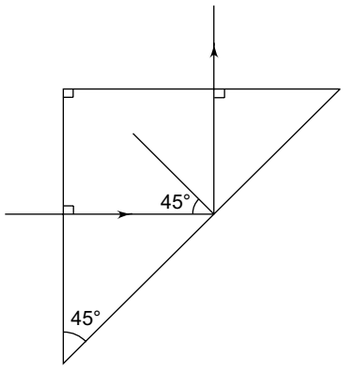


Fig. 26.36

41. A plano convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of the size of the object?

(a) 20 cm (b) 30 cm

(c) 60 cm (d) 80 cm

42. The two lenses of an achromatic doublet should have

(a) equal powers

(b) equal dispersive powers

(c) equal ratio of their power and dispersive power

(d) equal product of their power and dispersive power.

43. Rays from the sun subtend an angle θ (in radians) at the pole of a concave mirror of focal length f. If the diameter of the sun is D, the diameter of the image of the sun formed by the mirror is

(a) $D\ \theta$ (b) $2\ D\theta$

(c) $f\ \theta$ (d) $2\ f\theta$

44. A point source of light S is placed at a distance L in front of the centre of a plane mirror PQ of width d hung vertically on a wall as shown in Fig. 26.37. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown. The greatest distance over which he can see the image of the light source in the mirror is

(a) $\frac{d}{2}$ (b) d

(c) $2d$ (d) $3d$

IIT, 2000

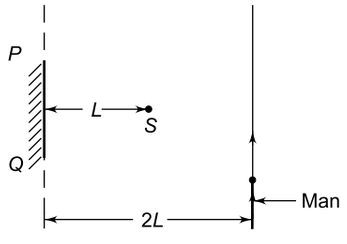


Fig. 26.37

45. A thin glass prism of refractive index 1.5 produces a deviation of 4° of a ray incident at a small angle. What will be the deviation of the same incident ray by the same prism if it is immersed in water of refractive index 4/3?

(a) 1° (b) 2°

(c) 8° (d) 16°

46. An air bubble in a glass slab ($\mu = 1.5$) is 5 cm deep when viewed through one face and 2 cm deep when

viewed through the opposite face. What is the thickness of the slab?

(a) 7.0 cm (b) 7.5 cm
(c) 10.0 cm (d) 10.5 cm

47. The refracting angle of a prism is A and the refractive index is cot (A/2). The angle of minimum deviation is

(a) $180° - A$ (b) $180° - 2A$
(c) $180° - 3A$ (d) $180° - 4A$

48. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in Fig. 26.38. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have

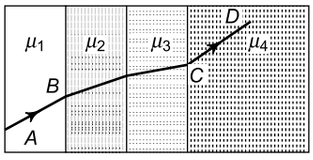


Fig. 26.38

(a) $\mu_1 = \mu_2$ (b) $\mu_2 = \mu_3$
(c) $\mu_3 = \mu_4$ (d) $\mu_4 = \mu_1$

IIT, 2001

49. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prisms Q and R of identical shape and of the same material as P are now added as shown in Fig. 26.39. The ray will now suffer

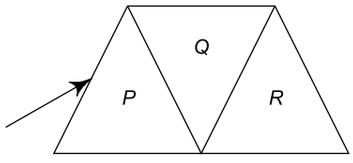


Fig. 26.39

(a) greater deviation
(b) no deviation
(c) same deviation as before
(d) total internal reflection

50. An observer can see through a pin-hole the top end of a thin rod of height h, placed as shown in Fig. 26.40. The beaker height is $3h$ and its radius h. When the beaker is filled with a liquid upto a height $2h$, he can see the lower end of the rod. Then the refractive index of the liquid is

(a) 5/2
(b) $\sqrt{(5/2)}$
(c) $\sqrt{(3/2)}$
(d) 3/2

IIT, 2002

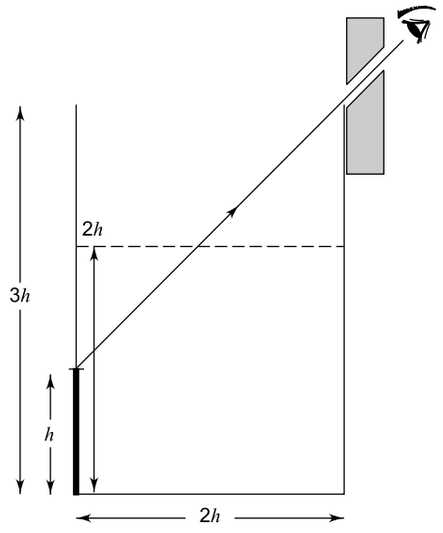


Fig. 26.40

51. Which one of the spherical lenses shown in Fig. 26.41 does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams.

IIT, 2002

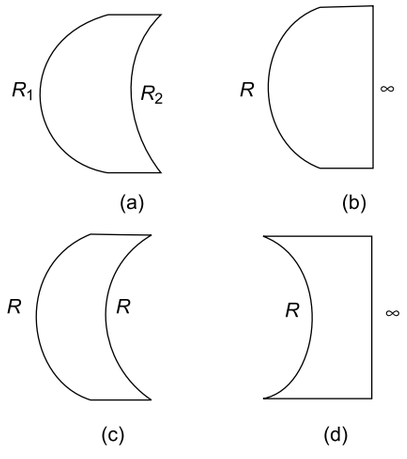


Fig. 26.41

52. Two plane mirrors A and B are aligned parallel to each other, as shown in Fig. 26.42. A light ray is incident at an angle of 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure.

The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is

(a) 28 (b) 30
(c) 32 (d) 34

IIT, 2002

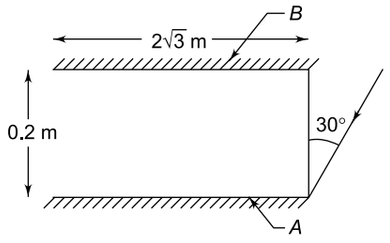


Fig. 26.42

53. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 1.6 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance at 26 cm from the convex lens, the size of the final image will be

(a) 0.8 cm (b) 1.2 cm

(c) 2.0 cm (d) 2.4 cm

IIT, 2003

54. A ray of light is incident at the glass-water interface at an angle i. It emerges finally parallel to the surface of water as shown in the Fig. 26.43. The value of μ_g would be

(a) $\left(\frac{4}{3}\right)\sin i$ (b) $\frac{1}{\sin i}$

(c) $\frac{2}{\sqrt{3}\sin i}$ (d) 1.5

IIT, 2003

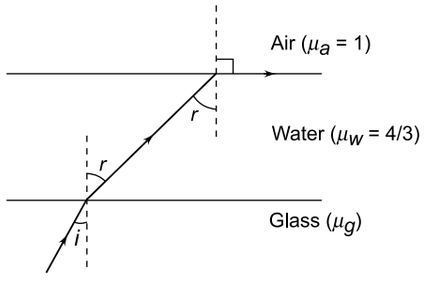


Fig. 26.43

55. A motor car is fitted with a convex driving mirror of focal length 20 cm. A second motor car 2.8 m behind the first car is overtaking at a relative speed of 15 ms^{-1}. The speed of the image of the second car as seen in the mirror of the first is

(a) $\frac{1}{10}$ ms^{-1} (b) $\frac{1}{15}$ ms^{-1}

(c) 10 ms^{-1} (d) 15 ms^{-1}

56. A short linear object of length b lies on the axis of a concave mirror of focal length f at a distance u from the pole. The lenght of the image will be

(a) $\left(\frac{f}{u-f}\right)b$ (b) $\left(\frac{u-f}{f}\right)b$

(c) $\left(\frac{f}{u-f}\right)^2 b$ (d) $\left(\frac{u-f}{f}\right)^2 b$

57. A point source of light S is placed at the bottom of a vessal containing a liquid of refractive index $\sqrt{2}$. A person is viewing the source from above the surface. There is an opaque disc of radius r floating on the surface. The centre of the disc is vertically above the source S. The liquid is gradually drained out from the vessel through a tap. The maximum height of the liquid for which the source cannot be seen from above is

(a) r (b) $\sqrt{2}\,r$

(c) $\sqrt{3}\,r$ (d) $2r$

58. Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index n. The least value of n so that the light entering the rod does not emerge from the curved suface of the rod for any value of α is

(a) $\frac{4}{3}$ (b) $\sqrt{2}$

(c) 1.5 (d) $\sqrt{3}$

59. A ray of light is incident at an angle of 60° on one face of a prism of refracting angle 30°. The ray emerges out of the prism making an angle of 30° with the incident ray. The refractive index of the material of the prism is

(a) $\sqrt{2}$ (b) 1.5

(c) $\sqrt{3}$ (d) $\frac{1}{2}\left(1+\sqrt{3}\right)$

60. A square wire of side 3.0 m is placed 25 cm from a concave mirror of focal length 10 cm. The area enclosed by the image of the wire is

(a) 1 cm^2 (b) 4 cm^2

(c) 16 cm^2 (d) 25 cm^2

61. A ray of light PQ is incident on an isosceles glass prims placed on a horizontal table. If the prism is in the minimum deviation position for the ray PQ, which of the following is true? (See Fig. 26.44).

IIT, 2004

(a) $\alpha = \beta$ (b) $\alpha > \beta$

(c) $\alpha < \beta$ (d) $\alpha + \beta = 90°$

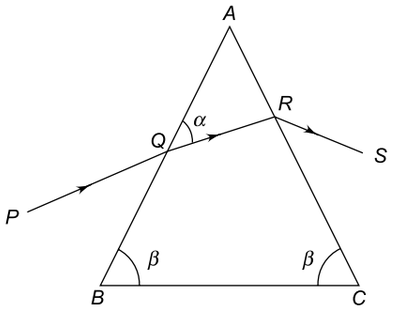


Fig. 26.44

62. White light is incident on face AB of a glass prism. The path of the green component is shown in the figure. If the green light is just totally internally reflected at face AC as shown in Fig. 26.45 the light emerging from face AC will contain

(a) yellow, orange and red colours
(b) violet, indigo and blue colours
(c) all colours
(d) all colours except green.

IIT, 2004

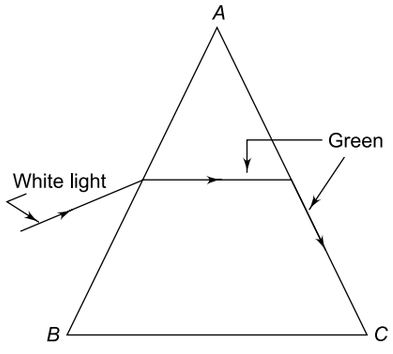


Fig. 26.45

63. A point object is placed at the centre of a glass sphere of diameter 12 cm and refractive index 1.5. What is the distance of the virtual image from the surface of the sphere?

(a) 4 cm (b) 6 cm
(c) 9 cm (d) 12 cm

IIT, 2004

64. A beaker contains water ($\mu = 4/3$) filled to a height of 32 cm. A concave mirror is fixed 6 cm above the surface of water as shown in Fig. 26.46. An object is placed at the bottom of the beaker and its image is formed 14 cm below the surface of water. The focal length of the mirror is

(a) 8 cm (b) 12 cm
(c) 16 cm (d) 20 cm

IIT, 2005

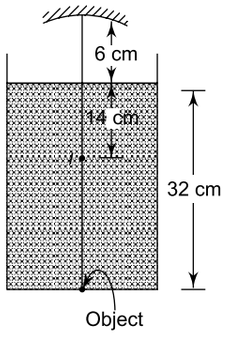


Fig. 26.46

65. A convex lens and a concave lens are placed in contact. The ratio of the magnitude of the power of the convex lens to that of the concave lens is 4 : 3. If the focal length of the convex lens is 12 cm, the focal length of the combination will be

(a) 16 cm (b) 24 cm
(c) 32 cm (d) 48 cm

IIT, 2005

66. A right-angled prism is to be made by selecting a proper material and angles A and B ($B \le A$), as shown in Fig. 26.47. It is desired that a ray of light incident on face AB emerges parallel to the incident direction after two internal reflections. What should be the minimum refractive index n for this to be possible?

IIT, 1987

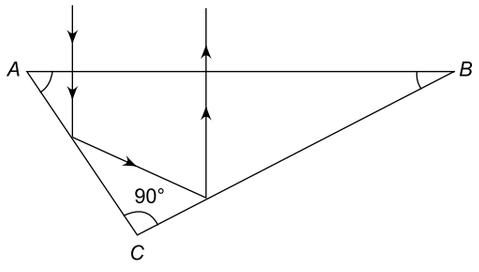


Fig. 26.47

(a) $n_{\min} = \dfrac{1}{\sin A}$

(b) $n_{\min} = \dfrac{1}{\sin B}$

(c) $n_{\min} = \dfrac{\sin A}{\sin B}$

(d) $n_{\min} = \sqrt{\sin A \times \sin B}$

67. A plano-convex lens has thickness 4 cm. When placed on a horizontal table with the curved face in

contact with it, the apparent depth of the bottom-most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face of the lens is found to be $\frac{25}{8}$ cm. The focal length of the lens is

IIT, 1984

(a) 25 cm (b) 50 cm
(c) 75 cm (d) 100 cm

68. Figure 26.48 shows a lens having radii of curvature R_1 and R_2 and $\mu_1 < \mu_2 < \mu_3$. If the thickness of the lens is negligible and $R_1 = R_2 = R$, the focal length of the lens will be

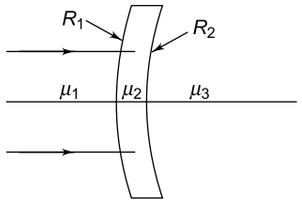


Fig. 26.48

(a) $f = \frac{\mu_3 R}{(\mu_3 - \mu_1)}$ (b) $f = \frac{\mu_2 R}{(\mu_3 - \mu_1)}$

(c) $f = \frac{\mu_1 R}{(\mu_3 - \mu_2)}$ (d) $f = \frac{(\mu_2 - \mu_1)R}{(\mu_3 - \mu_1)}$

IIT, 2003

69. A parallel sides slab *ABCD* of refractive index 2 is sandwiched between two slabs of refractive indices $\sqrt{2}$ and $\sqrt{3}$ as shown in the Fig. 26.49. The minimum value of angle θ such that the ray *PQ* suffers total internal reflection at both the surfaces *AB* and *CD* is

IIT, 2005

(a) 30° (b) 45°
(c) 60° (d) 75°

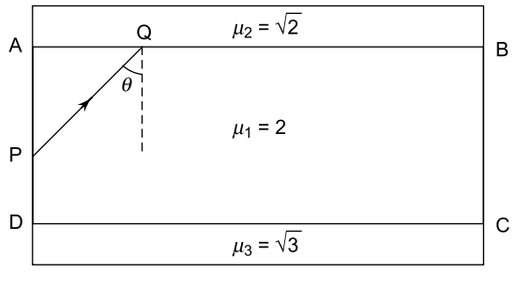


Fig. 26.49

70. Two identical thin isosceles prisms of refracting angle α (in radian) and refractive index μ are placed with their bases touching each other as shown in Fig. 26.50. A ray of light is incident on the prism at a small height *h*. The focal length of this crude converging lens is

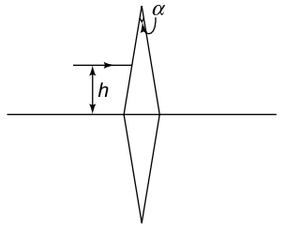


Fig. 26.50

(a) $f = \frac{h\alpha}{\mu}$

(b) $f = \frac{h\alpha}{(\mu - 1)}$

(c) $f = \frac{h}{\alpha\mu}$ (d) $f = \frac{h}{\alpha(\mu - 1)}$

IIT, 2005

71. A ray of light is incident at an angle of 45° on a square slab of a transparent material. What should be the refractive index of the material of the slab so that total internal reflection occurs at the vertical face *AB* (see Fig. 26.51)?

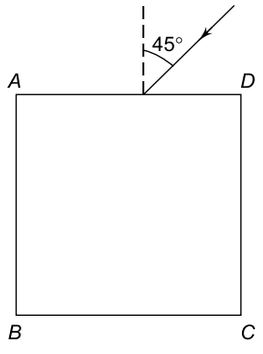


Fig. 26.51

(a) $\sqrt{2}$ (b) $\sqrt{\frac{3}{2}}$

(c) 1.5 (d) $\sqrt{\frac{5}{3}}$

72. Parallel rays from a distant object fall on a solid transparent sphere of radius *R* and refractive index μ. The distance of the image from the sphere is

(a) $\frac{R(2-\mu)}{2(\mu-1)}$ (b) $\frac{R\mu}{(\mu-1)}$

(c) $\frac{R}{(\mu-1)}$ (d) $R\ (\mu - 1)$

73. An object *O* is placed at a distance of 20 cm from a thin plano-convex lens of focal length 15 cm. The plane surface of the lens is silvered as shown in Fig. 26.52. The image is formed at a distance of

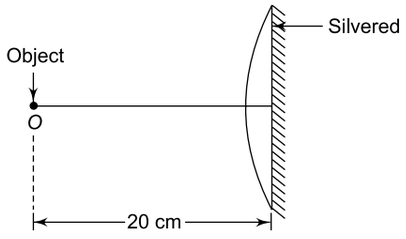


Fig. 26.52

(a) 60 cm to the right of the lens
(b) 30 cm to the left of the lens
(c) 24 cm to the right of the lens
(d) 12 cm to the left of the lens

IIT, 2006

74. Figure 26.53 shows the graph between the image distance v (in cm) and the object distance u (in cm) for a thin convex lens. The focal length of the lens is

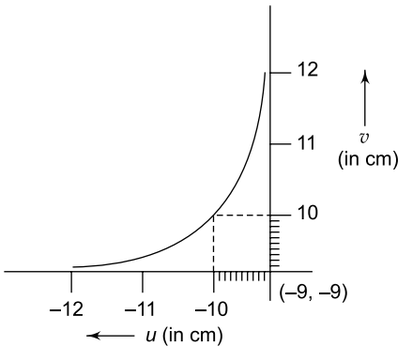


Fig. 26.53

(a) (5.00 ± 0.05) cm (b) (5.00 ± 0.10) cm
(c) (10.0 ± 0.10) cm (d) (0.50 ± 0.05) cm

IIT, 2006

75. Image of the sun is formed by a biconvex lens of focal length f. The image is a circular patch of radius r and is formed on the focal plane of the lens. Choose the correct statement from the following.
(a) The area of the image is πr^2 and it is directly proportional to f.
(b) The area of the image is πr^2 and it is directly proportional to f^2.
(c) The intensity of the image will increase if f is increased.
(d) If the lower half of the lens is covered with black paper, the area of the image will become half.

IIT, 2006

76. In an experiment to determine the focal length (f) of a concave mirror by the $u - v$ method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then,
(a) $x < f$ (b) $f < x < 2f$
(c) $x = 2f$ (d) $x > 2f$

IIT, 2007

77. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be (Fig. 26.54)
(a) only a reflected ray and no refracted ray
(b) only a refracted ray and no reflectd ray
(c) a reflected ray and a refracted ray and the angle between them would be less than $180° - 2\theta$
(d) a reflected ray and a refracted ray and the angle between them would be greater than $180° - 2\theta$

IIT, 2007

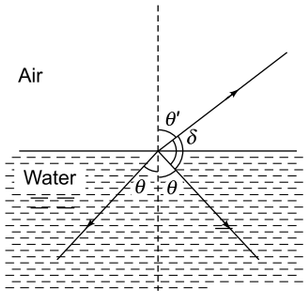


Fig. 26.54

78. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be
(a) 30° for both the colours
(b) greater for the violet colour
(c) greater for the red colour
(d) equal but not 30° for both the colours

IIT, 2008

79. A light beam is traveling from Region I to Region IV (Refer to Fig. 26.55). The refractive indices in Regions I, II, III and IV are n_0, $\frac{n_0}{2}, \frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering region IV is

(a) $\sin^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{8}\right)$
(c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{3}\right)$

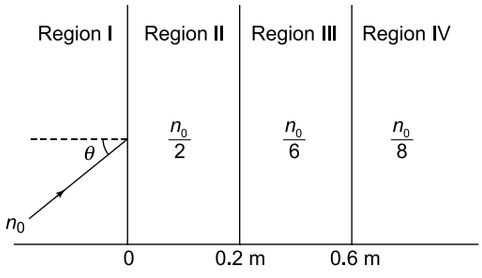
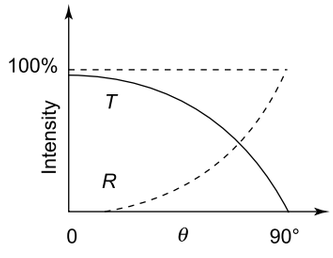


Fig. 26.55

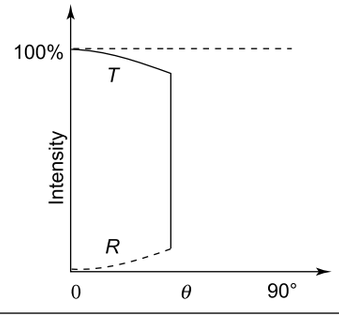
IIT, 2008

80. A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is [refer to Fig. 26.56]

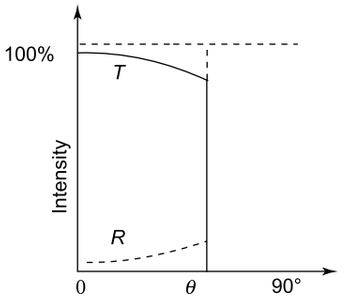
(a)



(b)



(c)



(d)

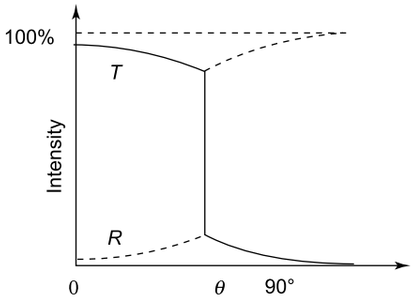


Fig. 26.56

IIT, 2011

ANSWERS

1. (c)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (a)
7. (c)	**8.** (a)	**9.** (c)	**10.** (c)	**11.** (a)	**12.** (b)
13 (b)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (d)
19. (c)	**20.** (d)	**21.** (c)	**22.** (c)	**23.** (c)	**24.** (b)
25. (b)	**26.** (a)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (d)
31. (a)	**32.** (b)	**33.** (a)	**34.** (a)	**35.** (d)	**36.** (b)
37. (a)	**38.** (b)	**39.** (c)	**40.** (b)	**41.** (a)	**42.** (d)
43. (d)	**44.** (d)	**45.** (a)	**46.** (d)	**47.** (b)	**48.** (d)
49 (c)	**50.** (b)	**51.** (c)	**52.** (b)	**53.** (c)	**54.** (b)
55. (b)	**56.** (c)	**57.** (a)	**58.** (b)	**59.** (c)	**60.** (b)
61. (a)	**62.** (a)	**63.** (b)	**64.** (b)	**65.** (d)	**66.** (b)
67. (c)	**68.** (a)	**69.** (c)	**70.** (d)	**71.** (b)	**72**. (a)
73. (d)	**74.** (b)	**75.** (b)	**76.** (b)	**77.** (c)	**78.** (a)
79. (b)	**80.** (c)				

SOLUTIONS

1. Refer to Fig. 26.57.

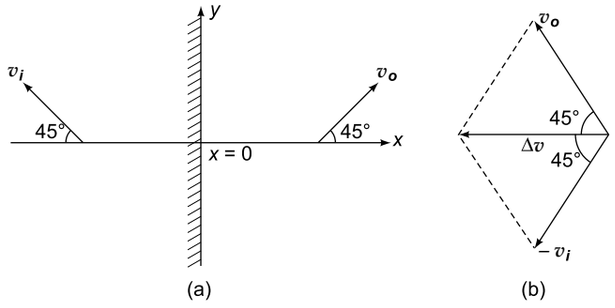


Fig. 26.57

Velocity of the object is $v_o = (2\ \hat{\mathbf{i}} + 2\ \hat{\mathbf{j}})\ \text{ms}^{-1}$

$\therefore$ Speed of object is $vo = \sqrt{2^2 + 2^2} = 2\sqrt{2}\ \text{ms}^{-1}$

= speed of the image (v_i). The velocity v_i of the image will be as shown in Fig. (a). The relative velocity of the image with respect to the object is

$$\Delta v = v_i - v_o = v_i + (-v_o)$$

The magnitude of Δv is given by [see Fig. (b)]

$$\Delta v = [v_0^2 + v_i^2 - 2v_o\ v_i \cos 90°]^{1/2}$$

$$= \left[\left(2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2\right]^{1/2}$$

$$= 4\ \text{ms}^{-1} \text{ along } -x \text{ axis.}$$

2. Refer to Fig. 26.58.

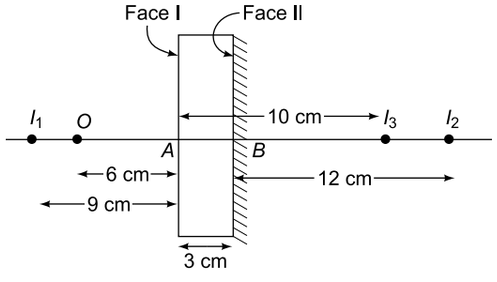


Fig. 26.58

I_1 is the image of O due to refraction at face I

$$AI_1 = \mu\ (OA) = 1.5 \times 6 = 9\ \text{cm}$$

I_2 is the image of I_1 due to reflection at face II. Since $I_1 B = 9 + 3 = 12$ cm, $I_2 B = 12$ cm.

I_3 is the image of I_2 due to refraction at Face I again.

$$\therefore \qquad AI_3 = \frac{I_2 A}{\mu} = \frac{15}{1.5} = 10\ \text{cm}$$

$\therefore$ Distance of I_3 from $B = 10 - 3 = 7$ cm.

3. For a spherical mirror $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$. For a concave mirror $u = -u$ and $f = -f$. Hence

$$\frac{1}{v} = \frac{1}{u} - \frac{1}{f} \quad \Rightarrow v = \frac{uf}{f-u}$$

If $u < f$, the image is virtual. Hence v is positive for u lying between zero and f. If $u > f$, the image is real. Hence v is negative for u lying between f and infinity. When $u \to \infty$, $v \to -f$. When $u \to f$, $v \to \pm\infty$. Hence the correct graph is (b).

4. Refer to Fig. 26.59.

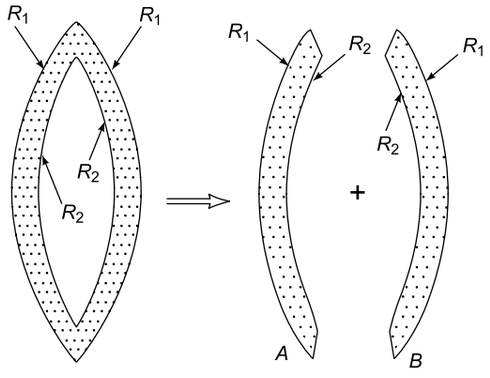


Fig. 26.59

The hollow lens can be considered to be a combination of two lens A and B. For a lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For lens A, $R_1 = +R_1$ and $R_2 = +R_2$. Hence

$$\frac{1}{f_A} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For lens B, $R_1 = -R_1$ and $R_2 = -R_2$. Hence

$$\frac{1}{f_B} = (\mu - 1)\left(-\frac{1}{R_1} + \frac{1}{R_2}\right) = -\frac{1}{f_A}$$

Thus $f_B = -f_A$. The focal length of the hollow lens is given by

$$\frac{1}{f_h} = \frac{1}{f_A} + \frac{1}{f_B} = 0 \qquad (\because f_B = -f_A)$$

or $f_h = \infty$. Hence a hollow lens behaves like a glass slab.

5. $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) = \frac{1}{R} \qquad (\because R_1 = -R_2 = R)$

which gives $f = R$. When the space between the lenses is filled with water, we have a concave water lens of $\mu_\omega = 4/3$ surrounded by a medium of $\mu_g = 3/2$. Therefore, for the water lens,

$$\frac{1}{f'} = \frac{\left(\frac{3}{2}-\frac{4}{3}\right)}{4/3}\left(-\frac{2}{R}\right) \quad (\because R_1 = -R \text{ and } R_2 = +R)$$

$$= -\frac{1}{4R} = -\frac{1}{4f}$$

The focal length of the combination of the three lenses is given by

$$\frac{1}{F} = \frac{1}{f}+\frac{1}{f}+\frac{1}{f'}$$

$$= \frac{2}{f}-\frac{1}{4f} = \frac{7}{4f}$$

$\Rightarrow F = \frac{4f}{7}$, which is choice (d).

6. Refer to Fig. 26.60.

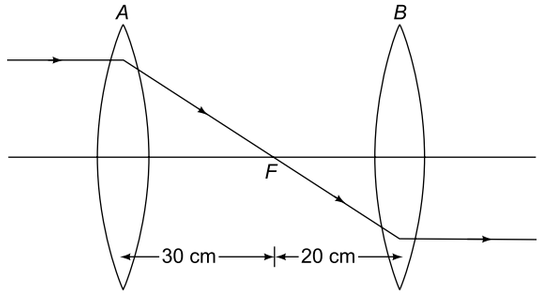


Fig. 26.60

For no deviation, the ray must emerge from lens B parallel to the principal axis. For this to happen, point F must be at the second focus of lens A and at the first focus of lens B. Hence $d = 30 + 20 = 50$ cm, which is choice (a).

7. $u = -3f$. The distance v of the image I is given by

$$\frac{1}{v}-\frac{1}{-3f} = \frac{1}{f} \Rightarrow v = 1.5\,f$$

The distance of the eye from I is $= 3f - 1.5f = 1.5f$ (see Fig. 26.61)

Triangles ABI and CDI are similar. Hence

$$\frac{CD}{AB} = \frac{DI}{BI} \Rightarrow \frac{h}{d/2} = \frac{1.5f}{1.5f}$$

or $h = \frac{d}{2}$. So the correct choice is (c).

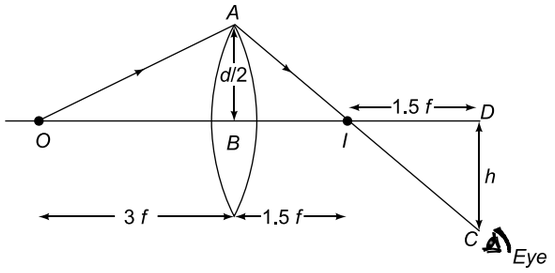


Fig. 26.61

8. The ray falling normally on face AB is refracted undeviated into the prism and is incident on face AC at an angle of incidence $i = \theta$. For total reflection at face AC

$$\sin i \geq \frac{\mu_w}{\mu_g} \text{ or } \sin\theta \geq \frac{4}{3\times 1.5} \text{ or } \sin\theta \geq \frac{8}{9}$$

Hence the correct choice is (a).

9. The ray does not suffer any deviation on entering the lens. Hence $\mu_1 = \mu$. The ray leaves the second surface of the lens bending towards the normal. Hence $\mu_2 > \mu$. Thus the correct choice is (c).

10. When a glass plate of thickness t is introduced, the image shifts by an amount $t\left(1-\frac{1}{\mu}\right)$. Hence

$$d = t\left(1-\frac{1}{\mu}\right) \text{ or } t = \frac{d}{\left(1-\frac{1}{\mu}\right)}$$

Thus the correct choice is (c).

11. For the first position of the lens, $m_1 = \frac{v}{u}$. For the conjugate (second) position, since u and v are interchanged, we have $m_2 = \frac{u}{v}$.

Therefore

$$m_1 - m_2 = \frac{v}{u} - \frac{u}{v} = \frac{v^2-u^2}{uv}$$

$$= \frac{(v-u)(v+u)}{uv}$$

But $f = \frac{uv}{u+v}$ and $v - u = d$.

Hence $m_1 - m_2 = \frac{d}{f}$

or $f = \frac{d}{(m_1 - m_2)}$.

Thus the correct choice is (a).

12. Refer to Fig. 26.62.

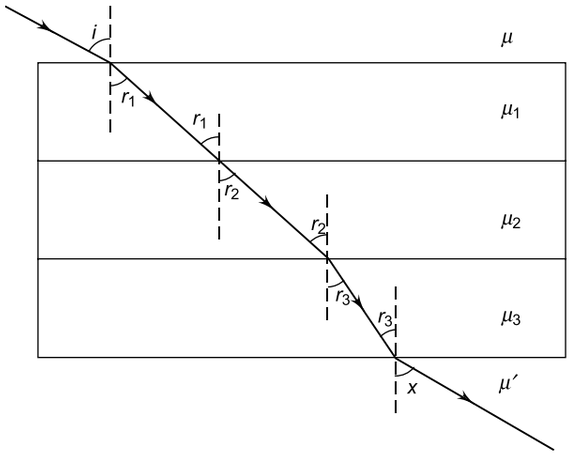


Fig. 26.62

Applying Snell's law to successive refractions, we have

$$\frac{\sin i}{\sin r_1} = \frac{\mu_1}{\mu}; \qquad \frac{\sin r_1}{\sin r_2} = \frac{\mu_2}{\mu_1}$$

$$\frac{\sin r_2}{\sin r_3} = \frac{\mu_3}{\mu_2}; \qquad \frac{\sin r_3}{\sin x} = \frac{\mu'}{\mu_3}$$

Multiplying these equations we get

$$\frac{\sin i}{\sin x} = \frac{\mu'}{\mu} \text{ or } \sin x = \frac{\mu}{\mu'} \sin i$$

13. The path of the ray is shown in Fig. 26.63.

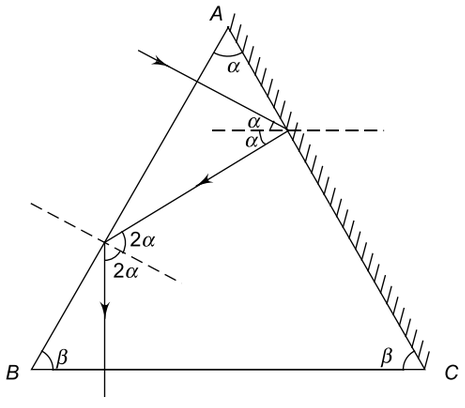


Fig. 26.63

It is clear that

$$\alpha + 2\beta = 180° \text{ and } \beta = 2\alpha$$

Hence, $\alpha = 36°$, which is choice (b).

14. Given $m = \frac{v}{u} = -3$ ($\because$ the image is inverted)

or $v = -3u$

Now $u = -10$ cm, therefore $v = +30$ cm.

If F is the focal length of the combination, we have

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u} = \frac{1}{30} + \frac{1}{10} \text{ or } F = \frac{15}{2} \text{ cm}$$

Focal length of the concave lens $B = f_1 = -30$ cm.
If f_2 is the focal length of lens A, we have

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F} \text{ or } \frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1} = \frac{2}{15} + \frac{1}{30}$$

which gives $f_2 = 6$ cm.
Since f_2 is positive, the lens is convex. Hence the correct choice is (c).

15. In the first case we have ($\because \quad f_m = \infty$)

$$\frac{1}{28} = \frac{2}{f_1} + \frac{1}{f_m} = \frac{2}{f_1} \qquad \text{(i)}$$

In the second case we have

$$\frac{1}{10} = \frac{2}{f_1} + \frac{1}{f_m} \qquad \text{(ii)}$$

where f_m is the focal length of the curved silvered surface. Hence $f_m = \frac{R}{2}$ where R is the radius of curvature of the curved surface.
Subtracting (i) from (ii) we get

$$\frac{1}{f_m} = \frac{1}{10} - \frac{1}{28} \text{ or } f_m = \frac{140}{9} \text{ cm.}$$

Therefore $R = 2 f_m = \frac{280}{9}$ cm

From (i) we have $f_1 = 28 \times 2 = 56$ cm. Now

$$\frac{1}{f_1} = (\mu - 1)\frac{1}{R}$$

$$\text{or } \mu - 1 = \frac{R}{f_1} = \frac{280}{9 \times 56} = 0.55 \text{ or } \mu = 1.55$$

16. The value of the effective focal length F is given by

$$\left|\frac{1}{F}\right| = \frac{1}{f_1} + \frac{1}{f_m} + \frac{1}{f_1} = \frac{2}{f_1} + \frac{1}{f_m}$$

$$= \frac{2}{20} + \frac{2}{22}$$

$$\text{or } |F| = \frac{110}{21} \text{ cm}$$

Since the convex lens with a silvered surface behaves as a concave mirror of effective focal length F, we have

$$F = -\frac{110}{21} \text{ cm and } u = -10 \text{ cm}$$

Substituting these values in the mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{F}$$

we have $\frac{1}{v} = -\frac{21}{110} + \frac{1}{10}$ or $v = -11$ cm.

The negative sign shows that the image is in front of the effective mirror and hence is real.

17. For an achromatic combination

$$\frac{f}{f'} = -\frac{\omega}{\omega'} \quad \therefore \quad \frac{1}{f'} = -\frac{\omega}{\omega'}\frac{1}{f}$$

where $\frac{\omega}{\omega'} = \frac{2}{3}$. Therefore,

$$\frac{1}{f'} = -\frac{2}{3f} \qquad \text{(i)}$$

The focal length of the combination is

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f'}$$

where $F = +20$ cm. Therefore

$$\frac{1}{20} = \frac{1}{f} + \frac{1}{f'} \qquad \text{(ii)}$$

From Eqs (i) and (ii) we get

$$f = 6.67 \text{ cm and } f' = -\frac{3f}{2} = -10 \text{ cm}$$

18. Since $u >> f_0$, $v = f_0 = 19$ m. Now $u = -3.8 \times 10^8$m. Therefore, magnification produced by the objective is

$$m_0 = \frac{v}{u} = -\frac{19}{3.8 \times 10^8} = -0.5 \times 10^{-7}$$

$\therefore$ Diameter of the image of the moon is

$$3.5 \times 10^6 \times 0.5 \times 10^{-7} = 0.175 \text{ m} = 17.5 \text{ cm}$$

Hence the correct choice is (d).

19. The focal length F of the combination is given

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

In terms of powers we have

$P = P_1 + P_2 - d\,P_1P_2$ for $P = 0$,

$$d = \frac{P_1 + P_2}{P_1\,P_2} = \frac{1}{P_2} + \frac{1}{P_1} = f_2 + f_1.$$

Hence the correct choice is (c).

20. The focal length of a lens does not change if a part of it is blcoked. If the central part of the aperture upto $d/2$ is blocked, the exposed area of the aperture reduces by one-fourth the earlier area because

$$\frac{\pi\left(\frac{d}{2}\right)^2}{\pi d^2} = \frac{1}{4}$$

Hence the intensity of the image reduces by a factor of 4. Thus the intensity becomes $I - I/4 = 3I/4$. Hence the correct choice is (d).

21. Refer to Fig. 26.64. Here $AB = 6$ cm. Therefore, $a = AE = 3$ cm. Let C be the centre of curvature of the lens. The radius of curvature of the lens is

$$R = AC = BC = CD$$

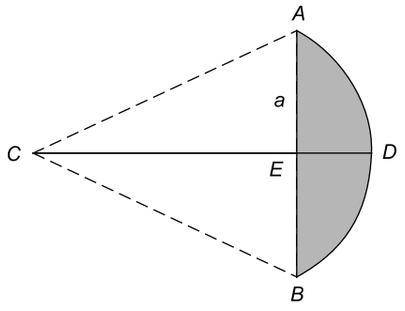


Fig. 26.64

Now $ED = 3$ mm $= t$, say. In triangle AEC, we have

$$(AC)^2 = (AE)^2 + (CE)^2$$

or $R^2 = a^2 + (R - t)^2 = a^2 + R^2 - 2Rt + t^2$

or $2Rt\left(1 - \frac{t}{2R}\right) = a^2$

Since $t << R$, the term $t/2R$ can be neglected compared to 1. Hence

$$2Rt = a^2$$

or $R = \frac{a^2}{2t} = \frac{3\text{ cm} \times 3\text{ cm}}{2 \times 0{\cdot}3\text{ cm}} = 15$ cm

Now the refractive index $\mu = \frac{c}{v} = \frac{3 \times 10^8 \text{ ms}^{-1}}{2 \times 10^8 \text{ ms}^{-1}}$ $= 1.5$. Therefore, the focal length of the plano-convex lens is given by

$$\frac{1}{f} = (\mu - 1) \times \frac{1}{R} = (1.5 - 1) \times \frac{1}{15} = \frac{1}{30}$$

or $f = 30$ cm. Hence the correct choice is (c).

22. For a prism with a very small refracting angle A, the deviation is given by (Fig. 26.65)

$$\delta = (\mu - 1)\,A$$

$\therefore$ Deviation produced by the first prism is

$$\delta_1 = (\mu_1 - 1)\,A_1$$

and that produced by the second prism is

$$\delta_2 = (\mu_2 - 1)\,A_2$$

The total deviation will be zero if $\delta_1 + \delta_2 = 0$. The emergent ray will then the parallel to the incident ray (see Fig. 26.65). Thus

$$(\mu_2 - 1)\, A_2 = -(\mu_1 - 1)\, A_1$$

The negative sign shows that the refracting angles of the two prisms are in opposite directions. Thus

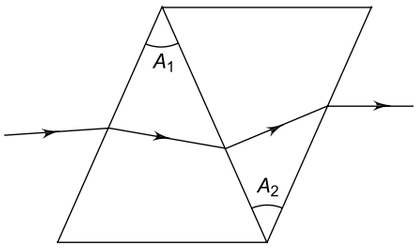


Fig. 26.65

$$|A_2| = \frac{(\mu_1 - 1)\, A_1}{(\mu_2 - 1)} = \frac{(1.54 - 1) \times 4^\circ}{(1.72 - 1)} = 3^\circ$$

23. Deviation produced by a prism having a small refracting angle is given by

$$\delta = (\mu - 1)\, A$$

Also $A + \delta = i + e = i$ ($\because$ $e = 0$, since the ray emerges normally from the opposite face. Thus

$$\delta = i - A \text{ or } (\mu - 1)\, A = i - A \text{ or } i = \mu A$$

24. A complete image will be formed but the intensity of the image will decrease due to decrease in aperture. Hence the correct choice is (b).

25. Refer to Fig. 26.66. The refracted ray BC will retrace its path if it falls normally on the silvered face PQ of the prism, i.e. $\angle PCB = 90^\circ$. Therefore, angle α in triangle PBC is $\alpha = 90^\circ - A$. Hence $r = A$. Now

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 2A}{\sin A}$$

$$= \frac{2 \sin A \cos A}{\sin A} = 2 \cos A$$

Hence the correct choice is (b).

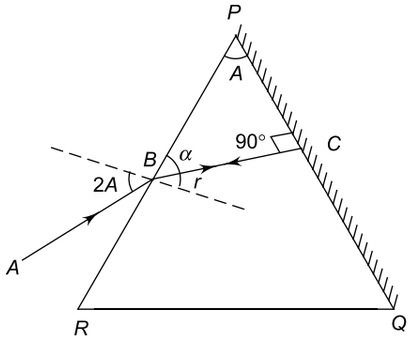


Fig. 26.66

26. Since the refractive index of glass is greater than that of air, the speed of light is less in glass than in air. The frequency of light never changes due to reflection or refraction. Since $v = \nu\lambda$ or $\lambda = v/\nu$, wavelength λ decreases because speed v decreases.

27. As the beam falls normally on face AB, it goes through undeviated. The critical angles for red, green and blue colours respectively are

$$i_r = \sin^{-1}\left(\frac{1}{1.39}\right) = 46^\circ,\ i_g = \sin^{-1}\left(\frac{1}{1.44}\right) = 44^\circ$$

and $$i_b = \sin^{-1}\left(\frac{1}{1.47}\right) = 43^\circ$$

The angle of incidence at face AC is $i = 45^\circ$. Since i is greater than i_g and i_b but less than i_r, the red colour will be refracted out from face AC but green and blue colours will be totally reflected at AC towards the base BC. Hence the correct choice is (a).

28. Using monochromatic light eliminates chromatic aberration. Using a doublet combination minimizes chromatic aberration. Increasing the size of the lens increases its resolving power. To reduce spherical aberration, the aperture (i.e. exposed portion of the lens) must be decreased. Hence the correct choice is (c).

29. Spherical aberration is reduced if the total deviation is distributed over the two surfaces of the lens. If the plane surface of the lens faces the object, all the deviation takes place at the curved surface. Hence spherical aberration is not reduced. Hence the correct choice is (b).

30. Figure 26.67 shows the ray diagram for the image formation in the two cases. When the mirror is filled with water, the image is real and located at C' which is between O and C. Hence the correct choice is (d).

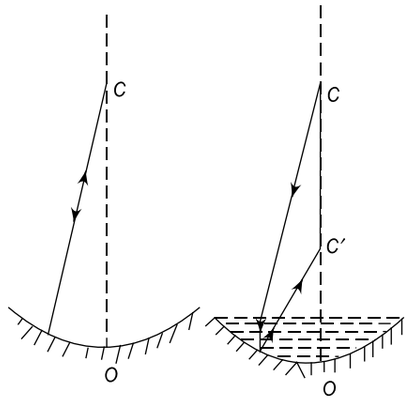


Fig. 26.67

31. Using the formula for a spherical surface

$$\frac{\mu_a}{u} + \frac{\mu_g}{v} = \frac{\mu_g - \mu_a}{R}$$

we have $$\frac{1.0}{u} + \frac{1.5}{u} = \frac{1.5 - 1.0}{R} \text{ (since } v = u)$$

which gives $u = 5R$.

32. Power of the lens combination is

$$P = P_1 + P_2 = \frac{1}{f_1\,(\text{in m})} + \frac{1}{f_2\,(\text{in m})}$$

$$= \frac{1}{+0.40\text{ m}} + \frac{1}{-0.25\text{ m}}$$

$$= -1.5\text{ m}^{-1} = -1.5\text{ D}$$

33. The focal length f of a lens of refractive index μ_2 surrounded by a medium of refractive index μ_1 is given by

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Now, for a concave lens, $f = -f$. Given $R_1 = R$ and $R_2 = -R_1 = -R$.

Also $\mu_2 = 1.5$ and $\mu_1 = 1.75$. Hence, we have

$$-\frac{1}{f} = \left(\frac{1.5 - 1.75}{1.75}\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \left(\frac{-0.25}{1.75}\right)\left(\frac{2}{R}\right)$$

which gives $f = 3.5\ R$. Since the focal length is positive, the lens acts like a convergent lens. Hence the correct choice is (a).

34. In a compound microscope, the object is placed just beyond the focus of the objective. Hence the image formed by the object is real, inverted and highly magnified.

35. Refer to the solution of Q. 33 above. Since the lens is concave $f = -f$. Also $R_2 = -R_1$. Therefore, we have

$$-\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where n_2 is the refractive index of the liquid filling the lens and n_1 that of of the liquid in which the lens is immersed.

It will act as a divergent lens (i.e. f will remain negative) if $n_2 > n_1$. Hence the correct choice is (*d*).

36. When a ray of light passes through a glass slab with parallel faces, it does not suffer any deviation; it is only displaced parallel to itself. Therefore, the direction of the beam remains unchanged after passing through the glass slab. However, the rays are displaced slightly towards the outer side. Hence the divergence angle of the emergent beam will be the same as that of the incident beam.

37. The ray will emerge from side CD of the slab if the ray refracted in the slab suffers repeated total internal reflections at faces AD and BC of the slab as shown in Fig. 26.68. From Snell's law, we have

$$\frac{\sin r}{\sin \alpha_{\max}} = \frac{n_2}{n_1} \text{ or } \sin \alpha_{\max} = \frac{n_1}{n_2} \sin r$$

The critical angle i_c is given by

$$\sin i_c = \frac{n_2}{n_1}$$

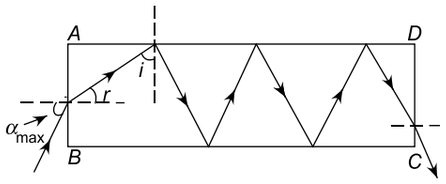


Fig. 26.68

Thus angle i must at least be equal to i_c. Therefore $i_c = 90° - r$ or $r = 90° - i_c$.

$$\therefore \quad \sin \alpha_{\max} = \frac{n_1}{n_2} \sin (90° - i) = \frac{n_1}{n_2} \cos i_c$$

$$= \frac{n_1}{n_2} \cos\left(\sin^{-1}\frac{n_2}{n_1}\right) \qquad \left(\because \sin i_c = \frac{n_2}{n_1}\right)$$

or $$\alpha_{\max} = \sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\frac{n_2}{n_1}\right)\right]$$

38. The refractive index of the prism with respect to the liquid in which it is immersed is given by

$$\mu = \frac{\sin\left\{\frac{1}{2}(A + \delta_m)\right\}}{\sin\frac{1}{2}(A)}$$

$$= \frac{\sin\left(\frac{60° + 30°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$$

The critical angle i_c is given by

$\sin i_c = \frac{1}{\sqrt{2}}$, which give $i_c = 45°$. Hence the correct choice is (b).

39. Mean angular deviations produced by crown and flint glass prisms respectively are

$$d = (\mu - 1)\ A \text{ and } d' = (\mu' - 1)\ A'$$

Their dispersive powers are

$$\omega = \frac{(\mu_v - \mu_r)}{(\mu - 1)} \text{ and } \omega' = \frac{(\mu'_v - \mu'_r)}{(\mu' - 1)}$$

Their angular dispersions respectively are

$$D = (\mu_v - \mu_r)\ A \text{ and } D' = (\mu'_v - \mu'_r)\ A'$$

When the prisms are combined, the dispersion by the combination will be zero if

$$D + D' = 0$$

or $(\mu_v - \mu_r)\,A + (\mu'_v - \mu'_r)\,A' = 0$

or $\dfrac{(\mu_v - \mu_r)}{(\mu - 1)}\,(\mu - 1)\,A + \dfrac{(\mu'_v - \mu'_r)}{(\mu - 1)}\,(\mu' - 1)A' = 0$

or $\omega d + \omega' d' = 0$. which is choice (c).

40. The critical angle for total internal reflection is given by

$$\sin i_c = \frac{1}{n}$$

If $n = \sqrt{2}$, $\sin i_c = \dfrac{1}{\sqrt{2}}$ or $i_c = 45°$. For total internal reflection the angle of incident (i) must be greater than 45°. The figure shows that the ray suffers total internal reflection when $i = 45°$. Hence the critical angle for the given prism is less than 45°. Therefore, n is greater than $\sqrt{2}$. Thus the correct choice is (b).

41. If the curved surface of a plano-convex lens is silvered, the focal length of the lens is given by

$$\frac{1}{F} = \frac{2\mu}{R} \text{ or } F = \frac{R}{2\mu} = \frac{30 \text{ cm}}{2 \times 1.5} = 10 \text{ cm}$$

The real image will be equal to the size of the object if the object distance $u = 2F = 2 \times 10$ cm = 20 cm. Hence the correct choice is (a).

42. For an achromatic doublet

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}$$

In terms of powers of the lenses, we have

$$\frac{P_2}{P_1} = -\frac{\omega_1}{\omega_2}$$

or $P_1\omega_1 = -P_2\omega_2$. Hence the correct choice is (d).

43. Diameter of image = radius of curvature × angle in radians

$$= R\theta = 2f\theta \qquad (\because R = 2f)$$

Hence the correct choice is (d).

44. Refer to Fig. 26.69. It is clear that the greatest distance is AB. Now, since $PR = RD = L$, from triangles PRT and PBD we have

$$BD = 2RT = 2RS = 2 \times \frac{d}{2} = d$$

and $OD = \dfrac{d}{2}$. Therefore, $OB = OD + BD$

$$= \frac{d}{2} + d = \frac{3d}{2}$$

$$\therefore \quad AB = 2OB = 2 \times \frac{3d}{2} = 3d,$$

which is choice (d).

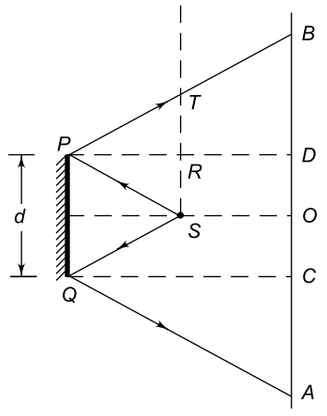


Fig. 26.69

45. For a small-angled prism and for a small angle of incidence, the deviation is given by

$$\delta = (\mu_g - 1)A \qquad \text{(i)}$$

The refractive index of the prism, when it is dipped in water is

$$\mu' = \frac{\mu_g}{\mu_w} = \frac{1.5}{4/3} = \frac{9}{8}$$

$$\therefore \quad \delta' = (\mu' - 1)A \qquad \text{(ii)}$$

From (i) and (ii) we have

$$\delta' = \left(\frac{\mu' - 1}{\mu_g - 1}\right)\delta = \frac{\left(\frac{9}{8} - 1\right)}{(1.5 - 1)} \times 4° = 1°$$

Hence the correct choice is (a).

46. Real thickness = $\mu \times$ apparent thickness

$$= 1.5 \times (5 + 2) = 10.5 \text{ cm}$$

Hence the correct choice is (d)

47. $\mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$. Given $\mu = \cot\left(\dfrac{A}{2}\right)$. Thus

$$\cot\left(\frac{A}{2}\right) = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

or $\quad \cos\left(\dfrac{A}{2}\right) = \sin\left(\dfrac{A + \delta_m}{2}\right)$

or $\quad \sin\left(90° - \dfrac{A}{2}\right) = \sin\left(\dfrac{A + \delta_m}{2}\right)$

which gives $\dfrac{A + \delta_m}{2} = 90° - \dfrac{A}{2}$

or $\delta_m = (180° - 2A)$ which is choice (b).

48. Refer to Fig. 26.70. From Snell's law, we have

$$\mu_1 \sin i = \mu_2 \sin r_1$$
$$\mu_2 \sin r_1 = \mu_3 \sin r_2$$
$$\mu_3 \sin r_2 = \mu_4 \sin e$$

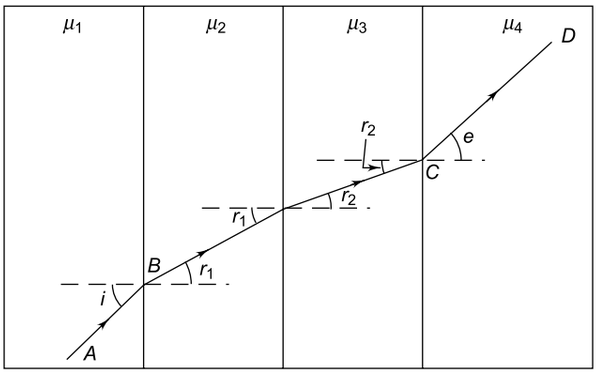


Fig. 26.70

From these equations, it follows that

$$\mu_1 \sin i = \mu_4 \sin e \qquad (1)$$

Ray CD will be parallel to ray AB, if $e = i$. Hence $\mu_1 = \mu_4$, which is choice (d).

49. The ray will undergo minimum deviation in prism P if the angle of incidence on P is such that the refracted ray in prism P is parallel to its base. Since prisms P, Q and R are of identical shape and of the same material, the refracted ray in prism P suffers no deviation at the inter faces between P and Q and Q and R. Hence the ray emerging out of R will suffer the same deviation as was produced by prism P. Thus the correct choice is (c).

50. When beaker is filled upto a height $2h$, the bottom Q of the rod will be visible if the ray QD travelling in the liquid refracts along DB in air. It follows from Fig. 26.71 that D is the mid-point of diagonal PB of square $ABPR$. Hence $DE = PE = h$. Also $\angle BDF = 45°$ since $\angle DPE = 45°$. From Snell's law, we have (since the object is in a denser medium)

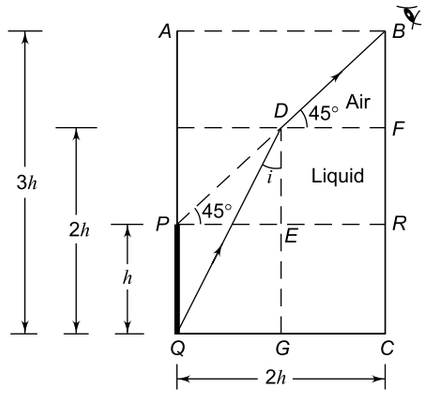


Fig. 26.71

$$\frac{1}{\mu} = \frac{\sin i}{\sin r} = \frac{QG/QD}{\sin 45°} = \frac{h/\sqrt{5}h}{1/\sqrt{2}} = \sqrt{\frac{2}{5}}$$

or $\mu = \sqrt{\frac{5}{2}}$, which is choice (b).

51. If the radii of curvature of the two faces of a lens are equal, the incident and the emergent rays will be parallel, i.e. there is no deviation produced by the lens and hence there is no dispersion. Thus the correct choice is (c).

52. It is clear from Fig. 26.72 that the first reflected ray AB shifts by a distance $x = BC$ which is given by $x = AC \tan 30°$

$$= 0.2 \text{ m}/\sqrt{3} = 0.2/\sqrt{3} \text{ m}$$

$\therefore$ Number of reflections required to a cover a distance of $2\sqrt{3}$ m is

$$n = \frac{2\sqrt{3} \text{ m}}{0.2/\sqrt{3} \text{ m}} = 30$$

Hence the correct choice is (b).

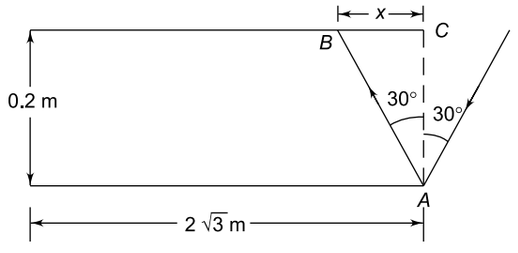


Fig. 26.72

53. Refer to the Fig. 26.73. AB is the image of size 1.6 cm formed by the convex lens at its focal plane. It serves as the virtual object for the concave lens. $A'B'$ is the final image. For concave lens, $f = -20$ cm, $u = +4$ cm; $v = ?$ Now

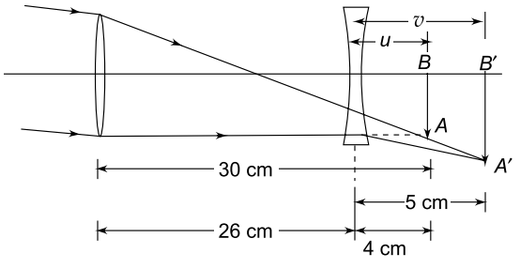


Fig. 26.73

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or $$\frac{1}{v} - \frac{1}{4} = -\frac{1}{20}$$

which gives $v = 5$ cm. Thus the final image at a distance of 5 cm from the concave lens. Now

$$\frac{A'B'}{AB} = \frac{v}{u}$$

or $$\frac{A'B'}{1.6\text{ cm}} = \frac{5\text{ cm}}{4\text{ cm}}$$

or $A'B' = 2.0$ cm. Hence the correct choice is (c).

54. For refraction at glass-water interface, we have from Snell's law

$$\mu_g \sin i = \mu_\omega \sin r \quad \text{(i)}$$

For refraction at water-air interface, we have

$$\mu_\omega \sin r = \mu_a \sin 90° = \mu_a = 1 \quad \text{(ii)}$$

Using (ii) in (i), we get

$$\mu_g \sin i = 1 \text{ or } \mu_g = \frac{1}{\sin i},$$

which is choice (b).

55. Using $f = +20$ cm and $u = -280$ cm in $\frac{1}{v}+\frac{1}{u} = \frac{1}{f}$, we get $v = +\frac{56}{3}$ cm. Differentiating $\frac{1}{v}+\frac{1}{u} = \frac{1}{f}$, we have

$$-\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0 \quad \text{or} \quad \frac{dv}{dt}=-\frac{v^2}{u^2}\frac{du}{dt}$$

Speed of image $= \frac{v^2}{u^2} \times$ (speed of object). The correct choice is (b).

56. The concave mirror formula is

$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f} \quad (1)$$

Since, for a given concave mirror, focal length f is fixed, we partially differentiate Eq. (1) to get

$$-\frac{\delta v}{v^2}=-\frac{\delta u}{u^2}=0 \quad \text{or} \quad \delta v = -\left(\frac{v}{u}\right)^2 \delta u \quad (2)$$

Multiplying Eq. (1) by u, we get

$$\frac{u}{v}+1=\frac{u}{f} \quad \text{or} \quad \frac{v}{u}=\frac{f}{u-f} \quad (3)$$

Using Eq. (3) in Eq. (2), we get

$$\delta v = -\left(\frac{f}{u-f}\right)^2 \delta u$$

Given $\delta u = b$. Therefore

$$\delta v = -\left(\frac{f}{u-f}\right)^2 b \quad (4)$$

The negative sign shows that image is longitudinally inverted. The magnitude of the size of the image is

$$|\delta v| = b\left(\frac{f}{u-f}\right)^2,$$ which is choice (c).

57. Referring to Fig. 26.74, the source S cannot be seen at all from above when the water level attains a critical maximum height x so that rays such as SA and SB suffer internal relection. The critical angle i_c is given by

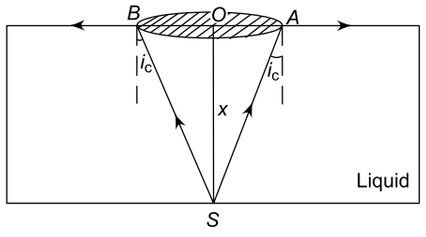


Fig. 26.74

$\sin i_c = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$, which gives $i_c = 45°$.

In triangle OAS, we have

$$\frac{r}{x} = \tan i_c \text{ or } x = \frac{r}{\tan i_c} = \frac{r}{\tan 45°} = r.$$

Hence the correct choice is (a).

58. Refer to Fig. 26.75.

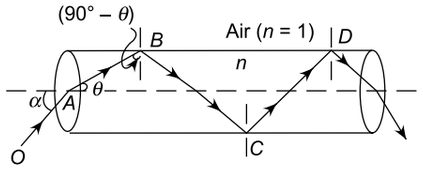


Fig. 26.75

Ray OA is incident at an angle α at the planar face of the cylindrical rod. Let θ be the angle of refraction. From Snell's law, we have

$$n = \frac{\sin\alpha}{\sin\theta} \quad \text{or} \quad \sin\theta = \frac{\sin\alpha}{n} \quad (1)$$

The ray AB is incident at point B of the curved surface of the cylinder at an angle $(90° - \theta)$. This ray is travelling in a denser medium of refractive index n and is incident at the cylinder-air interface at point B. The ray will not emerge from the curved surface if it suffers total internal reflection at B. For this to happen $(90° - \theta) \geq i_c$, the critical angle or

$$\sin(90° - \theta) \geq \sin i_c \text{ or } \cos\theta \geq \sin i_c$$

or $$(1 - \sin^2\theta)^{1/2} \geq \sin i_c$$

or $$1 - \sin^2\theta \geq \sin^2 i_c \quad (2)$$

The critical angle is given by

or $\sin i_c = \dfrac{1}{n}$ (3)

Using Eqs. (1) and (3) in Eq. (2) we get

$$1 - \frac{\sin^2 \alpha}{n^2} \geq \frac{1}{n^2} \text{ or } n^2 - \sin^2 \alpha \geq 1$$

or $n^2 \geq (1 + \sin^2 \alpha)$

Since the maximum value of $\sin^2 \alpha = +1$, it follows that

$$n^2_{\min} \geq 2 \quad \text{or} \quad n_{\min} \geq \sqrt{2}$$

This is the minimum value of refractive index of the cylindrical rod for the ray AB to suffer total internal reflection at point B. By symmetry, ray BC will be totally reflected along CD suffering another total internal reflection at D and so on until the ray finally emergs from the opposite planar face of the rod.

59. Given $i = 60°$, $\delta = 30°$ and $A = 30°$. Using $\delta = i + e - A$, we get $e = 0$. i.e. the emergent ray is perpendicular to face AC of the prism (see Fig. 26.76).

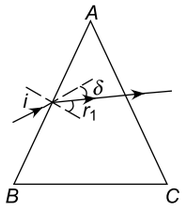


Fig. 26.76

Now $r_1 = i - \delta = 60° - 30° = 30°$. Hence

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}.$$

Hence the correct choice is (c).

60. $u = -25$ cm and $f = -10$ cm. The distance of the image is given by

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{-25} \quad \text{or} \quad v = -\frac{3}{50} \text{ cm}$$

Area of the object wire is $3.0 \times 3.0 = 9.0$ cm^2. The area magnification is given by

$$\frac{\text{area of image}}{\text{area of object}} = (\text{linear magnification})^2$$

$$= \frac{v^2}{u^2} = \left(\frac{50}{3 \times 25}\right)^2 = \frac{4}{9}$$

Therefore, the area enclosed by image is

$\dfrac{4}{9} \times 9$ cm$^2 = 4$ cm^2, which is choise (b).

61. At minimum deviation, the incident ray PQ and emergent ray RS are symmetrical with respect to the refracting faces AB and AC of the prism, i.e. angle of incident i_1 = angle of emergence i_2 (see Fig. 26.77). Therefore, $r_1 = r_2 = r$. Also

$r_1 + r_2 = A$, which gives $r_1 = r_2 = \dfrac{A}{2}$.

In triangle ABC, $A + \beta + \beta = 180°$

or $A + 2\beta = 180°$

or $\dfrac{A}{2} = 90° - \beta$

But $\dfrac{A}{2} = r_1 = 90° - \alpha$

Hence $\alpha = \beta$, which is choice (a).

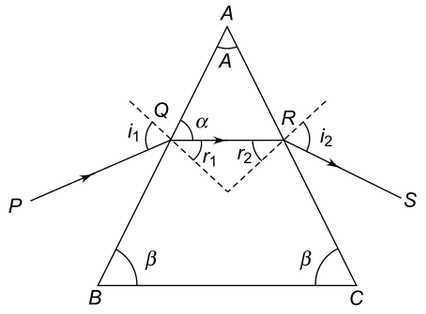


Fig. 26.77

62. The refractive index of glass is inversely proportional to the square of the wavelength of light, i.e.

$$\mu \propto \frac{1}{\lambda^2}$$

Now the wavelength of violet light is the least and that of the red light is the highest. Hence the refractive index of the prism is the highest for violet light and the least for red light. Therefore, violet light is deviated the most and red light is deviated the least as shown in Fig. 26.78. Since the green light is just totally internally reflected at face AC, its angle of incidence on this face is given by

$$\sin i_c = \frac{1}{\mu_G}$$

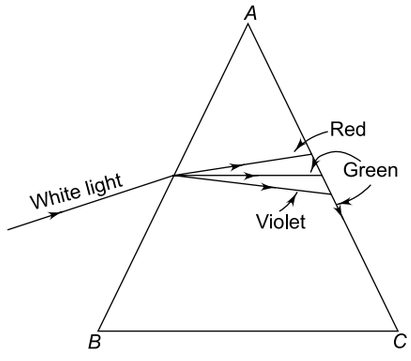


Fig. 26.78

where μ_G is the refractive index of the prism for green light, and i_c is the critical angle for green light. Since the refractive indices for violet, indigo and blue lights are greater than that for green light (this follows from VIBGYOR), the critical angles for total internal reflection are smaller than that for

green light. Since lights of these colours are incident on face AC at angles greater than their respective critical angles, the violet, indigo and blue lights will be totally reflected at face AC back into the prism. The lights of yellow, orange and red colours are incident on face AC at angles less than their respective critical angles. Hence these colours will not suffer total internal reflection at face AC and will refract out and emerge from face AC. Thus the correct choice is (a).

63. Two rays 1 and 2 from the object placed at O fall normally on the spherical surface and go through undeviated (see Fig. 26.79). The divergent rays $1'$ and $2'$ appear to come from O. Hence the virtual image is formed at the centre O, which is at a distance of 6 cm from the surface of the sphere. Hence the correct choice is (b).

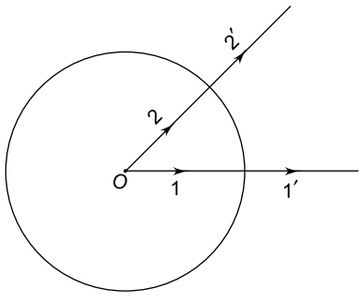


Fig. 26.79

64. The apparent depth of the object is (see Fig. 26.80)

$$d = \frac{32}{4/3} = 24 \text{ cm}$$

Thus I' is the image of object due to refraction. This image serves as the virtual object for the concave mirror forming the final image I due to reflection.

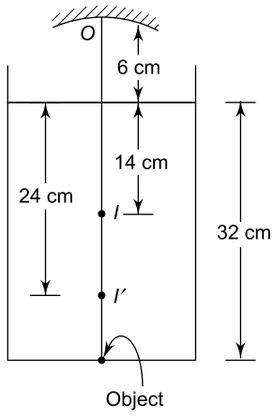


Fig. 26.80

Now $u = OI' = -(24 + 6) = -30$ cm

and $v = OI = -(14 + 6) = -20$ cm

Using these values in the spherical mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

we have

$$\frac{1}{f} = \frac{1}{-20} + \frac{1}{-30} = -\frac{1}{12}$$

or $f = -12$ cm.

Hence the correct choice is (b).

65. Given $f_1 = +12$ cm and $\frac{|P_1|}{|P_2|} = \frac{4}{3}$ or $\frac{|f_2|}{|f_1|} = \frac{4}{3}$

Since f_2 is negative, $\frac{f_2}{f_1} = -\frac{4}{3}$. Hence

$$f_2 = -\frac{4}{3} f_1 = -\frac{4}{3} \times 12 = -16 \text{ cm}$$

The focal length F of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{12} + \frac{1}{-16} = \frac{1}{48}$$

which gives $F = 48$ cm. Hence the correct choice is (d).

66. Referring to Fig. 26.81, the path of the ray is $PQRS$ suffering internal reflections at Q and R. It is clear from the figure that angles α and β should be greater than the critical angle given by

$$\sin i_c = \frac{1}{n}$$

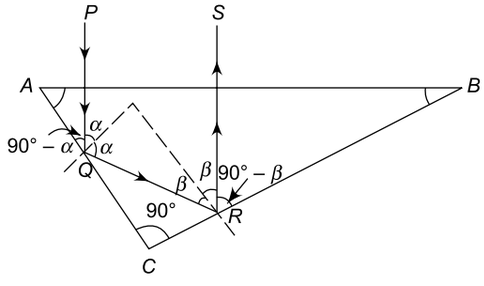


Fig. 26.81

Also angle $A = \alpha$ and angle $B = \beta$. Since $A \geq B$; $\beta \leq \alpha$, the minimum value of n is given by

$$\frac{1}{n} \leq \sin\beta \;\therefore\; n_{\min} = \frac{1}{\sin\beta} = \frac{1}{\sin B}$$

So the correct choice is (b).

67. Refer to Fig. 26.82.

When the curved face of the lens is in contact with the table, the virtual image of the bottom-most

point O of the lens is formed at I_1 due to refraction at the plane surface as shown in Fig. 26.82 (a).

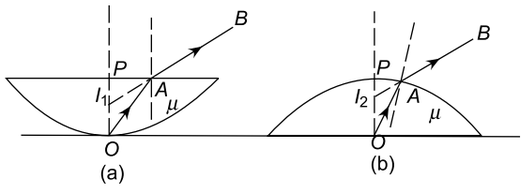


Fig. 26.82

Real depth = 4 cm and apparent depth = 3 cm. Hence

$$\mu = \frac{4}{3}$$

When the plane face of the lens is in contact with the table, the image of the centre O of the plane face of the lens is formed at I_2 due to refraction at the curved face as shown in Fig. 26.82 (b). For refraction at this face, we have

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

where $\mu_1 = 1$, $\mu_2 = \mu$, $u = OP = -4$ cm, and $v = OI_2 = -\frac{25}{8}$ cm. Putting these values, we get

$$-\frac{8}{25} - \frac{\mu}{-4} = \frac{1-\mu}{R}$$

Putting $\mu = \frac{4}{3}$ and solving we get $R = 25$ cm. Now, the focal length of the plano-convex lens is given by

$$\frac{1}{f} = (\mu - 1)\ \frac{1}{R} = \left(\frac{4}{3} - 1\right) \times \frac{1}{25} = \frac{1}{75}$$

or $f = 75$ cm.

68. Refer to Fig. 26.83.

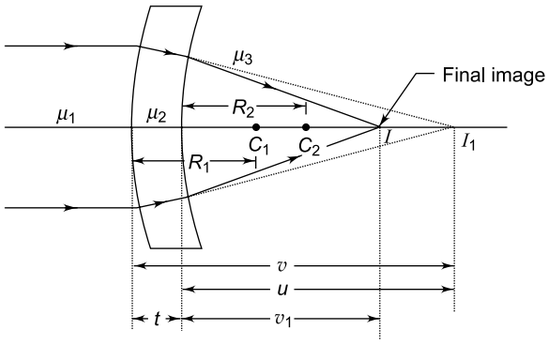


Fig. 26.83

Refraction at the first surface: $u = -\infty$, $v = +v$ and $R = +R_1$. We have

$$\frac{\mu_2}{v} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{R_1}$$

or $$\frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R_1} \quad \text{(i)}$$

Refraction at the second surface: $u = v - t \simeq v$ ($\because$ t is negligible),

$v = +v_1$ and $R = +R_2$. We have

$$\frac{\mu_3}{v_1} - \frac{\mu_2}{v} = \frac{\mu_3 - \mu_2}{R_2} \quad \text{(ii)}$$

Since the incident ray is parallel to the principal axis, $v_1 = f$, the focal length of the lens and using $v_1 = f$ (i) in (ii), we get

$$\frac{\mu_3}{f} - \frac{\mu_2 - \mu_1}{R_1} = \frac{\mu_3 - \mu_2}{R_2}$$

or $$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_3}\right).\frac{1}{R_1} + \left(\frac{\mu_3 - \mu_2}{\mu_3}\right).\frac{1}{R_2} \quad \text{(iii)}$$

This is the expression for the focal length. If $R_1 = R_2$ we get [put $R_1 = R_2 = R$ in (iii)]

$$\frac{1}{f} = \left(\frac{\mu_3 - \mu_1}{\mu_3}\right)\frac{1}{R}$$

So the correct choice is (a).

69. Refer to Fig. 26.84.

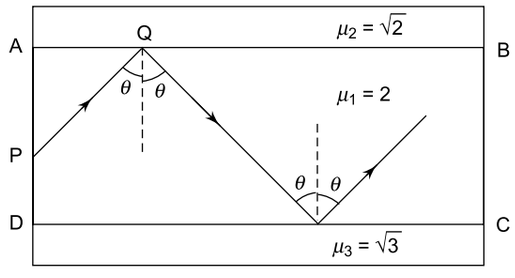


Fig. 26.84

For total internal reflection at surface AB, angle θ must be greater than or equal to the critical angle i_1 given by

$$\sin i_1 = \frac{\mu_2}{\mu_1} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

which gives $i_1 = 45^\circ$

For total internal reflection at surface CD, angle θ must be greater than or equal to the critical angle i_2 given by

$$\sin i_2 = \frac{\mu_3}{\mu_1} = \frac{\sqrt{3}}{2}$$

which gives $i_2 = 60^\circ$.

Hence, for total internal reflection at both the surfaces AB and CD, the minimum value of $\theta = 60^\circ$. Thus the correct choice is (c).

70. Two isosceles prisms ABC and DBC are placed with their bases BC touching each other as shown in Fig. 26.85, which shows the path of parallel incident rays which come to a focus at point F. Distance $OF = f$ is the focal length of the system. It follows from figure that

$$\tan \delta = \frac{h}{f}$$

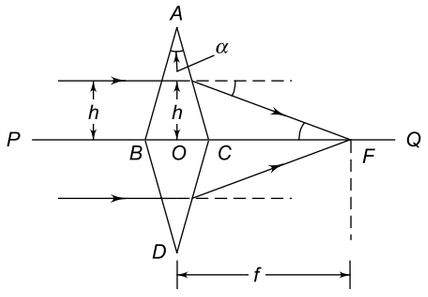


Fig. 26.85

Since the prism is very thin and if h is small, i.e. the incident rays are close to the axis PQ of the system, then δ will be very small and we can replace $\tan \delta$ by δ where δ is measured in radian. Thus

$$\delta = \frac{h}{f}$$

Now, for a prism having a small refracting angle α and for a small h, the deviation produced by a prism is given by (here angle α is expressed in radian)

$$\delta = (\mu - 1)\alpha$$

Thus $$\frac{h}{f} = (\mu - 1)\alpha \text{ or } f = \frac{h}{(\mu - 1)\alpha}$$

So the correct choice is (d).

71. Refer to Fig. 26.86. The ray will be totally reflected at E if it is incident at an angle equal to or greater than the critical angle i_c given by

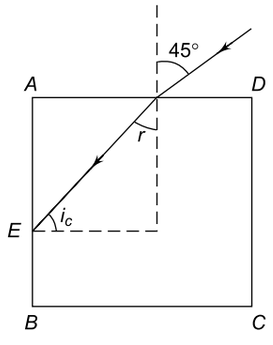


Fig. 26.86

$$\sin i_c = \frac{1}{\mu} \quad (1)$$

Angle r is given by

$$\mu = \frac{\sin 45^\circ}{\sin r} \text{ or } \sin r = \frac{1}{\mu\sqrt{2}} \quad (2)$$

It follows from the figure that $i_c + r = 90^\circ$ or $i_c = 90^\circ - r$. Using this in Eq. (1), we get $\cos r = \frac{1}{\mu}$ which gives

$$\sin r = (1 - \cos^2 r)^{1/2}$$

$$= \left(1 - \frac{1}{\mu^2}\right)^{1/2} = \frac{1}{\mu}(\mu^2 - 1)^{1/2}$$

Using this in Eq. (2), we get

$$\frac{1}{\mu}(\mu^2 - 1)^{1/2} = \frac{1}{\mu\sqrt{2}} \text{ which gives } \mu = \sqrt{\frac{3}{2}},$$

which the correct choice is (b).

72. Refer to Fig. 26.87. For refraction at face I,

$$\frac{1}{u_1} + \frac{\mu}{v_1} = \frac{\mu - 1}{R}$$

Since $u_1 = \infty$, we have

$$\frac{\mu}{v_1} = \frac{\mu - 1}{R}$$

or $$v_1 = \frac{\mu R}{(\mu - 1)} \quad (1)$$

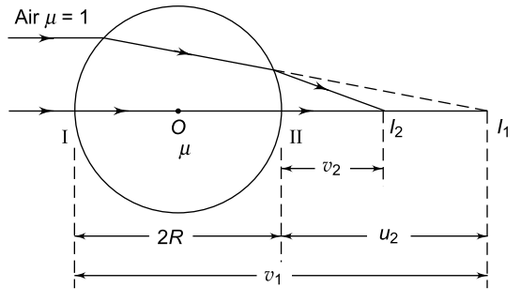


Fig. 26.87

For refraction at face II, $u_2 = -(v_1 - 2R) = 2R - v_1$. Using Eq. (1), we get

$$u_2 = 2R - \frac{\mu R}{(\mu - 1)} = \frac{R(\mu - 2)}{(\mu - 1)} \quad (2)$$

The image distance v_2 is given by

$$\frac{\mu}{u_2} + \frac{1}{v_2} = \frac{\mu - 1}{R} \quad (3)$$

Using Eq. (2) in Eq. (3) and simplifying, we get

$$v_2 = \frac{R(2-\mu)}{2(\mu-1)}.$$

The correct choice is (a).

73. The effective focal length of the silvered lens is given by

$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m} = \frac{2}{15} + \frac{1}{\infty} = \frac{2}{15}$, which gives

$F = \frac{15}{2}$ cm. The silvered lens behaves like a concave mirror. Using the spherical mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{F}$, we have

$$\left(\because u = -20 \text{ cm and } F = -\frac{15}{2}\text{cm}\right)$$

$$\frac{1}{v} + \frac{1}{-20} = \frac{2}{-15}$$

which gives $v = -12$ cm. The negative sign indicates that the image is formed to the left of the lens. Hence the correct choice is (d).

74. It follows from the graph that when $u = -10$ cm, $v = +10$ cm. The focal length of the lens is given by

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{10} - \frac{1}{-10}$, which gives $f = 5.0$ cm

The maximum error Δf in the measurement of f is given by

$$\frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}$$

where Δu and Δv are the least counts of u and v scales of the graph. If follows from the graph that $\Delta u = \Delta v = 1$ mm $= 0.1$ cm. Hence

$$\Delta f = \left(\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right) \times f^2$$

$$= \left[\frac{0.1}{(10)^2} + \frac{0.1}{(10)^2}\right] \times (5)^2$$

$$= \frac{5}{100} \text{ cm} = 0.05 \text{ cm}$$

Hence $f = (5.00 \pm 0.05)$ cm, which is choice (a).

75. Refer to the Fig. 26.88.

The angular diameter of the sun is 2θ. From the figure it follows that $\tan\theta = r/f$. Since θ is small, $\tan\theta \simeq \theta$, where θ is expressed in radian. Hence, we have

$$\theta = \frac{r}{f} \quad \text{or} \quad r = f\theta$$

$\therefore$ Area of image $= \pi r^2 = \pi\theta^2 f^2$. Thus area $\propto f^2$, which is choice (b).

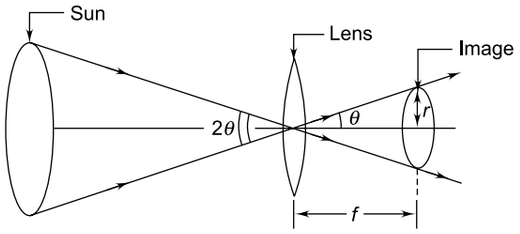


Fig. 26.88

76. It is given that the image is inverted and it does not coincide with the object pin. If $x < f$, the image is erect and if $x = 2f$, the image will coincide with the object. Hence choices (a) and (c) are wrong. It is given that when the student shifts his eye towards left, the image appears to move to the right of the object pin. This can happen if the image is closer to the eye than the object. Hence choice (d) is also false. If x lies between f and $2f$, the image is closer to the eye than the object as shown in Fig. 26.89.

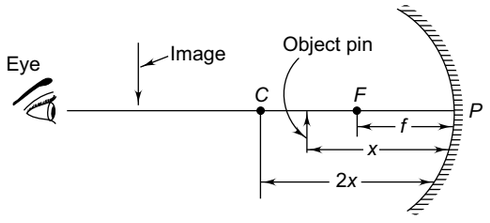


Fig. 26.89

77. If the angle of incidence in the denser medium is less than the critical angle for the two media, the ray is party reflected back into the denser medium and partly refracted into the rarer medium.

Angle between the reflected ray and the refracted ray is

$$\delta = 180° - (\theta + \theta')$$

Since $\theta' > \theta$, $\delta < 180° - 2\theta$. Hence the correct choice is (c).

78. When the deviation produced by a prism is minimum, the angle of refraction $r = A/2 = 60°/2 = 30°$ for lights of all wavelengths. However the corresponding angle of incidence and the value of angles of minimum deviation are different for different colours. The correct choice is (a).

79. The beam will not enter region IV if the angle refraction in region IV equals 90°. Apply Snell's law at the interfaces, we have [See Fig. 26.90]

$$n_0 \sin\theta = \frac{n_0}{2} \sin\theta_1$$

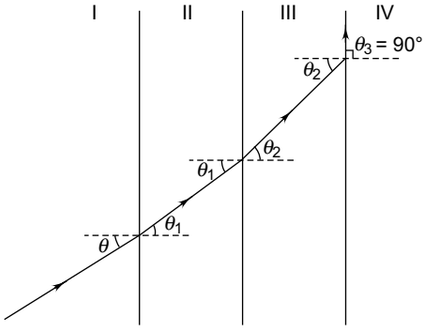


Fig. 26.90

$$= \frac{n_0}{6} \sin \theta_2 = \frac{n_0}{8} \sin 90°$$

which gives $\sin \theta = \frac{1}{8}$.

80. If I is the intensity of the incident light then $R + T = I$. Since the incident ray is travelling in a denser medium (glass), it will be totally reflected at a certain critical angle θ_c. For $\theta < \theta_c$ a part of the incident intensity is reflected and the remaining part is transmitted. But for $\theta > \theta_c$, there is no refracted ray. Hence for $\theta > \theta_c$, the value of R is 100 %. Hence the only correct option is (c).

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. A motor car is fitted with a convex mirror of focal length 20 cm. A second car 2.1 m broad and 1.05 m high is 4 m behind the first car. The breadth and height of the image of the second car as seen in the mirror of the first car are x and y respectively. If the second car is overtaking at a relative speed of 21 ms^{-1}, the speed of the image is v. Then
- (a) x = 20 m
- (b) y = 5 cm
- (c) $v = \frac{1}{21}$ ms^{-1}, moving towards the mirror
- (d) v = 21 ms^{-1}, moving away from the mirror.

2. The deviation produced by a prism depends on
- (a) the refractive index of the material of the prism
- (b) the refractive index of the medium surrounding the prism
- (c) the refracting angle of the prism
- (d) the angle of incidence of the ray falling on the prism.

3. The angle of minimum deviation of a prism placed in air depends on
- (a) the refractive index of the material of the prism
- (b) the refracting angle of the prism
- (c) the angle of incidence
- (d) the intensity of the incident light.

4. The dispersion of light in a medium implies that
- (a) lights of differents wavelengths travel with different speeds in the medium
- (b) lights of all frequencies travel with the same speed in the medium
- (c) the refractive index of the medium is different for different wavelengths of light.
- (d) the refraction index of the medium is the same for all frequencies of light.

5. For a given angle of incidence, the angle of deviation by a prism is greater for
- (a) violet light than for yellow light
- (b) red light than for green light
- (c) blue light than for red light
- (d) yellow light than for green light.

6. A lens is placed in air. The focal length of the lens depends on
- (a) the object and image distances
- (b) the refractive index of the material of the lens
- (c) the radii of curvature of its faces
- (d) the wavelength of light used.

7. The focal lenght of a lens is
- (a) greater for violet light than for yellow light
- (b) less for blue light than for red light
- (c) greater for green light than for blue light
- (d) the same for lights of all colours.

8. An illuminated object is placed at a distance $D = 100$ cm from a screen. A convex lens of focal length 21 cm is placed between them. A real image of the object is formed on the screen for two conjugate positions A and B of the lens separated by a distance d (see Fig. 26.91). The respective linear magnifications of the image are m_1 and m_2 for positions A and B of the lens. Then

(a) $d = 40$ cm (b) $d = 42$ cm

(c) $m_1 = \frac{7}{3}, m_2 = \frac{3}{7}$ (d) $m_1 = \frac{7}{5}, m_2 = \frac{5}{7}$

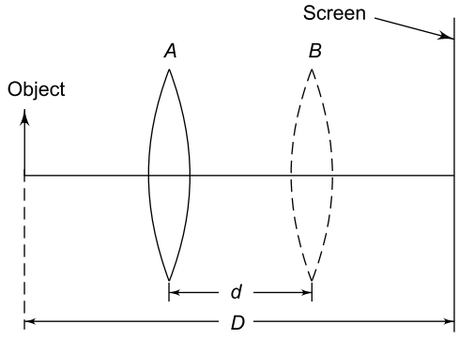


Fig. 26.91

9. When light travelling in air is reflected from a glass slab

(a) the wavelength of the reflected light decreases.

(b) the frequency of the reflected light is the same as that of the incident light.

(c) the reflected light undergoes a phase chage of π.

(d) the intensity of the reflected light is less than that of the incident light.

10. When light travelling in air enters a glass slab,

(a) its wavelength decreases

(b) its frequency increases

(c) its velocity decreases

(d) it undergoes a phase change of π.

11. The radius of curvature of a thin planoconvex lens is 10 cm and the refractive index of its glass is 1.5. Its focal length is f_1. If the plane surface is silvered, its focal length becomes f_2. Then

(a) $f_1 = 5$ cm (b) $f_1 = 40$ cm

(c) $f_2 = 10$ cm (d) $f_2 = 20$ cm

12. The focal length of a thin planoconvex lens is 60 cm and the refractive index of its glass is 1.5. Its radius of curvature is R. When the plane surface of the lens is silvered, it behaves like a concave mirror of focal length f. Then

(a) $R = 30$ cm (b) $R = 120$ cm

(c) $f = 10$ cm (d) $f = 30$ cm

13. A convex lens made of glass of refractive index 1.5 has both surfaces of the same radius of curvature R. In case (i) the lens is immersed in a medium of refractive index 1.25 and in case (ii) it is immersed in a medium of refractive index 1.75. Then

(a) in case (i) the lens will behave as a convergent lens of focal length $2.5R$

(b) in case (i) the lens will behave as a divergent lens of focal length $3.0R$

(c) in case (ii) the lens will behave as a convergent lens of focal length $3.0R$

(d) in case (ii) the lens will behave as a divergent lens of focal length $3.5R$

14. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole. The size of the image is a. If the object begins to move with a speed V_o, the speed with which the image moves is V_i. Then

(a) $a = b\left(\frac{f}{u-f}\right)$ (b) $a = b\left(\frac{f}{u-f}\right)^2$

(c) $V_i = V_o\left(\frac{f}{u-f}\right)$ (d) $V_i = V_o\left(\frac{f}{u-f}\right)^2$

IIT, 1987

15. A thin rod AB of length $f/3$ is placed along the axis of a concave mirror of focal length f, such that its image $A'B'$ which is real and elongated just touches the rod. The magnification produced is m. Then

(a) $A'B' = \frac{f}{3}$ (b) $A'B' = \frac{f}{2}$

(c) $m = 1$ (d) $m = 3/2$

IIT, 1991

16. A quarter cylinder of radius R is made of glass of refractive index 1.5. It is placed on a table and a point object P is kept at a distance mR from it as shown in Fig. 26.92. The value of m for which a ray from P will emerge parallel to the table as shown in the figure is

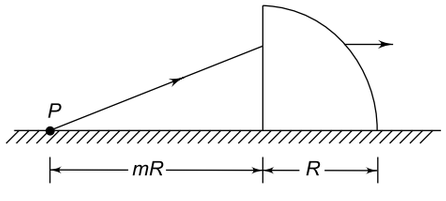


Fig. 26.92

(a) either $\frac{4}{3}$ or $\frac{3}{2}$ (b) neither $\frac{4}{3}$ nor $\frac{3}{2}$

(c) equal to $\frac{4}{3}$ (d) equal to $\frac{3}{2}$

IIT, 1999

17. Two identical equilateral prisms *ABC* and *DCE*, each of refractive index $\sqrt{3}$ are placed as shown in Fig. 26.93. A light ray *PQ* is incident on face *AB* at an angle *i*. Prism *DCE* is fixed at point *C* and can rotate about the axis passing through *C* and perpendicular to the plane of the page. The value of *i* for which the deviation produced by the prism *ABC* is minimum is i_0. The angle through which the prism *DCE* should be rotated about *C* so that the final emergent ray also has minimum deviation is θ. Then

(a) $i_0 = 45°$ (b) $i_0 = 60°$

(c) $\theta = 45°$ (d) $\theta = 60°$

IIT, 2005

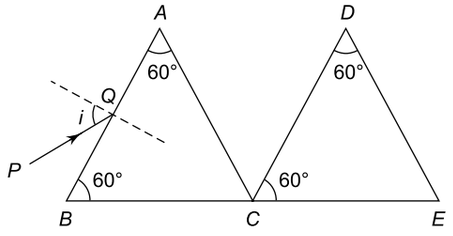


Fig. 26.93

ANSWERS AND SOLUTIONS

1. The correct choices are (a) and (c).

2. All the four choices are correct.

3. The correct choices are (a) and (b).

4. The correct choices are (a) and (c).

5. In a prism, the greater the wavelength of light the smaller is the deviation. In VIBGYOR, violet has the smallest wavelength and red has the highest wavelength. Hence the correct choices are (a) and (c).

6. The correct choices are (b), (c) and (d).

7. In a prism, the angle of deviation is proportional to the refractive index. The refractive index of a prism is more for violet light than for red light. Hence the correct choices are (b) and (c).

8. Refer to Fig. 26.94. If *O* and *I* are the object and the screen respectively and L_1 and L_2 are the two conjugate positions of the lens, then

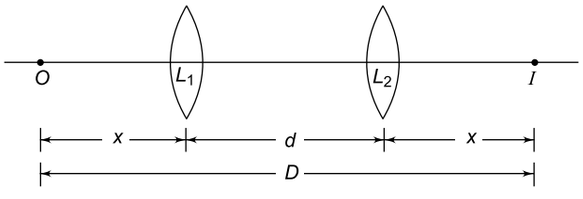


Fig. 26.94

For lens at L_1, $u = -x = -\frac{D-d}{2}$ and $v = \frac{D+d}{2}$.

Using these in $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, we get

$$f = \frac{D^2 - d^2}{4D}$$

Putting f = 21 cm and D = 100 cm, we get d = 40 cm.

Magnification $m = \frac{v}{u} = -\frac{D+d}{D-d}$

$$= -\frac{100+40}{100-40} = -\frac{7}{3}.$$

Thus $|m| = \frac{7}{3}$. For lens at L_2, $|m| = \frac{3}{7}$. So the correct choices are (a) and (c).

9. The frequency of light never changes on reflection and refraction. The speed of light in a medium depends on the permittivity and permeability of the medium. Since the reflected light travels in the same medium as the incident light, the speed of reflected light is the same as that of the incident light. From $v = \nu\lambda$, we conclude that the wavelength of reflected is the same as that of the incident light. When light falls on the glass slab, it is partly reflected back into air and partly refracted into the slab. Hence the intensity of reflected is always less than that of incident light. Furthermore, when light travelling in a rarer medium is reflected from the boundary of a denser medium, it undergoes a phase change of π. Hence the correct choices are (b), (c) and (d).

10. The speed of light in glass is less than in air. Since the frequency of light remains the same, we find from $v = \nu\lambda$, that the wavelength of refracted light is less than that of the incident light. The refracted light never undergoes a phase change. Hence the correct choices are (a) and (c).

11. $\frac{1}{f_1} = (\mu - 1) \times \frac{1}{R} = (1.5 - 1) \times \frac{1}{10} \Rightarrow f_1 = 20$ cm.

When the plane surface is silvered, the focal length is given by

$$\frac{1}{f_2} = \frac{2(\mu-1)}{R} = \frac{2\times(1.5-1)}{10} \quad f_2 = 10 \text{ cm}.$$

Thus the only correct choice is (c).

12. For a plano-convex lens of $\mu = 1.5$, $f_1 = 2\ R \Rightarrow R = f_1/2 = 30$ cm. When the plane surface is silvered, the focal length becomes

$$f = \frac{R}{2(\mu-1)} = \frac{30}{2(1.5-1)} = 30 \text{ cm}$$

Thus the correct choices are (a) and (d).

13. The correct choices are (a) and (d).

14. The concave mirror formula is

$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f} \qquad (1)$$

Since, for a given concave mirror, focal length f is fixed, we partially differentiate Eq. (1) to get

$$-\frac{\delta v}{v^2} = -\frac{\delta u}{u^2} = 0 \text{ or } \delta v = -\left(\frac{v}{u}\right)^2 \delta u \qquad (2)$$

Multiplying Eq. (1) by u, we get

$$\frac{u}{v} + 1 = \frac{u}{f} \text{ or } \frac{v}{u} = \frac{f}{u-f} \qquad (3)$$

Using Eq. (3) in Eq. (2), we get

$$\delta v = -\left(\frac{f}{u-f}\right)^2 \delta u$$

Given $\delta u = b$. Therefore

$$\delta v = -\left(\frac{f}{u-f}\right)^2 b \qquad (4)$$

The negative sign shows that the image is longitudinally inverted. The magnitude of the size of the image is

$$a = b\left(\frac{f}{u-f}\right)^2$$

Dividing Eq. (4) by δt, we have

$\frac{\delta v}{\delta t} = -\left(\frac{f}{u-f}\right)^2 \frac{\delta u}{\delta t}$. Given $\frac{\delta u}{\delta t} = V_0$. The speed of the image is $V_i = \frac{\delta v}{\delta t} = -\left(\frac{f}{u-f}\right)^2 V_0$

The negative sign shows that the image moves in a direction opposite to that of the object. The magnitude of the speed of the image is

$$V_i = V_0\left(\frac{f}{u-f}\right)^2$$

So the correct choices are (b) and (d).

15. Refer to Fig. 26.95. $A'B'$ is the image of the rod AB. B' is the image of end B of the rod. Given that B' coincides with B.

Location of B': Let x be the distance of point B or B' from the pole O of the mirror. Thus $u = -x$, $v = -x$ and $f = -f$. Using these in the spherical mirror formula

$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f}$$

We have $-\frac{1}{x}-\frac{1}{x} = -\frac{1}{f}$

which gives $x = 2f$. Thus $OB = OB' = 2f$

Location of A': A' is the image of A. Since $AB = \frac{f}{3}$, $AO = y = 2f - \frac{f}{3} = \frac{5f}{3}$.

Thus for point A, $u = -y = -\frac{5f}{3}$. The image distance $A'O$ is obtained from

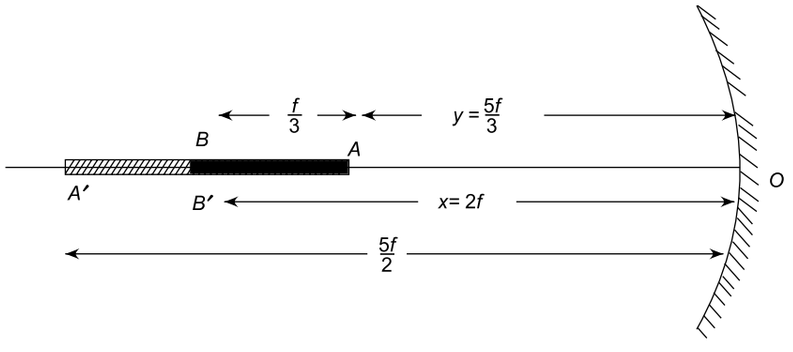


Fig. 26.95

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad \frac{1}{v}-\frac{3}{5f}=\frac{1}{f}$$

which gives $v=-\frac{5f}{2}$. Thus $A'O=\frac{5f}{2}$ as shown in the figure. Therefore, size of the image is

$$A'B'=\frac{5f}{2}-2f=\frac{f}{2}$$

The size of the object is $AB=\frac{f}{3}$

$$\therefore \quad \text{Magnification}=\frac{A'B'}{AB}=\frac{f/2}{f/3}=\frac{3}{2}.$$

Thus the correct choices are (b) and (d).

16. Refer to Fig. 26.96 which shows the ray diagram. Q is the image of P due to refraction at the plane surface. Ray PA refracts along AB and appears to come from Q which is the virtual image of P. Since, the radius of curvature of the plane surface in infinity, we have

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{\infty}$$

Here $\mu_1=1$, $\mu_2=1.5$, $u=-mR$. Therefore,

$$\frac{1.5}{v}-\frac{1}{-mR}=0 \text{ or } v=-1.5\,mR$$

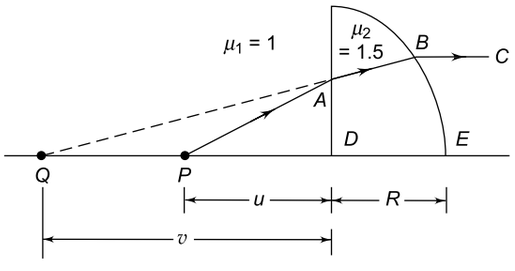


Fig. 26.96

After refraction at the curved surface, the ray BC is to emerge parallel to the table. Image Q serves as the object for the curved surface, for which the object distance $u'=-(v+R)=-(1.5\,mR+R)=-R\,(1.5m+1)$. The image distance $v'=-m\infty$ ($\because$ ray BC is parallel to the table) and $R=-R$. Therefore, we have

$$\frac{\mu_2}{u'}-\frac{\mu_1}{v'}=\frac{\mu_2-\mu_1}{-R}$$

$$\text{or} \quad \frac{1.5}{-R\,(1.5m+1)}-0=\frac{1.5-1}{-R}$$

which gives $(1.5\,m+1)\times 0.5=1.5$

$$\text{or} \quad m=\frac{2}{1.5}=\frac{4}{3}.$$

The only correct choice is (c).

17. For minimum deviation in prism ABC, the ray QR is parallel to base BC. Hence

$$r_1=r_2 \text{ and } r_1+r_2=60^\circ$$

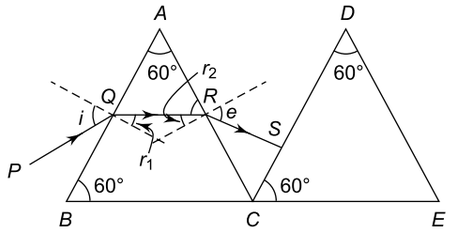


Fig. 26.97

which give $r_1=r_2=30^\circ$

From Snell's law,

$$\sin i=\mu \sin r_1$$

$$=\sqrt{3}\times \sin 30^\circ$$

$$=\frac{\sqrt{3}}{2}$$

which gives $i=60^\circ$

The angle of emergence $e=60^\circ$. If follows from Fig. 26.97 that the ray RS falls normally on face DC of the second prism. For ray RS to suffer minimum deviation in the second prism, the angle of incident of ray RS on face DC must be 60°. Therefore, the second prism DCE should be rotated about C through an angle of 60° in the clockwise or anticlockwise direction.

So the correct choices are (b) and (d).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 5 are based on the following passage

Passage I

The Compound Microscope

A microscope is a device which is used to view tiny objects. A compound microscope consists of two converging lenses called the objective and the eyepiece. The tiny object to be examined is placed just beyond the first focus of the objective. The position of the eyepiece is adjusted till the image due to the objective is within the first focus of the eyepiece. The highly enlarged final image is seen by the eye which is held close to the eyepiece.

1. In a compound microscope, the intermediate image (i.e. image of the object due to the objective) is
   (a) real, inverted and magnified
   (b) real, inverted and diminished
   (c) virtual, erect and magnified
   (d) virtual, erect and diminished

2. In a compound microscope, the final image is
   (a) real, inverted and magnified
   (b) real, erect and magnified
   (c) virtual, erect and magnified
   (d) virtual, inverted and magnified

3. The magnifying power of a compound microscope is high if
   (a) both the objective and the eyepiece have short focal lengths.
   (b) both the objective and the eyepiece have long focal lengths.
   (c) the objective has a short focal length and the eyepiece has a long focal length
   (d) the objective has a long focal length and the eyepiece has a short focal length.

4. The resolving power of a compound microscope is increased if
   (a) light of a shorter wavelength is used to illuminate the object
   (b) the objective of a bigger diameter is used
   (c) the objective of a higher focal length is used
   (d) the eyepiece of a shorter focal length is used.

5. If the aperture of the objective of a microscope is increased,
   (a) its resolving power will increase
   (b) its magnifying power will decrease
   (c) the intensity of the final image will increase
   (d) the intensity of the final image will decrease

SOLUTION

1. The correct choice is (a).

2. The correct choice is (d).

3. The correct choice is (a). The magnifying power is given by

$$M = \frac{L}{f_o} \cdot \frac{D}{f_e} \qquad (\text{for } D >> f_o)$$

where L = distance between the objective and the eyepiece, D = least distance of distinct vision, f_o = focal length of the objective and f_e = focal length of the eyepiece.

4. The resolving power of a microscope is given by (see Fig. 26.98)

$$\text{R.P.} = \frac{2\mu \sin\theta}{\lambda}$$

where 2θ = angle of the cone of light rays entering the objective, λ = wavelength of light used to illuminate the object and μ = refractive of the medium between the object and the objective. The value of θ increases if the objective of a bigger diameter AB is used. The resolving power does not depend on f_o or f_e. Hence the correct choices are (a) and (b).

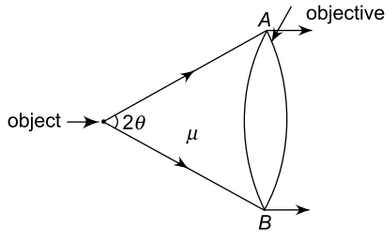


Fig. 26.98

5. If the aperture of the objective is increased, the value of θ increases. Therefore, the light gathering capacity of the objective increases. As a result, the intersity of the image increases. Hence the correct choices are (a) and (c).

Questions 6 to 10 are based on the following passage

Passage II

The Astronomical Telescope

A telescope is an optical instrument that is used to examine distant objects. Two types of telescopes are in use—refracting and reflecting telescopes. A refracting astronomical telescope consists of two converging lenses called the objective and the eyepiece. The objective faces the distant object. The image of the object is formed at the focal plane of the objective. The position of the eyepiece is adjusted till this image is within the first focus of the eyepiece. A highly magnified final image is formed which is seen by the eye held close to the eyepiece. If both the object and the final image are at infinity, the telescope is said to be in normal adjustment.

6. In a refracting astronomical telescope, the intermediate image is
- (a) real, inverted and magnified
- (b) real, inverted and diminished
- (c) virtual, erect and magnified
- (d) virtual, inverted and diminished

7. In a refracting astronomical telescope, the final image is
- (a) real, inverted and magnified
- (b) real, erect and magnified
- (c) virtual, erect and magnified
- (d) virtual, inverted and magnified

8. The magnifying power of a telescope is high if
- (a) both the objective and the eyepiece have short focal lengths
- (b) both the objective and the eyepiece have long focal lengths
- (c) the objective has a short focal length and the eyepiece has a long focal length
- (d) the objective has a long focal length and the eyepiece has a short focal length

9. The resolving power of a telescope is increased if
- (a) the objective of a bigger diameter is used
- (b) the objective of a smaller diameter is used
- (c) the objective of a higher focal length is used
- (d) the eyepiece of a shorter focal length is used

10. If the aperture of the objective of a telescope is increased,
- (a) its resolving power will increase
- (b) its magnifying power will decrease
- (c) the intensity of the final image will increase
- (d) the intensity of the final image will decrease.

SOLUTION

6. The correct choice is (b).

7. The correct choice is (d).

8. The magnifying power of a telescope (if the object is at infinity) is given by

$$M = \frac{f_o}{f_e} \cdot \frac{D + f_e}{D}$$

where D = least distance of distinct vision, where the final image is formed. Hence the correct choice is (d).

9. The resolving power of a telescope is given by

$$\text{R.P.} = \frac{d}{1.22\lambda}$$

where d = diameter of the objective and λ = wavelength of light. The resolving power is independent of f_o or f_e. Hence the correct choice is (a).

10. The correct choices are (a) and (c).

Questions 11 to 13 are based on the following passage

Passage III

A ray of light travelling in air is incident at grazing angle (incident angle = 90°) on a long rectangular slab of a transparent medium of thickness $t = 1.0$ m (see Fig. 26.99). The point of incidence is the origin A (0, 0). The medium has a variable index of refraction $n(y)$ given by

$$n(y) = (ky^{3/2} + 1)^{1/2}$$

where $k = 1.0$ (metre)$^{-3/2}$. The refractive index of air is 1.0.

IIT, 1995

11. The relation between the slope of the trajectory of the ray at the point $B(x, y)$ in the medium and the incident angle i at that point is

(a) $\frac{dy}{dx} = \sin i$ (b) $\frac{dy}{dx} = \cos i$

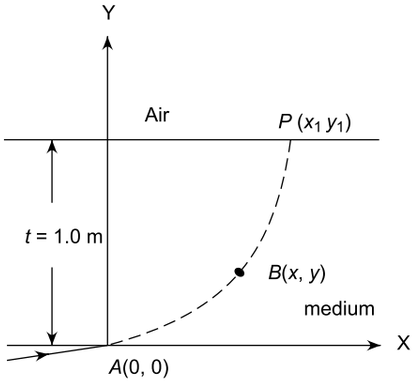


Fig. 26.99

(c) $\frac{dy}{dx} = \tan i$ (d) $\frac{dy}{dx} = \cot i$

12. The equation of the trajectory of the ray in the medium is

(a) $y = (x)^4$ (b) $y = \left(\frac{x}{2}\right)^4$

(c) $y = \left(\frac{x}{3}\right)^4$ (d) $y = \left(\frac{x}{4}\right)^4$

13. The coordinate x_1 of point P where the ray intersects the upper surface of the air-slab boundary is

(a) $x_1 = 4.0$ m (b) $x_1 = 3.0$ m

(c) $x_1 = 2.0$ m (d) $x_1 = 1.0$ m

SOLUTION

11. Refer to Fig. 26.100. The variation of refractive index is given by

$$n(y) = (ky^{3/2} + 1)^{1/2} \quad (1)$$

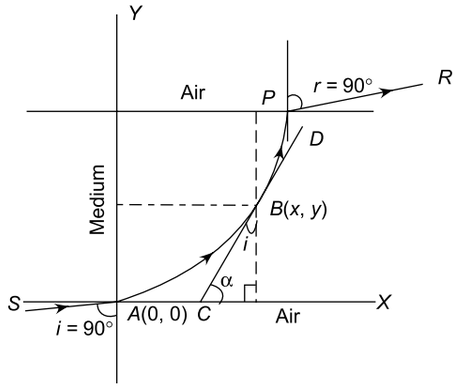


Fig. 26.100

It follows from relation (1) that the refractive index increases in the slab from $y = 0$ to $y = t$. Therefore, the ray SA entering the slab at A at grazing incidence ($i = 90°$) keeps bending towards the normal as it progresses in the slab, following a curved path ABP.

If the angle of incidence at point $B(x, y)$ is i, then the slope CD of the trajectory at B is

$$\frac{dy}{dx} = \tan \alpha = \tan (90° - i) = \cot i \quad (2)$$

The correct choice is (d).

12. If i is the angle of incident at a point in the medium of refractive index n, then applying Snell's law, we have

$$n \sin i = n_a \sin i_a = 1 \times \sin 90° = 1$$

because n_a = refractive index of air = 1 and i_a at $A = 90°$.

or $\sin i = \frac{1}{n}$

$$\therefore \quad \cot i = \frac{\left(1 - \frac{1}{n^2}\right)^{1/2}}{\left(\frac{1}{n}\right)} = (n^2 - 1)^{12} \quad (3)$$

Using Eq. (2) in Eq. (3) we get

$$\frac{dy}{dx} = (n^2 - 1)^{1/2} \text{ or } \left(\frac{dy}{dx}\right)^2 = n^2 - 1 \quad (4)$$

Using Eq. (1) in Eq. (4) we have

$$\left(\frac{dy}{dx}\right)^2 = ky^{3/2} + 1 - 1 = ky^{3/2}$$

or $\frac{dy}{dx} = k^{1/2} y^{3/4}$ or $\frac{dy}{y^{3/4}} = k^{1/2} dx$

Integrating, we get

$$\int \frac{dy}{y^{3/4}} = k^{1/2} \int dx \text{ or } 4y^{1/4} = k^{1/2} x$$

Given $k = 1.0$. Therefore, we have

$$y^{1/4} = \left(\frac{1}{4}\right)x \text{ or } y = \left(\frac{x}{4}\right)^4 \quad (5)$$

So the correct choice is (d).

13. For point P, $y = 1.0$ m. Using $y = 1.0$ m in Eq. (5) we get $x = 4.0$ m. Thus the coordinates of point P are $x_1 = 4.0$ m and $y_1 = 1.0$ m. Thus the correct choice is (a).

Questions 14 to 16 are based on the following passage

Passage IV

The x-z plane is the boundary between two transparent media, medium 1 with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium 2 with $z \leq 0$ has a refractive index $\sqrt{3}$. A ray of light in medium 1 given by the vector $\mathbf{A} = 6\sqrt{3}\ \hat{\mathbf{i}} + 8\sqrt{3}\ \hat{\mathbf{j}} - 10\hat{\mathbf{k}}$ is incident on the plane of separation.

IIT, 1999

14. The angle between vector A and the positive z-direction is

(a) 90° (b) 120°

(c) 135° (d) 150°

15. The angle of refraction in medium 2 is

(a) 30° (b) 45°

(c) 60° (d) 75°

16. The refracted ray is given by $\boldsymbol{A}_r = a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}} + c\,\hat{\mathbf{k}}$ where a and b are

(a) $a = 6\sqrt{3}$, $b = 8\sqrt{3}$

(b) $a = 3\sqrt{3}$, $b = 4\sqrt{3}$

(c) $a = 8\sqrt{3}$, $b = 6\sqrt{3}$

(d) $a = 4\sqrt{3}$, $b = 3\sqrt{3}$

SOLUTION

14. Refer to Fig. 26.101. Ray PQ in x-z plane travelling in medium 1 ($z \geq 0$) is incident at angle i on the boundary ($z = 0$ plane) and is refracted along QR in medium 2 ($z \leq 0$) at an angle of reflection r.

Let θ be the angle between vector $\mathbf{A}$ and the positive z-direction. Since $\hat{\mathbf{k}}$ is the unit vector along the positive z-axis, we have

$$\cos\theta = \frac{\mathbf{A}.\hat{\mathbf{k}}}{|\mathbf{A}|}$$

$$= \frac{(6\sqrt{3}\hat{\mathbf{i}} + 8\sqrt{3}\hat{\mathbf{j}} - 10\hat{\mathbf{k}})\cdot(\hat{\mathbf{k}})}{\left[(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10)^2\cdot\right]^{1/2}}$$

$$= \frac{-10}{20} = -\frac{1}{2}$$

$(\because\ \hat{\mathbf{i}}\ .\ \hat{\mathbf{k}} = \hat{\mathbf{j}}\ .\ \hat{\mathbf{k}} = 0)$

which gives $\theta = 120°$. which is choice (b).

15. Therefore, angle of incidence $i = 180° - \theta = 180° - 120° = 60°$. From Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

or $$\sin r = \frac{\mu_1}{\mu_2}\sin i = \frac{\sqrt{2}}{\sqrt{3}}\sin 60° = \frac{1}{\sqrt{2}}$$

which gives $r = 45°$. so the correct choice is (b).

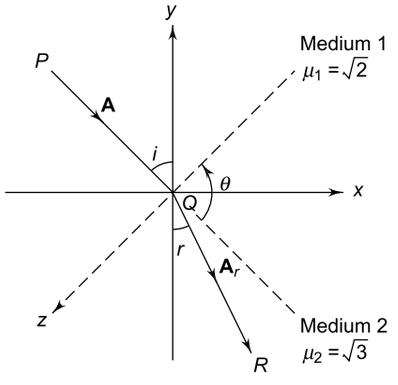


Fig. 26.101

16. Since the refracted ray lies in the same plane as the incident wave (which is the x-y plane), the refracted ray will be given by the vector

$$\mathbf{A}_r = a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}} + c\,\hat{\mathbf{k}}$$

with $a = 6\sqrt{3}$ and $b = 8\sqrt{3}$, the same as in the incident vector $\mathbf{A}$. Thus

$$\mathbf{A}_r = 6\sqrt{3}\ \hat{\mathbf{i}} + 8\sqrt{3}\ \hat{\mathbf{j}} + c\,\hat{\mathbf{k}}$$

This the corrcet choice is (a).

Questions 17 to 19 are based on the following passage

Passage V

The radius of curvature of the curved face of a thin plano-convex lens is 10 cm and it is made of glass of refractive index 1.5. A small object is approaching the lens with a speed of 1 cm s^{-1} moving along the principal axis.

IIT, 2004

17. The focal length of the lens is

(a) 5 cm (b) 10 cm

(c) 15 cm (d) 20 cm

18. When the object is at a distance of 30 cm from the lens, the magnitude of the speed of its image is

(a) 1 cm s^{-1} (b) 2 cm s^{-1}

(c) 3 cm s^{-1} (d) 4 cm s^{-1}

19. When the object is at a distance of 30 cm from the lens, the magnitude of the rate of change of the lateral magnification is

(a) 0.1 per second (b) 0.2 per second

(c) 0.3 per second (d) 0.4 per second

SOLUTION

17. $\frac{1}{f}=(\mu-1)\times\frac{1}{R}=(1.5-1)\times\frac{1}{10}$ gives $f=20$ cm. The correct choice is (d).

18. Lens formula is (here $u = -30$ cm and $f = +20$ cm)

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad (1)$$

or $\frac{1}{v}-\frac{1}{-30}=\frac{1}{20}$ which gives $v = 60$ cm

Differentiating (1) with respect to time t, we get

$$-\frac{1}{v^2}\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0$$

or $$\frac{dv}{dt}=\left(\frac{v^2}{u^2}\right)\frac{du}{dt}$$

or Speed of image $=\left(\frac{v^2}{u^2}\right)\times$ speed of object

$$=\left(\frac{60}{30}\right)^2\times 1 = 4 \text{ cm s}^{-1},$$

which is choice (d).

19. Linear magnification is given by

$$m=\frac{v}{u} \qquad (2)$$

Differentiating (2) with respect to time t, we have

$$\frac{dm}{dt}=-\frac{v}{u^2}\frac{du}{dt}+\frac{1}{u}\frac{dv}{dt}$$

$$=\frac{1}{u^2}\left(-v\frac{du}{dt}+u\frac{dv}{dt}\right)$$

$$=\frac{1}{(30)^2}(-60\times 1-30\times 4)$$

$$=-0.2 \text{ per second}$$

$\therefore$ Magnitude of $\frac{dm}{dt}=0.2$ per second.

So the correct choice is (b).

SECTION IV

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. Statement-1
When a concave mirror is held under water, its focal length will increase.
Statement-2
The focal length of a concave mirror is independent of the medium in which it is placed.

2. Statement-1
When a convex lens is held under water, its focal length increases.
Statement-2
The focal length of a lens depends on the medium in which it is placed.

3. Statement-1
A convex lens is made of glass of refractive index 1.45 and its two faces have the same radius of curvature equal to R. The focal length of the lens is more than R.
Statement-2
The above statement is false because the focal length of a lens is half the radius of curvature.

4. Statement-1
A ray of light is incident at the glass-water interface at an angle i. It emerges finally parallel to the surface of water as shown in Fig. 26.102. Then

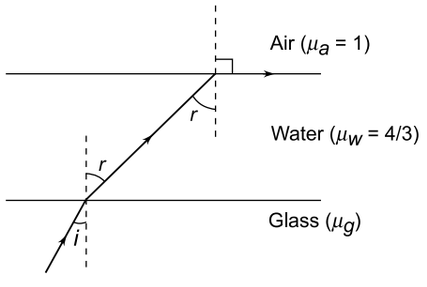


Fig. 26.102

$$\mu_g = \left(\frac{4}{3}\right) \sin i$$

Statement-2
The refracted ray follows a path in accordance with Snell's law of refraction.

SOLUTIONS

1. The correct choice is (d). The focal length of a concave mirror depends only on its radius of curvature.

2. The correct choice is (a). The focal length of a lens depends not only on the radii of curvature of its faces but also on the refractive index of its glass and the refractive index of the medium in which it is placed. The focal length f of a convex lens of refractive index μ_2 and radii of curvature R_1 and R_2 when placed in a medium of refractive index μ_1 is given by

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

It is easy to see that f will be more if μ_1 is different from unity ($\mu_1 = 1$ for air).

3. The correct choice is (c). The focal length f is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.45 - 1) \times \frac{2}{R}$$

which gives $f = \dfrac{R}{0.9}$, i.e. f is greater than R.

4. The correct choice is (d). For refraction at glass-water interface, we have from Snell's law

$$\mu_g \sin i = \mu_\omega \sin r \qquad \text{(i)}$$

For refraction at water-air interface, we have

$$\mu_\omega \sin r = \mu_a \sin 90° = \mu_a = 1 \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$\mu_g \sin i = 1 \text{ or } \mu_g = \frac{1}{\sin i}$$

SECTION V

Integer Answer Type

1. A telescope has an objective of focal length 50 cm and an eye-piece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focussed for distinct vision on a scale 200 cm away from the objective. Find the magnitude of magnification produced by the telescope.

IIT, 1980

2. The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 15 cm. The concave surface has a radius of curvature 50 cm. The convex surface is silvered and placed on a horizontal surface as shown in Fig. 26.103. It is found that the image of a pin at a distance of $\dfrac{100}{n}$ cm is at the same place. Find the value of n.

IIT, 1981

3. A ray of light is incident at an angle of 60° on one face of a prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. If the refractive index of the material of the prism is $\mu = \sqrt{a}$, find the value of a.

IIT, 1987

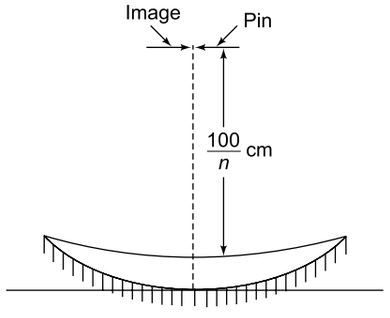


Fig. 26.103

4. A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. The image is formed due to refractions at the two opposite surfaces of the bubble. Find the distance (in mm) of the image from the centre of the bubble.

IIT, 1988

5. Water (with refractive index = $\dfrac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\dfrac{7}{4}$ lies on water

making a convex surface of radius of curvature 'R = 6 cm' as shown in Fig. 26.104. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is

IIT, 2011

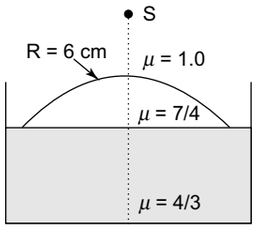


Fig. 26.104

SOLUTION

1. For the objective, $\frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}$. Putting u_0 = – 200 cm, and f_0 = 50 cm, we get $v_0 = \frac{200}{3}$ cm. Therefore, magnification by the objective is

$$m_0 = \frac{v_0}{u_0}=\frac{200/3}{-200}=-\frac{1}{3}$$

The negative sign shows that the image formed by the objective is inverted. For the eye-piece, we have

$$\frac{1}{v_e}-\frac{1}{u_e}=\frac{1}{f_e}$$

where v_e = – 25 cm and f_e = 5 cm. Using these values, we get $u_e = -\frac{25}{6}$ cm. Magnification by the eye-piece is

$$m_e = \frac{v_e}{u_e}=\frac{-25}{-25/6}=6$$

Total magnification by the telescope is

$$m = m_0 \times m_e = -\frac{1}{3}\times 6 = -2$$

$$\therefore \quad |m| = 2$$

2. For the image to be formed at the same place as the object, the ray from the object, after refraction at the concave surface must fall normally on the silvered convex surface so that it retraces its path. Hence the image formed by the concave surface must be situated at a distance = the radius of curvature of the convex surface. Therefore v = – 15 cm. For refraction at the concave surface

$$\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{R}\text{ , where } R = -50 \text{ cm}$$

$$\Rightarrow \quad \frac{1.5}{-15}-\frac{1}{u}=\frac{1.5-1}{-50} \Rightarrow u = \frac{100}{9} \text{ cm}$$

Hence n = 9.

3. Given i = 60°, δ = 30° and A = 30°. We have

$$\delta = i + e - A \qquad (1)$$

From Eq. (1), we get 30° = 60° + e – 30° or e = 0. Here also i = 60° and δ = 30°. [see Fig. 26.105]

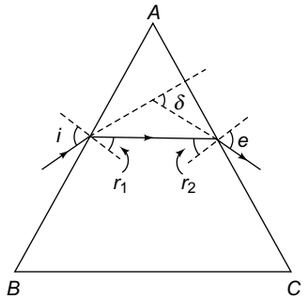


Fig. 26.105

Therefore, $r_1 = i - \delta = 60° - 30° = 30°$. Hence

$$\mu = \frac{\sin i}{\sin r_1}=\frac{\sin 60°}{\sin 30°}=\sqrt{3}$$

Hence the value of a = 3.

4. For refraction at the first surface, we use see Fig. 26.106]

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

where μ_1 = 4/3, μ_1 = 1 and $u = \infty$ and R = 2 mm. Thus

$$\frac{1}{v_1}-\frac{4/3}{\infty}=\frac{1-4/3}{2}$$

which gives v_1 = – 6 mm, the negative sign indicates that the image I_1 is virtual and is on the same side as the object at a distance of 6 mm from the first surface.

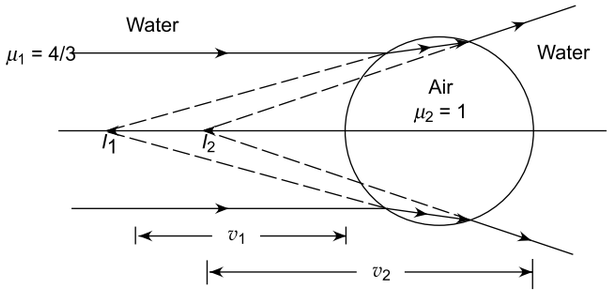


Fig. 26.106

For refraction at the second surface, the image I_1 serves as the virtual object which is at a distance of 6 mm + 4 mm = 10 mm from the second surface. For this refraction, we use

$$\frac{\mu_1}{v_2} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

where $u = -10$ mm and $R = -2$ mm. Thus

$$\frac{4/3}{v_2} - \frac{1}{-10} = \frac{4/3 - 1}{(-2)}$$

which gives $v_2 = -5$ mm. Therefore, the distance of the image I_2 from the centre of the bubble = 3 mm.

5. If the incident ray is in a medium of refractive index μ_1 and the refracted ray is in medium of refractive index μ_2, then

$$\frac{\mu_2}{v_2} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{(i)}$$

Applying Eq. (i) at the air-lens interface

$$\frac{7/4}{v} - \frac{1}{(-24)} = \frac{7/4 - 1}{6}$$

$$\Rightarrow \quad v = 21 \text{ cm}$$

Applying Eq. (i) at the lens-water interface (for this refraction the object distance $u = 21$ cm)

$$\frac{4/3}{v'} - \frac{7/4}{21} = 0$$

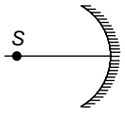
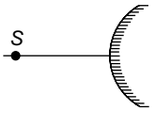
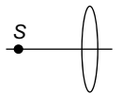
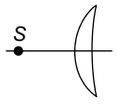
$$\Rightarrow \quad v' = 16 \text{ cm}$$

Hence the distance of the final image from the bottom of the tank = 18 – 16 = 2 cm

SECTION VI

Matrix Match Type

1. An optical component and an object S placed along its optic axis are given in **Column I**. The distance between the object and the component can be varied. The properties of images are given in **Column II**. Match all the properties of imges from **Column II** with the appropriate components given in **Column I**.

Column I	***Column II***
(a) 	(p) Real image
(b) 	(q) Virtual image
(c) 	(r) Magnified image
(d) 	(s) Image at infinity

IIT, 2009

SOLUTION

(a) A concave mirror forms real, virtual as well as magnified image. For an object on its focus, the image is at infinity. Hence the all the four choices in Column II are correct.
(b) A convex mirror forms only virtual images. Hence the correct choice is (q).
(c) For a convex lens, all the four choices are correct.
(d) The focal length of the lens is given by

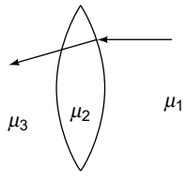
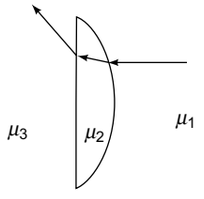
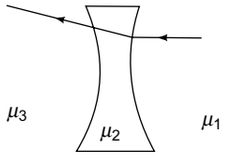
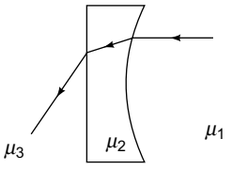
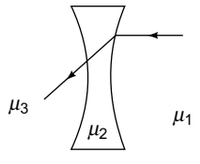
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where R_1 = radius of curvature of the first surface (the one closer to object S)
and R_2 = radius of curvature of the second surface. R_1 and R_2 are both positive. Sine $R_2 > R_1$, f is positive, i.e. the lens is a converging lens for which all the four choices are correct.

ANSWER

(a) $\rightarrow$ (p), (q), (r), (s) (b) $\rightarrow$ (q)
(c) $\rightarrow$ (p), (q), (r), (s) (d) $\rightarrow$ (p), (q), (r), (s)

2. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figure. In Column I different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in Column II.

Column I	***Column II***
(a) $\mu_1 < \mu_2$	(p) 
(b) $\mu_1 > \mu_2$	(q) 
(c) $\mu_2 = \mu_3$	(r) 
(d) $\mu_2 > \mu_3$	(s) 
	(t) 

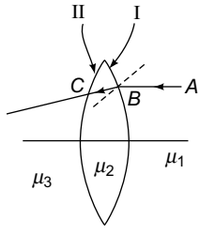
IIT, 2010

ANSWERS

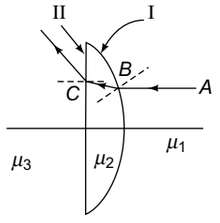
(a) $\rightarrow$ (p), (r) (b) $\rightarrow$ (q), (s), (t)
(c) $\rightarrow$ (p), (r), (t) (d) $\rightarrow$ (q), (s)

Explanation:

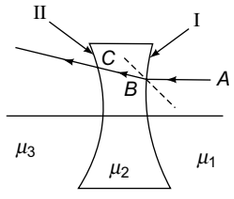
(p) Ray AB bends towards the normal on refraction at face I of the lens. Hence $\mu_2 > \mu_1$. Ray BC goes undeviated on refraction at face II. Hence $\mu_2 = \mu_3$.



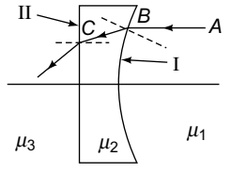
(q) Ray AB bends away from the normal on refraction at face I. Hence $\mu_2 < \mu_1$. The ray BC bends away from the normal on refraction at face II. Hence $\mu_3 < \mu_2$.



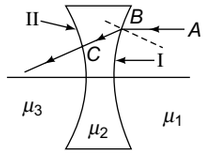
(r) Ray AB bends towards the normal on refraction at face I. Hence $\mu_2 > \mu_1$. Ray BC goes undeviated on refraction at face II. Hence $\mu_3 = \mu_2$.



(s) Ray AB bends away from the normal on refraction at face I. Hence $\mu_2 < \mu_1$. Ray BC bends away from the normal on refraction at face II. Hence $\mu_3 < \mu_2$.



(t) Ray AB bends away from the normal on refraction at face I. Hence $\mu_2 < \mu_1$. Ray BC goes undeviated on refraction at face II. Hence $\mu_3 = \mu_2$.



27 Chapter

Wave Optics

REVIEW OF BASIC CONCEPTS

27.1 WAVE NATURE OF LIGHT

Light is an electromagnetic wave which does not require a material medium for propagation. The electric and magnetic fields vary in space and time resulting in the propagation of an electromagnetic wave even in free space.

The electric field varies in space and time as

$$E = A \sin (\omega t - kx)$$

which represents a wave travelling along the $+x$ direction. A = amplitude, $\omega = 2\pi\nu$ (ω is angular frequency in rad s^{-1} and ν is frequency in Hz) and $k = \frac{2\pi}{\lambda}$; λ = wavelength. Also

$$v = \nu\lambda = \frac{\omega}{k}$$

where v is the wave velocity.

Phase

The phase ϕ of a wave at a point x and at time t is given by the argument of the harmonic function (sine or cosine) representing the wave, i.e.

$$\phi = \omega t - kx$$

Phase Difference

Suppose two waves meeting at a point P are represented by

$$E_1 = A_1 \sin (\omega_1 t - k_1 x_1)$$

and

$$E_2 = A_2 \sin (\omega_2 t - k_2 x_2)$$

where x_1 and x_2 are paths of the waves up to point P where they meet. The phase difference between them is

$$\Delta\phi = \phi_1 - \phi_2$$
$$= (\omega_1 - \omega_2)t - (k_1 x_1 - k_2 x_2)$$
$$\Delta\phi = (\phi_1 - \phi_2)$$
$$= 2\pi (\nu_1 - \nu_2)t - 2\pi \left(\frac{x_1}{\lambda_1} - \frac{x_2}{\lambda_2}\right)$$

1. If the two waves have different frequencies, i.e., $\nu_1 \neq \nu_2$ then $\lambda_1 \neq \lambda_2$ and $\Delta\phi$ depends on time t.
2. If $\nu_1 = \nu_2$, then $\lambda_1 = \lambda_2$. In this case

$$\Delta\phi = \frac{2\pi}{\lambda} (x_2 - x_1)$$

or Phase difference $= \frac{2\pi}{\lambda} \times$ (path difference)

i.e., the phase difference is independent of time and depends only on the path difference $(x_2 - x_1)$. This holds only if the two sources of wave are 'coherent', i.e., they have a constant fixed phase relationship.

Intensity The intensity of a wave at any point in its path is proportional to the square of its amplitude at that point.

27.2 REFLECTION AND REFRACTION OF LIGHT

When a light wave falls on a reflecting surface, it is reflected obeying the usual laws of reflection. When a wave travels from one medium into another, its velocity and wavelength undergo a change and the wave is said to suffer refraction. The frequency of the wave does not undergo any change in refraction (and reflection). If v_1 is the velocity of the wave in the medium in which the incident wave propagates and v_2 is the velocity of the wave in the medium in which the refracted wave propagates, then ${}^1\mu_2$, the reflective index of the second medium with respect to the first, is defined as

$$^1\mu_2 = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

where λ_1 and λ_2 are the wavelengths of the same wave in the two media. The frequency of the refracted wave remains the same as that of the incident wave.

When a wave, travelling in a rarer medium, is reflected at the boundary of a denser medium, the reflected wave suffers a phase change of 180° (or π radians) in relation to that of the incident wave. No phase change occurs if a wave, travelling in a denser medium, is reflected at the boundary of a rarer medium. The refracted wave, in both cases, does not undergo any phase change.

27.3 INTERFERENCE OF LIGHT

When two or more light waves meet (or superpose) at a point in a medium, the electric field of the resultant wave can be obtained by using the *principle of superposition* which states that *the resultant electric field is given by the algebraic sum of the individual electric fields, at that point, due to the individual waves, i.e.,*

$$E = E_1 + E_2 + \cdots$$

resulting in a change in amplitude (and hence in intensity) at that point. The phenomenon in which the intensity of light at a point is modified by the superposition of two or more waves is known as *interference.*

If two waves of intensities I_1 and I_2, differing in phase by ϕ, superpose, the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\ \phi$$

Constructive Interference The resultant intensity I is maximum if $\cos\ \phi = +1$, i.e.

$$\phi = 2n\pi;$$

where $n = 0, 1, 2, 3, \ldots$ etc. is an integer

or $$\frac{2\pi\Delta}{\lambda} = 2n\pi$$

or $$\Delta = n\lambda$$

where Δ is the path difference between the interfering waves. Then

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

The interference is said to be constructive. If the two interfering waves have equal intensities $I_1 = I_2 = I_0$, then

$$I_{max} = 4I_0$$

Destructive Interference The resultant intensity I is minimum if $\cos\ \phi = -1$, i.e.

$$\phi = (2n - 1)\ \pi$$

and $$\Delta = \left(n - \frac{1}{2}\right)\lambda\ ;\ n = 1, 2, 3, \text{.....etc.}$$

Then

$$I_{min} = I_1 + I_2 - 2I_1 I_2$$

$$= \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

At maxima, the waves reinforce each other and at minima they cancel out each other. These maxima and minima constitute the bright and dark fringes.

27.4 COHERENT LIGHT SOURCES

The resultant intensity of light at a point on the screen depends on the phase difference (ϕ) between the two interfering waves. This phase difference depends upon two factors—(1) the initial phase difference between the waves emitted by the two sources and (2) the phase difference resulting from the path difference for that point. The initial phase difference depends upon the time and hence remains constant only for about 10^{-8} to 10^{-10} second. Thus the resultant intensity changes so rapidly with time that, due to persistence of vision, we are unable to see the interference pattern. Thus, non-coherent sources cannot produce sustained interference effects. We conclude that, for a steady interference pattern, the following two conditions must be satisfied.

1. The sources must be coherent.
2. The wavelength of the interfering waves must be the same. Thus, *only monochromatic coherent light sources produce observable interference pattern.*

27.5 YOUNG'S DOUBLE SLIT EXPERIMENT

Monochromatic light from a source slit S illuminates two slits S_1 and S_2 which are very close together and equidistant from S (Fig. 27.1).

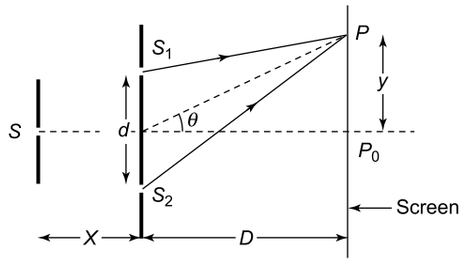


Fig. 27.1

Secondary waves from S_1 and S_2 interfere giving rise to bright and dark fringes on the screen. There is bright fringe at centre P_0 of the screen.

(i) The distance of the nth bright fringe from the centre of the fringe system is

$$y_n = \frac{n\lambda D}{d} \quad ; \; n = 0, 1, 2, \ldots \text{ etc.}$$

where λ = wavelength of light used, d = seperation between slits S_1 and S_2 and D = distance between the screen and the plane of the two slits.

(ii) The distance of the nth dark fringe from the centre of the fringe system is

$$y_n^* = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d}; \; n = 1, 2, 3, \ldots \text{ etc}$$

(iii) The separation between two consecutive bright or dark fringes is called fringe width (β) which is given by

$$\beta = \frac{\lambda D}{d}$$

(iv) Angular separation between nth bright fringe and the central fringe is

$$\theta_n = \frac{y_n}{D} = \frac{n\lambda}{d} \quad ; \quad \theta_n \text{ is in radian.}$$

(v) Angular separation between nth dark fringe and the central fringe is

$$\theta_n^* = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$$

27.6 DISPLACEMENT OF FRINGES

If a transparent plate of thickness t and refractive index μ is introduced in the path of one of the interfering waves, the entire fringe pattern is shifted by a distance

$$y_0 = (\mu - 1)\frac{tD}{d}$$

$$\text{Number of fringes shifted} = \frac{(\mu - 1)t}{\lambda}$$

27.7 DIFFRACTION AT A SLIT

When a parallel beam of monochromatic light falls normally on a narrow slit, the diffraction pattern on a screen has a bright central maximum bordered on both sides by secondary maxima of rapidly decreasing intensity.

If λ is the wavelength of light and a is the width of the slit, then

(i) For bright fringes : $\sin\theta = 0, \frac{3\lambda}{2a}, \frac{5\lambda}{2a}, \ldots$ etc

(ii) For dark fringes : $\sin\theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a}$ ... etc

(iii) Angular width of central maximum = $\frac{2\lambda}{a}$

(iv) Linear width of central maximum = $\frac{2f\lambda}{a}$, where f = focal length of the convex lens placed close to the slit.

27.8 SOME IMPORTANT POINTS ABOUT INTERFERENCE OF LIGHT

(1) In Young's double slit experiment, if monochromatic light is replaced by white light then central fringe will be white; all other fringes will be coloured. White light consists of colours between violet and rad (VIBGYOR). Wavelength λ is the shortest for violet light and longest for red light. At the central fringe, the path difference for all colours is zero. Hence at the central fringe, all colours superpose to give a white fringe. The first bright fringe after the central fringe will be violet colour.

(2) In Young's double slit experiment, if one of the slits is covered with a transparent film or sheet of thickness t and refractive index μ, then

(a) the path difference at the centre of the screen will not be zero, it will be equal to $(\mu - 1)t$.

(b) the entire fringe pattern will shift by an amount $y_0 = \frac{(\mu - 1)tD}{d}$.

(c) at the centre of the screen there will be a bright fringe if $(\mu - 1)\, t = n\lambda$; $n = 1, 2, 3, \ldots$ etc.

(d) At the centre of the screen there will be a dark fringe if

$$(\mu - 1)t = \left(n - \frac{1}{2}\right)\lambda; \; n = 1, 2, 3, \ldots \text{ etc.}$$

(e) the fringe width will remain the same.

(f) the intensity of light from the covered slit will decrease due to absorption by the film or sheet. Hence intensity of bright fringes will decrease and dark fringes will have some finite intensity (because the two interfering beam do not now have equal intensity). Hence the fringe pattern will become less distinct.

(3) If one of the slits in Young's double slit experiment is closed (or covered with black paper), the interference pattern is replaced by single slit diffraction pattern which has a bright central fringe bordered on both sides by fringes of decreasing intensity.

(4) If Young's interference experiment is performed in still water rather than in air, the fringe width

will decrease. Since the refractive index of water is greater than that of the air, the speed of light in water (v) will be less than that in air (c). Since the frequency of light is the same in all media, $\lambda_w = \frac{v}{\nu}$ and $\lambda_a = \frac{c}{\nu}$ which give $\frac{\lambda_w}{\lambda_a} = \frac{v}{c} = \frac{1}{\mu_w}$.

Now $\mu_w = 4/3$. Hence $\lambda_w < \lambda_a$. Fringe width $\beta \propto \lambda$. Hence β in water < β in air.

(5) In Young's interference experiment, if the beam of light has two wavelengths λ_1 and λ_2, their maxima will coincide if $n_1\lambda_1 = n_2\lambda_2$, where n_1 and n_2 are integers.

(6) In an interference experiment if the two coherent light sources have intensities in the ratio n : 1, i.e. $\frac{I_1}{I_2} = n$, then the ratio of the intensity of maxima and minima in the interference pattern is

$$\frac{I_{\max}}{I_{\max}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}}$$

$$= \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

$$= \left[\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} + 1}\right]^2 = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$$

(7) In an interference experiment with two coherent light sources, if the ratio of the intensities of maxima and minima in the interference pattern is n : 1, i.e. $\frac{I_{\max}}{I_{\min}} = n$, then ratio of the intensities of the coherent sources is

$$\frac{I_1}{I_2} = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2 \text{ because}$$

$$\frac{I_{\max}}{L_{\min}} = \left[\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right]^2$$

$$\Rightarrow \quad \sqrt{n} = \frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1} \Rightarrow \frac{I_1}{I_2} = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$$

(8) In an interference experiment, if the two coherent sources have intensities in the ratio n : 1, then in the interference pattern

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{n}}{(n+1)}$$

(9) The intensity of light emerging from a slit is proportional to its width. If the two slits in Young's interference experiment have widths in the ratio n : 1, then $\frac{I_1}{I_2} = n$ and

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$$

(10) In Young's double slit experiment (Fig. 27.1), if x is the width of the source slit S and X its distance from the plane of the slits, the interference fringes will not be seen (because the interference pattern becomes indistinct) if the condition

$$\frac{x}{X} < \frac{\lambda}{d}$$

is not satisfied.

27.9 RESOLVING POWER

Resolving power of an optical instrument is its ability to produce distinctly separate images of two objects very close together.

(a) *Resolving power of a microscope*

$$R.P. = \frac{2\mu \sin\theta}{1.22\lambda}$$

where 2θ = angle of the cone of light rays entering the objective of the microscope (Fig. 27.2), μ = refractive index of the medium between the object and the objective and λ = wavelength of light used to illuminate the object.

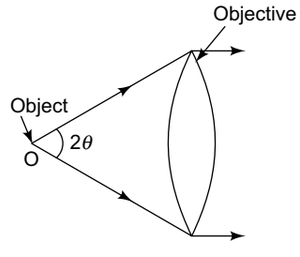


Fig. 27.2

(b) *Resolving power of a telescope*

$$R.P. = \frac{D}{1.22\lambda}$$

where D = diameter of objective and λ = wavelength of light

27.10 POLARIZATION OF LIGHT

The phenomena of reflection, refraction, interference and diffraction are common on both transverse and longitudinal waves, mechanical as well as electromagnetic. The distinguishing feature is that only transverse waves can be *polarized.*

In an unpolarized light, the electric field vector has all the possible orientations in a plane perpendicular to the direction of propagation. When this light is passed through a specially cut crystal of calcite or quartz, called a polaroid, we obtain a plane polarized light.

Only transverse waves can be polarized. Longitudinal waves cannot be poarlized.

Polarization by Reflection: Brewster's Angle

In 1808, the French physicist Brewster discovered that when a beam of ordinary unpolarized light is incident at a particular angle i_p on the surface of a transparent medium, the reflected light is polarized.

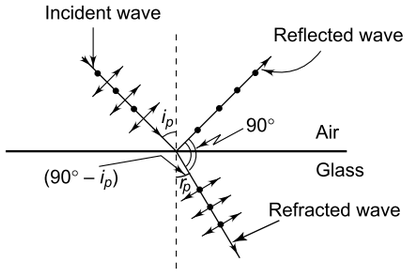


Fig. 27.3 Polarization by reflection; i_p is Brewster's angle

Figure 27.3 shows an unpolarized light incident at an angle i_p at the surface of glass. Brewster discovered that when $i = i_p$, the reflected and refracted rays are exactly 90° apart. The angle i_p when this happens is called the *polarizing* or the *Brewster angle*. If r_p is the corresponding angle of refraction, then from the geometry of Fig. 27.3.

$$i_p + r_p = 90° \text{ or } r_p = 90° - i_p$$

Now from Snell's law, the refractive index of glass is

$$n = \frac{\sin i_p}{\sin r_p} = \frac{\sin i_p}{\sin(90° - i_p)}$$

$$= \frac{\sin i_p}{\cos i_p} = \tan i_p$$

$$\therefore \quad n = \tan i_p$$

This equation is called the *Brewster law* and the special angle satisfying this equation is the *Brewster angle.*

EXAMPLE 27.1

In Young's double-slit experiment, the intesity of light at a point on the screen where the path difference is λ is K units. What is the intensity of light at a point where the path difference is $\lambda/3$; λ being the wavelength of light used?

SOLUTION

Path difference $\Delta = \lambda$. Therefore, phase difference $\phi = \frac{2\pi}{\lambda}\Delta = 2\pi$. Hence intensity at a point where $\Delta = \lambda$ or $\phi = 2\pi$ is

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos 2\pi$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= I + I + 2I = 4I = K \text{ units}$$

$$(\because \ I_1 = I_2 = I)$$

i.e. $I = K/4$. The intensity at a point where the path difference is

$$\Delta' = \frac{\lambda}{3} \text{ or } \phi' = \frac{2\pi}{\lambda}\Delta'$$

$$= \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3} \text{ is}$$

$$I' = I + I + 2I \cos \frac{2\pi}{3}$$

$$= 2I - I = I = \frac{K}{4} \text{ units}$$

EXAMPLE 27.2

Two coherent sources of intensity ratio 100 : 1, interfere. What is the ratio of the intensity between the maxima and minima in the interference pattern?

SOLUTION

Given $I_1/I_2 = 100$, i.e. $I_1 = 100$ units and $I_2 = 1$ unit.

Intensity at maxima is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= 100 + 1 + 2\sqrt{100 \times 1} = 121$$

Intensity at minima is

$$I' = I_1 + I_2 - 2\sqrt{I_1 I_2} = 81$$

$$\therefore \quad \frac{I}{I'} = \frac{121}{81} = 1.49$$

EXAMPLE 27.3

In Young's double slit experiment, the slits are separated by by 0.28 mm and the screen is placed 1.4 m away. The distance between the fourth bright fringe and the central bright fringe is measured to be 1.2 cm. What is the wavelength of light used in the experiment?

SOLUTION

The position of the nth bright fringe with respect to the central fringe is given by

$$y_n = \frac{n\lambda D}{d}$$

For the central bright fringe ($n = 0$), $y_0 = 0$. For the fourth bright fringe ($n = 4$), $y_4 = 4\ \lambda D/d$. Therefore

$$y_4 - y_0 = \frac{4\lambda D}{d} \text{ or } \lambda = (y_4 - y_0)\frac{d}{4D} \qquad \text{(i)}$$

It is given that $(y_4 - y_0) = 1.2$ cm $= 1.2 \times 10^{-2}$ m, $D = 1.4$ m and $d = 0.28$ mm $= 0.28 \times 10^{-3}$ m. Substituting these values in (i) and solving, we get

$\lambda = 6 \times 10^{-7}$ m(or 600 nm or 6000 Å).

EXAMPLE 27.4

The ratio of the intensities of the maxima and minima in an interference pattern is 49 : 9. What is the ratio of the intensities of the two coherent sources employed in the interference experiment?

SOLUTION

Given $\quad \frac{I_{max}}{I_{min}} = \frac{49}{9}$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

and $\quad I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{x+1}{x-1}\right)^2$$

where $\quad x = \sqrt{\frac{I_1}{I_2}}$.

$$\Rightarrow \quad \frac{49}{9} = \left(\frac{x+1}{x-1}\right)^2 \Rightarrow \frac{7}{3} = \frac{x+1}{x-1}$$

which gives $x = \frac{5}{2}$. Therefore

$$\frac{I_1}{I_2} = x^2 = \frac{25}{4}$$

EXAMPLE 27.5

In Young's double slit experiment, find the ratio of the intensities at points P and Q on the screen where the path difference between the interfering waves is (a) zero and (b) $\frac{\lambda}{4}$, where λ is the wavelength of light used.

SOLUTION

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

where $\quad \phi = \frac{2\pi\Delta}{\lambda}$; Δ = path difference

In Young's experiment $I_1 = I_2 = I_0$. Therefore,

$$I = I_0 + I_0 + 2I_0 \cos\phi$$
$$= 2I_0\ (1 + \cos\phi)$$

(a) For $\Delta = 0$, $\phi = 0°$. Hence

$$I_1 = 2I_0\ (1 + \cos 0°) = 4I_0$$

(b) For $\Delta = \frac{\lambda}{4}$, $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$. Hence

$$I_2 = 2I_0 \left(1 + \cos\frac{\pi}{2}\right) = 2I_0$$

$$\therefore \quad \frac{I_1}{I_2} = 2$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. Monochromatic light of wavelength λ in air is refracted into a glass slab of refractive index μ. The wavelength of light in glass is

(a) λ (b) $\mu\lambda$

(c) $\frac{\lambda}{\mu}$ (d) $\frac{\lambda}{\mu^2}$

2. Young's double slit experiment is first performed in air and then in a liquid. It is observed that the 10th bright fringe in liquid is replaced by 8th dark fringe in air. The refractive index of the liquid is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$

(c) $\frac{5}{3}$ (d) $\frac{20}{17}$

3. In Young's double slit experiment, the fringes are displaced by a distance y when a glass plate of refractive index 1.5 is introduced in the path of one of the interfering beams. If this plate is replaced by another plate of the same thickness but of refractive index 1.75, the fringes will be displaced by

(a) $\frac{3y}{2}$ (b) $\frac{2y}{3}$

(c) $\frac{7y}{6}$ (d) $\frac{6y}{7}$

4. Young's double slit experiment is performed using monochromatic light of wavelength λ. The separation between the slits is $d = 50\ \lambda$ and the distance between the screen and the slits is $D = 1000\ \lambda$. If I_0 is the intensity of each interfering beam, the intensity at the point on the screen directly in front of one of the slits is

(a) $\frac{I_0}{2}$ (b) I_0

(c) $2I_0$ (d) $4I_0$

5. What will happen if one of the slits in Young's double slit experiment is covered with cellophane paper which absorbs a fraction of the intensity of light from the slit?

(a) The fringe width will decrease

(b) The fringes will become more distinct

(c) The bright fringes will become less bright and the dark fringes will not be completely dark

(d) No fringes will be observed

6. In Young's double slit experiment, if the slit widths are in the ratio of 1 : 2, the ratio of the intensities at minima and maxima will be

(a) 1 : 2 (b) 1 : 3

(c) 1 : 4 (d) 1 : 9

7. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude A and of wavelength λ. In another experiment with the same set up, the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the intensity of light at the midpoint of the screen in the first case to that in the second case is

(a) 1 : 1 (b) 1 : 2

(c) 2 : 1 (d) $\sqrt{2}$: 1

8. How is the interference pattern affected if the Young's experiment was performed in still water than in air?

(a) Fewer fringes will be visible

(b) Fringes will be broader

(c) Fringes will be narrower

(d) No fringes will be observed.

9. How is the interference pattern in Young's experiment affected if one of the slits is covered with black opaque paper?

(a) The bright fringes become fainter

(b) The fringe width decreases

(c) There will be uniform illumination all over the screen

(d) There will be a bright central fringe bordered on both sides by fringes of decreasing intensity.

10. What is the effect on the interference fringes in Young's double slit experiment if the source slit is moved closer to the double slit plane?

(a) The fringe width increases

(b) The fringe width decreases

(c) The fringes become more distinct

(d) The fringes become less distinct.

11. What is the effect on the interference fringes in Young's double slit experiment if the width of the source slit is increased?

(a) The fringe width increases

(b) The fringe width decreases

(c) The fringes become more distinct

(d) The fringes become less distinct.

12. What is the effect on the interference fringes in Young's double slit experiment if the widths of the two slits are increased?

(a) The fringe width increases

(b) The fringe width decreases

(c) The bright fringe are equally bright and equally spaced

(d) The bright fringes are no longer equally bright and equally spaced.

13. In Young's double slit experiment the slits are 0.5 mm apart and interference is observed on a screen placed at a distance of 100 cm from the slits. It is found that the 9th bright fringe is at a distance of 9.0 mm from the second dark fringe from the centre of the fringe pattern. What is the wavelength of light used.

(a) 2000 Å (b) 4000 Å
(c) 6000 Å (d) 8000 Å

14. A screen is placed at a certain distance from a narrow slit which is illuminated by a parallel beam of monochromatic light. What will you observe if you scan the screen with the help of a microscope?
(a) The whole screen is uniformly illuminated.
(b) Equally spaced and equally bright fringes are observed.
(c) One bright fringe is observed at the centre of the screen.
(d) A bright central fringe bordered on both sides with fringes of rapidly decreasing intensity will be observed.

15. If the wavelength of light used in the experiment above is λ and if d is the width of the slit, what is the angular width of the central maximum?
(a) $\sin^{-1}(\lambda/d)$ (b) $2\sin^{-1}(\lambda/d)$
(c) $\sin^{-1}(2\lambda/d)$ (d) $\sin^{-1}(\lambda/2d)$

16. A single-slit diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light?
(a) There is no change in the diffraction pattern.
(b) Diffraction fringes become narrower and crowded together.
(c) Diffraction fringes become broader and farther apart.
(d) The diffraction pattern disappears.

17. A parallel beam of light of wavelength 6000 Å is incident normally on a slit of width 0.2 mm. The diffraction pattern is observed on a screen which is placed at the focal plane of a convex lens of focal length 50 cm. If the lens is placed close to the slit, the distance between the minima on both sides of the central maximum will be
(a) 1 mm (b) 2 mm
(c) 3 mm (d) 4 mm

18. In Young's double slit experiment the distance d between the slits S_1 and S_2 is 1.0 mm. What should the width of each slit be so as to obtain 10 maxima of the two slit interference pattern within the central maximum of the single slit diffraction pattern?
(a) 0.1 mm (b) 0.2 mm
(c) 0.3 mm (d) 0.4 mm

19. Monochromatic light is refracted from air into glass of refractive index μ. The ratio of the wavelengths of the incident and refracted waves is
(a) 1 : 1 (b) 1 : μ
(c) μ : 1 (d) μ^2 : 1

20. In Young's double slit experiment, the intensity of the maxima is I. If the width of each slit is doubled the intensity of the maxima will be
(a) $\frac{I}{2}$ (b) I
(c) $2I$ (d) $4I$

21. In Young's double slit experiment, the 10th maximum of wavelength λ_1 is at a distance y_1 from its central maximum and the 5th maximum of wavelength λ_2 is at a distance y_2 from its central maximum. The ratio y_1/y_2 will be
(a) $\frac{2\lambda_1}{\lambda_2}$ (b) $\frac{2\lambda_2}{\lambda_1}$
(c) $\frac{\lambda_1}{2\lambda_2}$ (d) $\frac{\lambda_2}{2\lambda_1}$

22. White light is used to illuminate the two slits in Young's double slit experiment. The separation between the slits is d and the distance between the screen and the slit is D ($>> d$). At a point on the screen directly in front of one of the slits, certain wavelengths are missing. The missing wavelengths are (here $m = 0, 1, 2, \ldots$ is an integer)
(a) $\lambda = \frac{d^2}{(2m+1)D}$ (b) $\lambda = \frac{(2m+1)d^2}{D}$
(c) $\lambda = \frac{d^2}{(m+1)D}$ (d) $\lambda = \frac{(m+1)d^2}{D}$

33. In Young's double slit experiment using two identical slits, the intensity of the maximum at the centre of the screen is I. What will be the intensity at the centre of the screen if one of the slits is closed?
(a) I (b) $\frac{I}{2}$
(c) $\frac{I}{4}$ (d) none of these

24. A mixture of violet light of wavelength 3800 Å and blue light of wavelength 4000 Å is incident normally on an air film of 0.00029 mm thickness. The colour of the reflected light is
(a) red (b) blue
(c) violet (d) green

25. Light of wavelength λ is incident on a slit of width d. The resulting diffraction pattern is observed on a screen at a distance D. The linear width of the principal maximum is equal to the width of the slit if D equals

(a) $\frac{d}{\lambda}$ (b) $\frac{2\lambda}{d}$

(c) $\frac{d^2}{2\lambda}$ (d) $\frac{2\lambda^2}{d}$

26. Two waves of intensities I and $4I$ superpose, then the maximum and minimum intensities are

(a) $5I$, $3I$ (b) $9I$, I

(c) $9I$, $3I$ (d) $5I$, I

IIT, 1982

27. Electromagnetic radiation of frequency n, wavelength λ, travelling with velocity v in air, enters a glass slab of refractive index μ. The frequency, wavelength and velocity of the wave in glass slab respectively are

(a) $\frac{n}{\mu}, \frac{\lambda}{\mu}, \frac{v}{\mu}$ (b) $n, \frac{\lambda}{\mu}, \frac{v}{\mu}$

(c) $n, \lambda, \frac{v}{\mu}$ (d) $\frac{n}{\mu}, \frac{\lambda}{\mu}, v$

28. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1.0 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is

(a) 1.2 cm (b) 1.2 mm

(c) 2.4 cm (d) 2.4 mm

IIT, 1994

29. A parallel beam of monochromatic light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is

(a) zero (b) $\frac{\pi}{2}$

(c) π (d) 2π

IIT, 1998

30. A string of length 0.4 m and mass 10^{-2} kg is rigidly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time Δt. The minimum value of Δt which allows constructive interference between successive pulses is

(a) 0.05 s (b) 0.10 s

(c) 0.20 s (d) 0.40 s

IIT, 1998

31. Yellow light is used in a single slit diffraction experiment with a slit of width 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal

(a) that the central maximum is narrower

(b) more number of fringes

(c) less number of fringes

(d) no diffraction pattern

IIT, 1999

32. In Young's double slit interference experiment the wavelength of light used is 6000 Å. If the path difference between waves reaching a point P on the screen is 1.5 microns, then at that point P:

(a) Second bright band occurs

(b) Second dark band occurs

(c) Third dark band occurs

(d) Third bright band occurs

33. The difference in the number of wavelengths, when yellow light (of wavelength 6000 Å in vacuum) propagates through air and vacuum columns of the same thickness is one. If the refractive index of air is 1.0003, the thickness of the air column is

(a) 1.8 mm (b) 2 mm

(c) 2 cm (d) 2.2 cm

34. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double slit experiment is

(a) infinite (b) five

(c) three (d) zero

35. An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a dielectric medium of relative permittivity $\varepsilon_r = 4.0$. Then

(a) wavelength is doubled and the frequency remains unchanged.

(b) wavelength is doubled and frequency becomes half

(c) wavelength is halved and frequency remains unchanged

(d) wavelength and frequency both remain unchanged

36. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is

(a) $2I$ (b) $4I$

(c) $5I$ (d) $7I$

IIT, 2001

37. A double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of

1 mm and distance between the plane of slits and screen 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å. What is the fringe width?

(a) (1.33×0.63) mm (b) $\frac{0.63}{1.33}$ mm

(c) $\frac{0.63}{(1.33)^2}$ mm (d) 0.63 mm

IIT, 1996

38. In Young's experiment, the fringe width was found to be 0.4 mm. If the whole apparatus is immersed in water of refractive index 4/3, the new fringe width in mm is

(a) 0.25 (b) 0.30

(c) 0.40 (d) 0.53

39. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain region of the screen when light of wavelength 600 nm is used. If the light of wavelength 400 nm is used, the number of fringes observed in the same region of the screen will be

(a) 12 (b) 18

(c) 24 (d) 8

IIT, 2000

40. In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by a distance of 5×10^{-2} m towards the slits, the change in the fringe width is 3×10^{-5} m. If the separation between the slits is 10^{-3} m, the wavelength of light used is

(a) 5×10^{-7} m (b) 6×10^{-7} m

(c) 7×10^{-7} m (d) 6×10^{-6} m

41. In the ideal double-slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is

(a) 2λ (b) $2\lambda/3$

(c) $\lambda/3$ (d) λ

IIT, 2002

42. In Fig. 27.4, PQ represents a plane wavefront and AO and BP the corresponding extreme rays of monochromatic light of wavelength λ. The value of angle θ for which the ray BP and the reflected ray OP interfere constructively is given by

(a) $\cos\theta = \frac{\lambda}{2d}$ (b) $\cos\theta = \frac{\lambda}{4d}$

(c) $\sec\theta = \frac{\lambda}{3d}$ (d) $\sec\theta = \frac{2\lambda}{3d}$

IIT, 2003

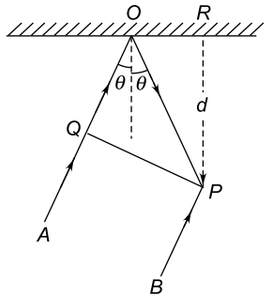


Fig. 27.4

43. In Young's double slit experiment, the fringe width is 2.0 mm, The separation between the 9th bright fringe and the second dark fringe from the centre of the fringe system will be

(a) 5.0 mm (b) 10 mm

(c) 15 mm (d) 20 mm

44. When one of the slits in Young's experiment is covered with a transparent sheet of thickness 3.6×10^{-3} cm the central fringe shifts to a position originally occupied by the 30th brigth fringe. If $\lambda = 6000$ Å, the refractive index of the sheet is

(a) 1.50 (b) 1.55

(c) 1.60 (d) 1.65

45. A beam of light, consisting of two wavelengths 6500 Å and 5200 Å is used to obtain interference fringes in Young's double slit experiment. The separation between the slits is 2.6 mm and the distance between the plane of the slits and the screen is 1.0 m. The least distance from the central maximum where the bright fringes due to both the wavelengths coincide is

(a) 1.0 mm (b) 1.5 mm

(c) 2.0 mm (d) 2.5 mm

46. Two coherent light sources of intensity ratio n are employed in an interference experiment. The ratio of the intensities of the maxima and minima in the interference pattern is

(a) $\left(\frac{n+1}{n-1}\right)$ (b) $\left(\frac{n+1}{n-1}\right)^2$

(c) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$ (d) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$

47. Two coherent light sources are employed in an interference experiment. The ratio of the intensities of the maxima and minima in the interference pattern is n. The ratio of the intensities of the two coherent sources is

(a) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$ (b) $\left(\frac{n+1}{n-1}\right)^2$

(c) $\left(\frac{n^2+1}{n^2-1}\right)$ (d) $\left(\frac{n+1}{n-1}\right)$

48. The two slits in Young's interference experiment have widths in the ratio n:1. The ratio of the intensities of the maxima and minima in the interference pattern is

(a) $\frac{\sqrt{n}+1}{\sqrt{n}}$ (b) $\frac{\sqrt{n}}{\sqrt{n}-1}$

(c) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$ (d) $\left(\frac{n+1}{n}\right)^2$

49. In Young's double slit experiment, the angular width of a fringe formed on a distant screen is 0.1°. If the wavelength of light used is 628 nm, the spacing between the slits is

(a) 0.9×10^{-4} m (b) 1.8×10^{-4} m

(c) 3.6×10^{-4} m (d) 7.2×10^{-4} m

50. Interference pattern is obtained with two coherent light sources of intensity ratio n. In the interference pattern, the ratio $\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$ will be

(a) $\frac{\sqrt{n}}{(n+1)}$ (b) $\frac{2\sqrt{n}}{(n+1)}$

(c) $\frac{\sqrt{n}}{\left(\sqrt{n}+1\right)^2}$ (d) $\frac{2\sqrt{n}}{\left(\sqrt{n}+1\right)^2}$

51. Interference pattern is obtained with two coherent light sources of intensities I and $4I$. The intensity at a point where the phase difference is $\pi/2$ is

(a) I (b) $2I$

(c) $3I$ (d) $5I$

52. Young's double slit expetiment is performed using light of wavelength λ. One of the slits is covered by a thin glass sheet of refractive index μ at this wavelength. The smallest thickness of the sheet to bring the adjacent minimum to the centre of the screen is

(a) $\frac{\lambda}{2(\mu-1)}$ (b) $\frac{\lambda}{(\mu-1)}$

(c) $\frac{\lambda}{2\mu}$ (d) $\frac{\lambda}{\mu}$

53. Monochromatic light of wavelength 500 nm is incident on two parallel slits separated by a distance of 5×10^{-4} m. The interference pattern is obtained on a screen at a distance of 1.0 m from the slits. The intensity of the central maximum is I_0. When one of the slits is covered by a glass sheet of thickness 1.5×10^{-6} m and refractive index 1.5, the intensity at the centre of the screen will be equal to

(a) $\frac{I_0}{2}$ (b) $\frac{I_0}{3}$

(c) $\frac{I_0}{4}$ (d) zero

54. In Q. 53 above, the lateral shift of the central maximum is

(a) 1.5 mm (b) 4 mm

(c) 3 mm (d) 2 mm

55. In an interference experiment, 20th order maximum is observed at a point on the screen when light of wavelength 480 nm is used. If this light is replaced by light of wavelength 600 nm, the order of the maximum at the same point will be

(a) 16 (b) 14

(c) 12 (d) 10

56. A beam of light consisting of two wavelengths 4500 Å and 7500 Å is used to obtain interference fringes in Young's double slit experiment. The distance between the slits is 1 mm and the distance between the plane of the slits and the screen is 120 cm. What is the minimum distance between two successive regions of complete darkness on the screen?

(a) 4.5 mm (b) 5.4 mm

(c) 2.7 mm (d) 1.2 mm

IIT, 2004

57. In Young's double slit experiment, the intensity at a point P on the screen is half the maximum intensity in the interference pattern. If the wavelength of light used is λ and d is the distance between the slits, the angular separation between point P and the centre of the screen is

(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$

(c) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

IIT, 2005

58. A parallel beam of fast moving electrons is incident normally on a narrow slit. A screen is placed at a large distance from the slit. If the speed of the electrons is increased, which of the following statements is correct?

(a) Diffraction pattern is not observed on the screen in the case of electrons

(b) The angular width of the central maximum of the diffraction pattern will increase

(c) The angular width of the central maximum will decrease

(d) The angular width of the central maximum will remain the same.

IIT, 2007

59. In Young's double slit experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a transparent sheet of thickness t and refractive index μ is introduced in the path of one of the interfering waves. The sheet is then removed and the distance between the screen and the slits is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift when the sheet was introduced. The wavelength of light used is

IIT, 1983

(a) $\lambda = (\mu - 1)t$ (b) $\lambda = \frac{1}{2}(\mu - 1)t$

(c) $\lambda = (\mu + 1)t$ (d) $\lambda = \frac{1}{2}(\mu + 1)t$

60. In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen 1 m away. The wavelength of light used is 6000 Å. What will be the angular width of a fringe if the entire experimental arrangement is immersed in water? Refractive index of water $= \frac{4}{3}$.

(a) 0.15° (b) 0.18°
(c) 0.2° (d) 0.27°

61. A coherent parallel beam of microwaves of wavelength 0.5 mm falls normally on Young's double slit apparatus. The separation between the slits is 1.0 mm and the screen is placed at a distance of 1.0 m from the slits. The number of minima in the interference pattern observed on the screen is

IIT, 1998

(a) 3
(b) 4
(c) 5
(d) much greater than 5.

62. In Young's double slit experiment sodium light composed of two wavelengths λ_1 and λ_2 close to each other (with λ_2 greater than λ_1) is used. The order n up to which the fringes can be seen on the screen is given by

(a) $n = \frac{\lambda_2}{\lambda_2 - \lambda_1}$ (b) $n = \frac{\lambda_1}{\lambda_2 - \lambda_1}$

(c) $n = \frac{\lambda_2}{2(\lambda_2 - \lambda_1)}$ (d) $n = \frac{\lambda_1}{2(\lambda_2 - \lambda_1)}$

63. Monochromatic light of wavelength λ emerging from slit S illuminates slits S_1 and S_2 which are placed with respect to S as shown in Fig. 27.5. The distances x and D are large compared to the separation d between the slits. If $x = D/2$, the minimum value of d so that there is a dark fringe at the centre P of the screen is

(a) $\sqrt{\frac{\lambda D}{3}}$ (b) $\sqrt{\frac{2\lambda D}{3}}$

(c) $\sqrt{\lambda D}$ (d) $2\sqrt{\frac{\lambda D}{3}}$

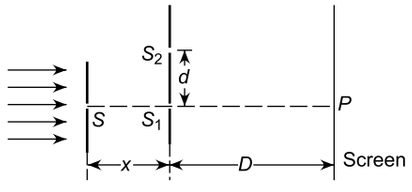


Fig. 27.5

ANSWERS

1. (c)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (d)
7. (c)	**8.** (c)	**9.** (d)	**10.** (d)	**11.** (d)	**12.** (d)
13. (c)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (b)
19. (c)	**20.** (c)	**21.** (a)	**22.** (a)	**23.** (c)	**24.** (c)
25. (c)	**26.** (b)	**27.** (b)	**28.** (d)	**29.** (d)	**30.** (b)
31. (a)	**32.** (c)	**33.** (b)	**34.** (c)	**35.** (c)	**36.** (b)
37. (c)	**38.** (b)	**39.** (b)	**40.** (a)	**41.** (d)	**42.** (a)
43. (c)	**44.** (a)	**45.** (a)	**46.** (d)	**47.** (a)	**48.** (c)
49. (c)	**50.** (b)	**51.** (d)	**52.** (a)	**53.** (d)	**54.** (a)
55. (a)	**56.** (c)	**57.** (d)	**58.** (c)	**59.** (b)	**60.** (a)
61. (b)	**62.** (d)	**63.** (a)			

SOLUTIONS

1. Since the frequency ν of light remains the same, we have

$$c = \nu\,\lambda \quad \text{and} \quad v = \nu\,\lambda'$$

where c = speed of light in air and v = speed of light in glass. Therefore

$$\frac{c}{v} = \frac{\lambda}{\lambda'} \quad \Rightarrow \quad \mu = \frac{\lambda}{\lambda'} \Rightarrow \lambda' = \frac{\lambda}{\mu}.$$

So the correct choice is (c).

2. $y_{10} = \dfrac{10\lambda' D}{d}$ and $y_8 = \left(8 - \dfrac{1}{2}\right)\dfrac{\lambda D}{d}$

$$\therefore \quad \frac{y_{10}}{y_8} = \frac{10}{7.5} \times \frac{\lambda'}{\lambda} = \frac{4}{3\mu} \qquad (\because \lambda = \mu\lambda')$$

Given $y_{10} = y_8$. Hence $\mu = \dfrac{4}{3}$, which is choice (b).

3. Displacement of fringes = $(\mu - 1)\,\dfrac{tD}{d}$

$$y = (1.5 - 1)\,\frac{tD}{d}$$

$$y' = (1.75 - 1)\,\frac{tD}{d}$$

$$\therefore \quad \frac{y'}{y} = \frac{0.75}{0.5} = \frac{3}{2} \Rightarrow y' = \frac{3y}{2}$$

4. Path difference $\Delta = \dfrac{yd}{D}$

For the point P on the screen directly in front of one of the slits,

$$y = \frac{d}{2} = \frac{50\lambda}{2} = 25\,\lambda$$

For point P, $\Delta = \dfrac{25\lambda \times 50\lambda}{1000\,\lambda} = \dfrac{5\lambda}{4}$

Phase difference $\phi = \dfrac{2\pi\Delta}{\lambda} = \dfrac{2\pi}{\lambda} \times \dfrac{5\lambda}{4} = \dfrac{5\pi}{2}$

Intensity at P is

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$= 4I_0 \cos^2\left(\frac{5\pi}{4}\right) = 2I_0$$

5. The intensity (and hence the amplitude) of the light from the covered slit will decrease resulting in a difference in the intensities of the two virtual sources. Hence the correct choice is (c).

6. The amplitudes of the two coherent waves will be $A_1 = 2A$ and $A_2 = A$. Therefore

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\frac{2A + A}{2A - A}\right) = 9$$

Hence the correct choice is (d).

7. If the two sources are coherent, the resultant amplitude at the midpoint of the screen due to interference = $A + A = 2A$. Therefore, intensity is $I_1 \propto (2A)^2$ or $I_1 = k \times 4A^2$ where k is a constant of proportionality. But if the sources are not coherent, their intensities simply add up at the midpoint, i.e.

$$I_2 \propto (A^2 + A^2) \text{ or } I_2 = k \times 2A^2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{4kA^2}{2kA^2} = 2$$

Hence the correct choice is (c).

8. Since the refractive index of water is greater than that of the air, the speed of the light used in the experiment will be less in water than in air. Since the frequency of light is the same in water and in air, it follows from the relation $\lambda = v/\nu$ that the wavelength λ in water is less than in air. Since fringe width $\beta \propto \lambda$, the value of β will decrease. Hence the correct choice is (c).

9. We then obtain a single slit diffraction pattern on the screen. Hence the correct choice is (d).

10. and **11.** Let x be the width of the source slit and X the distance between the source slit and the plane of the two slits. For interference fringes to be distinctly visible, the condition $x/X < \lambda/d$ should be satisfied. If x is too large (i.e. the source slit is too wide) or if X is too small (X is the distance between the source slit and the two slits) the requirement $x/X < \lambda/d$ may be violated and fringes will no longer be distinct. The reason is that the interference patterns due to various parts of the source slit overlap. Consequently, the minima will not be totally dark and fringe pattern becomes indistinct. However, as long as the fringe pattern remains visible, a change in x or X has no effect on the fringe width β.

12. The single slit diffraction effects at the two slits becomes important and as a result, the interference fringe pattern will be modified. The bright fringes will not now be equally bright and equally spaced.

13. The distance of the mth bright fringe from the central fringe is

$$y_m = \frac{m\lambda D}{d} = m\beta$$

where $\beta = \lambda D/d$ is the fringe width.

$$\therefore \qquad y_9 = 9\beta \qquad \text{(i)}$$

The distance of the mth dark fringe from the central fringe is

$$y'_m = \left(m - \frac{1}{2}\right)\frac{\lambda D}{d} = \left(m - \frac{1}{2}\right)\beta$$

$$\therefore \qquad y'_2 = \frac{3}{2}\beta \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get $y_9 - y'_2 = 9\beta - \frac{3}{2}\beta$

$$= \frac{15}{2}\beta$$

It is given that $y_9 - y'_2 = 9.0$ mm. Hence

$$\beta = \frac{9.0 \times 2}{15} = 1.2 \text{ mm}$$

$$= 1.2 \times 10^{-3} \text{ m}$$

Now $\lambda = \beta d/D$. Substituting for β, d and D, we get

$$\lambda = 6 \times 10^{-7} \text{ m} = 6000 \text{ Å}$$

14. A single slit diffraction pattern is characterised by a bright central fringe bordered on both sides with fringes of rapidly decreasing intensity.

15. The intensity falls to zero on both sides of the central maximum at an angle θ given by

$$\sin\theta = \frac{\lambda}{d}$$

where θ is the angular separation between the central maximum and the minimum on either side of it so that 2θ is the angular width of the central maximum. Now

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) \quad \therefore \; 2\theta = 2\sin^{-1}\left(\frac{\lambda}{d}\right)$$

Hence the correct choice is (b). If $\lambda << d$, then $\sin\theta \simeq \theta$, where θ is in radians. In that case, the width of the central maximum is $2\theta = 2\lambda/d$.

16. The wavelength of blue light is less than that of the red light. Hence the angular width of the maxima will decrease which means that the fringes become narrower and crowded together.

17. The angular separation of the minima on both sides of the central maximum is 2θ where θ is given by

$$\sin\theta = \frac{\lambda}{d} = \frac{6000 \times 10^{-10}}{0.2 \times 10^{-3}} = 3 \times 10^{-3}$$

since θ is small, $\sin\theta \simeq \theta$. Therefore, $\theta = 3 \times 10^{-3}$ rad. If the lens is placed close to the slit then

$$x = f\tan\theta \simeq f\theta \qquad (\because \; \theta \text{ is small, } \therefore \tan\theta \simeq \theta)$$

where x is the distance of the first minimum from the central maximum. Therefore, the distance between two minima on both sides of the central maximum is

$$2x = 2f\theta = 2 \times 0.5 \times 3 \times 10^{-3}$$

$$= 3 \times 10^{-3} \text{ m} = 3 \text{ mm}.$$

18. Let the width of each slit be a. The linear separation between m bright fringes in the double slit experiment is $y_m = \frac{m\lambda D}{d}$

Since $y << D$, the angular separation between m bright fringes will be

$$\theta_m = \frac{y_m}{D} = \frac{m\lambda}{d}$$

For 10 bright fringes we have

$$\theta_{10} = \frac{10\lambda}{d} \qquad \text{(i)}$$

Now the angular width of the principal maximum in the diffraction pattern due to a slit of width a is

$$2\theta_1 = \frac{2\lambda}{a} \qquad \text{(ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{10\lambda}{d} = \frac{2\lambda}{a} \text{ or } a = \frac{d}{5} = \frac{1.0\,\text{mm}}{5}$$

$$= 0.2 \text{ mm}.$$

19. Since the frequency n of the light does not change as light travels from air into glass, we have

$$v_a = n\,\lambda_a \text{ and } v_g = n\,\lambda_g$$

Therefore $\frac{\lambda_a}{\lambda_g} = \frac{v_a}{v_g} = \mu$

Hence the correct choice is (c).

20. $I = I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

When the width of each slit is doubled, I_1 becomes $2I_1$ and I_2 becomes $2I_2$. Therefore,

$$I' = I'_{\max} = 2I_1 + 2I_2 + 2\sqrt{2I_1 \times 2I_2}$$

$$= 2(I_1 + I_2 + 2\sqrt{I_1 I_2}) = 2\,I_{\max} = 2I.$$

Hence the correct choice is (c).

21. We know that $y_m = \dfrac{m\lambda D}{d}$. Therefore, for wavelength λ_1, $y_1 = \dfrac{10\lambda_1 D}{d}$ and for wavelength λ_2,

$$y_2 = \frac{5\lambda_2 D}{d}, \quad \therefore \quad \frac{y_1}{y_2} = \frac{2\lambda_1}{\lambda_2}$$

Hence the correct choice is (a).

22. The distance of the mth minimum from the centre of the screen is given by

$$y = \left(m + \frac{1}{2}\right)\frac{\lambda D}{d}$$

For a point on the screen directly in front of one of the slits, $y = d/2$. Hence, for minima (zeros) of intensity, we have

$$\frac{d}{2} = \left(m + \frac{1}{2}\right)\frac{\lambda D}{d}$$

or $$\lambda = \frac{d^2}{(2m+1)D}$$

Hence the correct choice is (a).

23. Let I_0 be the intensity at the centre of the screen due to each slit. Then, for the central maximum, the intensity is

$$I = I_0 + I_0 + 2\sqrt{I_0 I_0} = 4I_0$$

or $I_0 = \dfrac{I}{4}$. Hence, the correct choice is (c).

24. When a wave of wavelength λ falls at an angle of incidence i on a film of refractive index μ and thickness t, then the condition for constructive interference in the reflected system is

$$2\,\mu t \cos r = \left(m + \frac{1}{2}\right)\lambda \; ; \; m = 0, 1, 2, 3, \ldots$$

where r is the angle of refraction in the film. For normal incidence $i = 0$, hence $r = 0$. Therefore ($\because \mu = 1$ for air)

$$2t = \left(m + \frac{1}{2}\right)\lambda = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots \text{ etc.}$$

or $$\lambda = \frac{2t}{\left(m + \frac{1}{2}\right)} = 4t, \frac{4t}{3}, \frac{4t}{5}, \ldots \text{ etc.}$$

Now $t = 0.00029$ mm $= 2.9 \times 10^{-5}$ cm
$= 2900$ Å. Therefore,
$\lambda = 11600$ Å, 3867 Å, 2320 Å, … etc.

Wave of $\lambda = 11600$ Å is in the infrared region and wave of $\lambda = 2320$ Å is in the ultraviolet. These waves are not visible. Hence the visible wave in the reflected system has a wavelength $\lambda = 3867$ Å which is close to violet light ($\lambda = 3800$ Å). Hence the correct choice is (c).

25. If $D >> d$, the linear width of the central principal maximum = angular width × distance $D = \dfrac{2\lambda D}{d}$, where d is the width of the slit. The linear width of the principal maximum will be equal to slit width for a value of D given by

$$\frac{2\lambda D}{d} = d \text{ or } D = \frac{d^2}{2\lambda}$$

Hence the correct choice is (c).

26. Given $I_1 = I$ and $I_2 = 4I$. Now

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= I + 4I + 2\sqrt{4I^2} = 9I$$

and $$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$
$$= 5I - 4I = I$$

Hence the correct choice is (b).

27. The frequency of the wave remains unchanged on refraction or reflection. The wavelength changes from λ to $\lambda' = \dfrac{\lambda}{\mu}$. Now the velocity of the wave in glass is

$$v' = n\lambda' = \frac{n\lambda}{\mu} = \frac{v}{\mu}$$

Hence the correct choice is (b).

28. The angular separation between the mth dark fringe and the central bright fringe is given by

$$a \sin \theta_m = m\lambda$$

For the first dark fringe, $m = 1$. Therefore

$$a \sin \theta_1 = \lambda$$

or $\sin \theta_1 = \lambda/a$. Since $\lambda << a$, $\sin \theta_1 \simeq \theta_1$. Hence $\theta_1 = \lambda/a$. This is also the angular separation between the central bright fringe and the first dark fringe on the other side of the central bright fringe. Hence, the angular separation between the first dark fringes on either side of the central bright fringe $= 2\theta_1 = 2\lambda/a$. Therefore, their separation at distance $d = 2$ m is

$$\frac{2\lambda d}{a} = \frac{2 \times (600 \times 10^{-9}) \times 2}{1.0 \times 10^{-3}}$$
$$= 2.4 \times 10^{-3} \text{ m} = 2.4 \text{ mm}$$

29. If follows from Fig. 27.6 that the path difference at the first minimum between rays coming from A and B is

$$\Delta = BC = a \sin \theta_1$$

But $\sin \theta_1 = \lambda/a$. Therefore, $\Delta = a \times \lambda/a = \lambda$. A path difference of λ corresponds to a phase difference of 2π.

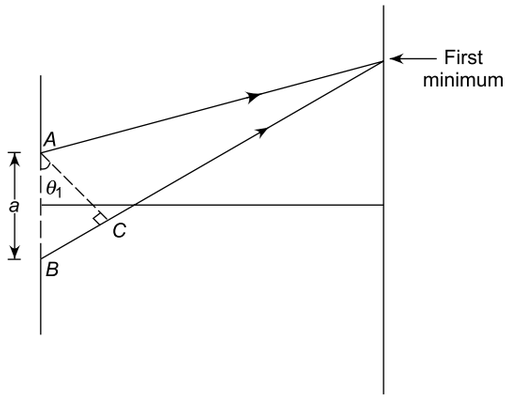


Fig. 27.6

30. Mass per unit length of the string is

$m = 10^{-2}$ kg/0.4 m = 2.5×10^{-2} kg m^{-1}.

$\therefore$ Speed of each pulse is $v = \sqrt{\frac{T}{m}}$

$$= \sqrt{\frac{1.6}{2.5 \times 10^{-2}}} = 8 \text{ ms}^{-1}$$

To have a constructive interference, the pulses must arrive at the same end of the string in the same phase after reflection from the other end. If L is the length of the string, the minimum value of Δt is

$$(\Delta t)_{\min} = \frac{2L}{v} = \frac{2 \times 0.4}{8} = 0.1 \text{ s}$$

31. The wavelength of X-rays is of the order of 1 Å $\simeq 10^{-10}$ m. Now, the angular width of the central maximum is

$$\theta_0 = \frac{2\lambda D}{a}$$

where D is the distance of the screen from the slit. Since λ for X-rays is very small compared to that for yellow light, it follows that the angular width of the central maximum becomes extremely small. Hence, the central maximum is narrower.

32. Given λ = 6000 Å = 6000×10^{-10} m = 6×10^{-7} m and Δ = 1.5 microns = 1.5×10^{-6} m. For bright fringes: $\Delta = n\lambda$; where n is an integer.

$n = \frac{\Delta}{\lambda} = \frac{1.5 \times 10^{-6}}{6 \times 10^{-7}} = \frac{5}{2}$, which is not an integer. Hence, path difference of 1.5×10^{-6} m does not correspond to a bright fringe. For dark fringes, we have

$$\Delta = \left(n - \frac{1}{2}\right) \lambda$$

or $1.5 \times 10^{-6} = \left(n - \frac{1}{2}\right) \times (6 \times 10^{-7})$

which gives $n - \frac{1}{2} = \frac{5}{2}$ or n = 3. Hence a path difference of 1.5×10^{-6} m corresponds to the third dark fringe. Thus the correct choice is (c).

33. Let λ_a and λ be the wavelengths of yellow light in air and vacuum respectively and v_a and c be their respective speeds in air and vacuum. Since the frequency of light is the same in both media, we have

$$\nu = \frac{v_a}{\lambda_a} = \frac{c}{\lambda}$$

or $\frac{c}{v_a} = \frac{\lambda}{\lambda_a}$. But $\frac{c}{v_a} = \mu_a$ (by definition)

$$\therefore \quad \mu_a = \frac{\lambda}{\lambda_a} \text{ or } \lambda_a = \frac{\lambda}{\mu_a} \qquad (1)$$

Now, if t is the thickness of each column, then the number of wavelengths in the two media are

$n_a = \frac{t}{\lambda_a}$ and $n = \frac{t}{\lambda}$. Given $(n_a - n) = 1$. Hence

$$1 = \frac{t}{\lambda_a} - \frac{t}{\lambda} = \frac{t}{\lambda}\left(\frac{\lambda}{\lambda_a} - 1\right) \qquad (2)$$

Using (1) in (2), we have

$$1 = \frac{t}{\lambda}(\mu_a - 1) \qquad (3)$$

Given μ_a = 1.0003 and λ = 6000 Å = 6000×10^{-8} cm. Using these values in (3), we have

$$1 = \frac{t}{6000 \times 10^{-8}} (1.0003 - 1)$$

or $$t = \frac{6000 \times 10^{-8}}{0.0003} = 0.2 \text{ cm} = 2 \text{ mm}$$

Hence the correct choice is (b).

34. Referring to Fig. 27.7, the direction θ along which we have the nth interference maximum is given by

$$d \sin \theta = n\lambda$$

or $$n = \frac{d \sin \theta}{\lambda}$$

If $d = 2\lambda$, we have

$$n = 2 \sin \theta$$

Since the maximum value of θ is 90°, $n_{\max}$ = 2 sin 90° = 2. Thus, there are two interference maxima in addition to the central maximum (which corresponds to $\theta = 0$). Hence the maximum number of possible interference maxima is three, which is choice (c).

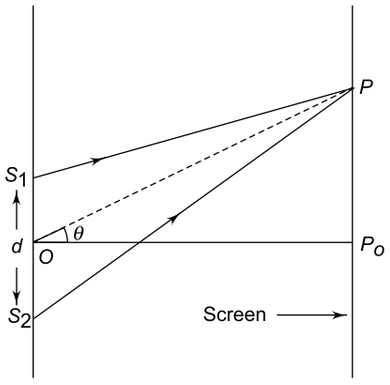


Fig. 27.7

35. According to the electromagnetic theory, the refractive index n of a dielectric medium is given by

$$n = \sqrt{\varepsilon_r} = \sqrt{4} = 2$$

If c is the speed of the electromagnetic wave in vacuum and v in the dielectric medium, then

$$n = \frac{c}{v} \text{ or } 2 = \frac{c}{v} \text{ or } v = \frac{c}{2}$$

When a wave passes from one medium into another, its frequency remains unchanged. If λ_0 and λ are the wavelengths of the wave in vacuum and in the medium respectively, then

$v = \nu\lambda$ and $c = \nu\lambda_0$, which give

$$\frac{v}{c} = \frac{\lambda}{\lambda_0} \text{ or } \frac{\lambda}{\lambda_0} = \frac{c/2}{c} = \frac{1}{2} \text{ or } \lambda = \frac{\lambda_0}{2}$$

Thus, the wavelength is halved but the frequency remains unchanged. Hence the correct choice is (c).

36. Resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$. Given $I_1 = I$ and $I_2 = 4I$.

$$I_A = I + 4I + 0 = 5I \qquad \left(\therefore \phi = \frac{\pi}{2}\right)$$

and $I_B = I + 4I + 2\sqrt{I \times 4I}\cos\pi$

$= 5I - 4I = I$

Hance $I_A - I_B = 5I - I = 4I$, which is choice (b).

37. The fringe width in liquid is

$$\beta_l = \frac{\beta_a}{n_l}$$

where $\beta_a = \dfrac{\lambda D}{d}$ is the fringe width in air. Thus

$$\beta_l = \frac{\lambda D}{n_l d} = \frac{\left(6300 \times 10^{-10}\right) \times 1.33}{1.33 \times \left(1 \times 10^{-3}\right)}$$

$= 0.63 \times 10^{-3}$ m $= 0.63$ mm

Hence the correct choice is (d).

38. $\beta_l = \dfrac{\beta_a}{n_l} = \dfrac{0.4 \text{ mm}}{4/3} = 0.3$ mm, which is choice (b).

39. Number of fringes $= \dfrac{\text{width of region}}{\text{fringe width}}$ or $n = \dfrac{L}{\beta}$.

Now, fringe width β is proportional to wavelength. Hence the new number of fringes will be

$$n' = n \times \frac{\beta}{\beta'} = 12 \times \frac{600}{400} = 18,$$

which is choice (b).

40. $\beta = \dfrac{\lambda D}{d}$. Therefore, $\Delta\beta = \dfrac{\lambda \Delta D}{d}$. Given $\Delta D = 5 \times 10^{-2}$ m, $\Delta\beta = 3 \times 10^{-5}$ m and $d = 10^{-3}$ m. Using these values, we have

$$3 \times 10^{-5} = \frac{\lambda \times 5 \times 10^{-2}}{10^{-3}}$$

which gives $\lambda = 6 \times 10^{-7}$ m which is choice (b).

41. When a transparent plate is introduced in the path of one of the interfering beams, the entire fringe pattern shifts by an amount $\Delta y = (\mu - 1)t$, where μ is the refractive index and t is the thickness of the plate. Since the path difference must change by λ for one maximum to be replaced by its neighbouring maximum, we have

$\Delta y = \lambda$ or $(\mu - 1)t = \lambda$ or $\left(\dfrac{3}{2} - 1\right)t = \lambda$ or $t = 2\lambda$.

Hence the correct choice is (a).

42. Since P and Q are points on the same wavefront, they are in the same phase. Therefore, the path difference at point P between the ray BP and the reflected ray OP is

$$\Delta = QO + OP \qquad \text{(i)}$$

Now, in triangle POR, $OP = \dfrac{PR}{\cos\theta} = \dfrac{d}{\cos\theta}$. Also in triangle QOP, $QO = OP \sin(90° - 2\theta) = OP\cos 2\theta$

$\therefore \quad \Delta = OP\cos 2\theta + OP$

$= OP(\cos 2\theta + 1) = 2OP\cos^2\theta$

$= 2 \times \dfrac{d}{\cos\theta} \times \cos^2\theta = 2d\cos\theta$

Since there is a sudden path change of $\frac{\lambda}{2}$ due to reflection, the condition of constructive interference at P is

$$\Delta = \frac{\lambda}{2}, \frac{3\lambda}{2} \text{ etc.}$$

or $$2d \cos\theta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \cdots \text{ etc.}$$

or $$\cos\theta = \frac{\lambda}{4d}, \frac{3\lambda}{4d}, \cdots \text{ etc.}$$

Hence the correct choice is (b).

43. The distance of the mth bright fringe from the central fringe is

$$y_m = m\lambda \frac{D}{d} = m\beta$$

where $\beta = \frac{\lambda D}{d}$ is the fringe width.

$$\therefore \quad y_9 = 9\beta \qquad (1)$$

The distance of the mth dark fringe from the central fringe is

$$y'_m = \left(m - \frac{1}{2}\right)\frac{\lambda D}{d} = \left(m - \frac{1}{2}\right)\beta$$

$$y'_2 = \frac{3}{2}\beta \qquad (2)$$

From Eqs. (1) and (2) we get

$$y_9 - y'_2 = 9\beta - \frac{3}{2}\beta = \frac{15}{2}\beta$$

$$= \frac{15}{2} \times 2.0 \text{ mm} = 15 \text{ mm}$$

Hence the correct choice is (c).

44. The position of the 30th bright fringe is given by

$$y_{30} = 30\ \frac{\lambda D}{d}$$

Hence the shift of the central fringe is

$$y_0 = 30\ \frac{\lambda D}{d}$$

But $$y_0 = \frac{D}{d}(\mu - 1)t$$

$$\therefore \quad 30\ \frac{\lambda D}{d} = \frac{D}{d}(\mu - 1)t$$

or $$(\mu - 1) = \frac{30\lambda}{t} = \frac{30 \times (6000 \times 10^{-10})}{(3.6 \times 10^{-5})} = 0.5$$

$$\therefore \quad \mu = 1.5$$

45. Let the nth bright fringe of wavelength λ_n and the mth bright fringe of wavelength λ_m coincide at a distance y from the central maximum, then

$$y = \frac{m\lambda_m D}{d} = \frac{n\lambda_n D}{d}$$

or $$\frac{m}{n} = \frac{\lambda_n}{\lambda_m} = \frac{6500}{5200} = \frac{5}{4}$$

The least integral values of m and n which satisfy the above condition are

$$m = 5 \text{ and } n = 4$$

i.e., the 5th bright fringe of wavelength 5200 Å coincides with the 4th bright fringe of wavelength 6500 Å. The smallest value of y at which this happens is

$$y_{\min} = \frac{m\lambda_m D}{d}$$

Substituting the values of m, λ_m, D and d, we get $y_{\min} = 1.0$ mm, which is choice (a).

46. Given $\frac{I_1}{I_2} = n$. Therefore, the amplitude ratio is

$$\frac{A_1}{A_2} = \sqrt{n}$$

Now $I_{\max} = (A_1 + A_2)^2$ and $I_{\min} = (A_1 - A_2)^2$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\frac{A_1}{A_2} + 1\right)^2}{\left(\frac{A_1}{A_2} - 1\right)^2}$$

$$= \frac{\left(\sqrt{n} + 1\right)^2}{\left(\sqrt{n} - 1\right)^2}$$

Hence the correct choice is (d).

47. Given $\frac{I_{\max}}{I_{\min}} = n$. Hence

$$\frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = n$$

or $$\frac{(A_1 + A_2)}{(A_1 - A_2)} = \sqrt{n}$$

which gives $$\frac{A_1}{A_2} = \frac{\sqrt{n} + 1}{\sqrt{n} - 1}$$

$$\therefore \quad \frac{I_1}{I_2} = \left(\frac{\sqrt{n} + 1}{\sqrt{n} - 1}\right)^2$$

Hence the correct choice is (a).

48. The intensity of light emerging from a slit is proportional to its width. Since the amplitude is proportional to the square-root of the intensity, we have

$$\frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{n}{1}} = \sqrt{n}$$

As shown in solution of Q. 46, the correct choice is (c).

49. Angular width of a fringe is given by

$$\theta = \frac{\lambda}{d} \quad \text{or} \quad d = \frac{\lambda}{\theta} \qquad (1)$$

Given λ = 628 nm = 628 × 10^{-9} m and θ = 0.1° = $\frac{0.1 \times \pi}{180}$ rad. Using these values in Eq. (1), we find that $d = 3.6 \times 10^{-4}$ m. Hence the correct choice is (c).

50. The correct choice is (b). Refer to the solution of Q. 46.

51. The correct choice is (d). Use

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

where $I_1 = I, I_2 = 4I$ and $\phi = \frac{\pi}{2}$.

52. If t is the thickness of the glass sheet, the fringes are displaced by an amount given by

$$\Delta y = (\mu - 1)\ \frac{tD}{d}$$

In order to bring the adjacent minimum to the centre of the screen (i.e. to bring the first dark fringe the central bright fringe), the fringes must be displaced by half the fringe width, i.e.

$$\Delta y = \frac{\beta}{2} = \frac{1}{2}\frac{\lambda D}{d}$$

Hence $(\mu - 1)\ \frac{tD}{d} = \frac{1}{2}\frac{\lambda D}{d}$

or $t = \frac{\lambda}{2(\mu - 1)}$, which is choice (a).

53. When a transparent plate of thickness t and refractive index μ is introduced in one of the interfering waves, the path difference at the centre of the screen is

$$\Delta = (\mu - 1)\ \frac{tD}{d}$$

$\therefore$ Phase difference $\phi = \frac{2\pi\Delta}{\lambda}$

$$= \frac{2\pi}{\lambda}(\mu - 1)\ \frac{tD}{d} \qquad (1)$$

Given λ = 5000 Å = 5 × 10^{-7} m, μ = 1.5, $t = 1.5 \times 10^{-6}$ m, D = 1 m and $d = 5 \times 10^{-4}$ m. Using these values in Eq. (1). we get $\phi = 3\pi$. If I is the intensity of each interfering wave, the resultant intensity at the centre of the screen is

$$I_r = I + I + 2\sqrt{I \times I}\ \cos 3\pi$$
$$= 2I - 2I = 0 \qquad (\because \cos 3\pi = -1)$$

Hence the intensity at the centre is zero, i.e. there is a dark fringe at the centre.

Hence the correct choice is (d).

54. The correct choice is (a). The lateral shift is given by

$$\Delta y = (\mu - 1)\ \frac{tD}{d}$$

55. The position of the nth order maximum is given by

$$y_n = \frac{n\lambda D}{d}$$

For a given point y_n is fixed. Since D and d are also fixed, $n\lambda$ = constant, i.e. $n_1 \lambda_1 = n_2 \lambda_2$. Hence $n_2 = n_1 \frac{\lambda_1}{\lambda_2} = \frac{20 \times 480}{600} = 16$, which is choice (a).

56. Let the nth dark fringe of wavelength λ_n and the mth dark fringe of wavelength λ_m coincide at a distance y from the centre of the screen, then

$$y = \left(n - \frac{1}{2}\right)\frac{\lambda_n D}{d} = \left(m - \frac{1}{2}\right)\frac{\lambda_m D}{d} \qquad (1)$$

At this position there is complete darkness on the screen. Eq. (1) gives

$$\frac{n - \frac{1}{2}}{m - \frac{1}{2}} = \frac{\lambda_m}{\lambda_n} = \frac{7500\text{Å}}{4500\text{Å}} = \frac{5}{3}$$

which gives $m = \frac{6n + 2}{10} = \frac{3n + 1}{5}$ (2)

Integral values of n and m which satisfy Eq. (2) n = 3, m = 2; n = 8, m = 5; and so on. Let n_1 = 3 and n_2 = 8, then from Eq. (1) the distances from the centre of the screen of the first and the second regions of darkness are given by

$$y_1 = \left(n_1 - \frac{1}{2}\right)\frac{\lambda_n D}{d}$$

and $$y_2 = \left(n_2 - \frac{1}{2}\right)\frac{\lambda_n D}{d}$$

$\therefore$ $$\Delta y = y_2 - y_1 = (n_2 - n_1) \times \frac{\lambda_n D}{d}$$

$$= (8 - 3) \times \frac{4500 \times 10^{-10} \times 1 \cdot 2}{1 \times 10^{-3}}$$

$$= 2.7 \times 10^{-3}\ \text{m} = 2.7\ \text{mm},$$

Hence the correct choice is (c).

57. If δ is the phase difference between the interfering waves at point P, then the intensity at point P is given by (see Fig. 27.8)

$$I = I_{\max} \cos^2 \left(\frac{\delta}{2}\right)$$

Given $I = \frac{I_{\max}}{2}$. Hence

$$\cos^2 \left(\frac{\delta}{2}\right) = \frac{1}{2}, \text{ which gives } \frac{\delta}{2} = \frac{\pi}{4}$$

or $$\delta = \frac{\pi}{2}$$

The angular separation θ between points P and O is given by $\tan \theta = y/D$. Since θ is very small, $\tan \theta \simeq \sin \theta$. Hence

$$\sin \theta = \frac{y}{D} \tag{1}$$

If ω is the fringe width, then

$$\frac{y}{\omega} = \frac{\pi/2}{2\pi} = \frac{1}{4} \tag{2}$$

This is so because the phase difference δ between two consecutive maxima is 2π. Now $\omega = \frac{\lambda D}{d}$. Using this in Eq. (2), we get

$$\frac{yd}{\lambda D} = \frac{1}{4}$$

or $$\frac{y}{D} = \frac{\lambda}{4d} \tag{3}$$

Using Eq. (1) in Eq. (3), we have

$\sin \theta = \frac{\lambda}{4d}$ or $\theta = \sin^{-1}\left(\frac{\lambda}{4d}\right)$, which is choice (d).

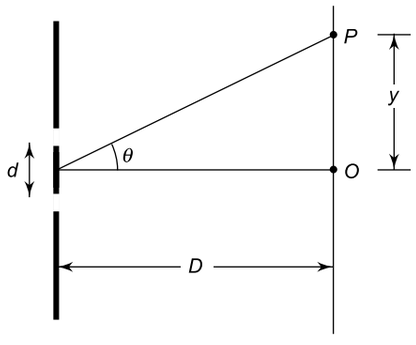


Fig. 27.8

58. De Broglie wavelength of electron is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

If speed v of electron is increased, momentum $p(= mv)$ will increase. Hence wavelength λ will decrease. Now, the angular width of the central maximum of the diffraction pattern is 2θ where θ is given by

$$\sin \theta = \frac{\lambda}{a}$$

where a is the width of the slit. Thus, if λ decreases, θ and hence 2θ will decrease. Therefore, the correct statement is (c).

59. When a sheet of thickness t and refractive index μ is introduced in one of the interfering waves, the distance y_0 through which the fringes shift is given by

$$y_0 = (\mu - 1)t \frac{D}{d} \tag{1}$$

The fringe width β, i.e., the distance between successive maxima (or minima) is given by

$$\beta = \frac{\lambda D}{d}$$

When the distance D between the slits and the screen is doubled, the new fringe width becomes

$$\beta' = 2 \frac{\lambda D}{d} \tag{2}$$

It is given that $y_0 = \beta'$. Equating Eqs. (1) and (2) we get

$$2 \frac{\lambda D}{d} = (\mu - 1)\ t \frac{D}{d}$$

$$\lambda = \frac{1}{2} (\mu - 1)t$$

which is choice (b)

60. The angular width of a fringe is given by

$$\theta = \frac{\lambda}{d}$$

In air: $$\theta_a = \frac{\lambda_a}{d}$$

In water: $$\theta_w = \frac{\lambda_w}{d}$$

$$\therefore \quad \frac{\theta_w}{\theta_a} = \frac{\lambda_w}{\lambda_a} \text{ or } \theta_w = \frac{\lambda_w \theta_a}{\lambda_a} \tag{1}$$

Now, refractive index is defined as

$$\mu_w = \frac{\text{speed of light in air}}{\text{speed of light in water}} = \frac{v_a}{v_w} = \frac{\nu \lambda_a}{\nu \lambda_w} = \frac{\lambda_a}{\lambda_w} \tag{2}$$

where ν is the frequency of light which remains unchanged.

Using Eq (2) in Eq (1) we have

$$\theta_w = \frac{\theta_a}{\mu_w} = \frac{0.2^\circ}{4/3} = 0.15^\circ$$

So the correct choice is (a).

61. Refer to Fig. 27.9. When the incident beam falls normally on the slits S_1 and S_2, the path difference at the central point P_0 of the screen is zero. Hence we have the central maximum at P_0.

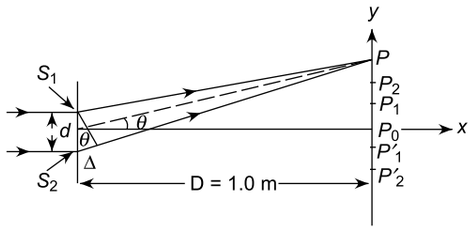


Fig. 27.9

Let the minima appear along directions θ with respect to the incident direction. Coherent waves from S_1 and S_2 along this direction are brought to a focus at P. It is clear that the path difference between the waves from S_1 and S_2 on reaching P is

$$\Delta = d \sin \theta$$

The interference minima will appear on the screen if

$$\Delta = \left(m + \frac{1}{2}\right)\lambda$$

or $$d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$$

where m is an integer. Thus the directions of minima are given by

$$\sin \theta = \left(m + \frac{1}{2}\right) \times \left(\frac{\lambda}{d}\right)$$

Given $d = 1.0$ mm and $\lambda = 0.5$ mm. Therefore

$$\sin \theta = \left(m + \frac{1}{2}\right) \times \left(\frac{0.5}{1.0}\right) = \frac{1}{2}\left(m + \frac{1}{2}\right)$$

The allowed values of m are those integers for which $\sin \theta$ is not more than + 1 or less than – 1. These values are $m = 1, 0, -1$ and -2. Hence four minima will be observed. The correct choice is (b)

62. If an interference experiment is performed using two wavelengths close to each other, two interference patterns corresponding to the two wavelengths are obtained on the screen. The fringe system remains distinct upto a point on the screen where the nth order maximum of one wavelength, say $\lambda_1 = 5890$ Å falls on the nth order minimum of the other wavelength $\lambda_2 = 5895$ Å. Thus, interference pattern can be seen upto a distance y_n from the centre of the screen if

$$y_n = \frac{n\lambda_1 D}{d}; \ (n\text{th maximum}) \qquad (1)$$

$$= \left(n - \frac{1}{2}\right) \frac{\lambda_2 D}{d}; \ (n\text{th minimum}) \qquad (2)$$

or $$n\lambda_1 = \left(n - \frac{1}{2}\right) \lambda_2 \text{ or } 2n\lambda_1 = (2n - 1)\lambda_2$$

which gives

$$n = \frac{\lambda_2}{2(\lambda_2 - \lambda_1)}, \text{ which is choice (d).}$$

63. Refer to Fig. 27.10. To reach point P, wave 1 has to travel a path ($SS_2 + S_2P$) while wave 2 has to travel a path ($SS_1 + S_1P$). Therefore, when the waves arrive at P, the path difference is

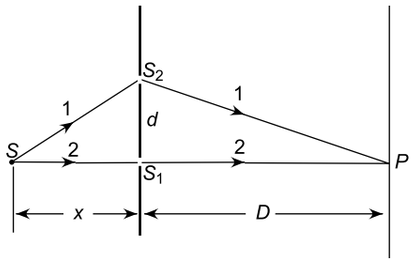


Fig. 27.10

$$\Delta = (SS_2 + S_2P) - (SS_1 + S_1P) \qquad (1)$$

Now, in triangle SS_2S_1, we have

$$SS_2 = (x^2 + d^2)^{1/2} = x\left(1 + \frac{d^2}{x^2}\right)^{1/2}$$

$$= \left(1 + \frac{d^2}{2x^2}\right) \qquad (\because d << x)$$

Similarly, $S_2P = (D^2 + d^2)^{1/2} = D\left(1 + \frac{d^2}{2D^2}\right)$

$(\because d << D)$

Also $(SS_1 + S_1P) = x + D$. Using these in Eq. (1), we have

$$\Delta = x\left(1 + \frac{d^2}{2x^2}\right) + D\left(1 + \frac{d^2}{2D^2}\right) - (x + D)$$

$$= x + \frac{d^2}{2x} + D + \frac{d^2}{2D} - x - D$$

or $$\Delta = \frac{d^2}{2}\left(\frac{1}{x} + \frac{1}{D}\right)$$

In order to have a dark fringe at P, $\Delta = \frac{\lambda}{2}$. Hence

$$\frac{\lambda}{2} = \frac{d^2}{2}\left(\frac{1}{x} + \frac{1}{D}\right) \qquad (1)$$

or $$d = \left[\frac{\lambda xD}{(x + D)}\right]^{1/2} \qquad (2)$$

Putting $x = \frac{D}{2}$ in Eq. (2), we find that the correct choice is (a).

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. When a light wave is reflected from a mirror, there is no change in its
(a) amplitude (b) frequency
(c) wavelength (d) speed

2. When a light wave travels from air into glass, which of the following will change?
(a) amplitude (b) frequency
(c) wavelength (d) speed

3. In Young's double slit experiment the fringe width with light of wavelength λ_1 is β_1 and with light of wavelength λ_2 is β_2. Using light of wavelength λ_1, the fringe width becomes β_3 if the entire apperatus is immersed in a transparent liquid of refractive index μ. Then
(a) $\beta_2 = \beta_1 \frac{\lambda_1}{\lambda_2}$ (b) $\beta_2 = \beta_1 \frac{\lambda_2}{\lambda_1}$
(c) $\beta_3 = \frac{\beta_1}{\mu}$ (d) $\beta_3 = \mu\beta_1$

4. In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then in the interference pattern, the intensity
(a) of maxima will increase
(b) of maxima will decrease
(c) of minima will increase
(d) of minima will decrease

IIT, 2000

5. In a single slit diffraction experiment, the width of the slit is made double its original width. Then the central maximum of the diffraction pattern will become
(a) narrower (b) fainter
(c) broader (d) brighter

6. In Young's double slit experiment, the 10th bright fringe is at a distance x from the central fringe. Then
(a) the 10th dark fringe is at a distance of $\frac{19x}{20}$ from the central fringe.
(b) the 10th dark fringe is at a distance of $\frac{21x}{20}$ from the central fringe.
(c) the 5th dark fringe is at a distance of $\frac{x}{2}$ from the central fringe.
(d) the 5th dark fringe is at a distance of $\frac{9x}{20}$ from the central fringe.

7. In Young's double slit experiment, the 10th bright fringe of wavelength λ_1 is at a distance x from the central fringe. Then
(a) the 5th bright fringe of wavelength λ_2 will be at a distance of $\frac{\lambda_1 x}{2\lambda_2}$ from the central fringe.
(b) the 5th bright fringe of wavelength λ_2 will be at a distance of $\frac{\lambda_2 x}{2\lambda_1}$ from the central fringe.
(c) the 5th dark fringe of wavelength λ_2 will be at a distance of $\frac{9\lambda_2 x}{20\lambda_1}$ from the central fringe.
(d) the 5th dark fringe of wavelength λ_2 will be at a distance of $\frac{9\lambda_1 x}{20\lambda_2}$ from the central fringe.

8. In Young's double slit experiment, the slits are separated by 0.3 mm and the interference pattern is observed on a screen placed at a distance of 100 cm from the slits. The wavelength of light used is 600 nm.
(a) the distance of the 4th bright fringe from the central fringe is 8 mm.
(b) the distance of the 4th dark fringe from the central fringe is 7 mm.
(c) the distance between the 9th dark fringe and the second bright fringe on the same side of the central fringe is 15 mm.
(d) the distance between the first dark and the second bright fringe on the opposite sides of the central fringe is 5 mm.

9. A narrow slit of width 1.3×10^{-6} m is illuminated by a parallel beam of light of wavelength 6500 Å incident normally on it. Then
(a) the angular width of the central maximum is 30°.

(b) the angular width of the central maximum is 60°.

(c) the angular separation between central maximum and the first order maximum is 45°.

(d) the angular separation between the central maximum and the first order maximum is $\sin^{-1}\left(\frac{3}{4}\right)$.

10. A parallel beam of monochromatic light of wavelength 600 nm is incident normally on a slit of width 0.3 mm. The diffraction pattern is observed on a screen which is placed at the focal plane of a convex lens of focal length 50 cm. The linear separations between the first minima and the first maxima on both sides of the central maximum are x_1 and x_2 respectively. Then

(a) $x_1 = 1$ mm (b) $x_1 = 2$ mm

(c) $x_2 = 3$ mm (d) $x_2 = 6$ mm

11. A beam of light consisting of two wavelengths 750 nm and 450 nm is used to obtain interference fringes in a Young's double slit experiment. The separation between the slits is 1 mm and the distance between the plane of the slits and the screen is 100 cm. The least distance from the central maximum where the bright fringes due to both the wavelengths coincide is y_{min} and y'_{min} is the corresponding distance where the dark fringes due to both the wavelengths coincide. Then

IIT, 2004

(a) $y_{min} = 2.25$ mm (b) $y_{min} = 2.0$ mm

(c) $y'_{min} = 4.5$ mm (d) $y'_{min} = 0.1$ mm

12. In Young's double slit experiment, the upper slit is covered with a thin glass sheet of thickness t and refractive index μ_1 while the lower slit is covered with another glass plate of the same thickness t but having a refractive index μ_2 ($> \mu_1$). Interference pattern is observed using light of wavelength λ. It is observed that the point P on the screen where the central maximum ($n = 0$) fell before the plates were introduced now has (3/4) the original intensity. The phase difference between the interfering waves at point P now is

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{3}$

IIT, 1997

13. In Young's double slit experiment the separation between the slits is 2 mm and the distance of the screen from the plane of the slits is 2.5 m. A light of wavelengths in the range 200 nm to 800 nm is allowed to fall on the slits. The wavelengths in the visible region that will be present on the screen at 1 mm from the central maximum are

(a) 400 nm (b) 500 nm

(c) 600 nm (d) 700 nm

14. A parallel beam of light containing two wavelengths λ and λ' is incident normally on a narrow slit. Fraunhofer diffraction pattern is obtained on a screen placed at the focal plane of a lens of focal length 0.5 m. It is observed that the second minimum of λ and the third minimum of λ' overlap at the same point on the screen 2.5 mm from the centre of the screen. If $\lambda = 6000$ Å,

(a) $\lambda' = 4000$ Å

(b) $\lambda' = 3000$ Å

(c) Slit width = 0.20 mm

(d) Slit width = 0.24 mm

15. In a Young double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intesity of light falling on slit 2. Choose the correct choice(s).

(a) If $d = \lambda$, the screen will contain only one maximum

(b) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen

(c) If the intesity of light falling on slit 1 is increased so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase.

(d) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intesities of the observed dark and bright frings will increase.

SOLUTIONS

1. The correct choices are (b), (c) and (d).

2. The correct choices are (a), (c) and (d).

3. $\beta_1 = \frac{\lambda_1 D}{d}$, $\beta_1 = \frac{\lambda_2 D}{d}$, Therefore,

$\frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1}$, which is choice (b).

Now $\lambda_1 = \frac{\lambda_a}{\mu} = \frac{\lambda_1}{\mu}$. Therefore,

$\beta_3 = \dfrac{\beta_1 \lambda_l}{\lambda_1} = \dfrac{\beta_1}{\mu}$, which is choice (c).

4. In the case when the slits are of equal width, the intensity of light emerging from the two slits is the same, say, I_0. Then

$$I_{max} = I_0 + I_0 + 2\sqrt{I_0 I_0} = 4I_0$$

and $$I_{min} = I_0 + I_0 - 2\sqrt{I_0 I_0} = 0$$

When one slit say S_1 is made twice as wide as the other, the intensity of light from S_1 is doubled, i.e. $I_1 = 2I_0$ but $I_2 = I_0$. Hence, in this case

$$I'_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= 2I_0 + I_0 + 2\sqrt{2 I_0 I_0}$$
$$= 3I_0 + 2\sqrt{2} I_0 = 5.83\ I_0$$

and $$I'_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$
$$= 2I_0 + I_0 - 2\sqrt{2 I_0 I_0}$$
$$= 3I_0 - 2\sqrt{2} I_0 = 0.17 I_0$$

Thus $I'_{max} > I_{max}$ and $I'_{min} > I_{min}$,

so the correct choice are (a) and (c).

5. The angular width of the central maximum is $2\lambda / a$ where a is the width of the slit. If the value of a is doubled, the angular width of the central maximum decreases to half its earlier value. This implies that the central maximum becomes much sharper. Furthermore if a is doubled, the intensity of the central maximum becomes four times. Thus the central maximum becomes much sharper and brighter.
The correct choice are (a) and (d).

6. The correct choices are (a), (c) and (d). The distance of the nth bright fringe from the central fringe is given by

$$x_n = \frac{n\lambda D}{d}$$

The distance of the nth dark fringe from the central fringe is given by

$$x_n^* = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d}$$

7. The correct choices are (b) and (c).

8. All the four choices are correct.

9. The correct choices are (b) and (d). The angular width of the central maximum is $2\theta_1$ where θ_1 is given by

$$\sin \theta_1 = \frac{\lambda}{a}$$

where λ = wavelength and a = width of the slit. The angular separation between the central maximum and the first order maximum is θ_2, where θ_2 is given by

$$\sin \theta_1 = \frac{3\lambda}{2a}$$

10. The correct choices are (b) and (d). The linear separation between the first minima = $\dfrac{2f\lambda}{a}$ and between the first maxima = $\dfrac{3f\lambda}{a}$.

11. Let nth bright fringe of wavelength λ_n and the mth bright fringe of wavelength λ_m coincide at a distance y from the centre of the screen. Then

$$y = \frac{n\lambda_n D}{d} = \frac{m\lambda_m D}{d}$$

or $$n\lambda_n = m\lambda_m$$

or $$\frac{\lambda_n}{\lambda_m} = \frac{m}{n}$$

or $$\frac{750}{450} = \frac{m}{n}$$

or $\dfrac{m}{n} = \dfrac{5}{3}$. The minimum integral values of m and n that satisfy this equation are $m = 5$ and $n = 3$. Therefore, the minimum value of y is

$$y_{min} = \frac{n\lambda_n D}{d} = \frac{3 \times 750 \times 10^{-9} \times 1}{10^{-3}}$$
$$= 2.25 \times 10^{-3}\ \text{m} = 2.25\ \text{mm}$$

The only correct choice is (a). All other choices are wrong.

12. Refer to Fig. 27.11. In Young's double slit experiment, slits S_1 and S_2 are equidistant from the source S and the slits have equal widths. Hence light emerging from S_1 and S_2 has the same intensity, say I_0. Before the plates are introduced, the path difference (and hence the phase difference ϕ) at the central point P between the interfering waves is zero. Hence the intensity at point P is

$$I = I_0 + I_0 + 2\sqrt{I_0 I_0} \cos 0° = 4I_0$$

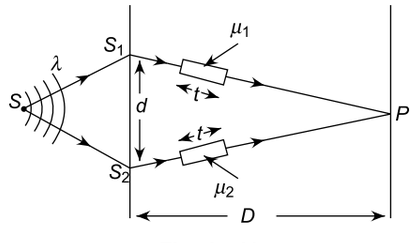


Fig. 27.11

When the two plates are introduced the path difference at the central point P between the interfering waves becomes

$$\Delta = (\mu_2 - 1)t - (\mu_1 - 1)t = (\mu_2 - \mu_1)t$$

$$\therefore \quad \text{Phase difference } \phi = \frac{2\pi\Delta}{\lambda} = \frac{2\pi}{\lambda}(\mu_2 - \mu_1)t$$

Hence the intensity at point P becomes

$$I' = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi$$
$$= 2I_0 + 2I_0\cos\phi$$

Given that $I' = \frac{3I}{4} = \frac{3}{4}\times 4I_0 = 3I_0$. Thus

$$3I_0 = 2I_0 + 2I_0\cos\phi$$

or $\quad \cos\phi = \frac{1}{2}$

or $\quad \phi = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \ldots$ etc.

Thus the correct choices are (a) and (d).

13. The distance of the nth maximum from the centre of the screen is given by

$$y_n = \frac{n\lambda D}{d} \quad \text{or} \quad \lambda = \frac{y_n d}{n D}$$

Given $d = 2\times 10^{-3}$ m, $D = 2.5$ m and $y_n = 10^{-3}$ m. Thus

$$\lambda = \frac{10^{-3}\times 2\times 10^{-3}}{n\times 2.5}$$

$$= \frac{1}{n}\times 8\times 10^{-7}\text{ m} = \frac{8000\text{ Å}}{n}$$

Here n is an integer having values 1, 2, 3, … etc.

For $\quad n = 1\ ;\ \lambda_1 = 8000$ Å

For $\quad n = 2\ ;\ \lambda_2 = 4000$ Å

For $\quad n = 3\ ;\ \lambda_3 = \frac{8000\text{ Å}}{3} \approx 2667$ Å

For $\quad n = 4\ ;\ \lambda_4 = 2000$ Å

Wavelength 8000 Å is in the infrared region, wave-length 4000 Å is in the visible region and wavelengths 2667 Å and 2000 Å are both in the ultraviolet region of the electromagnetic spectrum. So the only correct choice is (a).

14. The nth minimum lies along a direction θ_n given by

$$a\sin\theta_n = n\lambda;\ n = 1, 2, 3, \ldots \text{ etc.}$$

or $a\,\theta_n = n\lambda \qquad (\because\ \theta_n \text{ is small})$

or $\quad \theta_n = \frac{n\lambda}{a} \qquad (1)$

The second minimum of λ lies along a direction θ_2 which is given by putting $n = 2$ in Eq. (1).

$$\theta_2 = \frac{2\lambda}{a} \qquad (2)$$

The third minimum of λ' lies along a direction θ_3' which is given by putting $n = 3$ in Eq. (1).

$$\theta_3' = \frac{3\lambda'}{a} \qquad (3)$$

Given $\theta_2 = \theta_3'$. Hence, equating Eqs (2) and (3), we have

$$\frac{2\lambda}{a} = \frac{3\lambda'}{a}$$

or $\quad \lambda' = \frac{2\lambda}{3} = \frac{2\times 6000\text{ Å}}{3} = 4000$ Å

If y is the distance of the second minimum of λ (or third minimum of λ'), then, using Eq. (1), we have

$$y = f\theta_2 = \frac{2f\lambda}{a} \quad \text{or} \quad a = \frac{2f\lambda}{y} \qquad (4)$$

Given $f = 0.5$ m, $\lambda = 6000$ Å $= 4\times 10^{-7}$ m and $y = 2.5$ mm $= 2.5\times 10^{-3}$ m. Using these values in Eq. (4) we get $a = 2.4\times 10^{-4}$ m $= 0.24$ mm.

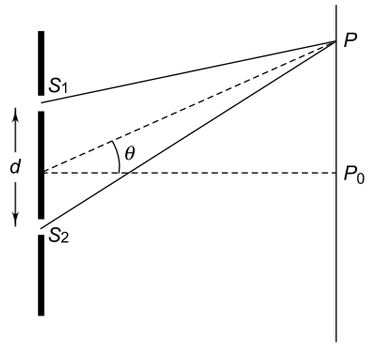
So the correct choices are (a) and (d).

15. Refer to following figure.

Path difference $\Delta = d\sin\theta$. For maxima $\Delta = n\lambda$; $n = 0, 1, 2, \ldots$ etc. For the central maximum ($n = 0$), $d\sin\theta_0 = 0$ which gives $\theta_0 = 0$. The central maximum appears at point P_0. For the next maximum ($n = 1$), $d\sin\theta_1 = \lambda$. For $d = \lambda$, we have $\sin\theta_1 = 1$ or $\theta_1 = 90°$. Hence the next maximum (after the central maximum) is not seen on the screen. So choice (a) is correct.

If $\lambda < d < 2\lambda$, $\sin\theta_1$ lies between $\frac{1}{2}$ and 1, i.e. θ_1 lies between 30° and 90°. So at least one more maximum (besides the central maximum) will be seen choice (b) is also correct.

The resultant intensity is given by



$$I_r = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

$$\therefore \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

and $$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

For $I_1 = 4I$ and $I_2 = I$, we have $I_{max} = 9I$ and $I_{min} = I$.

If $I_1 = I = I_2$, than $I_{max} = 4I$ and $I_{min} = 0$. So the intensity of maxima and minima both will decrease. Hence choice (c) is incorrect.

If $I_1 = 4I = I_2$, then $I_{max} = 16\ I$ and $I_{min} = 0$. Hence the intensity of maxima increases while the intensity of minima decreases. So choice (d) is also incorrect. Choice (a) and (b) are correct.

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage

Passage I

In a modified Young's double slit experiment, a monochromatic and parallel beam of light of wavelength 6000 Å and intensity $\frac{10}{\pi}$ Wm^{-2} is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength 6000 Å is placed in front of aperture A (Fig. 27.12).

IIT, 1989

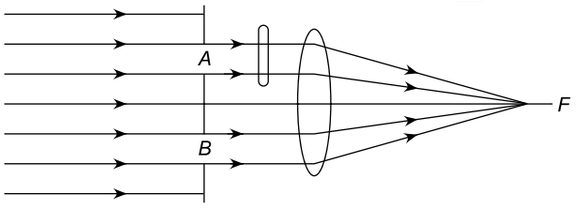


Fig. 27.12

1. The ratio of the powers received at aperture A to that at aperture B is

(a) 1 : 2 (b) 1 : 4

(c) 1 : 8 (d) 1 : 16

2. The phase difference between the interfering waves at point F is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

3. If 10% of the power received by each aperture goes in the original direction, the resultant power at point F will be

(a) 5 μW (b) 6 μW

(c) 7 μW (d) 8 μW

SOLUTION

1. Intensity of the beam $(I) = \frac{10}{\pi}$ Wm^{-2}

Power received at aperture $A = I \times$ cross-sectional area of A

$$= \frac{10}{\pi} \times \pi \times (0.001)^2 = 10^{-5}\ W$$

Power received at aperture $B = \frac{10}{\pi} \times \pi \times (0.002)^2$

$$= 4 \times 10^{-5}\ W$$

So the correct choice is (b).

2. The phase difference at F is

$$\delta = (\mu - 1) \times t \times \frac{2\pi}{\lambda}$$

$$= \frac{(1.5-1) \times (2000 \times 10^{-8}) \times 2\pi}{(6000 \times 10^{-8})} = \frac{\pi}{3}\ \text{rad}$$

The correct choice is (b).

3. Since 10% the power received at each aperture goes in the original direction, the power at point F due to the two apertures respectively is

$$P_A = 10\%\ \text{of}\ 10^{-5}\ W = 10^{-6}\ W$$

$$P_B = 10\%\ \text{of}\ 4 \times 10^{-5}\ W = 4 \times 10^{-6}\ W$$

Now, intensity (and hence power) is proportional the square of the amplitude. If A_1 and A_2 are the amplitudes at F due to the two sources, we have $P_A = kA_1^2$ and $P_B = kA_2^2$, where k is the proportionality constant. Thus

$$A_1 = \sqrt{\frac{P_A}{k}} \text{ and } A_2 = \sqrt{\frac{P_B}{k}}$$

Resulting amplitude

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\delta}$$

Substituting the values of A_1, A_2 and δ, we get

$$A = \sqrt{\frac{7\times10^{-6}}{k}}$$

$\therefore$ Resultant power at $F = kA^2$

$$= k\times \frac{7\times10^{-6}}{k} = 7\times10^{-6}\text{ W}$$

Hence the correct choices is (c).

Questions 4 to 6 are based on the following passage

Passage II

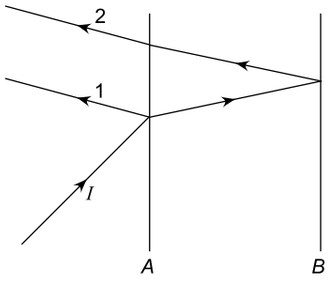


Fig. 27.13

A narrow monochromatic beam of light of intensity I is incident on a glass plate A as shown in Fig. 27.13. Another identical glass plate B is kept close to A and parallel to it. Each plate reflects 25% of the light intensity incident on it and transmits the remaining. Interference pattern is formed by beams 1 and 2 obtained after reflection at each plate.

IIT, 1990

4. The intensity of beam 2 is

(a) $\frac{3I}{16}$ (b) $\frac{3I}{32}$

(c) $\frac{9I}{32}$ (d) $\frac{9I}{64}$

5. The ratio of the intensities of beams 1 and 2 is

(a) $\frac{16}{9}$ (b) $\frac{4}{3}$

(c) $\frac{25}{16}$ (d) $\frac{5}{4}$

6. The ratio of the maximum and minimum intensities in the interference pattern is

(a) 16 : 1 (b) 25 : 1

(c) 36 : 1 (d) 49 : 1

SOLUTION

4. A beam of light of intensity I is incident on plate A. Since the plate reflects 25% of I, the intensity of the reflected beam 1 (see Fig. 27.14) is

$$I_1 = I\times\frac{25}{100} = \frac{I}{4}$$

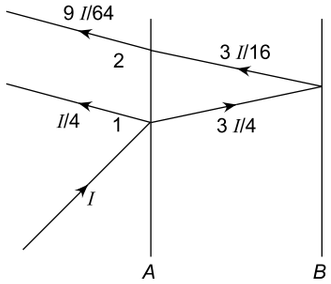


Fig. 27.14

The remaining intensity $3I/4$ falls on plate B which reflects 25% of the intensity incident on it. Hence intensity of beam reflected from B is

$$\frac{3I}{4}\times\frac{25}{100} = \frac{3I}{16}$$

A beam of intensity $3I/16$ falls on plate A which transmits 75% of this intensity. Hence the intensity of beam 2 is

$$I_2 = \frac{3I}{16}\times\frac{75}{100} = \frac{9I}{64}$$

So the correct choice is (d).

5. $\frac{I_1}{I_2} = \frac{I/4}{9I/64} = \frac{16}{9}$, which is choice (a).

6. The ratio of amplitudes is $\frac{a_1}{a_2} = \sqrt{\frac{16}{9}} = \frac{4}{3}$. Thus .

$a_1 = 4$ units and $a_2 = 3$ units.

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2 = \left(\frac{4+3}{4-3}\right)^2 = 49$$

Thus the correct choice is (d).

Questions 7 to 9 are based on the following passage

Passage III

Two parallel beams of light P and Q (separation d) each containing radiations of wavelengths 4000 Å and 5000 Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in Fig. 27.15. The refractive index of the prims as a function of wavelength is given by

$$\mu = 1.20 + \frac{b}{\lambda^2}$$

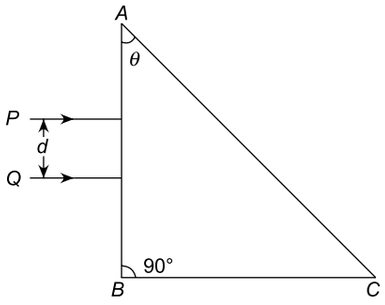


Fig. 27.15

where λ is in Å and b is a positive constant. The value of b is such that the condition of total internal reflection at the face AC is just satisfied for one wavelength and is not satisfied for the other.

IIT, 1991

7. The angle of deviation produced by the prism is ($\lambda = 4000$ Å, $\lambda' = 5000$ Å)

(a) greater for beam λ than for beam λ'

(b) less for beam λ than for beam λ'

(c) equal for both the wavelengths

(d) zero for both the wavelengths.

8. The value of b is (given $\sin\theta = 0.8$)

(a) 2×10^5 (Å)^2 (b) 4×10^5 (Å)^2

(c) 8×10^5 (Å)^2 (d) 1×10^4 (Å)^2

9. The refractive index of the prism for 5000 Å wavelength is

(a) 1.5 (b) $\sqrt{2}$

(c) greater than 1.25

(d) less than 1.25.

SOLUTION

7. Given $\lambda = 4000$ Å and $\lambda' = 5000$ Å. Also

$$\mu = 1.20 + \frac{b}{\lambda^2} \qquad (1)$$

It follows from this relation that the value of μ is greater for λ than for λ'. The angle of deviation is greater for greater value of μ. Hence the correct choice is (a).

8. The relation between refractive index μ and critical angle i_c is

$$\sin i_c = \frac{1}{\mu}$$

Thus, the smaller the value of μ, the greater is the angle i_c. It follows from relation (1) that μ for λ is greater than μ' for λ'. Hence i_c for λ is less than i_c' for λ'. It follows from Fig. 27.16 that the angle of incidence at face AC is the same ($= \theta$) for both beams. It is given that the condition of total internal reflection at face AC is just satisfied for one of the wavelengths. Since i_c for λ (= 4000 Å) is less than i'_c for λ'(= 5000 Å), it is obvious that the radiation of wavelength $\lambda = 4000$ Å is just totally reflected and the other radiation of wavelength $\lambda' = 5000$ Å is transmitted through the face AC (see Fig. 27.16)

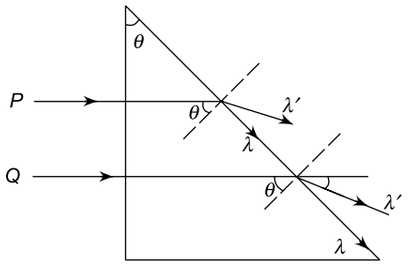


Fig. 27.16

$$\mu = \frac{1}{\sin i_c} = \frac{1}{\sin\theta} = \frac{1}{0.8} = 1.25$$

[$\because \sin\theta = 0.8$ (given)]

Substituting $\mu = 1.25$ and $\lambda = 4000$ Å in relation (1), we have

$$1.25 = 1.20 + \frac{b}{(4000)^2}$$

which gives $b = 8 \times 10^5$ (Å)^2, which is choice (c).

9. Using $b = 8 \times 10^5$ (Å)^2 and $\lambda' = 5000$ Å in Eq. (1), we get

$$\mu' = 1.20 + \frac{8 \times 10^5}{(5000)^2} = 1.232$$

So the correct choice is (d).

Questions 10 to 13 are based on the following passage

Passage IV

A glass plate of refractive index $\mu_3 = 1.5$ is coated with a thin layer of thickness t and refractive index $\mu_2 = 1.8$. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere.

IIT, 2000

10. The two reflected waves interfere constructively if (n is an integer)

(a) $t = \dfrac{n\lambda}{2\mu_2}$ (b) $t = \dfrac{n\lambda}{2(\mu_2 - \mu_3)}$

(c) $t = \dfrac{\left(n+\frac{1}{2}\right)\lambda}{2\mu_2}$ (d) $t = \dfrac{\left(n+\frac{1}{2}\right)\lambda}{2\mu_3}$

11. If $\lambda = 648$ nm, the least value of t for which the waves interface constructively is

(a) 90 nm (b) 180 nm

(c) 108 nm (d) 216 nm

12. The two reflected waves interfere destructively if

(a) $t = \dfrac{n\lambda}{2\mu_2}$ (b) $t = \dfrac{n\lambda}{2\mu_3}$

(c) $t = \dfrac{\left(n+\frac{1}{2}\right)\lambda}{2\mu_2}$ (d) $t = \dfrac{\left(n+\frac{1}{2}\right)\lambda}{2\mu_3}$

13. If $\lambda = 648$ nm, the least value of t for which the waves interfere destructively is

(a) 90 nm (b) 180 nm

(c) 108 nm (d) 216 nm

SOLUTION

10. Refer to Fig. 27.17. A ray of light travelling in air ($\mu_1 = 1$) falls normally on a thin layer ($\mu_2 = 1.8$) of thickness t. It is partly reflected at point P as wave 1 and partly refracted as wave 2. Wave 2 on meeting the surface of the glass plate ($\mu_3 = 1.5$) is reflected at point Q and travels along QP.

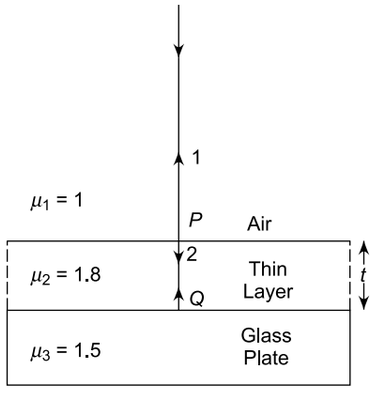


Fig. 27.17

Waves 1 and 2 meet at point P where they interfere. We know that when a wave is travelling in a rarer medium and gets reflected at the boundary of a denser medium, it undergoes a phase change of π or a path change of $\lambda/2$. Thus wave 1 has an optical path of $\Delta_1 = \lambda/2$. Wave 2 travelling from P to Q in the layer of refractive index 1.8 gets reflected at Q from the boundary of glass of refractive index 1.5. Thus wave 2 travelling in a denser medium is reflected from the boundary of a rarer medium undergoes no phase change due to reflection. Thereore, Optical path for wave 2 from P to Q and from Q to P in the layer is

$$\Delta_2 = \text{refractive index of layer} \times 2(PQ)$$
$$= \mu_2 \times 2t = 2\mu_2 t$$

$\therefore$ Optical path difference between waves 1 and 2 at point p is

$$\Delta = \Delta_2 - \Delta_1 = 2\mu_2 t - \frac{\lambda}{2}$$

Now, for constructive interference, $\Delta = n\lambda$; $n = 0, 1, 2, \ldots$

or $$2\mu_2 t - \frac{\lambda}{2} = n\lambda \text{ or } 2\mu_2 t = \left(n+\frac{1}{2}\right)\lambda$$

or $$t = \frac{\left(n+\frac{1}{2}\right)\lambda}{2\mu_2}$$

So the correct choice is (c).

11. The minimum value of t corresponds to $n = 0$. Hence

$$t_{\min} = \frac{\lambda}{4\mu_2} = \frac{648 \text{ nm}}{4 \times 1.8} = 90 \text{ nm}.$$

So the correct choice is (a)

12. For destructive interference $\Delta = \left(n-\frac{1}{2}\right)\lambda$. Hence

$$2\mu_2 t - \frac{\lambda}{2} = \left(n-\frac{1}{2}\right)\lambda$$

which gives $t = \dfrac{n\lambda}{2\mu_2}$, which is choice (a).

13. The minimum value of t corresponds to $n = 1$. Hence

$$t_{\min} = \frac{\lambda}{2\mu_2} = \frac{648\text{nm}}{2\times 1.8} = 180 \text{ nm}$$

Thus the correct choice is (b).

Questions 14 to 16 are based on the following passage

Passage V

Figure 27.18 shows a surface XY separating two transparent media, medium-1 and medium-2. The lines ab and cd represent wavefronts of a light wave traveling in medium-1 and incident on XY. The lines of ef and gh represent wavefronts of the light wave in medium-2 after refraction.

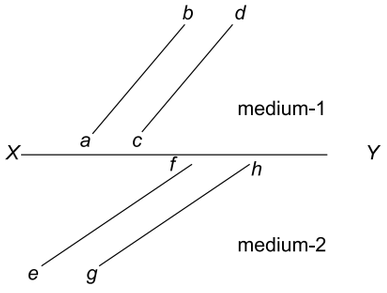


Fig. 27.18

IIT, 2007

14. Light travels as a
- (a) parallel beam in each medium
- (b) convergent beam in each medium
- (c) divergent beam in each medium
- (d) divergent beam in one medium and convergent beam in the other medium

15. The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$.
- (a) ϕ_c cannot be equal to ϕ_d
- (b) ϕ_d can be equal to ϕ_e
- (c) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$
- (d) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

16. Speed of light is
- (a) the same in medium-1 and medium-2
- (b) larger in medium-1 than in medium-2
- (c) larger in medium-2 than in medium-1
- (d) different at b and d

SOLUTION

14. The rays of light are perpendicular to the wavefront. Since the wavefronts in both the media are plane and parallel, the corresponding beam of light in each medium will be parallel. Therefore, the correct choice is (a).

15. All points on a wavefronts are in the same phase of oscillation. Therefore $\phi_c = \phi_d$ and $\phi_f = \phi_e$

$$\therefore \qquad (\phi_d - \phi_f) = (\phi_c - \phi_e)$$

Hence the correct choice is (c).

16. The direction of the rays is perpendicular to the wavefront. It is clear from the diagram that the refracted rays bend towards the normal ($\because$ angle r is less than angle i) as the beam travels from medium-1 into medium-2. Therefore medium-2 is denser than medium-1. Hence the speed of light is smaller in medium-2 than in medium-1. Thus the correct choice is (b).

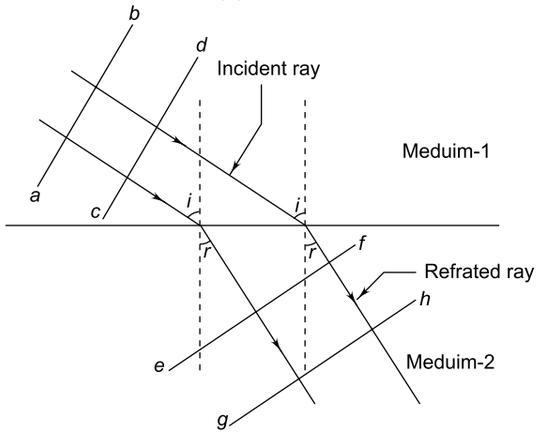


Fig. 27.19

SECTION IV

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. **Statement-1**
   Red light travels faster in glass than green light.
   Statement-2
   The refractive index of glass is less for red light than for green light.
2. **Statement-1**
   In Young's double slit experiment, if the width of the source slit is increased, the fringe pattern becomes indistinct.
   Statement-2
   The angular width of interference maxima increases if the width of the source slit is increased.
3. **Statement-1**
   In a single slit diffraction experiment, if the width of the slit is increased, the diffraction maxima become sharper and brighter.
   Statement-2
   The angular width the diffraction maxima is inversely proportional to the width of the slit.
4. **Statement-1**
   When light travels from a rarer to a denser medium, its speed decreases.
   Statement-2
   Energy carried by the refracted light is reduced.
5. **Statement-1**
   When a light wave travels from one medium to another, its frequency remains unchanged.
   Statement-2
   The speed of the wave undergoes a change.
6. **Statement-1**
   When a light wave is reflected from a mirror, it undergoes a phase change of π.
   Statement-2
   The direction of the propagation of light is changed due to reflection.

SOLUTIONS

1. The correct choice is (a). Refractive index of a medium is defined as

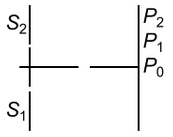
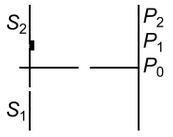
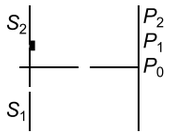
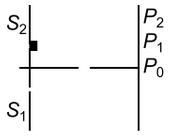
$$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in the medium}}$$

   The refractive index of glass is less for light of longer wavelength. The wavelength of red light is more than that of green light. Hence $\mu_R < \mu_G$ which implies that the speed of red light is more than that of green light in glass.
2. The correct choice is (c). If the source slit is wide, the interference pattern becomes indistinct because the interference patterns due to various parts of the source slit overlap. Consequently, the minima will not be totally dark and the fringe pattern becomes indistinct.
3. The correct choice is (a).
4. The correct choice is (b). The energy of a wave is determined by the square of its amplitude; it does not depend on the speed of the wave.
5. The correct choice is (c). The frequency of a wave does not depend on its speed or wavelength; it depends on the frequency of the source which produces that wave.
6. The correct choice is (c). The phase change is due to the reversal of amplitude of the wave on reflection from the mirror.

SECTION V

Matrix Match Type

1. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases b, c and d, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in **Column I** with the statement(s) in **Column II** valid for that situation.

Column I		***Column II***
(a)		(p) $\delta(P_0) = 0$
(b) $(\mu - 1)\, t = \lambda/4$		(q) $\delta(P_1) = 0$
(c) $(\mu - 1)\, t = \lambda/2$		(r) $I(P_1) = 0$
(d) $(\mu - 1)\, t - 3\lambda/4$		(s) $I(P_0) > I(P_1)$
		(t) $I(P_2) > I(P_1)$

IIT, 2009

ANSWERS

(a) $\rightarrow$ (p), (s) (b) $\rightarrow$ (q)
(c) $\rightarrow$ (t) (d) $\rightarrow$ (r), (s), (t)

Explanation:

$I(P) = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$ where I_0 is the intensity of each interfering beam and ϕ is the phase difference given by

$$\phi = \frac{2\pi}{\lambda}\delta(P)$$

(a) At P_0, $\delta(P_0) = 0$, therefore $\phi(P_0) = 0$ and $I(P_0) = 4I_0$.

At P_1, $\delta(P_1) = \frac{\lambda}{4} \Rightarrow \phi = \frac{\pi}{2}$. Therefore

$$I(P_1) = 4I_0 \cos^2\left(\frac{\pi}{4}\right) = 2I_0$$

Hence $I(P_0) > I(P_1)$.

At P_2, $\delta(P_2) = \frac{\lambda}{3} \Rightarrow \phi = \frac{2\pi}{3}$. Therefore

$$I(P_2) = 4I_0 \cos^2\left(\frac{\pi}{3}\right) = I_0$$

Hence $I(P_2) < I(P_1)$

So the correct choices are (p) and (s).

(b) At P_0, $\delta(P_0) = \frac{\lambda}{4} \Rightarrow \phi = \frac{\pi}{2}$. Therefore, $I(P_0) = 2I_0$

At P_1, $\delta(P_1) = 0$. Therefore, $I(P_1) = 4I_0$

At P_2, $\delta(P_2) = \frac{\lambda}{3} - \frac{\lambda}{4} = \frac{\lambda}{12} \Rightarrow \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{12} = \frac{\pi}{6}$

Therefore $I(P_2) = 4\, I_0 \cos^2\left(\frac{\pi}{12}\right) \simeq 3.7\, I_0$

So $I(P_1) > I(P_2)$. So the only correct choice is (q).

(c) At P_0, $\delta(P_0) = \frac{\lambda}{4}$. Therefore $I(P_0) = 0$

At P_1, $\delta(P_1) = \frac{\lambda}{2} - \frac{\lambda}{4} = \frac{\lambda}{4}$. Therefore $I(P_1) = 2\, I_0$

At P_2, $\delta(P_2) = \frac{\lambda}{2} - \frac{\lambda}{3} = \frac{\lambda}{6} \Rightarrow \phi = \frac{\pi}{3}$.

Therefore $I(P_2) = 4I_0 \cos^2\left(\frac{\pi}{6}\right) = 3I_0$

So the only correct choice is (t).

(d) At P_0, $\delta(P_0) = \frac{3\lambda}{4} \Rightarrow I(P_0) = 2\, I_0$

At P_1, $\delta(P_1) = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2} \Rightarrow I(P_1) = 0$

At P_2, $\delta(P_2) = \frac{3\lambda}{4} - \frac{\lambda}{3} = \frac{5\lambda}{12}$

$\Rightarrow \phi = \frac{2\pi}{\lambda} \times \frac{5\lambda}{12} = \frac{5\pi}{6}$

Therefore, $I(P_2) = 4I_0 \cos^2\left(\frac{5\pi}{12}\right) = 0.27\, I_0 \neq 0$

So the correct choices are (r), (s) and (t).

SECTION VI

Integer Answer Type

1. A narrow slit is placed in front of a convex lens of focal length 25 cm and is illuminated normally with a parallel beam of light of wavelength 600 nm. The first diffraction minima on either side of the central maximum are separated by 6×10^{-3} m. Find the width of the slit (in mm).

IIT, 1997

2. A coherent parallel beam of microwaves of wavelength 0.5 mm falls normally on Young's double slit apparatus. The separation between the slits is 1.0 mm and the screen is placed at a distance of 1.0 m from the slits. Find the number of minima in the interference pattern observed on the screen.

IIT, 1998

SOLUTION

1. For a slit of width a, the angular separation between the nth minimum and the central maximum is given by

$$\sin\,\theta_n = \frac{n\lambda}{a}$$

For first minimum, $n = 1$. Hence $\sin\theta_1 = \frac{\lambda}{a}$. Since $\lambda << a$, $\sin\theta_1 = \theta_1$ (in radian) and $\theta_1 = \frac{\lambda}{a}$. Therefore, the angular separation between the first minima on either side of the central maximum is

$$2\theta_1 = \frac{2\lambda}{a}$$

The linear separation $\Delta y = \frac{2f\lambda}{a}$. Hence

$$a = \frac{2f\lambda}{\Delta y} = \frac{2 \times 0.25 \times 600 \times 10^9}{6 \times 10^{-3}}$$

$$= 5 \times 10^{-3}\ \text{m} = 5\ \text{mm}$$

2. When the incident beam falls normally on the slits S_1 and S_2, the path difference at the central point P_0 of the screen is zero. Hence we have the central maximum at P_0. (Fig. 27.20)

Let the minima appear along directions θ with respect to the incident direction. Coherent waves from S_1 and S_2 along this direction are brought

to a focus at P. It is clear that the path difference between the waves from S_1 and S_2 on reaching P is

$$\Delta = d \sin \theta$$

The interference minima will appear on the screen if

$$\Delta = \left(m + \frac{1}{2}\right)\lambda$$

or $$d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$$

where m is an integer. Thus the directions of minima are given by

$$\sin \theta = \left(m + \frac{1}{2}\right) \times \left(\frac{\lambda}{d}\right)$$

Given $d = 1.0$ mm and $\lambda = 0.5$ mm. Therefore

$$\sin \theta = \left(m + \frac{1}{2}\right) \times \left(\frac{0.5}{1.0}\right) = \frac{1}{2}\left(m + \frac{1}{2}\right)$$

The allowed values of m are those integers for which $\sin \theta$ is not more than + 1 or less than – 1. These values are $m = 1, 0, -1$ and -2. Hence four minima will be observed.

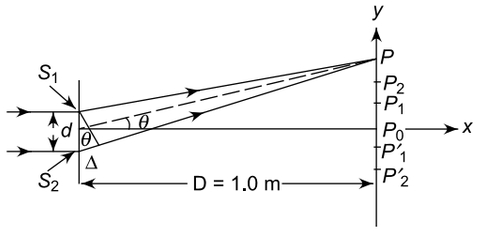


Fig. 27.20

28 Chapter

Atomic Physics

REVIEW OF BASIC CONCEPTS

28.1 PHOTOELECTRIC EFFECT

When electromagnetic radiation of appropriate frequency falls on a metal, electrons are emitted. This phenomenon is called photoelectric effect and the emitted electrons are called photoelectrons because they are liberated by means of light.

Einstein's Photoelectric Equation

The classical electromagnetic wave theory of light, which successfully explained interference, diffraction, and polarization of light, could not account for the observations related to photoelectric effect. In 1900 Planck postulated that light waves consist of tiny bundles of energy called *photons* or *quanta*. The energy of a light wave of frequency ν is given by $E = h\nu$, where h is Planck's constant. Photon is simply a light wave of energy E.

Following Planck's idea, Einstein proposed a theory for photoelectric effect. According to him, when a photon of light falls on a metal, it is absorbed, resulting in the emission of a photoelectron. The maximum kinetic energy $K_{max} = 1/2\ mv^2_{max}$ of the emitted electron is given by

$$\frac{1}{2} mv^2_{max} = h\nu - W_0 \qquad (1)$$

This is the famous Einstein's photoelectric equation. The term $h\nu$ represents the total energy of the photon incident on the metal surface. The photon penetrates a distance of about 10^{-8} m before it is completely absorbed. In disappearing, the photon imparts all its energy to a single electron. Part of this energy is used up by the electron in freeing itself from the atoms of the metal. This energy designated by W_0 in Eq. (1) is called the *work-function* of the metal and is a characteristic of it. The rest of the energy is used up in giving the electron kinetic energy.

The work function W_0, i.e. the energy required to pull an electron away from the surface of the metal, is large for heavier elements like platinum whereas for other elements like alkali metals, W_0 is quite small. The minimum, or threshold, energy which a photon must have to free the electron from the surface of the metal should be equal to its work function. If the threshold frequency is ν_0 the threshold energy is $h\nu_0$. Thus

$$W_0 = h\nu_0$$

Einstein's photoelectric equation therefore becomes

$$\frac{1}{2} mv^2_{max} = h\ (\nu - \nu_0) \qquad (2)$$

It is evident that when $\nu < \nu_0$, no electron is emitted for any intensity of light. When $\nu > \nu_0$, the energy of the electron increases linearly with the frequency ν of light. Since intensity of light is a measure of the number of photons and since each photon emits a photoelectron on absorption, the intensity of photoelectrons is proportional to the intensity of light.

Below a certain negative voltage V_0, no photoelectrons are emitted no matter what the intensity of light is. This voltage is called the cut-off or stopping potential. Since there is no photoelectric emission at potentials less than V_0, the maximum velocity v_{max} acquired by the photoelectrons is given by

$$K_{max} = \frac{1}{2} mv^2_{max} = eV_0$$

where K_{max} is the maximum kinetic energy. V_0 is given by

$$eV_0 = h\ (\nu - \nu_0)$$

or

$$V_0 = \frac{h}{e}\ (\nu - \nu_0) \qquad (3)$$

Laws and graphs of photoelectric effect

(1) For a given emitter illuminated by radiation of a given frequency, the photoelectric current is proportional to the intensity of radiation (Fig. 28.1)

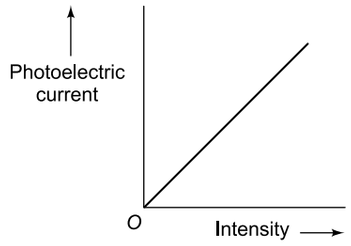


Fig. 28.1

(2) The maximum kinetic energy (K_{max}) of photoelectrons is proportional to the frequency (ν) of the incident radiation and is independent of intensity of the radiation (Fig. 28.2).

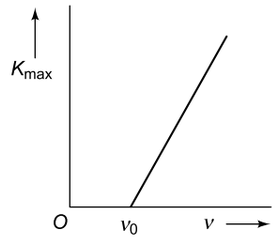


Fig. 28.2

$$K_{max} = h(\nu - \nu_0)$$

Slope of graph = h(Plank's constant).

$K_{max} = 0$ when $\nu \le \nu$ (threshold frequency)

(3) For every emitter there is definite threshold frequency (ν_0) below which no photoelectrons are emitter no matter what the intensity of radiation is.

(4) Graph of stopping potential (V_0) versus frequency ν of incident radiation (Fig. 28.3).

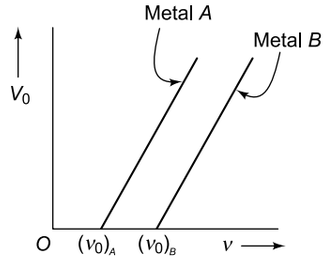


Fig. 28.3

$$V_0 = \frac{h}{e}(\nu - \nu_0)$$

Slope of graph = $\frac{h}{e}$, which is the same for all metals.

(5) Graph of photoelectric current (i) versus voltage (V) for radiations of different intensities ($I_1 > I_2$) but of the same frequency (Fig. 28.4).

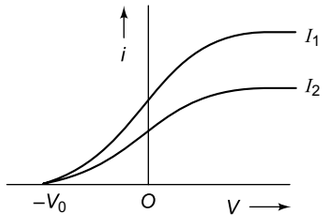


Fig. 28.4

(6) Graph of photoelectric current (i) versus voltage (V) for radiations of different frequencies ($\nu_1 > \nu_2$) but of the same intensity (Fig. 28.5)

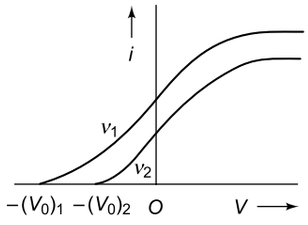


Fig. 28.5

(7) Threshold wavelength is $\lambda_0 = \frac{c}{\nu_0}$. For photoelectric emission $\lambda < \lambda_0$.

EXAMPLE 28.1

A metal of work function 3.3 eV is illuminated by light of wavelength 300 nm. Find (a) the threshold frequency of photoelectric emission, (b) the maximum kinetic energy of photoelectrons and (c) the stopping potential. Take $h = 6.6 \times 10^{-34}$ Js.

SOLUTION

(a) $W_0 = 3.3 \text{ eV} = 3.3 \times 1.6 \times 10^{-19}$ J

Threshold frequency is

$$\nu_0 = \frac{W_0}{h} = \frac{3.3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$= 8 \times 10^{14} \text{ Hz}$$

(b) Frequency of incident radiation is

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{300 \times 10^{-9}}$$

$$= 10 \times 10^{14} \text{ Hz}$$

$$\therefore \quad K_{max} = h(\nu - \nu_0)$$

$$= (6.6 \times 10^{-34}) \times (10 \times 10^{14} - 8 \times 10^{14})$$

$$= 1.32 \times 10^{-19} \text{ J}$$

(c) $K_{max} = eV_0 \quad \Rightarrow \quad V_0 = \dfrac{K_{max}}{e} = \dfrac{1.32 \times 10^{-19}}{1.6 \times 10^{-9}}$

$= 0.825$ V

EXAMPLE 28.2

Photoelectric emission from a metal begins at a frequency of 6×10^{14} Hz. The emitted electrons are fully stopped by a retarding potential of 3.3 V. Find the wavelength (in nm) of the incident radiation. Take $h = 6.6 \times 10^{-34}$ Js.

SOLUTION

$$eV_0 = h(\nu - \nu_0)$$

$\Rightarrow \quad (1.6 \times 10^{-19}) \times 3.3 = (6.6 \times 10^{-34}) \times (\nu - 6 \times 10^{14})$

$\Rightarrow \quad \nu = 1.4 \times 10^{15}$ Hz

$\therefore \lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^8}{1.4 \times 10^{15}} = 2.14 \times 10^{-7}$ m $= 214$ nm

EXAMPLE 28.3

Light of wavelength 300 nm is incident on two metals A and B whose work functions are respectively 4 eV and 2 eV. Which of the two metals will emit photoelectrons?

SOLUTION

Energy of incident radiation is

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{300 \times 10^{-9}}$$

$$= 6 \times 10^{-19} \text{ J}$$

$$= \frac{6 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.75 \text{ eV}$$

Photoelectrons will be emitted from the metal if E is greater than the work function of the metal. Hence metal B will emit photoelectrons but A will not.

EXAMPLE 28.4

For photoelectric affect in a metal, the graph of stopping potential V_0 (in volt) versus frequency ν (in Hz) of the incident radiation is shown in Fig. 28.6. From the graph find,

(a) threshold frequency

(b) Planck's constant and

(c) work function of the metal.

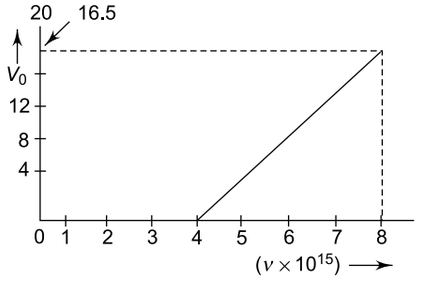


Fig. 28.6

SOLUTION

$$eV_0 = h(\nu - \nu_0)$$

$$\Rightarrow \quad V_0 = \frac{h}{e}(\nu - \nu_0) \qquad (1)$$

(a) It follows from Eq. (1) that $\nu = \nu_0$ if $V_0 = 0$. Hence $\nu_0 = 4 \times 10^{15}$ Hz

(b) The slope of V_0 versus ν graph $= \dfrac{h}{e}$

$$\text{Now slope} = \frac{16.5 - 0}{(8 - 4) \times 10^{15}} = 4.125 \times 10^{-15}$$

$$\therefore \quad h = e \times \text{slope}$$

$$= (1.6 \times 10^{-19}) \times (4.125 \times 10^{-15})$$

$$= 6.6 \times 10^{-34} \text{ Js}$$

(c) $W_0 = h\nu_0 = (6.6 \times 10^{-34}) \times (4 \times 10^{15})$

$$= 26.4 \times 10^{-19} \text{ J}$$

$$= 16.5 \text{ eV}$$

EXAMPLE 28.5

Calculate the number of photons emitted per second by a transmitter of power 10 kW sending radiowaves of frequency 6×10^5 Hz. Take $h = 6.63 \times 10^{-34}$ Js.

SOLUTION

Let N be the number of photons emitted in time t. Energy of 1 photon $= h\nu$. Therefore, energy of N photons $= Nh\nu$. Therefore, the power is

$$P = \frac{Nh\nu}{t} = nh\nu;$$

n = number of photons emitted per second.

or $$n = \frac{P}{h\nu} = \frac{10 \times 10^3}{(6.63 \times 10^{-34}) \times 6 \times 10^5}$$

$$= 2.5 \times 10^{31} \text{ photons per second}$$

EXAMPLE 28.6

The stopping potential of a metal is 3 V when it is illuminated by light of wavelength 500 nm. What will be the stopping potential of the metal when the wavelength is 600 nm? Also find the cut-off frequency and work function of the metal. Take $h = 6.6 \times 10^{-34}$ Js.

SOLUTION

$$eV_1 = h(\nu_1 - \nu_0) \quad (1)$$

$$eV_0 = h(\nu_2 - \nu_0) \quad (2)$$

Subtracting, we get

$$e(V_1 - V_2) = h(\nu_1 - \nu_2)$$

$$\Rightarrow \quad V_1 - V_2 = \frac{h}{e}(\nu_1 - \nu_2)$$

$$= \frac{h}{e}\left(\frac{c}{\lambda_1} - \frac{c}{\lambda_2}\right)$$

$$= \frac{hc}{e}\left(\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right)$$

$$= \frac{6.6\times10^{-3}\times3\times10^{8}}{1.6\times10^{-19}} \times \left(\frac{6\times10^{-7} - 5\times10^{-7}}{6\times5\times10^{-14}}\right)$$

$$= 0.4 \text{ V}$$

$$\therefore \quad V_2 = V_1 - 0.4 = 3 - 0.4 = 2.6 \text{ V}$$

From Eq. (1),

$$\nu_0 = \nu_1 - \frac{eV_1}{h}$$

$$= \frac{c}{\lambda_1} - \frac{eV_1}{h}$$

$$= \frac{3\times10^{8}}{6\times10^{-7}} - \frac{1.6\times10^{-19}\times3}{6.6\times10^{-34}}$$

$$= 4.27 \times 10^{14} \text{ Hz}$$

$$W_0 = h\nu_0 = 6.6 \times 10^{-34} \times 4.27 \times 10^{14}$$

$$= 2.82 \times 10^{-19} \text{ J} = 1.76 \text{ eV}$$

EXAMPLE 28.7

Ultraviolet light of wavelength 250 nm falls on the metal emitter of a photocell. If the stopping potential is 1.2 V, find the work functions of the metal. Will the photocell work if yellow light of wavelength 600 nm is used? Take $h = 6.6 \times 10^{-34}$ Js.

SOLUTION

$$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{250\times10^{-9}} = 1.2 \times 10^{15} \text{ Hz}$$

$$eV_0 = h(\nu - \nu_0) = h\nu - W_0$$

$$\therefore \quad W_0 = h\nu - eV_0$$

$$= 6.6\times10^{-34}\times1.2\times10^{15} - 1.6\times10^{-19}\times1.2$$

$$= 6 \times 10^{-19} \text{ J} = 3.75 \text{ eV}$$

Now $\nu_0 = \frac{W_0}{h} = \frac{6\times10^{-19}}{6.6\times10^{-34}} = 0.9 \times 10^{15}$ Hz

Frequency of yellow light is

$$\nu_y = \frac{c}{\lambda_y} = \frac{3\times10^{8}}{600\times10^{-9}} = 0.5 \times 10^{14} \text{ Hz}$$

Since $\nu_y < \nu_0$, the photocell will not work with yellow light as no photoelectrons will be emitted.

28.2 BOHR'S THEORY OF HYDROGEN LIKE ATOM

(a) Bohr's quantization condition: The magnitude of angular momentum of the electron in a circular orbit is

$$L = \frac{nh}{2\pi} \rightarrow mv_n r_n = \frac{nh}{2\pi}$$

where m = mass of electron, r_n = radius of nth circular orbit, v_n = orbital speed of electron in the nth orbit, h = Planck's constant and n is an integer called the principal quantum number.

(b) Speed of electron in nth orbit is

$$v_n = \left(\frac{e^2}{2\varepsilon_0 h}\right)\frac{Z}{n}$$

where Z = atomic number of atom. For hydrogen $Z = 1$. For a given atom $v_n \propto \frac{1}{n}$. Substituting the known values of e, ε_0 and h we get

$$v_n = (2.2 \times 10^{6} \text{ ms}^{-1}) \times \frac{Z}{n}$$

(c) Radius of nth orbit is

$$r_n = \left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right)\frac{n^2}{Z} = (0.53 \times 10^{-10} \text{ m})\ \frac{n^2}{Z}$$

$$= (0.53\text{Å})\ \frac{n^2}{Z}$$

For a given atom $r_n \propto n^2$.

(d) Total energy of electron in nth orbit

$$\text{K.E.} = \frac{Z e^2}{8\pi\varepsilon_0 r_n};\ \text{P.E.} = -\frac{Z e^2}{4\pi\varepsilon_0 r_n}$$

$\therefore$ P.E. = –2K.E.

Total energy of electron in nth object is

$$E_n = \text{K.E.} + \text{P.E.} = \text{K.E.} - 2\text{K.E.} = -\text{K.E.}$$

$$\Rightarrow \quad E_n = -\frac{Z e^2}{8\pi\varepsilon_0 r_n}$$

Putting the value of r_n, we get

$$E_n = \left(-\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{Z^2}{n^2}$$

$$= (-21.76 \times 10^{-19}\ \text{J})\ \frac{Z^2}{n^2}$$

$$= (-13.6\text{eV})\ \frac{Z^2}{n^2}$$

(e) Time period of revolution of electron in nth orbit is

$$T_n = \frac{2\pi r_n}{v_n} = \left(\frac{4\varepsilon_0^2 h^3}{me^4}\right)\frac{n^3}{Z^2} = (1.51\times10^{-16}\,\text{s})\ \frac{n^3}{Z^2}$$

(f) Frequency of revolution of the electron in nth orbit is

$$\nu_n = \frac{1}{T_n} = (6.6 \times 10^{15}\ \text{Hz})\ \frac{Z^2}{n^3}$$

(g) Wavelength of emitted radiation: When an electron jumps from a higher energy state $n = n_2$ to a lower state $n = n_1$, a photon of energy $h\nu$ of radiation is emitted.

$$h\nu = E_{n_2} - E_{n_1}$$

$$\Rightarrow \quad \frac{hc}{\lambda} = \left(-\frac{me^4}{8\varepsilon_0^2 h^2}\right) Z^2 \left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda} = \left(\frac{me^4}{8\varepsilon_0^2 h^3 c}\right) Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where $R_H = \dfrac{me^4}{8\varepsilon_0^2 h^3 c} = 1.097 \times 10^7\ \text{m}^{-1}$ is called Rydberg constant.

(h) *Main Series of Hydrogen Spectrum* ($Z = 1$)

(1) *Lyman series:* $n_1 = 1$, $n_2 = 2, 3, 4, \ldots\infty$

$$\frac{1}{\lambda} = R_H \left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

$\lambda_{min} = 91.2$ nm, $\lambda_{max} = 121.6$ nm

Spectral lines in Lyman series lie in the ultraviolet region.

(2) *Balmer series:* $n_1 = 2$, $n_2 = 3, 4, 5, \ldots\infty$

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$$

$\lambda_{min} = 364.5$ nm, $\lambda_{max} = 656.1$ nm

Spectral lines in Balmer series lie in the visible region.

(3) *Paschen series:* $n_1 = 3$, $n_2 = 4, 5, 6, \ldots\infty$

$$\frac{1}{\lambda} = R_H \left(\frac{1}{3^2} - \frac{1}{n_2^2}\right)$$

$\lambda_{min} = 820.1$ nm, $\lambda_{max} = 1874.6$ nm

Spectral lines in Paschen series lie in the infrared region.

(4) *Brackett series:* $n_1 = 4$, $n_2 = 5, 6, 7, \ldots\infty$

$$\frac{1}{\lambda} = R_H \left(\frac{1}{4^2} - \frac{1}{n_2^2}\right)$$

$\lambda_{min} = 1458$ nm, $\lambda_{max} = 4050$ nm

These spectral lines also lie in the infrared region.

(i) The energy of electron in hydrogen atom in the ground state is –13.6 eV.

(j) The ionization potential of hydrogen atom in the ground state is 13.6 V. Ionization potential of a hydrogen like atom in nth state = $\dfrac{13.6Z^2}{n^2}$ volt.

EXAMPLE 28.8

The energy required to excite a hydrogen atom from $n = 1$ to $n = 2$ energy state 10.2 eV. What is the wavelength of the radiation emitted by the atom when it goes back to its ground state?

SOLUTION

Given $E_2 - E_1 = 10.2\ \text{eV} = 10.2 \times 1.6 \times 10^{-19}$ J.

Therefore, frequency the emitted radiation is

$$\nu = \frac{E_2 - E_1}{h}$$

and wavelength is

$$\lambda = \frac{c}{\nu} = \frac{ch}{E_2 - E_1}$$

$$= \frac{3\times10^8 \times 6.6\times10^{-34}}{10.2\times1.6\times10^{-19}}$$

$$= 1.22 \times 10^{-7}\ \text{m}$$

$$= 1220\ \text{Å} = 122\ \text{nm}$$

EXAMPLE 28.9

The ionization potential of the hydrogen atom is 13.6 V. Find the energy of the atom in $n = 2$ energy state.

SOLUTION

Energy of hydrogen atom in the ground state is ($\because$ ionization potential is 13.6 V)

$$E_1 = -13.6 \text{ eV}$$

Since $E_n \propto \frac{1}{n^2}$, the energy in the $n = 2$ state is

$$E_2 = \frac{E_1}{(2)^2} = \frac{-13.6}{4} = -3.4 \text{ eV}$$

EXAMPLE 28.10

The innermost orbit of hydrogen atom has a diameter of 1.06 Å. What is the diameter of the 10th orbit?

SOLUTION

Given $d_1 = 1.06$ Å. We know that $d_n \propto n^2 d_1$. Hence

$$d_{10} = (10)^2 \times 1.06 \text{ Å}$$
$$= 106 \text{ Å}$$

EXAMPLE 28.11

Find the ratio of longest and shortest wavelength in the Lyman series of hydrogen atom.

SOLUTION

For Lyman series

$$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

where $n = 2, 3, 4, \ldots \infty$. The shortest wavelength (λ_s) corresponds to $n = \infty$ and the longest wavelength (λ_l) corresponds to $n = 2$.

$$\frac{1}{\lambda_s} = R_H\left(1 - \frac{1}{\infty}\right) = R_H$$

$$\Rightarrow \quad \lambda_s = \frac{1}{R_H}$$

and

$$\frac{1}{\lambda_l} = R_H\left(1 - \frac{1}{4}\right) = \frac{3R_H}{4}$$

$$\Rightarrow \quad \lambda_l = \frac{4}{3R_H}$$

$$\therefore \quad \frac{\lambda_l}{\lambda_s} = \frac{4}{3}$$

EXAMPLE 28.12

The wavelength of the second line of Balmer series is 486.4 nm. What is the wavelength of the first line of Lyman series?

SOLUTION

Wavelengths in Balmer series for hydrogen are given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$
$$= R_H\left(\frac{1}{4} - \frac{1}{n^2}\right); \; n = 3, 4, 5 \ldots$$

The second line in Balmer series corresponds to $n = 4$. Hence

$$\frac{1}{\lambda_2} = R_H\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R_H}{16} \text{ or } \lambda_2 = \frac{16}{3R_H}$$

The wavelength of the first line ($n = 2$) in Lyman series is

$$\frac{1}{\lambda_1} = R_H\left(1 - \frac{1}{2^2}\right) = R_H\left(1 - \frac{1}{4}\right)$$
$$= \frac{3R_H}{4}$$

or $$\lambda_1 = \frac{4}{3R_H}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{4}{3R_H} \times \frac{3R_H}{16} = \frac{1}{4}$$

or $$\lambda_1 = \frac{\lambda_2}{4} = \frac{486.4}{4}$$
$$= 121.6 \text{ nm}$$

28.3 X-RAYS

X-rays are produced when energetic electrons fall on a suitable target. The apparatus used for the production of X-rays is called the Coolidge tube in which electrons are produced by thermionic emission. X-rays are electromagnetic waves of wavelength of the order of 1Å or 0.001 nm.

1. Duane-Hunt Law

The shortest X-ray wavelength emitted when electrons incident on the target are accelerated through a potential V volts is given by

$$\lambda_{\min} = \frac{hc}{eV} = \frac{1239.6}{V} \text{ nm}$$

This is called the Duane-Hunt Law.

2. Diffraction of X-rays

A crystal is a natural grating for diffraction of X-rays, since the spacing d between the crystal planes is of the order of the wavelength of X-rays. The angle θ of diffraction of X-rays is given by the relation

$$2d \sin \theta = n\lambda$$

where n is an integer having values 1, 2, 3, … etc. and λ is the wavelength of X-rays incident on the crystal. This relation is called Bragg's equation.

3. Absorption of X-rays

X-rays are absorbed by materials following the exponential relation

$$I = I_0 e^{-\mu x}$$

where I_0 is the initial intensity of X-rays, I their intensity after they have traversed a thickness x and μ is the absorption coefficient.

4. X-ray Spectra

X-ray spectra may be classified into two types—continuous spectrum and characteristic spectrum. X-ray spectrum consists of a series of discrete spectral lines superimposed on a continuous luminous background. The background spectrum consists of all sorts of wavelengths and is called the continuous spectrum which is the same for all target materials. The discrete line spectrum is a characteristic of the target metal and is, therefore, called the characteristic spectrum. The spectral lines in this spectrum are due to the transitions of electrons from the outer orbits to the inner K, L, M shells, etc. The emitted radiations are called K_α, K_β, K_γ, … radiations. These transitions are shown in Fig. 28.7.

The frequency (ν) of the characteristic X-rays is related to the atomic number (Z) of the target metal by the relation

$$\sqrt{\nu} = a\,(Z - b)$$

where a and b are constants. This relation is known as Moseley's law.

Some facts about X-rays can be summerized as follows:

1. X-rays are electromagnetic waves of wavelength of the order of 1 Å or 0·001 nm.
2. X-rays are not deflected by electric and magnetic fields.
3. X-rays travel in vacuum at the speed of light, $c = 3 \times 10^8$ ms^{-1}.
4. If electrons are accelerated through a potential difference V and then are made to fall on a target, X-rays of wavelength greater than hc/eV are produced. The shortest wavelength emitted is $\lambda_{min} = hc/eV$.
5. The characteristic X-rays consist of K and L series. The K_α X-rays are produced when electrons jump from $n = 2$ to $n = 1$ orbit. They are called L_α when electrons jump from $n = 3$ to $n = 2$ orbit. The wavelength of the K_α line of the X-ray spectrum is given by

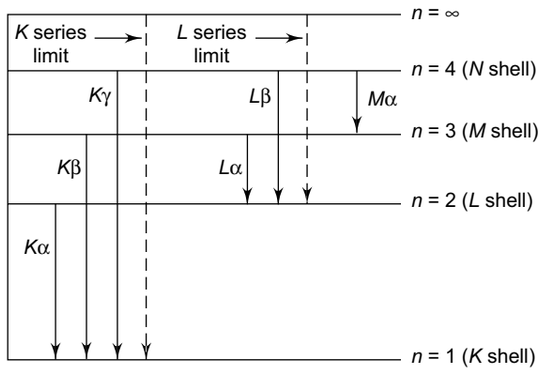


Fig. 28.7

$$\frac{1}{\lambda} = R_H(Z - 1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

where R is the Rydberg constant and Z is the atomic number. The wavelength of L_α line is

$$\frac{1}{\lambda} = R_H(Z - 7{\cdot}4)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

6. The frequency of the characteristics X-rays is proportional to $(Z - b)^2$ where b is a constant.
7. X-rays are absorbed by materials according to the relation

$$I = I_0 e^{-\mu x}$$

where I_0 = incident intensity, μ = constant for the given material and x = distance penetrated.

EXAMPLE 28.13

What is the maximum frequency of X-rays emitted from an X-ray tube operating at 50 kV?

SOLUTION

$eV = h\nu_{max}$. Hence

$$\nu_{max} = \frac{eV}{h} = \frac{(1.6 \times 10^{-19}) \times (50 \times 10^3)}{(6.63 \times 10^{-34})}$$

$$= 1.2 \times 10^{19} \text{ Hz}$$

EXAMPLE 28.14

An X-ray tube produces a continuous spectrum of radiation with its shortest wavelength end at 0.66 Å. What is the maximum energy of a photon of this radiation? Take $h = 6.6 \times 10^{-34}$ Js

SOLUTION

$$E_{max} = h\nu_{max} = \frac{hc}{\lambda_{min}} \qquad (\because c = \nu\lambda)$$
$$= \frac{(6.6\times10^{-34})\times(3\times10^{8})}{(0.66\times10^{-10})}$$
$$= 3 \times 10^{-15} \text{ J}$$

EXAMPLE 28.15

The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 mA. Find the number of electrons striking the target per second.

SOLUTION

$$n = \frac{I}{e} = \frac{3.2\times10^{-3}}{1.6\times10^{-19}} = 2\times10^{16} \text{ electrons per second}$$

28.4 WAVE NATURE OF MATTER

In 1924, Louis de Broglie, a French theoretical physicist, derived an equation which predicted that all atomic particles have associated with them waves of a definite wavelength. Under certain circumstances, a beam of electrons or atoms will behave like a group of waves. On the basis of theoretical considerations, de Broglie predicted that the wavelength λ of these waves is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where h is Planck's constant and p is the momentum of the particles. This equation is known as *de Broglie's wave equation.* For an electron moving at a high speed, the momentum is large and the wavelength λ is small. The faster the electron, the shorter is the wavelength. Notice that the particle need not have a charge to have an associated wave. This is why de Broglie waves are sometimes referred to as matter waves.

1. If the rest mass of a particle is m_0, its *de Broglie* wavelength is given by
$$\lambda = \frac{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}{m_0 v}$$
2. In terms of kinetic energy K, *de Broglie* wavelength is given by
$$\lambda = \frac{h}{\sqrt{2mK}}$$
3. If a particle of charge q is accelerated through a potential difference V, its de Broglie wavelength is given by
$$\lambda = \frac{h}{\sqrt{2mqV}}$$
For an electron,
$$\lambda = \left(\frac{150}{V}\right)^{1/2} \text{ Å}$$
4. For a gas molecule of mass m at temperature T kelvin, the *de Broglie* wavelength is given by
$$\lambda = \frac{h}{\sqrt{3mkT}}$$
where k is the Boltzmann constant.

EXAMPLE 28.16

Calculate the wavelength associated with a dust particle of mass 1 μg moving with a velocity of 10^6 ms^{-1}. Given $h = 6.6 \times 10^{-34}$ Js.

SOLUTION

$$m = 1 \text{ μg} = 10^{-6} \text{ g} = 10^{-9} \text{ kg}$$
$$\lambda = \frac{h}{mv} = \frac{6.6\times10^{-34}}{10^{-9}\times10^{6}}$$
$$= 6.6 \times 10^{-31} \text{ m}$$

EXAMPLE 28.17

Calculate de Broglie wavelength of an electron having kinetic energy of 1 BeV. Given mass of electron (m) = 9.1×10^{-31} kg, $h = 6.6 \times 10^{-34}$ Js and $e = 1.6 \times 10^{-19}$ C.

SOLUTION

$K = 1 \text{ BeV} = 10^9 \text{ eV} = 10^9 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-10}$ J

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-10}}}$$
$$= 3.87 \times 10^{-14} \text{ m}$$

EXAMPLE 28.18

Calculate de Broglie wavelength associated with an electron accelerated through a potential difference of 200 V. Given $m = 9.1 \times 10^{-31}$ kg and $h = 6.6 \times 10^{-34}$ Js.

SOLUTION

$K = 200 \text{ eV} = 200 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-17}$ J

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times3.2\times10^{-17}}}$$
$$= 0.86 \times 10^{-10} \text{ m} = 0.86 \text{ Å}$$

EXAMPLE 28.19

Calculate the wavelength of de Broglie waves associated with a neutron at room temperature of 27°C. Given mass of neutron (m) = 1.67×10^{-27} kg, Boltzman constant (k) = 1.38×10^{-23} JK^{-1} and $h = 6.63 \times 10^{-34}$ Js.

SOLUTION

Kinetic energy of neutron due to thermal speed is $K = \frac{3}{2}kT$, where $T = 273 + 27 = 300$ K.

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2m \times \frac{3}{2}kT}} = \frac{h}{\sqrt{3mkT}}$$

Substituting the values of h, m, k and T, we get $\lambda = 1.45 \times 10^{-10}$ m = 1.45 Å

EXAMPLE 28.20

A photon of wavelength 19.8 nm collides with an electron at rest. After the collision, the wavelength of the photon is found to be 30 nm. Is the collision elastic or inelastic? Calculate the energy of the scattered electron. Given $h = 6.6 \times 10^{-34}$ Js.

SOLUTION

Energy of photon before collision is

$$E_i = h\nu_1 = \frac{hc}{\lambda_1} = \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{19.8 \times 10^{-9}}$$

$$= 10 \times 10^{-18} \text{ J}$$

Energy of photon after collision is

$$E_f = \frac{hc}{\lambda_2} = \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{30 \times 10^{-9}}$$

$$= 6.6 \times 10^{-18} \text{ J}$$

Since there is a loss of kinetic energy ($\because E_f < E_i$), the collision is inelastic. The energy of the scattered electron = $E_i - E_f = 3.4 \times 10^{-18}$ J.

EXAMPLE 28.21

Ultraviolet light of wavelength 99 nm falls on a metal plate of work function 1.0 eV. Find the wavelength of the fastest photoelectron emitted. Mass of electron (m) = 9.1×10^{-31} kg and $h = 6.6 \times 10^{-34}$ Js.

SOLUTION

$$K_{max} = h\nu - W_0 = \frac{hc}{\lambda} - W_0$$

$$= \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{99 \times 10^{-9}} - 1.0 \times 1.6 \times 10^{-19}$$

$$= 2 \times 10^{-18} - 1.6 \times 10^{-19}$$

$$= 1.84 \times 10^{-18} \text{ J}$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mK_{max}}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times (9.1 \times 10^{-31}) \times 1.84 \times 10^{-18}}}$$

$$= 3.6 \times 10^{-10} \text{ m} = 0.36 \text{ nm}$$

EXAMPLE 28.22

A proton and an electron have equal kinetic energy. Which of the two has a greater de Broglie wavelength?

SOLUTION

$$\lambda_p = \frac{h}{\sqrt{2m_pK}}, \quad \lambda_e = \frac{h}{\sqrt{2m_eK}}$$

$$\therefore \quad \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} > 1 \qquad (\because m_p > m_e)$$

Hence the electron has the greater wavelength

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. Figure 28.8 shows the variation of photoelectric current (i) with voltage (V) between the electrodes in a photo-cell for two different radiations.

If I_a and I_b are the intensities of the incident radiations and ν_a and ν_b are the respective frequencies, then

(a) $I_a > I_b,\ \nu_b < \nu_a$ (b) $I_a < I_b,\ \nu_b > \nu_a$

(c) $I_a > I_b,\ \nu_a = \nu_b$ (d) $I_a < I_b,\ \nu_b < \nu_a$

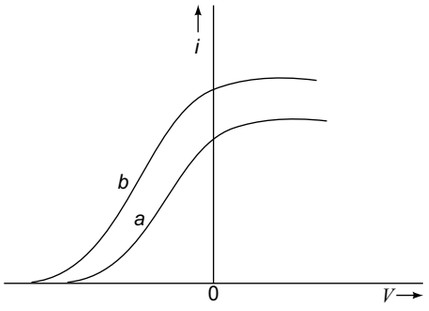


Fig. 28.8

2. When a radiation of energy 5 eV falls on a surface, the emitted photoelectrons have a maximum kinetic energy of 3 eV. The stopping potential is

(a) 2 V (b) 3 V

(c) 5 V (d) 8 V

3. When a radiation of wavelength λ_1 falls on a surface, the maximum kinetic energy of the emitted photoelectrons is K_1. For a radiation of wavelength λ_2, the maximum kinetic energy is K_2. If $\lambda_1 = \lambda_2/2$, then

(a) $K_1 = 2K_2$ (b) $K_2 = 2K_1$

(c) $K_1 > 2K_2$ (d) $K_1 < 2K_2$

4. Figure 28.9 represents the observed intensity (I) of X-rays emitted by an X-ray tube as a function of wavelength (λ).

(a) Peaks A and B represent K_α lines

(b) Peaks A and B represents K_β lines

(c) Peak A represents K_α line and peak B represents K_β line

(d) Peak A represents K_β line and peak B represents K_α line

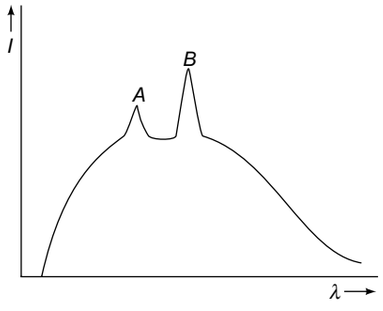


Fig. 28.9

5. A particle at rest disintegrates into two fragments of masses in the ratio of 1 : 2 having non-zero velocities. The ratio of the de Broglie wavelength of the lighter particle to that of the heavier particle is

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) $\frac{1}{4}$

6. A potential difference V is applied across an X-ray tube. If e/m is the charge to mass ratio of an electron and c the speed of light in vacuum, the ratio of the de Broglie wavelength of the incident electrons to the shortest wavelength of X-rays produced is given by

(a) $\frac{1}{c}\sqrt{\frac{eV}{2m}}$ (b) $\frac{1}{c}\sqrt{\frac{eV}{m}}$

(c) $\frac{1}{c}\sqrt{\frac{2eV}{m}}$ (d) $\frac{2}{c}\sqrt{\frac{eV}{m}}$

7. The de Broglie wavelength of the electron in the nth energy state of a hydrogen atom is proportional to

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$

(c) n (d) n^2

8. The magnitude of the magnetic moment of the electron in the nth energy state of a hydrogen atom is proportional to

(a) $\frac{1}{n}$ (b) $\sqrt{n}$

(c) n (d) n^2

9. The shortest wavelength in the Brackett series of a hydrogen like atom of atomic number Z is equal to the shortest wavelength in the Balmer series of hydrogen atom. The value of Z is

(a) 2 (b) 3

(c) 4 (d) 5

10. The difference between $(n + 1)$th Bohr radius and nth Bohr radius is equal to the $(n - 1)$th Bohr radius. The value of n is

(a) 1 (b) 2

(c) 3 (d) 4

11. The magnetic field at the nucleus of a hydrogen atom due to the motion of an electron in the nth orbit is inversely proportional to

(a) n^2 (b) n^3

(c) n^4 (d) n^5

12. The magnitude of the angular momentum of an electron revolving in a circular orbit of radius r in a hydrogen atom is proportional to

(a) $r^{1/2}$ (b) r

(c) $r^{3/2}$ (d) r^2

13. The wavelength in air associated with a photon of energy E is (c is the speed of light in air and h is the Planck's constant)

(a) $\frac{hc}{E}$ (b) $\frac{h}{cE}$

(c) $\frac{c}{hE}$ (d) hcE

14. An image of the sun is formed by a lens of focal length 30 cm on the metal surface of a photoelectric cell and it produces a current I. The lens forming the image is then replaced by another lens of the same diameter but of focal length 15 cm. The photoelectric current in this case will be

(a) $I/2$ (b) $2\ I$

(c) I (d) $4\ I$

15. The momentum of a photon of wavelength λ is

(a) $\frac{h}{c\lambda}$ (b) $\frac{hc}{\lambda}$

(c) $h\lambda$ (d) $\frac{h}{\lambda}$

16. Violet light can cause photoelectric emission from a metal but blue light cannot. If sodium light is incident on the metal, then

(a) the photoelectric current decreases
(b) the number of photoelectrons ejected per second increases
(c) the velocity of photoelectrons increases
(d) no photoelectric emission occurs.

17. 10^{20} photons of wavelength 660 nm are emitted per second from a lamp. What is the wattage of the lamp. Planck's constant = $6{\cdot}6 \times 10^{-34}$ Js.

(a) 30 W (b) 60 W

(c) 100 W (d) 500 W

18. When a certain photosensitive surface is illuminated with monochromatic light of frequency ν, the stopping potential for photoelectric current is $V_0/2$. When the same surface is illuminated by monochromatic light of frequency $\nu/2$, the stopping potential is V_0. The threshold frequency for photoelectric emission is

(a) $\frac{2\nu}{3}$ (b) $\frac{3\nu}{2}$

(c) $\frac{3\nu}{5}$ (d) $\frac{5\nu}{3}$

19. When a certain photosensitive surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current is $2V_0$. When the same surface is illuminated with monochromatic light of wavelength 2λ, the stopping potential is $V_0/2$. The threshold wavelength for photoelectric emission is

(a) 1.5 λ (b) 2.0 λ

(c) 2.5 λ (d) 3 λ

20. The threshold frequency for a certain photosensitive metal is ν_0. When it is illuminated by light of frequency $\nu = 2\nu_0$, the stopping potential for photoelectric current is V_0. What will be the stopping potential when the same metal is illuminated by light of frequency $\nu = 3\nu_0$?

(a) $1{\cdot}5\ V_0$ (b) $2\ V_0$

(c) $2{\cdot}5\ V_0$ (d) $3\ V_0$

21. The threshold frequency for a certain photosensitive metal is ν_0. When it is illuminated by light of frequency $\nu = 2\nu_0$, the maximum velocity of photoelectrons is v_0. What will be the maximum velocity of the photoelectrons when the same metal is illuminated by light of frequency $\nu = 5\nu_0$?

(a) $\sqrt{2}\ v_0$ (b) $2\ v_0$

(c) $2\sqrt{2}\ v_0$ (d) $4\ v_0$

22. The threshold wavelength for a metal whose work function is W_0 is λ_0. What is the threshold wavelength for a metal whose work function is $W_0/2$?

(a) $\frac{\lambda_0}{4}$ (b) $\frac{\lambda_0}{2}$

(c) $2\ \lambda_0$ (d) $4\ \lambda_0$

23. The minimum wavelength of X-rays emitted from an X-ray tube operating at a voltage of 10^4 volts is roughly equal to

(a) 1 Å (b) 1.5 Å

(c) 2 Å (d) 2.5 Å

24. The energy in monochromatic X-rays of wavelength 1 Å is roughly equal to

(a) 2×10^{-15} J (b) 2×10^{-16} J

(c) 2×10^{-17} J (d) 2×10^{-18} J

25. When the accelerating voltage applied on the electrons, in an X-ray tube, is increased beyond a critical value

(a) the spectrum of white radiation is unaffected
(b) only the intensities of various wavelengths are increased
(c) only the wavelength of characteristic radiation is affected
(d) the intensities of characteristic lines relative to the white spectrum are increased but there is no change in their wavelength.

26. In the continuous part of the spectrum of X-rays the limiting frequency is
(a) inversely proportional to the potential through which electrons have been accelerated
(b) is directly proportional to the accelerating potential
(c) not dependent upon the accelerating potential
(d) is dependent upon the nature of the target material.

27. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation,
(a) the intensity increases
(b) the minimum wavelength increases
(c) the intensity decreases
(d) the minimum wavelength decreases.

IIT, 1988

28. In an X-ray tube, electrons accelerated through a very high potential difference strike a metal target. If the potential difference is increased, the speed of the emitted X-rays
(a) increases
(b) decreases
(c) remains unchanged
(d) is always equal to 3×10^8 ms^{-1}.

29. An X-ray tube is operated at 66 kV. Then, in the continuous spectrum of the emitted X-rays
(a) wavelengths 0.01 nm and 0.02 nm will both be present
(b) wavelengths 0.01 nm and 0.02 nm will both be absent
(c) wavelengths 0.01 nm will be present but wavelength 0.02 nm will be absent
(d) wavelength 0.01 nm will be absent but wavelength 0.02 nm will be present.

30. The minimum wavelength of X-rays produced in an X-ray tube is λ when the operating voltage is V. What is the minimum wavelength of the X-rays when the operating voltage is $V/2$?
(a) $\frac{\lambda}{2}$ (b) λ
(c) 2λ (d) 4λ

31. The maximum frequency of X-rays produced in an X-ray tube is ν when the operating voltage is V. What is the maximum frequency of the X-rays when the operating voltage is $V/2$?
(a) $\frac{\nu}{2}$ (b) ν
(c) 2ν (d) 4ν

32. X-rays are incident normally on a crystal of lattice constant 0.6 nm. The first order reflection on diffraction from the crystal occurs at an angle of 30°. What is the wavelength of X-rays used?
(a) 0.3 nm (b) 0.6 nm
(c) 1.2 nm (d) 2.4 nm

33. X-rays of wavelength λ are incident normally on a crystal and the second order reflection on diffraction from the crystal is observed at an angle of 45°. The lattice constant of the crystal is
(a) $\frac{\lambda}{\sqrt{2}}$ (b) $\sqrt{2}\ \lambda$
(c) λ (d) 2λ

34. The frequency of K_α line of a source of atomic number Z is proportional to
(a) Z^2 (b) $(Z-1)^2$
(c) $1/Z$ (d) Z

35. The wavelength of K_α line from an element of atomic number 41 is λ. Then the wavelength of K_α line of an element of atomic number 21 is
(a) 4λ (b) $\lambda/4$
(c) 3.08λ (d) 0.26λ

IIT, 2005

36. The continuous X-ray spectrum is produced due to
(a) acceleration of electrons towards the nuclei of the target atoms
(b) retardation of energetic electrons when they approach the nuclei of the target atoms
(c) fall of the electrons of the target atoms from higher energy level to lower energy levels
(d) knocking out of the electrons from the target atoms by the fast moving incident electrons

37. An X-ray photon of wavelength λ and frequency ν collides with an electron and bounces off. If λ' and ν' are respectively the wavelength and frequency of the scattered photon, then
(a) $\lambda' = \lambda\ ;\ \nu' = \nu$ (b) $\lambda' < \lambda\ ;\ \nu' > \nu$
(c) $\lambda' > \lambda\ ;\ \nu' > \nu$ (d) $\lambda' > \lambda\ ;\ \nu' < \nu$

38. The binding energy of the innermost electron in tungsten is 40 keV. To produce characteristic X-rays using a tungsten target in an X-ray tube, the accelerating voltage should be greater than
(a) 4 kV (b) 40 kV
(c) 400 kV (d) 4000 kV

39. The shortest wavelength of X-rays emitted from an X-ray tube depends upon
(a) the current in the tube
(b) the voltage applied to the tube

(c) the nature of the gas in the tube
(d) the atomic number of the target material

IIT, 1982

40. Which one of the following parameters of the emitted X-rays increases when the potential difference between the electrodes of an X-ray tube is increased?
(a) intensity (b) frequency
(c) wavelength (d) speed

41. A proton, when accelerated through a potential difference of V volts, has a wavelength λ associated with it. If an alpha particle is to have the same wavelength λ, it must be accelerated through a potential difference of
(a) $V/8$ volts (b) $V/4$ volts
(c) $4\ V$ volts (d) $8\ V$ volts

42. Two particles of masses m and $2m$ have equal kinetic energies. Their de Broglie wavelengths are in the ratio of
(a) 1 : 1 (b) 1 : 2
(c) $1 : \sqrt{2}$ (d) $\sqrt{2} : 1$

43. The de Broglie wavelength of a neutron at 927°C is λ. What will be its wavelength at 27°C?
(a) $\frac{\lambda}{2}$ (b) λ
(c) $2\ \lambda$ (d) $4\ \lambda$

44. The de Broglie wavelength of a neutron when its kinetic energy is K is λ. What will be its wavelength when its kinetic energy is $4K$?
(a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$
(c) $2\ \lambda$ (d) $4\ \lambda$

45. What is the de Broglie wavelength of an electron of energy 180 eV? Mass of electron = 9×10^{-31} kg and Planck's constant = 6.6×10^{-34} Js.
(a) 0.5 Å (b) 0.9 Å
(c) 1.3 Å (d) 1.8 Å

46. Moving with the same velocity, which of the following has the longest de Broglie wavelength?
(a) β-particle (b) α-particle
(c) proton (d) neutron

47. The de Broglie wavelength of an electron moving with a velocity 1.5×10^8 ms^{-1} is equal to that of a photon. The ratio of the kinetic energy of the electron to that of the photon is
(a) 2 (b) 4
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

IIT, 2004

48. The shortest wavelength in the Lyman series is 911·6 Å. Then the longest wavelength in the Lyman series is
(a) 1215 Å (b) ∞
(c) 2430 Å (d) 600 Å

49. If electron orbits with principal quantum numbers $n > 3$ were not allowed, the number of possible elements would be
(a) 28 (b) 90
(c) 32 (d) 64

50. Pauli's exclusion principle states that no two electrons in an atom can have identical values for
(a) one of the four quantum numbers
(b) two of the four quantum numbers
(c) three of the four quantum numbers
(d) all four quantum numbers

51. Which energy state of the triply ionized beryllium (Be^{+++}) has the same electron orbital radius as that of the ground state of hydrogen? Given Z for beryllium = 4.
(a) $n = 1$ (b) $n = 2$
(c) $n = 3$ (d) $n = 4$

52. In Q. 51, what is the ratio of the energy state of beryllium and that of hydrogen?
(a) 1 (b) 2
(c) 3 (d) 4

53. Which energy state of doubly ionized lithium (Li^{++}) has the same energy as that of the ground state of hydrogen? Given Z for lithium = 3.
(a) $n = 1$ (b) $n = 2$
(c) $n = 3$ (d) $n = 4$

54. In Q. 53, what is the ratio of the electron orbital radius of Li^{++} to that of hydrogen?
(a) 1 (b) 2
(c) 3 (d) 4

55. The total energy of the electron in the first excited state of hydrogen is – 3·4 eV. What is the kinetic energy of the electron in this state?
(a) + 1.7 eV (b) + 3.4 eV
(c) + 6.8 eV (d) + 13.4 eV

56. In Q. 55, the potential energy of the electron is
(a) – 1.7 eV (b) – 3.4 eV
(c) – 6.8 eV (d) – 13.4 eV

57. The wavelength of the first line in Balmer series in the hydrogen spectrum is λ. What is the wavelength of the second line.
(a) $\frac{20\lambda}{27}$ (b) $\frac{3\lambda}{16}$
(c) $\frac{5\lambda}{36}$ (d) $\frac{3\lambda}{4}$

58. The frequency of the first line in Lyman series in the hydrogen spectrum is ν. What is the frequency of the corresponding line in the spectrum of doubly ionized Lithium?
(a) ν (b) $3\ \nu$
(c) $9\ \nu$ (d) $27\ \nu$

59. The energy difference between the first two levels of hydrogen atom is 10.2 eV. What is the corresponding energy difference for a singly ionized helium atom?
(a) 10.2 eV (b) 20.4 eV
(c) 40.8 eV (d) 81.6 eV

60. The ionization energy of hydrogen atom is 13.6 eV. What is the ionization energy of helium atom?
(a) 3.4 eV (b) 13.6 eV
(c) 54.4 eV (d) 108.8 eV

61. The ionization energy of hydrogen atom is 13.6 eV. Hydrogen atoms in the ground state are excited by electromagnetic radiation of energy 12.1 eV. How many spectral lines will be emitted by the hydrogen atom?
(a) one (b) two
(c) three (d) four

62. If an orbital electron of the hydrogen atom jumps from the ground state to a higher energy state, its orbital speed reduces to half its initial value. If the radius of the electron orbit in the ground state is r, then the radius of the new orbit would be
(a) $2r$ (b) $4r$
(c) $8r$ (d) $16r$

63. The orbital speed of the electron in the ground state of hydrogen is v. What will be its orbital speed when it is excited to the energy state – 3.4 eV?
(a) $2\ v$ (b) $\frac{v}{2}$
(c) $\frac{v}{4}$ (d) $\frac{v}{8}$

64. In the Bohr model of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in a quantum state n is
(a) -1 (b) $+1$
(c) $\frac{1}{n}$ (d) $\frac{1}{n^2}$

65. The ratio of the wavelengths of the longest wavelength lines in the Lyman and Balmer series of hydrogen spectrum is
(a) $\frac{3}{23}$ (b) $\frac{5}{27}$
(c) $\frac{7}{29}$ (d) $\frac{9}{31}$

66. If a hydrogen atom at rest, emits a photon of wavelength λ, the recoil speed of the atom of mass m is given by
(a) $\frac{h}{m\lambda}$ (b) $\frac{mh}{\lambda}$
(c) $mh\lambda$ (d) none of these

67. If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be
(a) 60 (b) 32
(c) 4 (d) 64

IIT, 1983

68. When a monochromatic source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are respectively 0.6 V and 18 mA. If the same source is placed 0.6 m away from the cell, then
(a) the stopping potential will be 0.2 V
(b) the stopping potential will be 1.8 V
(c) the saturation current will be 6.0 mA
(d) the saturation current will be 2.0 mA

IIT, 1992

69. The energy of a photon of frequency ν is $E = h\nu$ and the momentum of a photon of wavelength λ is $p = h/\lambda$. From this statement one may conclude that the wave velocity of light is equal to
(a) $3 \times 10^8\ \text{ms}^{-1}$ (b) $\frac{E}{p}$
(c) Ep (d) $\left(\frac{E}{p}\right)^2$

70. When a centimetre thick surface is illuminated with light of wavelength λ, the stopping potential is V. When the same surface is illuminated by light of wavelength 2λ, the stopping potential is $V/3$. The threshold wavelength for the surface is
(a) $\frac{4\lambda}{3}$ (b) $4\ \lambda$
(c) $6\ \lambda$ (d) $\frac{8\lambda}{3}$

71. Energy levels A, B and C of a certain atom correspond to increasing values of energy, i.e. $E_A < E_B < E_C$. If λ_1, λ_2 and λ_3 are the wavelengths of radiations corresponding to transitions C to B, B to A and C to A respectively, which of the following relations is correct?
(a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$
(c) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (d) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

72. Figure 28.10 represents the observed intensity (I) of X-rays emitted by an X-ray tube, as a function of wavelength (λ). The sharp peaks A and B denote
(a) band spectrum
(b) continuous spectrum
(c) characteristic radiations
(d) white radiations.

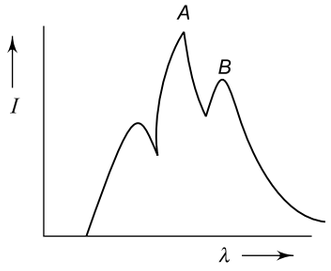


Fig. 28.10

73. In a photo-emissive cell, with exciting wavelength λ, the fastest electron has a speed v. If the exciting wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be
(a) $v\sqrt{\frac{3}{4}}$ (b) $v\sqrt{\frac{4}{3}}$
(c) less than $v\sqrt{\frac{4}{3}}$ (d) more than $v\sqrt{\frac{4}{3}}$

74. An energy of 24.6 eV is required to remove one of the electrons from the neutral helium atom. The energy (in eV) required to remove both the electron from a neutral helium atom is
(a) 38.2 (b) 49.2
(c) 51.8 (d) 79.0

IIT, 1995

75. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom ($Z = 3$) is
(a) 1.51 (b) 13.6
(c) 40.8 (d) 122.4

IIT, 1997

76. The K_α X-ray emission line of tungsten occurs at $\lambda = 0.021$ nm. The energy difference between K and L levels in this atom is about
(a) 0.51 MeV (b) 1.2 MeV
(c) 59 keV (d) 136 eV

IIT, 1997

77. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential is
(a) 2V (b) 4V
(c) 6V (d) 10V

IIT, 1997

78. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$ where n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is 8 times that in the final state. The possible values of n_1 and n_2 are
(a) $n_1 = 4$, $n_2 = 1$ (b) $n_1 = 8$, $n_2 = 2$
(c) $n_1 = 8$, $n_2 = 1$ (d) $n_1 = 6$, $n_2 = 3$

IIT, 1998

79. X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from
(a) 0 to ∞
(b) λ_{min} to ∞ where $\lambda_{min} > 0$
(c) 0 to λ_{max} where $\lambda_{max} < \infty$
(d) λ_{min} to λ_{max} where $0 < \lambda_{min} < \lambda_{max} < \infty$.

IIT, 1998

80. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately
(a) 540 nm (b) 400 nm
(c) 310 nm (d) 220 nm

IIT, 1998

81. A particle of mass M at rest decays into two particles of masses m_1 and m_2, having non zero velocities. The ratio of the de Broglie wavelengths of the particles, λ_1/λ_2, is
(a) $\frac{m_1}{m_2}$ (b) $\frac{m_2}{m_1}$
(c) 1.0 (d) $\sqrt{\frac{m_2}{m_1}}$

IIT, 1998

82. Imagine an atom made up of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength λ (given in terms of the Rydberg constant R for the hydrogen atom) equal to
(a) $\frac{9}{5R}$ (b) $\frac{36}{5R}$
(c) $\frac{18}{5R}$ (d) $\frac{4}{R}$

IIT, 2000

83. Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. K shell electrons of tungsten have – 72.5 keV energy. X-rays emitted by the tube contain
(a) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of about 0.155Å
(b) a continuous X-ray spectrum (Bremsstrahlung) with all wavelengths
(c) the characteristic X-ray spectrum of tungsten
(d) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of about 0.155 Å and the characteristics X-ray spectrum of tungsten.

IIT, 2000

84. Photoelectric emission is observed from a metallic surface for frequencies v_1 and v_2 of the incident light ($v_1 > v_2$). If the maximum values of kinetic energy of the photoelectrons emitted in the two cases are in the ratio 1 : n, then the threshold frequency of the metallic surface is

(a) $\frac{v_1 - v_2}{n-1}$ (b) $\frac{nv_1 - v_2}{n-1}$

(c) $\frac{nv_2 - v_1}{n-1}$ (d) $\frac{v_1 - v_2}{n}$

85. If λ_0 is the de Broglie wavelength for a proton accelerated through a potential difference of 100 V, the de Broglie wavelength for α-particle accelerated through the same potential difference is

(a) $2\sqrt{2}\lambda_0$ (b) $\frac{\lambda_0}{2}$

(c) $\frac{\lambda_0}{2\sqrt{2}}$ (d) $\frac{\lambda_0}{\sqrt{2}}$

86. The de-Broglie wavelength of a particle moving with a velocity 2.25×10^8 m/s is equal to the wavelength of a photon. The ratio of kinetic energy of the particle to the energy of the photon is

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$

(c) $\frac{5}{8}$ (d) $\frac{7}{8}$

87. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is

(a) $\frac{E}{c}$ (b) $\frac{2E}{c}$

(c) Ec (d) $\frac{E}{c^2}$

88. The kinetic energy of the most energetic photoelectrons emitted from a metal surface is doubled when the wavelength of the incident radiation is reduced from λ_1 to λ_2. The work function of the metal is

(a) $\frac{hc}{\lambda_1 \lambda_2}(2\lambda_2 - \lambda_1)$ (b) $\frac{hc}{\lambda_1 \lambda_2}(2\lambda_1 - \lambda_2)$

(c) $\frac{hc}{\lambda_1 \lambda_2}(\lambda_1 + \lambda_2)$ (d) $\frac{hc}{\lambda_1 \lambda_2}(\lambda_1 - \lambda_2)$

89. The slope of the graph of the frequency of incident light versus the stopping potential for a given metallic surface is

(a) h (b) $\frac{h}{e}$

(c) $\frac{e}{h}$ (d) eh

90. Lights of two different frequencies, whose photons have energies 2 eV and 10 eV respectively, successively illuminate a metal of work function 1 eV. The ratio of the maximum speeds of the emitted electrons will be
(a) 1 : 5 (b) 3 : 11
(c) 1 : 9 (d) 1 : 3

91. The mass of a photon of wavelength λ is given by

(a) $h\lambda c$ (b) $\frac{h}{\lambda c}$

(c) $\frac{hc}{\lambda}$ (d) $\frac{h\lambda}{c}$

92. The de-Broglie wavelength of an electron moving in the nth Bohr orbit of radius r is given by

(a) $\frac{2\pi r}{n}$ (b) $n\pi r$

(c) $\frac{nr}{2\pi}$ (d) $\frac{nr}{\pi}$

93. The momentum of a particle of mass m and charge q is equal to that of a photon of wavelength λ. The speed of the particle is given by

(a) $\frac{h}{m\lambda}$ (b) $\frac{h\lambda}{qm}$

(c) $qh\lambda$ (d) $\frac{mh}{\lambda}$

94. The kinetic energies of photoelectrons emitted from a metal are K_1 and K_2 when it is irradiated with lights of wavelength λ_1 and λ_2 respectively. The work function of the metal is

(a) $\frac{K_1\lambda_1 - K_2\lambda_2}{\lambda_2 - \lambda_1}$ (b) $\frac{K_1\lambda_1 + K_2\lambda_2}{\lambda_2 + \lambda_1}$

(c) $\frac{K_1\lambda_2 - K_2\lambda_1}{\lambda_2 - \lambda_1}$ (d) $\frac{K_1\lambda_2 + K_2\lambda_1}{\lambda_2 + \lambda_1}$

95. When the energy of the incident radiation is increased by 20%, the kinetic energy of the photoelectrons emitted from a metal increased from 0.5 eV to 0.8 eV. The work function of the metal is

(a) 0.65 eV (b) 1.0 eV
(c) 1.3 eV (d) 1.5 eV

96. An electron of mass m is moving such that its momentum is equal to that of a photon of wavelength λ. The velocity of the electron is (h is the Planck's constant)

(a) $\frac{h}{m\lambda}$ (b) $\frac{mh}{\lambda}$

(c) $\frac{\sqrt{2}h}{m\lambda}$ (d) $mh\lambda$

97. The radius of hydrogen atom in the ground state is 0.53 Å. After collision with an electron, it is found to have a radius of 2.12 Å. What is the principal quantum number n of the final state of the atom?

(a) $n = 1$ (b) $n = 2$
(c) $n = 3$ (d) $n = 4$

IIT, 2003

98. Figure 28.11 shows the variation of photoelectric current (i) with anode potential (V) for a photosensitive surface for two radiations of intensities I_a and I_b and frequencies ν_a and ν_b for the curves a and b respectively. It follows from the graph that

(a) $\nu_a = \nu_b$, $I_b < I_a$ (b) $\nu_a = \nu_b$, $I_b > I_a$
(c) $\nu_a < \nu_b$, $I_b > I_a$ (d) $\nu_a < \nu_b$, $I_b = I_a$

IIT, 2004

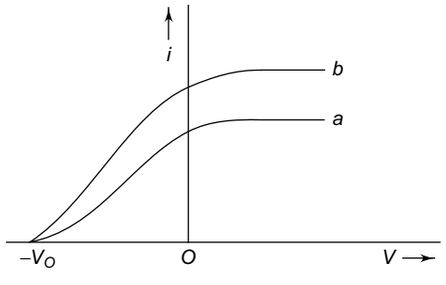


Fig. 28.11

99. A photon of energy 10.2 eV undergoes an inelastic collision with a hydrogen atom in the ground state. After a few microseconds, another photon of energy 14.6 eV collides inelastically with the same hydrogen atom. In these processes

(a) a photon of energy 3.4 eV and an electron of energy 1.0 eV are released.
(b) a photon of energy 10.2 eV and an electron of energy 1.0 eV are released
(c) two photons of energy 10.2 eV are released
(d) two photons of energy 3.4 eV and 1.0 eV are released.

IIT, 2005

100. Ultraviolet light of wavelengths λ_1 and λ_2 (with $\lambda_2 > \lambda_1$) when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with kinetic energies E_1 and E_2 respectively. The value of the Planck's constant can be found from the relation

IIT, 1983

(a) $h = \frac{1}{c}(\lambda_2 - \lambda_1)(E_1 - E_2)$

(b) $h = \frac{1}{c}(\lambda_2 + \lambda_1)(E_1 + E_2)$

(c) $h = \frac{(E_1 - E_2)\lambda_1\lambda_2}{c(\lambda_2 - \lambda_1)}$

(d) $h = \frac{(E_1 + E_2)\lambda_1\lambda_2}{c(\lambda_2 + \lambda_1)}$

101. The wavelength of the characteristic X-ray K_α line emitted by a hydrogen like element is 0.32 Å. The wavelength of K_β line emitted by the same element will be

IIT, 1990

(a) 0.21 Å (b) 0.27 Å
(c) 0.34 Å (d) 0.40 Å

102. The potential energy U of a moving particle of mass m varies with x as shown in Fig. 28.12. The de-Broglie wavelengths of the particle in the regions $0 \le x \le 1$ and $x > 1$ are λ_1 and λ_2 respectively. If the total energy of the particle is nE, the ratio λ_1/λ_2 is

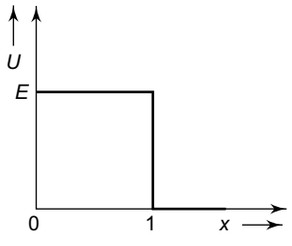


Fig. 28.12

IIT, 2005

(a) $\frac{n}{(n-1)}$ (b) $\sqrt{\frac{n}{(n-1)}}$

(c) $\sqrt{\frac{n^2}{(n^2-1)}}$ (d) $\frac{n^2}{(n^2-1)}$

103. The wavelength of K_α line from an element of atomic number 41 is λ. Then the wavelength of K_α line of an element of atomic number 21 is

(a) 4λ (b) $\lambda/4$

(c) 3.08λ (d) 0.26λ

IIT, 2005

104. Electrons with de-Broglie wavelength λ fall on the target in an X- ray tube. The cut-off wavelength of the emitted X-ray is

(a) $\lambda_0 = \dfrac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \dfrac{2h}{mc}$

(c) $\lambda_0 = \dfrac{2m^2c^2\lambda^2}{h^2}$ (d) $\lambda_0 = \lambda$

IIT, 2007

105. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is

(a) 802 nm (b) 823 nm

(c) 1882 nm (d) 1648 nm

IIT, 2007

106. Which one of the following statements is Wrong in the context of X-rays generated from a X-ray tube?

(a) Wavelength of characteristic X-ray decreases when the atomic number of the target increases.

(b) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target

(c) Intensity of the characteristic X-rays depends on the electric power given to the X-rays tube

(d) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube.

IIT, 2008

107. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV. $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is (see Fig. 28.13) [Take $hc = 1240$ eV nm]

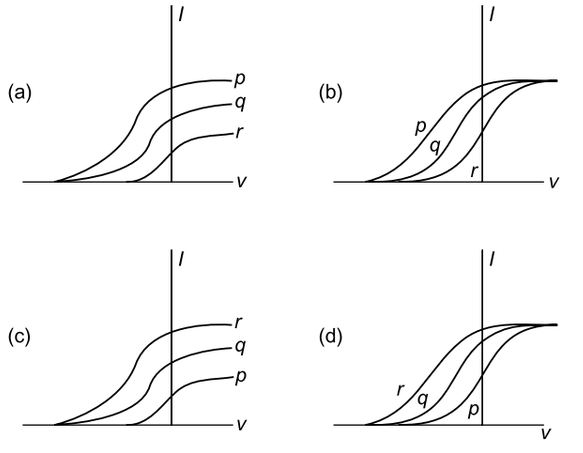


Fig. 28.13

IIT, 2009

108. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is

(a) 1215 Å (b) 1640 Å

(c) 2430 Å (d) 4687 Å

IIT, 2011

ANSWERS

1. (b)	**2.** (b)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (a)
7. (c)	**8.** (c)	**9.** (d)	**10.** (d)	**11.** (d)	**12.** (a)
13. (a)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (a)	**18.** (b)
19. (d)	**20.** (b)	**21.** (b)	**22.** (c)	**23.** (a)	**24.** (a)
25. (d)	**26.** (b)	**27.** (d)	**28.** (d)	**29.** (d)	**30.** (c)
31. (a)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (b)	**36.** (b)
37. (d)	**38.** (b)	**39.** (b)	**40.** (b)	**41.** (a)	**42.** (d)
43. (c)	**44.** (b)	**45.** (b)	**46.** (a)	**47.** (d)	**48.** (a)
49. (a)	**50.** (d)	**51.** (b)	**52.** (d)	**53.** (c)	**54.** (c)
55. (b)	**56.** (c)	**57.** (a)	**58.** (c)	**59.** (c)	**60.** (c)
61. (c)	**62.** (b)	**63.** (b)	**64.** (a)	**65.** (b)	**66.** (a)
67. (a)	**68.** (d)	**69.** (b)	**70.** (b)	**71.** (b)	**72.** (c)

73. (d)	**74.** (d)	**75.** (d)	**76.** (c)	**77.** (b)	**78.** (d)
79. (b)	**80.** (c)	**81.** (c)	**82.** (c)	**83.** (d)	**84.** (b)
85. (c)	**86.** (b)	**87.** (b)	**88.** (a)	**89.** (c)	**90.** (d)
91. (b)	**92.** (a)	**93.** (a)	**94.** (a)	**95.** (b)	**96.** (a)
97. (b)	**98.** (b)	**99.** (b)	**100.** (c)	**101.** (b)	**102.** (b)
103. (b)	**104.** (a)	**105.** (b)	**106.** (a)	**107.** (a)	**108.** (a)

SOLUTIONS

1. The saturation current is proportional to the intensity of incident radiation. The slopping potential (the magnitude of V when $i = 0$) increases with increase in frequency. Hence $I_b > I_a$ and $\nu_b > \nu_a$. So the correct choice is (b).

2. Stopping potential is the negative potential needed to stop the fastest moving electrons. $K_{max} = eV_0$.
Thus $3\,eV = eV_0 \Rightarrow V_0 = 3\ V$.

3. $K_1 = \dfrac{hc}{\lambda_1} - W$ (1)

$K_2 = \dfrac{hc}{\lambda_2} - W$ (2)

Putting $\lambda_1 = \lambda_2/2$ in Eq. (1) and using Eq. (2),

$$K_1 = \frac{2hc}{\lambda_2} - W$$

$$= 2\,(K_2 + W) - W$$

$$\Rightarrow \quad K_1 = 2K_2 + W$$

Hence $K_1 > 2K_2$, which is choice (c).

4. K_α X-rays are produced when electrons jump from $n = 2$ to $n = 1$ state and K_β X-rays are produced in a transition $n = 3$ to $n = 1$. Hence the energy of K_β rays is greater than that of K_α rays. Since $E = \dfrac{hc}{\lambda}$; the wavelength K_β rays is less than that of K_α rays. Hence peak A represents K_β line and peak B represents K_α line.

5. From the conservation of linear momentum, the two fragments will have equal and opposite momenta. Now de Broglie wavelength $\lambda = h/p$. Hence

$$\frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1}$$

Since $p_1 = p_2$, $\dfrac{\lambda_1}{\lambda_2} = 1$.

6. de Broglie wavelength of electrons is

$$\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2\,meV}} \quad (1)$$

The shortest X-ray wavelength (λ_2) corresponds to continuous X-ray spectrum and is given by

$E = h\nu \quad \Rightarrow \quad eV = \dfrac{hc}{\lambda_2}$, which gives

$$\lambda_2 = \frac{hc}{eV} \quad (2)$$

From Eqs. (1) and (2), we get

$$\frac{\lambda_1}{\lambda_2} = \frac{1}{c}\sqrt{\frac{eV}{2\,m}}$$

7. Speed of the electron in the nth orbit is

$$v = \frac{e^2}{2\varepsilon_0\, hn}$$

$\therefore$ Momentum $p = mv = \dfrac{me^2}{2\varepsilon_0\, hn}$

de Broglie wavelength $\lambda = \dfrac{h}{p} = \left(\dfrac{2h^2\,\varepsilon_0}{me^2}\right)n$. Hence $\lambda \propto n$, which is choice (c).

8. Magnitude of magnetic moment of an electron moving in an orbit of radius r with angular frequency ω is

$$M = \frac{\text{charge} \times \text{area}}{\text{time period}}$$

$$= \frac{e \times \pi r^2}{T} = \frac{1}{2}\, e\omega r^2 \qquad \left(\because\ T = \frac{2\pi}{\omega}\right)$$

Magnitude of angular momentum is

$$L = m\omega r^2$$

$$\therefore \quad \frac{M}{L} = \frac{e}{2\,m}$$

$$\Rightarrow \quad M = \frac{eL}{2\,m}$$

From Bohr's postulate $L = \dfrac{nh}{2\pi}$. Hence

$$M = \left(\frac{eh}{4\pi\, m}\right)n$$

So $M \propto n$, which is choice (c).

9. The wavelengths in Brackett series are given by

$$\frac{1}{\lambda} = R_H Z^2 \left(\frac{1}{4^2} - \frac{1}{n^2}\right); \; n = 5, 6, 7, \ldots$$

where R_H is the Rydberg constant. The shortest wavelength corresponds to $n = \infty$. Thus

$$\frac{1}{\lambda} = \frac{1}{16} R_H Z^2$$

The shortest wavelength in Balmer series of hydrogen ($Z = 1$) is given by

$$\frac{1}{\lambda'} = \frac{1}{4} R_H$$

Given $\lambda = \lambda'$. Therefore $Z^2 = \frac{16}{4} = 4 \Rightarrow Z = 2$. So the correct choice is (a).

10. $r_n \propto n^2$ or $r_n = kn^2$ where k is a constant. Hence

$$r_{n+1} = k(n+1)^2$$
$$r_n = kn^2$$
$$r_{n-1} = k(n-1)^2$$

Given

$$r_{n+1} - r_n = r_{n-1}$$
$$\Rightarrow \quad k(n+1)^2 - kn^2 = k(n-1)^2$$
$$\Rightarrow \quad (n+1)^2 - n^2 = (n-1)^2$$

which gives $n = 4$.

11. Current due to circulating electron is

$$I = \frac{e}{T_n} = \frac{e}{2\pi r_n / v_n} = \frac{e v_n}{2\pi r_n}$$

Magnetic field at the nucleus is

$$B = \frac{\mu_0 I}{2 r_n} = \frac{\mu_0 e v_n}{2\pi r_n^2} \qquad (1)$$

Now $r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$ and $v_n = \frac{e^2}{2\varepsilon_0 hn}$

Using these in Eq. (1), we find that $B \propto \frac{1}{n^5}$. So the correct choice is (d).

12. $r = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$

$$L = \frac{nh}{2\pi} \quad \Rightarrow \quad nh = 2\pi L$$

$$\therefore \quad r = \frac{\varepsilon_0 (2\pi L)^2}{\pi m e^2}$$

$$\Rightarrow \quad L = \left(\frac{me^2}{4\pi\varepsilon_0}\right)^{1/2} \times r^{1/2}$$

Thus $L \propto r^{1/2}$, which is choice (a)

13. Energy of a photon is $E = h\nu$. Now $\nu = \frac{c}{\lambda}$. Hence

$$E = \frac{hc}{\lambda} \quad \text{or} \quad \lambda = \frac{hc}{E}$$

Hence the correct choice is (a).

14. Lenses of equal diameters collect the same amount of lighter so that the intensity remains the same; hence the photoelectric current also remains the same.

15. Energy of photon $h\nu = mc^2$. Therefore,

$$m = \frac{h\nu}{c^2} = \frac{h}{c\lambda} \qquad (\because c = \nu\lambda)$$

$\therefore$ Momentum of photon $= mc = \frac{h}{\lambda}$. Hence the correct choice is (d).

16. The wavelength of blue light is longer than that of violet light and the wavelength of sodium light (yellow light) is longer than that of blue light. Since no photoelectric emission occurs for blue light, it follows that the wavelength threshold ($\lambda_{\min}$) is less than the wavelength of blue light and hence less than that of yellow light. Hence the correct choice is (d).

17. Energy of photons of frequency (ν) $= h\nu$. If the lamp emits n photons per second, then the power of the lamp is $P = nh\nu$. Now, $\nu = c/\lambda$. Therefore

$$P = \frac{nhc}{\lambda}$$

$$= \frac{10^{20} \times 6.6 \times 10^{-34} \times 3 \times 10^8}{660 \times 10^{-9}} = 30 \text{ W}$$

18. $h\nu = h\nu_0 + eV$, where V is the stopping potential. For frequency ν, we have

$$h\nu = h\nu_0 + \frac{eV_0}{2} \qquad \text{(i)}$$

and for frequency $\nu/2$, we have

$$\frac{h\nu}{2} = h\nu_0 + eV_0 \qquad \text{(ii)}$$

From (i) and (ii) on eliminating V_0, we get $\nu_0 = 3\nu/2$. Hence the correct choice is (b).

19. $h\nu = h\nu_0 + eV$. Since $\nu = c/\lambda$, we have $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eV$, where λ_0 is the threshold wavelength. For wavelength λ, we have

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + 2\, eV_0 \qquad \text{(i)}$$

and for wavelength 2λ, we have

$$\frac{hc}{2\lambda} = \frac{hc}{\lambda_0} + \frac{eV_0}{2} \qquad \text{(ii)}$$

Eliminating V_0 from (i) and (ii) we get $\lambda_0 = 3\lambda$. Hence the correct choice is (d).

20. For light of frequency $2\nu_0$, we have

$$2\,h\nu_0 = h\nu_0 + eV_0 \text{ or } h\nu_0 = eV_0 \qquad \text{(i)}$$

For light of frequency $3\nu_0$, we have

$$3\,h\nu_0 = h\nu_0 + eV \text{ or } 2h\nu_0 = eV \qquad \text{(ii)}$$

From (i) and (ii) we get $V = 2V_0$. Hence the correct choice is (b).

21. $\frac{1}{2}mv_{\max}^2 = h(\nu - \nu_0)$. For light of frequency $\nu = 2\nu_0$ we have

$$\frac{1}{2}mv_0^2 = h(2\nu_0 - \nu_0) = h\nu_0 \qquad \text{(i)}$$

For light of frequency $\nu = 5\nu_0$, we have

$$\frac{1}{2}mv^2 = h(5\nu_0 - \nu_0) = 4h\nu_0 \qquad \text{(ii)}$$

Dividing (ii) by (i) we get $v^2 = 4v_0^2$ or $v = 2v_0$. Hence the correct choice is (b).

22. $W_0 = h\nu_0 = \frac{hc}{\lambda_0}$. For a metal of work function $W_0/2$, the threshold wavelength λ is given by

$$\frac{W_0}{2} = \frac{hc}{\lambda}$$

Thus $\lambda = 2\lambda_0$. Hence the correct choice is (c).

23. Now $\nu_{\max} = \frac{eV}{h}$. Since $\nu = \frac{c}{\lambda}$, we have

$$\lambda_{\min} = \frac{c}{\nu_{\max}} = \frac{ch}{eV}$$

$$= \frac{3\times10^8 \times 6.6\times10^{-34}}{1.6\times10^{-19}\times10^4}$$

$$\simeq 1 \times 10^{-10}\text{ m} = 1\text{ Å}$$

Hence the correct choice is (a).

24. $E = h\nu = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{10^{-10}}$

$$= 1.98\times10^{-15}\text{ J} \simeq 2\times10^{-15}\text{ J}$$

Hence the correct choice is (a).

25. The correct choice is (d) because the emission of characteristic X-rays starts only when the incident electrons are accelerated beyond a certain critical value.

26. The maximum frequency is given by

$$\nu_{\max} = \frac{eV}{h}, \text{ i.e. } \nu_{\max} \propto V$$

Hence the correct choice is (b).

27. $\lambda_{\min} = \frac{c}{\nu_{\max}} = \frac{ch}{eV}$. Thus $V \propto \frac{1}{\lambda_{\min}}$. Thus if V is increased, $\lambda_{\min}$ decreased. Hence the correct choice is (d).

28. X-rays, being electromagnetic waves, always travel with the speed of light i.e., $3\times10^8\text{ ms}^{-1}$ in vacuum. It is only the energy of the incident electrons (and, therefore, the energy of the emitted X-rays) that depends on the accelerating p.d. The speed of X-rays has nothing to do with this p.d., it is a characteristic only of the medium of propagation. Unlike the case of visible light, the speed of X-rays changes but little with a change in medium (the refractive index of different media for X-rays is very close to unity)

29. $V = 66\text{ kV} = 66{,}000\text{ V}$. Now

$$\lambda_{\min} = \frac{hc}{eV}$$

$$= \frac{6.6\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times66{,}000}$$

$$= 1.87\times10^{-11}\text{ m} = 0.0187\text{ nm}$$

Since wavelength 0.01 nm is less than $\lambda_{\min}$, it will be absent from the continuous spectrum. Hence the correct choice is (d).

30. $\lambda_{\min} = \frac{hc}{eV}$, i.e. $\lambda_{\min} \propto \frac{1}{V}$. Hence the correct choice is (c).

31. $\nu_{\max} = \frac{eV}{h}$. Thus $\nu_{\max} \propto V$. Hence the correct choice is (a).

32. We use Bragg's equation $2d\sin\theta = n\lambda$. Here order $n = 1$, $\theta = 30°$ and $d = 0.6$ nm. Therefore,

$$\lambda = 2\times0.6\times\sin 30° = 0.6\text{ nm}$$

33. Here order $n = 2$ and $\theta = 45°$. From Bragg's equation, we have

$$2d\sin 45° = 2\lambda$$

or $$d = \sqrt{2}\,\lambda$$

34. According to Moseley's law $\sqrt{\nu} = a(Z - b)$. Hence $\nu \propto (Z-b)^2$. For K_α line, $b = 1$. Hence the correct choice is (b).

35. From Moseley's law, we have

$$\frac{\nu_{41}}{\nu_{21}} = \left(\frac{41-1}{21-1}\right)^2 = 4. \text{ Hence}$$

$$\lambda_{21} = \frac{\lambda_{41}}{4} = \frac{\lambda}{4}$$

36. The correct choice is (b).

37. Due to collision, the energy of the scattered photon will be less than that of the incident photon as some energy is lost in the collision. Now $E = h\nu = \dfrac{hc}{\lambda}$. Thus if E decreases ν decreases and λ increases. Hence $\nu' < \nu$ and $\lambda' > \lambda$.

38. To produce characteristic X-rays, the energy of the incident electrons must be greater than the binding energy of the innermost electron. Thus the accelerating voltage must be greater than 40 kV, then the accelerated electrons will have energy greater than 40 keV. Hence the correct choice is (b).

39. $\lambda_{\min} = \dfrac{hc}{eV}$. Hence the correct choice is (b).

40. If V is increased, the energy of X-rays $E = eV$ also increases. But $E = h\nu$. Hence frequency ν is increased.

41. The wavelength associated with a particle of charge q, mass m and accelerated through a potential difference V is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

or $$V = \frac{h^2}{2mq\lambda^2}$$

for proton: $$V = \frac{h^2}{2m_p q_p \lambda^2}$$

For α-particle : $$V' = \frac{h^2}{2m_\alpha q_\alpha \lambda^2}$$

$$\therefore \quad \frac{V'}{V} = \frac{m_p}{m_\alpha} \times \frac{q_p}{q_\alpha} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$(\because m_\alpha = 4\, m_p$ and $q_\alpha = 2q_p)$

Thus $V' = V/8$. Hence the correct choice is (a).

42. The de Broglie wavelength of a particle of mass m moving with a speed v is given by,

$$\lambda = \frac{h}{mv}$$

Kinetic energy $K = \dfrac{1}{2} mv^2$ or $mv^2 = 2K$ or $m^2v^2 = 2Km$ or $mv = \sqrt{2Km}$. Therefore

$$\lambda = \frac{h}{\sqrt{2Km}}$$

Since K is the same for both particles, we have

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{2m}{m}} = \sqrt{2}$$

Hence the correct choice is (d).

43. We have seen above that $\lambda \propto \dfrac{1}{\sqrt{T}}$. Hence

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{927 + 273}{27 + 273}} = 2$$

$\therefore \quad \lambda' = 2\lambda$. Hence the correct choice is (c).

44. In terms of kinetic energy $K = \dfrac{1}{2} mv^2$, de Broglie wavelength is

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Thus $\lambda \propto \dfrac{1}{\sqrt{K}}$. If K is increased by a factor of 4, λ is decreased by a factor of 2. Hence the correct choice is (b).

45. $K = 180$ eV $= 180 \times 1.6 \times 10^{-19}$ J $= 2.88 \times 10^{-17}$ J.

Now $$\lambda = \frac{h}{\sqrt{2mK}}$$

$$\therefore \quad \lambda = \frac{6.6 \times 10^{-34}}{\left[2 \times 9 \times 10^{-31} \times 2.88 \times 10^{-17}\right]^{1/2}}$$

$$= \frac{6.6 \times 10^{-34}}{7.2 \times 10^{-24}} = 0.9 \times 10^{-10} \text{ m}$$

46. β-particle is an electron. Since electron has the smallest mass, its de Broglie wavelength is the longest as the velocity of all particles is the same.

47. The speed of photon $= c = 3 \times 10^8$ ms^{-1}. Wavelength of photon is, say, λ.

Wavelength of electron $= \dfrac{h}{mv} = \lambda$. Now

$$\frac{\text{KE of electron}}{\text{KE of photon}} = \frac{\frac{1}{2}mv^2}{h\nu} = \frac{1}{2}\frac{mv^2}{hc}\lambda$$

$$= \frac{mv^2}{2hc} \cdot \frac{h}{mv} \qquad \left(\because \lambda = \frac{h}{mv}\right)$$

$$= \frac{v}{2c} = \frac{1}{4} \qquad (\because v = c/2).$$

48. For Lyman series, we have $\dfrac{\lambda_l}{\lambda_s} = \dfrac{4}{3}$. Hence $\lambda_l = \dfrac{4}{3} \times 911.6 = 1215$ Å.

49. The maximum number of electrons that can be accommodated in orbits with $n = 3$ is

$$2 \times 1^2 + 2 \times 2^2 + 2 \times 2^3 = 28.$$

50. The correct choice is (d).

51. For an atom of atomic number Z, the radius of the nth orbit is given by [see Eq. (4)]

$$r_n = \frac{K n^2}{Z} \qquad \text{(i)}$$

where $K = \dfrac{\varepsilon_0 h^2}{\pi m e^2}$ is a constant. For the ground state of hydrogen ($Z = 1$), $n = 1$ so that

$$r_1 = K$$

Let n be the energy state of Be^{+++} for which the orbital radius is r_1. Putting $Z = 4$ and $r_n = r_1 = K$ in Eq. (i) we get

$$K = K\,\frac{n^2}{4} \text{ or } n^2 = 4 \text{ or } n = 2$$

Hence the correct choice is (b).

52. The energy in the nth state is given by [see Eq. (5)]

$$E_n = -\frac{me^2}{8\varepsilon_0 h^2} \cdot \frac{Z^2}{n^2} = \frac{CZ^2}{n^2}$$

where $C = -\dfrac{me^2}{8\varepsilon_0 h^2}$ is a constant. For the ground state $n = 1$ of hydrogen and for $n = 2$ state of Be^{+++}, we have

$$E_1 = C \text{ and } E_2' = \frac{C(4)^2}{(2)^2} = 4C$$

$$\therefore \quad \frac{E_2'}{E_1} = \frac{4C}{C} = 4.$$

Hence the correct choice is (d).

53. $E_n = \dfrac{CZ^2}{n^2}$. For $n = 1$ state of hydrogen, we have $E_1 = C$ and for the nth state of Li^{++}, we have $E_n = \dfrac{C(3)^2}{n^2} = \dfrac{9C}{n^2}$. For $E_1 = E_n$, we require $C = \dfrac{9C}{n^2}$ which gives $n = 3$. Hence the correct choice is (c).

54. Now $r_n = \dfrac{K n^2}{Z}$. Therefore, for hydrogen ($n = 1$ state), we have

$$r_1 = K \qquad (\because Z = 1)$$

and for Li^{++} ($n = 3$ state) we have ($\because Z = 3$)

$$r_3' = \frac{K(3)^2}{3} = 3K$$

$$\therefore \quad \frac{r_3'}{r_1} = \frac{3K}{K} = 3.$$

Hence the correct choice is (c).

55. The kinetic and potential energies of an electron in the nth excited state are given by

$$\text{KE} = \frac{1}{8\pi\varepsilon_0} \cdot \frac{e^2}{r_n} \qquad \text{(i)}$$

and $$\text{PE} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r_n} \qquad \text{(ii)}$$

Adding (i) and (ii) we get the total energy E which is

$$E = \text{KE} + \text{PE} = -\frac{1}{8\pi\varepsilon_0} \cdot \frac{e^2}{r_n} \qquad \text{(iii)}$$

Notice from (i) and (iii) that $E = -$ KE. Given $E = -3.4$ eV. Hence KE $= -E = -(-3.4) = +3.4$ eV. Thus the correct choice is (b).

56. From (ii) and (iii) we find that $E = \dfrac{\text{PE}}{2}$ or PE $= 2E$ $= 2 \times -3.4 = -6.8$ eV. Hence the correct choice is (c).

57. The first line corresponds to $n = 3$. Therefore,

$$\frac{1}{\lambda_1} = R_H\left(\frac{1}{4} - \frac{1}{3^2}\right)$$

$$= R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36}$$

or $\lambda_1 = \dfrac{36}{5R_H}$. We have seen above that $\lambda_2 = \dfrac{16}{3R_H}$. Hence

$$\frac{\lambda_2}{\lambda_1} = \frac{16}{3R_H} \times \frac{5R_H}{36} = \frac{20}{27}$$

or $$\lambda_2 = \frac{20}{27}\,\lambda_1 = \frac{20}{27}\,\lambda.$$

58. Now $\nu \propto Z^2$. For doubly ionized lithium $Z = 3$. Hence the correct choice is (c).

59. Energy difference $\Delta E \propto Z^2$. For a singly ionized helium atom $Z = 2$. Hence $\Delta E = 10.2 \times (2)^2 = 40.8$ eV.

60. Ionization energy $E \propto Z^2$. For helium $Z = 2$. Hence E for helium $= 13.6 \times (2)^2 = 54.4$ eV, which is choice (c).

61. When an electron in the ground state receives 12.1 eV of energy, it jumps to a level where its

energy = 13.6 – 12.1 = 1.5 eV. This corresponds to the third excited state corresponding to $n = 3$. It can have three transitions, namely from $n = 3$ to $n = 2$, from $n = 2$ to $n = 1$ and from $n = 3$ to $n = 1$. Hence three spectral lines will be emitted.

62. Orbital speed $v_0 \propto \frac{1}{n}$. Therefore, $n = 2$. Now $r_n \propto n^2$. Hence the radius of the new orbit $= (2)^2 \times r = 4r$.

63. Energy state – 3.4 eV corresponds to a level n given by $-13.6 \text{ eV}/n^2 = -3.4$ eV which gives $n = 2$. Now, orbital speed $v_0 \propto \frac{1}{n}$. Hence the orbital speed in the excited state is $v/2$.

64. The total energy of an electron bound to an atom is negative and is the sum of its P.E. and K.E. The magnitude of the P.E. is twice that of the K.E. (as per the Bohr model) but since the P.E. is – ve, we have
Total Energy = $(-2K) + (K) = -K$
(K = Kinetic energy). Hence the ratio of
total energy and kinetic energy = – 1 : 1 = – 1.

65. The longest wavelengths in the two series are given by

$$\frac{1}{\lambda_L} = R_H \cdot \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R_H \cdot \frac{3}{4}$$

and $$\frac{1}{\lambda_B} = R_H \cdot \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R_H \cdot \frac{5}{36}$$

$$\therefore \quad \frac{\lambda_B}{\lambda_L} = \frac{3}{4} \times \frac{36}{5} = \frac{27}{5} \text{ or } \lambda_L : \lambda_B = 5 : 27$$

66. Since the momentum of photon is $\frac{h}{\lambda}$, from the law of conservation of momentum, the recoil speed v of an atom of mass m is given by

$$mv = \frac{h}{\lambda} \text{ or } v = \frac{h}{m\lambda}$$

67. The maximum number of electrons allowed in an orbit is $2n^2$. Hence the number of possible elements is

$$2\,(1^2 + 2^2 + 3^2 + 4^2) = 60$$

68. The stopping potential depends on the frequency (or wavelength) of the incident electromagnetic wave and is independent of the distance of the source from photocell. Hence the stopping potential will still be 0.6 V. However, the saturation current varies as $1/r^2$, where r is the distance of the source from the photocell. Since r is increased by a factor of 3, the saturation current will decrease by a factor of $(3)^2 = 9$, i.e. it will be 18 mA/9 = 2 mA at $r = 0.6$ m. Hence the correct choice is (d).

69. We know that $c = \nu\lambda$. Now $\nu = \frac{E}{h}$ and $\lambda = \frac{h}{p}$. Therefore,

$$c = \nu\lambda = \frac{E}{h} \times \frac{h}{p} = \frac{E}{p}$$

Hence the correct choice is (b).

70. $h\nu = eV + W_0$ or $eV = h\nu - W_0 = h\nu - h\nu_0$. Now $\nu = c/\lambda$ and $\nu_0 = c/\lambda_0$. Thus, for wavelength λ, we have

$$eV = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \quad \text{(i)}$$

and for wavelength $\lambda' = 2\lambda$, we have ($\because V' = V/3$)

$$\frac{eV}{3} = hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)$$

or $$eV = 3hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) \quad \text{(ii)}$$

From (i) and (ii) we have

$$\frac{1}{\lambda} - \frac{1}{\lambda_0} = 3\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)$$

which gives $\lambda_0 = 4\lambda$. Hence the correct choice is (b).

71. When an electron falls from energy level E_2 to energy level E_1, the frequency ν of the emitted radiation is given by

$$h\nu = E_2 - E_1$$

or $$\frac{hc}{\lambda} = E_2 - E_1$$

or $$\frac{1}{\lambda} = \frac{1}{hc}(E_2 - E_1) = k\,(E_2 - E_1)$$

where $k = 1/h\,c$. For energy levels A, B and C, we have

$$\frac{1}{\lambda_1} = k\,(E_C - E_B) \quad \text{(i)}$$

$$\frac{1}{\lambda_2} = k\,(E_B - E_A) \quad \text{(ii)}$$

and $$\frac{1}{\lambda_3} = k\,(E_C - E_A) \quad \text{(iii)}$$

Adding (i), (ii) and (iii), we get

$$\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} = k\,\{(E_C - E_B) + (E_B - E_A) + (E_C - E_A)\}$$

$$= k\,(2E_C - 2E_A) = \frac{2}{\lambda_3}$$

or $$\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3} \quad \text{or} \quad \lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$

Hence the correct choice is (b).

72. Peaks A and B represent characteristic X-rays; they are different for different emitters. Hence the correct choice is (c).

73. For wavelength λ we have

$$\frac{1}{2}\, mv^2 = h\nu - W_0$$

$$= \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{hc\,(\lambda_0 - \lambda)}{\lambda\,\lambda_0} \qquad \text{(i)}$$

For wavelength λ', we have

$$\frac{1}{2}\, mv'^2 = \frac{hc\,(\lambda_0 - \lambda')}{\lambda'\,\lambda_0} \qquad \text{(ii)}$$

From (i) and (ii) we get

$$\frac{v'^2}{v^2} = \frac{\lambda}{\lambda'} \times \frac{(\lambda_0 - \lambda')}{(\lambda_0 - \lambda)}$$

Now $\lambda' = 3\lambda/4$. Hence

$$\frac{v'^2}{v^2} = \frac{4}{3}\,\frac{(\lambda_0 - 3\lambda/4)}{(\lambda_0 - \lambda)}$$

Therefore, $\dfrac{v'^2}{v^2}$ is greater than $\dfrac{4}{3}$. Hence $v' > v\sqrt{\dfrac{4}{3}}$.

74. Energy required to remove one electron is E_1 = 24.6 eV. The energy required to remove the second electron is

$$E_2 = Z^2\, R_{\mathrm{H}} hc \left(\frac{1}{1^2} - \frac{1}{(\infty)^2}\right)$$

$$= (2)^2 \times 13.6 \text{ eV} = 54.4 \text{ eV}$$

$\therefore$ Total energy required = $E_1 + E_2$

$$= 24.6 + 54.4 = 79.0 \text{ eV}.$$

75. $E = Z^2 R_{\mathrm{H}} hc \left(\dfrac{1}{1^2} - \dfrac{1}{(\infty)^2}\right) = (3)^2 \times 13.6\,\text{eV} = 122.4\,\text{eV}$

76. $\Delta E = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.021 \times 10^{-9}} = 59 \times 10^3$ eV

$$= 59 \text{ keV}.$$

77. The stopping potential in volts = kinetic energy of the emitted photoelectrons in eV. Hence the correct choice is (b).

78. $T = \dfrac{2\pi r}{v}$ and $m\,v\,r = nh/2\pi$. Therefore,

$$T = \frac{2\pi r}{nh/(2\pi m r)} = \left(\frac{4\pi^2 m}{nh}\right) r^2$$

Also $\quad r = \left(\dfrac{h^2 \varepsilon_0}{\pi m e^2}\right) n^2, \therefore\ T = \left(\dfrac{4h^3 \varepsilon_0^2}{me^4}\right) \times n^3$

$\therefore$ For the two orbits $\dfrac{T_1}{T_2} = \left(\dfrac{n_1}{n_2}\right)^3$. Given $T_1 = 8T_2$. Hence $n_1/n_2 = 2$. Hence the correct choice is (d).

79. For a given accelerating voltage, the wavelength of the continuous X-rays will vary from a minimum value $\lambda_{\min}$ to infinity where $\lambda_{\min}$ is greater than zero. Hence the correct choice is (b).

80. $\lambda_{\max} = \dfrac{hc}{W_0} = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^8}{4.0 \times 1.6 \times 10^{-19}}$

$$= 3.10 \times 10^{-7} \text{ m} = 310 \text{ nm}$$

81. Let v_1 and v_2 be the respective speeds of the two particles. The law of conservation of linear momentum gives

$m_1 v_1 + m_2 v_2 = 0$ or $\left|\dfrac{m_2\, v_2}{m_1\, v_1}\right| = 1.0$

Since de Broglie wavelength $\lambda = h/(m\,v)$, we will have

$$\frac{\lambda_1}{\lambda_2} = \frac{m_2\, v_2}{m_1\, v_1} = 1.0$$

82. The energy levels of the hypothetical particle of double the mass of the electron but having the same charge as the electron are given by

$$E_n = -\frac{2R_{\mathrm{H}} hc}{n^2} \qquad (\because\ Z = 2)$$

For the longest wavelength photon emission leaving the atom in the first excited state will involve transition from $n = 3$ state to $n = 2$ state. Hence

$$|\Delta E| = 2\, R_{\mathrm{H}} hc \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 2\, R_{\mathrm{H}} hc \left(\frac{5}{36}\right)$$

$$= \frac{5}{18} R_{\mathrm{H}} hc$$

The wavelength corresponding to this energy is

$$\lambda = \frac{hc}{|\Delta E|} = \frac{18}{5R_{\mathrm{H}}}, \text{ which is choice (c).}$$

83. The energy of incident electrons is

$E = 80$ keV $= 80 \times 10^3$ eV

$$= 80 \times 10^3 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 1.28 \times 10^{-14} \text{ J}$$

The minimum wavelength of the continuous X-ray spectrum is

$$\lambda_{\min} = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.28 \times 10^{-14}}$$

$$= 0.155 \times 10^{-10} \text{ m} = 0.155 \text{ Å}$$

Since the energy of the incident electrons is more than that of the K shell electrons in tungsten, the

characteristic X-ray spectrum of tungsten will appear as peaks on the background of the continuous X-ray spectrum. Hence the correct choice is (d).

84. $E_1 = h(\nu_1 - \nu_0)$ and $E_2 = h(\nu_2 - \nu_0)$. Dividing them, we get

$$\frac{E_2}{E_1} = \frac{\nu_2 - \nu_0}{\nu_1 - \nu_0}$$

Given $E_2 = nE_1$. Hence, we have

$$n = \frac{\nu_2 - \nu_0}{\nu_1 - \nu_0}$$

which gives $\nu_0 = \dfrac{n\nu_1 - \nu_2}{(n-1)}$, which is choice (b).

85. Let m_0 and e_0 be the mass and charge of a proton and m_1 and e_1 those of α-particle. Then

$$\lambda_0 = \frac{h}{\sqrt{2m_0\, e_0\, V}} \text{ and}$$

$$\lambda_1 = \frac{h}{\sqrt{2m_1\, e_1\, V}}$$

Dividing, we get $\dfrac{\lambda_1}{\lambda_0} = \sqrt{\dfrac{m_0}{m_1} \times \dfrac{e_0}{e_1}}$ (i)

Now, α-particle has twice the charge and 4 times the mass of a proton, i.e. $m_1 = 4\, m_0$ and $e_1 = 2\, e_0$. Using these in Eq. (i), we get

$$\lambda_1 = \frac{\lambda_0}{2\sqrt{2}}, \text{ which is choice (c).}$$

86. $\lambda = \dfrac{hc}{mv}$ is the de Broglie wavelength. The energy of a photon of this wavelength is

$$E = h\nu = \frac{hc}{\lambda} = \frac{hc}{h/mv} = mvc \qquad \text{(i)}$$

Kinetic energy of the particle is

$$E' = \frac{1}{2}\, mv^2 \qquad \text{(ii)}$$

From (i) and (ii), we have

$$\frac{E'}{E} = \frac{\frac{1}{2}mv^2}{mvc} = \frac{v}{2c} = \frac{2.25 \times 10^8}{2 \times \left(3 \times 10^8\right)}$$

$$= \frac{2.25}{6} = \frac{3}{8}$$

Hence the correct choice is (b).

87. The momentum of a photon of wavelength λ is

$$p = \frac{h}{\lambda} = \frac{h\nu}{\lambda\nu} = \frac{E}{c}$$

(because $E = h\nu$ and $c = \nu\lambda$). Since the photon is reflected back, the momentum after reflection is $(-p)$.

$\therefore$ Momentum transferred $= p - (-p) = 2p = \dfrac{2E}{c}$. Hence the correct choice is (b).

88. Given $K_{max} = \dfrac{hc}{\lambda_1} - W_0$ (i)

and $2K_{max} = \dfrac{hc}{\lambda_2} - W_0$ (ii)

Dividing (ii) by (i), we get

$$2 = \frac{\frac{hc}{\lambda_2} - W_0}{\frac{hc}{\lambda_1} - W_0}$$

which gives $W_0 = \dfrac{hc}{\lambda_1 \lambda_2}(2\lambda_2 - \lambda_1)$ which is choice (a).

89. $eV_0 = h\nu$. Therefore, $\nu = \dfrac{e}{h} V_0$. Hence the slope of ν versus V_0 graph is $\dfrac{e}{h}$ which is choice (c).

90. $E = \dfrac{1}{2}\, mv^2 = h\nu_0 - W_0$. Now $E_1 = 2 - 1 = 1$ eV and $E_2 = 10 - 1 = 9$ eV. Therefore $E_1/E_2 = 1/9$, i.e.

$$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{1}{9}$$

or $\dfrac{v_1}{v_2} = \dfrac{1}{3}$. Hence the correct choice is (d).

91. $h\nu = mc^2$ or $\dfrac{hc}{\lambda} = mc^2$ ($\because c = \nu\lambda$). Hence

$m = \dfrac{h}{\lambda c}$ which is choice (b).

92. For nth Bohr orbit, $mvr = \dfrac{nh}{2\pi}$. The de-Broglie wavelength is

$$\lambda = \frac{h}{mv}$$

But $mv = \dfrac{nh}{2\pi r}$. Therefore,

$$\lambda = h \times \frac{2\pi r}{nh} = \frac{2\pi r}{n} \text{ which is choice (a).}$$

93. Given $mv = \dfrac{h}{\lambda}$. Hence the correct choice is (a).

94. Given $\dfrac{hc}{\lambda_1} - W_0 = K_1$ (i)

and $\frac{hc}{\lambda_2} - W_0 = K_2$ (ii)

Eliminate hc from (i) and (ii). The correct choice is (a).

95. $h\nu = E + W_0 = 0.5 \text{ eV} + W_0$ (i)

When the energy of the incident photon is increased by 20%, we have

$\frac{6}{5} h\nu = E' + W_0 = 0.8 \text{ eV} + W_0$ (ii)

Subtracting (ii) from (i), we get $h\nu = 1.5$ eV. Hence $W_0 = h\nu - 0.5$ eV $= 1.5$ eV $- 0.5$ eV $= 1.0$ eV. Thus the correct choice is (b).

96. Momentum of photon is $p = \frac{h}{\lambda}$. Momentum of an electron moving with velocity v is mv. Given $\frac{h}{\lambda} = mv$ or $v = \frac{h}{m\lambda}$, which is choice (a).

97. We know that $r_n \propto n^2$ or $n \propto \sqrt{r_n}$. If r_1 and r_2 are the radii of the initial and final states respectively, then

$$n = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{2.12}{0.53}} = 2$$

Hence the correct choice is (b).

98. It follows from the figure that the stopping potential (V_0) is the same for the two radiations. We know that $eV_0 = E_{max}$ and $E_{max} = h\nu - W_0$. Since V_0 is the same, E_{max} and hence ν is the same for radiations a and b. Hence $\nu_a = \nu_b$. Since the saturation current is greater for radiation b than for radiation a, the intensity I_b is greater than I_a. Hence the correct choice is (b).

99. The energy required to excite a hydrogen atom from state ($n = 1$ state) to the $n = 2$ state is 10.2 eV. So when a photon of energy 10.2 eV undergoes an inelastic collision with a hydrogen atom in the ground state, the electron of the hydrogen atom jumps from $n = 1$ state to $n = 2$ state. The electron spends a time between 10^{-10} s to 10^{-8} s in the excited state before falling back to the ground state.In this process, a photon of energy 10.2 eV is released. After a few microseconds, when a another photon of energy 14.6 eV collides with the same hydrogen atom, it finds the atom in the ground state. Now, the ionization energy of hydrogen atom is 13.6 eV. The part 13.6 eV of the energy of the incident photon is used up in ionizing the atom, i.e. in knocking the electron from the atom and remaining energy = 14.6 – 13.6 = 1.0 eV is used up in imparting kinetic energy to the released electron. Hence the correct choice is (b).

100. The energy of a photon of wavelength λ is given by

$$E = \frac{hc}{\lambda}$$

The energies of radiations of wavelengths λ_1 and λ_2 are

$$E_1 = \frac{hc}{\lambda_1} \text{ and } E_2 = \frac{hc}{\lambda_2} \text{ respectively}$$

$$\therefore \quad E_1 - E_2 = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

The value of h is given by choice (c)

101. For a hydrogen like element, we have

$$\frac{1}{\lambda} = Z^2 R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For K_α-line: $\frac{1}{\lambda_\alpha} = Z^2 R_H \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3Z^2 R_H}{4}$

For K_β-line: $\frac{1}{\lambda_\beta} = Z^2 R_H \left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8Z^2 R_H}{9}$

Dividing, we have

$$\frac{\lambda_\beta}{\lambda_\alpha} = \frac{3}{4} \times \frac{9}{8} = \frac{27}{32}$$

$$\therefore \quad \lambda_\beta = \frac{27}{32} \lambda_\alpha = \frac{27}{32} \times 0.32 \text{ Å} = 0.27 \text{ Å}.$$

The correct choice is (b).

102. Total energy = KE + PE = $K + U$

In the region $0 \le x \le 1$; $U_1 = E$. Therefore, kinetic energy is

K_1 = total energy – $U_1 = nE - E = (n-1)E$

$$\therefore \quad \lambda_1 = \frac{h}{\sqrt{2mK_1}} = \frac{h}{\sqrt{2m(n-1)E}} \qquad (1)$$

In the region $x > 1$; $U_2 = 0$. Therefore, kinetic energy is

$$K_2 = nE - 0 = nE$$

$$\therefore \quad \lambda_2 = \frac{h}{\sqrt{2mK_2}} = \frac{h}{\sqrt{2mnE}} \qquad (2)$$

Dividing (1) by (2), we get

$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{n}{(n-1)}}$. So the correct choice is (b).

103. From Moseley's law, we have

$\frac{\nu_{41}}{\nu_{21}} = \left(\frac{41-1}{21-1}\right)^2 = 4$. Hence

$$\lambda_{21} = \frac{\lambda_{41}}{4} = \frac{\lambda}{4}$$

104. de-Broglie wavelength is

$$\lambda = \frac{h}{\sqrt{2mE}} \qquad (1)$$

where E is the kinetic energy of the electrons. The out-off wavelength is

$$\lambda_0 = \frac{hc}{E}$$

From Eq. (1) $E = \dfrac{h^2}{2m\lambda^2}$. Hence

$$\lambda_0 = \frac{2mc\,\lambda^2}{h}$$

105. $\dfrac{1}{\lambda} = R_H\left[\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right]$

The largest wavelength in the ultraviolet region of the hydrogen spectrum corresponds to the transition $n_1 = 2$ to $n_2 = 1$ (Lyman series). Thus

$$\frac{1}{122\,\text{nm}} = R_H\left(1 - \frac{1}{4}\right) = \frac{3R_H}{4}$$

which gives $\quad R_H = \dfrac{4}{3\times 122\,\text{nm}}$

The smallest wavelength λ in the infrared region of the hydrogen spectrum corresponds to $n_1 = \infty$ and $n_2 = 3$ (Paschen series). Therefore

$$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{\infty}\right) = \frac{R_H}{9}$$

$$\Rightarrow \qquad \lambda = \frac{9}{R_H} = \frac{9\times 3\times 122\text{nm}}{4} \simeq 823\ \text{nm}$$

106. $\lambda_{\min} = \dfrac{hc}{eV}$, which is independent. Hence choice (a) is wrong.

107. For photoelectric emission, the wavelength of the incident radiation must be less than the cut-off wavelength of the metal given by $\lambda_0 = hc/\phi_0$. For metals p, q and r, the cut-off wavelengths are

$$\lambda_p = \frac{hc}{\phi_p} = \frac{1240\ \text{eV nm}}{2.0\ \text{eV}} = 620\ \text{nm}$$

$$\lambda_q = \frac{1240}{2.5} = 496\ \text{nm}$$

$$\lambda_r = \frac{1240}{3.0} = 413.3\ \text{nm}$$

Hence metal plate p emits photoelectrons for all the three given radiations, metal plate q emits photoelectrons for radiation of wavelengths 450 nm and 350 nm and metal plate r emits photoelectrons only for wavelength 350 nm. Therefore, photoelectric current is maximum for metal p and minimum for r, i.e. $I_p > I_q > I_r$. So the correct choice is (a).

108. $\dfrac{1}{\lambda} = R_H Z^2\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

For singly ionized helium atom, $Z = 2$. For hydrogen atom $Z = 1$.

For Balmer series $n_1 = 2$.

For first spectral line of hydrogen :

$$\frac{1}{6561} = R_H \times (1)^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

$$\Rightarrow \qquad R_H = \frac{36}{5\times 6561}$$

For second spectral line of helium,

$$\frac{1}{\lambda} = R_H \times (2)^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R_H}{4}$$

$$= \frac{3}{4}\times\frac{36}{5\times 6561}$$

$$\Rightarrow \qquad \lambda = 1215\ \text{Å}$$

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. The threshold frequency for photoelectric emission from a material is 4.5×10^{14} Hz. Photoelectrons will be emitted when this material is illuminated with monochromatic light from a

(a) 50 watt infrared lamp
(b) 100 watt red neon lamp
(c) 60 watt sodium lamp
(d) 5 watt ultraviolet lamp

2. When monochromatic light from a bulb falls on a photosensitive surface, the number of photoelectrons emitted per second is n and their maximum kinetic energy is K_{max}. If the distance of the lamp from the surface is halved, then
(a) n is doubled
(b) n becomes 4 times
(c) K_{max} is doubled
(d) K_{max} remains unchanged

3. The maximum kinetic energy of photoelectrons in a photocell depends upon
(a) the frequency of the incident radiation
(b) the work function of the photosensitive material used in the cell
(c) the intensity of the incident radiation
(d) all the above parameters.

4. When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When X-rays are incident on the same cell, then
(a) V_0 will increase
(b) K_{max} will increase
(c) V_0 will decrease
(d) K_{max} will decrease

5. The work function of metal A is greater than that for metal B. The two metals are illuminated with appropriate radiation of frequency ν so as to cause photoelectric emission in both metals. If ν_0 is the threshold frequency and K_{max}, the maximum kinetic energy of photoelectrons, then
(a) ν_0 for metal A is greater than that for metal B.
(b) ν_0 for metal A is less than that for metal B.
(c) K_{max} for metal A is greater than that for metal B.
(d) K_{max} for metal A is less than that for metal B.

6. X-rays are used to cause photoelectric emission from sodium and copper. Then
(a) the stopping potential is more for copper than for sodium.
(b) the stopping potential is less for copper than for sodium.
(c) the threshold frequency is more for copper than for sodium.
(d) the threshold frequency is less for copper than for sodium.

7. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are respectively 0.6 volt and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then
(a) the stopping potential will be 0.2 volt
(b) the stopping potential will be 0.6 volt
(c) the saturation current will be 6.0 mA
(d) the saturation current will be 2.0 mA.

8. When a point light source, of power W emitting monochromatic light of wavelength λ is kept at a distance a from a photosensitive surface of work function ϕ, and area S, we will have
(a) number of photons striking the surface per unit time as $\dfrac{W\lambda S}{4\pi hca^2}$
(b) the maximum energy of the emitted photoelectrons as $\dfrac{1}{\lambda}(hc - \lambda\phi)$
(c) the stopping potential needed to stop the most energetic emitted photoelectrons as $\dfrac{e}{\lambda}(hc - \lambda\phi)$.
(d) photoemission occurs only if λ lies in the range $0 \le \lambda \le h\,c/\phi$

9. Which of the following statements are correct about photons?
(a) The rest mass of a photon is zero
(b) The energy of a photon of frequency ν is $h\nu$
(c) The momentum of a photon of frequency ν is $\dfrac{h\nu}{c}$
(d) Photons do not exert any pressure on a surface on which they are incident.

10. Figure 28.14 shows the stopping potential V_0 versus frequency ν for photoelectric emission from two metals A and B. Choose the correct statement(s) from the following.

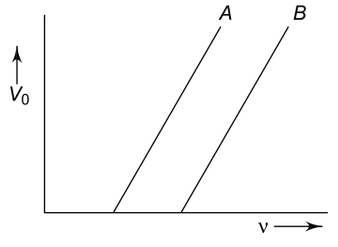


Fig. 28.14

(a) Work function of A is greater than that of B.
(b) Work function of B is greater than that of A.

(c) Threshold frequency of A is greater than that of B.

(d) Threshold frequency of B is greater than that of A

11. The intensity of X-rays from a Coolidge tube is plotted against wavelength as shown in Fig. 28.15. The minimum wavelength found is λ_C and the wavelength of K_α line is λ_k. If the accelerating voltage is increased

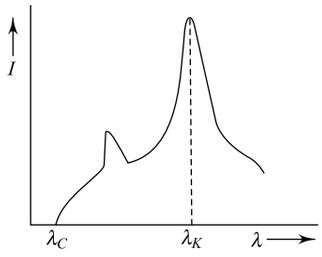


Fig. 28.15

(a) λ_C decreases

(b) λ_K increases

(c) $(\lambda_K - \lambda_C)$ increases

(d) λ_C and λ_K both decrease but $(\lambda_K - \lambda_C)$ remains unchanged

12. A hydrogen atom and a Li^{2+} ion are both in the second excited state. If l_H and l_{Li} are their respective electronic angular momenta, and E_H and E_{Li} their respective energies, then

(a) $l_H = l_{Li}$ (b) $l_H > l_{Li}$

(c) $E_H < E_{Li}$ (b) $E_H > E_{Li}$

13. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statements are not true?

(a) Its kinetic energy increases and its potential and total energies decrease.

(b) Its kinetic energy decreases, potential energy increases and its total energy remains the same

(c) Its kinetic and total energies decrease and its potential energy increases.

(d) Its kinetic, potential and tatal energies decrease.

IIT, 2000

14. Figure 28.16 shows graphs between cut-off voltage V_0 and $\frac{1}{\lambda}$ for three metals 1, 2 and 3, where λ is the wavelength of the incident radiation in nm.
If W_1, W_2 and W_3 are the work functions of metals 1, 2 and 3 respectively, then

(a) $W_1 : W_2 : W_3 = 1 : 2 : 4$

(b) $W_1 : W_2 : W_3 = 4 : 2 : 1$

(c) The graphs for metals 1, 2 and 3 are parallel to each other and the slope of each graph is hc/e, where h = Planck's contant, c = speed of light and e = charge of an electron.

(d) Ultraviolet light will eject photoelectrons from metals 1 and 2 and not from metal 3.

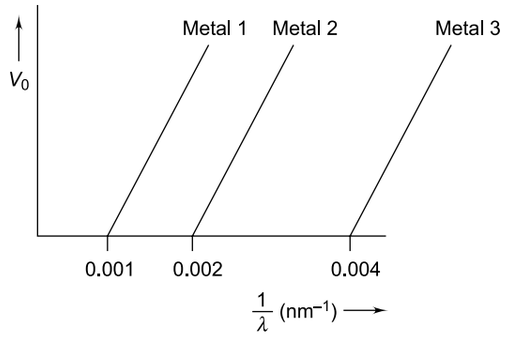


Fig. 28.16

IIT, 2006

15. In a hydrogen-like atom of atomic number $Z = 11$, an electron makes a transition from the nth orbit and emits a radiation in the Lyman series. What is the value of n if the de Broglie wavelength of the electron in the nth orbit is equal to the wavelength of the emitted radiation?

IIT, 2006

(a) $n < 10$ (b) n between 11 and 24

(c) $n = 25$ (d) $n > 30$

SOLUTIONS

1. Electrons will be emitted if the frequency of incident light is greater than 4.5×10^{14} Hz.
Wavelength of infrared light $\simeq$ 10,000 Å, its frequency is ν (infrared) $= \dfrac{3\times10^8}{10,000\times10^{-10}}$

$= 3 \times 10^{14}$ Hz.

Wavelength of red light is about 7800 Å, its frequency is about 3.8×10^{14} Hz. Frequency of sodium light is about 5×10^{14} Hz and the frequency of ultraviolet light is about 15×10^{14} Hz. Hence the correct choices are (c) and (d).

2. The value of n is proportional to the intensity of incident light. If the distance of the lamp is halved, intensity becomes four times. But K_{max} is independent of the intensity of light. Hence the correct choices are (b) and (d).

3. The correct choices are (a) and (b)

4. The frequency of X-rays is higher than that of ultraviolet light. Now $K_{max} = h(\nu - \nu_0)$. Hence K_{max} increases as ν is increased. Also $K_{max} = eV_0$, where V_0 is the stopping potential. Hence V_0 also increases with frequency. Hence the correct choices are (a) and (b).

5. Work function $W_0 = h\nu_0$ and $K_{max} = h(\nu - \nu_0)$. So the correct choices are (a) and (d).

6. The work function of sodium is smaller than that of copper. Since $W_0 = h\nu_0$, the threshold frequency for sodium is less than that for copper. So choice (c) is correct and choice (d) is incorrect.
Since the work function of sodium is lower than that of copper, it is easier to extract electrons from sodium than from copper. Therefore, the electrons ejected from sodium will have a greater kinetic energy and will hence need a greater stopping potential. So choice (a) is incorrect and choice (b) is correct.

7. The cut off potential remains the same as long as the frequency of the incident light remains unchanged.
The saturation current is proportion to the intensity of light. Since the distance has become 3 times its previous value, the intensity is only $\frac{1}{3^2}$ or $\frac{1}{9}$ of its previous value. Hence the new value of the saturation current is $\frac{18.0}{9}$ mA = 2.0 mA. Hence the correct choices are (b) and (d).

8. The energy of each photon is $\frac{hc}{\lambda}$ so that the number of photons released per unit time is $W \div \left(\frac{hc}{\lambda}\right)$.
These photons are spread out in all directions over an area $4\pi a^2$ so that the 'share' of an area S is a fraction $S/4\pi a^2$ of the total number of photons emitted.
The maximum energy of the emitted photoelectrons is
$E_{max} = h\nu - \phi = \frac{hc}{\lambda} - \phi = \frac{1}{\lambda}(hc - \lambda\phi)$.
The stopping potential is given by $eV_S = E_{max}$.
Hence $V_S = \frac{1}{e} E_{max} = \frac{1}{e\lambda}(hc - \lambda\phi)$.
Hence choice (c) is incorrect.
For photoemission to be possible, we must have $h\nu \geq \phi$. Hence $\frac{hc}{\lambda} \geq \phi$ or $\lambda \leq hc/\phi$
Thus the permitted range of values of λ is $0 \leq \lambda \leq hc/\phi$. Hence the correct choices are (a), (b) and (d).

9. The correct statements are (a), (b) and (c).

10. $h\nu = h\nu_0 + eV_0$ or $V_0 = \frac{h}{e}\nu - \frac{h}{e}\nu_0 = \frac{h}{e}(\nu - \nu_0)$.
Thus $V_0 = 0$ if $\nu = \nu_0$. If follow from graphs in Fig. 28.14 that the intercept ν_0 on ν-axis is less for metal A than for metal B. Hence choice (d) is correct. Also since $W_0 = h\nu_0$, the work function of metal A is less than that of metal B. Hence choice (b) is also correct.

11. The minimum wavelength is given by
$$\lambda_C = \frac{hc}{eV}$$
As V increases, λ_C decreases. Since the wavelength of K_α line is due to transition $n = 2$ to $n = 1$ in the element of the target in the tube, wavelength λ_K remains unchanged as V is increased. Hence the difference $(\lambda_K - \lambda_C)$ increases with increase in the accelerating voltage. Thus the correct choices are (a) and (c).

12. For a hydrogen-like atom, the energy in the nth excited state is
$$\therefore \qquad E \propto \left(-\frac{Z^2}{n^2}\right)$$
Since Z for Li^{2+} is greater than Z for H^+, $|E_{Li}| > |E_H|$.
Also $l = \sqrt{n(n+1)}\left(\frac{h}{2\pi}\right)$. Hence $l_{Li} = l_H$. Thus, the correct choices are (a) and (c).

13. Potential energy (PE) $= -\frac{1}{4\pi\varepsilon_0}\left(\frac{Ze^2}{r}\right)$
Kinetic energy (KE) $= \frac{1}{2} \times \frac{1}{4\pi\varepsilon_0}\left(\frac{Ze^2}{r}\right)$
$\therefore$ Total energy E = PE + KE
$$= -\frac{1}{2} \times \frac{1}{4\pi\varepsilon_0}\left(\frac{Ze^2}{r}\right)$$
When an electron makes a transition from an excited state to the ground state, the value of r decreases. From the above expressions it follows that the kinetic energy increases, while the potential energy and the total energy both decrease as they become more negative. Hence the only correct choice is (a).

14. Work function $W = h\nu_0 = \frac{hc}{\lambda_0}$, where λ_0 is the threshold wavelength. Hence
$$W_1 : W_2 : W_3 = \frac{hc}{(\lambda_0)_1} : \frac{hc}{(\lambda_0)_2} : \frac{hc}{(\lambda_0)_3}$$

$$= \frac{1}{(\lambda_0)_1} : \frac{1}{(\lambda_0)_2} : \frac{1}{(\lambda_0)_3}$$
$$= 0.001 : 0.002 : 0.004$$
$$= 1 : 2 : 4$$

Hence choice (a) is correct. In photoelectric emission, the relation between V_0 and λ is given by

$$eV_0 = h\nu - W = \frac{hc}{\lambda} - W$$

or
$$V_0 = \frac{hc}{e}\left(\frac{1}{\lambda}\right) - \frac{W}{e}$$

Hence the slope of the graph between V_0 and $\frac{1}{\lambda}$ is $\frac{hc}{e}$ which is the same for all metals. Therefore, choice (c) is correct. The threshold wavelength for the three metals are

$\frac{1}{(\lambda_0)_1} = 0.001\ \text{nm}^{-1}$, therefore $(\lambda_0)_1 = 1000$ nm $= 10{,}000$ Å

$\frac{1}{(\lambda_0)_2} = 0.002\ \text{nm}^{-1}$, therefore $(\lambda_0)_2 = 500$ nm $= 5{,}000$ Å

$\frac{1}{(\lambda_0)_3} = 0.004\ \text{nm}^{-1}$, therefore $(\lambda_0) = 250$ nm $= 2{,}500$ Å

For photoelectric emission, the wavelength of the incident radiation must be less than the threshold wavelength. Since the wavelength of ultraviolet light is about 1200 Å, it will eject photoelectrons from all the three metals. Hence the correct choices are (a) and (d).

15. According to Bohr's hypothesis, the momentum of the electron in the nth orbit is given by

$$p = \frac{nh}{2\pi r_n}$$

where r_n is the radius of the nth orbit. The de-Broglie wavelength of the electron in the nth orbit is

$$\lambda = \frac{h}{p} = \frac{2\pi r_n}{n}$$

Now $r_n = \frac{r_1 n^2}{Z}$, where $r_1 = 0.53 \times 10^{-10}$ m is the radius of the first orbit of the hydrogen atom. Therefore,

$$\lambda = \frac{2\pi r_1 n}{Z} \quad (1)$$

The wavelength λ' of the transition $n = n$ to $n = 1$ in Lyman series is given by

$$\frac{1}{\lambda'} = RZ^2\left(1 - \frac{1}{n^2}\right)$$

which gives $\lambda' = \frac{n^2}{RZ^2(n^2-1)}$ (2)

where $R = 1.097 \times 10^7\ \text{m}^{-1}$. Equating (1) and (2), we have

$$\frac{2\pi r_1 n}{Z} = \frac{n^2}{R_H Z^2 (n^2-1)}$$

or
$$n^2 - 1 = \frac{n}{2\pi r_1 R_H Z}$$

$$= \frac{n}{2 \times 3.14 \times (0.53 \times 10^{-10}) \times (1.097 \times 10^7) \times 11}$$
$$= 25\,n$$

or $n^2 - 25n - 1 = 0$

The two roots of n are

$$n = \frac{1}{2}\left(25 \pm \sqrt{629}\right) \simeq \frac{1}{2}(25 \pm 25)$$

i.e. $n = 25$ or $n = 0$. Now $n = 0$ is not possible. Hence the value of $n = 25$.

The only correct choice is (c).

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage

Passage I

A beam of light has three wavelengths 440 nm, 495 nm and 660 nm with a total intensity of $3.24 \times 10^{-3}\ \text{Wm}^{-2}$ equally distributed amongst the three wavelengths. The beam falls normally on an area of 1.0 cm^2 of a clean metallic surface of work function 2.2 eV. Assume that there is no loss of light by reflection and each energetically capable photon ejects one electron and take $h = 6.6 \times 10^{-34}$ Js.

IIT, 1989

1. Photoelectric emission is caused by
(a) light of wavelength 440 nm alone
(b) light of wavelength 660 nm alone

(c) lights of wavelengths 440 nm and 495 nm
(d) lights of wavelengths 495 nm and 660 nm

2. The incident energy (in Js^{-1}) of each wavelength is
(a) 3.24×10^{-7} (b) 1.62×10^{-7}
(c) 1.08×10^{-7} (d) 0.81×10^{-7}

3. The total number of photoelectrons liberated per second is
(a) 4.9×10^{11} (b) 5.1×10^{11}
(c) 5.3×10^{11} (d) 5.5×10^{11}

SOLUTION

1. The threshold wavelength is

$$\lambda_0 = \frac{hc}{W_0} = \frac{(6.6\times10^{-34})\times(3\times10^{8})}{2.2\times1.6\times10^{-19}}$$
$$= 6 \times 10^{-7} \text{ m} = 600 \text{ nm}$$

Out of the three given wavelengths, two wavelengths $\lambda_1 = 440$ nm and $\lambda_2 = 495$ nm will cause photoelectric emission as these wavelengths are less than λ_0. Thus the correct choice is (c).

2. Intensity of each wavelength is $I = \frac{1}{3} \times 3.24 \times 10^{-3} = 1.08 \times 10^{-3}$ W m^{-2}. Area of metal surface is $A = 1$ cm^2 $= 1 \times 10^{-4}$ m^2. Therefore, energy of each wavelength is $E = I \times A = 1.08 \times 10^{-7}$ J s^{-1}, which is choice (c).

3. Let n_1 be the number of photons of wavelength λ_1 incident per second. The energy of one photon $= hc/\lambda_1$. Hence

$$E = \frac{n_1 hc}{\lambda_1}$$

or $$n_1 = \frac{E\lambda_1}{hc} = \frac{(1.08\times10^{-7})\times(440\times10^{-9})}{(6.6\times10^{-34})\times(3\times10^{8})}$$
$$= 2.4 \times 10^{11}$$

Similarly $n_2 = 2.7 \times 10^{11}$

$\therefore$ Total number $n = n_1 + n_2 = 5.1 \times 10^{11}$, which is choice (b).

Questions 4 to 6 are based on the following passage

Passage II

In a photoelectric effect set-up, a point source of light of power 3.2×10^{-3} W emits monoenergetic photons of energy 5.0 eV. The source is located at a dsitance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and of radius 8.0×10^{-3} m. The efficiency of photoelectric emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral, and that photoelectrons are instantly swept away after emission.

IIT, 1995

4. The number of photoelectrons emitted per second is
(a) 10^5 (b) 10^7
(c) 10^9 (d) 10^{11}

5. The kinetic energy of the fastest electron is
(a) 8 eV (b) 5 eV
(c) 3 eV (d) 2 eV

6. The photoelectric emission stops when the sphere acquires a potential of
(a) 2 V (b) 3 V
(c) 5 V (d) 8 V

SOLUTION

4. Refer to Fig. 28.17.

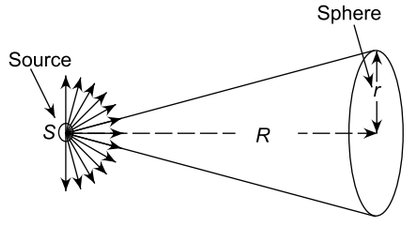


Fig. 28.17

Power of the source of light, $P = 3.2 \times 10^{-3}$ W. Energy of the emitted photon, $E = 5.0$ eV $= 5.0 \times 1.6 \times 10^{-19} = 8.0 \times 10^{-19}$ J.

If r is the radius of the metallic sphere and R its distance from the source S, the power received at the sphere is

$$P' = P \times \frac{\pi r^2}{4\pi R^2} = \frac{P r^2}{4R^2}$$
$$= \frac{3.2\times10^{-3}\times(8.0\times10^{-3})^2}{4\times(0.8)^2}$$
$$= 8 \times 10^{-8} \text{ W } (\text{Js}^{-1})$$

$\therefore$ Number of photons striking the sphere per second is

$$n' = \frac{P'}{E} = \frac{8\times10^{-8}\,\text{Js}^{-1}}{8.0\times10^{-19}\,\text{J}} = 10^{11} \text{ s}^{-1}$$

Since one photoelectron is emitted for every 10^6 incident photons, the number of photoelectrons emitted per second is

$$n = \frac{n'}{10^6} = \frac{10^{11}}{10^6} = 10^5 \text{ per second}$$

The correct choice is (a).

5. Kinetic energy of the fastest electron is

$$E_{max} = 5.0 - 3.0 = 2.0 \text{ eV},$$

which is choice (d).

6. Due to the emission of photoelectrons, the metallic sphere acquires a positive charge and it will oppose the ejection of photoelectrons due to attractive force exerted by the positive charge of the sphere on the electrons. The photoelectric emission will stop when the sphere acquires a positive potential equal to the stopping potential. In other words, the work function of the sphere keeps on increasing with time till it becomes equal to 5.0 eV (which is the energy of the incident photon). At this time, the emission of photoelectrons stops.

Increase in work function = 5.0 eV – 3.0 eV = 2.0 eV. This implies that the photoelectric emission will stop when the sphere has acquired a potential of 2.0 V due to accumulation of charge. Hence the correct choice is (a).

Questions 7 to 10 are based on the following passage

Passage III

A single electron orbits around a stationary nucleus of charge + Ze, where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to third Bohr orbit. The ionization energy of hydrogen atom = 13.6 eV.

IIT, 1981

7. The value of Z is
(a) 3 (b) 4
(c) 5 (d) 6

8. The ionization energy of the atom is
(a) 340 eV (b) 217.6 eV
(c) 122.4 eV (d) 13.6 eV

9. The potential energy of the electron in the first Bohr orbit is
(a) – 680 eV (b) – 340 eV
(c) – 170 eV (d) – 85 eV

10. The angular momentum of the electron in the first Bohr orbit is (h = Planck's constant)
(a) $\frac{h}{2\pi}$ (b) $\frac{h}{\pi}$
(c) $\frac{3h}{2\pi}$ (d) $\frac{5h}{2\pi}$

SOLUTION

7. The energy of the electron in the nth Bohr orbit is

$$E_n = -\frac{Z^2 Rhc}{n^2}$$

Given, ionization energy of hydrogen atom = Rhc = 13.6 eV. Therefore,

$$E_n = -\frac{(13.6\text{eV})Z^2}{n^2}$$

The energy required to excite the electron from $n = 2$ to $n = 3$, Bohr orbit is

$$E_3 - E_2 = -(13.6 \text{ eV})\, Z^2 \left(\frac{1}{3^2} - \frac{1}{2^2}\right)$$

$$= \frac{(13.6\text{eV}) \times 5Z^2}{36}$$

Given $E_3 - E_2 = 47.2$ eV. Hence

$$\frac{(13.6\text{eV}) \times 5Z^2}{36} = 47.2 \text{ eV}$$

which gives $Z = 5$. So the correct choice is (c).

8. Ionization energy of the atom is

$$E_\infty - E_1 = -(13.6 \text{ eV}) \left(\frac{1}{\infty} - \frac{1}{1^2}\right) Z^2$$

$$= (13.6 \text{ eV}) \times (5)^2 = 340 \text{ eV}$$

The correct choice is (a).

9. Kinetic energy of the electron in the first Bohr orbit is $E_1 = -340$ eV. The potential energy in this orbit = $2E_1 = -680$ eV, which is choice (a).

10. Angular momentum = $\frac{nh}{2\pi} = \frac{h}{2\pi}$ ($\because n = 1$). So the correct choice is (a).

Questions 11 to 14 are based on the following passage

Passage IV

Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Ionization potential of hydrogen is 13.6 V and the mass of hydrogen atom = 1.67×10^{-27} kg.

IIT, 1992

11. The energy of the photons causing the photoelectric emission is
(a) 1.168×10^{-19} J (b) 2.912×10^{-19} J
(c) 1.744×10^{-19} J (d) 4.08×10^{-19} J

12. The quantum numbers of the two levels in the emission of the photons are
(a) $n = 1, n = 4$ (b) $n = 1, n = 3$
(c) $n = 2, n = 4$ (d) $n = 3, n = 4$

13. In the transition in Q. 12, the change in the angular momentum of the electron is (h = Planck's constant)
(a) $\frac{h}{2\pi}$ (b) $\frac{h}{\pi}$
(c) $\frac{2h}{\pi}$ (d) $\frac{3h}{2\pi}$

14. The recoil speed of the emitting atom (assuming that it is at rest before the transition) is of the order of
(a) 1 cm s^{-1} (b) 1 ms^{-1}
(c) 10^2 ms^{-1} (d) 10^4 ms^{-1}

SOLUTION

11. Given, E_{max} = 0.73 eV, W = 1.82 eV, ionization potential of hydrogen atom = 13.6 V and mass of hydrogen atom, $m = 1.67 \times 10^{-27}$ kg.

(a) From Einstein's photoelectric equation,

$$h\nu = W + E_{max} = 1.82 + 0.73$$
$$= 2.55 \text{ eV or } 4.08 \times 10^{-19} \text{ J}$$

The correct choice is (d).

12. We are given that the ionization potential of hydrogen atom = 13.6 V. Therefore, the ionization energy = 13.6 eV. The energy levels of hydrogen atom are given by

$$E_n = -\frac{R_H hc}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

Hence the values of energy the electron in n = 1, 2, 3 and 4 energy levels are

$$E_1 = \frac{13.6}{1^2} = -13.6 \text{ eV}$$

$$E_2 = \frac{13.6}{2^2} = -3.4 \text{ eV}$$

$$E_3 = \frac{13.6}{3^2} = -1.51 \text{ eV and}$$

$$E_4 = \frac{13.6}{4^2} = -0.85 \text{ eV}$$

The quantum numbers for which the energy difference is 2.55 eV are n = 2 and n = 4 since

$$E_4 - E_2 = -(0.85) - (-3.4) = 2.55 \text{ eV}$$

So the correct choice is (c).

13. According to Bohr's theory, the angular momentum of the electron in the nth energy state is

$$L_n = \frac{nh}{2\pi}. \text{ For } n = 4, L_4 = \frac{4h}{2\pi} = \frac{2h}{\pi}$$

For $n = 2, L_2 = \frac{2h}{2\pi} = \frac{h}{\pi}$

$\therefore$ Change in angular momentum is

$$\Delta L = L_4 - L_2 = \frac{2h}{\pi} - \frac{h}{\pi} = \frac{h}{\pi}$$

Thus the correct choice is (b).

14. The recoil speed v of the emitting atom of mass m can be found by using the principle of conservation of linear momentum. The momentum p of a photon of wavelength λ is

$$p = \frac{h}{\lambda} = \frac{h}{c/\nu} = \frac{h\nu}{c}$$

where ν is the frequency of the emitted radiation. Hence

$$mv = p = \frac{h\nu}{c} \text{ or } v = \frac{h\nu}{mc}$$

We have seen above that $h\nu = 4.08 \times 10^{-19}$ J. Therefore,

$$v = \frac{4.08 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^8} = 0.81 \text{ ms}^{-1}$$

The correct choice is (b).

Questions 15 to 17 are based on the following passage

Passage V

Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2 Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d.

IIT, 1997

15. The wavelength of the de Broglie wave associated with the electron is
(a) 0.5 Å (b) 1.0 Å
(c) 2 Å (d) 2.5 Å

16. The minimum value of d for the formation of the standing wave is
(a) 2.25 Å (b) 1.5 Å
(c) 0.75 Å (d) 0.5 Å

17. The energy of the electron (in eV) is
(a) 151 (b) 15.1
(c) 1.51 (d) 0.51

SOLUTION

15. From the condition of standing wave formation, we have

$$n\frac{\lambda}{2} = 2\text{Å}$$

and $(n+1)\frac{\lambda}{2} = 2.5$ Å

where n is an integer. These equations on subtraction give $\frac{\lambda}{2} = 2.5 - 2 = 0.5$ Å $\Rightarrow \lambda = 1.0$ Å

16. The minimun value of d = distance between consecutive nodes = $\frac{\lambda}{2} = 0.5$ Å

17. $\lambda = \frac{h}{p} = \frac{h}{mv}$ and $E = \frac{1}{2}mv^2$ and $\phi = \sqrt{2mE}$.

Hence $\lambda^2 = \frac{h^2}{p^2} = \frac{h^2}{2mE}$

$$\Rightarrow \quad E = \frac{h^2}{2m\lambda^2}$$

$$= \frac{\left(6.63\times10^{-34}\right)^2}{2\times\left(9.1\times10^{-31}\right)\times\left(10^{-10}\right)^2}$$

$$= 2.42 \times 10^{-17} \text{ J}$$

$$= \frac{2.42\times10^{-17}}{1.6\times10^{-19}} = 151 \text{ eV}$$

Questions 18 to 20 are based on the following passage
Passage VI

A hydrogen-like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of energy 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. The ground state energy of hydrogen atom = – 13.6 eV.

IIT, 2000

18. The atomic number Z of the atom is
(a) 1 (b) 2
(c) 3 (d) 4

19. The quantum number n is
(a) 1 (b) 2
(c) 3 (d) 4

20. The ground state energy (in eV) of the atom is
(a) – 217.6 (b) – 108.8
(c) – 54.4 (d) – 13.6

SOLUTION

18,19. The atom will emit maximum energy for a transition $n_1 = 2n$ to $n_2 = 1$, we have

$$E_{2n} - E_1 = 204 \text{ eV} \qquad (1)$$

where E_1 is the ground state energy of the atom, Also

$$E_{2n} - E_n = 408 \text{ eV} \qquad (2)$$

Now $E_{2n} = -(13.6 \text{ eV}) \times \frac{Z^2}{(2n)^2}$,

$$E_n = -(13.6 \text{ eV}) \times \frac{Z^2}{n^2}$$

and $E_1 = -(13.6 \text{ eV})\frac{Z^2}{1^2}$

Using these in Equations (1) and (2) and solving, we get $Z = 4$ and $n = 2$

20. $E_1 = -(13.6 \text{ eV}) \times (4)^2 = -217.6$ eV

Questions 21 to 23 are based on the following passage
Passage VII

When a beam of 10.6 eV photons of intensity 2.0 W/m^2 falls on a platinum surface of area 1.0×10^{-4} m^2 and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons.

IIT, 2001

21. The maximum kinetic energy of the photoelectrons is
(a) 10.6 eV (b) 8.1 eV
(c) 5.0 eV (d) 0.53 eV

22. The minimum kinetic energy of the photoelectrons is
(a) zero (b) 0.53 eV
(c) 1.0 eV (d) 5.0 eV

23. The number of photoelectrons emitted per second is
(a) 6.25×10^{19} (b) 6.25×10^{16}
(c) 6.25×10^{11} (d) 6.25×10^{6}

SOLUTION

21. From Einstain's photolectric equation,

$$K_{max} = h\nu - W$$

$$= 10.6 \text{ eV} - 5.6 \text{ eV} = 5.0 \text{ eV}$$

22. The emitted photoelectrons have kinetic energies ranging from zero to a maximum value.

23. Number of photoelectrons emitted per second is

$$N = \frac{\text{Intensity} \times \text{area}}{\text{energy of photon}} \times \frac{0.53}{100}$$

$$= \frac{2.0 \times \left(1.0 \times 10^{-4}\right)}{10.6 \times \left(1.6 \times 10^{-19}\right)} \times \frac{0.53}{100}$$

$$= 6.25 \times 10^{11}$$

Questions 24 to 26 are based on the following passage

Passage VIII

A hydrogen-like atom (described by the Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between – 0.85 eV and – 0.544 eV (*including* both these values). Given $hc = 1240$ eV-nm and ground state energy of hydrogen atom = – 13.6 eV.

IIT, 2002

24. The quantum number of the lowest energy level is
(a) 1 (b) 6
(c) 12 (d) 15

25. The atomic number of the atom is
(a) 1 (b) 2
(c) 3 (d) 4

26. The smallest wavelength emitted in the possible transitions is very nearly equal to
(a) 4 nm (b) 40 nm
(c) 400 nm (d) 4000 nm

SOLUTION

24,25. A total of four energy levels will involve six elecronic transitions as shown in Fig. 28.18

It is given that

$$E_n = - 0.85 \text{ eV}$$

$$E_{n+3} = - 0.544 \text{ eV}$$

Also, form Bohr's theory we know that

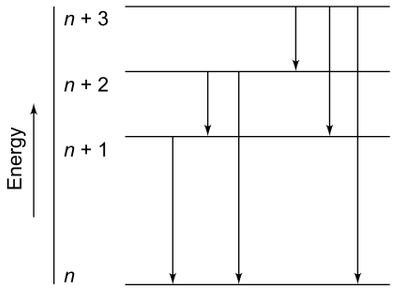


Fig. 28.18

$$E_n = \frac{Z^2}{n^2} (- 13.6 \text{ eV})$$

$$E_{n+3} = -\frac{Z^2}{(n+3)^2} (- 13.6 \text{ eV})$$

Hence, $\frac{E_n}{E_{n+3}} = \frac{(n+3)^2}{n^2}$

The given value is

$$\frac{E_n}{E_{n+3}} = \frac{0.85}{0.544}$$

Hence, $\left(\frac{n+3}{n}\right)^2 = \left(\frac{0.85}{0.544}\right) = 1.5625$

or $\frac{n+3}{n} = 1.25$ or $n = 12$

The atomic number of the atom may be computed as follows.

$$\frac{Z^2}{n^2}(- 13.6 \text{ eV}) = - 0.85 \text{ eV}$$

$$Z^2 = \left(\frac{0.85}{13.6}\right)(n^2) = \left(\frac{0.85}{13.6}\right)(12^2) = 9$$

which gives $Z = 3$

26. The smallest wavelength corresponds to maximum energy difference.

$$(\Delta E)_{max} = E_{max} - E_n = - 0.544 - (- 0.85)$$

$$= 0.306 \text{ eV}$$

$$\lambda_{min} = \frac{hc}{(\Delta E)_{max}} = \frac{1240 \text{ eV nm}}{0.306 \text{ eV}}$$

$$= 4052 \text{ nm}$$

Questions 27 to 29 are based on the following passage

Passage IX

In a mixture of H–He^+ gas (He^+ is singly inoized He atom). H atoms and He^+ ions excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

IIT, 2007

27. The quantum number n of the state finally populated in He^+ ions is
(a) 2 (b) 3
(c) 4 (d) 5

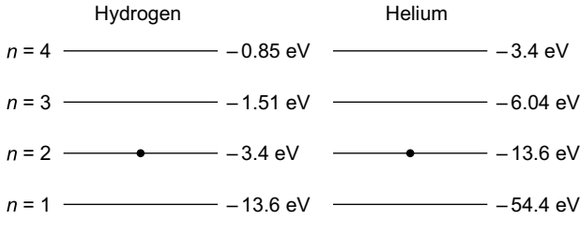
28. The wavelength of light emitted in the visible region by He^+ lons after collisions with H atoms is
(a) 6.5×10^{-7}m (b) 5.6×10^{-7}m
(c) 4.8×10^{-7}m (d) 6.5×10^{-7}m

29. The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of He^+ ion is
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

SOLUTION

27. $E_n = -(13.6\text{ eV}) \times \frac{Z^2}{n^2}$

For hydrogen $Z = 1$ and for helium $Z = 2$.



We notice that the hydrogen atom in $n = 2$ state has the same energy as the helium atom in the $n = 4$ state. If all the excitation energy of hydrogen in $n = 2$ state is transferred to helium, the energy transfer is 10.2 eV. Due to this the helium atom gets excited to $n = 4$ state.

28. $\Delta E = -(13.6\text{ eV})\, Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\lambda = \frac{hc}{\Delta E} = -\frac{hc}{(13.6\text{ eV})Z^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)^{-1}$$

Given $h = 4.1 \times 10^{-15}$ eVs and $c = 3 \times 10^8$ ms^{-1}. For He, $Z = 2$, using these values, we get

$$\lambda = \frac{(4.1\times10^{-15}\text{ eV}s)\times(3\times10^8\text{ ms}^{-1})}{(13.6\text{ eV})\times(2)^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)^{-1}$$

$$= (-0.226 \times 10^{-7}\text{m}) \times \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)^{-1}$$

The wavelength in the visible region corresponds to transitions $n_1 = 4$ to $n_2 = 3$. Thus

$$\lambda = (-0.226 \times 10^{-7}\text{m}) \times \left(\frac{1}{16} - \frac{1}{9}\right)^{-1}$$
$$= 4.65 \times 10^{-7}\text{m}$$

The closest choice is (*c*).

29. For *a* given state *n*, kinetic energy is proportional to Z^2. Hence the correct choice is (*a*).

Questions 30 to 32 are based on the following passage

Passage X

When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometre dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1, 2, 3, ... ($n = 1$, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ J s and $e = 1.6 \times 10^{-19}$ C.

IIT, 2009

30. The allowed energy for the particle for a particular value of n is proportional to
(a) a^{-2} (b) $a^{-3/2}$
(c) a^{-1} (d) a^2

31. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to
(a) 0.8 meV (b) 8 meV
(c) 80 meV (d) 800 meV

32. The speed of the particle, that can take discrete values, is proportional to
(a) $n^{-3/2}$ (b) n^{-1}
(c) $n^{1/2}$ (d) n

SOLUTION

30. $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$

$$\therefore \quad E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \qquad (1)$$

For a standing wave, the distance between consecutive node = $\lambda/2$. If there are n nodes between $x = 0$ and $x = a$, then

$$a = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2a}{n} \qquad (2)$$

Using (2) in (1), we get

$$E = \frac{h^2 n^2}{8ma^2} \qquad (3)$$

i.e. $E \propto a^{-2}$, which is choice (a).

31. From Eq. (3), setting $n = 1$, we have

$$E_1 = \frac{h^2}{8ma^2}$$

$$= \frac{\left(6.6\times10^{-34}\right)^2}{8\times\left(1.0\times10^{-30}\right)\times\left(6.6\times10^{-9}\right)^2}$$

$$= 1.25 \times 10^{-21} \text{ J}$$

$$= \frac{1.25\times10^{-21}}{1.6\times10^{-19}} = 7.8 \times 10^{-3} \text{ eV}$$

$$= 7.8 \text{ meV}$$

The closest choice is (b).

32. $E = \frac{1}{2} mv^2 \Rightarrow v^2 = \frac{2E}{m} = \frac{2h^2n^2}{8m^2a^2}$

Thus $v \propto n$. So the correct choice is (*d*).

Questions 33 to 35 are based on the following passage

Passage XI

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

IIT, 2010

33. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the nth level ($n = 0$ is not allowed) is

(a) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (b) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(c) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (d) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

34. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi} \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its centre of mass is close to (Take $h = 2\pi \times 10^{-34}$ Js)

(a) 2.76×10^{-46} kg m^2

(b) 1.87×10^{-46} kg m^2

(c) 4.67×10^{-47} kg m^2

(d) 1.17×10^{-47} kg m^2

35. In a CO molecule, the distance between C (mass = 12 a.m.u) and O (mass = 16 a.m.u.), where 1 a.m.u. $= \frac{5}{3} \times 10^{-27}$ kg, is close to

(a) 2.4×10^{-10} m (b) 1.9×10^{-10} m

(c) 1.3×10^{-10} m (d) 4.4×10^{-11} m

SOLUTION

33. Bohr's quantization condition is

$$L = \frac{nh}{2\pi} \rightarrow I\omega = \frac{nh}{2\pi}$$

$$\text{Rotational K.E.} = \frac{1}{2} I\omega^2 = \frac{1}{2} I \times \left(\frac{nh}{2\pi I}\right)^2$$

$$= \frac{n^2 h^2}{8\pi^2 I}, \text{ which is choice (d).}$$

34. We have shown in Q.33 that

$$\text{Rotational K.E.} = \frac{n^2 h^2}{8\pi^2 I} \qquad (1)$$

For ground state, $n = 1$ and for the first excited state, $n = 2$. Putting $n = 2$ and $n = 1$ in Eq. (1), we have

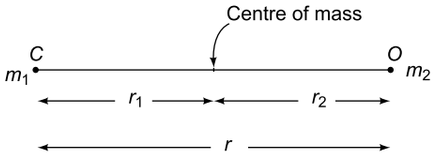
$$(\Delta\text{K.E.})_{\text{rot}} = \frac{h^2}{8\pi^2 I}(2^2 - 1^2) = \frac{3h^2}{8\pi^2 I}$$

This is the excitation energy which is equal to $h\nu$. Hence

$$\frac{3h^2}{8\pi^2 I} = h\nu \Rightarrow I = \frac{3h}{8\pi^2 \nu}$$

Substituting the given values of h and ν, we get $I = 1.87 \times 10^{-46}$ kgm^2. So the correct choice is (b).

35.



$$r_1 = \frac{m_2 r}{m_1 + m_2} \text{ and } r_2 = \frac{m_1 r}{m_1 + m_2}$$

$$I = m_1 r_1^2 + m^2 r_2^2$$

$$= m_1\left(\frac{m_2 r}{m_1+m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1+m_2}\right)^2$$

$$= \frac{m_1 m_2 r^2}{(m_1+m_2)}$$

$$\Rightarrow \quad r = \left[\frac{I(m_1+m_2)}{m_1 m_2}\right]^{1/2}$$

$$= \left[\frac{1.87\times10^{-46}\times(12+16)}{12\times16\times\frac{5}{3}\times10^{-27}}\right]^{1/2}$$

$= 1.3 \times 10^{-10}$ m, which is choice (c).

SECTION IV

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followd by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

1. Statement-1
A particle of mass M at rest decays into two particles of masses m_1 and m_2 which move with velocities v_1 and v_2 respectively. Their respective de Broglie wavelengths are λ_1 and λ_2. If $m_1 > m_2$, then $\lambda_1 > \lambda_2$.
Statement-2
The de Broglie wavelength of a particle having momentum p is $\lambda = h/p$.

2. Statement-1
The de-Broglie wavelength of an electron of mass m moving in the nth Bohr orbit of radius r is $2\pi r/n$.
Statement-2
According to Bohr's theory, the magnitude of the angular momentum of an electron moving with velocity v in the nth orbit is $L = nh/2\pi$.

3. Statement-1
Figure 28.19 shows the graphs of K_{max} (maximum kinetic energy) of the emitted photoelectrons versus the frequency ν of the incident light for two different metals A and B. The lines for metals A and B are always parallel to each other.
Statement-2
In photoelectric emission $K_{max} = h\nu - W_0$, where W_0 is the work function of the metal.

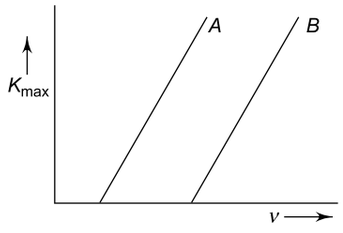


Fig. 28.19

4. Statement-1
From the graph in Q.3 above it follows that the work function of metal B is greater than that of metal A.
Statement-2
The work function does not depend upon the slope of the graph.

5. Statement-1
When an electron in a hydrogen atom makes a transition from an excited state to the ground state, its kinetic energy increases, its potential energy decreases and its total energy remains constant.
Statement-2
In a given orbit, the total energy of the electron consists of its kinetic energy and the electrostatic potential energy of the electron and the proton in the hydrogen atom.

6. Statement-1
When monochromatic light falls on a photosensitive material, the number of photoelectrons emitted per second is n and their maximum kinetic energy is K_{max}. If the intensity I of the incident light is doubled, n is doubled but K_{max} remains the same.
Statement-2
The value of n is directly proportional to I but K_{max} is independent of I.

7. **Statement-1**
When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, K_{max} increases but V_0 decreases.
Statement-2
Photoelectrons are emitted with speeds ranging from zero to a maximum value. Below a certain negative voltage V_0, no photoelectrons are emitted in a photocell.

8. **Statement-1**
If the voltage of an X-ray tube is increased, the minimum wavelength of the emitted radiation decreases.
Statement-2
The maximum frequency of the radiation in an *X*-ray tube is directly proportional to the voltage.

SOLUTIONS

1. The correct choice is (d). The law of conservation of linear momentum gives
$$m_1v_1 + m_2v_2 = 0 \text{ or } \left|\frac{m_2v_2}{m_1v_1}\right| = 1.0$$
Since de Broglie wavelength $\lambda = h_1/mv$, we will have
$$\frac{\lambda_1}{\lambda_2} = \frac{m_2v_2}{m_1v_1} = 1.0$$

2. The correct choice is (a). For nth Bohr orbit, $mvr = \frac{nh}{2\pi}$. The de-Broglie wavelength is
$$\lambda = \frac{h}{mv}$$
But $mv = \frac{nh}{2\pi r}$. Therefore,
$$\lambda = h \times \frac{2\pi r}{nh} = \frac{2\pi r}{n}$$

3. The correct choice is (a). The slope of each graph = h, the Planck's constant.

4. The correct choice is (c). The intercept of the line on the ν-axis gives the threshold frequency ν_0 and work function $W_0 = h\nu_0$. Thus work function = slope × intercept. The value of slope is the same for metals A and B but ν_0 for B is greater than that for A.

5. The correct choice is (d).
Potential energy (P.E.) = $-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}$
Kinetic energy (K.E.) = $\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_n}$

6. The correct choice is (a).

7. The correct choice is (d). The frequency of X-rays is higher than that of ultraviolet light. Since $K_{max} = h\nu - W_0$, K_{max} will increase if ν is increased. Also, $K_{max} = eV_0$. Hence V_0 will also increase if ν is increased.

8. The correct choice is (a). The maximum frequency is given by
$$\nu_{max} = \frac{eV}{h}. \text{ Thus } \nu_{max} \propto V$$
Hence $$\lambda_{min} = \frac{c}{\nu_{max}} = \frac{ch}{eV}$$
Thus $$\lambda_{min} \propto \frac{1}{V}$$

SECTION V

Integer Answer Type Questions

1. Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975 Å. How many different lines are possible in the resulting spectrum? The ionization energy of hydrogen atom is 13.6 eV.
IIT, 1982

2. A hydrogen-like atom of atomic number Z is in an excited state of quantum number $n = 6$. This excited atom makes a transition to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV respectively. Find the value of Z. The ionization energy of hydrogen is 13.6 eV.
IIT, 1994

3. The electric potential between a proton and an electron is given by

$$V = V_0 \ln\left(\frac{r}{r_0}\right)$$

where V_0 and r_0 are constants and r is the radius of the electron orbit around the proton. Assuming Bohr's model to be applicable, it is found that r is proportional to n^x, where n is the principal quantum number. Find the value of x.

IIT, 2003

4. The de Broglie wavelength of an electron moving with a velocity of 1.5×10^8 ms^{-1} is equal to that of a photon. Find the ratio of the kinetic energy of the photon to that of the electron.

IIT, 2004

5. An element of atomic number 9 emits K_α X-ray of wavelength λ. Find the atomic number of the element which emits K_α X-ray of wavelength 4λ.

IIT, 2004

6. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

7. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (where $1 < A < 10$). The value of 'Z' is

IIT, 2011

SOLUTION

1. Energy of monochromatic radiation is

$$E = h\nu = \frac{hc}{\lambda} = \frac{\left(6.63 \times 10^{-34}\right) \times \left(3 \times 10^8\right)}{\left(975 \times 10^{-10}\right)}$$

$$= 20.4 \times 10^{-19} \text{ J} = 12.75 \text{ eV}$$

Let n be the number of the possible spectral lines. Then

$$12.75 = 13.6\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\Rightarrow \quad \frac{12.75}{13.6} = 1 - \frac{1}{n^2}$$

$$\Rightarrow \quad \frac{1}{n^2} = \frac{0.85}{13.6} = \frac{1}{16} \Rightarrow n = 4$$

2. For the first excited state, $n = 2$. Hence

$$-13.6 \text{ eV}\left(\frac{1}{6^2} - \frac{1}{2^2}\right)Z^2 = 10.20 + 17.00 = 27.20 \text{ eV}$$

$$\Rightarrow \quad \left(\frac{1}{36} - \frac{1}{4}\right)Z^2 = \frac{27.20}{13.6}$$

$$\Rightarrow \quad Z^2 = 9 \quad \text{or} \quad Z = 3.$$

3. $V = V_0 \ln\left(\frac{r}{r_0}\right)$. Therefore,

$$F = -\frac{dV}{dr} = \frac{V_0}{r}$$

If v is the orbital speed of the electron and m its mass

$$\frac{mv^2}{r} = \frac{V_0}{r} \Rightarrow mv^2 = V_0 \qquad (1)$$

Bohr's quantum condition is

$$mvr = \frac{nh}{2\pi}$$

$$\Rightarrow \quad v = \frac{nh}{2\pi mr} \qquad (2)$$

Using (2) in the (1) we get

$$r^2 = \left(\frac{h^2}{4\pi^2 m V_0}\right)n^2$$

$$\Rightarrow \quad r \propto n. \quad \text{Hence } x = 1$$

4. Speed of photon (c) = 3×10^8 ms^{-1}. Let λ be the wavelength of the photon. The de Broglie wavelength of the electron = $\frac{h}{mv}$.

Given $\lambda = \frac{h}{mv}$. Now

$$\frac{\text{K.E. of photon}}{\text{K.E. of electron}} = \frac{h\nu}{\frac{1}{2}mv^2} = \frac{2hc}{mv^2\lambda} \quad \left(\because \nu = \frac{c}{\lambda}\right)$$

$$= \frac{2c}{v} \quad \left(\because \lambda = \frac{h}{mv}\right)$$

$$= \frac{2 \times 3 \times 10^8}{1.5 \times 10^8} = 4$$

5. For K_α X-ray, $(Z-1)^2\ \lambda$ = constant. Hence

$$(9-1)^2\ \lambda = (Z-1)^2\ (4\lambda)$$

$$\Rightarrow\quad (Z-1)^2 = \frac{64}{4} = 16$$

$$\Rightarrow\qquad Z-1 = 4 \qquad \text{or} \qquad Z = 5$$

6. $\lambda = \dfrac{h}{\sqrt{2\,mqV}}$

For proton $m = m_p$, $q = q_p = e$

For α-particle $m = m_\alpha = 4\ m_p$ and $q = q_\alpha = 2e$

$$\therefore \qquad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p} \times \frac{q_\alpha}{q_p}}$$

$$= \sqrt{4 \times 2} = \sqrt{8} = 2.83$$

The integer nearest to 2.83 is 3.

7. Energy of incident radiation = $h\nu = \dfrac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{200 \times 10^{-9}}$$

$$= 9.945 \times 10^{-17}\ \text{J}$$

$$= 6.2\ \text{eV}$$

Work function $W_0 = 4.7$ eV

$\therefore$ stopping potential is $V_0 = 6.2 - 4.7 = 1.5$ eV

Now $\qquad V_0 = \dfrac{q}{4\pi\varepsilon_0 r}$

$$\Rightarrow \qquad 1.5 = \frac{ne}{4\pi\varepsilon_0 r} = \frac{n \times 1.6 \times 10^{-19} \times 9 \times 10^9}{1 \times 10^{-2}}$$

$\Rightarrow \qquad n = 1.04 \times 10^7$. Hence $Z = 7$.

29 Chapter

Nuclear Physics

REVIEW OF BASIC CONCEPTS

29.1 RUTHERFORD'S α-PARTICLE SCATTERING EXPERIMENT

A beam of fast α-particles was made to fall on a thin metal foil. Rutherford observed that most of the α-particles passed through the foil without any appreciable deflection. The distance of the closest approach of an α-particle is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{E_k}$$

where $E_k = \frac{1}{2}mv^2$ is the initial kinetic energy of particle and Z is the atomic number of the nucleus. The impact parameter is given by

$$b = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2\cot(\theta/2)}{mv^2}$$

where θ is the scattering angle.

29.2 COMPOSITION OF THE NUCLEUS

The nucleus of an atom contains protons and neutrons which are collectively called nucleons. The total number of nucleons is called the mass number and it is denoted by A. The number of protons in a nucleus is called its atomic number Z. The number of neutrons is denoted by N, so that

$$A = Z + N$$

29.3 ISOTOPES, ISOBARS AND ISOTONES

Nuclei with the same Z but different A are called isotopes, such as ${}^{16}_{8}O$, ${}^{17}_{8}O$, ${}^{18}_{8}O$; ${}^{235}_{92}U$, ${}^{238}_{92}U$, ${}^{236}_{92}U$; ${}^{206}_{82}Pb$, ${}^{207}_{82}Pb$, ${}^{208}_{82}Pb$.

Nuclei with the same A but different Z are called isobars, such as ${}^{3}_{1}H$, ${}^{3}_{2}He$; ${}^{7}_{3}Li$, ${}^{7}_{4}Be$; ${}^{40}_{18}Ar$, ${}^{40}_{20}Ca$.

Nuclei with the same $(A - Z) = N$ but different Z are called isotones such as ${}^{3}_{1}H$, ${}^{3}_{2}H$; ${}^{3}_{1}H$, ${}^{4}_{2}He$; ${}^{17}_{7}N$, ${}^{18}_{8}O$, ${}^{19}_{9}F$

29.4 THE ATOMIC NUCLEUS

The first correct description of the distribution of positive and negative charges within the atom was given by Ernest Rutherford. His α-particle scattering experiments suggest that most of the mass of an atom is located in a very small volume of radius of the order of 10^{-14} m, called the nucleus. The electrons reside outside the nucleus. The nucleus of every atom carries a positive charge. The radius of an atom is of the order of 10^{-10} m. The radius R of a nucleus of mass number A is given by the relation

$$R = R_0 A^{1/3}$$

where R_0 is a quantity which varies slightly from one nucleus to another.

29.5 MASS DEFECT AND BINDING ENERGY

The mass of a nucleus which contains Z protons and $(A-Z)$ neutrons is always less than the sum of the masses of these particles in the free state. The difference is called the mass defect of the given nucleus and is given by

$$\Delta m = Z\,m_p + (A - Z)\,m_n - m$$

where m = mass of the nucleus, m_p = mass of a proton and m_n = mass of a neutron. The binding energy of the nucleus is given by

$$\text{BE} = (\Delta m)c^2 = [Z\,m_p + (A - Z)\,m_n - m]c^2$$

where c is the speed of light in free space. Since A is the total number of nucleons,

$$\text{BE per nucleon} = \frac{\left[Z\,m_p + (A-Z)\,m_n - m\right]c^2}{A}$$

29.6 NUCLEAR FISSION

The splitting of a heavy nucleus into two or more fragments of moderate and comparable sizes is called nuclear fission. The fission of uranium-235 is represented by the reaction.

$$^{235}_{92}\text{U} \longrightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 2{}^{1}_{0}\text{n} + \text{energy}$$

The energy released per fission is about 200 MeV– much more than the energy released in the usual nuclear reactions. This makes the *fission reaction* a particularly suitable source of energy.

The fission reaction given above has a unique feature. Apart from the fission fragments, the reaction results in the release of 2 to 3 neutrons—the very particles that initiated the reaction. So fission after fission, the neutrons present in a bulk sample of uranium increase in geometric ratio.

The rate of energy release also increases similarly in a geometric ratio. The fission reaction is thus a *self sustaining chain reaction*.

When the number of neutrons released per fission is limited to one per fission by *absorption* of excess neutrons, the chain reaction is a *controlled* one and is used in nuclear reactors. When there is no such control on the number of released neutrons, we have an uncontrolled chain reaction and this is the source of energy in the atom bomb.

An essential part of a controlled fission reaction is known as the moderator. The role of the moderator is to slow down the neutrons released in fissions so that they may be easily absorbed by another $^{235}{}_{92}\text{U}$ nucleus. Media which contain nuclei of masses comparable to the neutron are found to act as efficient moderators.

29.7 NUCLEAR FUSION

The process of nuclear fusion consists in the 'combination' of two light nuclei to form a stable nucleus of mass less than the total initial mass. It is believed to be the main source of energy for the sun and the stars. The fusion reaction in stars is believed to occur either *via* the *proton-proton* cycle or the *carbon-cycle*. The proton-proton cycle is as follows:

$$^{1}_{1}\text{H} + {}^{1}_{1}\text{H} \longrightarrow {}^{2}_{1}\text{H} + \text{e}^{+} + n$$
$$^{1}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{2}\text{He} + \nu$$
$$^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \longrightarrow {}^{4}_{2}\text{He} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H}$$

The energy released in this sequence works out to be 24.7 MeV.

Nuclear fusion occurs at very high temperatures of about 10^7 K and under extremely high pressures.

29.8 RADIOACTIVITY

The phenomenon of self-emission of radiations from a nucleus is called radioactivity and substances which emit these radiations are called radioactive substances. The radiations emitted from a radioactive element are of three types.

1. *Alpha rays*: These rays consist of α-particles. An alpha particle is a helium nucleus having two protons and two neutrons. It has a positive charge equal to the charge of two protons. It has an initial speed of about 10^7 ms^{-1}. They have very little penetrating power.
2. *Beta rays*: These rays consist of electrons. Their speed is very nearly equal to the speed of light. They have more penetrating power than alpha particles.
3. *Gamma rays*: These are high frequency electromagnetic waves having a high penetrating power.

Alpha Decay

The process of emission of an alpha particle from a nucleus is called alpha decay. When a nucleus emits an alpha particle $\left({}^{4}_{2}\text{He}\right)$, it loses two protons and two neutrons which means that the daughter nucleus has its mass number reduced by 4 and its atomic number reduced by 2. When a nucleus ${}^{A}_{Z}\text{X}$ of mass number A and atomic number Z emits an α-particle $\left({}^{4}_{2}\text{He}\right)$, it is transformed into a nucleus ${}^{A-4}_{Z-2}\text{Y}$ whose mass number is $(A-4)$ and atomic number is $(Z-2)$. Alpha decay is represented by

$$^{A}_{Z}\text{X} \xrightarrow{\text{minus } {}^{4}_{2}\text{He}} {}^{A-4}_{Z-2}\text{Y} + \text{energy}$$

Beta Decay

The process of the emission of an electron from a nucleus is called *beta decay*. In this process also, the nucleus achieves greater stability by emitting an electron. A neutron inside the nucleus is changed into a proton by the emission of an electron. Because the mass of an electron is negligibly small, the mass number of the resulting nucleus remains unaltered but its atomic number is increased by one. For example, when a radium nucleus $^{228}_{88}\text{Ra}$ emits a β-particle, the resulting element is an isotope of actinium $^{228}_{89}\text{Ac}$. Thus in β-decay also, a new element is formed.

The transformation of a nucleus ${}^{A}_{Z}\text{X}$ into the nucleus ${}^{A}_{Z+1}\text{Y}$ by β-decay is represented by an equation

$$^{A}_{Z}\text{X} \xrightarrow{\text{minus } \text{e}^{-}} {}^{A}_{Z+1}\text{Y}$$

Gamma Decay

Gamma rays are high frequency electromagnetic radiations (i.e. photons) which do not carry any charge. Hence in γ-decay, the mass number and atomic number of the nucleus remain unchanged so that no new element is formed.

Radioactive Decay Law

If N is the number of radioactive nuclei present in a sample at a given instant of time, then the rate of decay at that instant is proportional to N, i.e.

$$\frac{dN}{dt} = -\lambda N$$

The proportionality constant λ is called the disintegration constant. If N_0 is the number of radioactive nuclei at time $t = 0$, then the number of radioactive nuclei at a later time t is given by

$$N = N_0 e^{-\lambda t}$$

Half life: The half life of a radioactive element is the time in which half the number of nuclei decay. It is given by

$$T = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Average life: The average life of a radioactive sample is the reciprocal to its disintegration constant, i.e.

$$T_{av} = \frac{1}{\lambda}$$

Radioactivity decay rate or Activity: It is useful to use the concept of the decay rate R which is defined as the number of radioactive disintegrations taking place in a sample per second, which is given by

$$R = \frac{dN}{dt} = -\lambda N \text{ or } |R| = \lambda N = \frac{0.693}{T} N$$

As N decreases exponentially with time, R will also decrease exponentially with time. The SI unit of the decay rate R is called *curie* (symbol Ci) in honour of Madame M.S. Curie (1867–1934). It is defined as the decay rate of 3.7×10^{10} disintegrations per second, i.e.

1 Ci (curie) = 3.7×10^{10} disintegrations/s

1 mCi (millicurie) = 3.7×10^{7} disintegrations/s

1 μCi (microcurie) = 3.7×10^{4} disintegrations/s

EXAMPLE 29.1

Calculate the distance of closest approach when a 5 MeV proton approaches a gold nucleus. Atomic number of gold is $Z = 79$.

SOLUTION

Kinetic energy of porton is

$$K = 5 \text{ MeV} = 5 \times 10^6 \text{ MeV}$$

$$= 5 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 8 \times 10^{-13} \text{ J}.$$

The distance of closest approach is

$$r_0 = \frac{1}{4\pi E_0} \frac{Ze^2}{K}$$

$$= \frac{9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{8 \times 10^{-13}}$$

$$= 2.3 \times 10^{-14} \text{ m}$$

EXAMPLE 29.2

Calculate the ratio of the radii of two nuclei of mass numbers 1 and 27.

SOLUTION

$$R_1 = R_0 A_1^{1/3} \text{ and } R_2 = R_0 A_2^{1/3}$$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{1}{27}\right)^{1/3} = \frac{1}{3}$$

EXAMPLE 29.3

16 g of radioactive radon is kept in a container. How much radon will disintegrate in 19 days? Half life of radon is 3.8 days.

SOLUTION

Number of half lives in 19 days is

$$n = \frac{t}{T} = \frac{19 \text{ days}}{3.8 \text{ days}} = 5$$

$\therefore$ Number of atoms left undecayed after 5 half lives is

$$N = N_0 \left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^5 = \frac{N_0}{32}$$

$\therefore$ Mass of radon left undecayed after 19 days

$$= \frac{16 \text{ g}}{32} = 0.5 \text{ g}$$

$\therefore$ Mass of radon disintegrated = 16 – 0.5 = 15.5 g

EXAMPLE 29.4

The half life of a radioactive substance is 30 days. What is the time taken for 3/4 of its original mass to disintegrate?

SOLUTION

$$T = 30 \text{ days}$$

$$N = N_0 - \frac{3N_0}{4} = \frac{N_0}{4}$$

$$N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{4} = \left(\frac{1}{2}\right)^n \Rightarrow n = 2$$

$\therefore$ Time taken = 2 half lives

$= 2 \times 30 = 60$ days.

EXAMPLE 29.5

The activity of a radioactive element reduces to $\frac{1}{16}$ th of its original value in 30 years. Find the half life and the decay constant of the element.

SOLUTION

$|R| = \lambda N$, i.e. the activity of proportional to the number atoms present in the sample.

Given $N = \frac{N_0}{16}$

From $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, we have

$$\frac{1}{16} = \left(\frac{1}{2}\right)^n \Rightarrow n = 4$$

i.e. there are 4 half lives in 30 years. Therefore the half life of the element is

$$T = \frac{30 \text{ years}}{4} = 7.5 \text{ years}$$

Decay constant $\lambda = \frac{0.693}{T} = \frac{0.693}{7.5 \text{ years}}$

$= 0.0924$ per year.

EXAMPLE 29.6

Two radioactive substances A and B initially contain equal number of atoms. The half lives of A and B are 1 hour and 2 hours respectively. Find the ratio of their rates of disintegration at the end of 2 hours.

SOLUTION

Given $T_A = 1$ hour and $T_B = 2$ hours. At $t = 0$, number of atoms of A = number of atoms of $B = N_0$.

$\therefore$ Number of atoms of A after 2 hours (i.e. after 2 half lives of A) is

$$N_A = N_0\left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$$

Number of atoms of B after 2 hrs (i.e. after 1 half life of B) is

$$N_B = \frac{N_o}{2}$$

Now $|R| = \lambda N = \frac{0.693N}{T}$. Therefore.

$$R_A = \frac{0.693N_A}{T_A} \text{ and } R_B = \frac{0.693N_B}{T_B}$$

$\therefore$ $\frac{R_A}{R_B} = \frac{N_A}{N_B} \times \frac{T_B}{T_A} = \frac{N_0/4}{N_0/2} \times \frac{2}{1} = 1$

EXAMPLE 29.7

A sample contains 2.3 g of radioactive $^{230}_{90}$Th of half life 2.4×10^{11} seconds. How many disintegrations per second occur in the sample? Take Avogadro number $= 6 \times 10^{23}$ atoms per mole.

SOLUTION

1 mole of a substance has a mass equal to its atomic mass expressed in grams. Hence, number of moles in 2.3 g of $^{230}_{90}$Th is

$$\frac{2.3 \text{ g}}{230 \text{ g/mole}} = 10^{-2} \text{ mole}$$

Also Avogdro number = number of atoms in 1 mole of the substance. Hence number of atoms in 10^{-2} mole is

$$N = 6 \times 10^{23} \times 10^{-2} = 6 \times 10^{21} \text{ atoms}$$

$\therefore$ $|R| = \frac{0.693N}{T} = \frac{0.693 \times 6 \times 10^{21}}{2.4 \times 10^{11}}$

$= 1.73 \times 10^{10}$ disintegrations per second.

EXAMPLE 29.8

A radioactive substance of half life of 69.3 days is kept in a container. After a certain lapse of time, it was found that 20% of the substance is left undecayed. Find the time elapsed. Given ln (5) = 1.61.

SOLUTION

$$T = 69.3 \text{ days}$$

$\therefore$ $\lambda = \frac{0.693}{T} = \frac{0.693}{69.3 \text{ days}}$

$= 10^{-2}$ per day

$N = 20\%$ of $N_0 = 0.2\ N_0$. Therefore

$$\frac{N_0}{N} = 5$$

Now $N = N_0\, e^{-\lambda t} \Rightarrow \dfrac{N}{N_0} = e^{-\lambda t}$

$\Rightarrow \dfrac{N_0}{N} = e^{\lambda t} \Rightarrow \ln\left(\dfrac{N_0}{N}\right) = \lambda t$

$\therefore t = \dfrac{\ln(N_0/N)}{\lambda} = \dfrac{\ln(5)}{10^{-2}\text{ per day}}$

$= \dfrac{1.61}{10^{-2}}\text{ days} = 161\text{ days}$

EXAMPLE 29.9

1 gram of caesuim $\left({}^{137}_{55}\text{Cs}\right)$ decays by β-emission with a half life of 30 years. (a) Name the resulting isotope. (b) Write the equation of decay. (c) If the initial activity of caesuim is 1.0 millicurie (mCi), what is its activity after 60 years.

SOLUTION

(a) In β-decay, the mass number remains unchanged and the atomic number increases by 1. The resulting isotope has a mass number 137 and atomic number 56. This corresponds to the isotope ${}^{137}_{56}\text{Ba}$.

(b) The equation of β-decay is

$${}^{137}_{55}\text{Cs} \xrightarrow{\beta\text{-decay}} {}^{137}_{56}\text{Ba} + {}^{0}_{-1}e + \bar{\nu} + Q$$

where $\bar{\nu}$ is an antineutrino and Q is the energy released in β-decay.

Initial activity is $R_0 = 1.0$ mCi

Since there are 2 half lives in 60 years and since the activity is proportional to the number of atoms, the activity at the end of 60 years is

$$R = R_0 \times \left(\frac{1}{2}\right)^2 = \frac{R_0}{4} = \frac{1.0\,\text{mCi}}{4}$$

$$= 0.25\text{ mCi}$$

SECTION I

Multiple Choice Questions with Only One Choice Correct

1. The distance of the closest approach of an alpha particle fired at a nucleus with kinetic energy K is r_0. The distance of the closest approach when the alpha particle is fired at the same nucleus with kinetic energy $2K$ will be

(a) $2r_0$ (b) $4r_0$

(c) $\dfrac{r_0}{2}$ (d) $\dfrac{r_0}{4}$

2. The distance of the closest approach of an alpha particle fired at a nucleus with momentum p is r_0. The distance of the closest approach when the alpha particle is fired at the same nucleus with momentum $2p$ will be

(a) $2r_0$ (b) $4r_0$

(c) $\dfrac{r_0}{2}$ (d) $\dfrac{r_0}{4}$

3. When high energy alpha-particles $({}^{4}_{2}\text{He})$ pass through nitrogen gas, an isotope of oxygen is formed with the emission of particles named x. The nuclear reaction is

$${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + x$$

What is the name of x?

(a) electron (b) proton

(c) neutron (d) positron

4. What is particle x in the following nuclear reaction?

$${}^{9}_{4}\text{Be} + {}^{4}_{2}\text{He} \rightarrow {}^{12}_{6}\text{C} + x$$

(a) electron (b) proton

(c) neutron (d) photon

5. When aluminium is bombarded with fast neutrons, it changes into sodium with emission of particle x according to the equation

$${}^{27}_{13}\text{Al} + {}^{1}_{0}\text{n} \rightarrow {}^{24}_{11}\text{Na} + x$$

What is x?

(a) electron (b) proton

(c) neutron (d) alpha-particle

6. In the equation

$${}^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \rightarrow {}^{30}_{15}\text{P} + X,$$

The correct symbol for X is

(a) ${}^{0}_{-1}\text{e}$ (b) ${}^{1}_{1}\text{H}$

(c) ${}^{4}_{2}\text{He}$ (d) ${}^{1}_{0}\text{n}$

7. The energy released by the fission of one uranium atom is 200 MeV. The number of fissions per second required to produce 3.2 W of power is

(a) 10^7 (b) 10^{10}
(c) 10^{15} (d) 10^{17}

8. What is the energy released in the fission reaction

$$^{236}_{92}\text{U} \rightarrow {}^{117}_{46}\text{X} + {}^{117}_{46}\text{Y} + 2\ {}^{1}_{0}\text{n}$$

given that the binding energy per nucleon of X and Y is 8.5 MeV and that of $^{236}_{92}$U is 7.6 MeV?
(a) 20 MeV (b) 180 MeV
(c) 200 MeV (d) 2000 MeV

9. The binding energy of deuteron ($^2_1$H) is 1.15 MeV per nucleon and an alpha particle ($^4_2$He) has a binding energy of 7.1 MeV per nucleon. Then in the reaction

$$^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He} + \text{Q}$$

the energy Q released is
(a) 1 MeV (b) 11.9 MeV
(c) 23.8 MeV (d) 931 MeV

10. If M is the mass of a nucleus and A its atomic mass, then the packing fraction is
(a) $\dfrac{M-A}{M+A}$ (b) $\dfrac{M-A}{M}$
(c) $\dfrac{M-A}{A}$ (d) $\dfrac{M+A}{M-A}$

11. The half life of a certain radio isotope is 10 minutes. The number of radioactive nuclei at a given instant of time is 10^8. Then the number of radioactive nuclei left 5 minutes later would be
(a) $\dfrac{10^8}{2}$ (b) 10^4
(c) $\sqrt{2} \times 10^7$ (d) $\dfrac{10^8}{\sqrt{2}}$

12. The half life of Pa – 218 is 3 minutes. What mass of a 16 g sample of Pa – 218 will remain after 15 minutes?
(a) 3.2 g (b) 2.0 g
(c) 1.6 g (d) 0.5 g

13. The radioactivity of a sample is X at a time t_1 and Y at a time t_2. If the mean life of the specimen is τ, the number of atoms that have disintegrated in the time interval $(t_2 - t_1)$ is
(a) $X\,t_1 - Y\,t_2$ (b) $X - Y$
(c) $(X - Y)/\tau$ (d) $(X - Y)\,\tau$

14. The decay constant of a radioactive sample is λ. The half-life and mean-life of the sample are (respectively) given by:
(a) $1/\lambda$ and $(\ln 2)/\lambda$ (b) $(\ln 2)/\lambda$ and $1/\lambda$
(c) $1/\lambda$ and $\lambda\ (\ln 2)$ (d) $\lambda\ (\ln 2)$ and $1/\lambda$

15. The ionising power and the penetration range of radioactive radiations increase in the order
(a) γ, β, α and γ, β, α respectively
(b) γ, β, α and α, β, γ respectively
(c) α, β, γ and α, β, γ respectively
(d) α, β, γ and γ, β, α respectively

IIT, 1994

16. A radioactive element X has atomic number Z and atomic mass number A. It decays by the emission of an alpha particle and a gamma ray. The new element is
(a) $^{A-2}_{Z-1}Y$ (b) $^{A-4}_{Z-2}Y$
(c) $^{A+1}_{Z}Y$ (d) $^{A+4}_{Z+2}Y$

17. The radioactive decay of uranium into thorium is represented by the equation

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + x$$

What is x?
(a) an electron (b) a proton
(c) an alpha particle (d) a neutron

18. A carbon nucleus emits a particle x and changes into nitrogen according to the equation

$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + x$$

What is x?
(a) an electron (b) a proton
(c) an alpha particle (d) a photon

19. The radioactive decay of an element X to elements Y and K is represented by the equation

$$^{A}_{Z}\text{X} \rightarrow {}^{A}_{Z+1}\text{Y} \rightarrow {}^{A-4}_{Z-1}\text{K} \rightarrow {}^{A-4}_{Z-1}\text{K}$$

The sequence of the emitted radiations is
(a) α, β, γ (b) β, α, γ
(c) γ, α, β (d) β, γ, α

20. The half-life of a radioactive substance is 10 days. This means that
(a) the substance completely disintegrates in 20 days
(b) the substance completely disintegrates in 40 days
(c) 1/8 part of the mass of the substance will be left intact at the end of 40 days
(d) 7/8 part of the mass of the substance disintegrates in 30 days.

21. A rate-meter measures the number of disintegrations per second from a radioactive source. It gives a count of 320 counts per second. Ninety minutes later, it gives 40 counts per second. What is the half-life of the source?
(a) 30 minutes (b) 45 minutes
(c) 60 minutes (d) 75 minutes

22. Beta rays emitted by a radioactive material are
(a) electromagnetic radiations
(b) the electrons orbiting around the nucleus
(c) charged particles emitted by the nucleus
(d) neutral particles

IIT, 1983

23. The equation
$4\,{}_1^1H^+ \rightarrow {}_2^4He^+ + 2e^+ + 26$ MeV represents
(a) β-decay (b) γ-decay
(c) fusion (d) fission

24. What is the number of α and β particles emitted in the following radioactive decay?

$${}_{90}^{200}X \rightarrow {}_{80}^{168}Y$$

(a) 8 and 6 (b) 6 and 8
(c) 8 and 8 (d) 6 and 6

25. A freshly prepared radioactive source of half life 2 hours emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with the source is
(a) 6 hours (b) 12 hours
(c) 24 hours (d) 128 hours

IIT, 1988

26. A beam of fast-moving alpha particles was directed towards a thin gold film. The parts A', B' and C' of the transmitted and reflected beams corresponding to the incident parts A, B and C of the beam are shown in the Fig. 29.1. The number of alpha particles in
(a) B' will be minimum and in C' maximum
(b) A' will be maximum and in B' minimum
(c) A' will be minimum and in B' maximum
(d) C' will be minimum and in B' maximum

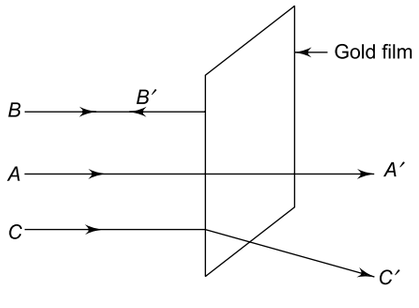


Fig. 29.1

27. A radioactive sample with a half life of 1 month carries a label "Activity = 2 microcuries on 1.8.1991". What was the activity two months later?
(a) 1.0 μCi (b) 0.5 μCi
(c) 4 μCi (d) 8 μCi

28. If the binding energy per nucleon in ^{7}Li and ^{4}He nuclei is respectively 5.60 MeV and 7.06 MeV, then the energy of the reaction

$$^7\text{Li} + \text{p} \longrightarrow 2\ {}_2^4\text{He} \text{ is}$$

(a) 19.6 MeV (b) 2.4 MeV
(c) 8.4 MeV (d) 17.3 MeV

29. In nuclear reactions, there is conservation of
(a) mass only
(b) energy only
(c) momentum only
(d) mass, energy and momentum.

30. If the half life of a radioactive atom is 2.3 days, its decay constant would be
(a) 0.1 (b) 0.2
(c) 0.3 (d) 2.3

31. A radioactive substance disintegrates $\frac{1}{64}$ of initial value in 60 seconds. The half life of the substance is
(a) 5 s (b) 10 s
(c) 30 s (d) 20 s

32. The atomic weight of boron is 10.81 and it has two isotopes ${}_5^{10}$B and ${}_5^{11}$B. The ratio of ${}_5^{10}$B : ${}_5^{11}$B in nature would be
(a) 19 : 81 (b) 10 : 11
(c) 15 : 16 (d) 81 : 19

33. The mass number of a nucleus is
(a) always less than its atomic number
(b) always more than its atomic number
(c) always equal to its atomic number
(d) sometimes more and sometimes equal to its atomic number.

IIT, 1986

34. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of
(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm

IIT, 1981

35. A star initially has 10^{40} deuterons. It produces energy via the processes

$${}_1^2\text{H} + {}_1^2\text{H} \longrightarrow {}_1^3\text{H} + p$$

and $${}_1^2\text{H} + {}_1^3\text{H} \longrightarrow {}_2^4\text{He} + n$$

where the masses of the nuclei are : $m\,({}^2\text{H}) = 2.014$ amu, $m(p) = 1.007$ amu, $m(n) = 1.008$ amu and $m\,({}^4\text{He}) = 4.001$ amu. If the average power radiated

by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of

(a) 10^6 s (b) 10^8 s
(c) 10^{12} s (d) 10^{16} s

IIT, 1993

36. A nucleus ruptures into two nuclear parts which have their velocities in the ratio of 2 : 1. What will be the ratio of their nuclear sizes (radii)?

(a) $2^{1/3} : 1$ (b) $1 : 2^{1/3}$
(c) $3^{1/2} : 1$ (d) $1 : 3^{1/2}$

37. A free neutron decays into a proton, an electron and

(a) a neutrino (b) an antineutrino
(c) an α-particle (d) a β-particle

38. A radioactive element $^{238}_{90}$X decays into $^{222}_{83}$Y. The number of β-particles emitted is

(a) 4 (b) 6
(c) 2 (d) 1

39. Fast neutrons can easily be slowed down by

(a) the use of lead shielding
(b) passing them through water
(c) elastic collisions with heavy nuclei
(d) applying a strong electric field

IIT, 1994

40. Masses of two isobars $^{64}_{29}$ Cu and $^{64}_{30}$ Zn are 63.9298 u and 63.9292 u respectively. It can be concluded from these data that

(a) both the isobars are stable
(b) ^{64}Zn is radioactive, decaying to ^{64}Cu through β-decay
(c) ^{64}Cu is radioactive, decaying to ^{64}Zn through γ-decay
(d) ^{64}Cu is radioactive, decaying to ^{64}Zn through β-decay.

IIT, 1997

41. The half-life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that

(a) no nucleus will decay before $t = 4$ days
(b) no nucleus will decay before $t = 8$ days
(c) all nuclei will decay before $t = 16$ days
(d) a given nucleus may decay any time after $t = 0$.

IIT, 1998

42. The order of magnitude of density of uranium nucleus is, ($m_p = 1.67 \times 10^{-27}$ kg)

(a) 10^{20} kg m^{-3} (b) 10^{17} kg m^{-3}
(c) 10^{14} kg m^{-3} (d) 10^{11} kg m^{-3}

IIT, 1999

43. ^{22}Ne nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is

(a) nitrogen (b) carbon
(c) boron (d) oxygen

IIT, 1999

44. Binding energy per nucleon versus mass number curve for nuclei is shown in Fig. 29.2. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is

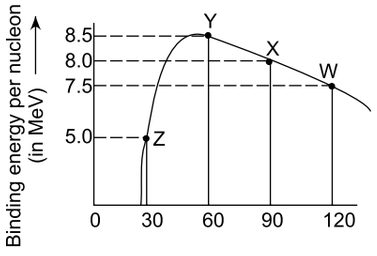


Fig. 29.2

(a) Y $\rightarrow$ 2Z (b) W $\rightarrow$ X + Z
(c) W $\rightarrow$ 2Y (d) X $\rightarrow$ Y + Z

IIT, 1999

45. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be 1/e after a time

(a) $\frac{1}{10\lambda}$ (b) $\frac{1}{11\lambda}$
(c) $\frac{11}{10\lambda}$ (d) $\frac{1}{9\lambda}$

IIT, 2000

46. A heavy nucleus at rest breaks into two fragments which fly off with velocities in the ratio of 8 : 1. The ratio of the radii of the fragments (assumed spherical) is

(a) 1 : 2 (b) 1 : 4
(c) 4 : 1 (d) 2 : 1

47. A nucleus at rest splits into two nuclear parts having radii in the ratio 1 : 2. Their velocities are in the ratio

(a) 8 : 1 (b) 6 : 1
(c) 4 : 1 (d) 2 : 1

48. If the radius of a nucleus ^{256}X is 8 fermi, the radius of ^{4}He nucleus will be

(a) 1 fermi (b) 2 fermi
(c) 3 fermi (d) 4 fermi

49. The binding energy per nucleon of C-12 is 7.68 MeV and of C-13 is 7.48 MeV. The energy (in MeV) required to remove the extra neutron from C-13 is very nearly equal to

(a) 0.2 (b) 3.7

(c) 3.9 (d) 5

50. A radioactive element of mass number 208 at rest disintegrates by emitting an α-particle. If E is the energy of the emitted α-particle, the energy of disintegration is

(a) $\frac{52}{51}E$ (b) $\frac{51}{52}E$

(c) $52\,E$ (d) E

51. A 1 MeV positron and a 1 MeV electron meet each moving in opposite directions. They annihilate each other by emitting two photons. If the rest mass energy of an electron is 0.51 MeV, the wavelength of each photon is

(a) 5.1×10^{-3} Å (b) 10.2×10^{-3} Å

(c) 8.2×10^{-3} Å (d) 6.2×10^{-3} Å

52. A gamma ray photon creates an electron-positron pair. If the total kinetic energy of the electron-positron pair is 0.78 MeV, the energy of the gamma ray photon is (given the rest mass energy of electron = 0.51 MeV)

(a) 0.27 MeV (b) 0.78 MeV

(c) 1.29 MeV (d) 1.80 MeV

53. A neutral π-meson at rest disintegrates to form two identical photons. The mass of π-meson is 264.2 m_0, where m_0 is the rest mass of an electron. The energy of each photon (in MeV) is very nearly equal to

(a) 67.4 (b) 132.1

(c) 200 (d) 931

54. The half life of a substance is 20 minutes. What is the time interval between 33% decay and 67% decay?

(a) 40 min (b) 20 min

(c) 30 min (d) 25 min

55. The electron emitted in beta radiation originates from

(a) inner orbits of atoms

(b) free electrons existing in nuclei

(c) decay of a neutron in a nucleus

(d) photon escaping from the nucleus

IIT, 2001

56. A radioactive sample consists of two distinct species having equal number of atoms initially. The mean life time of one species is τ and that of the other is 5τ. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figures best represents the form of this plot? (see Fig. 29.3).

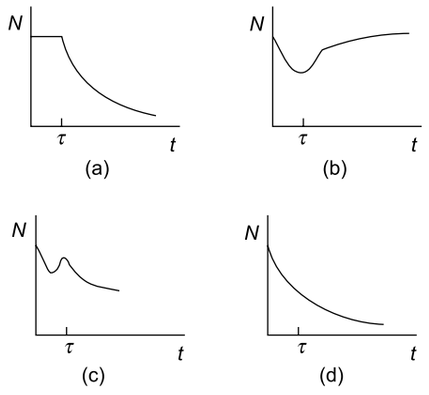


Fig. 29.3

57. The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is

(a) 400 μs (b) 6.3 μs

(c) 40 μs (d) 300 μs

IIT, 2002

58. Which of the following processes represents a gamma-decay?

(a) ${}^{A}_{Z}X + \gamma \rightarrow {}^{A}_{Z-1}X + a + b$

(b) ${}^{A}_{Z}X + {}^{1}_{0}n \rightarrow {}^{A-3}_{Z-2}X + c$

(c) ${}^{A}_{Z}X \rightarrow {}^{A}_{Z}X + f$

(d) ${}^{A}_{Z}X + {}_{-1}e \rightarrow {}^{A}_{Z-1}X + g$

IIT, 2002

59. For a certain radioactive substance, it is observed that after 4 hours, only 6.25% of the original sample is left undecayed. Choose the only wrong statement from the following.

(a) the half life of the sample is 1 hour

(b) the mean life of the sample 1/2 hour

(c) the decay constant of the sample is (ln 2) hour^{-1}

(d) after a further 4 hours, the amount of the substance left over would be only 0.39% of the original amount.

60. If the end A of a wire is irradiated with alpha rays and the end B is irradiated with beta rays, then
(a) there will be no current in the wire
(b) a current will flow from A to B
(c) a current will flow from B to A
(d) a current will flow from each end to the midpoint of the wire.

61. During a negative beta decay:
(a) an atomic electron is ejected
(b) an electron which is already present within the nucleus is ejected
(c) a neutron in the nucleus decays emitting an electron
(d) a part of the binding energy of the nucleus is converted into an electron.

62. The nucleus of $^{230}_{90}\text{Th}$ decays to $^{226}_{88}\text{Ra}$ and $^{4}_{2}\text{He}$ with the emission of energy. If the original nucleus was at rest, the ratio of kinetic energies of He and Ra nuclei will be very nearly equal to
(a) 22 (b) 44
(c) $\frac{113}{2}$ (d) 113

63. A nucleus X, initially at rest, undergoes α-decay according to the equation,

$$^{A}_{92}\text{X} \rightarrow {}^{228}_{Z}\text{Y} + \alpha$$

The values of A and Z are
(a) 232 and 90 (b) 234 and 94
(c) 230 and 88 (d) 232 and 92

64. At a given instant there are 25% undecayed radioactive nuclei in a sample. After 69.3 s, the number of undecayed nuclei reduces to 12.5%. The mean life of the sample is
(a) 1 s (b) 10 s
(c) 100 s (d) 1000 s

65. In Q. 64 above, the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number will be
(a) 6.93 s (b) 69.3 s
(c) 693 s (d) 277.2 s

66. A uranium nucleus $^{238}_{92}\text{U}$ emits two alpha particles and two beta particles and transforms into a thorium nucleus. The mass number and atomic number of thorium nucleus so formed are
(a) 230 and 90 (b) 232 and 90
(c) 234 and 92 (d) 234 and 88

67. A radioactive substance of half life 69.3 days is kept in a container. The time in which 80% of the substance will disintegrate will be [take ln(5) = 1.61]
(a) 1.61 days (b) 16.1 days
(c) 161 days (d) 1610 days

68. The sequence of decays of a radioactive nucleus is

$$\text{D} \xrightarrow{\alpha} \text{D}_1 \xrightarrow{\beta} \text{D}_2 \xrightarrow{\alpha} \text{D}_3 \xrightarrow{\alpha} \text{D}_4$$

If the mass number and atomic number of D_2 are 176 and 71 respectively, the corresponding values of D will be
(a) 180, 72 (b) 176, 70
(c) 172, 69 (d) 168, 67

69. The mass m of a uranium nucleus varies with its volume V as
(a) $m \propto \sqrt{V}$ (b) $m \propto \frac{1}{V}$
(c) $m \propto V$ (d) $m \propto V^2$

IIT, 2003

70. A nucleus of mass number 220, initially at rest, emits an α-particle. If the Q value of the reaction is 5.5 MeV, the energy of the emitted α-particle will be
(a) 4.8 MeV (b) 5.4 MeV
(c) 6.0 MeV (d) 6.8 MeV

IIT, 2003

71. After 24 hours; the activity of a radioactive sample is 2000 dps (disintegrations per second). After another 12 hours, the activity reduces to 1000 dps. The initial activity of the sample in dps is
(a) 1000 (b) 2000
(c) 4000 (d) 8000

IIT, 2004

72. If all the Helium nuclei in the core of a star get converted into oxygen nuclei, then the energy released per oxygen nucleus is (mass of $^{4}_{2}$ He-nucleus = 4.0026 amu, mass of $^{16}_{8}$O-nucleus = 15.9994 amu)

IIT, 2005

(a) 931 MeV (b) 200 MeV
(c) 10.24 MeV (d) 7.56 MeV

73. $^{221}_{87}\text{Ra}$ undergoes radioactive decay with a half life of 4 days. The probability that a Ra nucleus will disintegrate in 8 days is
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

IIT, 2006

74. The half life of a radioactive sample is 6.93 days. After how many days will only one-twentieth of the sample be left over? Take $\log_e (20) = 3.0$.

IIT, 1981

(a) 20 days (b) 27 days

(c) 30 days (d) 35 days

75. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is

(a) $E\left(^{236}_{92}\mathrm{U}\right) > E\left(^{137}_{53}\mathrm{I}\right) + E\left(^{97}_{39}\mathrm{Y}\right) + 2E\,(n)$

(b) $E\left(^{236}_{92}\mathrm{U}\right) < E\left(^{137}_{53}\mathrm{I}\right) + E\left(^{97}_{39}\mathrm{Y}\right) + 2E\,(n)$

(c) $E\left(^{236}_{92}\mathrm{U}\right) < E\left(^{140}_{56}\mathrm{Ba}\right) + E\left(^{94}_{36}\mathrm{Kr}\right) + 2E\,(n)$

(d) $E\left(^{236}_{92}\mathrm{U}\right) = E\left(^{140}_{56}\mathrm{Ba}\right) + E\left(^{94}_{36}\mathrm{Kr}\right) + 2E\,(n)$

IIT, 2007

76. A radioactive sample S_1 having an activity of 5 μ Ci has twice the number of nuclei as another sample S_2 which has an activily 10 μ Ci. The half lives of S_1 and S_2 can be

(a) 20 years and 5 years, respectively
(b) 20 years and 10 years, respectively
(c) 10 years each
(d) 5 years each

IIT, 2008

ANSWERS

1. (c)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (d)
7. (d)	**8.** (c)	**9.** (c)	**10.** (c)	**11.** (d)	**12.** (d)
13. (b)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (a)
19. (b)	**20.** (d)	**21.** (a)	**22.** (c)	**23.** (c)	**24.** (a)
25. (b)	**26.** (b)	**27.** (b)	**28.** (d)	**29.** (d)	**30.** (c)
31. (b)	**32.** (a)	**33.** (d)	**34.** (c)	**35.** (c)	**36.** (b)
37. (b)	**38.** (d)	**39.** (b)	**40.** (d)	**41.** (d)	**42.** (b)
43. (b)	**44.** (c)	**45.** (d)	**46.** (a)	**47.** (a)	**48.** (b)
49. (d)	**50.** (a)	**51.** (c)	**52.** (d)	**53.** (a)	**54.** (b)
55. (c)	**56.** (d)	**57.** (a)	**58.** (c)	**59.** (b)	**60.** (b)
61. (c)	**62.** (c)	**63.** (a)	**64.** (c)	**65.** (d)	**66.** (a)
67. (c)	**68.** (a)	**69.** (c)	**70.** (b)	**71.** (d)	**72.** (c)
73. (d)	**74.** (c)	**75.** (a)	**76.** (a)		

SOLUTIONS

1. The distance of the closest approach is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{K}$$

where $K = \frac{1}{2}\,mv^2$. Thus $r_0 \propto \frac{1}{K}$. When K is doubled, r_0 becomes half. Hence the correct choice is (c).

2. $K = \frac{1}{2}\,mv^2 = \frac{1}{2m}\times(mv)^2 = \frac{p^2}{2m}$. Therefore,

$$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4mZe^2}{p^2}$$

Thus $r_0 \propto \frac{1}{p^2}$. When p is doubled, r_0 becomes one-fourth. Hence the correct choice is (d).

3. The charge number and the mass number must be the same on both sides of a nuclear reaction. Particle x must have charge number + 1 and mass number 1. Hence it is a proton.

4. Particle x must have zero charge and mass number 1. Hence it is a neutron.

5. The mass number of $x = 27 + 1 - 24 = 4$ and its atomic number = 13 + 0 – 11 = 2. Hence particle x is helium nucleus, which is called alpha particle.

6. The mass number of X is $A = 27 + 4 - 30 = 1$ and its atomic number $Z = 13 + 2 - 15 = 0$. Hence particle X is a neutron and its symbol is $^{A}_{Z}n$ which is $^{1}_{0}n$.

7. Energy released = 200 MeV = 200×10^6 eV = $200 \times 10^6 \times 1.6 \times 10^{-19}$ J. Therefore, the required number is

$$\frac{3.2}{200\times10^6\times1.6\times10^{-19}} = 10^{17}$$

8. The energy released is of the order of
$(8.5 - 7.6)\ \text{MeV} \times (117 + 117) \simeq 200\ \text{MeV}$

9. Binding energy of ${}_1^2\text{H} = 1.15 \times$ number of nucelons $= 1.15 \times 2 = 2.3$ MeV. Total binding energy of reactants $= 2.3 + 2.3 = 4.6$ MeV. Binding energy of ${}_2^4\text{He} = 7.1 \times$ number of nucleons $= 7.1 \times 4 = 28.4$ MeV. Therefore, $Q = 28.4 - 4.6 = 23.8$ MeV. Hence the correct choice is (c).

10. The correct choice is (c).

11. We have $T_{1/2} = 10$ minutes. Therefore 5 minutes = $\frac{1}{2}$ of half-life. It follows that the number of nuclei left after 5 minutes will be

$$\frac{1}{(2)^{1/2}} = \frac{1}{\sqrt{2}} \text{ of the original number} = \frac{10^8}{\sqrt{2}}$$

Hence the correct choice is (d).

12. Since 15 minutes $= 5 \times 3$ minutes $= 5$ half lives, the number of nuclei left after 15 minutes $= \frac{1}{2^5} = \frac{1}{32}$ of the original number. Therefore, the mass of 16 g sample left after 15 minutes $= \frac{16}{32} = 0.5$ g. Hence the correct choice is (d).

13. There were X atoms at time $t = t_1$ and we are left with Y atoms at time $t = t_2$. Therefore, the number of atoms that have disintegrated in the time interval $(t_2 - t_1)$ is X-Y. Hence the correct choice is (b).

14. For a radioactive material of disintegration constant λ,

$$\text{Half-life} = T_{1/2} = \frac{1\text{n}\,2}{\lambda}, \text{ and}$$

$$\text{mean-life} = \tau = \frac{1}{\lambda}.$$

Hence the correct choice is (b).

15. Out of the three radioactive radiations, the α-rays are the most ionising but least penetrative while the γ-rays are the least ionising but most penetrative.

16. Alpha particle has mass number 4 and atomic number 2. Thus A decreases to $A - 4$ and Z decreases to $Z - 2$. Hence the correct choice is (b).

17. The mass number of x must be $238 - 234 = 4$ and atomic number of x must be $92 - 90 = 2$. Hence the correct choice is (c).

18. The charge of x is opposite to that of a proton. Hence x is an electron.

19. In transition ${}_Z^A\text{X} \longrightarrow {}_{Z+1}^{A}\text{Y}$, the atomic (or charge) number increases by unity; mass number remaining the same. Hence an electron (β-particle) is emitted. In transition ${}_{Z+1}^{A}\text{Y} \longrightarrow {}_{Z-1}^{A-4}\text{K}$, the mass number decreases by 4 and charge number decreases by 2. Hence an α-particle is emitted. In the third transition, mass and charge numbers do not change. Hence a γ-ray is emitted. Hence the correct choice is (b).

20. In 30 days (i.e. 3 half-lives) $1/2^3 = 1/8$ of the sustance is left. Hence the correct choice is (d).

21. The count decreases by a factor of $320/40 = 8$ in 90 minutes. Now $(2)^3 = 8$. Hence 90 minutes = 3 half-lives. Hence the half life of the radioactive source is 30 minutes.

22. The correct choice is (c).

23. It represents a fusion of four ${}_1^1\text{H}$ nuclei with the emission a huge amount of energy. The correct choice is (c).

24. The mass number reduces by $200 - 168 = 32$. Hence 8 α-particles are emitted. The emission of 8 α-particles reduces the atomic number by 16. But the atomic number reduces by $90 - 80 = 10$. Hence the number of β–particle emitted $= 16 - 10 = 6$. Hence the correct choice is (a).

25. Since the half life is 2 hours, the intensity of the radiation falls by a factor of 2 every 2 hours. In 12 hours it will fall by a factor of $(2)^6 = 64$. Thus, in 12 hours the intensity attains the safe level. Hence the correct choice is (b).

26. The probability of a beam of α-particles passing through the film increases with the increase in the number of particles in the beam. Thus beam A' has the maximum number of particles and B' has the minimum number of particles. Hence the correct choice is (b).

27. Two months = 2 half lives. The activity of the sample will become $\frac{1}{2^2}$, i.e. one-fourth in 2 months. Hence the correct choice is (b).

28. Binding energy of ${}^7\text{Li}$ = (binding energy per nucleons) $\times$ (no. of nucleons)

$= 5.60 \times 7 = 39.20$ MeV. Similarly, binding energy of ${}^4\text{He} = 7.06 \times 4 = 28.24$ MeV. Therefore, binding energy of two ${}^4\text{He}$ nuclei $= 28.24 \times 2 = 56.48$ MeV. Hence, the energy of reaction is $(56.48 - 39.20) = 17.28$ MeV. Thus the closest choice is (d).

29. The correct choice is (d).

30. The decay constant λ is given by

$$\lambda = \frac{\log_e(2)}{T} = \frac{0.693}{2.3} \simeq 0.3$$

Hence the correct choice is (c).

31. Now $64 = (2)^6$. Therefore, half life $= \frac{60\text{s}}{6} = 10$ s.

32. Let a_1 and a_2 be the respective abundance of isotopes ^{10}B and ^{11}B of mass number Z_1 and Z_2 of boron of mass number Z, then

$$a_1Z_1 + a_2Z_2 = (a_1 + a_2)Z$$

or $\quad 10\ a_1 + 11\ a_2 = (a_1 + a_2) \times 10.81$

or $\quad 10.81\ a_1 - 10\ a_1 = 11\ a_2 - 10.81\ a_2$

or $\quad 0.81\ a_1 = 0.19\ a_2$

which gives $\frac{a_1}{a_2} = \frac{19}{81}$, which is choice (a).

33. Mass number Z is greater than atomic number A for all nuclei; the single exception being the hydrogen nucleus for which $A = Z$. Hence the correct choice is (d).

34. The distance of closest approach is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{E_k}$$

Here $Z = 92$ for uranium and $E_k = 5$ MeV $= 5 \times 10^6 \times 1.6 \times 10^{-19}$ J. Also $e = 1.6 \times 10^{-19}$ C and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2 C^{-2}. Using these values, we get $r_0 \simeq 10^{-12}$ cm, which is choice (c).

35. Let the mass of $^3_1$H be x amu. Then the mass defect in the first process is

$$(\Delta m)_1 = m\ (^2_1\text{H}) + m\ (^2_1\text{H}) - m\ (^3_1\text{H}) - m(p)$$
$$= 2.014 + 2.014 - x - 1.007$$
$$= (3.021 - x)\ \text{amu}$$

Mass defect in the second process will be

$$(\Delta m)_2 = m\ (^2_1\text{H}) + m\ (^3_1\text{H}) - m\ (^4_2\text{He}) - m\ (n)$$
$$= 2.014 + x - 4.001 - 1.008$$
$$= (x - 2.995)\ \text{amu}$$

$\therefore$ Total mass defect is

$$\Delta m = (\Delta\ m)_1 + (\Delta\ m)_2 = (3.021 - x) + (x - 2.995)$$
$$= 0.026\ \text{amu}$$

Now 1 amu = 931 MeV $= 931 \times 10^6 \times 1.6 \times 10^{-19}$ J. Hence, energy released is given by

$$E = 0.026 \times 931 \times 10^6 \times 1.6 \times 10^{-19}$$
$$= 3.87 \times 10^{-12}\ \text{J}$$

Now power $P = 10^{16}$ W. Therefore, the number of deutrons per second is $\frac{10^{16}}{3.87 \times 10^{-12}} = 2.58 \times 10^{27}$ per second

$\therefore$ Deutron supply will exhaust in $\frac{10^{40}}{2.58 \times 10^{27}}$

$$= 3.87 \times 10^{12}\ \text{seconds}$$

So the correct choice is (c).

36. Let m_1 and m_2 be the masses and v_1 and v_2 the velocities of the two parts. From the principle of conservation of momentum, we have

$$m_1v_1 = m_2v_2 \text{ or } \frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{1}{2} \text{ (given)}$$

Now, the radius of a nucleus $R \propto A^{1/3}$, where A is the atomic mass. Since $A \propto m$, the mass of the nucleus; $R \propto m^{1/3}$.

Thus $\frac{R_1}{R_2} = \left(\frac{m_1}{m_2}\right)^{1/3} = \left(\frac{1}{2}\right)^{1/3} = \frac{1}{2^{1/3}}$

Hence the correct choice is (b).

37. The conservation of spin requires that the third particle is an antinuetrino ($\bar{\nu}$). The decay reaction is represented as $^1_0n \longrightarrow {}^1_1p + {}^0_{-1}e + \bar{\nu}$

38. β-particles are electrons. When a nucleus emits an electron, its atomic number increases by 1 but its mass number remains unchanged. Here the mass number decreases by 238 – 222 = 16. The mass number of a helium nucleus is 4. Hence $16/4 = 4$ α-particles are emitted. An α-particle is $^4_2$He. Hence the atomic number decreases by 8 by the emission of 4 α-particles. But the atomic number decreases by 90 – 83 = 7. This is possible if only one β-particle is emitted.

39. The correct choice is (b).

40. The atomic mass of stable Cu is smaller than that of Zn. Since the given mass of Cu is greater than that of Zn, ^{64}Cu will be unstable. In β-decay, the atomic number is increased by one while the mass number remains unchanged. Thus

$$^{64}_{29}\text{Cu} \xrightarrow{\beta\text{-decay}} {}^{64}_{30}Z_n + {}^0_{-1}e$$

Hence the correct choice is (d).

41. A radioactive nucleus may decay any time after $t = 0$. The number of nuclei at any time t is given by $N = N_0\ e^{-\lambda t}$, where $\lambda = \frac{0.693}{T}$

where T is the half-life. Hence the correct choice is (d).

42. If A is the atomic number, the mass of uranium nucleus is

$m = (1.67 \times 10^{-27})A$ kg and its volume is

$$V = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \times \{1.25 \times 10^{-15}\text{ m } A^{1/3}]^3$$

$$\simeq 8.2 \times 10^{-45} A \text{ m}^3$$

$$(\because\ r = r_0 A^{1/3};\ r_0 = 1.25 \times 10^{-15}\text{ m})$$

$$\therefore \text{ Density } = \frac{m}{V} = \frac{1.67 \times 10^{-27} A \text{ kg}}{8.2 \times 10^{-45} A \text{ m}^3}$$

$$\simeq 2 \times 10^{17} \text{ kg m}^{-3}$$

Hence the correct choice is (b).

43. The given nuclear reaction is given by the equation

$$^{22}_{10}\text{Ne} \rightarrow {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He} + Z^{X}$$

Since the atomic number is conserved, we have

$$10 = 2 + 2 + Z$$

or $Z = 6$. The carbon nucleus has atomic number 6.

44. The binding energies of the reactant and the products in the given nuclear reactions are as follows:

Reaction	*Reactant*	*Products*
(a) $Y \rightarrow 2Z$	60×8.5 = 510 MeV	$2 \times 30 \times 5.0$ = 300 MeV
(b) $W \rightarrow X + Z$	120×7.5 = 870 MeV	$(90 \times 8.0 + 30 \times 5.0)$ = 900 MeV
(c) $W \rightarrow 2Y$	120×7.5 = 900 MeV	$2 \times 60 \times 8.5$ = 1020 MeV
(d) $X \rightarrow Y + Z$	90×8.0 = 660 MeV	$(60 \times 8.5 + 30 \times 5.0)$ = 720 MeV

The binding energy of the products in reaction (c) is greater than that of the reactant. Hence reaction (c) releases energy.

45. $N_1 = N_0 e^{-\lambda 1 t}$, $N_2 = N_0 e^{-\lambda 2 t}$. Therefore

$$\frac{N_1}{N_2} = \frac{e^{-\lambda_1 t}}{e^{-\lambda_2 t}} = e^{-(\lambda 1 - \lambda 2)t}$$

$$= e^{-(10\lambda - \lambda)t} = e^{-9\lambda t}$$

Given, $\dfrac{N_1}{N_2} = e^{-1}$.

Hence $-9\lambda t = -1$ or $t = \dfrac{1}{9\lambda}$.

46. Let m_1 and m_2 be the atomic masses of the fragments and v_1 and v_2 their velocities. From the principle of conservation of linear momentum, we have

$$m_1 v_1 + m_2 v_2 = 0 \qquad \text{or} \qquad \frac{m_1}{m_2} = \left|\frac{v_2}{v_1}\right| = \frac{1}{8}$$

Now $m_1 = A_0 (R_1)^{1/3}$ and $m_2 = A_0 (R_2)^{1/3}$. Hence

$$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^{1/3}$$

or $\dfrac{1}{8} = \left(\dfrac{R_1}{R_2}\right)^{1/3}$ or $\dfrac{R_1}{R_2} = \dfrac{1}{2}$

Thus, the correct choice is (a).

47. Let A_1 and A_2 be the mass numbers of the two nuclear parts. Their radii are given by

$$R_1 = R_0 (A_1)^{1/3} \text{ and } R_2 = (A_2)^{1/3}$$

Dividing, we get

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

or $\dfrac{A_1}{A_2} = \left(\dfrac{R_1}{R_2}\right)^3 = \left(\dfrac{1}{2}\right)^3 = \dfrac{1}{8}$

Hence the ratio of their masses is

$$\frac{m_1}{m_2} = \frac{1}{8}$$

From the principle of conservation of momentum, the magnitude of p_1 = magnitude of p_2 or $m_1 v_1 = m_2 v_2$, which gives

$$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{8}{1}, \text{ which is choice (a).}$$

48. $R = R_0 (A)^{1/3}$. Therefore

$$\frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3} = \left(\frac{4}{254}\right)^{1/3} = \frac{1}{4}$$

or $R_2 = \dfrac{R_1}{4} = \dfrac{8 \text{ fermi}}{4} = 2 \text{ fermi}$

Hence the correct choice is (b).

49. Energy required = $(7.48 \times 13 - 12 \times 7.68)$ MeV = 5.08 MeV. Hence the correct choice is (d).

50. Mass number of daughter nucleus (M) = 208 – 4 = 204

Now, total energy of disintegration = energy of daughter nucleus + energy of α-particle

or $E_t = \dfrac{p^2}{2m} + \dfrac{P^2}{2m}$

Since momentum is conserved, $p = P$. Hence

$$E_t = \frac{p^2}{2}\left(\frac{1}{m} + \frac{1}{M}\right) = \frac{p^2}{2}\left(\frac{m+M}{mM}\right)$$

Energy of α-particle = $\dfrac{p^2}{2m} = E$. Hence

$$E_t = \left(\frac{m+M}{M}\right)E$$

$$= \left(\frac{4+204}{204}\right)E = \frac{52}{51}E$$

Hence the correct choice is (a).

51. Energy released in annihilation = 0.51 + 0.51 = 1.02 MeV.
Initial energy = 1 + 1 = 2 MeV. Therefore, the energy of the two photons is = 1.02 + 2 = 3.02 MeV. Hence energy of each photon is E = 1.51 MeV. Now, according to Duane-Hunt law, the wavelength of a photon of energy E (in eV) is given by

$$\lambda = \frac{hc}{E \text{ in eV}} \text{ Å} = \frac{6.62\times10^{-34}\times3\times10^{8}}{1.51\times10^{6}} \text{ Å}$$

$= 8.2 \times 10^{-3}$ Å which is choice (c).

52. E = 0.51 + 0.51 + 0.78 = 1.80 MeV. Hence the correct choice is (d).

53. Mass of each photon (m) = $\frac{1}{2} \times 264.2\ m_0$

$= 132.1\ m_0$.

Energy of photon = $mc^2 = 132.1 \times m_0 c^2$

$= 132.1 \times 0.51$ MeV

($\because\ m_0 c^2 = 0.51$ MeV)

$= 67.4$ MeV

Hence the correct choice is (a).

54. Given T = 20 min. We know that

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$$

For 33% decay, $\frac{N}{N_0} = \frac{100-33}{100} = \frac{67}{100}$

If t_1 is the time for 33% decay, then

$$\frac{67}{100} = \left(\frac{1}{2}\right)^{t_1/T} \quad \text{(i)}$$

For 67% decay, $\frac{N}{N_0} = \frac{100-67}{100} = \frac{33}{100}$

If t_2 is the time for 67% decay, then

$$\frac{33}{100} = \left(\frac{1}{2}\right)^{t_2/T} \quad \text{(ii)}$$

Dividing (ii) by (i), we have

$$\frac{33}{67} = \left(\frac{1}{2}\right)^{(t_2-t_1)/T} \quad \text{(iii)}$$

Now $\frac{33}{67} \simeq \frac{1}{2}$. Hence Eq. (iii) reduces to

$$\left(\frac{1}{2}\right)^{1} = \left(\frac{1}{2}\right)^{(t_2-t_1)/T}$$

which gives $1 = \frac{t_2 - t_1}{T}$ or $t_2 - t_1 = T$ = 20 min.

Hence the correct choice is (b).

55. Beta particle is an electron which is emitted from a nucleus when a neutron decays into a proton and an electron within a nucleus. Hence the correct choice is (c).

56. For each species, the number of radioactive nuclei decreases exponentially with time. Hence, for both the species taken together, the total number of radioactive nuclei will decrease exponentially with time. This is best represented in plot (d).

57. Since $\frac{1}{16} = \frac{1}{2^4}$, it follows that the time taken for the radioactivity to decay to $\frac{1}{16}$th of its initial value = four times the half-life of the sample = $4t_{1/2}$ = 4×100 μs = 400 μs. Thus, the correct choice is (a).

58. During the emission of a gamma radiation, both the mass number and atomic number remain the same. Hence the correct choice is (c).

59. 6.25% = $\frac{6.25}{100} = \frac{1}{16} = \frac{1}{(2)^4}$. Hence 4 hours are equal to 4 half lives. Therefore, the half life of the substance is 1 hour, which is choice (a). The decay constant = $\frac{\ln(2)}{\text{half life}} = \ln(2)$ per hour, which is choice (c).

Mean life = $\frac{1}{\text{decay constant}}$

$$= \frac{1}{\ln(2)} \text{ hour} = \frac{1}{0.693} \text{ hour}$$

Hence choice (b) is incorrect.

After further 4 hours (i.e. after 8 hours), $\frac{1}{(2)^8} = \frac{1}{256} = \frac{100}{256}\% = 0.39\%$ of the substance remains undecayed, which is choice (d). Thus the only incorrect choice is (b).

60. Alpha particles are positively charged helium nuclei and beta rays are negatively charged electrons. Hence the correct choice is (b).

61. Negative β decay is expressed by the equation:

$$n \longrightarrow p + e^- + \bar{\nu}$$

Hence the correct choice is (c).

62. If the original nucleus Th is at rest, i.e. if the momentum of the system before α-decay is zero, the total momentum after the decay will also be zero. Thus Ra and He will have equal and opposite linear momenta.

$$\therefore \quad m_{He}\, v_{He} = -\, m_{Ra}\, v_{Ra}$$

or $$m_{He}^2 v_{He}^2 = m_{Ra}^2 v_{Ra}^2 \qquad \text{(i)}$$

Now $$\frac{\text{K.E.(He)}}{\text{K.E.(Ra)}} = \frac{1/2 m_{He} v_{He}^2}{1/2 m_{Ra} v_{Ra}^2}$$

$$= \left(\frac{m_{He} v_{He}}{m_{Ra} v_{Ra}}\right)^2 \times \left(\frac{m_{Ra}}{m_{He}}\right)$$

$$= \frac{m_{Ra}}{m_{He}} = \frac{226}{4} = \frac{113}{2} \quad \text{[use Eq. (i)]}$$

Hence the correct choice is (c).

63. The a-particle is a helium nucleus ${}_2^4\text{He}$. Hence

$${}_{92}^{A}\text{X} \rightarrow {}_{Z}^{228}\text{Y} + {}_2^4\text{He}$$

Hence the correct choice is (a).

64. In 69.3 s, the number of nuclei reduces to half (from 25% to 12.5%). Hence half life = 69.3 s

$$\text{Mean life} = \frac{\text{half life}}{\ln(2)} = \frac{69.3\,\text{s}}{0.693} = 100\ \text{s}$$

Hence the correct choice is (c).

65. Let the reduced number further reduce to 6.25% in n half lives. Then

$$\frac{6.25}{100} = \left(\frac{1}{2}\right)^n \quad \text{or} \quad \frac{1}{16} = \left(\frac{1}{2}\right)^n$$

which gives $n = 4$. Therefore, the time taken would be $t = 4T_{1/2}$. Hence the correct choice is (d).

66. In each alpha decay, the mass number decreases by 4 and atomic number decreases by 2. In each beta decay, the mass number remains unchanged, but atomic number increases by 1. Hence the correct choice is (a).

67. $T_{1/2}$ = 69.3 days. Therefore $\lambda = \dfrac{0.693}{T_{1/2}} = 10^{-2}$ per day.

$$N = 20\%\ \text{of}\ N_0 = 0.2\ N_0.$$

Now $$N = N_0\, e^{-\lambda t}, \text{ which gives}$$

$$\frac{N_0}{N} = e^{\lambda t}$$

or $$\ln\left(\frac{N_0}{N}\right) = \lambda t$$

or $$t = \frac{\ln\left(\frac{N_0}{N}\right)}{\lambda} = \frac{\ln(5)}{10^{-2}} = \frac{1.61}{10^{-2}}$$

$$= 161\ \text{days}$$

Hence the correct choice is (c).

68. The correct choice is (a).

69. If m_p is the mass of a proton and A, the atomic number of uranium nucleus, then the mass of a uranium nucleus is $m = m_p A$ and the volume of uranium nucleus is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r_0 A^{1/3})^3 = \frac{4}{3}\pi r_0^3 A \qquad (\because r = r_0 A^{1/3})$$

$$\therefore \quad \frac{m}{V} = \frac{m_p A}{\frac{4}{3}\pi r_0^3 A} = \frac{3 m_p}{4\pi r_0^3} = \text{constant}$$

Thus $m \propto V$, which is choice (c).

70. Kinetic energy $= \dfrac{(\text{momentum})^2}{2 \times \text{mass}}$.

Mass number of α-particle (m) = 4 units. Mass number of daughter nucleus (M) = 220 – 4 = 216 units. If P and p denote the momenta of the daughter nucleus and the α-particle respectively, then

$$Q = \frac{P^2}{2M} + \frac{p^2}{2m}$$

Since momentum is conserved, $P = p$. Hence

$$Q = \frac{p^2}{2}\left(\frac{1}{M} + \frac{1}{M}\right) = \frac{p^2}{2m}\left(\frac{m}{M} + 1\right)$$

Now $\dfrac{p^2}{2m}$ = KE of α-particle = E_α. Thus,

$$Q = E_\alpha \left(\frac{m + M}{M}\right)$$

or $$E_\alpha = \frac{QM}{(m + M)} = \frac{5.5\ \text{MeV} \times 216}{(4 + 216)}$$

$$= 5.4\ \text{MeV}$$

Hence the correct choice is (B).

71. In 12 hours, the activity of the sample decreases from 2000 dps to 1000 dps, i.e. it becomes half in 12 hours. Hence the half life of the sample is 12 hours. Now, 24 hours = 2 half lives. Hence the initial activity = 2000 dps $\times (2)^2$ = 8000 dps. Hence the correct choice is (d).

72. Four He nuclei fuse to produce one oxygen nucleus according to equation

$$4(^4_2\text{He}) \rightarrow {}^{16}_{8}\text{O}$$

Mass defect $\Delta m = 4m\,(^4_2\text{He}) - m(^{16}_{8}\text{O})$

$= 4 \times 4.0026 - 15.9994$

$= 0.011$ amu

Energy released per oxygen nucleus is

$E = \Delta mc^2$

$= 0.011 \times 931$ MeV

$= 10.24$ MeV

Hence the correct choice is (c).

73. The probability at time t that a radioactive nucleus will disintegrate is defined as

$$P = \frac{N_0 - N}{N_0}$$

where N_0 = number of nuclei present initially at time $t = 0$

N = number of nuclei left undecayed at time time t.

Now 8 days = 2 half lives. After 2 half lives

$$N = \frac{N_0}{(2)^2} = \frac{N_0}{4}$$

$$\therefore \qquad P = \frac{N_0 - \frac{N_0}{4}}{N_0} = \frac{3}{4}$$

Hence the correct choice is (d).

74. $\frac{N}{N_0} = e^{-\lambda t}$. Thus $\frac{1}{20} = e^{\lambda t}$ or $20 = e^{\lambda t}$ or $\lambda t = \log_e (20) = 3.0$

where $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{6.93} = 0.1$ per day

Therefore $t = \frac{3.0}{\lambda} = \frac{3.0}{0.1} = 30$ days. Thus the correct choice is (c).

75. From the binding energy per nucleon versus mass number curve, it follows that $^{137}_{53}\text{I}$, $^{97}_{39}\text{Y}$, $^{140}_{56}\text{Ba}$ and $^{94}_{36}\text{Kr}$ have higher binding energy than $^{236}_{92}\text{U}$. Hence the nuclei of I, Y, Ba and Kr are more stable than the nucleus of U. Therefore, the rest mass energy of U is greater than the sum of the rest mass energies of I and Y or Ba and Kr. Hence the correct choice is (a).

76. Activity $R = -\lambda N$ where λ = decay constant and N = number of radioactive nuclie present in the sample. Also

$$\lambda = \frac{0.693}{T}$$

where T = half life of the sample. Hence

$$T = \frac{0.693}{\lambda} = -\frac{0.693\,N}{R}$$

$$\frac{T_1}{T_2} = \frac{N_1}{N_2} \times \frac{R_1}{R_2} = \frac{2}{1} \times \frac{10}{5} = \frac{20}{5}$$

So the correct choice is (a).

SECTION II

Multiple Choice Questions with One or More Choices Correct

1. For a certain radioactive substance, it is observed that after 4 hours, only 6.25% of the original sample is left undecayed. It follows that
- (a) the half life of the sample is 1 hour
- (b) the mean life of the sample 1/(ln 2) hour
- (c) the decay constant of the sample is (ln 2) hour^{-1}
- (d) after a further 4 hours, the amount of the substance left over would be only 0.39% of the original amount.

IIT, 2003

2. The half-life of a radioactive substance does not depends upon
- (a) its temperature
- (b) the external pressure on it
- (c) the mass of the substance
- (d) the strength of the nuclear force between the nucleons of its atoms.

3. In nuclear reactions, which of the following are conserved?
- (a) mass (b) energy
- (c) momentum (c) charge

4. When a nucleus emits a photon
- (a) its actual mass decreases
- (b) its actual mass remains the same
- (c) its atomic number decreases
- (d) its atomic number remains the same.

5. A tiny positively charged particle is moving head-on towards a heavy nucleus. The distance of closest approach depends upon
(a) the number of protons in the nucleus
(b) the number of nucleons in the nucleus
(c) the mass of the incident particle
(d) the charge and velocity of the incident particle.

6. A uranium nucleus (atomic number 92, mass number 238) emits an α-particle and the resultant nucleus emits a β-particle. If Z and A are the atomic and mass numbers of the final nucleus,
(a) $Z = 90$ (b) $Z = 91$
(c) $A = 234$ (d) $A = 233$

IIT, 1982

7. The decay rate of a radioactive element is found to be 1600 disintegrations per second. If the half life of the element in 1 second, then
(a) after 1 second the decay rate will be 800 disintegrations per second.
(b) after 2 seconds the decay rate will be 400 disintegrations per second.
(c) after 3 seconds the decay rate will be 200 disintegrations per second.
(d) after 4 seconds the decay rate will be 100 disintegrations per second.

IIT, 1983

8. Pick the possible nuclear fusion reactions from the following:
(a) ${}^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{C} \rightarrow {}^{14}_{6}\mathrm{C} + 4.3\ \mathrm{MeV}$
(b) ${}^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{C} \rightarrow {}^{13}_{7}\mathrm{N} + 2\ \mathrm{MeV}$
(c) ${}^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{15}_{8}\mathrm{O} + 7\ \mathrm{MeV}$
(d) ${}^{235}_{92}\mathrm{U} + {}^{1}_{0}n \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + 2\left({}^{1}_{0}n\right) + 200\mathrm{MeV}$

IIT, 1984

9. In the uranium radioactive series the initial nucleus is ${}^{238}_{92}\mathrm{U}$ and the final nucleus is ${}^{206}_{82}\mathrm{Pb}$. If m is the number of α-particles and n the number of β-particles emitted when the uranium nucleus decays to lead, then
(a) $m = 8$ (b) $m = 6$
(c) $n = 10$ (d) $n = 6$

IIT, 1985

10. Which of the following statement (s) is/are correct?
(a) The rest mass of a stable nucleus is less then the sum of the rest masses of its separated nucleons.
(b) The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons.
(c) In nuclear fusion energy is released by fusing two nuclei of medium mass (approximately 100 amu).
(d) In nuclear fusion, energy is released by fragmentation of a very heavy nucleus.

IIT, 1994

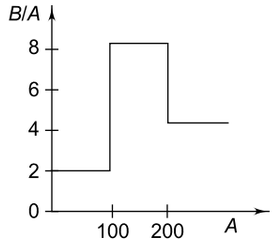
11. In the nuclear reaction
${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + Q$
the mass of ${}^{2}_{1}\mathrm{H} = 2.0141$ u, mass of ${}^{4}_{2}\mathrm{He} = 4.0024$ u and Q is the energy released.
(a) It is a fusion reaction
(b) It is a fission reaction.
(c) $Q = 200$ MeV (d) $Q = 24$ MeV

IIT, 1996

12. Let m_p be the mass of a proton m_n the mass of a neutron, M_1 the mass of ${}^{20}_{10}\mathrm{Ne}$ nucleus and M_2 the mass of ${}^{40}_{20}\mathrm{Ca}$ nucleus. Then
(a) $M_2 = 2M_1$ (b) $M_2 > 2M_1$
(c) $M_2 < 2M_1$ (d) $M_1 < 10(m_p + m_n)$

IIT, 1998

13. Assume that the nuclear binding energy per nucleon (B/A) versus mass number is as shown in the figure. Use this plot to choose the correct choice (s) given below.



(a) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy
(b) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy
(c) Fission of a nucleus lying in the mass number range of $100 < A < 200$ will release energy when broken into equal fragments
(d) Fission of a nucleus lying in the mass number range of $200 < A < 260$ will release energy when broken into equal fragments

IIT, 2008

SOLUTIONS

1. We have $6.25\% = \frac{6.25}{100} = \frac{1}{16} = \frac{1}{2^4}$

The given time of 4 hours thus equals 4 half lives so that the half life is 1 hour.

Since half life = $\frac{1n\,2}{\text{decay constant}}$ and mean life = $\frac{1}{\text{decay constant}}$, statements (b) and (c) are easily seen to be correct.

After a furthur 4 hours, the amount left over would be $\frac{1}{2^4} \times \frac{1}{2^4}$ i.e., $\frac{1}{256}$ or $\frac{100}{256}$ or 0.39% of the original amount.

Hence all the four choices are correct.

2. The correct choices are (a), (b) and (c).

3. All the four choices are correct.

4. A photon carries energy which is equivalent to mass. Hence the correct choices are (a) and (d).

5. The distance of closest approach is given by

$$r_0 = \frac{1}{2\pi\varepsilon_0}\frac{qZe}{mv^2}$$

where q, m and v respectively are the charge, mass and velocity of the incident particle and Z is the atomic number of the target nucleus. Hence the correct choices are (a), (c) and (d).

6. The correct choices are (b) and (c).

7. Decay rate at time $t = \frac{\text{decay rate at time } t=0}{2^n}$

$$= \frac{1600}{2^n}$$

where n = number of half lives in time t. Hence all the four choice are correct.

8. Reaction (a) is not possible because the atomic number is not conserved. Reactions (b), (c) and (d) are possible because both atomic and mass numbers are conserved, but reaction (d) is a fission reaction. Hence the possible fusion reactions are (b) and (c).

9. In α-decay, the mass number decreases by 4 and the atomic number decreases by 2. In β-decay, the mass number remains unchanged but the atomic number increases by 1. In the decay of uranium to lead, the mass number decreases by (238 -206) = 32. Hence the number of α-particles emitted is m = 32/4 = 8. But atomic number decreases by (92–82) = 10. In the emission of 8 α-particles, the mass number decreases by 16. Hence the number of β-particles emitted is n = 16 – 10 = 6. So the correct choices are (a) and (d).

10. The correct choices are (a) and (d).

11. It is a fusion reaction because two light nuclei $\left({}_1^2\text{H}\right)$ fuse to form a heavier nucleus $\left({}_2^4\text{He}\right)$

Mass defect Δm = 2 × mass of ${}_1^2\text{H}$ – mass of ${}_2^4\text{He}$

$= 2 \times 2.0141 - 4.0024$

$= 0.0258$ u

$= 0.0258 \times 931$ MeV

$= 24$ MeV

So the correct choices are (a) and (d).

12. The mass of a nucleus is less than the sum of the masses of its nucleons. In the nucleus ${}_{10}^{20}\text{Ne}$, there are 10 protons and 10 neutrons. Hence $M_1 < 10\,(m_p + m_n)$, which is choice (d). The heavier nucleus has more mass defect than the lighter nucleus, i.e.

Mass defect of ${}_{20}^{40}\text{Ca}$ > mass defect of ${}_{10}^{20}\text{Ne}$

$\Rightarrow \quad 20\,(m_p + m_n) - M_2 > 10\,(m_p + m_n) - M_1$

$\Rightarrow \quad M_2 < M_1 + 10\,(m_p + m_n)$

But $M_1 < 10\,(m_p + m_n)$

Hence $M_2 < 2M_1$, which is choice (c)

13. Energy is released if the total binding energy of the products is greater than the total binding energy of the reactants. This is not possible in choices (a) and (b) The correct choices are (b) and (d)

SECTION III

Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage

Passage I

At a given instant there are 25% undecayed radioactive nuclei in a sample. After 10 seconds the number of un decayed nuclei reduces to 12.5%

IIT, 1996

1. The half life of the sample is

(a) 10 s (b) 5 s

(c) 20 s (d) 30 s

2. The mean life of the nuclei is

(a) 6.93 s (b) 7.21 s

(c) 9.36 s (d) 14.43 s

3. The time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number is

(a) 10 s (b) 20 s
(c) 30 s (d) 40 s

SOLUTION

1. Since the number of undecayed nuclei reduces to half (from 25% to 12.5%) in 10 s, the half life is $T = 10$ s.

2. Mean life $= \dfrac{1}{\lambda} = \dfrac{T}{\log e(2)} = \dfrac{10}{0.693} = 14.43$ s

3. Let the reduced number further reduce to 6.25% in n half lives, then

$$\frac{6.25}{100} = \left(\frac{1}{2}\right)^n \rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^n \Rightarrow n = 4$$

$\therefore$ time taken $n\,T = 4 \times 10 = 40$ s.

Questions 4 to 6 are based on the following passage

Passage II

In an ore containing uranium, the ratio of U-238 to Pb – 206 is 3. Assume that all the lead present in the ore is the final stable product of U-238. The half life of uranium – 238 is 4.5×10^9 years. Take $\log_e(2) = 0.693$ and $\log_e\left(\dfrac{4}{3}\right) = 0.288$.

IIT, 1997

4. The decay constant is

(a) 2.22×10^{-9} per year
(b) 6.49×10^{-8} per year
(c) 3.11×10^{-9} per year
(d) 1.54×10^{-10} per year

5. If N_0 is the original number of radioactive nuclei and N is the number of nuclei left undecayed, then the ratio N/N_0 is

(a) $\dfrac{1}{3}$ (b) $\dfrac{2}{3}$
(c) $\dfrac{3}{4}$ (d) $\dfrac{4}{5}$

6. The age of the ore is of the order of

(a) 10^9 years (b) 10^{10} years
(c) 10^{11} years (d) 10^{12} years

SOLUTION

4. Decay constant $\lambda = \dfrac{\log_e(2)}{T} = \dfrac{0.693}{4.5 \times 10^9 \text{ years}}$

$= 1.54 \times 10^{-10}$ per year

5. If x nuclei of U-238 have decayed to Pb–206, then $N = N_0 - x$

Given $\dfrac{N_0 - x}{x} = 3 \Rightarrow x = \dfrac{N_0}{4}$

$\therefore \quad N = N_0 - x = N_0 - \dfrac{N_0}{4} = \dfrac{3N_0}{4}$

$\Rightarrow \quad \dfrac{N}{N_0} = \dfrac{3}{4}$

6. If the age of the ore is t years, then

$$\frac{3}{4} = e^{-\lambda t}$$

$$\Rightarrow \quad \frac{4}{3} = e^{\lambda t}$$

$$\Rightarrow \quad \lambda t = \log_e\left(\frac{4}{3}\right) = 0.288 \text{ (given)}$$

$$\therefore \quad t = \frac{0.288}{\lambda} = \frac{0.288}{1.54 \times 10^{-10}}$$

$= 1.87 \times 10^9$ years

Questions 7 to 9 are based on the following passage

Passage III

The element Curium $^{248}_{96}$Cm has a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The sample contains 10^{20}Cm atoms. The masses involved in α-decay are as follows:

$^{248}_{96}$Cm = 248.072220 u, $^{244}_{94}$Pu = 244.0641 u and $^{4}_{2}$He = 4.002603 u. (1 u = 931 Me V/c^2).

IIT, 1997

7. The energy released in each α-decay (in MeV) is

(a) 0.0514 (b) 0.514
(c) 5.14 (d) 51.4

8. The total energy released in one α-decay and one fission (in MeV) is

(a) 20.72 (b) 207.2
(c) 205.86 (d) 210.14

9. The power output from the sample of 10^{20} atoms is of the order of

(a) 10^5 W (b) 10 W
(c) 10^{-2} W (d) 10^{-5} W

SOLUTION

7. The equation governing the α-decay is

$$^{218}_{69}\text{Cm} \rightarrow {}^{244}_{94}\text{Pu} + {}^{4}_{2}\text{He}$$

Mass defect $\Delta m = m\,(\text{Pu}) + m\,(\text{He}) - m\,(\text{Cm})$

$$= 244.064100 + 4.002603 - 248.072220 = -0.005517\text{u}$$

$\therefore$ Energy released per α-decay is

$$E_\alpha = |\Delta m|c^2 = 0.005517 \times 931\,\text{MeV} \approx 5.136\,\text{MeV}$$

8. Energy released per fission is

$$E_f = 200\ \text{MeV (given)}$$

Since the probability of α-decay is 92% and that of fission is 8%, the total energy released in the transformation is

$$E = 0.92\ E_\alpha + 0.08\ E_f$$
$$= 0.92 \times 5.136\ \text{MeV} + 0.08 \times 200\ \text{MeV}$$
$$= 4.725 + 16 = 20.725\ \text{MeV}$$

9. Since there are 10^{20} atoms of Cm, the total energy released in 10^{20} reaction is

$$E_{\text{total}} = 20.725 \times 10^{20}\ \text{Me V}$$
$$= 20.725 \times 10^{26}\ \text{e V}$$
$$= 20.725 \times 10^{26} \times 1.6 \times 10^{-19}\ \text{J}$$
$$= 3.32 \times 10^{8}\ \text{J}$$

Given, mean life of Cm, $\tau = 10^{13}$ s. Therefore, the power output is

$$P = \frac{E_{\text{total}}}{\tau} = \frac{3.32 \times 10^8\,\text{J}}{10^{13}\,\text{s}}$$
$$= 3.32 \times 10^{-5}\ \text{Js}^{-1}\ (\text{or W})$$

So the correct choice is (d).

Questions 10 to 12 are based on the following passage

Passage IV

Nuclei of a radioactive element are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element.

IIT, 1998

10. If N is the number of nuclei at time t, the net decay rate of the element is

(a) α (b) $-\lambda N$
(c) $(\alpha - \lambda N)$ (d) $(\alpha + \lambda N)$

11. The number N of nuclei at time t is

(a) $\frac{1}{\lambda}(\alpha - \lambda N_0)e^{-\lambda t}$

(b) $\frac{1}{\lambda}(\alpha + \lambda N_0)e^{-\lambda t}$

(c) $\frac{1}{\lambda}\left[\alpha + (\alpha - \lambda N_0)e^{-\lambda t}\right]$

(d) $\frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}\right]$

12. If $\alpha = 2\,N_0\,\lambda$, the number N of nuclei as $t \rightarrow \infty$ is

(a) $\frac{N_0}{4}$ (b) $\frac{N_0}{2}$
(c) $2\,N_0$ (d) $4\,N_0$

SOLUTION

10. The rate of production of radioactive element is

$$-\frac{dN}{dt} = \alpha$$

The rate of decay of element is

$$-\frac{dN}{dt} = \lambda N \text{ or } \frac{dN}{dt} = -\lambda N$$

$\therefore$ The net rate is

$$\frac{dN}{dt} = \alpha - \lambda N$$

11. To calculate the number N of nuclei at time t (given that the number at $t = 0$ is N_0), we integrate the above expression from $N = N_0$ to $N = N$ and from $t = 0$ to $t = t$, We have

$$\int_{N_0}^{N} \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

or $$-\frac{1}{\lambda}\left|\log_e(\alpha - \lambda N)\right|_{N_0}^{N} = |t|_0^t$$

or $$-\frac{1}{\lambda}\log_e\left(\frac{\alpha - \lambda N}{\alpha - \lambda N_0}\right) = t$$

or $$\log_e\left(\frac{\alpha - \lambda N}{\alpha - \lambda N_0}\right) = -\lambda t \text{ or } \frac{\alpha - \lambda N}{\alpha - \lambda N_0} = e^{-\lambda t}$$

or $$N = \frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}\right]$$

12. If $\alpha = 2N_0\,\lambda$, then we have

$$N = \frac{1}{\lambda}\left[2N_0\lambda - (2N_0\lambda - \lambda N_0)e^{-\lambda t}\right]$$

$$= N_0\,(2 - e^{-\lambda t})$$

When $t \to \infty$

$$N = N_0\,(2 - e^{-\infty}) = N_0\,(2 - 0) = 2\,N_0.$$

Questions 13 to 15 are based on the following passage

Passage V

A nucleus at rest undergoes a decay emitting an α-particle of de Broglie wavelength $\lambda = 5.76 \times 10^{-15}$ m. The mass of the daughter nucleus D = 223.610 amu and the mass of an α-particle = 4.002 amu. Take 1 amu = 1.656×10^{-27} kg and Planck's constant $h = 6.63 \times 10^{-34}$ Js.

IIT, 2001

13. The momentum of the α-particle (in kg ms^{-1}) is

(a) 1.15×10^{-19} (b) 1.6×10^{-19}

(c) 8.69×10^{-21} (d) 4.002×10^{-27}

14. The total kinetic energy of the two particles is of the order of

(a) 10^{-6} J (b) 10^{-8} J

(c) 10^{-10} J (d) 10^{-12} J

15. The mass of the parent nucleus is

(a) (223.610 + 4.002) amu

(b) (223.610 – 0.0067) amu

(c) (223.610 + 4.002 + 0.0067) amu

(d) (223.610 + 4.002 – 0.0067) amu

SOLUTION

13. The given decay reaction may be represented as

$${}^{A}\mathrm{P} \rightarrow {}^{A-4}\mathrm{D} = {}^{4}\alpha$$

The given de-Brogile wavelength of α-particle is

$$\lambda = 5.76 \times 10^{-15}\ \text{m}$$

The momentum of α-particle will be

$$p_\alpha = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{5.76\times10^{-15}}$$

$$= 1.151 \times 10^{-19}\ \text{kg m/s}$$

14. Due to the conservation of linear momentum, we will also have

$$p_D = 1.151 \times 10^{-19}\ \text{kg m/s}$$

The total kinetic energy of these two particles is

$$E = \frac{p_\alpha^2}{2m_\alpha} + \frac{p_D^2}{2m_D} \qquad (1)$$

Now $m_\alpha = 4.002$ amu

$$= 4.002 \times 1.656 \times 10^{-27}$$

$$= 6.637 \times 10^{-27}\ \text{kg}$$

$$m_D = 223.610 \times 1.656 \times 10^{-27}$$

$$= 370.3 \times 10^{-27}\ \text{kg}$$

Substituting the values in eq. (1) and solving we get

$$E = 1.02 \times 10^{-12}\ \text{J}$$

15. The mass equivalent of energy E is

$$\Delta m = \frac{E}{c^2} = \frac{1.02\times10^{-12}}{\left(3\times10^{8}\right)^2}$$

$$= 1.113 \times 10^{-29}\ \text{kg}$$

$$= \frac{1.113\times10^{-29}}{1.656\times10^{-27}}$$

$$= 0.0067\ \text{amu}$$

$\therefore$ Mass of parent nucleus is

$$m = m_D + m_\alpha + \Delta_m$$

$$= (223.610 + 4.002 + 0.0067)\ \text{amu}$$

Questions 16 to 18 are based on the following passage

Passage VI

A radioactive element decays by beta emission. A detector records n beta particles in the first 2 seconds and 0.75 n beta particles in the nest 2 seconds.

IIT, 2003

16. The decay constant of the element is

(a) $\lambda = \frac{1}{2}\ln\left(\frac{4}{3}\right)$ (b) $\lambda = \frac{1}{2}\ln\left(\frac{3}{4}\right)$

(c) $\lambda = \frac{1}{2}\ln(2)$ (d) $\lambda = \frac{3}{4}\ln(2)$

17. The mean life of the element is (given ln (2) = 0.693 and ln (3) = 1.1)

(a) 5 s (b) 6 s

(c) 7 s (d) 8 s

18. The half life of the element is

(a) 4.8 s (b) 5.8 s

(c) 6.8 s (d) 7.8 s

SOLUTION

16. $N_t = N_0\, e^{-\lambda t}$ Therefore

$$N_2 = N_0\, e^{-2\lambda} \text{ and } N_4 = N_0\, e^{-4\lambda}$$

Given $n = N_0 - N_2 = N_0\,(1 - e^{-2\lambda})$ (1)

and $0.75\, n = N_2 - N_4 = N_0\, e^{-2\lambda} - N_0\, e^{-4\lambda}$

$\Rightarrow$ $0.75\, n = N_0\,(e^{-2\lambda} - e^{-4\lambda})$

$\Rightarrow$ $0.75\, n = N_0\, e^{-2\lambda}\,(1 - e^{-2\lambda})$ (2)

From Eqs. (1) and (2) we get

$$0.75 = e^{-2\lambda} \quad \Rightarrow e^{2\lambda} = \frac{4}{3}$$

$$\Rightarrow \quad \lambda = \frac{1}{2}\ln\left(\frac{4}{3}\right)$$

17. $\lambda = \frac{1}{2}(\ln 4 - \ln 3) = \frac{1}{2}(2\ln 2 - \ln 3)$

$$= \frac{1}{2}[2 \times 0.693 - 1.1]$$

$$= 0.143 \text{ s}^{-1}$$

$\therefore$ Mean life $= \frac{1}{\lambda} = \frac{1}{0.143} = 7$ s

18. Half life $= \frac{\ln(2)}{\lambda} = \frac{0.693}{0.143} = 4.8$ s

Questions 19 to 21 are based on the following passage

Passage VII

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}^2_1\text{H}$, known as deuteron and denoted by D, can be thought of as a candidate for fusion reactor. The D-D reaction is ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + n$ + energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}^2_1\text{H}$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm^3.

It may be helpful to use the following: Boltzmann constant $k = 8.6 \times 10^{-5}$ eV/K; $\frac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm.

IIT, 2009

19. In the core of nuclear fusion reactor, the gas becomes plasma because of

(a) strong nuclear force acting between the deuterons

(b) Coulomb force acting between the deuterons

(c) Coulomb force acting between deuteron-electron pairs

(d) the high temperature maintained inside the reactor core

20. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(a) 1.0×10^9 K $< T < 2.0 \times 10^9$ K

(b) 2.0×10^9 K $< T < 3.0 \times 10^9$ K

(c) 3.0×10^9 K $< T < 4.0 \times 10^9$ K

(d) 4.0×10^9 K $< T < 5.0 \times 10^9$ K

21. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?

(a) deuteron density $= 2.0 \times 10^{12}$ cm^{-3}, confinement time $= 5.0 \times 10^{-3}$ s

(b) deuteron density $= 8.0 \times 10^{14}$ cm^{-3}, confinement time $= 9.0 \times 10^{-1}$ s

(c) deuteron density $= 4.0 \times 10^{23}$ cm^{-3}, confinement time $= 1.0 \times 10^{-11}$ s

(d) deuteron density $= 1.0 \times 10^{24}$ cm^{-3}, confinement time $= 4.0 \times 10^{-12}$ s

SOLUTION

19. The correct choice is (d).

20. At a large separation, potential energy is zero and total energy $= 1.5\,kT + 1.5\,kT = 3\,kT$.

From conservation of energy,

$$3\,kT = -\Delta U = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad T = \frac{e^2}{4\pi\varepsilon_0} \times \frac{1}{3kr}$$

$$= \frac{1.44\times10^{-9}}{3\times(8.6\times10^{-5})\times(4\times10^{-5})}$$

$$= 1.4 \times 10^9 \text{ K}$$

So the correct choice is (a).

21. According to Lawson, deuteron density (n) and confinement time t_0 must satisfy the criterion

$$n\ t_0 > 5 \times 10^{14} \text{ cm}^{-3} \text{ s}$$

This condition is satisfied only by choice (b) for which $nt_0 = (8.0 \times 10^{14}) \times (9.0 \times 10^{-1}) = 7.2 \times 10^{14}$ cm^{-3} s. Hence the correct choice is (b).

SECTION IV

Matching

1. Match the processes given in Column I with the nuclear reactions given in Column II. Symbol Q stands for energy released.

Column I	***Column II***
(a) Alpha decay	(p) ${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\left({}^{1}_{0}\text{n}\right) + Q$
(b) Beta decay	(q) ${}^{3}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + Q$
(c) Nuclear fission	(r) ${}^{230}_{90}\text{Th} \rightarrow {}^{226}_{90}\text{Ra} + {}^{4}_{2}\text{He} + Q$
(d) Nuclear fusion	(s) ${}^{137}_{55}\text{Cs} \rightarrow {}^{137}_{56}\text{Ba} + \text{e}^- + \bar{\nu} + Q$

ANSWER

(a) → (r) (b) → (s)
(c) → (p) (d) → (q)

2. Match the processes in column I with their properties in Column II.

Column I	***Column II***
(a) Nuclear fission	(p) involves weak nuclear forces
(b) Nuclear fusion	(q) involves conversion of matter into energy
(c) β-decay	(r) atoms of higher atomic number are used
(d) Exothermic nuclear reaction	(s) atoms of lower atomic number are used

IIT, 2006

ANSWERS

(a) → (q), (r) (b) → (q), (s)
(c) → (p) (d) → (q)

SECTION V

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

1. Statement-1
A nucleus at rest splits into two nuclear parts having radii in the ratio 1 : 2. Their velocities will be in the ratio 8 : 1.

Statement-2
The radius of a nucleus is proportional to the cube root of its mass number.

2. Statement-1
The half life of radioactive sample is T. It will decay to $\frac{1}{16}$ of its initial value in a time $8T$.

Statement-2
The half life of a radioactive sample is the time in which half of the number of nuclei decay.

3. Statement-1
A freshly prepared radioactive sample has a half life of 3 hours and emits radiation of intensity which is 64 times the permissible safe value. The minimum time after which it would be safe to work with the sample is 18 hours.

Statement-2
The intensity of the radiation falls by a factor of 2 every 3 hours.

4. Statement-1
Two radioactive sources A and B initially contain equal number of radioactive nuclei. Source A has a half life of 1 hour and source B has a half life of 2 hours. At the end of 2 hours, they will have the same rate of disintegration.

Statement-2
The rate of disintegration is defined as the number of disintegrations taking place in the source per second.

5. Statement-1
The nucleus ^{22}Ne absorbs energy and decays into two alpha particles and an unknown nucleus. The unknown nucleus must be carbon.

Statement-2
In a nuclear reaction, the atomic number is conserved.

6. Statement-1
The number of α and β particles emitted when $^{238}_{92}$U decays into $^{206}_{82}$ Pb is 6 and 8 respectively.

Statement-2
In a nuclear reaction, the mass number and the atomic number are both conserved.

7. Statement-1
The radioactive decay of nucleus X to nuclei Y and K is represented by the equation

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} \rightarrow {}^{A-4}_{z-1}\mathrm{K} \rightarrow {}^{A-4}_{z-1}\mathrm{K}$$

The sequence of emitted radiations is β, α and γ.

Statement-2
In a nuclear reaction, the mass number and the atomic number are both conserved.

8. Statement-1
The distance of the closest approach of an alpha particle fired at a nucleus with momentum p is r_0. The distance of the closest approach when the alpha particle is fired at the same nucleus with momentum $2p$ will be $r_0/2$.

Statement-2
The distance of closest approach from a given target element is inversely proportional to the kinetic energy of the incident particle.

9. Statement-1
The binding energy of deuteron ($^{2}_{1}$H) is 1.15 MeV per nucleon and an alpha particle ($^{4}_{2}$He) has a binding energy of 7.1 MeV per nucleon. Then in the reaction

$$^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + \mathrm{Q}$$

the energy Q released is 23.8 MeV.

Statement-2
Total energy is conserved in a nuclear reaction.

10. Statement-1
A nucleus X, initially at rest, decays into a nucleus Y with the emission of an α-particle and energy Q is released. If m is the mass of an alpha particle and M that of nucleus Y, the energy of the emitted α-particle will be

$$E_\alpha = \frac{QM}{(M-m)}$$

Statement-2
Momentum and energy are conserved in the decay process.

SOLUTIONS

1. The correct choice is (a). Let A_1 and A_2 be the mass numbers of the two nuclear parts. Their radii are given by

$$R_1 = R_0\,(A_1)^{1/3} \text{ and } R_2 = R_0\,(A_2)^{1/3}$$

Dividing, we get

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} \text{ or}$$

$$\frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Hence the ratio of their masses is

$$\frac{m_1}{m_2} = \frac{1}{8}$$

From the principle of conservation of momentum, the magnitude of p_1 = magnitude of p_2 or $m_1 v_1 = m_2 v_2$, which gives

$$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{8}{1}$$

2. The correct choice is (d). Since $\frac{1}{16} = \frac{1}{2^4}$, it follows that the time taken for the sample to decay to 1/16 of its initial value = four half lives = 4 T.

3. The correct choice is (a). Since the half life is 3 hours, the intensity of radiation falls by a factor of 2 every three hours. In 18 hours, it will fall by a factor of $(2)^6 = 64$.

4. The correct choice is (a). The rate of disintegration is proportional to the number of radioactive nuclei present initially in the source.

5. The correct choice is (a). The given nuclear reaction is given by the equation

$$^{22}_{10}\text{Ne} \rightarrow {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He} + \text{Z}^X$$

Since the atomic number is conserved, we have

$$10 = 2 + 2 + \text{Z}$$

which gives Z = 6. The nucleus having Z = 6 is carbon.

6. The correct choice is (d). Let x and y respectively be the number of α and β particles emitted. The equation of the decay is

$$^{238}_{92}\text{U} \rightarrow {}^{206}_{82}\text{Pb} + x({}^{4}_{2}\text{He}) + y\,({}^{0}_{-1}\text{e})$$

From conservation of charge, we have

$$92 = 82 + 2x - y \qquad \text{(i)}$$

Conservation of mass number gives

$$238 = 206 + 4x \qquad \text{(ii)}$$

which gives $4x = 32$ or $x = 8$. Using this value of x in (i) we get $y = 6$.

7. The correct choice is (b). In transition $^{A}_{Z}\text{X} \rightarrow {}^{A}_{Z+1}\text{Y}$, the atomic (or charge) number increases by unity; mass number remaining the same. Hence an electron (β-particle) is emitted. In transition $^{A}_{Z+1}\text{Y} \rightarrow {}^{A-4}_{Z-1}\text{K}$, the mass number decreases by 4 and charge number decreases by 2. Hence an α-particle is emitted. In the third transition, mass and charge numbers do not change. Hence a γ-ray is emitted.

8. The correct choice is (d). The distance of the closest approach is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K}$$

where $K = \frac{1}{2} mv^2$.

$K = \frac{1}{2} mv^2 = \frac{1}{2m} \times (mv)^2 = \frac{p^2}{2m}$. Therefore,

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4mZe^2}{p^2}$$

Thus $r_0 \propto \frac{1}{p^2}$. When p is doubled, r_0 becomes one-fourth

9. The correct choice is (a). Binding energy of $^{2}_{1}\text{H}$ = 1.15 × number of nucelons = 1.15 × 2 = 2.3 MeV. Total binding energy of reactants = 2.3 + 2.3 = 4.6 MeV. Binding energy of $^{4}_{2}\text{He}$ = 7.1 × number of nucleons = 7.1 × 4 = 28.4 MeV. Therefore, Q = 28.4 – 4.6 = 23.8 MeV. Hence the correct choice is (c).

10. The correct choice is (d). $K = p^2/2m$. Therefore, from conservation of energy, $Q = \frac{P^2}{2M} + \frac{p^2}{2m}$ (P = momentum of Y, p = momentum of α-particle) Since momentum is conserved, P = p. Hence

$$Q = \frac{p^2}{2}\left(\frac{1}{M} + \frac{1}{m}\right) = \frac{p^2}{2m}\left(\frac{m}{M} + 1\right)$$

$$= E_\alpha \left(\frac{m}{M} + 1\right)$$

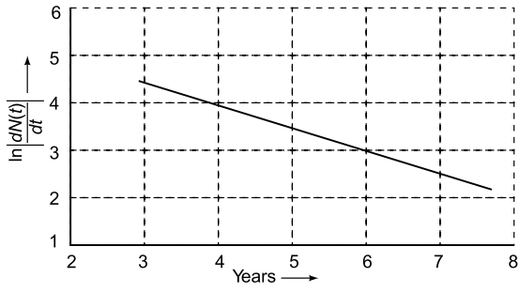
where E_α = energy of α-particle. Hence

$$E_\alpha = \frac{QM}{(M+m)}$$

SECTION VI

Integer Answer Type Questions

1. To determine the half life of radioactive element, a student plots a graph of $\ln \left|\frac{dN(t)}{dt}\right|$ versus t.



Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, find the value of p.

IIT, 2010

2. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

SOLUTIONS

1. $N = N_0 e^{-\lambda t} \Rightarrow \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$. Therefore

$$\left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t}$$

$$\Rightarrow \quad \ln \left|\frac{dN}{dt}\right| = \ln (\lambda N_0) - \lambda t$$

The slope of the graph is $= \frac{4-3}{4-6} = -\frac{1}{2}$

$\therefore \quad \lambda = \frac{1}{2}$ per year

Half life $T_{1/2} = \frac{0.693}{1/2} = 1.386$ year

$\therefore$ Number of half lives $= \frac{4.16}{1.386} = 3$

Hence $p = (2)^3 = 8$.

2. Activity $|R| = N\lambda = \frac{N}{\tau}$; τ = mean life

$\therefore \qquad N = \tau\,|R| = 10^9 \times 10^{10} = 10^{19}$ atoms

$\therefore$ Total mass $= 10^{19} \times 10^{-25} = 10^{-6}$ kg $= 1$ mg.

MODEL TEST PAPER – I

The questions in the practice papers are based on questions asked in previous years' Physics Question Papers of IIT-JEE. Answer key and complete solutions of questions are provided at the end of each practice paper. Each paper contains 30 questions to be answered in 45 minutes.

SECTION I

(Single Correct Answer Type)

This section contains 13 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

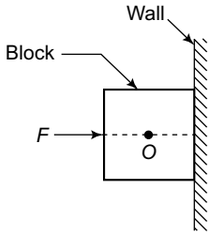
1. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table.
Least count for length = 0.1 cm
Least count for time = 0.1 s

Student	*Length of the pendulum (cm)*	*Number of oscillations (n)*	*Total time for (n) oscillations (s)*	*Time period (s)*
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	6	36.0	6.0

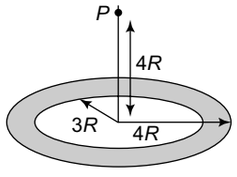
If E_I, E_{II} and E_{III} are the percentage errors in g. i.e. $\left(\frac{\Delta g}{g}\times 100\right)$ for student I, II and III, respectively,

(a) $E_I = 0$ (b) E_I is minimum
(c) $E_I = E_{II}$ (d) E_{II} is minimum

2. A block of mass m is held stationary against a wall by applying a horizontal force F on the block. Which of the following statements is false?
(a) The frictional force acting on the block is $f = mg$
(b) The normal reaction force acting on the block is $N = F$
(c) No net torque acts on the block
(d) N does not produce any torque.

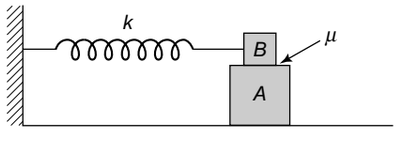


3. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is



(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$
(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$

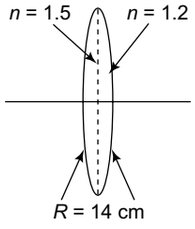
4. A block A of mass m is placed on a frictionless horizontal surface. Another block B of the same mass is kept on A and connected to the wall with the help of a spring of force constant k, as shown in the figure. The coefficient of friction between blocks A and B is μ. The blocks move together executing simple harmonic motion of amplitude a. The maximum value of frictional force between A and B is



(a) ka (b) $ka/2$
(c) zero (d) μmg

5. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surface are of the same radius of curvature R = 14 cm. For this

bi-convex lens, for an object distance of 40 cm, the image distance will be



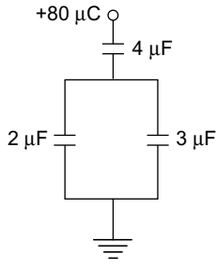
(a) −280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm

6. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e. second) plate under steady state condition is

(a) $\left(\frac{65}{2}\right)^{1/4} T$ (b) $\left(\frac{97}{4}\right)^{1/4} T$

(c) $\left(\frac{97}{2}\right)^{1/4} T$ (d) $(97)^{1/4}\ T$

7. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is



(a) +32 μC (b) +40 μC
(c) +48 μC (d) +80 μC

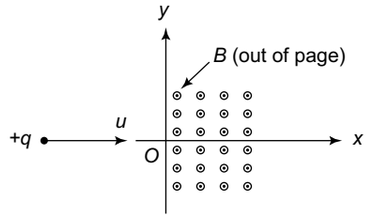
8. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

(a) 14.0 cm (b) 15.2 cm
(c) 16.4 cm (d) 17.6 cm

9. An RC circuit consists of a resistance R = 5 MΩ. and a capacitance C = 1.0 μF connected in series with a battery. In how much time will the potential difference across the capacitor become 8 times that across the resistor? (Given $\log_e$ (3) = 1.1)

(a) 5.5 s (b) 11 s
(c) 44 s (d) 88 s

10. A proton moving with a speed u along the positive x-axis enters at $y = 0$ a region of uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$ which exists to the right of y-axis as shown in the figure. The proton leaves the region after some time with a speed v at co-ordinate y. Then

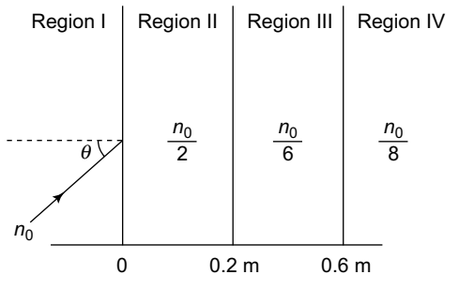


(a) $v > u, y < 0$ (b) $v = u, y > 0$
(c) $v > u, y > 0$ (d) $v = u, y < 0$

11. A cylindrical conducting rod is kept with its axis along the x-axis. Also there exists a uniform magnetic field parallel to the x-axis. The current induced in the cylinder is

(a) clockwise as seen from the + x axis
(b) anticlockwise as seen from the + x axis
(c) along the axis towards – x direction
(d) zero

12. A light beam is traveling from Region I to Region IV (Refer to figure). The refractive indices in Regions I, II, III and IV are n_0, $\frac{n_0}{2}$, $\frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering region IV is



(a) $\sin^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{8}\right)$

(c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{3}\right)$

13. Electrons with de-Broglie wavelength λ fall on the target in an *X*-ray tube. The cut-off wavelength of the emitted *X*-ray is

(a) $\lambda_0 = \dfrac{2mc\,\lambda^2}{h}$ (b) $\lambda_0 = \dfrac{2h}{mc}$

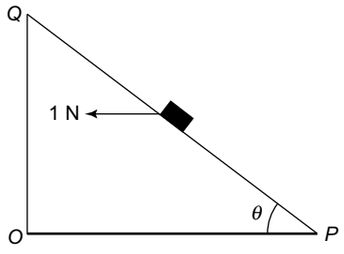
(c) $\lambda_0 = \dfrac{2m^2c^2\lambda^2}{h^2}$ (d) $\lambda_0 = \lambda$

SECTION II

(Multiple Correct Answer Type)

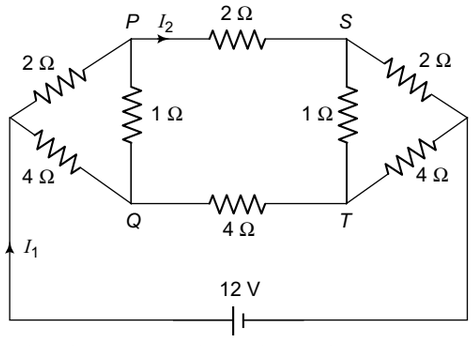
This section contains 7 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE than one choice/choices** is/are correct**.**

14. A small block of mass of 0.1 kg lies on a fixed inclined plane *PQ* which makes an angle θ with the horizontal. A horizontal force of 1 N acts of the block through its centre of mass as shown in the figure. The block remains stationary if (take $g = 10\ \text{m/s}^2$).



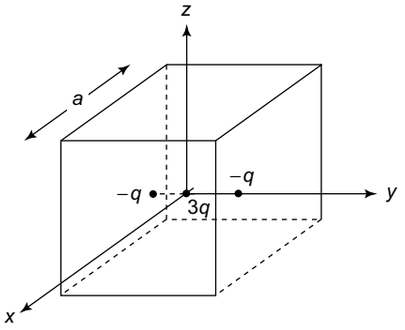
(a) $\theta = 45°$
(b) $\theta > 45°$ and a frictional forces acts on the block towards *P*.
(c) $\theta > 45°$ and a frictional forces acts on the block towards *Q*.
(d) $\theta < 45°$ and a frictional forces acts on the block towards *Q*.

15. For the resistance network shown in the figure, choose the correct option(s).



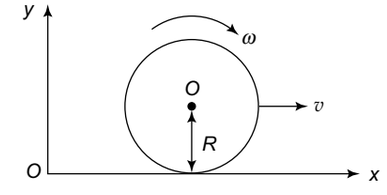
(a) The current through *PQ* is zero.
(b) I_1 = 3A
(c) The potential at *S* is less than that at *Q*.
(d) I_2 = 2A

16. A cubical region of side *a* has its centre at the origin. It encloses three fixed point charges, $-q$ at (0, $-a/4$, 0), $+3q$ at (0, 0, 0) and $-q$ at (0, $+a/4$, 0). Choose the correct options(s).



(a) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$
(b) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$.
(c) The net electric flux crossing the entire region is $\dfrac{q}{\varepsilon_0}$.
(d) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$.

17. A disc of mass *M* and radius *R* is rolling with angular speed ω on a horizontal surface as shown in the figure. The magnitude of angular momentum of the disc about the origin *O* is (here *v* is the linear velocity of the disc)



(a) $\frac{3}{2}MR^2\omega$ (b) $MR^2\omega$

(c) MRv (d) $\frac{3}{2}MRv$

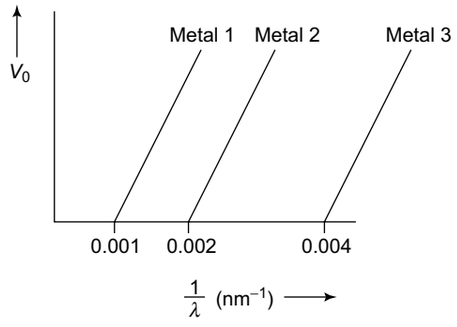
18. The displacement x of a particle varies with time t as $x - A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$. For what values of A, B and C is the motion simple harmonic?

(a) All values of A, B and C with $C \neq 0$.
(b) $A = B$, $C = 2B$
(c) $A = -B$, $C = 2B$
(d) $A = B$, $C = 0$

19. A beam of light consisting of two wavelengths 750 nm and 450 nm is used to obtain interference fringes in a Young's double slit experiment. The separation between the slits is 1 mm and the distance between the plane of the slits and the screen is 100 cm. The least distance from the central maximum where the bright fringes due to both the wavelengths coincide is y_{min} and y'_{min} is the corresponding distance where the dark fringes due to both the wavelengths coincide. Then

(a) $y_{min} = 2.25$ mm
(b) $y_{min} = 2.0$ mm
(c) $y'_{min} = 4.5$ mm
(d) $y'_{min} = 1.125$ mm

20. The following figure shows graphs between cut-off voltage V_0 and $\frac{1}{\lambda}$ for three metals 1, 2 and 3, where λ is the wavelength of the incident radiation in nm.



If W_1, W_2 and W_3 are the work functions of metals 1, 2 and 3 respectively, then

(a) $W_1 : W_2 : W_3 = 1 : 2 : 4$
(b) $W_1 : W_2 : W_3 = 4 : 2 : 1$
(c) The graphs for metals 1, 2 and 3 are parallel to each other and the slope of each graph is hc/e, where h = Planck's contant, c = speed of light and e = charge of an electron.
(d) Ultraviolet light will eject photoelectrons from metals 1 and 2 and not from metal 3.

SECTION III

(Linked Comprehension Type)

This section contains 4 questions based on a paragraph. Each question has four choices (a), (b), (c) and (d) out of which only **ONE** choice is correct**.**

Questions 21 and 22 are based on the following paragraph.

A light rod of length L having a body of mass M attached to its end hangs vertically. It is turned through 90° so that it is horizontal and then released.

21. The centripetal acceleration when the rod makes an angle θ with the vertical is

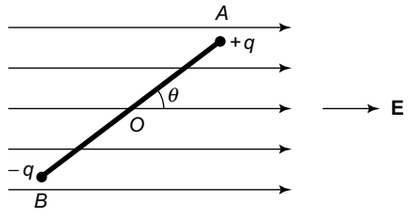
(a) $g\cos\theta$ (b) $2g\cos\theta$
(c) $g\sin\theta$ (d) $2g\sin\theta$

22. The tension in the rod when it makes an angle θ with the vertical is

(a) $Mg\cos\theta$ (b) $2\,Mg\cos\theta$
(c) $3\,Mg\cos\theta$ (d) zero

Questions 23 and 24 are based on the following passage.

A point particle of mass M is attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-q$. This arrangement is held in a region of a uniform electric field **E** such that the rod makes a small angle θ (say of about 5°) with the field direction.



23. When the rod is released, it will rotate with an angular frequency ω equal to

(a) $\left(\frac{qE}{ML}\right)^{1/2}$ (b) $\left(\frac{2qE}{ML}\right)^{1/2}$

(c) $\left(\frac{qE}{2ML}\right)^{1/2}$ (d) $\frac{1}{2}\left(\frac{qE}{ML}\right)^{1/2}$

24. The minimum time taken by the rod to align itself parallel to the electric field after it is set free is given by

(a) $\frac{\pi}{2}\left(\frac{ML}{2qE}\right)^{1/2}$ (b) $2\pi\left(\frac{ML}{qE}\right)^{1/2}$

(c) $2\pi\left(\frac{2ML}{qE}\right)^{1/2}$ (d) $2\pi\left(\frac{ML}{2qE}\right)^{1/2}$

SECTION IV

(Assertion-Reason Type)

This section contains 2 questions. In each question, Statement-1 is followed by Statement-2. Each question has the following four choices (a), (b), (c) and (d) out of which only **ONE** choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

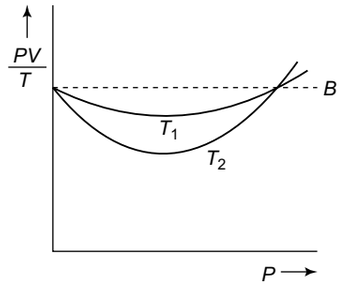
(b) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-1 is true.

25. Statement-1

The following figure shows $\frac{PV}{T}$ versus P graph for a certain mass of oxygen gas at two temperatures T_1 and T_2. It follows from the graph that $T_1 > T_2$.



Statement-2

At higher temperature, real gas behaves more like an ideal gas.

26. Statement-1

A particle of mass M at rest decays into two particles of masses m_1 and m_2 which move with velocities v_1 and v_2 respectively. Their respective de Broglie wavelengths are λ_1 and λ_2. If $m_1 > m_2$, then $\lambda_1 > \lambda_2$.

Statement-2

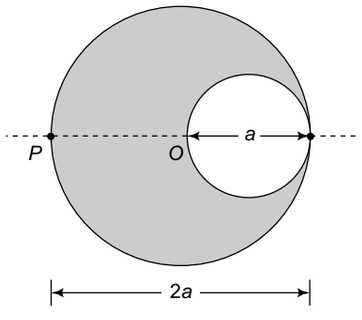
The de Broglie wavelength of a particle having momentum p is $\lambda = h/p$.

SECTION V

(Integer Answer Type)

This section contains 3 questions. The answer to each question is single digit integer, ranging from 0 to 9 (both inclusive)

27. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, find the value of N.



28. A binary star consists of two stars A (mass 2.2 M_s) and B (mass 11 M_s), where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. Find the ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass.

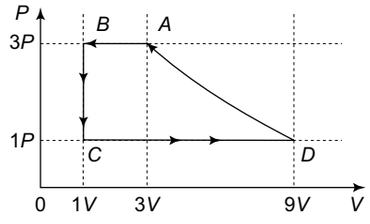
29. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

SECTION VI

(Matrix Match Type)

This section contains **1 question**. Each question has four statements (a, b, c and d) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

30. One mole of a monatomic gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I.



Column I		Column II	
(a)	Process A → B	(p)	Internal energy decreases
(b)	Process B → C	(q)	Internal energy increases
(c)	Process C → D	(r)	Heat is lost
(d)	Process D → A	(s)	Heat is gained
		(t)	Work is done on the gas

Answers

Section-I

1. (b) **2.** (d) **3.** (b)
4. (b) **5.** (b) **6.** (c)
7. (c) **8.** (b) **9.** (b)
10. (d) **11.** (d) **12.** (b)
13. (a)

Section-II

14. (a, c) **15.** (a, b, c, d) **16.** (a, c, d)
17. (a, d) **18.** (a, b, c) **19.** (a, d)
20. (a, d)

Section-III

21. (b) **22.** (c) **23.** (b)
24. (a)

Section-IV

25. (a) **26.** (d)

Section-V

27. (5) **28.** (6) **29.** (9)

Section-VI

30. (a) → (p, r, t); (b) → (p, r), (c) → (q, s), (d) → (r, t).

Solutions

Section-I

1. $T = 2\pi\sqrt{\dfrac{L}{g}}$

Time period $T = \dfrac{t}{n}$, when n = number of oscillation and t is the total time for n oscillation. In terms of measured quantities,

$\dfrac{t}{n} = 2\pi\sqrt{\dfrac{L}{g}} \Rightarrow g = \dfrac{4\pi^2 L\, n^2}{t^2}$. Therefore,

$\dfrac{\Delta g}{g} = \dfrac{\Delta L}{L} + \dfrac{2\,\Delta t}{t}$ ($\because \Delta n = 0$; there is no error in counting the number of oscillation)

For student I, $E_{\mathrm{I}} = \dfrac{\Delta g}{g} \times 100 = \left(\dfrac{0.1}{64} + \dfrac{2 \times 0.1}{128}\right) \times 100$

$= \dfrac{5}{16}\%$

For student II, $E_{II} = \left(\frac{0.1}{64} + \frac{2 \times 0.1}{64.0}\right) \times 100$

$= \frac{15}{32}\%$

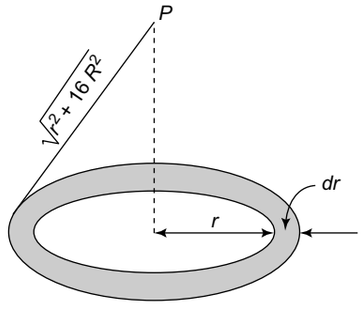
For student III, $E_{III} = \left(\frac{0.1}{20.0} + \frac{2 \times 0.1}{36}\right) \times 100$

$= \frac{19}{18}\%$

Thus the percentage error in the measurement of g is minimum for student I.

2. Since the block is held stationary, it is in translational as well as rotational equilibrium. Hence no net force and no net torque acts on the block. No net force will act on the block if $f = mg$ and $N = F$. No net torque will act on the block, if torque by frictional force f about centre O = counter torque by normal reaction N about centre O. Hence choice (d) is false.

3. By definition, the work required to take a unit mass from P to infinity $= -V_p$, where V_P is the gravitational potential at P due to the disc. To find V_P, we divide the disc into small elements, each of thickness dr. Consider one such element at a distance r from the centre of the disc, as shown in the figure.



Mass of the element $dm = \frac{M\,(2\pi r dr)}{\pi(4R)^2 - \pi(3R)^2}$

$= \frac{2M\,rdr}{7R^2}$

$V_P = -\int_{3R}^{4R} \frac{G\,dm}{\sqrt{r^2 + 16R^2}}$

$= -\frac{2\,MG}{7R^2} \int_{3R}^{4R} \frac{rdr}{(r^2 + 16R^2)^{1/2}}$

Putting $r^2 + 16\,R^2 = x^2$, we get $2r\,dr = 2x\,dx$ or $rdr = x\,dx$.

When $r = 3$ R, $x = \sqrt{9R^2 + 16R^2} = 5R$

When $r = 4\,R$, $x = \sqrt{16R^2 + 16R^2} = 4\sqrt{2}\,R$

$\therefore \quad V_P = -\frac{2MG}{7R^2} \int_{5R}^{4\sqrt{2}\,R} dx = -\frac{2MG}{7R^2}(4\sqrt{2} - 5)\,R$

Hence $-V_P = -\frac{2MG}{7R}(4\sqrt{2} - 5)$, which is choice (b).

4. The blocks will move together as long as the frictional force of block B = mass of block $B \times$ maximum acceleration of its S.H.M., i.e.

$f = m\omega^2 a$

where $\omega = \sqrt{\frac{k}{(m+m)}} = \sqrt{\frac{k}{2m}}$

Thus $f = m \times \frac{k}{2m} \times a$

$= ka/2$

5. $\frac{1}{f_1} = \left(\frac{1.5-1}{1}\right)\left(\frac{1}{14} - \frac{1}{\infty}\right) = \frac{1}{28}\ \text{cm}^{-1}$

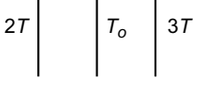
$\frac{1}{f_2} = \left(\frac{1.2-1}{1}\right)\left(\frac{1}{\infty} - \frac{1}{-14}\right) = \frac{1}{70}\ \text{cm}^{-1}$

$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{28} + \frac{1}{70} \Rightarrow F = 20\ \text{cm}$

Now $\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$

$\Rightarrow \quad \frac{1}{v} - \frac{1}{-40} = \frac{1}{20} \Rightarrow v = 40\ \text{cm}$

6. In the steady state, the rate at which the middle plate receives heat energy is equal to the rate at which heat energy is emitted by the other plates. Let A be the area of each plate and T_0 be the steady state temperature of the middle plate. Since both sides of the middle plate receive heat energy, the total area of the middle plate receiving energy is $2A$.



From Stefan's law

$$\sigma\,(2A)\,(T_o)^4 = \sigma A\,(2T)^4 + \sigma A\,(3T)^4$$

$$\Rightarrow \quad 2\,T_o^4 = 16\,T^4 + 81\,T^4 = 97\,T^4$$

$$\Rightarrow \quad T_o = \left(\frac{97}{2}\right)^{1/4} T$$

7. Let q μC be the charge on the upper plate of 3 μF capacitor. Then the charge on the upper plate of 2 μF capacitor will be $(80 - q)$ μC. Since potential difference across 2 μF capacitor = potential difference across 3 μF capacitors,

$$\frac{80-q}{2} = \frac{q}{3} \Rightarrow q = 48\ \mu\text{C}.$$

8. End correction $e = 0.3d = 0.3 \times 4 = 1.2$ cm

Wavelength $\lambda = \dfrac{v}{\nu} = \dfrac{336}{512} = 0.656$ m $= 65.6$ cm

Now $\quad L + e = \dfrac{\lambda}{4}$

$$\Rightarrow \quad L = \frac{\lambda}{4} - e = \frac{65.6}{4} - 1.2$$

$$= 16.4 - 1.2 = 15.2 \text{ cm}$$

9. At instant of time t, the charge on the capacitor is given by

$$q = q_0\,(1 - e^{-t/RC})$$

and the potential drop across the capacitor is given by ($\because\ V = q/C$)

$$V_C = V_0\,(1 - e^{-t/RC})$$

where V_0 is the voltage of the battery. The potential drop across the resistor is

$$V_R = V_0 - V_C = V_0 - V_0\,(1 - e^{-t/RC}) = V_0\,e^{-t/RC}$$

$$\therefore \quad \frac{V_C}{V_R} = \frac{1 - e^{-t/RC}}{e^{-t/RC}} = e^{t/RC} - 1$$

Given $\dfrac{V_C}{V_R} = 8$. Therefore,

$$8 = e^{t/RC} - 1$$

or $\quad e^{t/RC} = 9 = (3)^2$

$$\frac{t}{RC} = 2\log_e(3)$$

or $\quad t = RC \times 2\log_e(3)$

$$= (5 \times 10^6) \times (1 \times 10^{-6}) \times 2 \times 1.1$$

$$= 11 \text{ s}$$

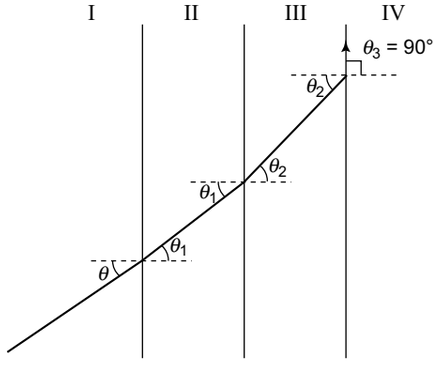
10. When the proton enters the region of the magnetic field, it will experience a force **F** given by

$$\mathbf{F} = q\,(\mathbf{u} \times \mathbf{B})$$

where q is the charge of the proton. The force **F** is perpendicular to both **u** and **B**. Since the force is perpendicular to the velocity of the particle, it does not do any work. Hence the magnitude of the velocity of the particle will remain unchanged; only the direction of the velocity changes. Hence $v = u$. Since **u** is perpendicular to **B**, the proton moves in a circular path. Since the charge of proton is positive, **u** is along positive x-axis and **B** is directed out of the page, the proton will move in a circle in the x-y plane in the clockwise direction. Hence its y coordinate will be negative, when it leaves the region. Thus the correct choice is (d).

11. Since the magnetic field is constant, the rate of change of magnetic flux is zero. Hence the induced emf and current are zero. So the correct choice is (d).

12. The beam will not enter region IV if the angle refraction in region IV equals 90°. Apply Snell's law at the interfaces, we have



$$n_0 \sin\theta = \frac{n_0}{2}\sin\theta_1$$

$$= \frac{n_0}{6}\sin\theta_2 = \frac{n_0}{8}\sin 90°$$

which gives $\sin\theta = \dfrac{1}{8}$.

13. de-Broglie wavelength is

$$\lambda = \frac{h}{\sqrt{2mE}}$$

where E is the kinetic energy of the electrons. The out-off wavelength is

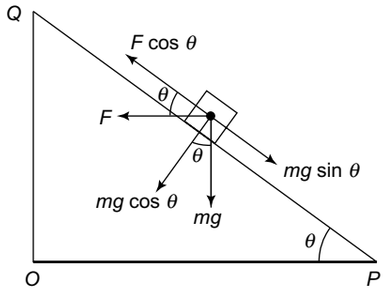
$$\lambda_0 = \frac{hc}{E}$$

From Eq. (1) $E = \frac{h^2}{2m\lambda^2}$. Hence

$$\lambda_0 = \frac{2mc\,\lambda^2}{h}$$

Section II

14.



Given $F = 1$ N, $m = 0.1$ kg and $g = 10$ ms^{-2}. Let f be the frictional force between the block and the plane surface PQ.

The block will be stationary if

$F \cos\theta = mg \sin\theta$

$\Rightarrow \quad 1 \times \cos\theta = 0.1 \times 10 \times \sin\theta$

$\Rightarrow \quad \tan\theta = 1 \Rightarrow \theta = 45°$ and $f = 0$

If $\theta > 45°$, $\sin\theta > \cos\theta$.

Hence $mg \sin\theta > F \cos\theta$ ($\because F = mg = 1$ N).

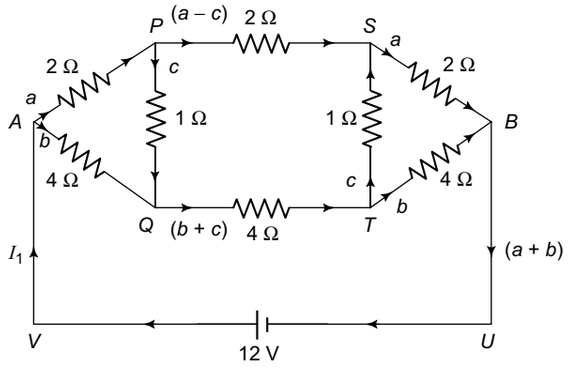
Therefore frictional force acts up the block towards Q.

If $\theta < 45°$, $\sin\theta < \cos\theta$

Hence $mg \sin\theta < F \cos\theta$.

Therefore, in this case, frictional force f acts down in the plane towards P.

15.



Applying Kirchhoff's loop rule to loops $APQA$, $PSTQP$ and $AQTBUVA$, we get

$$2a + c - 4b = 0 \qquad (1)$$

$$2(a - c) - c - 4(b + c) - c = 0$$

$$\Rightarrow \quad a - 2b - 4c = 0 \qquad (2)$$

and $\quad 4b + 4(b + c) + 4b - 12 = 0$

$$\Rightarrow \quad 3b + c = 3 \qquad (3)$$

Solving Eqs. (1), (2) and (3), we get

$a = 2$ A, $b = 1$ A and $c = 0$

Thus the current through PQ is zero. Also $I_1 = a + b = 3$ A and $I_2 = a - c = 2$ A.

Also $V_S - V_Q = -c - 4(b + c) = -4b = -4 \times 1 = -4$V ($\because c = 0$). Hence potential at S is less than that at Q.

16. According to Gauss's law, the electric flux cross the cubical region is

$$\phi = \frac{q_{net}}{\varepsilon_0} = \frac{3q - q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

By symmetry, the electric flux crossing the plane $x = a/2$ and the $x = -a/2$ is the same.

Further, the positions of charges with respect to $x = a/2$ and $z = a/2$ are the same; hence the flux through the planes $x = a/2$ and $z = a/2$ is the same

Also, by symmetry, the flux crossing the plan $y = a/2$ and $y = -a/2$ is the same.

17. The angular momentum about O is

$$\vec{L}_O = \vec{L}_{CM} + M(\vec{R} \times \vec{v})$$

Its magnitude is ($\because \vec{R} \perp \vec{v}$) and $L_{CM} = I\omega$

$$L_O = I\omega + MRv$$

$$= \left(\frac{1}{2}MR^2\right)\omega + MR \times R\omega \quad (\because v = R\omega)$$

$$= \frac{3}{2}MR^2\omega$$

$$= \frac{3}{2}MR^2 \times \left(\frac{v}{R}\right) = \frac{3}{2}MRv$$

Hence the correct choices are (a) and (d).

18. The displacement equation can be rewritten as

$$x = \frac{A}{2}(1 - \cos 2\omega t) + \frac{B}{2}(1 + \cos 2\omega t) + \frac{C}{2}\sin 2\omega t$$

or

$$x = \frac{1}{2}(A + B) + \frac{1}{2}(B - A)\cos 2\omega t + \frac{C}{2}\sin 2\omega t \qquad (1)$$

Choice (a): Equation (1) can be written as

$$x = x_0 + a\cos 2\omega t + b\sin 2\omega t \qquad (2)$$

where $x_0 = \frac{1}{2}(A+B), a = \frac{1}{2}(B-A)$

and $b = \frac{C}{2}$.

Equation (2) can be recast as

$$x = x_0 + A_0 \sin(2\omega + \phi) \qquad (3)$$

where $A_0 = (a^2 + b^2)^{1/2}$ and $\tan\phi = a/b$. Equation (3) represents a simple harmonic motion of angular frequency 2ω, amplitude $= x_0 + A_0$ and phase constant ϕ.

Choice (b): For $A = B$ and $C = 2B$, Eq. (1) becomes

$$x = B + B\sin 2\omega t = B(1 + \sin 2\omega t)$$

This equation represents a simple harmonic motion of amplitude $2B$ and angular frequency 2ω.

Choice (c): For $A = -B$ and $C = 2B$, Eq. (1) becomes

$$x = B\cos 2\omega t + B\sin 2\omega t$$

which represents a simple harmonic motion of amplitude $\sqrt{2}\,B$, angular frequency 2ω and phase constant $\pi/4$.

Choice (d): For $A = B$ and $C = 0$, Eq. (1) reduces to

$$x = A$$

which does not represent simple harmonic motion. Hence the correct choices are (a), (b) and (c).

19. Let nth bright fringe of wavelength λ_n and the mth bright fringe of wavelength λ_m coincide at a distance y from the centre of the screen. Then

$$y = \frac{n\lambda_n D}{d} = \frac{m\lambda_m D}{d}$$

or $n\lambda_n = m\lambda_m$

or $\frac{\lambda_n}{\lambda_m} = \frac{m}{n}$

or $\frac{750}{450} = \frac{m}{n}$

or $\frac{m}{n} = \frac{5}{3}$. The minimum integral values of m and n that satisfy this equation are $m = 5$ and $n = 3$.

Therefore, the minimum value of y is

$$y_{\min} = \frac{n\lambda_n D}{d} = \frac{3\times 750\times 10^{-9}\times 1}{10^{-3}}$$

$$= 2.25 \times 10^{-3}\ m = 2.25\ \text{mm}$$

For dark fringes to coincide, the condition is

$$y' = \left(n - \frac{1}{2}\right)\frac{\lambda_n D}{d} = \left(m - \frac{1}{2}\right)\frac{\lambda_m D}{d}$$

$$\Rightarrow \quad \frac{750}{450} = \frac{\left(m - \frac{1}{2}\right)}{\left(n - \frac{1}{2}\right)} \Rightarrow 5n = 3m + 1$$

The minimum integral values which satisfy this condition are $n = 2$ and $m = 3$. Hence

$$y'_{\min} = \left(n - \frac{1}{2}\right)\frac{\lambda_n D}{d}$$

$$= \frac{\left(2 - \frac{1}{2}\right)\times 750\times 10^{-9}\times 1}{10^3}$$

$$= 1.125 \times 10^{-3}\ \text{m} = 1.125\ \text{mm}$$

The correct choices are (a) and (d).

20. Work function $W = h\nu_0 = \frac{hc}{\lambda_0}$, where λ_0 is the threshold wavelength. Hence

$$W_1 : W_2 : W_3 = \frac{hc}{(\lambda_0)_1} : \frac{hc}{(\lambda_0)_2} : \frac{hc}{(\lambda_0)_3}$$

$$= \frac{1}{(\lambda_0)_1} : \frac{1}{(\lambda_0)_2} : \frac{1}{(\lambda_0)_3}$$

$$= 0.001 : 0.002 : 0.004$$

$$= 1 : 2 : 4$$

Hence choice (a) is correct. In photoelectric emission, the relation between V_0 and λ is given by

$$eV_0 = h\nu - W = \frac{hc}{\lambda} - W$$

or $V_0 = \frac{hc}{e}\left(\frac{1}{\lambda}\right) - \frac{W}{e}$

Hence the slope of the graph between V_0 and $\frac{1}{\lambda}$ is $\frac{hc}{e}$ which is the same for all metals. Therefore, choice (c) is correct. The threshold wavelength for the three metals are

$\frac{1}{(\lambda_0)_1} = 0.001\ \text{nm}^{-1}$, therefore $(\lambda_0)_1 = 1000\ \text{nm}$
$= 10{,}000\ \text{Å}$

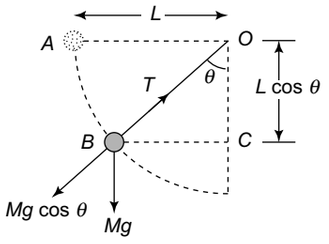
$\frac{1}{(\lambda_0)_2} = 0.002\ \text{nm}^{-1}$, therefore $(\lambda_0)_2 = 500\ \text{nm}$
$= 5{,}000\ \text{Å}$

$\frac{1}{(\lambda_0)_3} = 0.004\ \text{nm}^{-1}$, therefore $(\lambda_0) = 250\ \text{nm} = 2{,}500\ \text{Å}$

For photoelectric emission, the wavelength of the incident radiation must be less than the threshold wavelength. Since the wavelength of ultraviolet light is about 1200 Å, it will eject photoelectrons from all the three metals. Hence the correct choices are (a) and (d).

Section III

21.



The loss of PE when the body falls from A to B = $Mg \times OC = MgL \cos\theta$. If v is the velocity of the body at B, then

$$\frac{1}{2}Mv^2 = MgL\cos\theta \text{ or } v^2 = 2gL\cos\theta \quad (1)$$

$$\text{centripetal acceleration} = \frac{v^2}{L} = \frac{2gL\cos\theta}{L} = 2g\cos\theta,$$

which is choice (b).

22. The centripetal force when the body is at B is

$$F_c = \frac{Mv^2}{L}$$

Thus, we have

$$T - Mg\cos\theta = \frac{Mv^2}{L} \quad (2)$$

Using (1) in (2), we get

$$T - Mg\cos\theta = \frac{M}{L} \times 2gL\cos\theta = 2\,Mg\cos\theta$$

or $$T = 3\,Mg\cos\theta$$

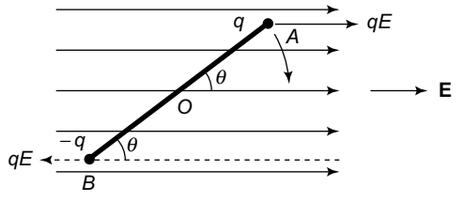
Thus the correct choice is (c).

23. A non-conducting rigid rod having equal and opposite charges at the ends is an electric dipole. When it is placed in a uniform electric field, it experiences a torque which tends to align it with the field lines. The electric forces $F = qE$ each acting at A and B constitute a couple whose torque is given by

τ = force × perpendicular distance

$= F \times AC = F \times AB \sin\theta = qEL \sin\theta$

So the correct choice is (a).



Since θ is small, $\sin\theta \simeq \theta$, where θ is expressed in radian. Thus $\tau = qEL\theta$

$\therefore$ Restoring torque $\tau = -qEL\theta$ (1)

If α is the angular acceleration of the rotatory motion,

$$\tau = I\alpha$$

where I is the moment of inertia of the two masses at A and B about an axis passing through the centre O and perpendicular to the rod. Since the rod is massless,

$$I = M \times (AO)^2 + M \times (BO)^2$$

$$= M \times \left(\frac{L}{2}\right)^2 + M \times \left(\frac{L}{2}\right)^2 = \frac{ML^2}{2}$$

Thus $$\tau = \frac{ML^2\alpha}{2} \quad (2)$$

Using Eq. (2) in Eq. (1), we get

$$\alpha = -\left(\frac{2qE}{ML}\right)\theta = -\omega^2\theta$$

where $$\omega = \left(\frac{2qE}{ML}\right)^{1/2},$$ which is choice (b).

24. The time period of oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi\left(\frac{ML}{2qE}\right)^{1/2}$$

Rotating in the clockwise sense, the minimum time taken by the rod to align itself parallel to the electric field is the time it takes to complete one-fourth of angular oscillation, i.e.

$$t_{\min} = \frac{T}{4} = \frac{\pi}{2}\left(\frac{ML}{2qE}\right)^{1/2}$$

So the correct choice is (a).

Section IV

25. The correct choice is (a). The line AB is parallel to the P-axis. This means that PV/T is a constant, independent of pressure. Hence line AB corresponds to an ideal gas for which PV/T = constant.

At higher temperatures, a real gas behave more like an ideal gas. Hence T_1 is greater than T_2.

26. The correct choice is (d). The law of conservation of linear momentum gives

$$m_1v_1 + m_2v_2 = 0 \text{ or } \left|\frac{m_2v_2}{m_1v_1}\right| = 1.0$$

Since de Broglie wavelength $\lambda = h/mv$, we will have

$$\frac{\lambda_1}{\lambda_2} = \frac{m_2v_2}{m_1v_1} = 1.0$$

Section V

27. Magnetic field at P due to complete cylinder is

$$B_1 = \frac{\mu_0(J\pi a^2)}{2\pi a} = \frac{\pi_0 Ja}{2}$$

Magnetic field at P due to cavity is

$$B_2 = \frac{\mu_0(J\pi a^2/4)}{2\pi\left(\frac{3a}{2}\right)} = \frac{\mu_0 Ja}{12}$$

$\therefore$ Net magnetic field at P is

$$B = B_1 - B_2 = \frac{\mu_0 Ja}{2} - \frac{\mu_0 Ja}{12} = \frac{5\mu_0 Ja}{12}$$

which gives $N = 5$.

28. Given $M_1 = 2.2\ M_s$ and $M_2 = 11\ M_s$. Let R_1 and R_2 be their respective distances from the centre of mass. The total angular momentum about the centre of mass is

$$L_{\text{total}} = (I_1 + I_2)\,\omega$$

and the angular momentum of B is

$$L_2 = I_2\ \omega$$

$$\therefore \quad \frac{L_{\text{tatal}}}{L_2} = \frac{I_1 + I_2}{I_2} = 1 + \frac{I_2}{I_1}$$

$$= 1 + \frac{M_1 R_1^2}{M_2 R_2^2}$$

$$= 1 + \frac{M_1 v_1^2}{M_2 v_2^2} \qquad (\because v = R\,\omega)$$

$$= 1 + \left(\frac{M_1 v_1}{M_2 v_2}\right)^2 \times \frac{M_2}{M_1}$$

$$= 1 + 1 \times \frac{11}{2.2} = 6$$

$$(\because\ M_1\ v_1 = M_2\ v_2)$$

29. From Wien's displacement law $\lambda_m\ T$ = constant, we have

$$\lambda_A\ T_A = \lambda_B\ T_B \Rightarrow \frac{T_A}{T_B} = \frac{\lambda_B}{\lambda_A}$$

From Stefan's law, $E = \sigma A \cdot T^4 = \sigma(4\ \pi R^2)\,T^4$, we have

$$\frac{E_A}{E_B} = \left(\frac{R_A}{R_B}\right)^2 \times \left(\frac{T_A}{T_B}\right)^4$$

$$= \left(\frac{R_A}{R_B}\right)^2 \times \left(\frac{\lambda_B}{\lambda_A}\right)^4$$

$$= \left(\frac{6}{18}\right)^2 \times \left(\frac{1500}{50}\right)^4$$

$$= 9$$

Section VI

30.(a) Process A $\rightarrow$ B is isobaric. Hence $V \propto T$. Therefore $T_A > T_B$. $\Delta U = nC_v\Delta T = nC_v(T_B - T_A)$. Since $T_B < T_A$, ΔU is negative, i.e. internal energy decreases.

$\Delta Q = nC_p\Delta T$ is also negative. Hence heat is lost $\Delta W = 3P(V_B - V_A) = -6PV$. Which is negative. Hence work is done on the gas.

$\therefore$ (a) $\rightarrow$ (p, r, t)

(b) Process B $\rightarrow$ C is isobaric. Hence $P \propto T$. Therefore $T_B > T_C$. $\Delta U = nC_v\Delta T = nC_v(T_C - T_B)$ is negative, i.e. internal energy decreases.

$\Delta W = P\Delta V = 0$

$(\because\ \Delta V = 0)$

From first law of thermodynamics ($\Delta Q = \Delta U + \Delta W$), $\Delta Q = \Delta U$. Since ΔU is negative, heat is lost.

$\therefore$ (b) $\rightarrow$ (p, r)

(c) Process C $\rightarrow$ D is isobaric, i.e. $V \propto T$. Hence $T_D > T_C$. $\Delta U = nC_v(T_D - T_C)$ is positive. Hence internal energy increases.

$\Delta Q = nC_p(T_D - T_C)$ is positive. Hence heat is gained by the gas.

$\therefore$ (c) $\rightarrow$ (q, s)

(d) In process D $\rightarrow$ A, the gas is returned to the initial state A. Hence $\Delta U = 0$. Therefore $\Delta Q = \Delta W$. Since the gas is compressed, work is done on the gas, i.e. ΔW is negative. Hence ΔQ is negative. Hence heat is lost by the gas.

$\therefore$ (d) $\rightarrow$ (r, t)

Answer: (a) $\rightarrow$ (p, r, t) (b) $\rightarrow$ (p, r)
(c) $\rightarrow$ (q, s) (d) $\rightarrow$ (r, t)

MODEL TEST PAPER – II

SECTION I

(Single Correct Answer Type)

This section contains 13 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length L = 2 m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension ℓ = 0.25 mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement.

(a) due to the errors in the measurement of d and ℓ are the same.

(b) due to the error in the measurement of d is twice that due to the error in the measurement of ℓ.

(c) due to the error in the measurement of ℓ is twice that due to the error in the measurement of d.

(d) due to the error in the measurement of d is four times that due to the error in the measurement of ℓ.

2. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{\text{rms}}(\text{helium})}{v_{\text{rms}}(\text{argon})}\right)$ is

(a) 0.32 (b) 0.45

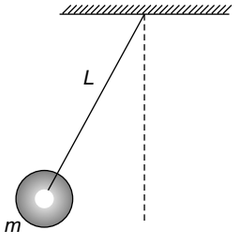
(c) 2.24 (d) 3.16

3. Two large vertical and parallel plates having a separation d are connected to a battery of voltage V. A particle of charge q is released at rest between the two plates. It is found to move at an angle θ to the vertical just after release. Then V is given by

(a) $\frac{mgd\sin\theta}{q}$ (b) $\frac{mgd\cos\theta}{q}$

(c) $\frac{mgd\tan\theta}{q}$ (d) $\frac{mgd}{q}$

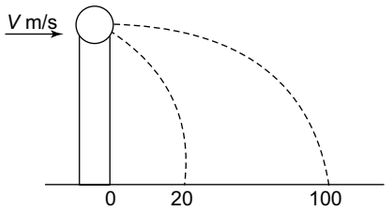
4. A ball of mass (m) 0.5 kg is attached to the end of string having (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is



(a) 9 (b) 18

(c) 27 (d) 36

5. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity V of the bullet is



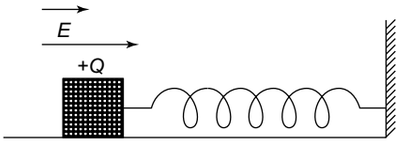
(a) 250 m/s (b) 250 m/s

(c) 400 m/s (d) 500 m/s

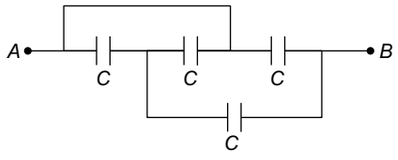
6. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown in the figure then the SHM of the block will be

(a) of the same frequency and with shifted mean position.

(b) of the same frequency and with the same mean position.
(c) of changed frequency and with shifted mean position.
(d) of changed frequency and with the same mean position.



7. In the network shown in the figure, each capacitor has capacitance C. The equivalent capacitance between A and B is

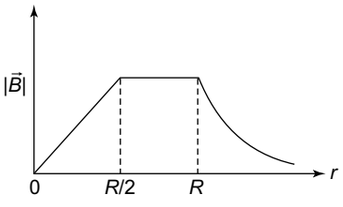


(a) C (b) $\frac{2C}{3}$

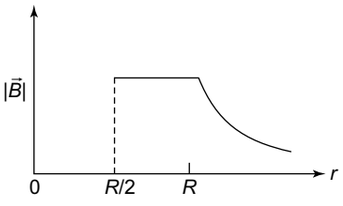
(c) $\frac{4C}{3}$ (d) $\frac{5C}{3}$

8. An infinitely long hollow conducting cylinder with inner radius $R/2$ and other radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by

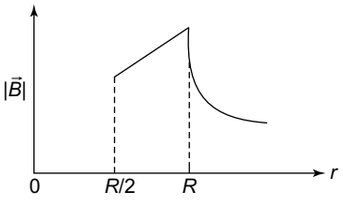
(a)



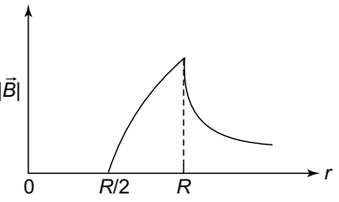
(b)



(c)



(d)



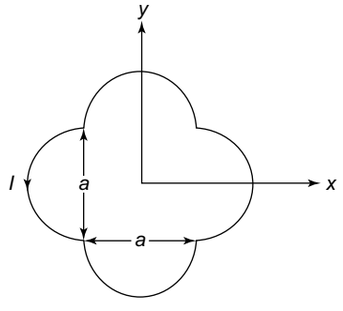
9. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

(a) 14.0 cm (b) 15.2 cm
(c) 16.4 cm (d) 17.6 cm

10. Young's double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe widths recorded are β_G, β_R and β_B respectively. Then,

(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$
(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

11. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

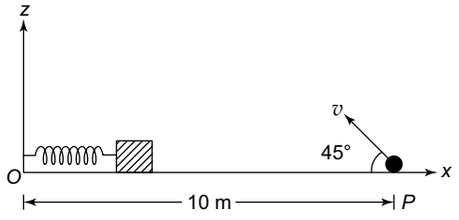


(a) $a^2 I\hat{k}$ (b) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

(c) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (d) $(2\pi+1)a^2 I\hat{k}$

12. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \pi/3$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown

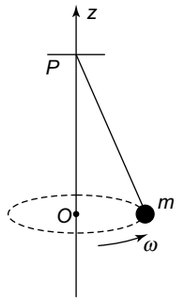
in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take $g = 10$ m/s^2)



(a) $\sqrt{50}$ m/st (b) $\sqrt{51}$ m/s

(c) $\sqrt{52}$ m/s (d) $\sqrt{53}$ m/s

13. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



(a) $\vec{L}_O$ and $\vec{L}_P$ do not very with time.

(b) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.

(c) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.

(d) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

SECTION II

(Multiple Correct Answer Type)

This section contains 7 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE** are correct.

14. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$?

(a) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.

(b) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis.

(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

15. A person blows into open-end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,

(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

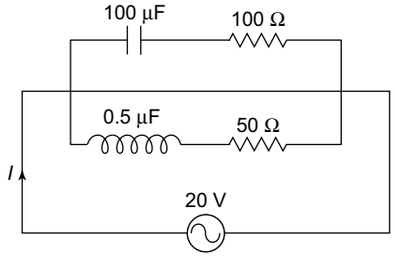
16. In the given circuit, the AC source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are)

(a) The current through the circuit, I is 0.3 A.

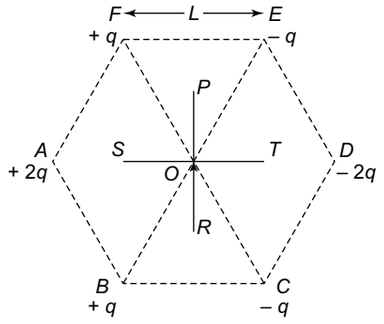
(b) The current through the circuit, I is $0.3\sqrt{2}$A .

(c) The voltage across 100 Ω resistor = $10\sqrt{2}$V .

(d) The voltage across 50 Ω resistor = 10 V.



17. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure, Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s), is (are) correct?



(a) The electric field at O is $6K$ along OD.
(b) The potential at O is zero.
(c) The potential at all points on the line PR is same.
(d) The potential at all points on the line ST is same.

18. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is (are) correct?
(a) Both cylinders P and Q reach the ground at the same time.
(b) Cylinders P has larger acceleration than cylinder Q.
(c) Both cylinders reach the ground with same translational kinetic energy.
(d) Cylinder Q reaches the ground with larger angular speed.

19. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it, the correct statement(s) is (are)
(a) The emf induced in the loop is zero if the current is constant.
(b) The emf induced in the loop is finite if the current is constant.
(c) The emf induced in the loop is zero if the current decreases at a steady rate.
(d) The emf induced in the loop is infinite if the current decreases at a steady rate.

20. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q and surface areas A and $4A$ respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are V_P, V_Q and V_R, respectively. Then

(a) $V_Q > V_R > V_P$ (b) $V_R > V_Q > V_P$

(c) $V_R/V_P = 3$ (d) $V_P/V_Q = \frac{1}{2}$

SECTION III
(Paragraph Type)

This section contains 4 multiple choice questions based on two paragraphs. Each question has four choices (a), (b), (c) and (d) out of which **only ONE** is correct.

Questions 21 and 22 are based on the following paragraph.

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay product of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e. $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy for the electron is 0.8×10^6 eV, The kinetic energy carried by the proton is only the recoil energy.

21. What is the maximum energy of the anti-neutrino?
(a) Zero
(b) Much less than 0.8×10^6 eV
(c) Nearly 0.8×10^6 eV
(d) Much larger than 0.8×10^6 eV

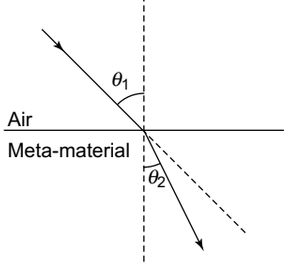
22. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?
(a) $0 \le K \le 0.8 \times 10^6$ eV
(b) 3.0 eV $\le K \le 0.8 \times 10^6$ eV
(c) 3.0 eV $< K \le 0.8 \times 10^6$ eV
(d) $0 < K \le 0.8 \times 10^6$ eV

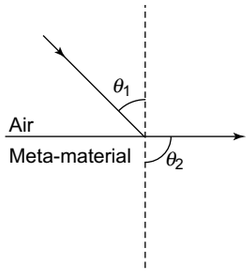
Questions 23 and 24 are based on the following paragraph.

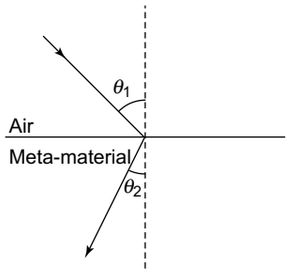
Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$ where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r, and μ_r, are relative permittivity and permeability of the medium respectively.

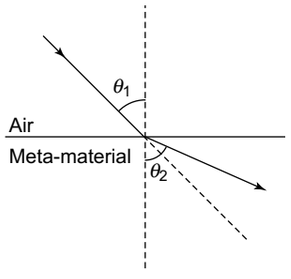
In normal materials, ε_r and μ_r, are positive, implying positive n for the medium. When both ε_r and μ_r, are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

23. For light incident from air on a meta-material. the appropriate ray diagram is

(a)


(b)


(c)


(d)


24. Choose the correct statement.

(a) The speed of light in the meta-material is $v = c|n|$

(b) The speed of light in the meta-material is $v = \frac{c}{|n|}$

(c) The speed of light in the meta-material is $v = c$.

(d) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}\,|n|$. where λ_{air} is wave1ength of the light in air.

SECTION IV

(Assertion-Reason Type)

This section contains 2 questions. Each question has statement-1 followed by statement-2. Only one of the following four choices (a), (b), (c) and (d) is correct.

(a) Statement-1 is true, statement-2 is true and is the correct explanation of statement-1.

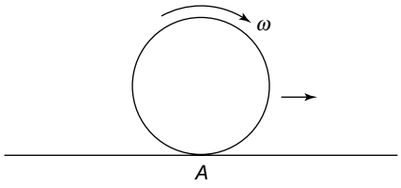
(b) Statement-1 is true and statement-2 is true but is not the correct explanation of statement-1.

(c) Statement-1 is true but statement-2 is false.

(d) Statement-1 is false but statement-2 is true.

25. Statement-1

A sphere is rolling on a rough surface in the direction of the arrow as shown in the figure. The force of friction at the point of contact will be in the direction of the arrow.



Statement-2
Friction opposes motion.

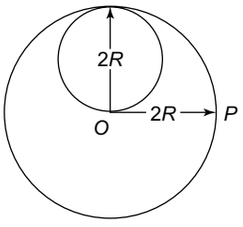
26. Statement-1
No induced emf is developed across the ends of a conductor if it is moved parallel to a magnetic field.
Statement-2
No force acts on the free electrons of the conductor.

SECTION V
(Integer Answer Type)

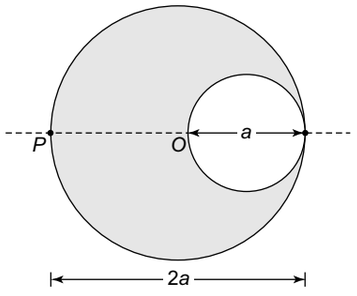
This section contains 3 questions. The answer to each question is single digit integer ranging from 0 to 9 (both inclusive)

27. A proton is fired from very far away towards a nucleus with charge $Q = 120e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg: $h/e = 4.2 \times 10^{-15}$ J.s/C; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F: 1 fm = 10^{-15} m)

28. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing though O and P is I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_P/I_O to the nearest integer is



29. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, the value of N is



SECTION VI
(Matrix Match Type)

30. In Column I are listed some charged bodies and current carrying conductors. Match them with the effects they produce listed in column II

Column I	**Column II**
(a) A uniformly charged stationary ring	(p) Electric field
(b) A uniformly charged ring rotating	(q) Magnetic field
(c) A coil carrying a current $I = I_0 \sin \omega$	(r) Magnetic moment
(d) A wire carrying a constant current	(s) Induced electric field

Answers

Section-I

1. (a) **2.** (d) **3.** (a)
4. (d) **5.** (d) **6.** (a)
7. (d) **8.** (d) **9.** (b)
10. (d) **11.** (b) **12.** (a)
13. (c)

Section-II

14. (c), (d) **15.** (b), (d)
16. (a), (c) **17.** (a), (b), (c)
18. (d) **19.** (a), (c) **20.** (b), (d)

Section-III

21. (c) **22.** (d) **23.** (c)
24. (b)

Section-IV

25. (a) **26.** (a)

Section-V

27. (7) **28.** (3) **29.** (5)

Section-VI

30. (a) → (p), (b) → (p), (q), (r),
(c) → (p), (r), (s), (d) → (q)

Solutions

Section-I

1. Least count of screw gauge $= \dfrac{0.5\text{ mm}}{100} = 0.005$ mm.

Since M and L are fixed and g is constant, $\Delta M = 0$, $\Delta L = 0$ and $\Delta g = 0$. Hence

$$\frac{\Delta Y}{Y} = \frac{\Delta \ell}{\ell} + \frac{2\Delta d}{d}$$

$$\frac{\Delta \ell}{\ell} = \frac{0.005\text{ mm}}{0.25\text{ mm}} = 0.02$$

$$\frac{2\Delta d}{d} = \frac{2 \times 0.005\text{ mm}}{0.5\text{ mm}} = 0.02$$

2. $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

$$\frac{v_{\text{rms}}(\text{helium})}{v_{\text{rms}}(\text{argon})} = \sqrt{\frac{M_{\text{Ar}}}{M_{\text{He}}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$$

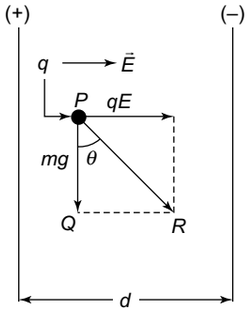
3. $\tan\theta = \dfrac{QR}{PQ} = \dfrac{qE}{mg}$

$$\Rightarrow E = \frac{mg\tan\theta}{q}$$

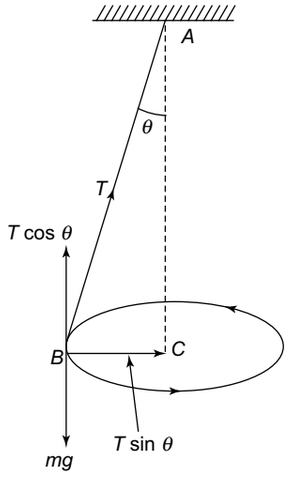
Also $E = \dfrac{V}{d}$. Hence

$$\frac{V}{d} = \frac{mg\tan\theta}{q}$$

$$\Rightarrow V = \frac{mgd\tan\theta}{q}$$



4. Radius of the circular path is $BC = r = L\sin\theta$, where $L = AB$ is the length of the string. The vertical component $T\cos\theta$ of tension T balances with the weight mg and the horizontal component $T\sin\theta$ provides the necessary centripetal force for circular motion. Hence



$$T\sin\theta = mr\omega^2 = m(L\sin\theta)\omega^2$$

$$\Rightarrow \quad T = mL\omega^2$$

$$\therefore \quad T_{\max} = mL\,\omega^2_{\max}$$

$$\Rightarrow \quad 324 = 0.5 \times 0.5 \times \omega^2_{\max}$$

$$\Rightarrow \quad \omega_{\max} = 36\text{ rad s}^{-1}$$

5. Time of flight $(t_f) = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 5}{10}} = 1\text{s}$

Horizontal range (R) = horizontal velocity × time of flight

∴ Horizontal velocities of the bullet and of the ball after the collision respectively are

$$(v)_{\text{bullet}} = \frac{100}{1} = 100\text{ ms}^{-1}$$

$$(v)_{\text{ball}} = \frac{20}{1} = 20\text{ ms}^{-1}$$

From conservation of momentum,
Total initial momentum = total final momentum

$\Rightarrow \quad (m)_{\text{bullet}} \times V = (m)_{\text{bullet}} \times (v)_{\text{bullet}} + (m)_{\text{ball}} \times (v)_{\text{ball}}$

$\Rightarrow \quad 0.01\ V = 0.01 \times 100 + 0.2 \times 20$

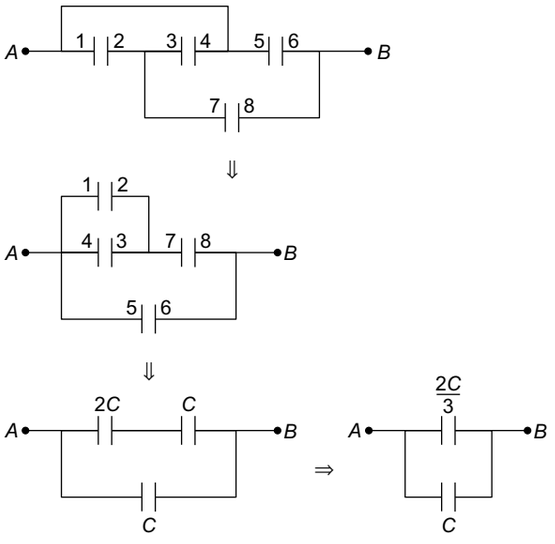
$\Rightarrow \quad V = 500\ \text{ms}^{-1}$

6. The force exerted on charge $+Q$ by the electric field is $\vec{E}$ is

$$\vec{F} = Q\vec{E}$$

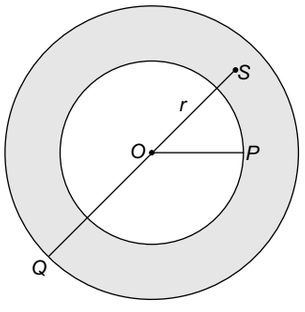
in the direction of $\vec{E}$. Since $\vec{F}$ is constant, a constant force is added to the applied force. Hence only the mean position will change and the frequency of oscillation will remain the same.

7. The circuit can be redrawn as follows.



$$C_{\text{eq}} = \frac{2C}{3} + C = \frac{5C}{3}$$

8.



$OP = \frac{R}{2}$, $OQ = R$, $OS = r$,

Inside the cavity (i.e. for r lying between zero and $\frac{R}{2}$); $B = 0$

Outside the cylinder. (i.e. for $r > R$),

$$B = \frac{\mu_0 I}{2\pi r}$$

In the shaded region (i.e. for $\frac{R}{2} < r < R$). From Ampere's circuital law,

$B \times 2\pi r = \mu_0 I$

where $J = \frac{I}{A}$ is the current density and A is the area of the shaded region. Now

$$A = \pi r^2 - \pi\left(\frac{R}{2}\right)^2$$

$$\therefore \quad B \times 2\pi r = \mu_0 J\left[\pi r^2 - \frac{\pi R^2}{4}\right]$$

$$\Rightarrow \quad B = \frac{\mu_0 J}{2}\left[\frac{r^2 - R^2/4}{r}\right]$$

$$= \frac{\mu_0 J}{2}\left[r - \frac{R^2}{4r}\right]$$

Hence the correct graph is (d).

9. End correction $e = 0.3d = 0.3 \times 4 = 1.2$ cm

Wavelength $\lambda = \frac{v}{\nu} = \frac{\cdot 336}{512} = 0.656$ m $= 65.6$ cm

Now $L + e = \frac{\lambda}{4}$

$$\Rightarrow \quad L = \frac{\lambda}{4} - e = \frac{65.6}{4} - 1.2$$

$$= 16.4 - 1.2 = 15.2 \text{ cm}$$

10. Since $\lambda_R > \lambda_G > \lambda_B$ and $\beta = \frac{\lambda D}{d}$, $\beta_R > \beta_G > \beta_B$.

11. Magnetic Moment $\vec{M}$ = current × area of the loop

$= I\vec{A}$

$$= I \times \left[a^2 + \pi\left(\frac{a}{2}\right)^2 \times 2\right]\hat{k}$$

$$= Ia^2\left[1 + \frac{\pi}{2}\right]\hat{k}$$

The direction of area vector $\vec{A}$ is along $\hat{k}$.

12. When the pebble hits the block, the distance travelled by the pebble (S_p) = distance travelled by the block (S_b).

$$S_p = 4.9 + 0.2 \cos \omega t$$

$$= 4.9 + 0.2 \cos\left(\frac{\pi}{3}\right) \qquad (\because\ t - 1 \text{ s})$$

$$= 4.9 + 0.2 \times \frac{1}{2} = 5.0 \text{ m}$$

S_b = 10 – horizontal range

$$= 10 - \frac{v^2 \sin(2\theta)}{g}$$

$$= 10 - \frac{v^2 \sin(90^\circ)}{10} \qquad (\because\ \theta = 45^\circ)$$

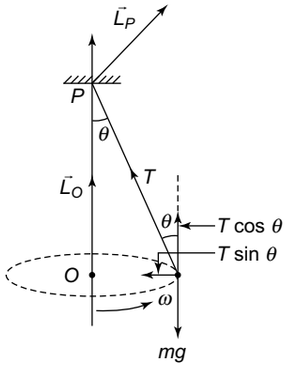
$$= 10 - \frac{v^2}{g}$$

Now $S_b = S_p$, Therefore,

$$10 - \frac{v^2}{g} = 5.0 \Rightarrow v^2 = 50$$

$$\Rightarrow \qquad v = \sqrt{50}\ \text{ms}^{-1}$$

13. $T\cos\theta = mg$



So $T\cos\theta$ balances with weight mg. $T\sin\theta$ produces no torque about O but produces a non-zero torque about P. Now

$\vec{L} = I\vec{\omega}$ and $\vec{\tau} = I\vec{\alpha} = I\dfrac{d\vec{\omega}}{dt}$

Since torque about O is zero, $\dfrac{d\vec{\omega}}{dt} = 0$, i.e. $\vec{\omega}$ = constant. Hence angular momentum $\vec{L}$ about O is constant. But torque about P is non-zero. Hence $\dfrac{d\vec{\omega}}{dt} \neq 0$, i.e. $\vec{\omega}$ changes with time. Hence $\vec{L}$ about P varies with time.

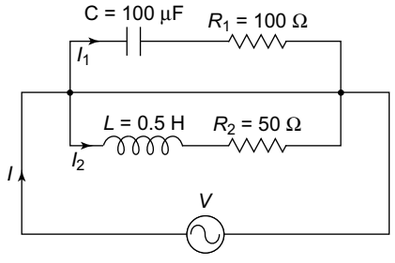
Section-II

14. Force on charge q due to electric field is $\vec{F}_e = q\vec{E}$. Since $\vec{E}$ is along $+y$ direction and q is positive, the charge will accelerate along the y-axis.

Force on charge q due to magnetic field is = $\vec{F}_m = q(\vec{v}\times\vec{B})$ which is perpendicular to both $\vec{v}$ and $\vec{B}$. If $\theta = 90°$, $\vec{v}$ will be parallel to $\vec{B}$ (which is along y-axis), $F_m = 0$ and the charge will accelerate along the y-axis due to the electric field. If θ lies between zero and 90°, the path is a helix with increasing pitch along the y-axis due to electric field $\vec{E}$.

15. When a compression reaches the open end of a pipe, it is reflected as a rarefaction due to openness of the medium just outside the open end. There is no reversal of amplitude of the pressure wave on reflection at the open end. But when a compression reaches the closed end of a pipe, it is reflected as a compression due to reversal of the amplitude of the pressure wave on reflection at the closed end. Hence the correct choices are (b) and (d).

16.



$$X_C = \frac{1}{\omega C} = \frac{1}{100\times100\times10^{-6}} = 100\ \Omega$$

$$X_L = \omega L = 100 \times 0.5 = 50\ \Omega$$

Since $X_C = R_1$, current I_1 leads V by 45°. Since $X_L = R_2$, current I_2 lags behind V by 45°. So the phase difference between I_1 and I_2 is $\phi = 90°$.

$$I_1 = \frac{V}{\sqrt{X_c^2 + R_1^2}} = \frac{20}{\sqrt{(100)^2 + (100)^2}} = \frac{\sqrt{2}}{10}\text{A}$$

and $$I_2 = \frac{V}{\sqrt{X_L^2 + R_2^2}} = \frac{20}{\sqrt{(50)^2 + (50)^2}}$$

$$= \frac{\sqrt{2}}{5}\text{A}$$

$$\therefore \quad I = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos 90^\circ}$$

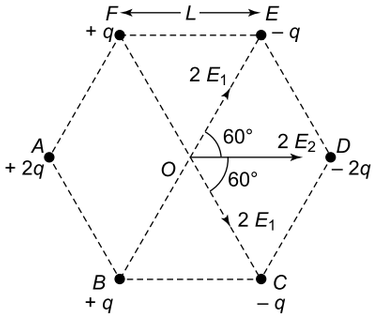
$$= \sqrt{\left(\frac{\sqrt{2}}{10}\right)^2 + \left(\frac{\sqrt{2}}{5}\right)^2} = \sqrt{0.1} = 0.316$$

$\simeq 0.3$ A (up to appropriate significant figure)

P.D. across $R_1 = I_1R_1 = \dfrac{\sqrt{2}}{10} \times 100 = 10\sqrt{2}$ V

P.D. across $R_2 = I_2R_2 = \dfrac{\sqrt{2}}{5} \times 50 = 10\sqrt{2}$ V

17.



E_1 = electric field at O due to $-q$ at E directed from O to E

= electric field at O due to $+q$ at B directed from O to E.

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$$

E_2 = electric field at O due to $+2q$ at A directed from O to D

= electric field at O due to $-q$ at D directed from O to D

$$= \frac{1}{4\pi\varepsilon_0}\frac{2q}{L^2}$$

The net electric field at O is

$$E = 2E_1 \cos 60° + 2E_1 \cos 60° + 2E_2$$
$$= E_1 + E_1 + 2E_2$$
$$= 2E_1 + 2E_2$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{2q}{L^2} + \frac{1}{4\pi\varepsilon_0}\frac{4q}{L^2}$$
$$= \frac{6}{4\pi\varepsilon_0}\frac{2q}{L^2} \text{ along } OD$$

So choice (a) is correct.

Net potential at O is

$$V = \frac{-q}{4\pi\varepsilon_0 L} - \frac{q}{4\pi\varepsilon_0 L} + \frac{q}{4\pi\varepsilon_0 L} + \frac{q}{4\pi\varepsilon_0 L} + \frac{2q}{4\pi\varepsilon_0 L} - \frac{2q}{4\pi\varepsilon_0 L} = 0$$

18. The linear acceleration of a body of mass M and radius R rolling down an inclined plane of inclination θ is given by

$$a = \frac{g\sin\theta}{1 + \dfrac{I}{MR^2}}$$

where I is the moment of intertia of the body about its centre of mass. Therefore,

$$a_P = \frac{g\sin\theta}{1 + \dfrac{I_P}{MR^2}}$$

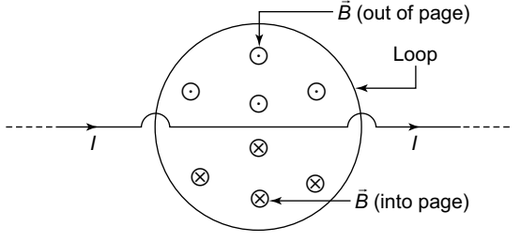
and $$a_Q = \frac{g\sin\theta}{1 + \dfrac{I_Q}{MR^2}}$$

Since cylinder P has most of its mass concentrated near its surface and Q has most of its mass concentrated near its axis, $I_P > I_Q$. Hence

$$a_P < a_Q$$
$$\therefore \quad \alpha_P < \alpha_Q \qquad (\because a = R\alpha)$$

Since $\omega^2 = 0^2 - 2\alpha(\theta)$; here θ = angular displacement and $\theta = \dfrac{1}{2}\alpha t^2$, it follows that $\omega_P < \omega Q$. So the only correct choice is (d).

19. The magnetic field above the wire is directed out of the plane of the loop and below the wire into the plane of the loop. Hence the net magnetic flux through the loop is zero. Therefore, the emf induced in the loop is zero, irrespective of whether the current is changing or not changing. Hence the correct choices are (a) and (c).



20. Let M be the mass of planet P and R its radius. Then Mass of P is $M = \rho \times \dfrac{4\pi}{3}R^3$

Mass of Q is $\rho \times \dfrac{4\pi}{3}(2R)^3 = 8M$ ($\because$ surface area of Q = 4 times that of P, therefore radius of Q = 2 (radius of P) $2R$

Mass of R = mass of P + mass of Q = $M + 8M$ = $9M$

$\therefore$ Radius of $R = 9^{1/3}R$

The escape velocities of P, Q and R are

$$V_P = \sqrt{\frac{2GM}{R}}$$

$$V_Q = \sqrt{\frac{2G\times 8M}{R}} = 2V_P$$

$$V_R = \sqrt{\frac{2G\times 9M}{9^{1/3}R}} = 9^{1/3}V_P$$

$\therefore \quad V_R > V_Q > V_P$

Also $\dfrac{V_Q}{V_P} = 2$

Section-III

21. The mass of a proton is very large compared to electron and antineutrino. So all the energy is shared by the electron and anti-neutrino. When the kinetic energy of anti-neutrino is zero, the maximum kinetic energy of electron is 0.8×10^6 eV and vice versa. Hence the total kinetic energy of electron + anti-neutrino is 0.8×106 eV.

22. If the anti-neutrino has a mass m = 3 eV/c^2, it will have kinetic energy = mc^2 = 3 eV.

Therefore, the maximum kinetic energy of the electron = $(0.8 \times 10^6 - 3)$ eV, which is only slightly less than 0.8×10^6 eV. The minimum kinetic energy of the electron is still zero. Hence correct choice is (d)

23. The refractive index n for meta-materials is negative. Hence $\dfrac{\sin\theta_1}{\sin\theta_2}$ is negative.

Thus if θ_1 is positive, θ_2 will be negative. So the current choice is (c).

24. $N = \dfrac{c}{v} \Rightarrow v = \dfrac{c}{|n|}$, which is choice (b)

Also frequency $\nu = \dfrac{v}{\lambda}$ Since ν remains unchanged,

$$\frac{v_{\text{air}}}{\lambda_{\text{air}}} = \frac{v_m}{\lambda_m}$$

$$\Rightarrow \quad \lambda_m = \lambda_{\text{air}} \times \frac{v_m}{v_{\text{air}}}$$

$$= \lambda_{\text{air}} \times \frac{v_m}{c} \times \frac{c}{v_{\text{air}}}$$

$$= \lambda_{\text{air}} \frac{n_{\text{air}}}{n_m} \qquad \left(\because v = \frac{c}{n}\right)$$

$$= \frac{\lambda_{\text{air}}}{|n|} \qquad (\because\ n_m = |n| \text{ and } n_{\text{air}} = 1)$$

So choice (d) is wrong.

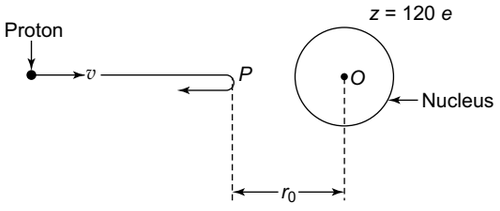
Section-IV

25. The correct choice is (a). The direction of the linear velocity at A, the point of contact is to the left (opposite to the direction of the arrow). Since friction opposes motion, the direction of the frictional force at A will be in the direction of the arrow, i.e. in the direction along which the sphere is rolling.

26. The correct choice is (a). Let $\vec{v}$ be the velocity of the conductor in a magnetic field $\vec{B}$. Since the free electrons in the conductor are moving with it, force $\vec{F} = e(\vec{v} = \vec{B})$ is zero because $\vec{v}$ is parallel to $\vec{B}$. Consequently, no induced emf is developed between the ends of the conductor.

Section-V

27.



The proton reaches a point P and is then repelled back by the nucleus.

Loss of kinetic energy = gain in potential energy.

$$\frac{1}{2}m_P v^2 = \frac{Ze}{4\pi\varepsilon_0 r_0} = \frac{120e^2}{4\pi\varepsilon_0 r_0}$$

If p is the linear momentum, then ($\because$ K.E. = $\dfrac{p^2}{2m}$)

$$\Rightarrow \quad \frac{p^2}{2m_p} = \frac{120e^2}{4\pi\varepsilon_0 r_0}$$

$$\Rightarrow \qquad p = \left(\frac{240e^2 m_p}{4\pi\varepsilon_0 r_0}\right)^{1/2}$$

Now $\lambda = \dfrac{h}{P} = \dfrac{h}{e}\left(\dfrac{4\pi\varepsilon_0 r_0}{240\,m}\right)^{1/2}$ (1)

Putting $\dfrac{h}{e} = 4.2 \times 10^{-15}$ Js/C, $\dfrac{1}{4\pi\varepsilon_n} = 9 \times 10^9$ m/F, 1 fm = 10^{-15} m, and r_0 = 10 fm = 10×10^{-15} m in Eq. (1), we get $r_0 = 7 \times 10^{-15}$ m = 7 fm

28. Let M be the mass of the complete disc. The mass of the cut-out disc is

$$m = \frac{M}{\pi(2R)^2} \times \pi R^2 = \frac{M}{4}$$

Moment of inertia of the complete disc about O

$$= \frac{1}{2} M \times (2R)^2 = 2MR^2$$

Moment of inertia of the cut-out disc about O

$$= \frac{1}{2} \times \frac{2MR^2}{4} = \frac{3MR^2}{8}$$

$$\therefore \qquad I_0 = 2MR^2 - \frac{3MR^2}{8} = \frac{13MR^2}{8}$$

Moment of inertia of the complete disc about

$$P = \frac{3}{2} M(2R)^2 = 6MR^2$$

Moment of inertia of the cut-out disc about P

$$= \frac{MR^2}{8} + \frac{M}{4}(R^2 + 4R^2) = \frac{11}{8} MR^2$$

$$\therefore \qquad I_p = 6MR^2 - \frac{11}{8} MR^2 = 37/8 \; MR^2$$

$$\therefore \qquad \frac{I_p}{I_0} = \frac{37}{13} \simeq 3 \text{ (to nearest integer)}$$

29. Magnetic field at P due to complete cylinder is

$$B_1 = \frac{\mu_0 (J\pi a^2)}{2\pi a} = \frac{\pi_0 Ja}{2}$$

Magnetic field at P due to cavity is

$$B_2 = \frac{\mu_0 (J\pi a^2/4)}{2\pi\left(\frac{3a}{2}\right)} = \frac{\pi_0 Ja}{12}$$

$\therefore$ Net magnetic field at P is

$$B = B_1 - B_2 = \frac{\mu_0 Ja}{2} - \frac{\mu_0 Ja}{12} = \frac{5\mu_0 Ja}{12}$$

Which gives $N = 5$.

30. (a) $\rightarrow$ (p), (b) $\rightarrow$ (p), (q), (r)
(c) $\rightarrow$ (p), (r), (s), (d) $\rightarrow$ (q)

IIT-JEE 2012: PAPER-I

SECTION I

(Single Correct Answer Type)

This section contains **10 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $\ell = 0.25$ mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

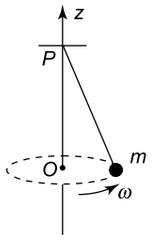
(a) due to the errors in the measurement of d and ℓ are the same.

(b) due to the error in the measurement of d is twice that due to the error in the measurement of ℓ.

(c) due to the error in the measurement of ℓ is twice that due to the error in the measurement of d.

(d) due to the error in the measurement of d is four times that due to the error in the measurement of ℓ.

2. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



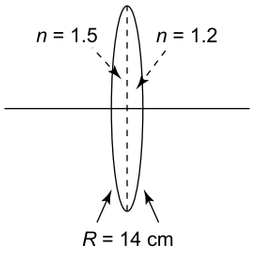
(a) $\vec{L}_O$ and $\vec{L}_P$ do not very with time.

(b) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.

(c) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.

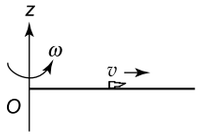
(d) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

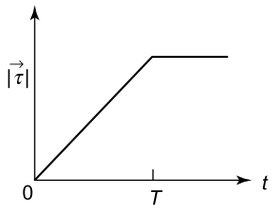
3. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surface are of the same radius of curvature $R = 14$ cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be

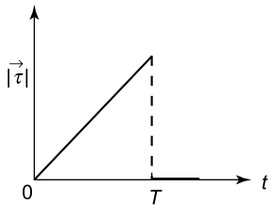


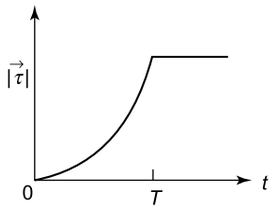
(a) – 280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm

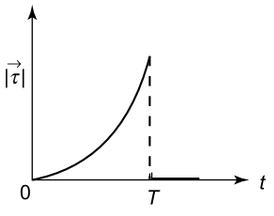
4. A thin uniform rod, pivoted at O is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v, with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ about O, as a function of time is best represented by which plot?



(a)


(b)


(c)


(d)


5. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{\text{rms}}(\text{helium})}{v_{\text{rms}}(\text{argon})}\right)$ is

(a) 0.32 (b) 0.45
(c) 2.24 (d) 3.16

6. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly

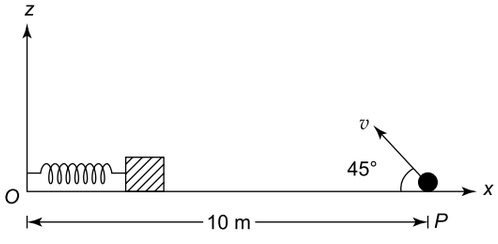
(a) 1×10^{-5} V (b) 1×10^{-7} V
(c) 1×10^{-9} V (d) 1×10^{-10} V

7. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e. second) plate under steady state condition is

(a) $\left(\frac{65}{2}\right)^{1/4} T$ (b) $\left(\frac{97}{4}\right)^{1/4} T$

(c) $\left(\frac{97}{2}\right)^{1/4}$ (d) $(97)^{1/4} T$

8. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \pi/3$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take $g = 10$ m/s^2)



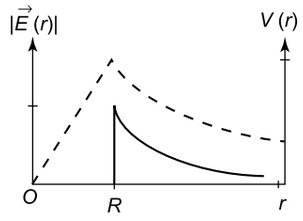
(a) $\sqrt{50}$ m/s (b) $\sqrt{51}$ m/s
(c) $\sqrt{52}$ m/s (d) $\sqrt{53}$ m/s

9. Young's double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe widths recorded are β_G, β_R and β_B respectively. Then,

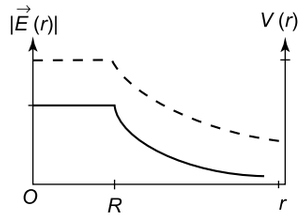
(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$
(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

10. Consider a thin spherical shell of radius R with centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph?

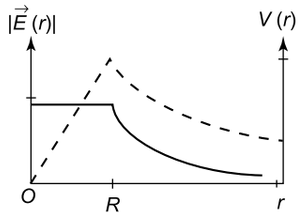
(a)



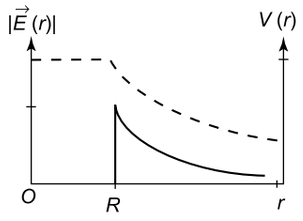
(b)



(c)



(d)



SECTION II

(Multiple Correct Answer(s) Type)

This section contains **5 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONE of MORE are correct.**

11. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0 \hat{j}$ and $\vec{B} = B_0 \hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$?

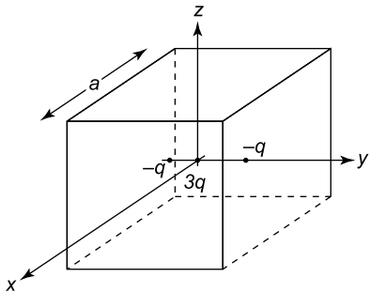
(a) If $\theta = 0°$ the charge moves in a circular path in the x-z plane.

(b) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis.

(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

12. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0,-a/4, 0)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $(0, +a/4, 0)$. Choose the correct options(s).



(a) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$

(b) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$.

(c) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$.

(d) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$.

13. A person blows into open-end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,

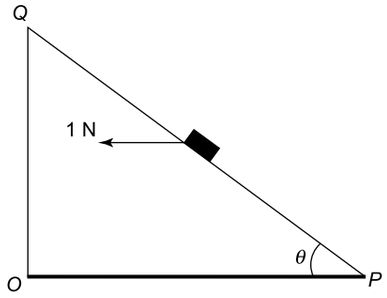
(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

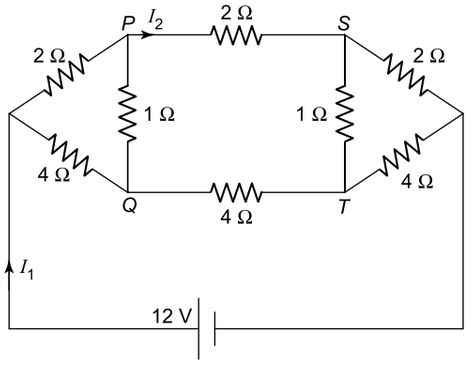
(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

14. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts of the block through its centre of mass as shown in the figure. The block remains stationary if (take $g = 10$ m/s^2).



(a) $\theta = 45°$
(b) $\theta > 45°$ and a frictional forces acts on the block towards P.
(c) $\theta > 45°$ and a frictional forces acts on the block towards Q.
(d) $\theta < 45°$ and a frictional forces acts on the block towards Q.

15. For the resistance network shown in the figure, choose the correct option(s).



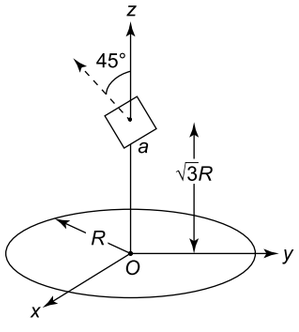
(a) The current through PQ is zero.
(b) $I_1 = 3$A
(c) The potential at S is less than that at Q.
(d) $I_2 = 2$A

SECTION III

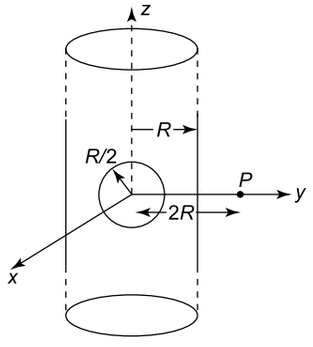
(Integer Correct Answer Type)

This section contains **5 questions.** The answer to each question is single digit integer, ranging from 0 to 9 (*both inclusive*)

16. A circular wire loop of radius R is placed in the x-y plane centered at the origin O.A square loop of side a ($a<<R$) having two turns is placed with its centre at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2}R}$, then the value of p is



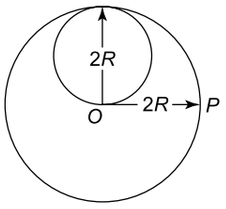
17. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



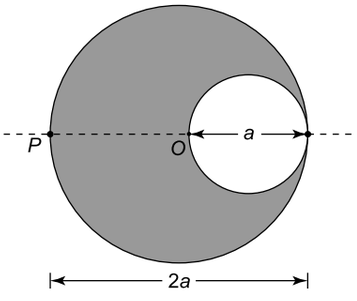
18. A proton is fired from very far away towards a nucleus with charge $Q = 120e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg; $h/e = 4.2 \times 10^{-15}$ J.s/C; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm = 10^{-15} m)

19. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass den-

sity and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing though O and P is I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_P/I_O to the nearest integer is



20. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, the value of N is



Answers

Section-I

1. (a)	**2.** (c)	**3.** (b)
4. (b)	**5.** (d)	**6.** (c)
7. (c)	**8.** (a)	**9.** (d)
10. (d)		

Section-II

11. (c, d)	**12.** (a, c, d)	**13.** (b, d)
14. (a, c)	**15.** (a, b, c, d)	

Section-III

16. 7	**17.** 6	**18.** 7
19. 3	**20.** 5	

Hints and Solutions

Section-I

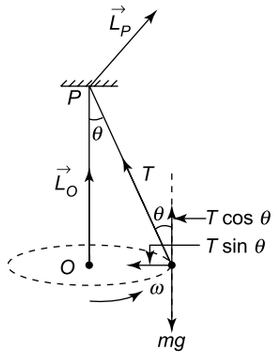
1. (a) Least count $= \frac{0.5\text{ mm}}{100} = 0.005$ mm

$$\frac{\Delta Y}{Y} = \frac{\Delta \ell}{\ell} + \frac{2\Delta d}{d}$$

$$\frac{\Delta \ell}{\ell} = \frac{0.005\text{ mm}}{0.25\text{ mm}} = 0.02$$

$$\frac{2\Delta d}{d} = \frac{2 \times 0.005\text{ mm}}{0.5\text{ mm}} = 0.02$$

2. (c) $T\cos\theta = mg$



So $T\cos\theta$ balances with weight mg. T sin θ produces no torque about O but produces a non-zero torque about P. Now

$$\vec{L} = I\vec{\omega} \quad \text{and} \quad \vec{\tau} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt}$$

Since torque about O is zero, $\frac{d\vec{\omega}}{dt} = 0$, *i.e.* $\vec{\omega}$ = constant. Hence angular momentum $\vec{L}$ about O is constant. But torque about P is non-zero. Hence $\frac{d\vec{\omega}}{dt} \neq 0$, i.e. $\vec{\omega}$ changes with time. Hence $\vec{L}$ about P varies with time.

3. (b) $\frac{1}{f_1} = \left(\frac{1.5-1}{1}\right)\left(\frac{1}{14} - \frac{1}{\infty}\right) = \frac{1}{28}$ cm^{-1}

$$\frac{1}{f_2} = \left(\frac{1.2-1}{1}\right)\left(\frac{1}{\infty} - \frac{1}{-14}\right) = \frac{1}{70} \text{ cm}^{-1}$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{28} + \frac{1}{70} \Rightarrow F = 20 \text{ cm}$$

Now $\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$

$$\Rightarrow \frac{1}{v} - \frac{1}{-40} = \frac{1}{20} \Rightarrow v = 40 \text{ cm}$$

4. (b) $L = I\omega$

$x = vt$

$I = mx^2 = mv^2t^2$

$\therefore \quad L = (mx^2)\ \omega = m\omega v^2 t^2$

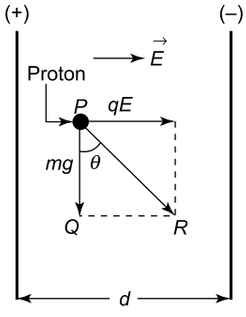
$\therefore \quad \tau = \dfrac{dL}{dt} = 2m\omega v^2 t$

Hence, as long as the insect is moving, the torque varies linearly with time. When the insect steps, $v = 0$, and torque becomes zero.

5. (d) $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

$\therefore \quad \dfrac{v_{\text{rms}}\ (\text{helium})}{v_{\text{rms}}\ (\text{argon})} = \sqrt{\dfrac{M_{\text{Ar}}}{M_{\text{He}}}} = \sqrt{\dfrac{40}{4}} = \sqrt{10} = 3.16$

6. (c) $\tan\theta = \dfrac{QR}{PQ} = \dfrac{qE}{mg}$



Given $\theta = 45°$. Therefore,

$\tan 45° = \dfrac{qE}{mg} \Rightarrow qE = mg$

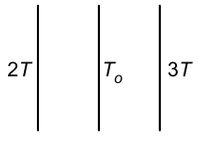
Also $E = \dfrac{X}{d}$. Hence

$\dfrac{qX}{d} = mg$

$\Rightarrow \quad X = \dfrac{mgd}{q}$

$= \dfrac{(1.67\times10^{-27})\times9.8\times(1\times10^{-2})}{1.6\times10^{-19}} \simeq 10^{-9}\ \text{V}$

7. (c) In the steady state, the rate at which the middle plate receives heat energy is equal to the rate at which heat energy is emitted by the other plates. Let A be the area of each plate and T_o be the steady state temperature of the middle plate. Since both sides of the middle plate receive heat energy, the total area of the middle plate receiving energy is $2A$.



From Stefan's law

$\sigma\ (2A)\ (T_o)^4 = \sigma A\ (2T)^4 + \sigma A\ (3T)^4$

$\Rightarrow \quad 2\ T_o^4 = 16T^4 + 81T^4 = 97\ T^4$

$\Rightarrow \quad T_o = \left(\dfrac{97}{2}\right)^{1/4} T$

8. (a) When the pebble hits the block, the distance travelled by the pebble (S_p) = distance travelled by the block (S_b).

$S_p = 4.9 + 0.2\cos\omega t$

$= 4.9 + 0.2\cos\left(\dfrac{\pi}{3}\right) \qquad (\because\ t = 1\ \text{s})$

$= 4.9 + 0.2\times\dfrac{1}{2} = 5.0\ \text{m}$

$S_b = 10 - \text{horizontal range}$

$= 10 - \dfrac{v^2\sin(2\theta)}{g}$

$= 10 - \dfrac{v^2\sin(90°)}{10} \qquad (\because\ \theta = 45°)$

$= 10 - \dfrac{v^2}{10}$

Now $S_b = S_p$. Therefore,

$10 - \dfrac{v^2}{10} = 5.0 \Rightarrow v^2 = 50 \Rightarrow v = \sqrt{50}\ \text{ms}^{-1}$

9. (d) Since $\lambda_R > \lambda_G > \lambda_B$ and $\beta = \dfrac{\lambda D}{d}$,

$\beta_R > \beta_G > \beta_B$.

10. (d) For a spherical shell

$$|\vec{E}(r)| = \begin{cases} 0 & \text{for } r < R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & \text{for } r > R \end{cases}$$

and

$$V(r) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 R} & \text{for } r < R \\ \dfrac{Q}{4\pi\varepsilon_0 r} & \text{for } r > R \end{cases}$$

Hence the correct graph is (d). The graph shown with broken lines represents $V(r)$ vs r graph.

Section-II

11. (c, d) Force on charge q due to electric field is $\vec{F}_e = q\vec{E}$. Since $\vec{E}$ is along $+y$ direction and q is positive, the charge will accelarate along the y-axis.

Force on charge q due to magnetic field is $\vec{F}_m = q\,(\vec{v}\times\vec{B})$ which is perpendicular to both $\vec{v}$ and $\vec{B}$. If $\theta = 90°$, $\vec{v}$ will be parallel to $\vec{B}$ (which is along y-axis), $F_m = 0$ and the charge will accelerate along the y-axis due to the electric field. If θ lies between zero and 90°, the path is a helix with increasing pitch along the y-axis due to electric field $\vec{E}$.

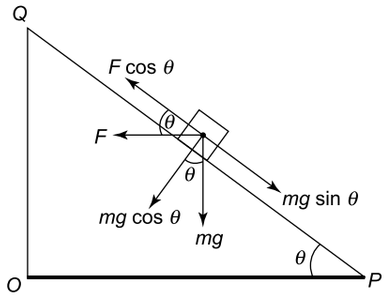
12. (a, c, d) According to Gauss's law, the electric flux cross the cubical region is

$$\phi = \frac{q_{net}}{\varepsilon_0} = \frac{3q - q - q}{\varepsilon_o} = \frac{q}{\varepsilon_0}$$

By symmetry, the electric flux crossing the plane $x = a/2$ and the $x = -a/2$ is the same.
Further, the positions of charges with respect to $x = a/2$ and $z = a/2$ are the same; hence the flux through the planes $x = a/2$ and $z = a/2$ is the same
Also, by symmetry, the flux crossing the plane $y = a/2$ and $y = -a/2$ is the same.

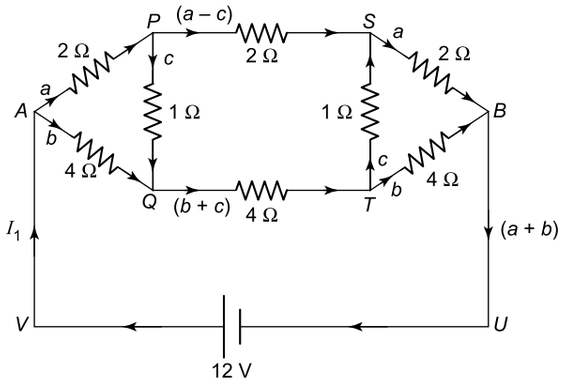
13. (b, d) When a compression reaches the open end of a pipe, it is reflected as a rarefaction due to openness of the medium just outside the open end. There is no reversal of amplitude of the pressure wave on reflection at the open end. But when a compression reaches the closed end of a pipe, it is reflected as a compression due to reversal of the amplitude of the pressure wave on reflection at the closed end. Hence the correct choices are (b) and (d).

14. (a, c)



Given $F = 1$ N, $m = 0.1$ kg and $g = 10$ ms^{-2}. Let f be the frictional force between the block and the plane surface PQ.
The block will be stationary if
$F \cos \theta = mg \sin \theta$
$\Rightarrow 1 \times \cos \theta = 0.1 \times 10 \times \sin \theta$
$\Rightarrow \tan \theta = 1 \Rightarrow \theta = 45°$ and $f = 0$
If $\theta > 45°$, $\sin \theta > \cos \theta$.
Hence $mg \sin \theta > F \cos \theta$ ($\because F = mg = 1$ N).
Therefore frictional force acts up the block towards Q.
If $\theta < 45°$, $\sin \theta < \cos\theta$.
Hence $mg \sin \theta < F \cos \theta$.
Therefore, in this case, frictional force f acts down in the plane towards P.

15. (a, b, c, d)



Applying Kirchhoff's loop rule to loops $APQA$, $PSTQP$ and $AQTBUVA$, we get

$$2a + c - 4b = 0, \quad (1)$$

$$2(a - c) - c - 4(b + c) - c = 0$$

$$\Rightarrow \quad a - 2b - 4c = 0 \quad (2)$$

and $4b + 4(b + c) + 4b - 12 = 0$

$$\Rightarrow \quad 3b + c = 3 \quad (3)$$

Solving Eqs. (1), (2) and (3), we get
$a = 2$ A, $b = 1$ A and $c = 0$
Thus the current through PQ is zero. Also $I_1 = a + b = 3$ A and $I_2 = a - c = 2$ A.
Also $V_S - V_Q = -c - 4(b + c) = -4b = -4 \times 1 = -4$V ($\because c = 0$). Hence potential at S is less than that at Q.

Section-III

16. (7) Magnetic flux $\phi = NBA \cos \theta$
Given $N = 2$, $A = a^2$, and $\theta = 45°$.

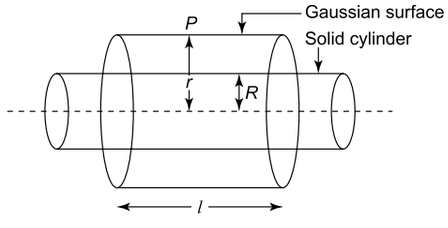
$$\text{Also } B = \frac{\mu_0}{4\pi} \frac{2\pi IR^2}{[R^2 + (\sqrt{3}R)^2]^{3/2}}$$

$$= \frac{\mu_0 I}{16R}$$

$$\therefore \phi = 2 \times \left(\frac{\mu_0 I}{16R}\right) \times a^2 \times \frac{1}{\sqrt{2}} = \left(\frac{\mu_0 a^2}{2^{7/2} R}\right) I$$

$$M = \frac{\phi}{I} = \frac{\mu_0 a^2}{2^{7/2} R} \Rightarrow p = 7$$

17. (6)



We take the Gaussian surface to be a cylinder of radius r and length l. From Gauss's law,

$$\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E \times 2\pi rl = \frac{q}{\varepsilon_0} = \frac{\rho \times \pi R^2 l}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r} \qquad (1)$$

The electric field at point P due to solid cylinder is (putting $r = 2R$)

$$E_c = \frac{\rho R^2}{2\varepsilon_0 (2R)} = \frac{\rho R}{4\varepsilon_0} \text{ along } +y \text{ direction}$$

The electric field at point P due to solid sphere (since $r > R$) is

$$E_s = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\rho \times \frac{4\pi}{3}(R/2)^3}{4\pi\varepsilon_0 (2R)^2}$$

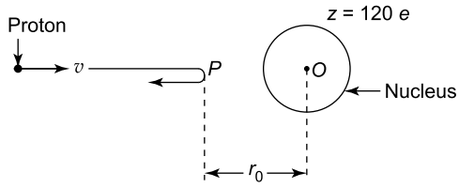
$$= \frac{\rho R}{96\varepsilon_0} \text{ along } -y \text{ direction.}$$

Therefore, net electric field at P is

$$E_p = E_c - E_s = \frac{\rho R}{4\varepsilon_0} - \frac{\rho R}{96\varepsilon_0} = \frac{23\rho R}{96\varepsilon_0}$$

which gives $k = 6$.

18. (7)



The proton reaches a point P and is then repelled back by the nucleus.

Loss of kinetic energy = gain in potential energy.

$$\frac{1}{2} m_p v^2 = \frac{Ze}{4\pi\varepsilon_0 r_0} = \frac{120e^2}{4\pi\varepsilon_0 r_0}$$

If p is the linear momentum, then ($\because$ K.E. $= \frac{p^2}{2m}$)

$$\Rightarrow \quad \frac{p^2}{2m_p} = \frac{120e^2}{4\pi\varepsilon_0 r_0}$$

$$\Rightarrow \quad p = \left(\frac{240e^2 m_p}{4\pi\varepsilon_0 r_0}\right)^{1/2}$$

Now $\lambda = \frac{h}{p} = \frac{h}{e}\left(\frac{4\pi\varepsilon_0 r_0}{240\, m}\right)^{1/2}$ (1)

Putting $\frac{h}{e} = 4.2 \times 10^{-15}$ Js/C, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F, 1 fm $= 10^{-15}$ m, and $r_0 = 10$ fm $= 10 \times 10^{-15}$ m in Eq. (1), we get $r_0 = 7 \times 10^{-15}$ m $= 7$ fm

19. (3)

Let M be the mass of the complete disc. The mass of the cut-out disc is

$$m = \frac{M}{\pi(2R)^2} \times \pi R^2 = \frac{M}{4}$$

Moment of inertia of the complete disc about O $= \frac{1}{2} M \times (2R)^2 = 2MR^2$

Moment of inertia of the cut-out disc about O $= \frac{1}{2} \times \frac{3MR^2}{4} = \frac{3MR^2}{8}$

$$\therefore \qquad I_0 = 2MR^2 - \frac{3MR^2}{8} = \frac{13MR^2}{8}$$

Moment of inertia of the complete disc about $P = \frac{3}{2} M (2R)^2 = 6\, MR^2$

Moment of inertia of the cut-out disc about P $= \frac{MR^2}{8} + \frac{M}{4}(R^2 + 4R^2) = \frac{11}{8} MR^2$

$$\therefore \quad I_P = 6MR^2 - \frac{11}{8} MR^2 = 37/8\, MR^2$$

$$\therefore \qquad \frac{I_P}{I_0} = \frac{37}{13} \simeq 3 \text{ (to nearest integer)}$$

20. (5)

Magnetic field at P due to complete cylinder is

$$B_1 = \frac{\mu_0 (J\pi a^2)}{2\pi a} = \frac{\pi_0 Ja}{2}$$

Magnetic field at P due to cavity is

$$B_2 = \frac{\mu_0 (J\pi a^2/4)}{2\pi\left(\frac{3a}{2}\right)} = \frac{\mu_0 Ja}{12}$$

$\therefore$ Net magnetic field at P is

$$B = B_1 - B_2 = \frac{\mu_0 Ja}{2} - \frac{\mu_0 Ja}{12} = \frac{5\mu_0 Ja}{12}$$

which gives $N = 5$.

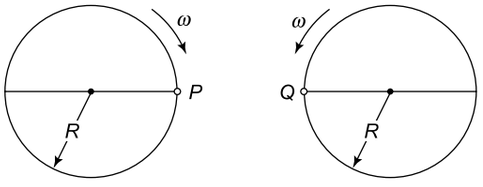
IIT-JEE 2012: PAPER-II

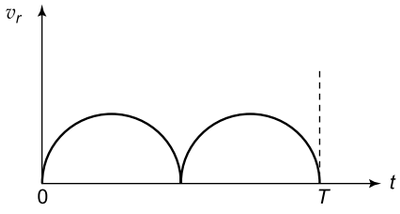
SECTION I

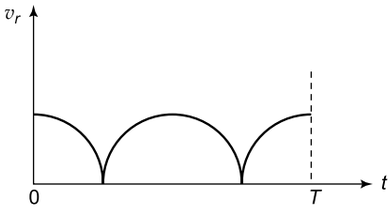
(Single Correct Answer Type)

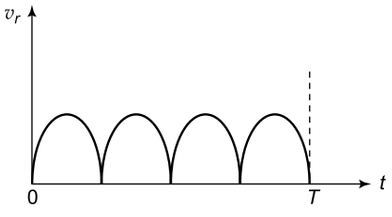
This section contains **8 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

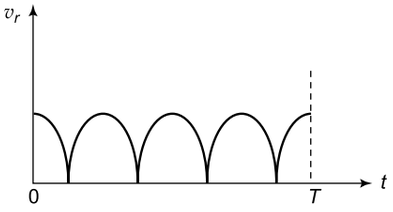
1. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the point P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by



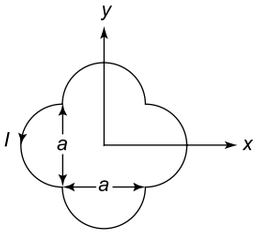
(a) 

(b) 

(c) 

(d) 

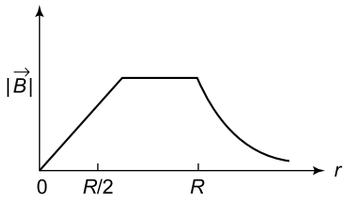
2. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is



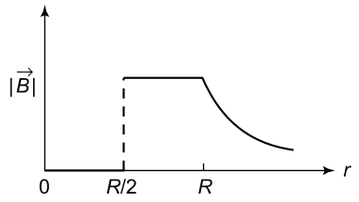
(a) $a^2 I\hat{k}$ (b) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

(c) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (d) $(2\pi+1)a^2 I\hat{k}$

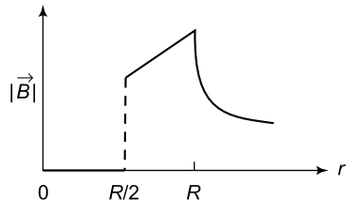
3. An infinitely long hollow conducting cylinder with inner radius $R/2$ and other radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by

(a) 

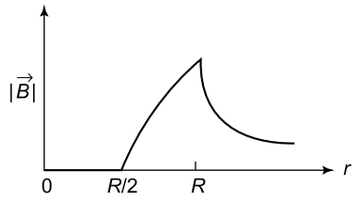
(b)



(c)



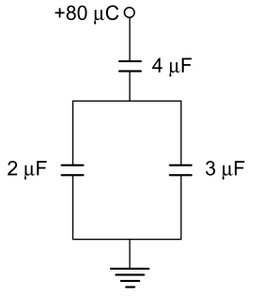
(d)



4. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

(a) more than half-filled if ρ_c is less than 0.5.
(b) more than half-filled if ρ_c is more than 1.0.
(c) half-filled if ρ_c is more than 0.5.
(d) less than half-filled if ρ_c is less than 0.5.

5. In the given circuit, a charge of + 80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is

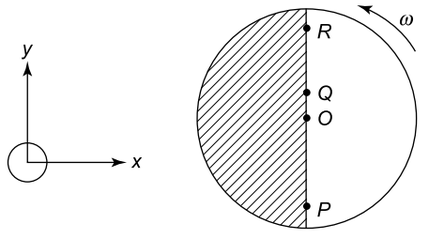


(a) + 32 μC (b) + 40 μC
(c) + 48 μC (d) + 80 μC

6. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol.K)

(a) 62 J (b) 104 J
(c) 124 J (d) 208 J

7. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then



(a) P lands in the shaded region and Q in the unshaded region.
(b) P lands in the unshaded region and Q in the shaded region.
(c) Both P and Q land in the unshaded region.
(d) Both P and Q land in the shaded region.

8. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

(a) 14.0 cm (b) 15.2 cm
(c) 16.4 cm (d) 17.6 cm

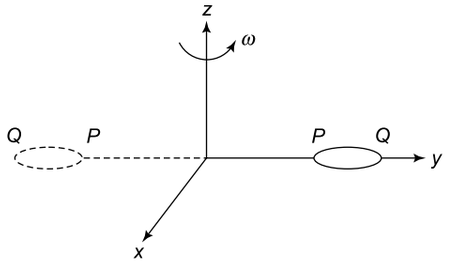
SECTION II

(Paragraph Type)

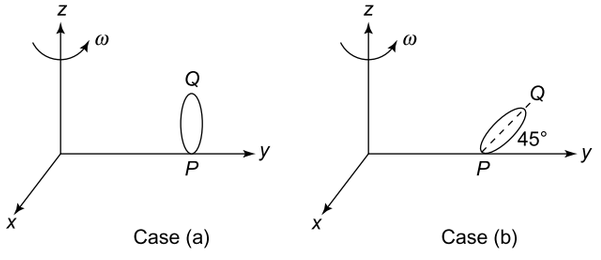
This section contains **6 multiple choice questions** relating to three paragraph with **two questions on each paragraph.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE is correct.**

Paragraph for Questions 9 and 10

The general motion of rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case



Now consider two similar system as shown in the figure: case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, disc is welded at point P, and the system are rotated with constant angular speed ω about the z-axis.



9. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is vertical for both the cases (a) and (b)

(b) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).

(c) It is horizontal of case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

(d) It is vertical for case (a); and is 45° to the x-z plane and is normal to the plane of the disc for case (b).

10. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is $\sqrt{2}\omega$ for both the cases.

(b) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b).

(c) It is ω for case (a); and $\sqrt{2}\omega$ for case (b).

(d) It is ω for both the cases.

Paragraph for Questions 11 and 12

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considerng a three-body decay process, i.e. $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy fo the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

11. What is the maximum energy of the anti-neutrino?

(a) Zero

(b) Much less than 0.8×10^6 eV

(c) Nearly 0.8×10^6 eV

(d) Much larger than 0.8×10^6 eV

12. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

(a) $0 \le K \le 0.8 \times 10^6$ eV
(b) 3.0 eV $\le K \le 0.8 \times 10^6$ eV
(c) 3.0 eV $< K \le 0.8 \times 10^6$ eV
(d) $0 \le K \le 0.8 \times 10^6$ eV

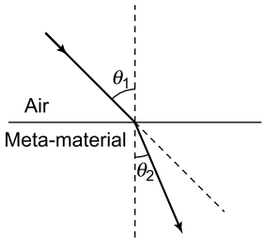
Paragraph for Questions 13 and 14

Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are relative permittivity and permeability of the medium respectively.

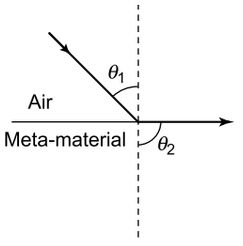
In normal materials, ε_r and μ_r, are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

13. For light incident from air on a meta-material, the appropriate ray diagram is

(a)



(b)



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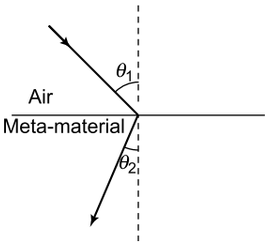

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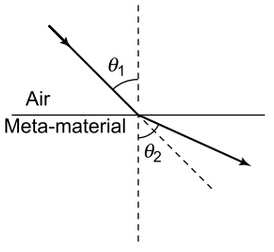


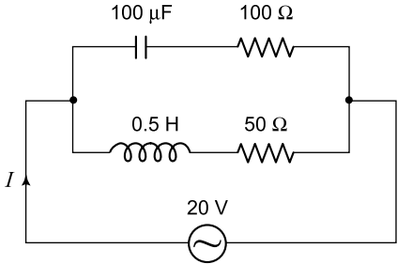
14. Choose the correct statement.
(a) The speed of light in the meta-material is $v = c|n|$
(b) The speed of light in the meta-material is $v = \frac{c}{|n|}$
(c) The speed of light in the meta-material is $v = c$.
(d) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}\,|n|$, where λ_{air} is wavelength of the light in air.

SECTION III

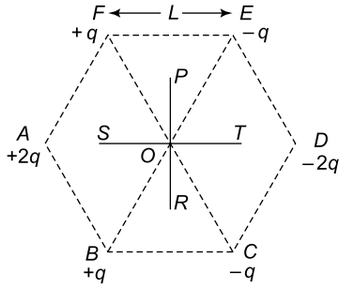
Multiple Correct Answer(s) Type

This section contains **6 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE are correct**.

15. In the given circuit, the AC source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are)
(a) The current through the circuit, I is 0.3 A.
(b) The current through the circuit, I is $0.3\sqrt{2}$ A
(c) The voltage across 100 Ω resistor = $10\sqrt{2}$ V.
(d) The voltage across 50 Ω resistor = 10 V.



16. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is (are) correct?



(a) The electric field at O is $6K$ along OD.
(b) The potential at O is zero.
(c) The poteltial at all points on the line PR is same.
(d) The potential at all points on the line ST is same.

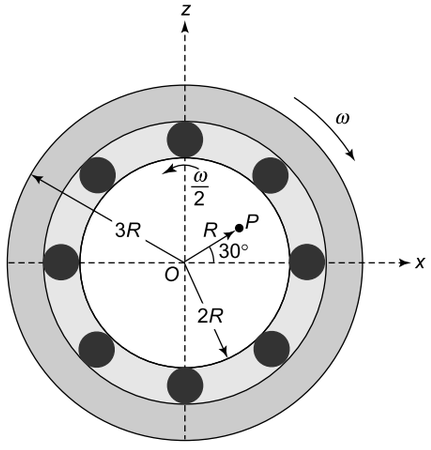
17. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q and surface areas A and $4A$ respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are V_P, V_Q and V_R, respectively. Then

(a) $V_Q > V_R > V_P$ (b) $V_R > V_Q > V_P$

(c) $V_R/V_P = 3$ (d) $V_P/V_Q = \frac{1}{2}$

18. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anticlockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface,

(a) the point O has linear velocity $3R\omega\hat{\mathbf{i}}$
(b) the point P has linear velocity
$\frac{11}{4}R\omega\hat{\mathbf{i}} + \frac{\sqrt{3}}{4}R\omega\hat{\mathbf{k}}$
(c) the point P has linear velocity
$\frac{13}{4}R\omega\hat{\mathbf{i}} - \frac{\sqrt{3}}{4}R\omega\hat{\mathbf{k}}$
(d) the point P has linear velocity
$\left(3 - \frac{\sqrt{3}}{4}\right)R\omega\hat{\mathbf{i}} + \frac{1}{4}R\omega\hat{\mathbf{k}}$



19. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is (are) correct?

(a) Both cylinders P and Q reach the ground at the same time.
(b) Cylinders P has larger linear acceleration than cylinder Q.
(c) Both cylinders reach the ground with same translational kinetic energy.
(d) Cylinder Q reaches the ground with larger angular speed.

20. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it, the correct statement(s) is (are)

(a) The emf induced in the loop is zero if the current is constant.
(b) The emf induced in the loop is finite if the current is constant.
(c) The emf induced in the loop is zero if the current decreases at a steady rate.
(d) The emf induced in the loop is infinite if the current decreases at a steady rate.

Answers

Section-I

1. (a)	**2.** (b)	**3.** (d)
4. (a)	**5.** (c)	**6.** (d)
7. (c)	**8.** (b)	

Section-II

9. (a)	**10.** (d)	**11.** (c)
12. (d)	**13.** (c)	**14.** (b)

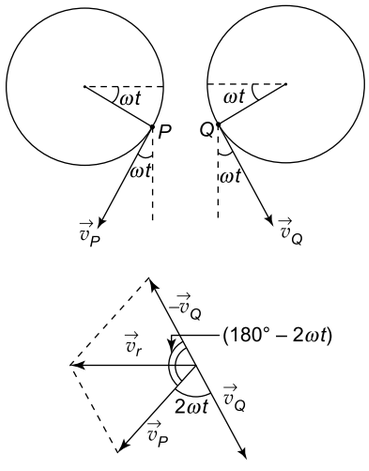
Section-III

15. (a,c)	**16.** (a, b, c)	**17.** (b,d))
18. (a, b)	**19.** (d)	**20.** (a, c)

Hints and Solutions

Section-I

1. (a)



$$\vec{v}_r = \vec{v}_P - \vec{v}_Q = \vec{v}_P + (-\vec{v}_Q)$$

Thus $\vec{v}_r$ is the resultant of $\vec{v}_P$ and $-\vec{v}_Q$.

$$v_r = \sqrt{v_P^2 + v_Q^2 + 2v_P v_Q \cos(180° - 2\omega t)}$$

$$= \sqrt{v^2 + v^2 - 2v^2 \cos 2\omega t}$$

$$= \sqrt{2v^2(1 - \cos 2\omega t)}$$

$$= \sqrt{4v^2 \sin^2 \omega t}$$

$$\Rightarrow v_r = 2v \sin \omega t = 2R\omega \sin \omega t \quad (\because v = R\omega)$$

The relative speed between points P and Q is

$$\therefore |v_r| = 2R\omega |\sin \omega t| = 2R\omega \left|\sin\left(\frac{2\pi t}{T}\right)\right|$$

Thus, $|v_r|$ becomes zero twice in one time period and becomes maximum twice in one time period. So the correct choice is (a).

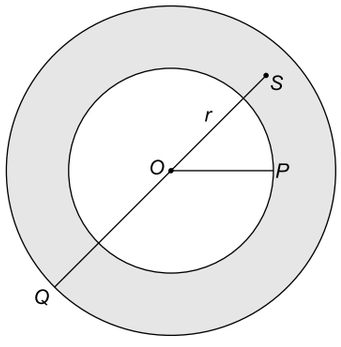
2. (b) Magnetic Moment $\vec{M}$ = current × area of the loop = $I\vec{A}$

$$= I \times \left[a^2 + \pi\left(\frac{a}{2}\right)^2 \times 2\right]\hat{k}$$

$$= Ia^2\left(1 + \frac{\pi}{2}\right)\hat{k}$$

The direction of area vector $\vec{A}$ is along $\hat{k}$.

3. (d)



$$OP = \frac{R}{2},\ OQ = R,\ OS = r,$$

Inside the cavity (i.e. for r lying between zero and $\frac{R}{2}$); $B = 0$

Outside the cylinder, (i.e. for $r > R$),

$$B = \frac{\mu_0 I}{2\pi r}$$

In the shaded region, (i.e. for $\frac{R}{2} < r < R$). From Ampere's circuital law,

$$B \times 2\pi r = \mu_0 I$$

$$= \mu_0 JA$$

where $J = \frac{I}{A}$ is the current density and A is the area of the shaded region. Now

$$A = \pi r^2 - \pi\left(\frac{R}{2}\right)^2$$

$$\therefore \quad B \times 2\pi r = \mu_0 J\left[\pi r^2 - \frac{\pi R^2}{4}\right]$$

$$\Rightarrow \qquad B = \frac{\mu_0 J}{2}\left[\frac{r^2 - R^2/4}{r}\right]$$

$$= \frac{\mu_0 J}{2}\left[r - \frac{R^2}{4r}\right]$$

Hence the correct graph is (d).

4. (a) Let V_c = Volume of the shell
d_c = density of the material of the shell = $\rho_c d_w$ where d_w is the density of water.
V_a = volume of air in the shell
V_w = volume of water in the shell
Since half the volume is submerged under water, from the law of floatation, weight of water displaced = upthrust, i.e.

$$\left(\frac{V_c + V_a + V_w}{2}\right) d_w g = V_c d_c g + V_w\, d_w g$$

$$= V_c\, \rho_c\, d_w\, g + V_w\, d_w\, g$$

$$\Rightarrow \quad V_w = V_c\,(1 - 2\rho_c) + V_a$$

$$= 2V_c \left(\frac{1}{2} - \rho_c\right) + V_a$$

If $\rho_c > \frac{1}{2}$, $V_w < V_a$

If $\rho_c < \frac{1}{2}$, $V_w > V_a$

Hence the correct choice is (a).

5. (c) Let q μC be the charge on the upper plate of 3 μF capacitor. Then the charge on the upper plate of 2 μF capacitor will be $(80 - q)$ μC. Since potential difference across 2 μF capacitor = potential difference across 3 μF capacitors,

$$\frac{80 - q}{2} = \frac{q}{3} \Rightarrow q = 48\ \mu\text{C}.$$

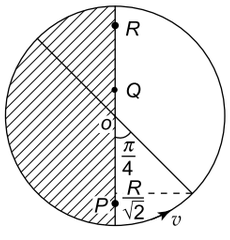
6. (d) $\Delta Q = nC_v\, \Delta T$

$$= 2 \times \frac{5R}{2} \times (35 - 30)$$

$$= 25\ R$$

$$= 25 \times 8.31 \approx 208\ \text{J}$$

7. (c)



At $t = \frac{T}{8}$, the horizontal distance travelled by P is

$$x = vt = \frac{\omega RT}{8} = \frac{2\pi}{T} \times \frac{RT}{8} = \frac{\pi R}{4}$$

where R is the radius of the disc. Since $\frac{\pi R}{4} < \frac{R}{\sqrt{2}}$, pebble P will land in the unshaded region.

For pebble Q, $\frac{\pi R}{4} < R$, it will also land in the unshaded region.

8. (b) End correction $e = 0.3d = 0.3 \times 4 = 1.2$ cm

Wavelength $\lambda = \frac{v}{\nu} = \frac{336}{512} = 0.656$ m $= 65.6$ cm

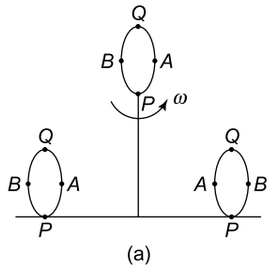
Now $\quad L + e = \frac{\lambda}{4}$

$$\Rightarrow \quad L = \frac{\lambda}{4} - e = \frac{65.6}{4} - 1.2$$

$$= 16.4 - 1.2 = 15.2\ \text{cm}$$

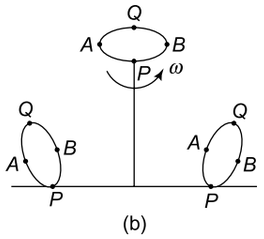
Section II

9. (a) In case (a), when the disc-stick system has rotated by half a cycle (or π radian), point B comes to the front and point A goes towards the back as shown in the figure (a).



(a)

Hence the disc is rotating about the axis in the vertical plane with angular frequency ω.



(b)

In case (b), when the disc-stick system has rotated by $\pi/2$, the points A and B have rotated by $\pi/2$ with respect to a vertical axis and after the system has rotated by half a cycle (π radian), A comes to the front and B goes towards the back as shown in Fig. (b). Hence in both cases, the disc is rotating about a vertical axis with the same frequency as the stick.

10. (d)

11. (c) The mass of a proton is very large compared to electron and antineutrino. So all the energy is shared by the electron and anti-neutrino. When the kinetic energy of anti-neutrino is zero, the

maximum kinetic energy of electron is 0.8×10^6 eV and vice versa. Hence the total kinetic energy of electron + anti-neutrino is 0.8×10^6 eV.

12. (d) If the anti-neutrino has a mass $m = 3$ eV/c^2, at will have kinetic energy = $mc^2 = 3$ eV.
Therefore, the maximum kinetic energy of the electron = $(0.8 \times 10^6 - 3)$ eV, which is only slightly less than 0.8×10^6 eV. The minimum kinetic energy of the electron is still zero. Hence correct choice is (d)

13. (c) The refractive index n for meta-materials is negative. Hence $\dfrac{\sin\theta_1}{\sin\theta_2}$ is negative.

Thus if θ_1 is positive, θ_2 will be negative. So the current choice is (c).

14. (b) $N = \dfrac{c}{v} \Rightarrow v = \dfrac{c}{|n|}$, which is choice (b)

Also frequency $\nu = \dfrac{v}{\lambda}$. Since ν remains unchanged,

$$\frac{v_{\text{air}}}{\lambda_{\text{air}}} = \frac{v_m}{\lambda_m}$$

$$\Rightarrow \quad \lambda_m = \lambda_{\text{air}} \times \frac{v_m}{v_{\text{air}}}$$

$$= \lambda_{\text{air}} \times \frac{v_m}{c} \times \frac{c}{v_{\text{air}}}$$

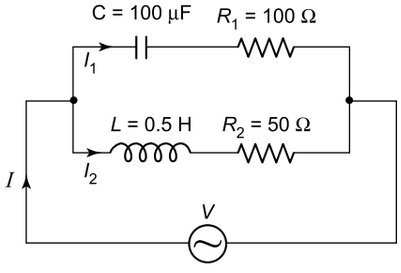
$$= \lambda_{\text{air}} \frac{n_{\text{air}}}{n_m} \qquad \left(\because \ v = \frac{c}{n}\right)$$

$$= \frac{\lambda_{\text{air}}}{|n|} \qquad (\because \ n_m = |n| \text{ and } n_{\text{air}} = 1)$$

So choice (d) is wrong.

Section III

15. (a, c)



$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\ \Omega$$

$$X_L = \omega L = 100 \times 0.5 = 50\ \Omega$$

Since $X_C = R_1$, current I_1 leads V by 45°. Since $X_L = R_2$, current I_2 lags behind V by 45°. So the phase difference between I_1 and I_2 is $\phi = 90°$.

$$I_1 = \frac{V}{\sqrt{X_c^2 + R_1^2}} = \frac{20}{\sqrt{(100)^2 + (100)^2}}$$

$$= \frac{\sqrt{2}}{10}\text{A}$$

and $$I_2 = \frac{V}{\sqrt{X_L^2 + R_2^2}} = \frac{20}{\sqrt{(50)^2 + (50)^2}}$$

$$= \frac{\sqrt{2}}{5}\text{A}$$

$$\therefore \quad I = \sqrt{I_1^2 + I_2^2 + 2I_1 I_2 \cos 90°}$$

$$= \sqrt{\left(\frac{\sqrt{2}}{10}\right)^2 + \left(\frac{\sqrt{2}}{5}\right)^2}$$

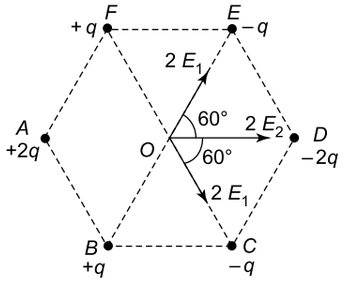
$$= \sqrt{0.1} = 0.316$$

$$\simeq 0.3 \text{ A} \quad \text{(up to appropriate significant figure)}$$

P.D. across $R_1 = I_1 R_1 = \dfrac{\sqrt{2}}{10} \times 100 = 10\sqrt{2}$ V

P.D. across $R_2 = I_2 R_2 = \dfrac{\sqrt{2}}{5} \times 50 = 10\sqrt{2}$ V

16. (a, b, c)



E_1 = electric field at O due to $-q$ at E directed from O to E
= electric field at O due to $+q$ at B directed from O to E.

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$$

E_2 = electric field at O due to + $2q$ at A directed from O to D
= electric field at O due to $-q$ at D directed from O to D

$$= \frac{1}{4\pi\varepsilon_0}\frac{2q}{L^2}$$

The net electric field at O is

$$E = 2E_1 \cos 60° + 2E_1 \cos 60° + 2E_2$$
$$= E_1 + E_1 + 2E_2$$
$$= 2E_1 + 2E_2$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{2q}{L^2} + \frac{1}{4\pi\varepsilon_0}\frac{4q}{L^2}$$
$$= \frac{6}{4\pi\varepsilon_0}\frac{q}{L^2} \text{ along } OD$$

So choice (a) is correct.

Net potential at O is

$$V = \frac{-q}{4\pi\varepsilon_0 L} - \frac{q}{4\pi\varepsilon_0 L} + \frac{q}{4\pi\varepsilon_0 L} + \frac{q}{4\pi\varepsilon_0 L} + \frac{2q}{4\pi\varepsilon_0 L} - \frac{2q}{4\pi\varepsilon_0 L} = 0$$

So choice (b) is also correct.

There are three electric dipoles ($+q$ at F, $-q$ at E), ($+q$ at B, $-q$ at C) and ($+2q$ at A, $-2q$ at D). Line PR is the perpendicular bisector of all the dipoles. Hence the potential at any point on line PR due to each dipole is zero.

17. (b, d)

Let M be the mass of planet P and R its radius.

Then Mass of P is $M = \rho \times \frac{4\pi}{3}R^3$

Mass of Q is $\rho \times \frac{4\pi}{3}(2R)^3 = 8M$ ($\because$ surface area of Q = 4 times that of P, therefore radius of Q = 2 (radius of P) = $2R$

Mass of R = mass of P + mass of Q

$= M + 8M = 9M$

$\therefore$ Radius of $R = 9^{1/3}R$

The escape velocities of P, Q and R are

$$V_P = \sqrt{\frac{2GM}{R}}$$

$$V_Q = \sqrt{\frac{2G \times 8M}{R}} = 2V_P$$

$$V_R = \sqrt{\frac{2G \times 9M}{9^{1/3}R}} = 9^{1/3}V_P$$

$\therefore$ $V_R > V_Q > V_P$

Also $\frac{V_Q}{V_P} = 2$

So the correct choices are b and d.

18. (a, b)

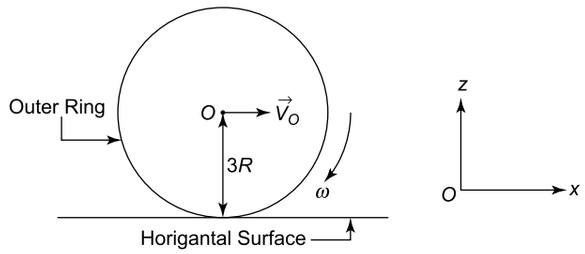


Fig. 1

With respect to the horizontal surface, point O has a linear velocity $V_O = 3\ R\omega$ along the x-axis. Hence $\vec{V}_O = 3\ R\omega\hat{\mathbf{i}}$ (see Fig. 1)

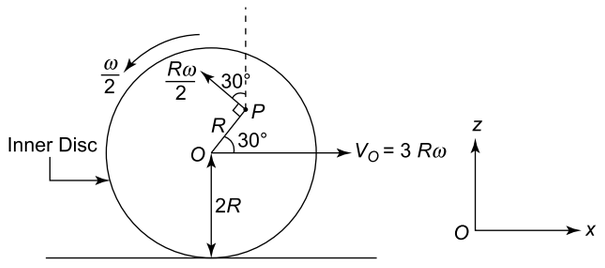


Fig. 2

With respect to the horizontal surface, the linear velocity of point P is

$$\vec{V}_P = V_O\hat{\mathbf{i}} + \frac{R\omega}{2}\cos 30°\,\hat{\mathbf{k}} - \frac{R\omega}{2}\sin 30°\,\hat{\mathbf{i}}$$
$$= 3R\omega\hat{\mathbf{i}} + \frac{R\omega}{2} \times \frac{\sqrt{3}}{2}\hat{\mathbf{k}} - \frac{R\omega}{2} \times \frac{1}{2}\hat{\mathbf{i}}$$
$$= \frac{11}{4}R\omega\hat{\mathbf{i}} + \frac{\sqrt{3}}{4}R\omega\hat{\mathbf{k}}$$

19. (d) The linear acceleration of a body of mass M and radius R rolling down an inclined plane of inclination θ is given by

$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

where I is the moment of intertia of the body about its centre of mass. Therefore,

$$a_P = \frac{g\sin\theta}{1 + \frac{I_P}{MR^2}}$$

and $$a_Q = \frac{g\sin\theta}{1 + \frac{I_Q}{MR^2}}$$

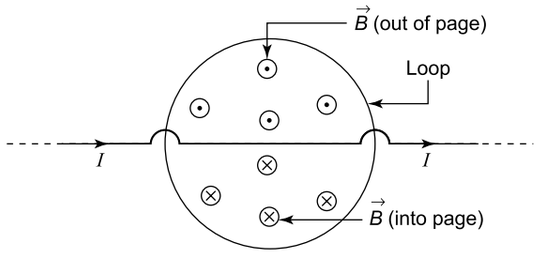
Since cylinder P has most of its mass concentrated near its surface and Q has most of its mass concentrated near its axis, $I_P > I_Q$. Hence

$$a_P < a_Q$$

$$\therefore \quad \alpha_P < \alpha_Q \qquad (\because\ a = R\alpha)$$

Since $\omega^2 = 0^2 - 2\alpha(\theta)$; here θ = angular displacement and $\theta = \frac{1}{2}\alpha t^2$, it follows that $\omega_P < \omega_Q$. So the only correct choice is (d).

20. The magnetic field above the wire is directed out of the plane of the loop and below the wire into the plane of the loop. Hence the net magnetic flux through the loop is zero. Therefore, the emf induced in the loop is zero, irrespective of whether the current is changing or not changing. Hence the correct choices are (a) and (c).



JEE ADVANCED 2013: PAPER-I

Instructions

Question Paper Format

Section 1 contains **10 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Section 2 contains **5 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE are correct.**

Section 3 contains **5 questions.** The answer to each question is a single-digit integer, ranging from 0 to 9 (both inclusive).

Marking Scheme

For each question in **Section 1**, you will be awarded **2 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. **No negative** marks will be awarded for incorrect answers in this section.

For each question in **Section 2**, you will be awarded **4 marks** if you darken **all** the bubble(s) corresponding to only the correct answer(s) and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded.

For each question in **Section 3**, you will be awarded **4 marks** if you darken the bubble corresponding to only the correct answer and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded.

SECTION-1

(Only One Option Correct Type)

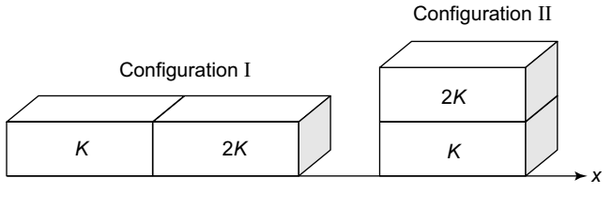
This section contains **10 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. The work done on a particle of mass m by a force, $K\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\hat{i}+\frac{y}{\left(x^2+y^2\right)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the x-y plane is:

(a) $\frac{2K\pi}{a}$ (b) $\frac{K\pi}{a}$

(c) $\frac{K\pi}{2a}$ (d) 0

2. Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity K and other $2K$. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is:



(a) 2.0s (b) 3.0s

(c) 4.5s (d) 6.0s

3. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is:

(a) 1 : 4 (b) 1 : 2

(c) 6 : 9 (d) 8 : 9

4. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another

identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}+\alpha$

(c) $\frac{\pi}{4}-\alpha$ (d) $\frac{\pi}{2}$

5. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 ms^{-1}. The final momentum (in kg ms^{-1}) of the object is

(a) 0.3×10^{-17} (b) 1.0×10^{-17}

(c) 3.0×10^{-17} (d) 9.0×10^{-17}

6. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is:

(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$

(c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$

7. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is:

(a) 1m (b) 2m

(c) 3m (d) 6m

8. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is:

(a) 0.25 (b) 0.50

(c) 2.00 (d) 4.00

9. A ray of light travelling in the direction $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$. The angle of incidence is:

(a) 30° (b) 45°

(c) 60° (d) 75°

10. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45cm. The 24th division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is:

(a) 5.112 cm (b) 5.124 cm

(d) 5.136 cm (d) 5.148 cm

SECTION-2

(One or More Options Correct Type)

This section contains **5 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE** are correct.

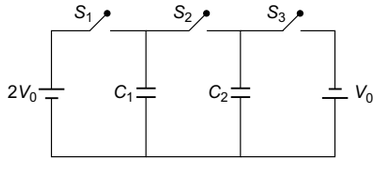
11. In the circuit shown in the figure, there are two parallel plate capacitors C_1 and C_2, each of capacitance C The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. Switch S_2 is then pressed so that the capaciton C_2 is also charged. After some time S_2 is released and switclh S_3 is pressed. Then

(a) the charge on the upper plate of C_1 is 2 CV_0

(b) the charge on the upper plate of C_1 is CV_0

(c) the charge on the upper plate of C_2 is zero

(d) the charge on the upper plate of C_2 is – CV_0



12. A particle of mass M and positive charge Q. moving with a constant velocity $\hat{u}_1 = 4\hat{i}$ ms^{-1} enters a region of uniform static magnetic field normal to the $x-y$ plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y, After passing through this rgion, the particle emerges on the other side after 10 milliseconds with a velocity $\hat{u}_2 = 2\left(\sqrt{3}\hat{i}+\hat{j}\right)$ms^{-1}. The correct statement(s) is (are)

(a) The direction of the magnetic field is –z direction.

(b) The direction of the magnetic field is + z direction.

(c) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.

(d) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

13. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x,t)=(0.01\text{m})\sin[(62.8\,\text{m}^{-1})x]\cos[628\,\text{s}^{-1})t]$. Assuming $\pi = 3.14$, the correct statements(s) is (are)

(a) The number of nodes is 5.

(b) The length of the string is 0.25 m.

(c) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01m.

(d) The fundamental frequency is 100 Hz

14. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. the complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are)

(a) the net elongation of the spring is $\frac{4\pi R^3\rho g}{3k}$

(b) the net elongation of the spring is $\frac{8\pi R^3\rho g}{3k}$

(c) the light sphere is partially submerged.

(d) the light sphere is completely submerged.

15. Two non-conducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero, the ratio $\frac{\rho_1}{\rho_2}$ can be.

(a) -4 (b) $-\frac{32}{25}$

(c) $\frac{32}{25}$ (d) 4

SECTION-3

(Integer Value Correct Type)

This section contains **5 questions**. The answer to each question is a **single digit integer,** ranging from 0 to 9 (*both inclusive*).

16. A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is

17. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is

18. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is

19. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that ln 2 = 0.693, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is

20. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is

Answers

Section-1

1. (d)	**2.** (a)	**3.** (d)
4. (a)	**5.** (b)	**6.** (b)
7. (c)	**8.** (c)	**9.** (a)
10. (b)		

Section-2

11. (b, c)	**12.** (a, c)	**13.** (b, c)
14. (a, d)	**15.** (b, d)	

Section-3

16. 5 **17.** 5 **18.** 1
19. 4 **20.** 8

Hints and Solutions

1. $W = \int \vec{F} \cdot \vec{dr}$

$= \int F_x\, dx + \int F_y\, dy$

$= K \int\limits_{(a,0)}^{(0,a)} \frac{x dx}{\left(x^2 + y^2\right)^{3/2}} + K \int\limits_{(a,0)}^{(0,a)} \frac{y dy}{\left(x^2 + y^2\right)^{3/2}}$

$= -\frac{K}{2}\left|\left(x^2 + y^2\right)^{-1/2}\right|_{(a,0)}^{(0,a)} - \frac{K}{2}\left|\left(x^2 + y^2\right)^{-1/2}\right|_{(a,0)}^{(0,a)}$

$= -0 - 0$

$= 0$

2. $\frac{Q}{t} = \frac{\Delta T}{R}$; ($R$ = thermal resistance)

$\Rightarrow \frac{Q}{\Delta T} = \frac{t}{R}$ (i)

Since Q and ΔT are the same, $\frac{t}{R}$ = constant. Hence

$\frac{t_1}{R_1} = \frac{t_2}{R_2}$

Configuration 1: $R_1 = \frac{l}{KA} + \frac{l}{2KA} = \frac{3l}{2KA}$

Configuration 2: $\frac{1}{R_2} = \frac{2KA}{l} + \frac{KA}{l} = \frac{3KA}{l}$

$\Rightarrow R_2 = \frac{l}{3KA}$

Using these in (i),

$\frac{9}{3l/2KA} = \frac{t_2}{l/3KA} \Rightarrow t_2 = 2s$

3. $PV = n\,RT \Rightarrow \frac{PM}{\rho} = n\,RT \Rightarrow PM = n\,\rho\,RT$

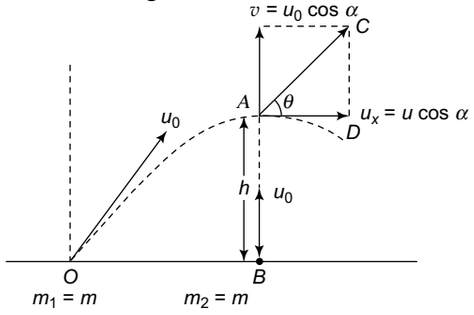
Since R and T are constant, then for given n

$\frac{PM}{\rho}$ = constant

or $\frac{P_1 M_1}{\rho_1} = \frac{P_2 M_2}{\rho_2}$

$\Rightarrow \frac{\rho_1}{\rho_2} = \left(\frac{P_1}{P_2}\right) \times \left(\frac{M_1}{M_2}\right) = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$

4. At the highest point A, both the particles have the same potential energy (= $mg\,h$). Since the total energy (K.E. + P.E) is conserved, the two particles has equal kinetic energy and hence equal speed at A. Speed of particle 1 at A is $u_x = u\cos\alpha$. Hence the speed of 2 at A will be $v = u_0 \cos\alpha$. The final velouity of the composite mass (since the particles stick after perfectly inelasric collision) will be along the resultant along AC. Therefore,



$\tan\theta = \frac{CD}{AD} = \frac{v}{u_x} = \frac{u_0 \cos\alpha}{u_0 \cos\alpha} = 1$

$\Rightarrow \theta = 45°$

5. Momentum = $\frac{E}{c} = \frac{\text{Power} \times \Delta t}{c}$

$= \frac{\left(30 \times 10^{-3}\right) \times \left(100 \times 10^{-9}\right)}{3 \times 10^8}$

$= 1.0 \times 10^{-17}$ kg ms^{-1}

6. $I = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$; $\Delta\phi$ = phase difference

$\therefore I_{\max} = 4I_0$. For $I = \frac{1}{2}\ I_{\max} = 2I_0$

$2I_0 = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$

$\Rightarrow \cos\left(\frac{\Delta\phi}{2}\right) = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\Delta\phi}{2} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, .. = (2n + 1)\frac{\pi}{4}$

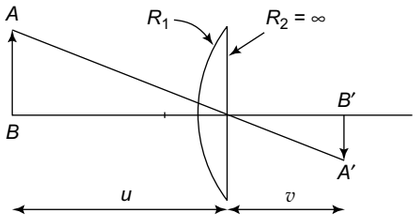
where $n = 0, 1, 2, \ldots$

or $\Delta\phi = (2n + 1)\frac{\pi}{2}$

$\Rightarrow \frac{2\pi}{\lambda}\Delta x = (2n + 1)\frac{\pi}{2}$; Δx = path difference

$\Rightarrow \Delta x = (2n + 1)\frac{\lambda}{4}$

7. Give $\frac{v}{u} = -\frac{1}{3} \Rightarrow u = -3v = -3 \times 8 = -24\text{m}$



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{8} - \frac{1}{-24} - \frac{1}{f} = \Rightarrow f = 6\text{m}$$

If λ is the wavelength of light in the lens and λ_0 in air, then

$$v = \nu\lambda$$

$$c = \nu\lambda_0$$

$$\therefore \quad \mu = \frac{c}{v} = \frac{\lambda_0}{\lambda} \Rightarrow \lambda = \frac{\lambda_0}{\mu}$$

Given $\lambda = \frac{2\lambda_0}{3\mu}$. Hence $\mu = \frac{3}{2} = 1.5$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right) = \frac{0.5}{R_1}$$

$$\Rightarrow \quad \frac{1}{6} = \frac{0.5}{R_1} \Rightarrow R_1 = 3\text{m}$$

8. $Y = \frac{FL}{A\Delta L} = \frac{FL}{\pi r^2 \Delta L}$

$$\Rightarrow \quad \Delta L = \frac{FL}{\pi r^2 Y}$$

Since Y and F is the same for both wires,

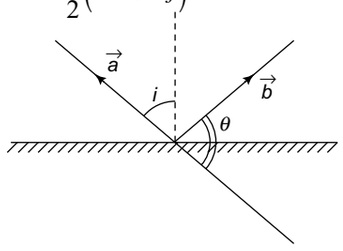
$$\frac{(\Delta L)_1}{(\Delta L)_2} = \left(\frac{L_1}{L_2}\right) \times \left(\frac{r_2}{r_1}\right)^2$$

$$= \left(\frac{2L}{L}\right) \times \left(\frac{r}{2r}\right)^2$$

$$= 2 \times \frac{1}{4} = \frac{1}{2}$$

$$\therefore \quad \frac{(\Delta L)_2}{(\Delta L)_1} = 2$$

9. Given $\vec{a} = \frac{1}{2}\left(\hat{i} + \sqrt{3}\,\hat{j}\right)$



$$\vec{b} = \frac{1}{2}\left(\hat{i} - \sqrt{3}\,\hat{j}\right)$$

Magnitude of $\vec{a}$ is $a = \frac{1}{2}(1+3)^{1/2} = 1$

Magnitude of $\vec{b}$ is $b = \frac{1}{2}(1+3)^{1/2} = 1$

If θ is the angle between $\vec{a}$ and $\vec{b}$ then

$$\cos\,\theta = \frac{\vec{a}.\vec{b}}{ab}$$

$$= \frac{\frac{1}{2}\left(\hat{i} + \sqrt{3}\,\hat{j}\right).\frac{1}{2}\left(\hat{i} - \sqrt{3}\,\hat{j}\right)}{1 \times 1}$$

$$= -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120°$$

$$\therefore \quad i = \frac{180° - \theta}{2} = \frac{180° - 120°}{2} = 30°$$

10. Least count = value of 1 MSD – value of 1 VSD

$$= 0.05\text{ cm} - \frac{2.45}{50}\text{ cm}$$

$$= 0.05 - 0.049 = 0.001\text{ cm}$$

Diameter = $5.10\text{ cm} + 24 \times 0.001\text{ cm}$

$$= 5.10\text{ cm} + 0.024\text{ cm}$$

$$= 5.124\text{ cm}$$

11. When only S_1 is pressed, capacitor C_1 is charged. Charge on the upper plate of C_1 is $+ 2CV_0$ (see Fig. 1.)

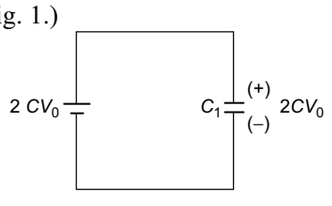


Fig. 1.

When S_1 is released and S_2 is pressed, charge will flow from C_1 to C_2 until they acquire equal charge

because they have the same capacitance (conservation of charge) as shown in Fig. 2.

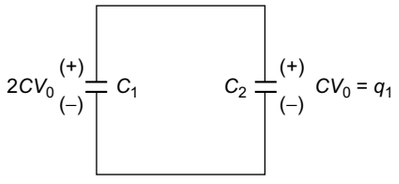
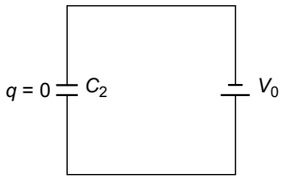


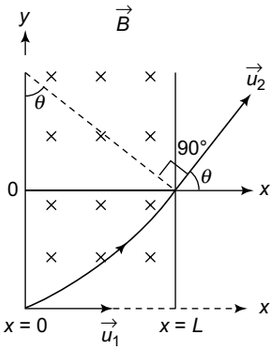
Fig. 2.

When S_2 is relased and S_3 is pressed (see Fig. 3),



the upper plate of C_2 acquires a charge $q_2 = -CV_0$ so that the net charge on this plate is $q = q_1 + q_2 = CV_0 - CV_0 = 0$

12. From Fleming's left-hand rule, the direction of the magnetic field $\vec{B}$ will be along negative z-direction.



$$\vec{u}_1 = 4\hat{i} \text{ ms}^{-1}$$

$$\vec{u}_2 = 2\left(\sqrt{3}\,\hat{i} + \hat{j}\right) \text{ms}^{-1}$$

$$\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^0 = \frac{\pi}{6} \text{ rad}$$

$$\therefore \qquad t = \frac{\theta M}{QB}$$

$$\Rightarrow 10 \times 10^{-13} = \frac{\left(\frac{\pi}{6}\right) \times M}{Q\,B}$$

$$\Rightarrow \qquad B = \frac{50\,\pi\,m}{3Q} \text{ units}$$

13. Comparing the given equation with the standing wave equation

$$y(x, t) = 2a \sin(kx) \cos(\omega t)$$

we get

$$k = 62.8 \Rightarrow \frac{2\pi}{\lambda} = 62.8$$

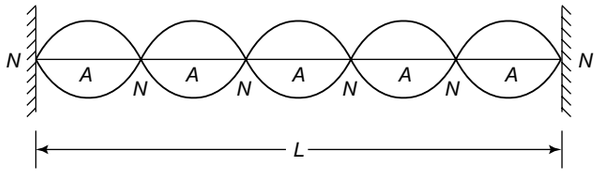
$$\Rightarrow \qquad \lambda = \frac{2 \times 3.14}{62.8} = 0.1\text{m}$$

and $\omega = 628 \Rightarrow 2\pi\nu = 628 \Rightarrow \nu = \dfrac{628}{2 \times 3.14} = 100$ Hz

Since the string is vibrating it the fifth harmonic, the number of nodes = 6. (see figure)

$$\frac{5\,\lambda}{2} = L \text{ (L = length of string)}$$

$$\Rightarrow \qquad L = \frac{5 \times 0.1}{2} = 0.25 \text{ m}$$



There is an antinode at the mid-point of the string where y is maximum. From the given equation,

$y_{max} = 0.01$ m

Fundemental frequenty $\nu_0 = \dfrac{v}{2L} = \dfrac{\nu\lambda}{2L} = \dfrac{100 \times 0.1}{2 \times 0.25}$

= 20 Hz

14. Volume of each sphere is $V = \dfrac{4\pi R^3}{3}$

Mass of sphere A is $m_1 = \rho V$ and of sphere B is $m_2 = 3\rho V$ (see Fig).

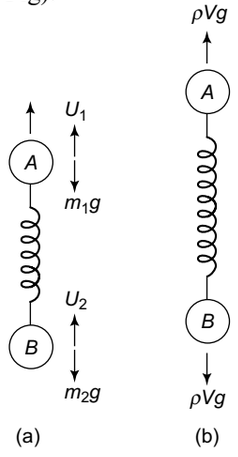


Fig (a)

The forces acting on m_1 are its weight m_1g and upthrust U_1 = weight of the liquid displaced by it = $2\rho Vg$. The forces acting on m_2 are its weight m_2g and upthrust $U_2 = 2\rho Vg$ (Fig. a). Net force acting on

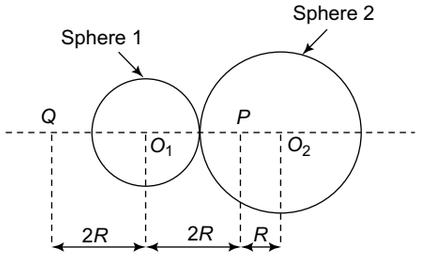
$A = U_1 - m_1 g = 2\rho Vg - \rho Vg = \rho Vg$ acting upwords. Net force acting on $B = m_2 g - U_2 = 3\rho Vg - 2\rho Vg = \rho Vg$ acting downwards (Fig. b). so the tension in the spring when equilibrium is attained = ρVg. If x is the extension of the spring, then

$$kx = \rho Vg = \rho \times \frac{4\pi R^3}{3} \times g$$

$$\Rightarrow \qquad x = \frac{4\pi R^3 \rho g}{3k}$$

In equilibrium, the total force acting on A is $F_A = kx + W_1 = \rho Vg + \rho Vg = 2\rho Vg$ which is equal to upthrust U_1. Hence it is completely submerged.

15. Let P and Q be the two points at a distance $2R$ on the line joining the centres O_1 and O_2 of the spheres.



The charges on spheres 1 and 2 are

$$Q_1 = \frac{4\pi}{3} R^3 \rho_1 \text{ and } Q_2 = \frac{4\pi}{3} (2R)^3 \rho_2$$

Electric field at P will be zero if

$$\frac{Q_1}{4\pi\varepsilon_0 (O_1P)^2} = \frac{\rho_2 R}{3\varepsilon_0}$$

$$\Rightarrow \qquad \frac{\frac{4\pi}{3} R^3 \rho_1}{4\pi\varepsilon_0 \times (2R)^2} = \frac{\rho_2 R}{3\varepsilon_0} \Rightarrow \frac{\rho_1}{\rho_2} = 4$$

Electric field at Q will be zero if

$$\frac{Q_1}{4\pi\varepsilon_0 (O_1Q)^2} + \frac{Q_2}{4\pi\varepsilon_0 (O_2Q)^2} = 0$$

$$\Rightarrow \frac{\frac{4\pi}{3} R^3 \rho_1}{4\pi\,\varepsilon_0\,(2R)^2} + \frac{\frac{4\pi}{3}(2R)^3 \rho_2}{4\pi\varepsilon_0 (5R)^2} = 0$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = -\frac{32}{25}$$

16. The minimum velocity which the bob must have is given by

$$\mu_1 = \sqrt{5gl_1}$$

It is then at the lowest point of the circle. From energy conservation, its velocity at the highest point will be

$$v_1 = \sqrt{gl_1}$$

Since the bobs are identical and the collision is elastic, their merely exchange their velocities on collision. Hence the second bob will have a velocity $u_2 = v_1 = \sqrt{gl_1}$. In order to complete the vertical circle,

$$\sqrt{gl_1} = \sqrt{5gl_2} \quad \Rightarrow \quad \frac{l_1}{l_2} = 5$$

17. Power $P = 0.5$ W. Work done in 5 s is

($\because$ P = constant)

$$W = \int_0^t P\,dt = P\int_0^t dt = P \times t = 0.5 \times 5 = 2.5 \text{ J}$$

From work energy theorem, work done = change in kinetic energy. Hence

$$\frac{1}{2} mv^2 = W$$

$$\Rightarrow \frac{1}{2} \times 0.2 \times v^2 = 2.5 \quad \Rightarrow v^2 = 25 \Rightarrow v = 5\,\text{ms}^{-1}$$

18 From Einstein's photoelectric equation

$$\hbar\nu = K_{\max} + W_0$$

$$= eV_0 + W_0$$

$$\Rightarrow \qquad V_0 = \left(\frac{\hbar}{e}\right)\nu - \frac{W_0}{e}$$

Therfore, the slope of V_0 versus ν graph is $\frac{\hbar}{e}$ which is constant independent of work function W_0.

19. Half life T is related to disintegration constant λ as

$$T = \frac{0.693}{\lambda}$$

$$\Rightarrow \qquad \lambda = \frac{0.693}{T} = \frac{0.693}{1386} = 5 \times 10^{-4}\ \text{s}^{-1}$$

Now $N = N_0\, e^{-\lambda t}$

The fraction of nuclei disintegrated in $t = 80$ s is

$$f = \frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0} = 1 - e^{-\lambda t} = 1 - e^{-5 \times 10^{-4} \times 80}$$

$$\Rightarrow \qquad f = 1 - e^{-0.04}$$

Using $e^{-a} = 1 - a$ for $a << 1$, we have

$$f = 1 - (1 - 0.04) = 0.04$$

Percentage disintegrated = $0.04 \times 100 = 4\%$

20. Moment of inertia of the disc before the rings are placed (Fig. a) about O is

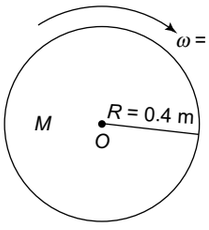


Fig. (a)

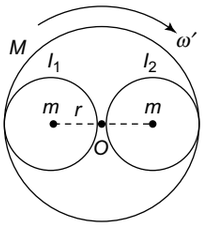


Fig. (b)

$$I = \frac{1}{2}MR^2$$

Using parallel-axes theorem, the moment of inertia of each ring about O is (Fig. b)

$$I_1 = I_2 = mr^2 + mr^2 = 2mr^2$$

Moment of inertia of the system about O after the rings are placed is

$$I' = I + I_1 + I_2 = \frac{1}{2}MR^2 + 2mr^2 + 2mr^2$$

$$= \frac{1}{2}MR^2 + 4mr^2$$

$$= \frac{1}{2}MR^2 + 4m\left(\frac{R}{2}\right)^2 \quad \left(\because r = \frac{R}{2}\right)$$

$$= \frac{1}{2}MR^2 + mR^2$$

$$= \frac{1}{2}(M + 2m)R^2$$

From conservation of angular momentum, we have

$$I\omega = I'\omega'$$

$$\Rightarrow \quad \omega' = \frac{I\omega}{I'} = \frac{\frac{1}{2}MR^2\omega}{\frac{1}{2}(M + 2m)R^2}$$

$$= \frac{M\omega}{M + 2m}$$

$$= \frac{50 \times 10}{50 + 2 \times 6.25}$$

$$= 8$$

JEE ADVANCED 2013: PAPER-II

Instructions

Question Paper Format

Section 1 contains **8 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE** are correct.

Section 2 contains **4 paragraphs.** Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE are correct.**

Section 3 contains **4 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE are correct.**

Marking Scheme

For each question in **Section 1**, you will be awarded **2 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. **No negative** marks will be awarded for incorrect answers in this section.

For each question in **Section 2**, you will be awarded **4 marks** if you darken **all** the bubble(s) corresponding to only the correct answer(s) and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded.

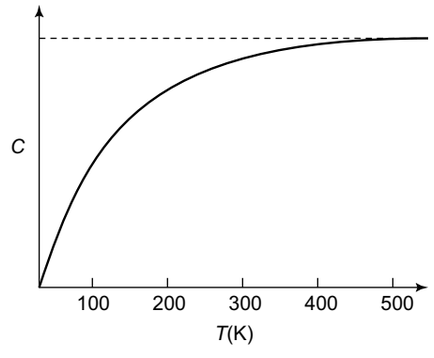
For each question in **Section 3**, you will be awarded **4 marks** if you darken the bubble corresponding to only the correct answer and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded.

SECTION-1

(One or More Options Correct Type)

This section contains **8 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE** are correct.

1. The figure below shows the variation of specific heat capacity (c) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation.



(a) the rate at which heat is absorbed in the range 0 – 100 K varies linearly with temperature T.

(b) heat absorbed in increasing the temperature from 0 – 100 K is less than the heat required for increasing the temperature from 400 – 500 K.

(c) there is no change in the rate of heat absorption in the range 400 – 500 K

(d) the rate of heat absorption increases in the range 200 – 300 K

2. The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0, where a_0, is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is/are

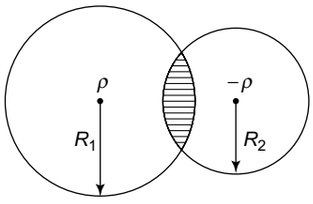
(a) $\frac{9}{32R}$ (b) $\frac{9}{16R}$

(c) $\frac{9}{5R}$ (d) $\frac{4}{3R}$

3. Using the expression $2d \sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0°,

(a) the absolute error in d remains constant
(b) the absolute error in d increases
(c) the fractional error in d remains constant
(d) the fractional error in d decreases

4. Two non-conducting spheres of radii R_1 and R_2 carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap as shown in the figure. At all points in the overlapping region,



(a) the electrostatic field is zero
(b) the electrostatic potential is constant
(c) the electrostatic field is constant in magnitude
(d) the electrostatic field has same direction

5. A Steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxilly inside an infinite solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. consider a point P at a distance r from the common axis. The correct statement(s) is (are)

(a) In the region $0 < r < R$, the magnetic field is non-zero
(b) In the region $R < r < 2R$, the magnetic field is along the common axis.
(c) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis.
(d) In the region $r > 2R$, the magnetic field is non-zero.

6. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are)

(a) If the wind blows from the observer to the source, $f_2 > f_1$.
(b) If the wind blows from the source to the observer, $f_2 > f_1$
(c) If the wind blows from observer to the source, $f_2 < f_1$
(d) If the wind blows from the source to the observer, $f_2 < f_1$

7. Two bodies, each of mass M, are kept fixed with a separation $2L$. A particle of mass m is projected from the midpoint of the line joining centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are)

(a) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$
(b) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$
(c) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$
(d) The energy of the mass m remains constant.

8. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5\ u_0$, it collides elastically with a rigid wall. After this collision

(a) the speed of the particle when it returns to its equilibrium position is u_0
(b) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$
(c) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$
(d) the time at which the particle passes through the equilibrium position for the second time $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

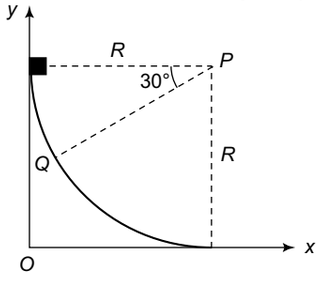
SECTION-2

(Paragraph Type)

This section contains **4 paragraphs** each describing theory, experiment, data etc. **Eight questions** related to four paragraphs with two questions on each paragraph. Each question of a paragraph has only **one correct answer** among the four choices (a), (b), (c) and (d).

Paragraph for Questions 9-10

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J. (Take the acceleration due to gravity, $g = 10\ \text{ms}^{-2}$)



9. The speed of the block when it reaches the point Q is

(a) $5\ \text{ms}^{-1}$ (b) $10\ \text{ms}^{-1}$

(c) $10\sqrt{3}\ \text{ms}^{-1}$ (d) $20\ \text{ms}^{-1}$

10. The magnitude of the normal reaction that acts on the block at the point Q is

(a) 7.5 N (b) 8.6 N

(c) 11.5 N (d) 22.5 N

Paragraph for Questions 11-12

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the currents and voltages mentioned are rms values.

11. If the direct transmission method with a cable of resistance $0.4\ \Omega\ \text{km}^{-1}$ is used, the power dissipation (in %) during transmission is.

(a) 20 (b) 30

(c) 40 (d) 50

12. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1:10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is.

(A) 200 : 1 (B) 150 : 1

(C) 100 : 1 (D) 50 : 1

Paragraph for Questions 13-14

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionally constant γ.

13. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is.

(a) $\frac{BR}{4}$ (b) $\frac{BR}{2}$

(c) BR (d) $2BR$

14. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change is.

(a) $-\gamma BQR^2$ (b) $-\gamma\frac{BQR^2}{2}$

(c) $\gamma\frac{BQR^2}{2}$ (d) γBQR^2

Paragraph for Questions 15-16

The mass of a nucleus ${}^{A}_{Z}\text{X}$ is less than the sum of the masses of $(A-Z)$ number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and

form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the talble below:

$m\left({}^{6}_{3}\text{Li}\right) = 6.015123\text{u}$, $m\left({}^{4}_{2}\text{He}\right) = 4.002603\text{u}$

$m\left({}^{2}_{1}\text{H}\right) = 2.014102\text{u}$, $m\left({}^{210}_{84}\text{Po}\right) = 209.982876\text{u}$

$m\left({}^{206}_{82}\text{Pb}\right) = 205.974455\text{u}$ and $1\text{u} = 932$ MeV

15. The correct statement is.

(a) The nucleus ${}^{6}_{3}\text{Li}$ can emit an alpha particle

(b) The nucleus ${}^{210}_{84}\text{Po}$ can emit a proton

(c) Deuteron and alpha particle can undergo complete fusion

(d) The nuclei ${}^{70}_{30}\text{Zn}$ and ${}^{82}_{34}\text{Se}$ can undergo complete fusion

16. The kinetic energy (in keV) of the alpha particle, when the nucleus ${}^{210}_{84}\text{Po}$ at rest undergoes alph decay, is

(a) 5319 (b) 5422

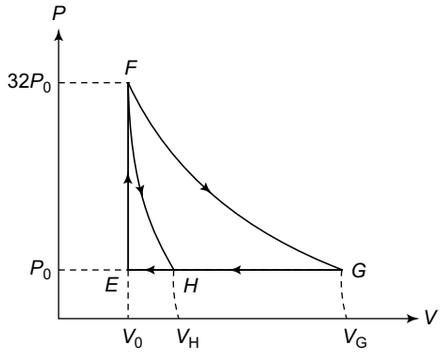
(c) 5707 (d) 5818

SECTION-3

(Matching List Type)

This section contains **4 multiple choice questions. Each question has matching lists.** The codes for the lists have choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

17. One mole of a monatomic ideal gas is taken along two cyclic processes $E \to F \to G \to E$ and $E \to F \to H \to E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



List I		List	
(P)	$G \to E$	**(1)**	$160\ P_0\ V_0 \ln 2$
(Q)	$G \to H$	**(2)**	$36\ P_0\ V_0$
(R)	$F \to H$	**(3)**	$24\ P_0\ V_0$
(S)	$F \to G$	**(4)**	$31\ P_0\ V_0$

Code:

	P	Q	R	S
(a)	4	3	2	1
(b)	4	3	1	2
(c)	3	1	2	4
(d)	1	3	2	4

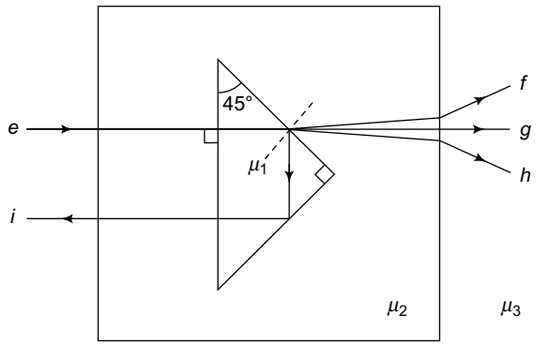
18. Match List-I of the nuclear processes with List-II containing parent nucleus and one of the end products of each process and then the correct answer using the codes given below the lists:

List-I		List-II	
(P)	Alpha decay	**(1)**	${}^{15}_{8}\text{O} \to {}^{15}_{7}\text{N} + ...$
(Q)	β^+ decay	**(2)**	${}^{238}_{92}\text{U} \to {}^{234}_{90}\text{Th} + ...$
(R)	Fission	**(3)**	${}^{185}_{83}\text{Bi} \to {}^{184}_{82}\text{Pb} + ...$
(S)	Proton emission	**(4)**	${}^{239}_{94}\text{Pu} \to {}^{140}_{57}\text{La} + ...$

Code:

	P	Q	R	S
(a)	4	2	1	3
(b)	1	3	2	4
(c)	2	1	4	3
(d)	4	3	2	1

19. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light '*e*' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths '*ef*', '*eg*', '*eh*' or '*ei*'.



Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists:

List I	List
[P] $e \to f$	**(1)** $\mu_1 > \sqrt{2}\,\mu_2$
[Q] $e \to g$	**(2)** $\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
[R] $e \to h$	**(3)** $\mu_1 = \mu_2$
[S] $e \to i$	**(4)** $\mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$

Codes:

	P	Q	R	S
(a)	2	3	1	4
(b)	1	2	4	3
(c)	4	1	2	3
(d)	2	3	4	1

20. Match List I and List II and select the correct answer using the codes given below the lists:

List I	List II
[P] Boltzmann constant	**(1)** $[ML^2T^{-1}]$
(Q) Coefficient of viscosity	**(2)** $[ML^{-1}\,T^{-1}]$
[R] Planck constant	**(3)** $[MLT^{-3}\,K^{-1}]$
[S] Thermal conductivity	**(4)** $[ML^2\,T^{-3}\,K^{-1}]$

Codes:

	P	Q	R	S
(a)	3	1	2	4
(b)	3	2	1	4
(c)	4	2	1	3
(d)	4	1	2	3

Answers

Section-1

1. (a, b, d) **2.** (a, c) **3.** (d)
4. (c, d) **5.** (a, d) **6.** (a, b)
7. (b, d) **8.** (a, d)

Section-2

9. (b) **10.** (a) **11.** (b)
12. (a) **13.** (a) **14.** (b)
15. (c) **16.** (A)

Section-3

17. (a) **18.** (c) **19.** (d)
20. (c)

Hints and Solutions

1. The heat energy ΔQ absorbed by a solid of mass m when its temperature is increased by ΔT is given by

$$\Delta Q = mC\Delta T$$

The rate at which heat is absorbed is given by

$$\frac{\Delta Q}{\Delta t} = mC\frac{\Delta T}{dt}$$

It is given that $\frac{\Delta T}{dt}$ is constant. In the range 0 to 100 K, C increases linearly with T. so choice (a) is correct. The slope of $C - T$ graph is higher in the range 0 – 100K than in the range 200 to 300 K. So choices (b) and (d) are also correct. In the range 400–500 K, C increases slightly will T. So choice (c) is only approximately correct.

2. Bohr's quantization condition is

$$L = \frac{nh}{2\pi}$$

It given that $L = \frac{3h}{2\pi}$. Hence $n = 3$.

Now $\quad \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$

Radius of n th orbit is given by

$$r_n = a_0 \times \frac{n^2}{Z^2}$$

Given $\quad r_n = 4.5\ a_0$. Therefore

$$4.5\ a_0 = a_0 \times \frac{3^2}{z^2} \qquad \Rightarrow Z = 2$$

$$\therefore \quad \frac{1}{\lambda} = 4R\left(\frac{1}{n^2} - \frac{1}{3^2}\right)$$

The possible wavelengths correspond to $n = 1$ and $n = 2$

$$\therefore \quad \frac{1}{\lambda_1} = 4R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) \Rightarrow \lambda_1 = \frac{9}{32R}$$

and $\quad \frac{1}{\lambda_2} = 4R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda_2 = \frac{9}{5R}$

3. $2d \sin\theta = \lambda$

$$\Rightarrow \quad d = \frac{\lambda}{2\sin\theta} \qquad (1)$$

Differentiating (i) w.r.t θ,

$$\frac{\Delta d}{\Delta\theta} = \frac{\lambda}{2}(-\text{cosec}\,\theta\ \cot\theta) = -\frac{\lambda\cot\theta}{\sin\theta}$$

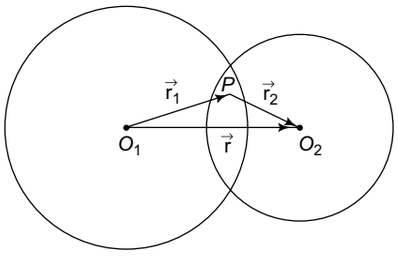
$$\Rightarrow \quad \Delta d = -\frac{\lambda\cot\theta}{\sin\theta}\Delta\theta \qquad (2)$$

Dividing (2) by (1)

$$\frac{\Delta d}{d} = -(\cot\theta)\ \Delta\theta$$

Since $\Delta\theta$ is given to be constant, the fractional error $\frac{\Delta d}{d}$ decreases.

4. Charge on sphere of radius R_1 is



$$Q = \frac{4\pi}{3} R_1^3 \rho$$

Charge on sphere of radius R_2 is

$$Q^1 = -\frac{4\pi}{3} R_2^3 \rho$$

$\therefore$ Electric field at point P due to charge Q is

$$\vec{E}_\rho = \frac{Q\ \vec{r}_1}{4\pi\varepsilon_0\ R_1^3} = \frac{\rho\,\vec{r}_1}{3\varepsilon_0} \text{ directed from } O_1 \text{ to } P$$

Electric field at ρ due to Q^1 is ($\because$ Q^1 is negative)

$$\vec{E}_p^1 = \frac{\rho\,\vec{r}_2}{3\,\varepsilon_0} \text{ directed from } P \text{ to } O_2.$$

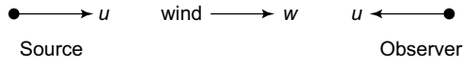
$\therefore$ Net electric field at P is

$$\vec{E} = \vec{E}_p + \vec{E}_p^1 = \frac{p}{3\varepsilon_0}\left(\vec{r}_1 + \vec{r}_2\right) = \frac{\rho\vec{r}}{3\varepsilon_0} \text{ (Triangle Law)}$$

Since $\vec{r} = \overrightarrow{O_1 O_2}$ is constant, $\vec{E}$ is constant in magnitude and direction.

5. From Ampere's Circuital law, it follow's that choices (a) and (d) are correct

6.



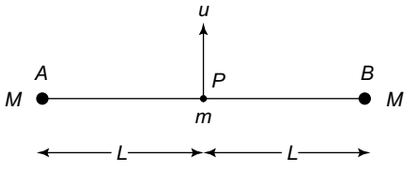
If the wind blows from source to observer (as shown in the figure), the apparent frequency is

$$f_2 = \left[\frac{(V+w)-(-u)}{(V+w)-(+u)}\right] f_1 = \left(\frac{V+w+u}{V+w-u}\right) f_1 \Rightarrow f_2 > f_1$$

If the wind blows from observer to sound,

$$f_2 = \left[\frac{(V-w)-(-u)}{V-w-(+u)}\right] f_1 = \left(\frac{V-w+u}{V-w-u}\right) f_1 \Rightarrow f_2 > f_1$$

7.



Minimum K.E. imparted to mass m at P to escape to infinity is

$$\text{K.E.} = \frac{1}{2}mu^2 \qquad (u = \text{minimum initial velocity of } m)$$

Total P.E. of the system of masses at A, P and B is

$$\text{P.E.} = -\frac{GmM}{L} - \frac{GmM}{L} - \frac{GM^2}{(2L)} = -\frac{2GmM}{L} - \frac{GM^2}{2L}$$

As mass m moves up, the P.E. of the masses at A and B which is $-GM^2/2L$ remains constant but P.E. between masses at A and P and B and P which is $-2GMm/L$ keeps increasing. From energy conservation,

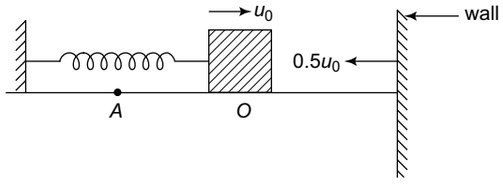
Loss fo K.E. = gain of P.E.

i.e. $$\frac{1}{2}mu^2 = \frac{2GMm}{L}$$

$$u = 2\sqrt{\frac{GM}{L}}$$

From the law of conservation of energy, the total energy of the system of three masses always remains constant.

8.



Since the particle passes through the mean position O at $t = 0$,

$$x = a \sin(\omega t) \qquad (1)$$

velocity is $V = \frac{dx}{dt} = A\omega \cos(\omega t) = u_0 \cos(\omega t)$ (2)

where $\omega = \sqrt{\frac{k}{m}} \Rightarrow$ Time period $T = 2\pi\sqrt{\frac{m}{k}}$

Putting $V = 0.5u_0$ in Eq. (2), the particle hits the wall at time t_1 given by

$$0.5u_0 = u_0 \cos(\omega t_1)$$

$$\cos\omega t_1 = \frac{1}{2} \Rightarrow \omega t_1 = \frac{\pi}{3} \qquad \Rightarrow t_1 = \frac{\pi}{3}\sqrt{\frac{m}{k}}$$

The time at which the particle passes the equilibrium position O the first time (moving to the left) is given by

$$t_2 = 2t_1 = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$$

The time at which the spring has maximum compression and reaches extreme position moving to the left in is

$$t_3 = t_2 + \frac{T}{4}$$

$$= \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{2\pi}{4}\sqrt{\frac{m}{k}} = \frac{7\pi}{6}\sqrt{\frac{m}{k}}$$

The time at which the particle passes the equilibrium position O the second time moving to the right is given by

$$t_4 = t_2 + \frac{T}{2}$$

$$= \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{k}} = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

Hence the correct choices are (a) and (d).

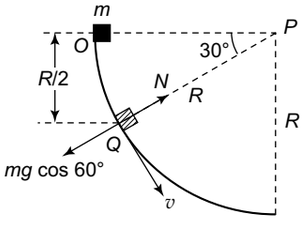
9. As the block slides from O to Q,

Loss in P.E. = gain in K.E. − work done against friction

$$\Rightarrow \quad mg \times \frac{R}{2} = \frac{1}{2}mv^2 + 150$$

$$\Rightarrow \quad 1 \times 10 \times \frac{40}{2} = \frac{1}{2} \times 1 \times v^2 + 150$$

$$\Rightarrow \quad v = 10 \text{ ms}^{-1}$$



10. If N is the normal reaction at Q, then

$$\Rightarrow \quad \frac{mv^2}{R} = N - mg \cos 60°$$

$$\Rightarrow \quad \frac{1 \times (10)^2}{40} = N - 1 \times 10 \times \frac{1}{2}$$

$$\Rightarrow \quad N = 7.5 \text{ N}$$

11. Resistance of cable is $R = 0.4\Omega \text{ km}^{-1} \times 20 \text{ km} = 8\ \Omega$
Power produced $P = VI$ which gives

$$I = \frac{P}{V} = \frac{600 \times 10^3}{4000} = 150\text{A}$$

Power dissipated is $P' = I^2R = (150)^2 \times 8 = 180$ kW

Percentage power dissipation

$$= \frac{p'}{p} \times 100 = \frac{180 \text{ kW}}{600 \text{ kW}} \times 100 = 30\%$$

12. $\frac{V_p}{V_s} = \frac{N_p}{N_s}$

At step-up stage : $\frac{4000}{V_p} = \frac{1}{10} \quad \Rightarrow V_p = 40000\,\text{V}$

At step-down stage: $V_s = 200$V. Hence

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{40000}{200} = 200$$

13. Magnetic flux $\phi = \int \vec{B}.\vec{ds} = B \times \pi R^2$

$$\therefore \quad \left|\frac{d\phi}{dt}\right| = \pi R^2 \frac{dB}{dt} = \pi R^2 \times \frac{B-0}{1} = \pi R^2 B$$

If E is the magnitude of induced electric field,

$$\int E \cdot \vec{dl} = \left|\frac{d\phi}{dt}\right| = \pi R^2 B$$

$$E \times 2\pi R = \pi R^2 B$$

$$E = \frac{BR}{2}$$

14. Given, magnetic moment = $\gamma \times$ angular momentum.

i.e. $M = \gamma L \quad ; \quad \gamma =$ constant

Change in magnetic moment is

$$\Delta M = \gamma \Delta L \qquad (1)$$

Now, torque $\tau = \frac{\Delta L}{\Delta t} \quad \Rightarrow \quad \Delta L = \tau \Delta t$

Also $\tau = RF = RQE \qquad (\because F = QE)$

$$\therefore \quad \Delta L = RQE\ \Delta t \qquad (2)$$

Using (1) in (2)

$$\Delta M = \gamma R Q E \Delta t$$

Putting $E = \frac{BR}{2}$ (see Q. 13 above) and $\Delta t = 1$s,

$$\Delta M = \gamma\, R\ Q \times \frac{BR}{2} \times 1 = \gamma\left(\frac{Q B R^2}{2}\right)$$

Since angular momentum increases in the negative z-direction,

$$\Delta M = -\gamma\left(\frac{Q B R^2}{2}\right)$$

15. When ^6_3Li emits an alpha particle, the reaction is represented as

$$^6_3\text{Li} \rightarrow ^4_2\text{He} + ^2_1\text{H}$$

Total mass of $^4_2\text{He} + ^2_1\text{H}$ = 4.002603 + 2.014102 = 6.016705u which is more than the mass of ^6_3Li.

Hence there is no mass defect. So choice (a) is not possible. Similarly the reaction mentioned in (b) is not possible. Also fusion is not possible by fusing heavy nuclei such as $^{70}_{30}\text{Zn}$ and $^{82}_{34}\text{Se}$. So choice (d) is also not possible. Hence the only correct choice is (b).

16. The reaction is represented as

$$^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He} + Q$$

Mass defects Δm = 209.982876 – (205.974455 + 4.002603 = 0.005818 u

$\therefore$ Energy released $Q = 0.005818 \times 932$

$= 0.5422$ MeV

From momentum conservation

$$|P_\alpha| = |P_{\text{Po}}|$$

$$\sqrt{2m_\alpha K_\alpha} = \sqrt{2m_{\text{Po}} K_{\text{Po}}}$$

$$m_\alpha K_\alpha = m_{\text{Po}} K_{\text{Po}} \qquad (2)$$

From (1) and (2) we have

$$K_a = \frac{m_{\text{Po}} Q}{m_{\text{Po}} + m_\alpha} = \frac{206 \times 542.2\ \text{keV}}{206 + 4} \simeq 5319\,\text{keV}$$

17. Process $G \rightarrow E$ is isobaric. Process $G \rightarrow H$ is also isobaric. Process $F \rightarrow E$ is isochoric. Process $F \rightarrow H$ has a higher slope at any point than process $F \rightarrow G$ at the corresponding point. Hence process $F \rightarrow H$ is adiabatic and process $F \rightarrow G$ is isothermal.

(1) *For item (P) in List I*

For isobaric process $G \rightarrow E$

$$W_{GE} = -P_0 (V_G - V_E) = -P_0 (V_{GE} - V_O) \quad \text{____(i)}$$

For isothermal process $F \rightarrow G$,

$$P_F V_F = P_G V_G$$

$$\Rightarrow \quad 32\ P_o V_o = P_o V_G$$

$$\Rightarrow \quad V_G = 32\ V_o \quad \text{____(ii)}$$

Using (ii) in (i),

$$\therefore \quad W_{GE} = -P_o (32\ V_o - V_o) = -31\ P_o V_o$$

So item (P) in List I corresponds to item (4) in List II.

(2) *For Item (Q) in List I*

$$W_{GH} = -P_o (V_G - V_H) = -P_o (32\ V_o - V_H) \quad \text{____(iii)}$$

Since process $F \rightarrow H$ is adiabatic,

$$PV^\gamma = \text{constant}$$

$$P_F V_o^\gamma = P_o V_H^\gamma$$

$$\Rightarrow 32\ P_o V_o^\gamma = P_o V_H^\gamma$$

$$\left(\frac{V_o}{V_H}\right)^\gamma = \frac{1}{32}$$

Now for a monoatomic gas, g = $\frac{5}{3}$. Therefore

$$\left(\frac{V_o}{V_H}\right)^{5/3} = \frac{1}{32} = \frac{1}{2^5}$$

$$\Rightarrow \quad V_H = 2^3 V_o = 8V_o \quad \text{____(iv)}$$

Using (iv) in (iii),

$$W_{GH} = -P_o(32V_o - 8V_o) = -24P_o V_o$$

So (Q) corresponds to (3).

(3) *For item (R) in List I*

Process $F \rightarrow H$ adiabatic. Therefore,

$$W_{FH} = \frac{f}{2} nR (T_F - T_H)$$

(f = no. of degrees of freedom)

$$= \frac{3}{2} nR (T_F - T_H)$$

$$= \frac{3}{2}(P_F V_F - P_H V_H) \quad (\because PV = nRT)$$

$$= \frac{3}{2}(32\ P_o V_o - 8P_o V_o)$$

$$= 36P_o V_o$$

So (R) corresponds to (2).

(4) *For item (S) in List I*

Process $F \rightarrow G$ is isothermal. Therefore,

$$W_{FG} = PV \ln\left(\frac{V_2}{V_1}\right)$$

$$= 32\ P_o V_o \ln\left(\frac{V_G}{V_F}\right)$$

$$= 32\ P_o V_o \ln\left(\frac{32V_o}{V_o}\right)$$

$$= 32\ P_o V_o \ln(32)$$

$$= 160\ P_o V_o \ln(2)$$

So (S) corresponds to (1).

Hence the answer is as follows:

(P) $\rightarrow$ (4)
(Q) $\rightarrow$ (3)
(R) $\rightarrow$ (2)
(S) $\rightarrow$ (1)

18. (P) : Alpha decay. In alpha decay the atomic number (Z) decreases by 2 and the mass number (A) decreases by 4.

So process (2) in List II represents alpha decay:

$$^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + ^{4}_{2}\text{He}$$

(Q) : β^+ decay. In β^+ decay, a proton decays into a positron $\left(^{0}_{+1}\text{e}\right)$ called β^+ particle with the emission of a neutrino (ν) which carries no charge. Hence in β^+ decay, the atomic number decreases by 1 and mass number remains unchanged. So process (1) in List II represents β^+ decay:

$$^{15}_{8}\text{O} \rightarrow ^{15}_{7}\text{N} + ^{0}_{+1}\text{e} + \nu$$

(R) : In nuclear fission, a heavy unstable nucleus splits into two nuclei of comparable masses, Process (4) in List II represents fission:

$$^{239}_{94}\text{Pu} \rightarrow ^{140}_{57}\text{La} + ^{99}_{37}\text{X}$$

(S) : In proton emission, the atomic number and mass number both decrease by 1. Process (3) in List II represents proton emission:

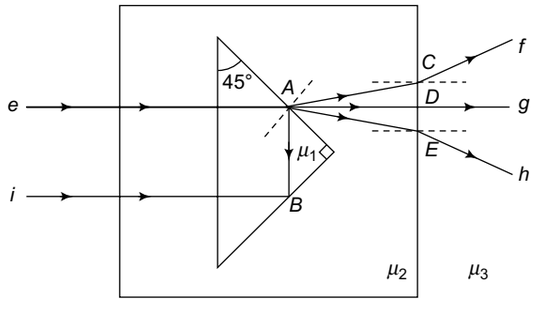
$$^{185}_{83}\text{Bi} \rightarrow ^{184}_{82}\text{Pb} + ^{1}_{1}\text{p}$$

So the correct answer is as follows:

(P) → (2), (Q) → (1), (R) → (4), (S) → (3).

19.

(p) *For ray eACf* : At A the ray bends towards the normal. Hence $\mu_2 > \mu_1$ (see figure). At C, ray AC bends away from the normal. Hence $\mu_3 < \mu_2$. So the correct choice is (2) is List II.



(Q) *For ray eADg* : This ray goes through without bending at A. Hence $\mu_1 = \mu_2$ so the correct choice is (3) in List II.

(R) *For ray eAEh* : At A this ray bends away from the normal.

Hence $\mu_2 < \mu_1$. At E, it bends away from the normal.

Hence $\mu_3 < \mu_2$. Applying Snell's law at A,

$\mu_1 \sin 45° = \mu_2 \sin r$

$$\frac{\mu_1}{\sqrt{2}} = \mu_2 \sin r \quad \Rightarrow \quad \mu_1 = \sqrt{2}\,\mu_2 \sin r$$

Since $\sin r < 1$, $\mu_1 < \sqrt{2}\ \mu_2$. So, for ray $eAEh$, the correct choice is (4) in List II.

(S) *For ray eABi*: This ray suffers total internal reflections at A and B. The critical angle $i_c = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

Also $\sin i_c = \frac{\mu_2}{\mu_1}$. Therefore

$\sin -1\left(\frac{\mu_2}{\mu_1}\right) < 45° \Rightarrow \mu_1 > \sqrt{2}\,\mu_2$. So the correction choice is (1) in List II. The correct answer is as follows:

(P) → (2), (Q) → (3), (R) → (4), (S) → (1).

20.

(P) According to the law of equipartition of energy, the energy per degree of freedom of a gas atom or molecule at a temperature θ kelvin is given by

$$E = \frac{1}{2}k\theta \quad or\ k = \frac{2E}{\theta}$$

where k is the Boltzmann's constant.

$$\therefore \quad \text{Dimension of } k = \frac{\text{dimensions of } E}{\text{dimension of } \theta}$$

$$= \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{K}]} = [\text{ML}^2\text{T}^{-2}\text{K}^{-1}]$$

(Q) The viscous force acting on a spherical body of radius r moving with a speed v in a fluid of coefficient of viscosity η is given by

$$F = 6\pi\,\eta\,r\,v \text{ or } \eta = \frac{F}{6\pi r v}$$

$$\therefore \text{ Dimensions of } \eta = \frac{\text{dimension of } F}{\text{dimension of } r \times \text{dimension of } v}$$

$$= \frac{[\text{MLT}^{-2}]}{[\text{L} \times \text{LT}^{-1}]} = [\text{ML}^{-1}\text{T}^{-1}]$$

(R) $E = h\nu$. Therefore

$$[h] = \frac{[E]}{\nu} = \frac{[\text{ML}^2\text{T}^{-2}]}{\text{T}^{-1}} = [\text{ML}^2\text{T}^{-1}]$$

(S) The rate of flow of heat energy $\frac{Q}{t}$ through a rod of cross-sectional area A, length L with its ends maintained at temperatures θ_1 and θ_2 (with $\theta_1 > \theta_2$) is given by

$$\frac{Q}{t} = \frac{kA(\theta_1 - \theta_2)}{L}; \qquad k = \text{thermal conductivity.}$$

$$\frac{[\text{ML}^2\text{T}^{-2}]}{[\text{T}]} = \frac{[k] \times [\text{L}^2] \times [\text{K}]}{[\text{L}]} \Rightarrow [k] = [\text{MLT}^{-3}\text{K}^{-1}]$$

PHYSICS JEE ADVANCED – 2014
PAPER-I – MODEL SOLUTIONS

SECTION I

(One or More than One Options Correct Type)

This section contains **10 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE** are correct.

1. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005)m, the gas in the tube is (**Useful information:** $\sqrt{167RT}$ = 640 $J^{1/2}$ $mole^{-1/2}$; $\sqrt{140RT}$ = 590 $J^{1/2}$ $mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

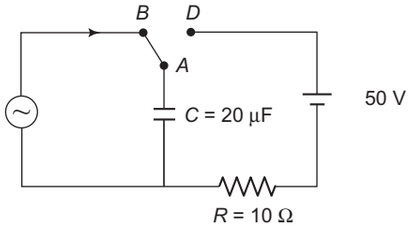
(a) Neon $\left(M = 20,\ \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(b) Nitrogen $\left(M = 28,\ \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(c) Oxygen $\left(M = 32,\ \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(d) Argon $\left(M = 36,\ \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

2. At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with I_0 = 1A and ω = 500 rad/s starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20 μF, R = 10Ω and the battery is ideal with emf of 50 V, identify the correct statement(s).



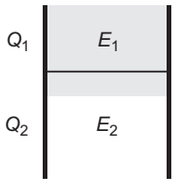
(a) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C.

(b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(c) Immediately after A is connected to D, the current in R is 10 A.

(d) $Q = 2 \times 10^{-3}$ C.

3. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1, and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.



(a) $\frac{E_1}{E_2} = 1$ (b) $\frac{E_1}{E_2} = \frac{1}{K}$

(c) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (d) $\frac{C}{C_1} = \frac{2+K}{K}$

4. One end of a taut string of length 3m along the x axis is fixed at $x = 0$. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are)

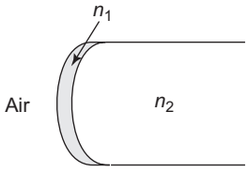
(a) $y(t) = A \sin \frac{\pi x}{6} \cos \frac{50\pi t}{3}$

(b) $y(t) = A \sin \frac{\pi x}{3} \cos \frac{100\pi t}{3}$

(c) $y(t) = A \sin \frac{5\pi x}{6} \cos \frac{250\pi t}{3}$

(d) $y(t) = A \sin \frac{5\pi x}{2} \cos 250\pi t$

5. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index. $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then

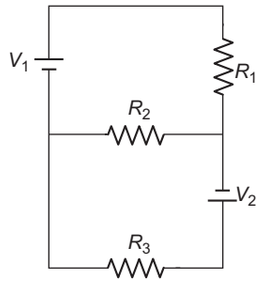


(a) $|f_1| = 3R$ (b) $|f_1| = 2.8R$
(c) $|f_2| = 2R$ (d) $|f_2| = 1.4R$

6. Heater of an electric kettle is made of a wire of length L and diameter d. It takes, 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter $2d$. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40K ?

(a) 4 if wires are in parallel
(b) 2 if wires are in series
(c) 1 if wires are in series
(d) 0.5 if wires are in parallel.

7. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if



(a) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(b) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(c) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(d) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

8. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(a) $Q = 4\sigma\pi r_0^2$

(b) $r_0 = \frac{\lambda}{2\pi\sigma}$

(c) $E_1(r_0/2) = 2E_3(r_0/2)$

(d) $E_2(r_0/2) = 4E_3(r_0/2)$

9. A light source, which emits two wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

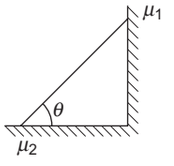
(a) $\beta_2 > \beta_1$

(a) $m_1 > m_2$

(c) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1.

(d) The angular separation of fringes for λ_1, is greater than λ_2

10. In the figure a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



(a) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$

(b) $\mu_1 \neq 0$ $\mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

(c) $\mu_1 \neq 0$ $\mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$

(d) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

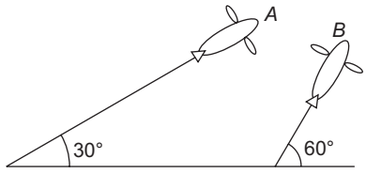
SECTION II

(One Integer Value Correct Type)

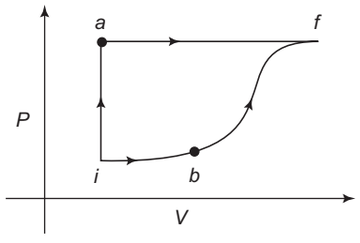
This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

11. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is

12. Airplanes A and B are flying with constant velocity in the same vertical plane at. angles 30° and 60° with respect to the horizontal respectively as shown in the figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time $t = 0$ an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is



13. A thermodynamic system is taken from an initial state i with internal energy $U_1 = 100$ J to the final state f along two different paths iaf and ibf as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J, the ratio $\boldsymbol{Q}_{bf}/\boldsymbol{Q}_{ib}$ is

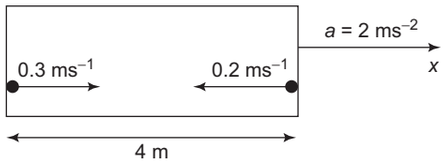


14. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$ the value of $\frac{R_1}{R_2}$ is

15. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$.The value of n is

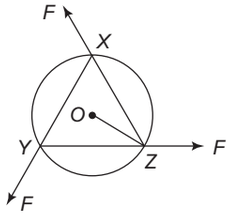
16. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along $+x$ direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in $-x$ direction with

a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is

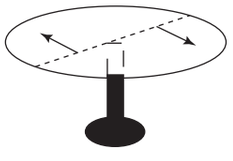


17. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 – 30 V. If connected to a $\frac{2n}{249}\,\Omega$ resistance, it becomes an ammeter of range 0 – 1.5 A. The Value of n is

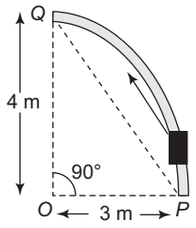
18. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is



19. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is



20. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is ($n \times 10$) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})



Answers

Section-I

1. (d)	**2.** (c, d)	**3.** (a, d)
4. (a, c, d)	**5.** (a, c)	**6.** (b, d)
7. (a, b, d)	**8.** (c)	**9.** (a, b, c)
10. (c, d)		

Section-II

11. 4	**12.** 5	**13.** 2
14. 3	**15.** 3	**16.** 2
17. 5	**18.** 2	**19.** 4
20. 5		

Hints and Solutions

Section-I

1. Speed of sound $v = \sqrt{\frac{\gamma RT}{M}}$. For a closed pipe, $\lambda = 4L$ (fundamental mode). Also $v = \nu\lambda$. Thus

$$v = \nu \times 4L$$

$$\Rightarrow \quad \sqrt{\frac{\gamma RT}{M}} = \nu \times 4L$$

$$\Rightarrow \quad L = \frac{1}{4\nu}\sqrt{\frac{\gamma RT}{M}} \qquad (1)$$

For Neon: $M = 20\,g = 20 \times 10^{-3}\,kg = 2 \times 10^{-2}\,kg$. Neon is a monatomic gas for which $\gamma = 1.67$. Substituting the given values in (1), we have [since $\sqrt{167RT} = 640\ J^{1/2}\ mol^{-1/2}$ and $\sqrt{\frac{10}{20}} = \sqrt{\frac{1}{2}} = \frac{7}{10}$]

$$L = \frac{1}{4\times 244}\sqrt{\frac{167RT}{2\times 10^{-2}}}$$

$$= \frac{1}{4\times 244}\sqrt{\frac{167RT}{2}}$$

$$= \frac{1}{4\times 244}\times 640\times\frac{7}{10}$$

$$= 0.459 \text{ m}$$

For Nitrogen: $M = 28\,\text{g} = 2.8 \times 10^{-2}$ kg. Since nitrogen is a diatomic gas, $\gamma = 1.4$. Given $\sqrt{140RT}$ = 590 $\text{J}^{1/2}$ $\text{mol}^{-1/2}$ and $\sqrt{\frac{10}{28}} = \frac{3}{5}$.

$$\therefore \quad L = \frac{1}{4\times 244}\sqrt{\frac{1.4RT}{2.8\times 10^{-2}}}$$

$$= \frac{1}{4\times 244}\times\sqrt{\frac{140RT\times 10}{28}}$$

$$= \frac{1}{4\times 244}\times 590\times\frac{3}{5}$$

$$= 0.363 \text{ m}$$

Similarly, for oxygen ($\gamma = 1.4$), we get $L = 0.340$ m and for argon ($\gamma = 1.67$), we get $L = 0.348$ m. It is given that $L = (0.350 \pm 0.005)$ m. Hence correct choice is (d)

2. Given $I = I_0 \cos(\omega t)$ where $I_0 = 1$A and $\omega = 500$ rad s^{-1}. Since the voltage lags behind the current, the voltage across the capacitor varies with t as

$$V = V_0 \cos\left(\omega t - \frac{\pi}{2}\right) = V_0 \sin \omega t$$

Now $\quad \frac{dq}{dt} = I_0 \cos(\omega t)$

$$\Rightarrow \quad dq = I_0 \cos(\omega t)dt$$

$$\Rightarrow \quad \int dq = I_0 \int \cos(\omega t)dt$$

$$\Rightarrow \quad q = \frac{I_0}{\omega}\sin(\omega t)$$

$$\Rightarrow \quad q = Q \sin \omega t, \text{ where } Q = \frac{I_0}{\omega}$$

$\therefore$ Magnitude of maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ (i.e., before A is connected to D) is

$$Q = \frac{I_0}{\omega} = \frac{1\text{A}}{500\,\text{s}^{-1}} = 2 \times 10^{-3} \text{ C}$$

So choice (a) is incorrect.

Current in the left past of the circuit just before $t = \frac{7\pi}{6\omega}$ is

$$I = I_0 \cos\left(\omega\times\frac{7\pi}{6\omega}\right)$$

$$= 1\times\cos\left(\frac{7\pi}{6}\right)$$

$$= -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}\text{A}$$

The negative sign shows that the current is anti-clockwise. So choice (b) is also wrong.

After A and D are connected, the battery of voltage $V = 50$V is in parallel with the capacitor, So the voltage V_C across the capacitor is also 50 V. Hence the current in the circuit is

$$I = \frac{V_c + V}{R} = \frac{50+50}{10} = 10\text{A}$$

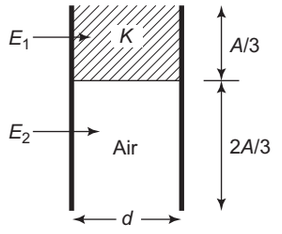
So choice (c) is correct.

Maximum charge $Q = CV_C = (20 \times 10^{-6}) \times 50$

$$= 2 \times 10^{-3}\text{C}$$

So choice (d) is also correct.

3. The capacitor is charged by connecting its plates to a battery of voltage, say, V. Since the battery is kept connected, the voltage between the plates is equal to V. Now $E = \frac{V}{d}$. Since d is not changed, $E_1 = E_2 = \frac{V}{d}$.



$$\therefore \quad \frac{E_1}{E_2} = 1$$

Capacitance of dielectric filled part is

$$C_1 = \frac{K\in_0 \frac{A}{3}}{d} = \frac{K\in_0 A}{3d}$$

Capacitance of the air filled part is

$$C_2 = \frac{\in_0 \frac{2A}{3}}{d} = \frac{2}{3}\frac{\in_0 A}{d}$$

Since the two capacitors are in parallel, the total capacitance is

$$C = C_1 + C_2$$

$$= C_1\left(1 + \frac{C_2}{C_1}\right)$$

$$= C_1\left(1 + \frac{\frac{2}{3}\in_0 \frac{A}{d}}{K\in_0 \frac{A}{3d}}\right)$$

$$= C_1\left(1+\frac{2}{K}\right)$$

$$\Rightarrow \quad \frac{C}{C_1} = \frac{K+2}{K}$$

Now $Q_1 = C_1V, \quad Q_2 = C_2V$

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{K}{2}$$

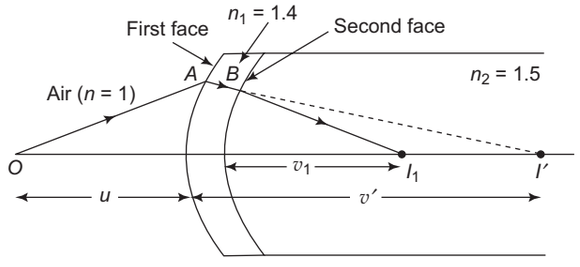
So the correct choices are (a) and (d).

4. Since the end at $x = 0$ of the string is fixed, it is a node, i.e. $y(t) = 0$ at $x = 0$. This condition is satisfied in all the four choices given in the question. The other end at $x = 3$m is free, it is an antinode, i.e. $y(t) \neq 0$ at $x = 3$ m. This condition is satisfied in choices (a), (c) and (d) but not in choice (b) for which $y(t) = 0$ at $x = 3$ m.

Hence the correct choices are (a), (c) and (d).

5. The incident ray suffers two refractions — one at each face.

Case (a) For refraction from air to glass



The ray OA from an object O, refracts along AB forming the image I'. For refraction at first face, we have

$$\frac{n_1}{v'} - \frac{n}{u} = \frac{n_1 - n}{R} \qquad (1)$$

The ray AB suffers refraction at the second face, forming the final image I_1. For refraction at second face, I' serves at the virtual object. Thus, we have

$$\frac{n_2}{v_1} - \frac{n_1}{v'} = \frac{n_2 - n_1}{R} \qquad (2)$$

Adding (1) and (2), we get

$$\frac{n_2}{v_1} - \frac{n}{u} = \frac{n_1 - n}{R} + \frac{n_2 - n_1}{R}$$

By difinition $v_1 = f_1$ when $u = \infty$. Therefore

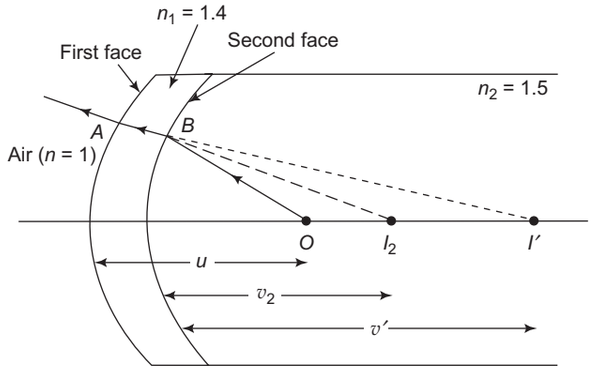
$$\frac{n_2}{f_1} = \frac{n_1 - n}{R} + \frac{n_2 - n_1}{R}$$

$$\Rightarrow \quad \frac{1.5}{f_1} = \frac{1.4-1}{R} + \frac{1.5-1.4}{R}$$

$$\Rightarrow \quad f_1 = 3R$$

According to sign convention, f_1 and R are both positive.

Case (b) For refraction from glass to air



For refraction second face, we have

$$\frac{n_1}{v'} - \frac{n_2}{u} = \frac{n_1 - n_2}{R} \qquad (3)$$

For refraction at first face, we have

$$\frac{n}{v_2} - \frac{n_1}{v'} = \frac{n - n_1}{R} \qquad (4)$$

Adding (3) and (4), we get

$$\frac{n}{v_2} - \frac{n_2}{u} = \frac{n_1 - n_2}{R} + \frac{n - n_1}{R}$$

Now $v_2 = -f_2$ when $u = -\infty$. Also $R = -R$. Therefore

$$\frac{1}{-f_2} = \frac{1.4-1.5}{-R} + \frac{1-1.4}{-R}$$

$$\Rightarrow \quad f_2 = -2R$$

$$\therefore \quad |f_2| = 2R$$

Hence the correct choices are (a) and (c).

6. Let R be the resistance of the wire of the kettle and let V be the voltage of the supply. The heat energy consumed in time t is

$$H = \frac{V^2 t}{R} \qquad (1)$$

If the heater is replaced by a new heater having two wires of the same material, the same length but twice the diameter, the resistance of each wire becomes $\frac{R}{4}$.

If the two wires are connected in series, the total resistance of the heater is $R_1 = \frac{R}{4} + \frac{R}{4} = \frac{R}{2}$. If t_1 is the time required to consume the same heat energy, then

$$H = \frac{V^2 t_1}{R_1} = \frac{V^2 t_1}{R/2} \qquad (2)$$

Equating (1) and (2),

$$\frac{V^2 t}{R} = \frac{V^2 t_1}{R/2}$$

$$\Rightarrow \qquad t_1 = t \times \frac{1}{2} = \frac{4\,\text{min}}{2} = 2\,\text{min}$$

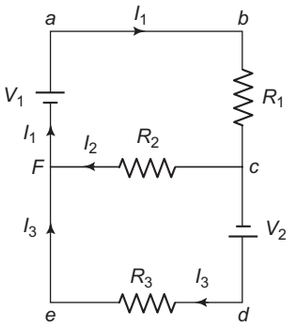
So choice (b) is correct.

If the two wires are connected in parallel, the total resistance $R_2 = \frac{R}{8}$. If t_2 is the required time,

$$\frac{V^2 t}{R} = \frac{V^2 t_2}{R/8}$$

$$\Rightarrow \qquad t_2 = \frac{t}{8} = \frac{4\,\text{min}}{8} = 0.5\,\text{min}$$, which is choice (d). So the correct choices are (b) and (d).

7. The following figure shows the currents I_1, I_2 and I_3 in the three branches of the circuit.



Using the junction rule, $I_2 + I_3 = I_1$. If $I_2 = 0$, then $I_1 = I_3$.

Applying the loop rule to loop *abcfa*, we have

$I_1 R_1 + I_2 R_2 - V_1 = 0$

Setting $I_2 = 0$, we get

$$V_1 = I_1 R_1 \qquad (1)$$

Applying the loop rule to loop *fcdef*, we get

$-I_2 R_2 - V_2 + I_3 R_3 = 0$

Putting $I_2 = 0$, we get

$$V_2 = I_3 R_3$$

But $I_3 = I_1$. Therefore

$$V_2 = I_1 R_3 \qquad (2)$$

From (1) and (2), we get

$$\frac{V_1}{V_2} = \frac{R_1}{R_3} \qquad (3)$$

So no current will flow through R_2 if condition (3) holds. This condition is does not contion R_2, i.e. R_2 can have any value. Choices (a), (b) and (d) satisfy condition (3) but choice (c) does not.

8. $E_1 (r_0) = \dfrac{Q}{4\pi \in_0 r_0^2}$

$E_2 (r_0) = \dfrac{\lambda}{2\pi \in_0 r_0}$

$E_3 (r_0) = \dfrac{\sigma}{2 \in_0}$

At $r = \dfrac{r_0}{2}$,

$$E_1\left(\frac{r_0}{2}\right) = \frac{Q}{4\pi \in_0 \left(\frac{r_0}{2}\right)^2} = \frac{Q}{\pi \in_0 r_0^2} = 4E_1 (r_0)$$

$$E_2\left(\frac{r_0}{2}\right) = \frac{\lambda}{2\pi \in_0 \left(\frac{r_0}{2}\right)} = \frac{\lambda}{\pi \in_0 r_0} = 2E_2 (r_0)$$

$$E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2 \in_0} = E_3(r_0)$$

Given $E_1 (r_0) = E_2 (r_0) = E_3 (r_0)$. It follows from the above equations that the only correct choice is (c).

9. Fringe width $\beta = \dfrac{\lambda D}{d}$. Since $\lambda_2 > \lambda_1$, $\beta_2 > \beta_1$.

Distance of *m*th bright fringe from the central maximum is

$$y_m = \frac{m\lambda D}{d}$$

$$\therefore \qquad y_{m_1} = \frac{m_1 \lambda_1 D_1}{d}$$

and $$y_{m_2} = \frac{m_2 \lambda_2 D_2}{d}$$

Since $y_{m_1} = y_{m_2}$ and $\lambda_2 > \lambda_1$, it follows that $m_1 > m_2$.

Distance of *m*th dark fringe from the central maximum is

$$y_m^* = \left(m - \frac{1}{2}\right)\frac{\lambda D}{d}$$

Now $y_3 (\text{for } \lambda_2) = \dfrac{3\lambda_2 D}{d}$

and $y_5^* (\text{for } \lambda_1) = \left(5 - \frac{1}{2}\right)\frac{\lambda_1 D}{d} = \frac{9\lambda_1 D}{2d}$

Putting $\lambda_1 = 400\,\text{nm}$ and $\lambda_2 = 600\,\text{nm}$, we find that

$$y_3 = \frac{3 \times (600\,\text{nm}) \times D}{d} = \frac{1800D}{d}\,\text{nm}$$

and $y_5^* = \dfrac{9\times(400\,\text{nm})\times D}{2d} = \dfrac{1800D}{d}\,\text{nm}$

Finally, angular separation θ of conscentive fringes is given by

$$\tan\theta = \frac{\lambda}{d} \Rightarrow \theta = \frac{\lambda}{d} \qquad (\because \theta \text{ is small})$$

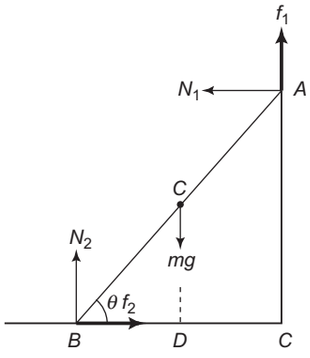
Since $\lambda_2 > \lambda_1$; $\theta_2 > \theta_1$.

Hence the correct choices are (a), (b) and (c).

10. The ladder AB of length ℓ is in contact with the wall at A where the normal reaction is N_1 and with the floor at B where the normal reaction is N_2. Let f_1 and f_2 the frictional force at A and B. Then

$$f_1 = \mu_1 N_1 \text{ and } f_2 = \mu_2 N_2$$

The ladder is in translational as well as rotational equilibrium.



For translational equilibruim

$$f_2 - N_1 = 0 \Rightarrow N_1 = f_2 = \mu_2 N_2 \quad (1)$$

and $f_1 + N_2 = mg \Rightarrow N_2 = mg - \mu_1 N_1$ (2)

Solving (1) and (2) we get

$$N_1 = \frac{\mu_2 mg}{1+\mu_1\mu_2} \quad (3)$$

and

$$N_2 = \frac{mg}{1+\mu_1\mu_2} \quad (4)$$

For rotational equilibruim

Total anticlockwise torque about B = total clockwise torque about B.

i.e. $N_1 \times AC + f_1 \times BC = mg \times BD$

$$\Rightarrow N_1\ell\sin\theta + \mu_1 N_1 \ell\cos\theta = mg \times \frac{\ell}{2}\cos\theta$$

$$\Rightarrow N_1\tan\theta + \mu_1 N_1 = \frac{mg}{2}$$

$$\Rightarrow N_1(\mu_1 + \tan\theta) = \frac{mg}{2} \quad (5)$$

Using (3) in (5) we get

$$\tan\theta = \frac{1-\mu_1\mu_2}{2\mu_2} \quad (6)$$

Choice a: $\mu_1 = 0$, $\mu_2 \neq 0$.

From (2), $N_2 = mg - \mu_1 N_1$

$\Rightarrow$ $N_2 = mg$ $(\because \mu_1 = 0)$

So $N_2 \tan\theta = mg\tan\theta$

Choice (b): $\mu_1 \neq 0$, $\mu_2 = 0$. In this case, $N_2 = mg$

Choice (c): $\mu_1 \neq 0$, $\mu_2 \neq 0$. Then from (4)

$$N_2 = \frac{mg}{1+\mu_1\mu_2}$$

Choice (d): $\mu_1 = 0$, $\mu_2 \neq 0$. Then from (3) and (6), we get

$$N_1 = \mu_2\, mg \text{ and } \tan\theta = \frac{1}{2\mu_2}$$

$\therefore$ $N_1\tan\theta = \dfrac{\mu_2 mg}{2\mu_2} = \dfrac{mg}{2}$

which agrees with (5) if $\mu_1 = 0$.

Hence the corrent choices are (c) and (d).

Section-II

11. Let x metre be the main scale reading and VC be the vernier constant.

First measurement of the change in length of the wire is

$$\ell_1 = x + 20 \times VC$$

Second measurement of the change in length of the wire is

$$\ell_2 = x + 45 \times VC$$

$\therefore$ Change in the length is

$$\ell = \ell_2 - \ell_1 = x + 45 \times VC - (x + 20 \times VC)$$
$$= 25 \times VC = 25 \times 1.0 \times 10^{-5}\text{m}$$

In the given experiment, only the change in the lengths is measured. Now

$$Y = \frac{FL}{\ell A}$$

$\therefore$ $\dfrac{\Delta Y}{Y} = \dfrac{\Delta\ell}{\ell}$

The maximum error ($\Delta\ell$) in the measurement of change in length = one $VC = 1.0 \times 10^{-5}$m. Hence, the maximum percentage error in Y is

$$\frac{\Delta Y}{Y}\times 100 = \frac{\Delta \ell}{\ell}\times 100$$

$$= \frac{1.0\times 10^{-5}}{25\times 1.0\times 10^{-5}}\times 100 = 4\%$$

12. Since $\left(\vec{V}_B - \vec{V}_A\right)$ is perpendicular to $\vec{V}_A$

$$\left(\vec{V}_B - \vec{V}_A\right)\cdot \vec{V}_A = 0$$

$$\Rightarrow \quad V_B V_A \cos\theta - V_A^2 = 0$$

$$\Rightarrow \quad V_B \cos\theta = V_A$$

where θ is the angle between $\vec{V}_A$ and $\vec{V}_B$. Given that $\theta = 60° - 30° = 30°$.

Therefore

$$V_B \cos 30° = V_A$$

$$\Rightarrow \quad V_B = \frac{V_A}{\cos 30°} = \frac{100\sqrt{3}}{\sqrt{3}/2} = 200\ \text{ms}^{-1}$$

$$t_0 = \frac{\text{relative distance between } A \text{ and } B \text{ at } t_0}{\text{relative velocity between } A \text{ and } B \text{ at } t_0}$$

$$\Rightarrow \quad t_0 = \frac{S_{AB}}{v_{AB}} = \frac{S_A - S_B}{v_A - v_B}$$

At time $t = t_0$, A just escapes being hit by B, therefore, $S_{AB} = 500\,\text{m} - 0 = 500\,\text{m}$.

$$v_{AB} = \left|\vec{v}_A - \vec{v}_B\right|$$

$$= \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

$$= \sqrt{(100\sqrt{3})^2 + (200)^2 - 2\times 100\sqrt{3}\times 200\times \cos 30°}$$

$$= 100\ \text{ms}^{-1}$$

$$\therefore \quad t_0 = \frac{S_{AB}}{v_{AB}} = \frac{500\,\text{m}}{100\,\text{ms}^{-1}} = 5\,\text{ms}^{-1}$$

13. Given $U_i = 100\,\text{J}$, $U_b = 200\,\text{J}$, $W_{af} = 200\,\text{J}$, $W_{\text{ib}} = 50\,\text{J}$, $W_{bf} = 100\,\text{J}$, and $Q_{iaf} = 500$ J.

In Process iaf,

$$W_{iaf} = W_{ia} + W_{af} = 0 + 200\,\text{J} = 200\,\text{J}$$

$$(\Delta U)_{iaf} = Q_{iaf} - W_{iaf} = 500\,\text{J} - 200\,\text{J} = 300\,\text{J}$$

$$\therefore \quad U_f = (\Delta_U)_{iaf} + U_i = 300\,\text{J} + 100\,\text{J} = 400\,\text{J}$$

In process ibf,

$$W_{ibf} = W_{ib} + W_{bf} = 50\,\text{J} + 100\,\text{J} = 150\,\text{J}$$

$$(\Delta U)_{ibf} = Q_{ibf} - W_{ibf}$$

$$= (Q_{ib} + Q_{bf}) - (W_{ib} + W_{bf})$$

$$= (Q_{ib} + Q_{bf}) - (50\,\text{J} + 100\,\text{J})$$

But $(\Delta U)_{ibf} = (\Delta U)_{iaf} = 300\,\text{J}$

$$\therefore \quad Q_{ib} + Q_{bf} = 300\,\text{J} + 150\,\text{J} = 450\,\text{J} \qquad (1)$$

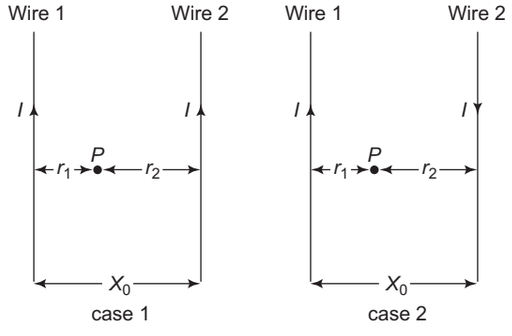
Also $$Q_{bf} - Q_{ib} = 150\,\text{J} \qquad (2)$$

From (1) and (2), $Q_{bf} = 300\,\text{J}$ and $Q_{ib} = 150\,\text{J}$.

Therefore

$$\frac{Q_{bf}}{Q_{ib}} = \frac{300\text{J}}{150\text{J}} = 2$$

14.



Given $r_1 = \frac{X_0}{3}$. Therefore $r_2 = \frac{2X_0}{3}$.

Case 1: Magnetic field at point P due to current I in wire 1 is

$$\frac{\mu_0 I}{2\pi r_1} = \frac{3\mu_0 I}{2\pi X_0}$$ directed into the page.

Magnetic field at P due to current I in wire 2 is

$$\frac{\mu_0 I}{2\pi r_2} = \frac{3\mu_0 I}{4\pi X_0}$$ directed out of the page

$\therefore$ Net magnetic field at P is

$$B_1 = \frac{3\mu_0 I}{2\pi X_0} - \frac{3\mu_0 I}{4\pi X_0} = \frac{3\mu_0 I}{4\pi X_0} \qquad (1)$$

Case 2: Magnetic field at P due to current I in wire 1 is

$$\frac{\mu_0 I}{2\pi r_1} = \frac{3\mu_0 I}{2\pi X_0}$$ directed into the page.

Magnetic field at P due to current I in wire 2 is

$$\frac{\mu_0 I}{2\pi r_2} = \frac{3\mu_0 I}{4\pi X_0} \text{ directed into the page.}$$

$\therefore$ Net megnetic field at P is

$$B_2 = \frac{3\mu_0 I}{2\pi X_0} + \frac{3\mu_0 I}{4\pi X_0} = \frac{9\mu_0 I}{4\pi X_0} \qquad (2)$$

As B_1 and B_2 depend on the values of r_1 and r_2 it is clear that B_1 and B_2 are not uniform in the region between the wires. Hence the trajectories of the charged particle are not circular. But the radius of curvature of the trajectory at point P is inversely proportional to magnetic field at that point. Thus

$$\frac{R_1}{R_2} = \frac{B_2}{B_1} = 3 \text{ [use (1) and (2)]}$$

15. Let $d = k\,\rho^a\, s^b\, f^c$

where k is a dimensionless constant.

$$[\rho] = [\mathrm{ML^{-3}}]$$

$$[s] = \frac{\text{power}}{\text{area}} = \frac{[\mathrm{ML^2T^{-3}}]}{\mathrm{L^2}} = [\mathrm{MT^{-3}}]$$

$$[f] = [\mathrm{T^{-1}}]$$

$$[d] = [\mathrm{L}]$$

$$[\mathrm{L}] = [\mathrm{ML^{-3}}]^a \times [\mathrm{MT^{-3}}]^b \times [\mathrm{T^{-1}}]^c$$

$$\Rightarrow \quad [\mathrm{L}] = [\mathrm{M}^{a+b}] \times [\mathrm{L}^{-3a}] \times [\mathrm{T}^{-3b-c}]$$

Equating powers of M, L and T, we have

$a + b = 0$, $\quad -3a = 1$ $\quad$ and $-3b - c = 0$

Solving, We get

$$a = -\frac{1}{3}, \quad b = +\frac{1}{3} \quad \text{and} \quad c = -1.$$

$$\therefore \quad d = k\ \rho^{-1/3}\, s^{+1/3} f^{-1}$$

Thus $\quad s^{1/n} = s^{1/3} \quad \Rightarrow \quad n = 3.$

16. The ball thrown from the left with a velocity $u_1 = 0.3\ \mathrm{ms^{-1}}$ will have an acceleration $a_1 = -2\,\mathrm{ms^{-2}}$ towards the left, i.e. it will be retarded and will come to rest after travelling a distance x given by

$$2a_1\, x = 0 - u_1^2$$

$$2 \times (-2) \times x = 0 - (0.3)^2$$

$$x = 0.0225\,\mathrm{m}$$

The second ball thrown from the right with a velocity $u_2 = 0.2\,\mathrm{ms^{-1}}$ will suffer a displacement $s = -4 + 0.0225 = -3.997\,\mathrm{m}$ to meet the first ball. Since its acceleration is $-2\,\mathrm{ms^{-2}}$

$$-3.997 = -0.02t - \frac{1}{2} \times 2 \times t^2$$

$$\Rightarrow \quad t^2 + 0.2t - 3.997 = 0$$

Since $\quad 3.997\,\mathrm{m} \simeq 4\,\mathrm{m}$, we have

$$t^2 + 0.2\,t - 4 = 0$$

The positive root of this equation is slightly greater than 1.9s. Thus t is approximately equal to 2s.

17. *For voltmeter,*

$$I_g = \frac{V}{R+G}$$

$$\Rightarrow G = \frac{V}{I_g} - R = \frac{30}{0.006} - 4990 = 10\,\Omega$$

For ammeter

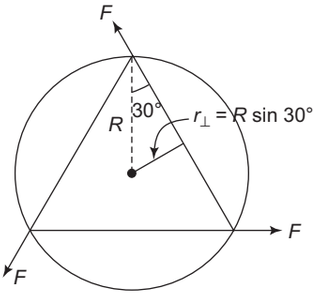
$$S = \frac{I_g G}{I - I_\gamma} = \frac{0.006 \times 10}{1.5 - 0.006}$$

$$= 0.04\,\Omega$$

Given $\quad S = \dfrac{2n}{249}\,\Omega$. Thus

$$0.4 = \frac{2n}{249} \quad \Rightarrow \quad n = 4.98 \simeq 5$$

18. Moment of inertia of disc is $I = \dfrac{1}{2}MR^2$. The total torque on the disc by the three forees is (see figure)



$$\tau = 3Fr_\perp = 3FR \sin 30°$$

Also $\quad \tau = I\alpha$

$$I\alpha = 3FR \sin 30°$$

$$\Rightarrow \quad \alpha = \frac{3FR \sin 30°}{I} \qquad (1)$$

Putting the values of M = 1.5 kg, F = 0.5 N, R = 0.5 m, $I = \frac{1}{2} \times 0.5 \times (0.5)^2$ in Eq. (1) and solving we get

$$\alpha = 2 \text{ rad } s^{-2}$$

Now $\omega = \omega_0 + \alpha\tau$

$$= 0 + 2 \times 1$$

$\Rightarrow$ $\omega = 2$ rad s^{-1}

19. Given R = 0.5 m, M = 0.45 kg, m = 0.05 kg, r = 0.25 m and v = 9 ms^{-1}.

Since no external torque acts, the angular momentum of the system about the centre of rotation is conserved, i.e.

$$L_i = L_f$$

If ω is the angular speed, then

i.e. $I\omega = 2mrv$

or $\frac{1}{2}MR^2 \times \omega = 2mrv$

$\Rightarrow$ $\frac{1}{2} \times 0.45 \times (0.5)^2 \times \omega = 2 \times 0.05 \times 0.25 \times 9$

$\Rightarrow$ $\omega = 4$ rad s^{-1}

20. Potential energy when the block is at Q is

$$U = mgh$$

Form work-energy principle,

Gain in KE = Work done in moving the block from P to Q – P.E. when the block reaches Q

$$= F \times PQ - U$$

$$= F \times 5 - mgh \quad \left(\because PQ = \sqrt{4^2 + 3^2} = 5\text{m}\right)$$

$$= 18 \times 5 - 1 \times 10 \times 4$$

$$= 90 - 40 = 50\text{ J}$$

Hence n = 5.

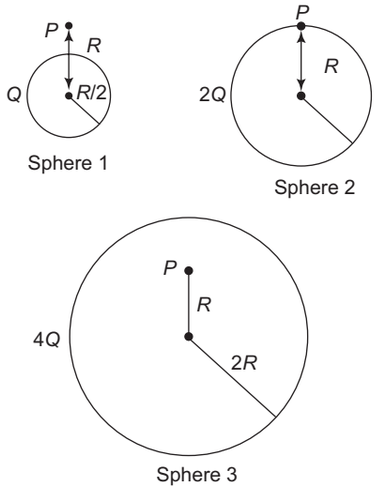
PHYSICS JEE ADVANCED – 2014
PAPER-II – MODEL SOLUTIONS

SECTION I

(Only One Option Correct Type)

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** option is correct.

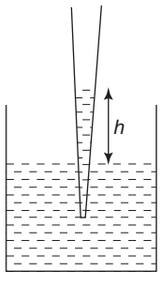
1. Charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2$, R and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_3 respectively, then



(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_1 > E_2$

(c) $E_2 > E_1 > E_3$ (d) $E_3 > E_2 > E_1$

2. A glass capillary tube is of the shape of truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ the value of h will be (g is the acceleration due to gravity)



(a) $\dfrac{2S}{b\rho g}\cos(\theta-\alpha)$

(b) $\dfrac{2S}{b\rho g}\cos(\theta+\alpha)$

(c) $\dfrac{2S}{b\rho g}\cos(\theta-\alpha/2)$

(d) $\dfrac{2S}{b\rho g}\cos(\theta+\alpha/2)$

3. If λ_{cu} is the wavelength of K_α X-ray line of copper (atomic number 29) and λ_{Mo} is the wavelength of the K_α X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{cu}/\lambda_{Mo}$ is close to

(a) 1.99 (b) 2.14

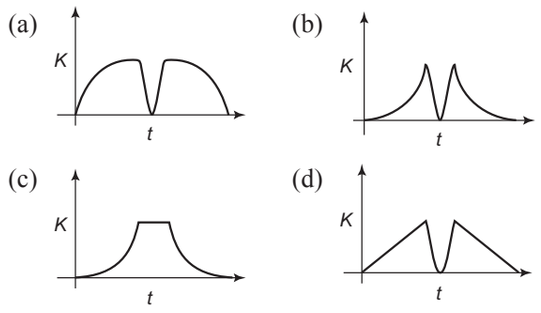
(c) 0.50 (d) 0.48

4. A planet of radius $R = \dfrac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\dfrac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity of Earth is 10 ms^{-2})

(a) 96 N (b) 108 N

(c) 120 N (d) 150 N

5. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale.

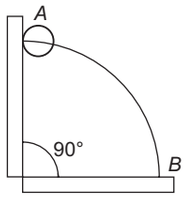


6. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and $hc = 1240$ eV nm, the work function of the metal is nearly

(a) 3.7 eV (b) 3.2 eV

(c) 2.8 eV (d) 2.5 eV

7. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is



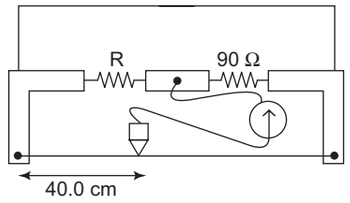
(a) always radially outwards.

(b) always radially inwards.

(c) radially outwards initially and radially inwards later.

(d) radially inwards initially and radially outwards later.

8. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance is



(a) 60 ± 0.15 Ω (b) 135 ± 0.56 Ω

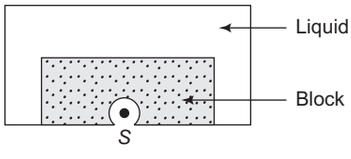
(c) 60 ± 0.25 Ω (d) 135 ± 0.23 Ω

9. Parallel rays of light of intensity $I = 912$ Wm^{-2} are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}$ Wm^{-2} K^{-4} and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to

(a) 330 K (b) 660 K

(c) 990 K (d) 1550 K

10. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is



(a) 1.21 (b) 1.30

(c) 1.36 (d) 1.42

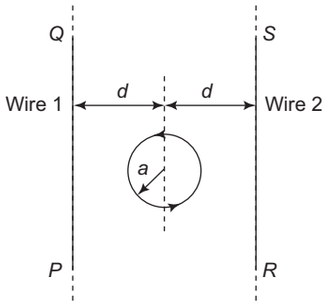
SECTION II

Comprehension Type (Only One Option Correct)

This section contains 3 paragraphs, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (a), (b), (c) and (d).

Paragraph for Questions 11 and 12

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above.



11. When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

(a) current in wire 1 and wire 2 is in the direction PQ and RS, respectively and $h \approx a$

(b) current in wire 1 and wire 2 is in the direction PQ and SR, respectively and $h \approx a$

(c) current in wire 1 and wire 2 is in the direction PQ and SR, respectively and $h \approx 1.2a$

(d) current in wire 1 and wire 2 is in the direction PQ and RS, respectively and $h \approx 1.2a$

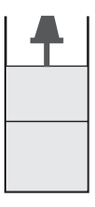
12. Consider $d >> a$, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(a) $\dfrac{\mu_0 I^2 a^2}{d}$ (b) $\dfrac{\mu_0 I^2 a^2}{2d}$

(c) $\dfrac{\sqrt{3}\,\mu_0 I^2 a^2}{d}$ (d) $\dfrac{\sqrt{3}\,\mu_0 I^2 a^2}{2d}$

Paragraph for Questions 13 and 14

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_v = \dfrac{3}{2}R, C_P = \dfrac{5}{2}R,$ and those for an ideal diatomic gas are $C_v = \dfrac{5}{2}R, C_P = \dfrac{7}{2}R.$



13. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(a) 550 K (b) 525 K

(c) 513 K (d) 490 K

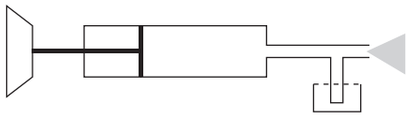
14. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be

(a) 250 R (b) 200 R

(c) 100 R (d) -100 R

Paragraph for Questions 15 and 16

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



15. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of

(a) 0.1 ms^{-1} (b) 1 ms^{-1}

(c) 2 ms^{-1} (d) 8 ms^{-1}

16. If the density of air is ρ_a and that of the liquid ρ_l, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(a) $\sqrt{\dfrac{\rho_a}{\rho_l}}$ (b) $\sqrt{\rho_a \rho_l}$

(c) $\sqrt{\dfrac{\rho_l}{\rho_a}}$ (d) ρ_l

SECTION III

Match List Type (Only One Option Correct)

This section contains **four questions,** each having two matching lists. Choices for the correct combination of elements from **List-I** and **List-II** are given as option (a), (b), (c) and (d) out of which one is correct.

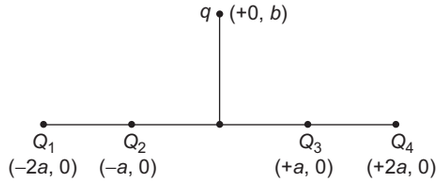
17. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists.

	List I		List II
P.	Lift is accelerating vertically up.	**1.**	d = 1.2 m
Q.	Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	**2.**	d > 1.2 m
R.	Lift is moving vertically up with constant speed.	**3.**	d < 1.2 m
S.	Lift is falling freely.	**4.**	No water leaks out of the jar

Code:

(a)	P-2,	Q-3,	R-2,	S-4
(b)	P-2,	Q-3,	R-l,	S-4
(c)	P-l,	Q-l,	R-l,	S-4
(d)	P-2,	Q-3,	R-1,	S-1

18. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a$, $-a$, $+a$ and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists.



	List I		List II
P.	Q_1, Q_2, Q_3 Q_4 all positive	**1.**	$+x$
Q.	Q_1, Q_2 positive; Q_3, Q_4 negative	**2.**	$-x$
R.	Q_1, Q_4 positive; Q_2, Q_3 negative	**3.**	$+y$
S.	Q_1, Q_3 positive; Q_2, Q_4 negative	**4.**	$-y$

Code:

(a)	P-3,	Q-1,	R-4,	S-2
(b)	P-4,	Q-2,	R-3,	S-1
(c)	P-3,	Q-1,	R-2,	S-4
(d)	P-4,	Q-2,	R-1,	S-3

19. Four combinations of two thin lenses are given in List I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List I with their focal length in List II and select the correct answer using the code given below the lists.

	List I		List II
P.		**1.**	$2r$
Q.		**2.**	$r/2$
R.		**3.**	$-r$
S.		**4.**	r

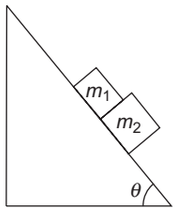
Code:

(a)	P-1,	Q-2,	R-3,	S-4
(b)	P-2,	Q-4,	R-3,	S-1
(c)	P-4,	Q-1,	R-2,	S-3
(d)	P-2,	Q-1,	R-3,	S-4

20. A block of mass $m_1 = 1$ kg another mass $m_2 = 2$ kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1, and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List II expressions for the friction on the block m_2 are

given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g.

[**Useful information:** tan (5.5°) ≈ 0.1; tan (11.5°) ≈ 0.2; tan (16.5°) ≈ 0.3]



	List I		List II
P.	$\theta = 5°$	**1.**	$m_2 g \sin\theta$
Q.	$\theta = 10°$	**2.**	$(m_1 + m_2) g \sin\theta$
R.	$\theta = 15°$	**3.**	$\mu m_2 g \cos\theta$
S.	$\theta = 20°$	**4.**	$\mu(m_1 + m_2) g \cos\theta$

Code:

(a)	P-1,	Q-1,	R-1,	S-3
(b)	P-2,	Q-2,	R-2,	S-3
(c)	P-2,	Q-2,	R-2,	S-4
(d)	P-2,	Q-2,	R-3,	S-3

Answers

Section I

1. (c)	**2.** (d)	**3.** (b)
4. (b)	**5.** (b)	**6.** (a)
7. (d)	**8.** (c)	**9.** (a)
10. (c)		

Section II

11. (c)	**12.** (b)	**13.** (d)
14. (d)	**15.** (c)	**16.** (a)

Section III

17. (c)	**18.** (a)	**19.** (b)
20. (d)		

Hints and Solutions

1. For Sphere 1

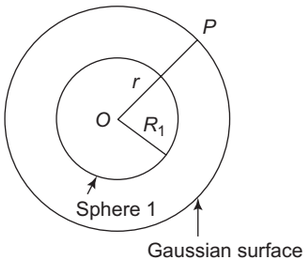
Let R_1 be the radius of sphere 1 and $r (> R_1)$ be the distance of P from O.

From Gauss's law,

$$\oint \vec{E}_1 \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad E_1 \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \qquad E_1 = \frac{q}{4\pi\varepsilon_0 r^2}$$



Given $q = Q$ and $r = R$. Hence

$$E_1 = \frac{Q}{4\pi\varepsilon_0 R^2} = k \qquad (1)$$

where $\quad k = \dfrac{Q}{4\pi\varepsilon_0 R^2}$

For Sphere 2

Let R_2 be the radius of sphere 2. Then from Gauss's law, the electric field E_2 at a point on its surface is given by

$$E_2 \times 4\pi R_2^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \qquad E_2 = \frac{q}{4\pi\varepsilon_0 R_2^2}$$

Given $q = 2Q$ and $R_2 = R$. Therefore

$$E_2 = \frac{2Q}{4\pi\varepsilon_0 R^2} = 2k \qquad (2)$$

For Sphere 3

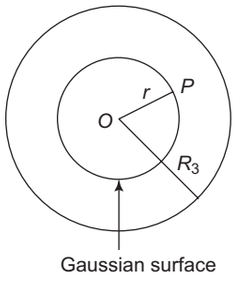
Let R_3 be the radius of the sphere and Q_3 be the charge.

Charge per unit volume $= \dfrac{Q_3}{\frac{4\pi}{3} R_3^3}$

Charge in the Gaussian sphere is

$$q = \frac{Q_3}{\frac{4\pi}{3} R_3^3} \times \frac{4\pi}{3} r^3$$

$$= \frac{Q_3 r^3}{R_3^3}$$



Given $Q_3 = 4Q$, $r = R$ and $R_3 = 2R$.

$$\therefore \qquad q = \frac{4Q \times R^3}{(2R)^3} = \frac{Q}{2}$$

From Gauss's law,

$$\therefore \quad E_3 \times 4\pi r^2 = \frac{q}{\varepsilon_0} = \frac{Q}{2\varepsilon_0}$$

$$\Rightarrow \qquad E = \frac{Q}{8\pi\varepsilon_0 r^2} = \frac{Q}{8\pi\varepsilon_0 R^2} = \frac{k}{2} \qquad (3)$$

From (1), (2) and (3) we find that $E_2 > E_1 > E_3$. So the correct choice is (c).

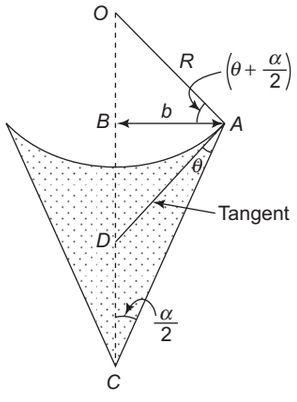
2. It is clear from the figure that $\angle OAB + \angle BAD = 90°$ because $OA = R$ is the radius of the meniscus and AD is the tangent to the meniscus at point A.

But $\angle ADB = \theta + \frac{\alpha}{2}$.

Therefore $\angle BAD = 90° - \angle ADB$.

Thus $\angle OAB + 90° - \angle ADB = 90°$

or $\angle OAB = \angle ADB = \theta + \frac{\alpha}{2}$



Now $b = R\cos\left(\theta + \frac{\alpha}{2}\right)$ and excess pressure on the concave side of the meniscus is

$$p = \frac{2S}{R}$$

$$\Rightarrow \qquad h\,\rho\,g = \frac{2S}{R} = \frac{2S\cos\left(\theta + \frac{\alpha}{2}\right)}{b}$$

$$\Rightarrow \qquad h = \frac{2S\cos\left(\theta + \frac{\alpha}{2}\right)}{b\rho g}$$

So the correct choice is (d).

3. The K_α X-ray line corresponds to the transition $n = 2$ to $n = 1$. For an element of atomic number Z, the wavelength λ of the K_α line is given by

$$\frac{1}{\lambda} = R_H(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R_H}{4}(Z-1)^2$$

$$\therefore \qquad \frac{\lambda_{cu}}{\lambda_{Mo}} = \left(\frac{Z_{Mo}-1}{Z_{cu}-1}\right)^2$$

$$= \left(\frac{42-1}{29-1}\right)^2 = 2.14$$

4. At a depth r below the surface of the planet,

$$g_r = g_s\frac{r}{R}$$

where g_s = acceleration due to gravity on the surface of the planet.

$$g_s = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4\pi}{3} R^3 \rho = \frac{4\pi G\rho R}{3}$$

$$g_r = \frac{4\pi G\rho r}{3}$$

If F is the force needed to keep the wire at rest, then

F = weight of the wire

$$= \int_{\frac{4R}{5}}^{R} (\lambda\, dr)\left(\frac{4\pi G\rho r}{3}\right)$$

$$= \frac{4\pi G\rho\lambda}{3}\left|\frac{r^2}{2}\right|_{\frac{4R}{5}}^{R}$$

$$\Rightarrow \qquad F = \frac{4\pi G\rho\lambda}{3} \times \frac{9R^2}{50} \qquad (1)$$

On the surface of the earth,

$$g_e = \frac{G M_e}{R_e^2} \Rightarrow G = \frac{g_e R_e^2}{M_e} \qquad (2)$$

Also $$\rho = \frac{M_e}{\frac{4\pi}{3} R_e^3} \qquad (3)$$

Given $$R = \frac{R_e}{10} \qquad (4)$$

Using (2), (3) and (4) in (1), we get

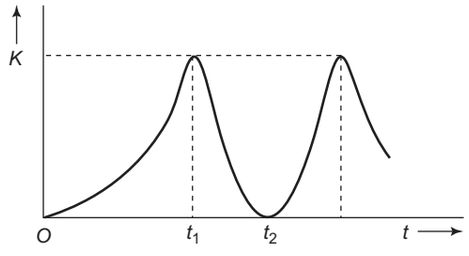
$$F = \frac{9 g_e R_e \lambda}{5 \times 10^3}$$

$$= \frac{9 \times 10 \times (6 \times 10^6) \times 10^{-3}}{5 \times 10^{+3}} = 108 \text{ N}$$

5. The ball is dropped from rest at say $t = 0$. It hits the ground at $t = t_1$ when its velocity $v = gt$. Its kinetic energy in time interval 0 to t_1 is given by

$$K = \frac{1}{2} mv^2 = \frac{1}{2} mg^2 t^2$$

i.e. $K \propto t^2$. So the slope of the K-t graph increases with time and the graph is not linear as shown in the figure.



At $t = t_1$, the velocity is reversed and the ball begins to rise upwards with initial velocity v which decreases with time and its K-t graph has a negative slope until the ball reaches the highest point at time t_2 when it is momentarily at rest and its kinetic energy $K = 0$.

After $t = t_2$, although the velocity is reversed, the speed is v and $K = \frac{1}{2} mv^2$ which is positive. So the ball reaches the same height from which it was dropped. Hence the correct graph is (b)

6. $h\nu = K_{\max} + W_0 = \frac{1}{2} m v_{\max}^2 + W_0$

(W_0 = work function)

$$\Rightarrow \quad \frac{hc}{\lambda} = \frac{1}{2} m v_{\max}^2 + W_0$$

$$\therefore \quad v_{\max}^2 = \frac{2hc}{m\lambda} - \frac{2W_0}{m}$$

$$\therefore \quad u_1^2 = \frac{2hc}{m\lambda_1} - \frac{2W_0}{m} \qquad (1)$$

and $$u_2^2 = \frac{2hc}{m\lambda_2} - \frac{2W_0}{m} \qquad (2)$$

Dividing (1) and (2) we get

$$\frac{u_1^2}{u_2^2} = \frac{\frac{hc}{\lambda_1} - W_0}{\frac{hc}{\lambda_2} - W_0} \qquad (3)$$

Given $\lambda_1 = 248$ nm, $\lambda_2 = 310$ nm, $hc = 1240$ eV nm

and $\frac{u_1}{u_2} = 2$. Using these values in (3), we have

$$4 = \frac{\frac{1240 \text{ eV nm}}{248 \text{ nm}} - W_0}{\frac{1240 \text{ eV nm}}{310 \text{ nm}} - W_0}$$

$$\Rightarrow \quad 4 = \frac{5 \text{eV} - W_0}{4 \text{eV} - W_0}$$

$$\Rightarrow \quad W_0 = 3.7 \text{ eV}$$

7. Initially the bead exerts an inward radial force (centripetal force) on the wire and the wire exerts a normal reaction N radially outwards. At a certain instant during the motion, the normal reaction N becomes zero. After that instant, the normal reaction N will act radially outwards. So the correct choice is (d).

8. $R = \left(\frac{l}{100 - l}\right) \times 90 = \frac{40 \times 90}{60} = 60\ \Omega$

Since 90 Ω is exact, the fractional error in R is

$$\frac{\Delta R}{R} = \frac{0.1 \text{ cm}}{40 \text{ cm}} + \frac{0.1 \text{ cm}}{60 \text{ cm}} = \frac{0.1}{40} + \frac{0.1}{60}$$

Since $R = 60\ \Omega$,

$$\Rightarrow \quad \Delta R = \frac{0.1}{40} \times 60 + \frac{0.1}{60} \times 60 = 0.15 + 0.1 = 0.25\ \Omega$$

$\therefore \quad R \pm \Delta R = (60 \pm 0.25)\ \Omega$

The correct choice is (c).

9. When the steady state is reached, the rate of energy lost by the sphere = rate at which the energy is incident on it, i.e. (here R = radius of sphere)

$$\sigma \times 4\pi R^2 [T^4 - (300)^4] = 912 \times \pi R^2$$

$$\Rightarrow \quad 5.7 \times 10^{-8} \times 4\left[T^4 - (300)^4\right] = 912$$

$$\Rightarrow \quad T^4 - (300)^4 = \frac{912}{4 \times 5.7 \times 10^{-8}} = 40 \times 10^8$$

$$\Rightarrow \quad T^4 - 81 \times 10^8 = 40 \times 10^8$$

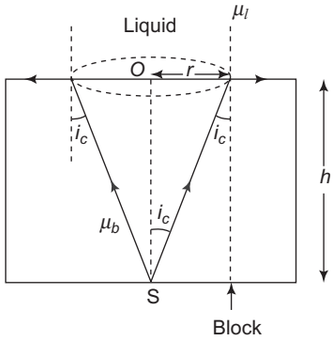
$$\Rightarrow \quad T = (121 \times 10^8)^{1/4} = 332\,\text{K} \simeq 330\,\text{K}$$

10. $\tan i_c = \dfrac{r}{h} = \dfrac{5.77}{10} = \sqrt{3}$

$$\Rightarrow \quad i_c = 30°$$

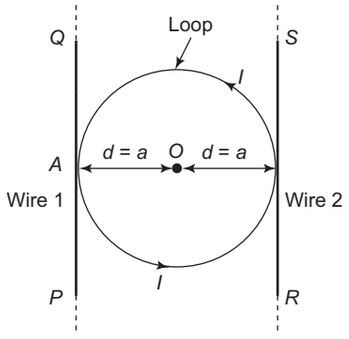
Also $\quad \sin i_c = \dfrac{\mu_l}{\mu_b}$

$$\Rightarrow \quad \mu_l = \mu_b \sin i_c$$
$$= 2.72 \times \sin 30° = 1.36$$



So the correct choice is (c).

11. If the currents in wires 1 and 2 are in the same direction and since the distance of point C (not shown in figure) at a height h above the centre O of the loop is the same from the two wires, the magnitudes of magnetic field at C are equal for both wires, but their directions are opposite. Hence, the magnetic field due to current I in the wires will be zero at C but the magnetic field due current I is not zero at point C. Hence the net magnetic field at C due to the net magnetic field at C due to the wires and due to the loop cannot be zero. Therefore, options (a) and (d) are wrong.



The distance of point C from the centre O of the loop is

$$r = \sqrt{h^2 + a^2}$$

The magnet field at the height h due to each wire is $\dfrac{\mu_0 I}{2\pi r}$. The direction of this field is along the tangent at the given point to the field. The total field due to both wires is

$B_w = \dfrac{2\mu_0 I}{2\pi r}$ along the tangent at the given point. But the magnetic field due the loop is

$B_l = \dfrac{\mu_0 I a^2}{2r^3}$ out of the page.

The net field at the given point will be zero if component of B_w along the axis of the loop = B_l, i.e.

$B_w \sin\theta = B_l$, where θ = angle between AC and CO.

$$\frac{2\mu_0 I}{2\pi r} \times \frac{a}{r} = \frac{\mu_0 I a^2}{2r^3}$$

$$\Rightarrow \quad \frac{2}{\pi r^2} = \frac{a}{r^3}$$

$$\Rightarrow \quad r = \frac{\pi a}{2}$$

$$\Rightarrow \sqrt{h^2 + a^2} = \frac{\pi a}{2}$$

$$\Rightarrow \quad h = a\left(\frac{\pi^2}{4} - 1\right)^{1/2} \simeq 1.2\,a$$

So the correct choice is (c)

12. If $d >> a$; the torque on the loop is

$$\tau = IBA \sin\theta$$

$$= I \times \frac{2\mu_0 I}{2\pi d} \times \pi a^2 \times \sin 30°$$

$$= \frac{\mu_0 I^2 a^2}{2d}$$

13. Let T be the equilibrium temperature. Since the partition between then two compartments is regid, the transfer of heat from the lower compartment (which contains a monoatomic gas at 700 K) to the upper compartment takes place at constant volume.

$\therefore$ Heat transferred from lower to upper compartment is

$$Q_1 = n C_v \Delta T = 2 \times \frac{3R}{2} \times (700 - T) \qquad (1)$$

Since the piston and the top of the upper compartment are frictionless, heat Q_2 is gained by the diatomic gas in the upper compartment at constant pressure, i.e.

$$Q_2 = n C_p \Delta T = 2 \times \frac{7R}{2} \times (T - 400) \qquad (2)$$

Equating (1) and (2),

$$2 \times \frac{3R}{2} \times (700 - T) = 2 \times \frac{7R}{2} \times (T - 400)$$

$\Rightarrow$ $3 \times (700 - T) = 7 \times (T - 400)$

$\Rightarrow$ $T = 490$ K

14. If the partition between the two compartments is frictionless, the heat exchange between the gases takes place at constant pressure. If T' is the equilibrium temperature now, then

$$2 \times \frac{5R}{2} \times (700 - T') = 2 \times \frac{7R}{2} \times (T' - 400)$$

$\Rightarrow$ $T' = 525$ K

Since the temperature of the gas in the lower compartment falls, the work done by the gas is

$W_1 = -nR \Delta T = -2 \times R \times (700 - 525) = -350$ R

Since the temperature of the gas in the upper compartment increases, the work done by the gas is

$W_2 = +nR \Delta T = +2 \times R \times (525 - 400) = +250$ R

$\therefore$ Net work done $= W_1 + W_2 = -350\ R + 250\ R = -100\ R$

So the correct choice is (d).

15. From the equation of continuity of flow,

$$a_1 v_1 = a_2 v_2$$

$\Rightarrow$ $(\pi r_1^2) v_1 = (\pi r_2^2) v_2$

$\Rightarrow$ $$v_2 = v_1 \times \left(\frac{r_1}{r_2}\right)^2$$

$$= 5 \text{ mm s}^{-1} \times \left(\frac{20 \text{ mm}}{1 \text{ mm}}\right)^2$$

$$= 2000 \text{ mm s}^{-1}$$

$$= 2 \text{ ms}^{-1}$$

So the correct choice is (c).

16. From Bernoulli's principle,

$$P_0 + \frac{1}{2} \rho_a v_a^2 = P_0 + \frac{1}{2} \rho_l v_l^2$$

$\Rightarrow$ $$v_l^2 = v_a^2 \frac{\rho_a}{\rho_l}$$

$\Rightarrow$ $$v_l = v_a \sqrt{\frac{\rho_a}{\rho_l}}$$

For given v_a, $v_l \propto \sqrt{\frac{\rho_a}{\rho_l}}$, which is choice (a).

17. Velocity of efflux is $v = \sqrt{2gh}$ where h is the depth of the hole below the surface of water in the jar. The time taken by water emerging from the hole to hit the floor of the lift is given by

$$t = \sqrt{\frac{2(H - h)}{g}}$$

where H is the height of water in the jar.

$\therefore$ Horizontal range $d = vt = \sqrt{2h(H - h)}$. Thus d is independent of g. Hence in P, Q and R, d remains the same = 1.2 m. But in S, $g_{\text{eff}} = 0$. Hence $v = 0$. Thus in case S, no water emerges out of the hole. So, the correct answer is P $\rightarrow$ 1, Q $\rightarrow$ 1, R $\rightarrow$ 1 and S $\rightarrow$ 4, which is option (c).

18. **Case P:** In this case, the net force F exerted on q by Q_1 and Q_4 and also by Q_2 and Q_3 are in the $+y$ direction. So P $\rightarrow$ 3.

Case Q: In this case, the net force F exerted on q by Q_1 and Q_4 and also by Q_2 and Q_3 are in the $+x$ direction. So Q $\rightarrow$ 1.

Case R: In this case, the net force F_1 exerted on q by Q_1 and Q_4 is in the $+y$ direction and the net force F_2 exerted on q by Q_2 and Q_3 is in the $-y$ direction. Since Q_2 and Q_3 are closer to q than Q_1 and Q_4, $F_2 > F_1$. Hence the net force on q due to all the four charges is in the $-y$ direction. So R $\rightarrow$ 4.

Case S: In this case, the net force F_1 exerted on q by Q_1 and Q_4 is in the $+x$ direction and the net force F_2 exerted on q by Q_2 and Q_3 is in the $-x$ direction. Since Q_2 and Q_3 are closer to q than

Q_1 and Q_4, $F_2 > F_1$. Hence the net force on q due to all the four charges is in the $-x$ direction. So S $\rightarrow$ 2. Thus the correct choice is (a).

19. $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

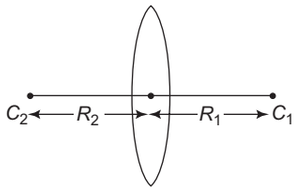


Fig. 1

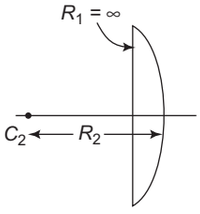


Fig. 2

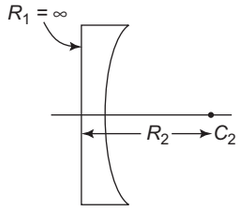


Fig. 3

For an equi-convex lens (Fig. 1)

$R_1 = +r$, $R_2 = -r$. Given $\mu = 1.5$.

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{r} - \frac{1}{-r}\right) = 0.5 \times \frac{2}{r} = \frac{1}{r}$$

$\Rightarrow \quad f = r$

For a plano-convex lens (Fig. 2)

$R_1 = \infty$, $R_2 = -r$

$$\therefore \quad \frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{-r}\right) = \frac{1}{2r}$$

$\Rightarrow \quad f = 2r$

For a plano-convex lens (Fig. 3)

$R_1 = \infty$, $R_2 = +r$

$$\therefore \quad \frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{r}\right) = -\frac{1}{2r}$$

$\Rightarrow \quad f = -2r$

For a combination of lens, $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$.

Case P: $\dfrac{1}{F} = \dfrac{1}{r} + \dfrac{1}{r} = \dfrac{2}{r} \Rightarrow F = \dfrac{r}{2}$. Therefore P $\rightarrow$ 2.

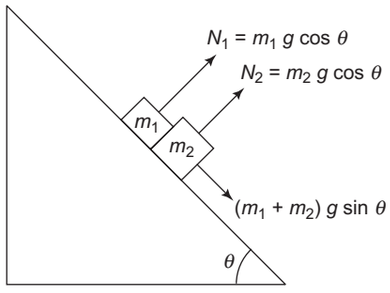
Case Q: $\dfrac{1}{F} = \dfrac{1}{2r} + \dfrac{1}{2r} = \dfrac{1}{r} \Rightarrow F = r$. Therefore Q $\rightarrow$ 4

Case R: $\dfrac{1}{F} = -\dfrac{1}{2r} - \dfrac{1}{2r} = -\dfrac{1}{r} \Rightarrow F = -r$. Therefore R $\rightarrow$ 3

Case S: $\dfrac{1}{F} = \dfrac{1}{r} - \dfrac{1}{2r} = \dfrac{1}{2r} \Rightarrow F = 2r$. Therefore S $\rightarrow$ 1.

So the correct option is (b).

20.



The block m_2 will not slide down the plane if the frictional force on it has a maximum value f_{max} given by

$f_{max} \geq (m_1 + m_2)\, g \sin \theta$

$\Rightarrow \quad \mu N_2 \geq (m_1 + m_2)\, g \sin \theta$

$\Rightarrow \quad \mu m_2 g \cos \theta \geq (m_1 + m_2)\, g \sin \theta$

$\Rightarrow \quad 0.3 \times 2 \times g \cos \theta \geq (1 + 2)\, g \sin \theta$

$\Rightarrow \quad 0.2 \geq \tan \theta$

Thus θ must be less than $\tan^{-1}(0.2) = 11.5°$.

For cases P and Q, θ is less than 11.5°. Hence the force of friction on m_2 is $f = (m_1 + m_2)\, g \sin \theta$. But for cases R and S, θ is greater than 11.5°. Hence the force of friction on m_2 must be

$$f = f_{max} = \mu m_2 g \sin \theta$$

So the correct answer is: P $\rightarrow$ 2, Q $\rightarrow$ 2, R $\rightarrow$ 3, S $\rightarrow$ 3, which is option (d).

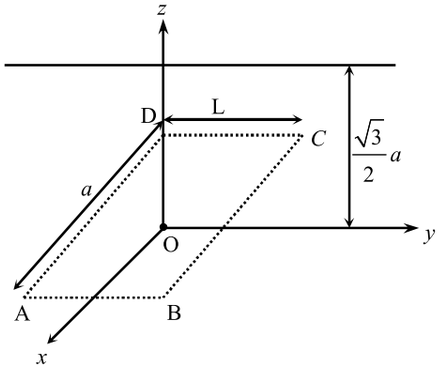
JEE ADVANCED 2015: PAPER–I (MODEL SOLUTIONS)

SECTION I

(Single Digit Integer Type)

This section contains **EIGHT** questions
The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive

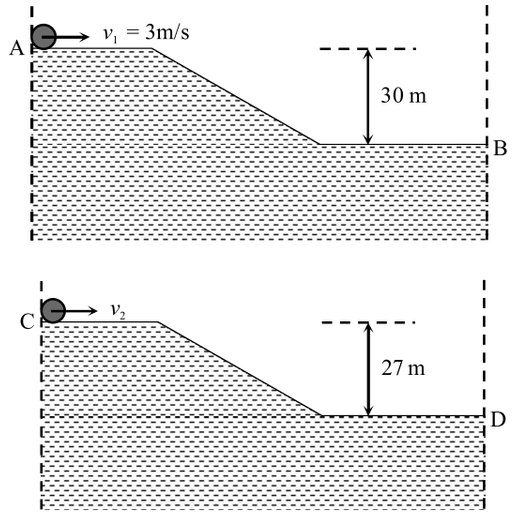
1. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its center at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is



2. Consider a hydrogen atom with its electron in the nth orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is (hc = 1242 eV nm)

3. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is 1/4th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

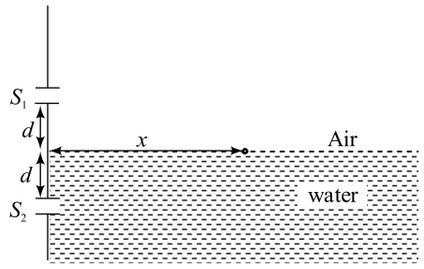
4. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and v_1 = 3 m/s, then v_2 in m/s is (g = 10 m/s^2)



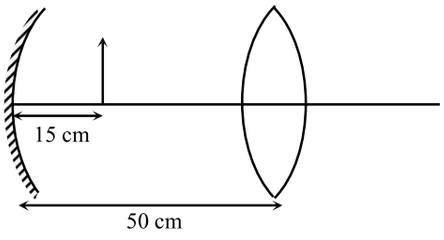
5. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 104 times the power emitted from B. The ratio (λ_A/λ_B) of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

6. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5 % of the electrical power available form the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is

7. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), $2d$ is the separation between the slits and m is an integer. The value of p is



8. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set- up is kept in a medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



SECTION II

(One or More than One Options Correct Type)

This section contains **TEN** questions
Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four options(s) is (are) correct

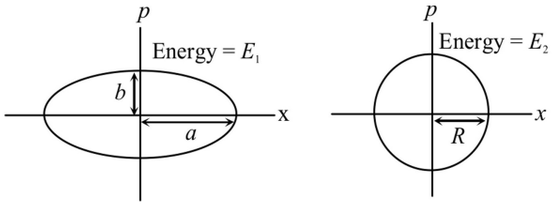
9. Consider a vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then:

(a) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.
(b) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.
(c) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.
(d) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

10. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is(are)

(a) $M \propto \sqrt{c}$ (b) $M \propto \sqrt{G}$
(c) $L \propto \sqrt{h}$ (d) $L \propto \sqrt{G}$

11. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If a $\frac{a}{b}n^2$ and $\frac{a}{R}n$, then the correct equation(s) is(are)



(a) $E_1\omega_1 = E_2\omega_2$ (b) $\frac{\omega_2}{\omega_1} = n^2$
(c) $\omega_1\omega_2 = n^2$ (d) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

12. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing

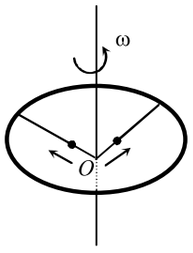
through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is

(a) $\frac{2R}{3}$

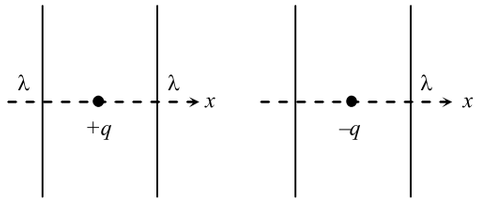
(b) $\frac{1R}{3}$

(c) $\frac{3R}{5}$

(d) $\frac{4R}{5}$



13. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are)



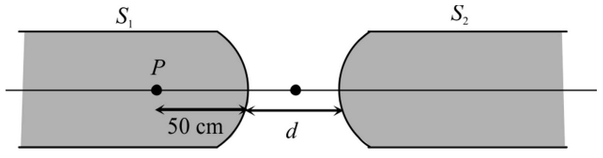
(a)Both charges execute simple harmonic motion.

(b) Both charges will continue moving in the direction of their displacement.

(c)Charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement.

(d) Charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement.

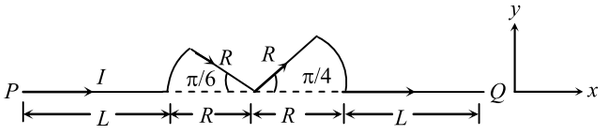
14. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is



(a)60 cm (b) 70 cm

(c)80 cm (d) 90 cm

15. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is(are)



(a)If $\vec{B}$ is along z, $F \propto (L + R)$

(b) If $\vec{B}$ is along x, $F = 0$

(c)If $\vec{B}$ is along y, $F \propto (L + R)$

(d) If $\vec{B}$ is along z, $F = 0$

16. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is(are)

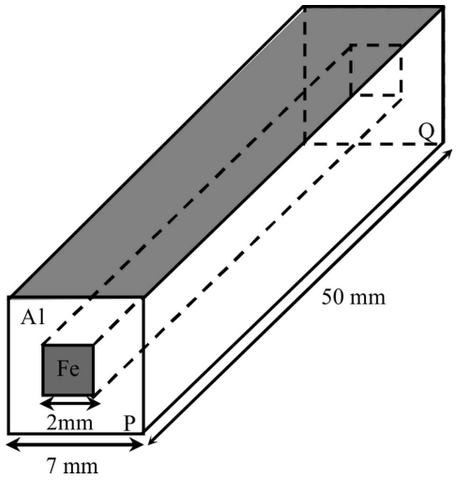
(a)The average energy per mole of the gas mixture is $2RT$.

(b) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

(c)The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(d) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

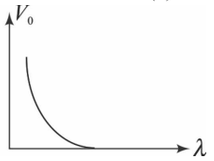
17. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are 2.7×10^{-8} Ω m and 1.0×10^{-7} Ω m, respectively. The electrical resistance between the two faces P and Q of the composite bar is

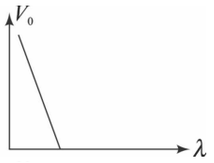


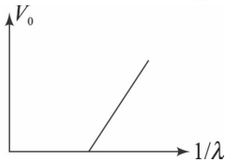
(a) $\frac{2475}{64}\mu\Omega$ (b) $\frac{1875}{64}\mu\Omega$

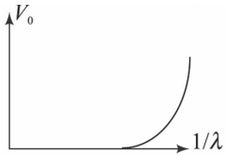
(c) $\frac{1875}{49}\mu\Omega$ (d) $\frac{2475}{132}\mu\Omega$

18. For photoelectric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $1/\lambda$.

(a) 

(b) 

(c) 

(d) 

SECTION III

This section contains **TWO** questions
Each question contains two columns, **Column I** and **Column II**
Column I has **four** entries (a), (b), (c) and (d)
Column II has **five** entries (p), (q), (r), (s) and (t)
Match the entries in **Column I** with the entries in **Column II**
One or more entries in **Column I** may match with one or more entries in **Column II**

19. Match the nuclear processes given in column I with the appropriate option(s) in column II

Column I	**Column II**
(a) Nuclear fusion	(p) Absorption of thermal neutrons by $^{235}_{92}U$
(b) Fission in a nuclear reactor	(q) $^{60}_{27}Co$ nucleus
(c) β-decay	(r) Energy production in stars via hydrogen conversion to helium
(d) γ-ray emission	(s) Heavy water
	(t) Neutrino emission

20. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

Column I	**Column II**
(a) $U_1(x)=\frac{U_0}{2}\left[1=\left(\frac{x}{a}\right)^2\right]^2$	(p) The force acting on the particle is zero at $x = a$.
(b) $U_2(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(q) The force acting on the particle is zero at $x = 0$.

(c) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$

(d) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$

(r) The force acting on the particle is zero at $x = -a$.

(s) The particle experiences an attractive force towards $x = 0$ in the region $|x| < a$.

(t) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$.

Answers

Section-I

1. 6	**2.** 2	**3.** 2
4. 7	**5.** 2	**6.** 3
7. 3	**8.** 7	

Section-II

9. (b) and (c) **10.** (a), (c) and (d)
11. (b) and (d) **12.** (d)
13. (c) **14.** (b)
15. (a), (b) and (c)
16. (a), (b) and (d)
17. (b) **18.** (a) and (c)

Section-III

19. (a) → (r), (b) → (p, s), (c) → (q, t), (d) → (q, r, t)
20. (c)

Hints and Solutions

Section-I

1. Every point on *ABCD* is at the same distance $z = \frac{\sqrt{3}}{2}\ a$ from the line of charge. Hence the electric field due to the linear charge distribution is constant inside the closed rectangular surface. According to Gauss's law, the electric flux through this surface is

$$\phi = \frac{q}{\varepsilon_0}$$

where q = charge enclosed in *ABCD* = charge of line *AB* of lengh *L*. If λ is the charge per unit length, $q = \lambda L$. Since the closed rectangular surface has 6 sider, the flux through each sides (such as *ABCD*).
is

$$\frac{\phi}{6} = \frac{q}{6\varepsilon_0} = \frac{\lambda L}{6\varepsilon_0}$$

Thus $n = 6$.

2. Ionization energy of hydrogen atom with the electron in the *n*th orbit is

$$E = h\nu - \frac{13.6\,\text{eV}}{n^2}$$

$$\Rightarrow E = \frac{hc}{\lambda} - \frac{13.6\,\text{eV}}{n^2}$$

Given $E = 10.4$ eV, $\lambda = 90$ nm and $hc = 1242$ eV nm. Hence

$$10.4\text{eV} = \frac{1242\,\text{eVnm}}{90\,\text{nm}} - \frac{13.6\,\text{eV}}{n^2}$$

$$\Rightarrow 10.4 = 13.8 - \frac{13.6}{n^2}$$

$$\Rightarrow \quad n^2 = 4. \text{ Hence } n = 2.$$

3. Let *PA* be the maximum height attained.

It is given that g at $P = \frac{1}{4}$ (g at A), *i.e.*

$$\frac{GM}{r^2} = \frac{1}{4}\frac{GM}{R^2}$$

$\Rightarrow r = 2R$
For energy conservation, total energy at A = total energy at P, i.e.,

$$\frac{1}{2}mv^2 - \frac{GmM}{R} = 0 - \frac{GmM}{r}$$

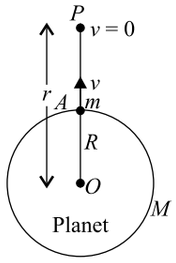
$$\Rightarrow \frac{1}{2}v^2 - \frac{GM}{R} = -\frac{GM}{2R} \quad (\because r = 2R)$$

$$\Rightarrow v = \sqrt{\frac{GM}{R}} \qquad \text{...(i)}$$

We know that the escape velocity from the surface of the planet is given by

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2}v \text{ [Use Eq. (i)]}$$

It is given that $v_e = \sqrt{N}\, v$. Hence $N = 2$.



4. Let M be the mass of each disc and R be its radius. the total kinetic energy of a disc rolling without slipping is given by

Total K.E. = rotational K.E. + Translation K.E.

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2$$

$$= \frac{1}{2}\left(\frac{1}{2} MR^2\right) \times \left(\frac{v}{R}\right)^2 + \frac{1}{2} mv^2$$

$$= \frac{1}{4} mv^2 + \frac{1}{2} mv^2 = \frac{3}{4} mv^2$$

As the disc rolls down the inclined plane, it gains kinetic energy and loses potential energy. Since the final kinetic energy of the discs is the same,

Increase in K.E. = decrease in P.E.

$$\frac{3}{4} mv_2^2 - \frac{3}{4} Mv_1^2 = Mg(h_2 - h_1)$$

$$\Rightarrow \frac{3}{4}(v_2^2 - 3^2) = 10 \times (30 - 27) = 30$$

$$\Rightarrow v_2^2 = 30 \times \frac{4}{3} + 9 = 49$$

$\Rightarrow v_2 = 7$ ms^{-1}

5. According to Wien's displacement law,

$\lambda_A = T_A = \lambda_B = T_B$

where T_A and T_B are the temperatures of stars A and B. Thus

$$\frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} \qquad ...(i)$$

According to Stefan's law, the energy radiated per second from the surface of A and B are

$$E_A \quad T_A^4 \; A_A^2$$

and $E_B = \sigma T_B^4 \; A_B^2$

where $A_A = 4\pi R_A^2$ and $A_B = 4\pi R_A^2$ are their respective surface areas. Therefore

$$\frac{E_A}{E_B} = \left(\frac{T_A}{T_B}\right)^4 \times \left(\frac{R_A}{R_B}\right)^2$$

Given $\frac{E_A}{E_B} = 10^4$ and $\frac{R_A}{R_B} = 400$. Substituting these values, we have

$$10^4 = \left(\frac{T_A}{T_B}\right)^4 \times (400)^2$$

$$\Rightarrow \frac{T_A}{T_B} = \frac{1}{2} \qquad ...(ii)$$

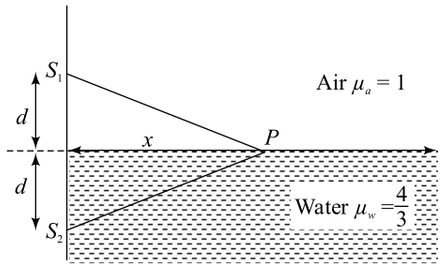
Using (2) in (i), we get $\frac{\lambda_A}{\lambda_B} = 2$

6. Let P be the power generated by the plant and P' be the power requirement of the village.

P' = 12.5% of $P = \frac{12.5}{100} \times P = \frac{P}{8} = \frac{P}{2^3}$

Therefore, the number of half lives when P' becomes $\frac{P}{8}$ is 3. So the time reqired = $3T$. Thus $n = 3$.

7. Optical path in a medium = refractive index of the medium × actual path.



$$S_1P = S_2P = \left(x^2 + d^2\right)^{1/2}$$

On reaching P, the optical path difference between the waves from S_2 and S_1 is

$\Delta x = \mu_w S_2P - \mu_a S_1P$

$$= \frac{4}{3} \times \left(x^2 + d^2\right)^{1/2} - 1 \times \left(x^2 + d^2\right)^{1/2}$$

$$= \frac{1}{3} \times \left(x^2 + d^2\right)^{1/2}$$

There will be a maximum (i.e. bright fringe) of order m at point P if

$\Delta x = m\lambda$

or $-\times \left(x^2 + d^2\right)^{1/2} = m$

$$\Rightarrow \frac{1}{9} \times \left(x^2 + d^2\right) = m^2\lambda^2$$

$\Rightarrow x^2 = 9m^2\lambda^2 - d^2$ ...(i)

Comparing Eq. (i) with the given equation $x^2 = p^2 m^2 \lambda^2 - d^2$,
we get, $p^2 = 9$ or $p = 3$.

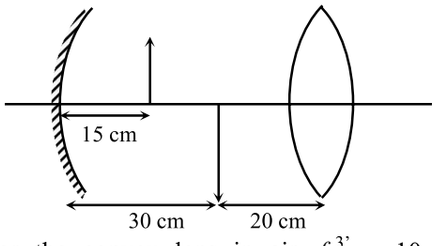
8. For the concave mirror, $u_1 = -15$ cm and $f_2 = -10$ cm.

$$\frac{1}{v_1}+\frac{1}{u_1}=\frac{1}{f_1}$$

$$\Rightarrow \quad \frac{1}{v_1}+\frac{1}{-15}=\frac{1}{-10} \Rightarrow v_1 = -30 \text{ cm}$$

Magnification produced by mirror is

$$m_1 = -\frac{u_1}{v_1} = -\frac{-30}{-15} = -2$$



For the convex lens in air, $f_2^{3'} = 10$ cm and $u_2 = 20$ cm

$$\frac{1}{v_2}=\frac{1}{u_2}=\frac{1}{f_2}$$

$$\Rightarrow \quad \frac{1}{v_2}-\frac{1}{20}=\frac{1}{10} \Rightarrow v_2 = +20 \text{ cm}$$

Magnification produced by lens is

$$m_2 = \frac{v_2}{u_2}=\frac{+20}{-20}=-1$$

Total magnification produced by the combination is

$$M_1 = m_1 \times m_2 = -2 \times 1 = +2$$

When the equipment is immersed in a medium of $\mu = \frac{7}{6}$, the focal length of the mirror remains unchanged hence $m_1 = -2$, but the focal length of the lens becomes f_2' given by

$$\frac{1}{f_2'}=\left(\frac{\frac{7}{6}-1.5}{1.5}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(i)$$

In air

$$\frac{1}{f_2}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{R_1}-\frac{1}{R_2}=\frac{1}{0.5f_2}=\frac{1}{0.5\times 10}=\frac{1}{5}$$ Using this in (i) we have,

$$\frac{1}{f_2'}=\left(\frac{1.5-\frac{7}{6}}{\frac{7}{6}}\right)\times\frac{1}{5} \Rightarrow f_2' = \frac{35}{2}\text{cm}$$

Using $u_2 = -20$ cm and $f_2' = \frac{35}{2}$cm in $\frac{1}{v_2'}-\frac{1}{u_2'}=\frac{1}{f_2'}$, we have

$$\frac{1}{v_2'}+\frac{1}{20}=\frac{2}{35} \Rightarrow v_2' = 140\,\text{cm}$$

$$\therefore \quad m_2' = \frac{140}{-20} = -7$$

$$\therefore \quad M_2 = -2 \times -7 = 14$$

$$\therefore \quad \frac{M_2}{M_1}=\frac{14}{-2}=-7$$

or $$\left|\frac{M_2}{M_1}\right| = 7$$

Section-II

9. For vernier callipers

1 M.S.D. = $\frac{1}{8}$ cm

Also 5 V.S.D. = 4 M.S.D.

$$\therefore \text{ 1 V.S.D. } = \frac{4}{5} \text{ M.S.D. } = \frac{4}{5}\times\frac{1}{8}=\frac{1}{10} \text{ cm}$$

Vernier constant V.C. = 1 M.C.D. – 1 V.S.D.

$$= \frac{1}{8}-\frac{1}{10}=\frac{1}{40} \text{ cm} = 0.25 \text{ cm}$$

Case (i): If pitch of screw gauge = 2 × V.C. = 2 × 0.025 cm = 0.05 cm, then least count of screw gauge is

$$\text{L.C.} = \frac{0.05\,\text{cm}}{100} = 0.005 \text{ cm} = 0.005 \text{ mm}$$

So choice (a) is wrong and choice (b) is correct.
Case (ii): If the least count of the linear scale of the screw gauge is twice the vernier constant, i.e. if 1 M.C.D. = 2 × 0.25 cm = 0.05 cm = 0.5mm, then the pitch of screw gauge = 2 × 0.5mm = 1 mm and its least count will be = $\frac{1\,\text{mm}}{100}$ = 0.01 mm. So choice (c) is correct and choice (d) is wrong. Hence the correct choices are (b) and (c).

10. **Case (i)** If h, c and G are used to form a unit of length to L then,

$L \propto h^a\ c^b\ G^c$

or $L = k\ [ML^2T^{-1}]^a \times (LT^{-1}]^b \times [M^{-1}\ L^3T^{-2}]^C$

where k is a dimensionless constant. Equating the power of M, L and T we get

$a - c = 0$, $2a + b\ 3c = 1$ and $-a - b - 2c = 0$.

These equations give $a = \frac{1}{2}$, $b = -\frac{3}{2}$ and $c = \frac{1}{2}$. Thus

$L = k^{1/2}\ h^{-1/2}\ c^{3/2}\ g^{1/2}$

So choices (c) and (d) are correct.

Case (ii) : If h, c and G are used to form a unit of mass M, then

$M = kh^ac^bG^c$

Substituting the dimensions of h, c and G and equating the powers of M, L and T, we get $a = \frac{1}{2}$, $b = \frac{1}{2}$, and $c = -\frac{1}{2}$.

Thus $M = kh^{1/2}c^{1/2}G^{-1/2}$

So the correct choice is (a). Hence the correct choices are (a), (c) and (d).

11. It follows from the figure that, for the first oscillator, the amplitude is equal to a because the maximum value of displacement x is a.

The velocity of the oscillator is given by

$$v = a\omega\left(1-\frac{x^2}{a^2}\right)^{1/2}$$

and its momentum is (m = mass of oscillator)

$$p = mv = ma\omega\left(1-\frac{x^2}{a^2}\right)^{1/2}$$

For the first oscillator, when $x = 0$, $p = p$. Hence $b = ma\omega_1$

$$\Rightarrow \frac{a}{b} = m\omega_1\ \ n^2 = m\omega_1 \qquad ...(i)$$

For the second oscillator, it follows the second figure, that for $x = 0$, $p = R$ and amplitude = R. Hence

$R = mR\omega_2$

$\Rightarrow\ 1 = mR\omega_2$ ...(ii)

From (1) and (2) it follows that $\frac{\omega_2}{\omega_1} = n^2$, which is choice (b).

The energy of the oscillator is

$$E_1 = \frac{1}{2}ma^2\omega_1^2$$

and $E_2 = \frac{1}{2}mR^2\omega_2^2$

$$\therefore\ \frac{E_1}{E_2} = \left(\frac{a}{R}\right)^2 \times \left(\frac{\omega_1}{\omega_2}\right)^2$$

$$= n^2 \times \frac{1}{n^2} \times \frac{\omega_1}{\omega_2} \qquad \left(\because \frac{\omega_1}{\omega_2} = \frac{1}{n^2}\right)$$

$$= \frac{\omega_1}{\omega_2}$$

or $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$, which is choice (d). So the correct choices are (b) and (d).

12. Let the required distance of the other mass from O be x. From conservation of angular momentum, $I\omega = I'\omega'$

$$\Rightarrow\ MR^2\omega = \left[MR^2 + \left(\frac{M}{8}\right) \times \left(\frac{3R}{5}\right)^2 + \frac{Mx^2}{8}\right] \times \frac{8\omega}{9}$$

$$\Rightarrow\ R^2 = \left(R^2 + \frac{9R^2}{200} + \frac{x^2}{8}\right) \times \frac{8}{9}$$

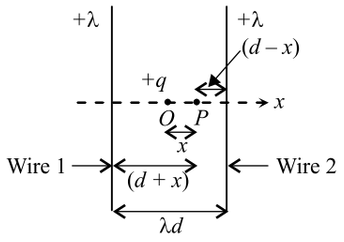
$$\Rightarrow\ 9R^2 = 8R^2 + \frac{9R^2}{25} + x^2$$

$$\Rightarrow\ x^2 = \frac{16R^2}{25} \Rightarrow x = \frac{4R}{5},$$ which is choice (d)

13. Electric field due to an infinitely long wire carrying a uniform linear charge density λ at a distance r from it is given by

$$\vec{E} = \frac{\lambda\hat{n}}{2\pi\varepsilon_0 r}$$

where $\hat{n}$ is a unit outward normal away from the wire if λ is positive.



Case 1: When q is positive

Charge $+q$ is displaced from its equilibrium positive O to a positive P by a small amount x

$<< d$ and released. Electric field at P due to wire 1 is

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0(d+x)} \text{ along } +x \text{ direction}$$

Therefore, force on charge $+q$ due to this electric field is

$F_1 = qE_1$ along $+x$ direction

Similarly, force on charge $+q$ due to wire 2 is

$F_2 = qE_2$ along $-x$ direction

where, $E_2 = \frac{\lambda}{2\pi\varepsilon_0(d-x)}$ along $-x$ direction.

Since $E_2 > E_1$, F_2 will be greater than F_1 and net restoring force on $+q$ is

$$F = -(F_2 - F_1)$$

$$= \frac{q\lambda}{2\pi\varepsilon_0}\left(\frac{1}{d-x} - \frac{1}{d+x}\right)$$

or $$F = -\left(\frac{2q\lambda}{2\pi\varepsilon_0{}^2}\right)x$$

Since $x << d$,

$$F = -\left(\frac{2q\lambda}{2\pi\varepsilon_0 d^2}\right)x$$

Since $f \propto (-x)$, the motion of charge $+q$ will be simple harmonic.

Case-2: When q is negative.

In this case $E_1 = \frac{\lambda}{2\pi\varepsilon_0(d+x)}$ will be along the $+x$ direction and force on charge $(-q)$ is

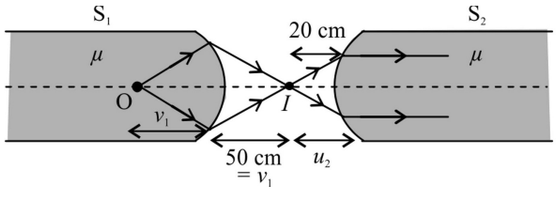
$F_1 = -qE_1$ along the $-x$ direction

Also force $F_2 = -qE_2$ will be along $+x$ direction,

where $E_2 = \frac{\lambda}{2\pi\varepsilon_0(d-x)}$

Since $F_2 > F_1$ the charge $-q$ will keep on moving along $+x$ direction (i.e. in the direction of displacement). Hence it cannot return to its equilibrium position O and will, therefore, not execute simple harmonic motion. So, the correct choice is (c).

14.



For refraction at slap S_1

$u_1 = -50$ cm, $R = -10$ cm, μ 1.5, $v_1 = ?$

$$\frac{1}{v_1} - \frac{1.5}{-50} = \frac{1-1.5}{-10}$$

$$\Rightarrow \frac{1}{v_1} + \frac{3}{100} = \frac{1}{20} \Rightarrow v_1 = +50 \text{ cm}$$

For refraction at slap S_2

$u_2 = -(d - 50)$ cm,

$R = +10$ cm,

$v_2 = \times$, $\mu = 1.5$

$$\frac{1.5}{\infty} - \frac{1}{-(d-50)} = \frac{1.5-1}{10}$$

$$\Rightarrow \frac{1}{(d-50)} = \frac{1}{20}$$

$$\Rightarrow d - 50 = 20 \Rightarrow d = 70 \text{ cm}$$

15. The magnetic force on a current element $\vec{l}$ in a magnetic field $\vec{B}$ is given by

$$\vec{F} = I(\vec{l} \times \vec{B}) = I(\overrightarrow{PQ} \times \vec{B})$$

The magnitude of $\overrightarrow{PQ}$ is $(L + R + R + L) = 2(L + R)$ and its direction is along the $+x$ axis. Hence $\overrightarrow{PQ} = 2(L + R)\hat{i}$.

(a) If $\vec{B}$ is along z-axis, $\vec{B} = B\hat{k}$. In this case

$$\vec{F} = I\left[2(L+R)\hat{i} \times B\hat{k}\right]$$

$$= 2I(L+R)B(-\hat{j}) \qquad (\because \hat{i} \times \hat{k} = -\hat{j})$$

Thus the direction of $\vec{F}$ is along negative y-axis and its magnitude is $2I\ (L + R)\ B$. So choice (a) correct.

(b) If $\vec{B}$ is along x-axis, $\vec{B} = B\hat{i}$. In this case

$$\vec{F} = 2I(L+R)B(\hat{i} \times \hat{i}) = 0 \qquad (\because \hat{i} \times \hat{i} = 0)$$

So choice (b) is also correct

(c) In this case $\vec{B} = B\hat{j}$ and

$$\vec{F} = 2I(L+R)B(\hat{i} \times \hat{j})$$

$$= 2I(L+R)B\hat{k}$$

So choice (c) is also correct

(d) In this case $\vec{B} = B\hat{k}$ and

$$\vec{F} = 2I(L+R)B(\hat{i} \times \hat{k})$$

$$= 2I(L+R)(-\hat{j})$$

So choice (d) is also correct

16. Average energy per mole of an ideal gas = $\frac{1}{2}RT$ per degre of freedom. Helium (being mono-atomic) has 3 degrees of freedom and hydrogen (being diatomic) has 5 degrees of freedom. Hence the total average of one mole of helium and one mole of hydrogen = $\frac{3}{2}RT+\frac{5}{2}RT = 4RT$. Therefore average energy per mole of the mixture = $\frac{4RT}{2} = 2RT$. So choice (a) is correct.

Speed of sound $v=\sqrt{\frac{\gamma RT}{M}}$

γ of mixture = $\frac{n_{He}(C_p)_{He}+m_{H_2}(C_P)_{H_2}}{n_{He}(C_v)_{He}+n_{H_2}(C_v)_{H_2}}$

$$= \frac{1\times\frac{5}{2}RT+1\times\frac{7}{2}RT}{1\times\frac{3}{2}RT+1+\frac{5}{2}RT} = \frac{3}{2}$$

M of mixture = $\frac{1\times2+1\times4}{2} = 3$

$$\therefore \frac{v \text{ in mixture}}{v \text{ in helium}} = \sqrt{\frac{r \text{ of mixture}}{r \text{ of helium}}\times\frac{M_{He}}{M_{mixture}}}$$

$$= \sqrt{\frac{3/2}{5/3}\times\frac{4\times1}{3}}$$

$$= \sqrt{\frac{6}{5}}$$

So, choice (b) is also current.

Finally $\frac{v_{rms} \text{ of mixture}}{v_{rms} \text{ of hydrogen}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$, which is choice (d).

17. $R=\frac{\rho l}{A}$

The resistance of aluminium bar is

$$R_1 = \frac{(2.7\times10^{-8})\times(0.05)}{(0.007)^2-(0.002)^2} = 3\times10^{-5}\Omega$$

The resistance of iron bar is

$$R_2 = \frac{(1.0\times10^{-7})\times(0.05)}{(0.002)^2} = \frac{5}{4}\times10^{-3}\Omega$$

Since the bars are in parallel, the equivalent resistance of the composite slab is

$$R_{eq} = \frac{R_1R_2}{R_1+R_2}$$

$$= \frac{3\times10^{-5}\times\frac{5}{4}\times10^{-3}}{3\times10^{-5}+\frac{5}{4}\times10^{-3}}$$

$$= \frac{150\times10^{-6}}{5.12} = \frac{1875}{64}\mu\Omega$$

So, the correct choice is (b).

18. Einstein's photoelectric equation is

$$h\nu = eV_0 + \phi$$

$$\Rightarrow \frac{hc}{\lambda} = eV_0+\phi$$

$$\Rightarrow \frac{hc}{e}\left(\frac{1}{\lambda}\right)-\frac{\phi}{e}$$

Hence the graph of V_0 against $\frac{1}{\lambda}$ is a straight line having a negative slope and the graph of V_0 agains λ is a curve with V_0 tending to zero λ as λ tends to infinity. So the correct choices are (a) and (c).

Section-III

19. Answer

(a) → (r), (b) → (p, s), (c) → (q, t), (d) → (q, r, t)

Explanation:

(a) Nuclear fusion is the process which is responsible for energy production in stars.

(b) In a nuclear reactor, thermal neutrons are made to hit a U-235 nucleus which breaks it up into stable nuclei with the release of energy. Heavy water is used as a moderator in a nuclear reactor.

(c) $^{60}_{27}$Co undergoes β-decay. In negative β-decay, neutrons are emitted.

(d) β-decay and nuclear fusion emit γ-rays.

20. (a) $F_x = -\frac{dU_1}{dx} = -\frac{d}{dx}\left[\frac{U_0}{2}\left\{1-\frac{x^2}{a^2}\right\}^2\right]$

$$= \frac{2U_0}{a^2}(x-a)^x(x+a)$$

So the correct choice are (p, q, r, t)

(b) $F_x = -\frac{dU_2}{dx} = -\frac{d}{dx}\left[\frac{U_0}{2}\frac{x^2}{a^2}\right]$

$$= -\frac{U_0\times2x}{2a^2} = -\frac{U_0x}{a^2}$$

Since $F_x \propto -x$, the correct choices are (q) and (s)

(c) $F_x = -\dfrac{dU_3}{dx} = -\dfrac{d}{dx}\left[\dfrac{U_0}{2}\dfrac{x^2}{a^2}\exp\left(-\dfrac{x^2}{a^2}\right)\right]$

$= U_0 \dfrac{\exp\left(-\dfrac{x^2}{a^2}\right)}{a^2} \times (x)(x-a)(x+a)$

So, the correct choices are (p, q, r and s)

(d) $F_x = -\dfrac{dU_4}{dx} = -\dfrac{d}{dx}\left[\dfrac{U_0}{2}\left(\dfrac{x}{a} - \dfrac{1}{3}\dfrac{x^3}{a^3}\right)\right]$

$= -\dfrac{U_0}{2a^3}[(x-a)(x+a)]$

So, the correct choices are (p, r, and t)

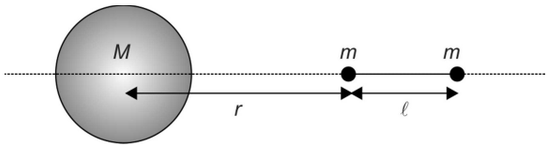
JEE ADVANCED 2015: PAPER–II
(MODEL SOLUTIONS)

SECTION I

(Single Digit Integer Type)

This section contains **EIGHT** questions
The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive

1. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is



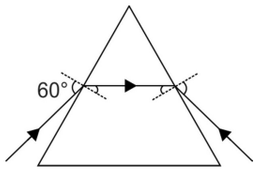
2. The energy of a system as a function of time t is given as $E(t) = A^2\exp(-at)$, where $a = 0.2\ s^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50 %, the percentage error in the value of $E(t)$ at $t = 5$ s is

3. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is:

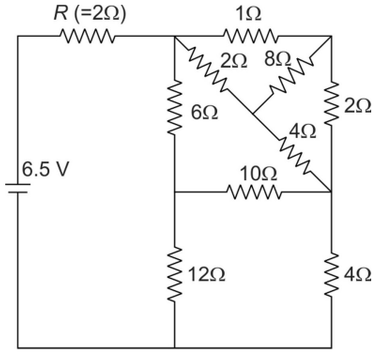
4. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\frac{\pi}{3}, \frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is:

5. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = \frac{-dN}{dt}$ and $R = \frac{-dA}{dt}$, where $N(t)$ is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rates of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is:

6. A monochromatic beam of light is incident at 600 on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is [2015]



7. In the following circuit, the current through the resistor R (= 2Ω) is I Amperes. The value of I is



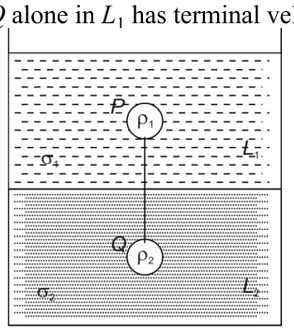
8. An electron in an excited state of Li^{2+} ion has angular momentum = $\frac{3h}{2\pi}$. The de-Broglie wavelength of the electron in this state is $\lambda = p\pi a_0$, where a_0 is the Bohr radius. The value of p is

SECTION II

(One or More than One Options Correct Type)

This section contains 10 questions
Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four options(s) is (are) correct

9. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity V_P and Q alone in L_1 has terminal velocity V_Q, then



(a) $\left|\frac{\mathbf{V}_P}{\mathbf{V}_Q}\right| = \frac{\eta_1}{\eta_2}$ (b) $\left|\frac{\mathbf{V}_P}{\mathbf{V}_Q}\right| = \frac{\eta_2}{\eta_1}$

(c) $\mathbf{V}_P.\mathbf{V}_Q > 0$ (d) $\mathbf{V}_P.\mathbf{V}_Q < 0$

10. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are)

(a) $\mu_0 I^2 = \varepsilon_0 V^2$ (b) $\varepsilon_0 I = \mu_0 V$

(c) $I = \varepsilon_0 cV$ (d) $\mu_0 cI = \varepsilon_0 V$

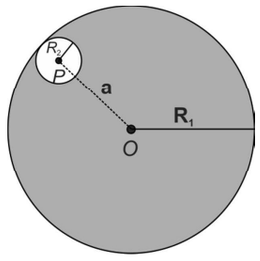
11. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\mathbf{r}$ is $\mathbf{E}(\mathbf{r})$, then the correct statement(s) is(are)

(a) $\mathbf{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\mathbf{r}$

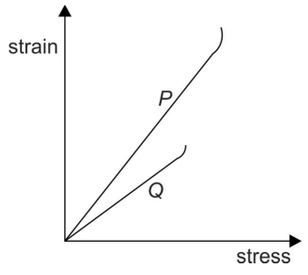
(b) $\mathbf{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\mathbf{r}$

(c) $\mathbf{E}$ is uniform, its magnitude is independent of a but its direction depends on $\mathbf{a}$

(d) $\mathbf{E}$ is uniform and both its magnitude and direction depend on $\mathbf{a}$



12. In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is(are)



(a) P has more tensile strength than Q

(b) P is more ductile than Q

(c) P is more brittle than Q

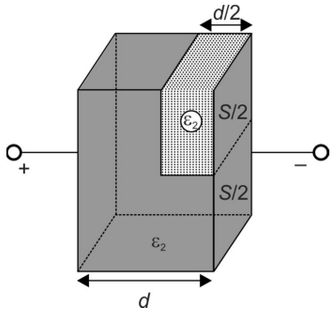
(d) The Young's modulus of P is more than that of Q

13. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $P(r)$ is the pressure at $r(r < R)$, then the correct option(s) is(are)

(a) $P(r = 0) = 0$ (b) $\frac{P\left(r = \frac{3R}{4}\right)}{P\left(r = \frac{2R}{3}\right)} = \frac{63}{80}$

(c) $\frac{P\left(r = \frac{3R}{5}\right)}{P\left(r = \frac{2R}{5}\right)} = \frac{16}{21}$ (d) $\frac{P\left(r = \frac{R}{2}\right)}{P\left(r = \frac{R}{3}\right)} = \frac{20}{27}$

14. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is



(a) $\frac{6}{5}$ (b) $\frac{5}{3}$

(c) $\frac{7}{5}$ (d) $\frac{7}{3}$

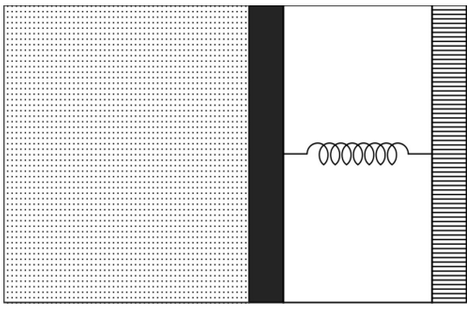
15. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are)

(a) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(b) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(c) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(d) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$



16. A fission reaction is given by ${}^{236}_{92}\mathrm{U} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + x + y$, where x and y are two particles. Considering ${}^{236}_{92}\mathrm{U}$ to be at rest, the kinetic energies of the products are denoted by K_{Xo}, K_{Sr}, K_{x}(2MeV) and K_{y}(2MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}\mathrm{U}$, ${}^{140}_{54}\mathrm{Xe}$ and ${}^{94}_{38}\mathrm{Sr}$ be 7.5 MeV, 8.5 MeV and 8.5 MeV respectively. Considering different conservation laws, the correct option(s) is(are)

(a) $x = n$, $y = n$, $K_{\mathrm{Sr}} = 129$MeV, $K_{\mathrm{Xe}} = 86$ MeV
(b) $x = p$, $y = e$, $K_{\mathrm{Sr}} = 129$ MeV, $K_{\mathrm{Xe}} = 86$ MeV
(c) $x = p$, $y = n$, $K_{\mathrm{Sr}} = 129$ MeV, $K_{\mathrm{Xe}} = 86$ MeV
(d) $x = n$, $y = n$, $K_{\mathrm{Sr}} = 86$ MeV, $K_{\mathrm{Xe}} = 129$ MeV

SECTION III

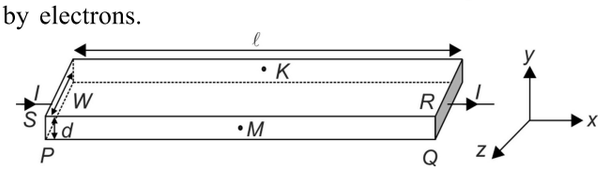
This section contains **TWO** Paragraphs
Based on each paragraph, there will be **TWO** questions
Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct

Questions 17 and 18 are based on the following paragraph.

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d, respectively.

A uniform magnetic field **B** is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation

continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



17. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are)

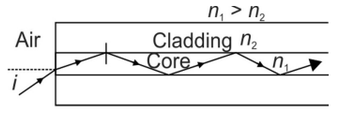
(a) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$
(b) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$
(c) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$
(d) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

18. Consider two different metallic strips (1 and 2) of same dimensions (lengths ℓ, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are)

(a) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$
(b) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$
(c) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$
(d) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

Questions 19 and 20 are based on the following paragraph.

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as sin i_m.



19. For two structures namely S_1 with $n_1 = \frac{\sqrt{45}}{4}$ and n_2, and S_2 with $n_1 = \frac{8}{5}$ and $n_2 = \frac{7}{5}$ and taking the refractive index of water to be $\frac{4}{3}$ and that of air to be 1, the correct option(s) is(are)

(a) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$
(b) NA of S_1 immersed in liquid of refractive index $\frac{16}{\sqrt{15}}$ is the same as that of S_2 immersed in water
(c) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$
(d) NA of S_1 placed in air is the same as that of S_2 placed in water

20. If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 \leftarrow NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

(a) $\frac{NA_1 + NA_2}{NA_1 + NA_2}$ (b) $NA_1 + NA_2$
(c) NA_1 (d) NA_2

Answers

Section-I

1. 7	**2.** 4	**3.** 6
4. 3	**5.** 2	**6.** 2
7. 1	**8.** 2	

Section-II

9. (a), (d)	**10.** (a), (c)	**11.** (d)
12. (a), (b)	**13.** (b), (c)	**14.** (d)
15. (a), (b), (c)	**16.** (a)	

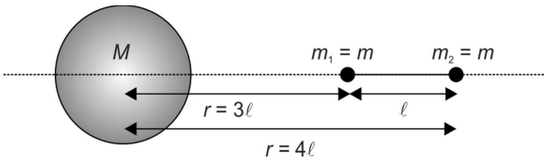
Section-III

17. (a), (d)		
18. (a), (c)	**19.** (a), (c)	**20.** (d)

Hints and Solutions

Section-I

1.



Mass m_1 at $r = 3l$ will be attracted by mass m_2 to the right and by mass M to the left. The net force acting of m_1 is

$$F_1 = \frac{Gm_1M}{(3l)^2} - \frac{Gm_1m_2}{l^2}$$

Mass m_2 at $r = 4l$ will be attracted by m_1 to the left and by M also to the left. The net force acting on m_2 is

$$F_2 = \frac{Gm_2M}{(4l)^2} + \frac{Gm_1m_2}{l}$$

The tension in the rod connecting m_1 and m_2 will be zero if $F_1 = F_2$, i.e. if

$$\frac{Gm_1M}{(3l)^2} - \frac{Gm_1m_2}{l^2} = \frac{Gm_2M}{(4l)^2} + \frac{Gm_1m_2}{l^2}$$

It is given that $m_1 = m_2 = m$. Therefore,

$$\frac{GmM}{9l^2} - \frac{Gm^2}{l^2} = \frac{GmM}{16l^2} + \frac{Gm^2}{l^2}$$

$$\Rightarrow \quad \frac{M}{9} - m = \frac{M}{16} + m$$

$$\Rightarrow \quad 2m = \frac{M}{9} - \frac{M}{16} = \frac{7M}{144} \Rightarrow m = \frac{7M}{288}$$

Given $m = \frac{kM}{288}$. Hence $k = 7$.

2. $E = A^2\ e^{-at}$

Taking logarithm of both sides,

$$\log E = 2 \log A - at$$

$$\therefore \quad \frac{\Delta E}{E} = \pm\frac{2\Delta A}{A} \pm a\Delta t$$

$$\Rightarrow \quad \frac{\Delta E}{E} = \frac{2\Delta A}{A} \pm a\frac{\Delta t}{t} \times t \qquad ...(1)$$

Given $\frac{\Delta A}{A} = 1.25\%$, $a = 0.2s^{-1}$ and $\Delta t = 1.5\%$

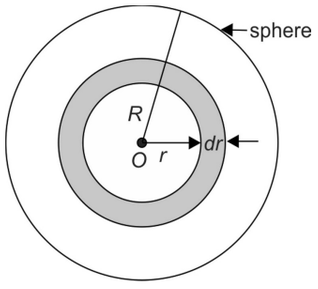
Substituting these values in Eq. (1), we have

$$\frac{\Delta E}{E} = \pm\ 2 \times 1.25\% \pm 0.2 \times 1.5\% \times 5$$

$$= \pm\ 2.5\% \pm 1.5\% = \pm\ 4\%$$

So the correct choice is 4.

3. Divide the sphere into very small spherical elements, each of a very small thickness dr. Consider one such element at a distance r from the centre O of the sphere.



Volume of element = $4\pi r^2\ dr$

Mass of element $dm = \rho \times 4\pi r^2 dr$

Moment of inertia of the element about an axis passing through O is

$$dI = \frac{2}{3}dmr^2$$

Moment of inertia of the sphere of radius R about O is

$$I = \int dI = \frac{2}{3} \times \frac{4\pi}{R}\int_0^R \rho r^2 dr\ r^2$$

$$= \frac{8\pi}{3}\int_0^R \rho r^4 dr$$

$$I_A = \frac{8\pi}{3}\int_0^R k\left(\frac{r}{R}\right) \times r^4 dr = \frac{8\pi k}{3R} \times \frac{R^6}{6}$$

$$= \frac{8\pi k}{18}R^5$$

Similarly,

$$I_B = \frac{8\pi}{3}\int_0^R k\left(\frac{r}{R}\right)^5 \times r^4 dr = \frac{8\pi k}{3R^5} \times \frac{R^{10}}{10}$$

$$= \frac{8\pi k}{30}R^5$$

$$\therefore \quad \frac{I_B}{I_A} = \frac{18}{30} = \frac{6}{10} \text{ which gives } n = 6.$$

4. Let a be the amplitude and ω be the angular frequency of each wave.

Intensity $\propto$ (amplitude)^2 or $I_0 = ka^2$ where k is a constant. The four harmonic waes are given by

$$y_1 = a \sin(\omega t - kx) = a \sin\theta,$$

where $\theta = (\omega t - kx)$

$$y_2 = a \sin\left(\omega t - kx + \frac{\pi}{3}\right)$$

$$= a \sin\left(\theta + \frac{\pi}{3}\right)$$

$$y_3 = a \sin\left(\omega t - kx + \frac{2\pi}{3}\right)$$

$$= a \sin\left(\theta + \frac{2\pi}{3}\right)$$

and $y_4 = a \sin(\omega t - kx + \pi) = a \sin(\theta + \pi)$

From superposition principle, the resultant wave is given by

$$y = y_1 + y_2 + y_3 + y_4$$

$$= a\left[\sin\theta + \sin\left(\theta + \frac{\pi}{3}\right) + \sin\left(\theta + \frac{2\pi}{3}\right) + \sin(\theta + \pi)\right]$$

$$= a\left[\sin\left(\theta + \frac{\pi}{3}\right) + \sin\left(\theta + \frac{2\pi}{3}\right)\right] \quad [\because \sin(\theta + \pi) = -\sin\theta]$$

$$= 2a \sin\left(\theta + \frac{\pi}{2}\right)\cos\left(\frac{\pi}{6}\right)$$

$$\Rightarrow \quad y = 2a\cos\left(\frac{\pi}{6}\right)\sin\left(\omega t - kx + \frac{\pi}{2}\right)$$

or $\quad y = A \sin\left(\omega t - kx + \frac{\pi}{2}\right)$

where A is the amplitude of the resultant wave.

$$A = 2a \times \frac{\sqrt{3}}{2} = \sqrt{3}a$$

$\therefore$ Intensity of the resultant wave is

$$I = kA^2 = 3ka^2 = 3I_0$$

Given $I = n\, I_0$, Therefore $n = 3$

Note : The superposition principle holds only if the waves are collinear. This is not stated in the question.

5. $A_P = A_0 e^{\frac{-t}{\tau}}$ and $A_Q = A_0 e^{\frac{-t}{2\tau}}$

$$\therefore \quad R_P = \frac{dA_P}{dt} = -\frac{A_0}{\tau} e^{\frac{-t}{\tau}}$$

and $\quad R_Q = \frac{dA_Q}{dt} = -\frac{A_0}{2\tau} e^{\frac{-t}{2\tau}}$

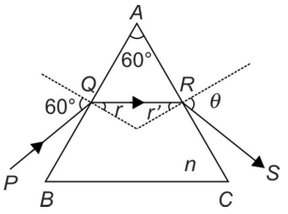
At $\quad t = 2\tau$

$$R_P = -\frac{A_0}{\tau} e^{-2}$$

and $\quad R_Q = -\frac{A_0}{2\tau} e^{-1}$

$$\therefore \quad \frac{R_P}{R_Q} = 2e^{-1} = \frac{2}{e}.$$ Hence $n = 2$

6.



$r + r' = A \Rightarrow r' = A - r = 60° - r$

Applying Snell's law at Q,

$\sin 60° = n \sin r$ ...(1)

Differentiating (1) w.r.t. n we have

$$0 = \sin r + n \cos r \frac{dr}{dn}$$

$$\Rightarrow \quad \frac{dr}{dn} = -\frac{\sin r}{n \cos r} \quad ...(2)$$

Applying Snell's law at R, we have

$n \sin r' = \sin\theta$

$\Rightarrow n \sin(60° - r) = \sin\theta$ ...(3)

Differentiating (3) w.r.t. n we have

$$\sin(60° - r) - n \cos(60° + r)\frac{dr}{dn} = \cos\theta \frac{d\theta}{dn}$$

$$\Rightarrow \frac{d\theta}{dn} = \frac{1}{\cos\theta}\left[\sin(60° - r) - n\cos(60° + r)\frac{dr}{dn}\right] \quad ...(4)$$

Using (2) in (4), we get

$$\frac{d\theta}{dn} = \frac{1}{\cos\theta}\left[\sin(60° - r) + n\cos(60° + r)\frac{\sin r}{n\cos r}\right]$$

$$\Rightarrow \frac{d\theta}{dn} = \frac{1}{\cos\theta}\left[\sin(60° - r) + \cos(60° + r)\frac{\sin r}{\cos r}\right] \quad ...(5)$$

If $n = \sqrt{3}$ eq. (1) gives $\sin r = \frac{\sin 60°}{\sqrt{3}} = \frac{1}{2}$

$\Rightarrow r = 30°$

Using $r = 30°$ in (3) gives $\sqrt{3}\sin 30° = \sin\theta$

$\Rightarrow \theta = 60°$

Using $r = 30°$ and $\theta = 60°$ in (5), we get

$$\frac{d\theta}{dn} = \frac{1}{\cos 60^\circ}\left[\sin 30^\circ + \cos 30^\circ \times \frac{\sin 30^\circ}{\cos 30^\circ}\right]$$

$$= \frac{2\sin 30^\circ}{\cos 60^\circ} = \frac{2\times\frac{1}{2}}{\frac{1}{2}} = 2$$

So $m = 2$

7. The circuit can be simplified as follows:

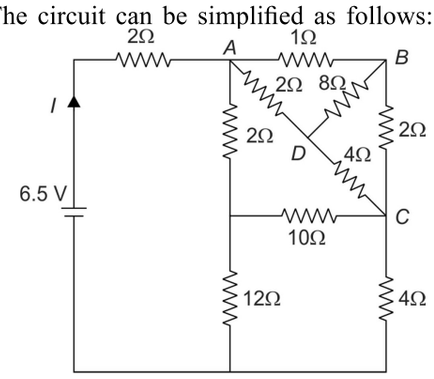


Fig.1

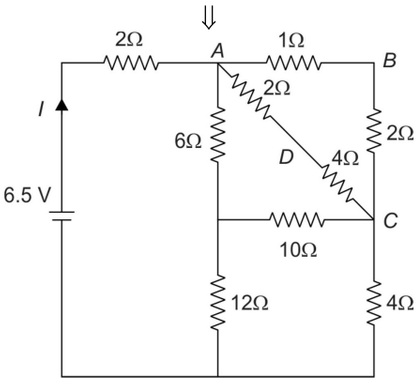


Fig.2

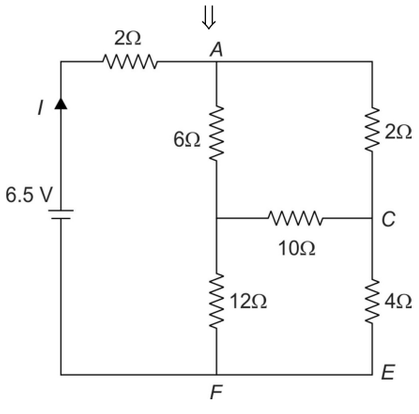


Fig.3

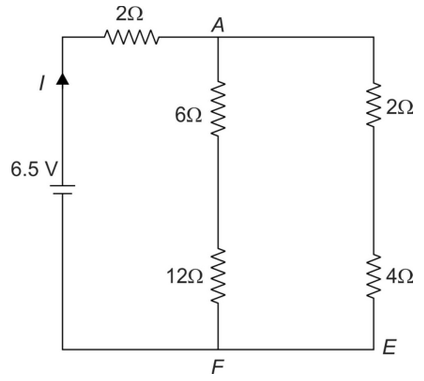


Fig.4

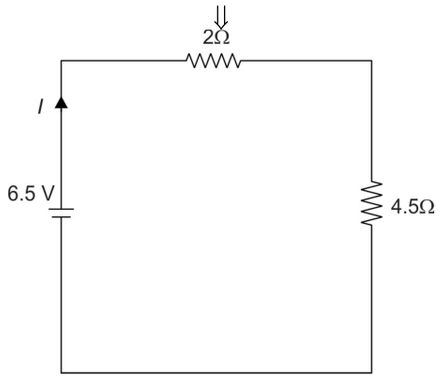


Fig.5

The circuit in fig. 1 reduces to fig. 2 because *ABCD* is a balanced Wheatstone's bridge and the 8Ω resistor is redundant. In fig. 2, the resistance between *A* and *C* is a parallel combination of 1Ω + 2Ω = 3Ω and 2Ω + 4Ω = 6Ω which is 2Ω and circuit 2 reduces to circuit 3.

In circuit 3 we again have a balanced Wheatstone's Bridge and the 10Ω resistor becomes redundant. So, circuit 3 reduces to circuit 4 which further simplifies to fig. 5. Hence the current through the 2Ω resistor is

$$I = \frac{6.5V}{2\Omega + 4.5\Omega} = 1\text{A}$$

8. Bohr's quantization condition is

$$L = \frac{nh}{2\pi} = \frac{3h}{2\pi} \qquad \text{(given)}$$

$\Rightarrow n = 3$

Also, $mvr = \frac{3h}{2\pi} \Rightarrow mv = \frac{3h}{2\pi r}$

de-Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{h}{\frac{3h}{2\pi r}} = \frac{2\pi r}{3}$$

Bohr's radius of the electron in the n-orbit is

$$r = a_0\frac{n^2}{Z} = \frac{a_0(3)^2}{Z} \qquad (\because n = 3)$$

For L_i, $Z = 3$,

$$\lambda = \frac{2\pi}{3}\times\frac{a_0(3)^2}{3} = 2\pi a_0$$

comparing with $\lambda = p\pi a_0$, we get $p = 2$

Section-II

9. Let r be the radius of each sphere and v the volume. Then for floatation.

$$\rho_1 vg = \sigma_1 vg \text{ and } \rho_2 vg = \sigma_2 vg$$

Adding, we have

$$(\rho_1 + \rho_2)vg = (\sigma_1 + \sigma_2)g$$

$$\Rightarrow \quad \rho_1 + \rho_2 = \sigma_1 + \sigma_2$$

or $\quad \sigma_2 - \rho_1 = \rho_1 - \sigma_2 = -(\sigma_2 - \rho_1)$

Since the string is taut, $\rho_1 < \sigma_1$ and $\rho_2 > \sigma_2$. The terminal velocities are

$$\mathbf{V}_P = \frac{2}{9}\frac{r^2(\sigma_2 - \rho_1)\vec{g}}{\eta_2}$$

and $\mathbf{V}_Q$ and $\dfrac{2}{9}\dfrac{r^2(\sigma_1 - \rho_2)\vec{g}}{\eta_1}$

Dividing, we get

$$\frac{\mathbf{V}_P}{\mathbf{V}_Q} = \frac{\sigma_2 - \rho_1}{\sigma_1 - \rho_2}\times\frac{\eta_1}{\eta_2}$$

Using (1)

$$\frac{\mathbf{V}_P}{\mathbf{V}_Q} = -\frac{\eta_1}{\eta_2}$$

Also $\mathbf{V}_P \times \mathbf{V}_Q < 0$ because $\mathbf{V}_P$ and $\mathbf{V}_Q$ are in opposite directions. So the correct choices are (a) and (d).

10. (i) Every stored in an inductor is

$$E = \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right)I^2$$

$$\therefore\ \mu_0 I^2 = \frac{2El}{N^2 A}$$

$$\therefore\ [\mu_0 I^2] = \frac{[\mathrm{ML^2\,T^{-2}}]\times[\mathrm{L}]}{[\mathrm{L^2}]} = [\mathrm{MLT^{-2}}]$$

Energy stored in a capacitor is

$$E = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{l}\right)V^2$$

$$\therefore\ \varepsilon_0 V^2 = \frac{2El}{A}$$

$$\varepsilon_0 V^2 = \frac{[\mathrm{ML^2T^{-2}}]\times[\mathrm{L}]}{[\mathrm{L^2}]} = [\mathrm{MLT^{-2}}]$$

So choice (a) is correct and choice (b) is wrong.

(ii) $Q = CV = \left(\dfrac{\varepsilon_0 A}{l}\right)\times V$

or $\varepsilon_0 V = \dfrac{Ql}{A}$

or $\varepsilon_0 cV = \dfrac{Ql}{A}$

or $[\varepsilon_0 eV] = \dfrac{[AT]\times[L]\times[LT^{-1}]}{[L^2]}$

$= [A]$

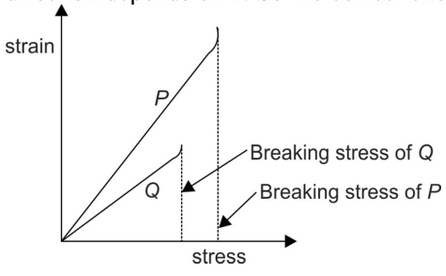
So $\varepsilon_0 cV = I$, which is choice (c) choice (d) is wrong.

11. The electric field inside the cavity at a position r is given by

$$\mathbf{E(r)} = \frac{\rho}{3\varepsilon_0} = \mathbf{(OP)} = \frac{\rho\mathbf{a}}{3\varepsilon_0}$$

Where and ρ is the volume charge density, Hence E(r) is uniform and its magnitude as well a's its direction depends on a. So the correct choice is (d).

12.



The breaking stress of P is higher than that of Q. Hence P is more ductile than Q.

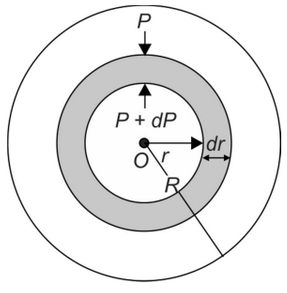
The reciprocal of the straight line portion of strain–stress graph = Young's modulus. The slope of P is greater than that of Q. So the correct choice are (a) and (b).

13. Consider a point at a distance $r < R$. from the centre of the sphere. If ρ is the density of fluid in the sphere the mass of the sphere of radius r is

$$M = \frac{4\pi}{3}r^3\rho$$

Gravitational intensity at a distance r is

$$I = \frac{GM}{r^2} = \frac{G\times\frac{4\pi}{3}r^3\rho}{r^2} = \frac{4\pi G\rho r}{3}$$



Consider a small spherical element of thickness dr and area of A at distance r. The mass of this element is

$$m = \rho \times V = \rho dAdr$$

The pressure on the concave side of the fluid is greater than that on the convex side. If dP is the excess press the force on the element of area of dA is

$$dF = dP \times dA \qquad ...(i)$$

Since gravitational field intensity is force per unit mass dF is also given by

$$dF = Im = \frac{4\pi G\rho A}{3} \times \rho dAdr \qquad ...(2)$$

Equating (1) and (2) we get

$$dp = \frac{4\pi G\rho^2 rdr}{3}$$

Integrating,

$$P = \frac{4\pi G\rho^2}{3}\int_r^R rdr$$

$$\Rightarrow P = \frac{4\pi G\rho^2}{2\times 3}(R^2 - r^2)$$

$$\Rightarrow P = k\,(r^2 - r^2) \qquad ...(3)$$

$k = \dfrac{2\pi G\rho^2}{3}$ is a constant

Choice (a) : If $r = 0$, Eq. (3) gives $P = kR^2$. So choice (a) is wrong.

Choice (b) : For $r = \dfrac{3R}{4}, P_1 = k\left(R^2 - \dfrac{9R^2}{16}\right) = \dfrac{7kR^2}{16}$

For $r = \dfrac{2R}{3}$, $P_2 = k\left(R^2 - \dfrac{4R^2}{9}\right) = \dfrac{5kR^2}{9}$

$$\therefore \frac{P_1}{P_2} = \frac{7}{16}\times\frac{9}{5} = \frac{63}{80}$$

So choice (b) is correct.

Choice (c) : For $r = \dfrac{3B}{5}$,

$$P_3 = k\left(R^2 - \frac{9R^2}{25}\right) = \frac{16kR^2}{25}$$

For $r = \dfrac{2R}{5}$, $P_4 = \dfrac{21kR^2}{25}$

$$\therefore \frac{P_3}{P_4} = \frac{16}{25}\times\frac{25}{21} = \frac{16}{21}$$

So choice (c) is correct.

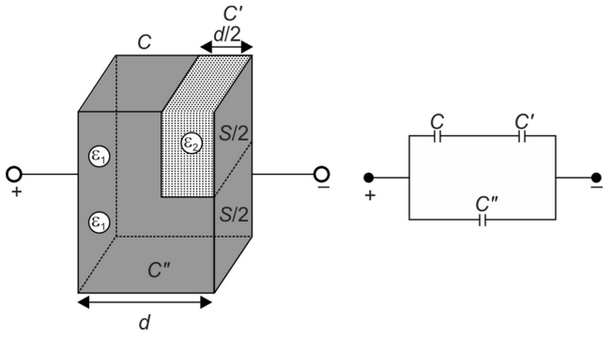
Choice (d) : Similarly we can show that

$$\frac{P \text{ at } r = \frac{R}{2}}{P \text{ at } r = \frac{R}{3}} = \frac{21}{32}$$

So choice (d) is incorrect.

Hence the correct choices are (b) and (c)

14. $C_1 = \dfrac{\varepsilon_0 S}{d}$



$$C_1 = \frac{\varepsilon_0 S}{d}$$

The capacitance of S the capacitor C is

$$C = \frac{\varepsilon_1\varepsilon_0\frac{S}{2}}{\frac{d}{2}} = \frac{2\varepsilon_0 S}{d} = 2C_1$$

$$C' = \frac{\varepsilon_2\varepsilon_0\frac{S}{2}}{\frac{d}{2}} = \frac{4\varepsilon_0 S}{d} = 4C_1$$

$$C'' = \frac{\varepsilon_1\varepsilon_0\frac{S}{2}}{d} = \frac{2\varepsilon_0\frac{S}{2}}{d} = C_1$$

C is in series with C' and this series combination is in parallel with C″. Therefore,

$$C_2 = \frac{CC'}{C+C'} + C''$$

$$= \frac{2C_1\times 4C_1}{2C_1 + 4C_1} + C_1$$

$$= \frac{4C_1}{3} + C_1 = \frac{7C_1}{3}$$

$\therefore \dfrac{C_2}{C_1} = \dfrac{7}{3}$. So the correct choice is (d)

15. Case (a) : $\frac{P_1v_1}{T_1} = \frac{P_2v_2}{T_2}$

If $v_2 = 2v_1$ and $I_2 = 3T_1$, then

$$\frac{P_1v_1}{T_1} = \frac{P_2 \times 2v_1}{T_2} \Rightarrow P_2 = \frac{3P_1}{2}$$

If A is the cross-sectional area of the piston and k is the spring constant, then the pressure of the gas increases by $\frac{kx}{A}$. Hence

$$P_2 = P_1 + \frac{kx}{A}$$

$$\text{or} \quad \frac{3P_1}{2} = P_1 + \frac{kx}{A}$$

$$\Rightarrow \quad kx = \frac{P_1A}{2}$$

Now increase in volume of gas = xA. Therefore, $v_2 - v_1 = xA$

$$2v_1 - v_2 = x_A \text{ or } v_1 = x_A \text{ or } x = \frac{v_1}{A}$$

$$\therefore \text{ Energy stored in spring} = \frac{1}{2}kx^2$$

$$= \frac{1}{2}kx \times x$$

$$= \frac{1}{2} \times \frac{P_1A}{2} \times \frac{v_1}{A} = \frac{P_1v_1}{A}$$

So choice (*a*) is correct.

Case (b) : Change in internal energy is

$$\Delta U = nC_v\Delta T$$

$$= n \times \frac{3R}{2} \times (3T_1 - T_1)$$

$$= n \times \frac{3R}{2} \times 2T_1$$

$$= n \times \frac{3R}{2} \times \frac{2P_1v_1}{nR} = 3P_1V_1,$$

which is choice (*b*)

Case (c) : In this case, $P_2 = \frac{4P_1}{3}, kx = \frac{P_1A}{3}$ and $x - \frac{2v_1}{A}$

Work done by the gas is

$$W = PAx + -kx \times x$$

$$= P_1A + \frac{2v_1}{A} + \frac{1}{2} \times \frac{P_1A}{3} \times \frac{2v_1}{A}$$

$$= 2P_1v_1 + \frac{P_1v_1}{3} = \frac{7P_1v_1}{3}$$

so choice (c) is also correct.

Case (d) : In this case, $\Delta U = \frac{9}{2}P_1v_1$ and

$$\Delta Q = W + \Delta U = \frac{7P_1v_1}{3} + \frac{9}{2}P_1v_1 = \frac{41P_1v_1}{6}$$

16. If $x = y = n$, the fission section is given by

$${}^{236}_{92}\text{U} \longrightarrow {}^{140}_{54}\text{X}_e + {}^{94}_{38}\text{Sr} + {}^{1}_{0}\text{n} + {}^{1}_{0}n$$

In this case, the atomic number and mass number are conserved. Hence laws of charge and mass conservations are obeyed. Further, the energy released in this nuclear reaction is

$$Q = (140 \times 8.5\,\text{MeV} + 94 \times 8.5\,\text{MeV}) - 236 \times 75\text{MeV}$$
$$= (1190 + 799)\,\text{MeV} - 1770\,\text{MeV}$$
$$= 219\,\text{MeV}$$

In choice (a) the total kinetic energy of the products is

$$Q^1 = K_{xe} + K_{sr} + K_x + K_y$$
$$= 129\text{MeV} + 86\text{MeV} + 2\text{MeV} + 2\text{MeV}$$
$$= 219\,\text{MeV}$$

Since $Q = Q^1$, the energy conservation law is also obeyed. In choice (d), the above three conservation laws (namely, mass charge and energy conservation) are also obeyed. In choice (d), the momentum conservation law is not obeyed because $K_{xe} > K_{sr} + K_x + K_y$ and $P = \sqrt{2mK}$. Choices (b) and (c) do not obey any conservation law. Hence the only correct choice is (a).

Section-III

17. Let q be the charge accumulated when magnetic force becomes equal to electric force, i.e. $F_m = Fe$ or $q\nu B = qE$

Where ν is teh drift speed of electrons and E is the electric field between points K and M. Now

$$E = \frac{V}{KM} = \frac{V}{w}$$

where V is the potential difference between K and M and w is the distance between them. Therefore

$$q\nu B = \frac{qV}{w}$$

$$\Rightarrow \quad \nu B = \frac{V}{w}$$

$$\Rightarrow \quad \nu w = \frac{V}{B} \qquad \text{...(i)}$$

The current I is given by

$$I = neA\nu$$

Where $A = wd$ is the cross-sectional area.

So, $I = ne\,(wd)v$

$$\Rightarrow \quad vw = \frac{I}{ned} \qquad ...(2)$$

For (1) and (2) we get

$$V = \frac{B}{ned} \qquad ...(3)$$

So V is independent of w.

Choice (a) : When $d = d_1$, Eq. (3) gives

$$V_1 - \frac{B}{ned_1}$$

and when $d = d_2$,

$$V_2 - \frac{B}{ned_2}$$

$\Rightarrow V_2 = 2V_1$, So choice (a) is correct and choice (b) is wrong.

Choice (c) : when $d_1 = d_2$, it follows that $V_1 = V_2$. So choice (d) is correct and choice (c) is wrong. So the correct choices (a) and (d) are correct.

18. $V = \dfrac{B}{ned}$

Choice (a) : $V_1 = \dfrac{B_1}{n_1 ed}$

$$V_2 = \frac{B_2}{n_2 ed}$$

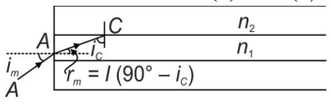
$$\therefore \frac{V_1}{V_2} = \frac{B_1}{B_2} \times \frac{n_2}{n_1}$$

$$= \frac{B_2}{B_2} \times \frac{n_2}{2n_2} = \frac{1}{2}$$

$\Rightarrow V_2 = 2V_1$, which is choice (a).

It is easy to check that choice (c) is correct. So the correct choices are (a) and (c)

19.



The critical angle i_c at the core-cladding interface is given by

$$\sin i_c = \frac{n_2}{n_1}$$

For total internal reflection at C, the angle of incidence or ray BC must exceed i_c. The ray AB in medium n is refracted along BC in n_1. The angle of refraction at B must have a maximum value $r_m = (90^\circ - i_c)$ so that the ray BC suffer total interval reflection at C. Let i_m be the corresponding maximum angle of incidence in n. Applying Snell's law at B, we have

$$n = sin\, i_m = n_1 \sin r_m$$

$$= n_1\left(1 - co^2 s\frac{r}{m}\right)^{1/2}$$

$$= n_1\left(1 - \frac{n_2^2}{n_1^2}\right)^{1/2}$$

$$= \left(n_1^2 = n_2^2\right)^{1/2}$$

Now $NA = \sin i_m = \dfrac{1}{n} \quad \left(n_1^2 = n_2^2\right)^{1/2}$

Choice (a): When S_1 is immersed in water $= \left(n = \dfrac{4}{3}\right)$,

$$NA = \frac{1}{\frac{4}{3}}\left(\frac{45}{16} - \frac{9}{4}\right)^{1/2} = \frac{9}{16}$$

For S_2 immersed in a liquid of refractive index $n = \dfrac{16}{3\sqrt{15}}$,

$$NA = \frac{3\sqrt{15}}{16}\left[\left(\frac{8}{5}\right)^2 - \left(\frac{7}{5}\right)^2\right]^{\frac{1}{2}} = \frac{9}{16}$$

So choice (a) is correct

Choice (b): For S_1 immersed in a liquid of refractive index $n = \dfrac{16}{\sqrt{15}}$,

$$NA = \frac{\sqrt{15}}{16}\left(\frac{45}{16} - \frac{9}{4}\right)^{\frac{1}{2}} = \frac{3\sqrt{15}}{64}$$

For S_2 immersed in water ($n = \dfrac{4}{3}$),

$$NA = \frac{3}{4}\left(\frac{64}{25} - \frac{49}{25}\right)^{\frac{1}{2}} = \frac{3\sqrt{15}}{20}$$

So choice (b) is wront

Choice (c): For S_1 in air ($n = 1$),

$$NA = \frac{1}{1}\left(\frac{45}{16} - \frac{9}{4}\right)^{\frac{1}{2}} = \frac{3}{4}$$

For S_2 immersed in a liquid of refractive index $n = \dfrac{4}{\sqrt{15}}$,

$$NA = \frac{\sqrt{15}}{4}\left(\frac{64}{25} - \frac{49}{25}\right)^{\frac{1}{2}} = \frac{3}{4}$$

So choice (c) is wront

Choice (d): For S_1 in air ($n = 1$),

$$NA = \frac{3}{4}$$

For S_2 in water $\left(n = \frac{4}{3}\right)$

$$NA = \frac{3\sqrt{15}}{20}$$

So choicd (d) is wrong

Hence the correct choices are (a) and (c).

20. The following figure shows structures S_1 and S_2 joined longitudinally

Structure S_1	**Structure S_2**
$n_2 = \frac{3}{2}$	$n_2 = \frac{7}{5}$
$n_1 = \frac{\sqrt{45}}{4}$	$n_1 = \frac{8}{5}$

For S_1, the incident ray is in air, Therefore

$$NA_1 = \frac{1}{1}\left(\frac{45}{16} - \frac{9}{4}\right)^{\frac{1}{2}} = \frac{3}{4} = 0.75$$

For, S_2 the incident ray in $n_1 = \frac{\sqrt{45}}{9}$. Hence

$$NA_2 = \frac{4}{\sqrt{45}}\left(\frac{64}{25} - \frac{49}{25}\right)^{\frac{1}{2}} = \frac{4}{5\sqrt{3}} = 0.46$$

It can be proved that the numerical aperature of combined structure is equal to the numerical aperture of the structure whose NA is smaller, which is NA_2.

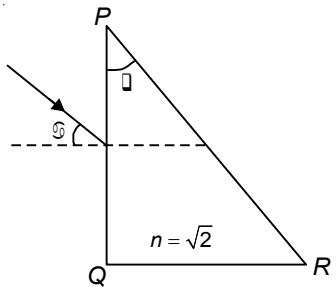
Hence the correct choice is (d)

JEE ADVANCED 2016: PAPER-I
(MODEL SOLUTIONS)

SECTION I
(Single Correct Answer Type)

This section contains **FIVE** questions.
Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.

1. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. the angle θ of the prism is



(a) 15° (b) 22.5^o (c) 30° (d) 45°

2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

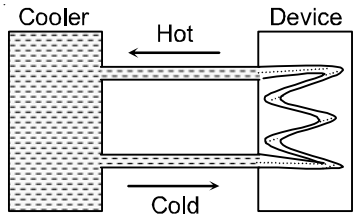
$\lambda(\mu m)$	V_0(Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ ms^{-1} and $e = 1.6 \times 10^{-19}$C, Planck's constant (in units of J s) found from such an experiment is

(a) 6.0×10^{-34} (b) 6.4×10^{-34} (c) 6.6×10^{-34} (d) 6.8×10^{-34}

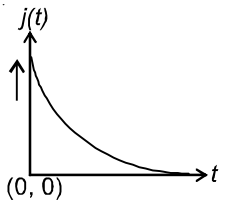
3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30^oC and the entire stored 120 litres of water is initially cooled to 10^oC.

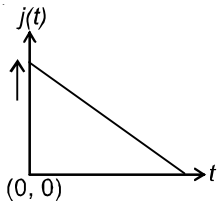
The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is: (Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

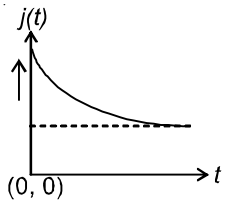


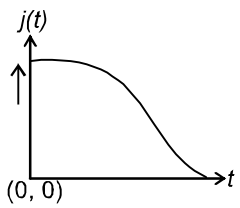
(a) 1600 (b) 2067 (c) 2533 (d) 3933

4. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time $t = 0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material?

(a) 

(b) 

(c) 

(d) 

5. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height $h(<\ell)$ such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are (g = 10 ms^{-2})

(a) $\frac{h}{\ell}=\frac{\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}\text{N}$ (b) $\frac{h}{\ell}=\frac{3}{16}, f=\frac{16\sqrt{3}}{3}\text{N}$

(c) $\frac{h}{\ell}=\frac{3\sqrt{3}}{16}, f=\frac{8\sqrt{3}}{3}\text{N}$ (d) $\frac{h}{\ell}=\frac{3\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}\text{N}$

SECTION II
(One or More than One Options Correct Type)

This section contains **EIGHT** questions.

Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.

6. The position vector $\vec{r}$ of a particle of mass m is given by the following equation: $\vec{r}(t)=\alpha t^3\hat{i}+\beta t^2\hat{j}$ where $\alpha = 10/3$ ms^{-3}, $\beta = 5$ ms^{-2} and m = 0.1 kg. At t = 1s, which of the following statement(s) is(are) true about the particle?

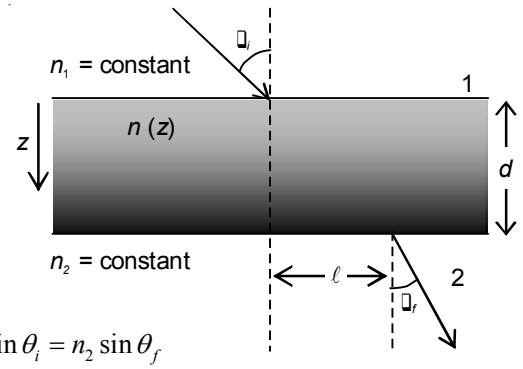
(a) The velocity $\vec{\upsilon}$ is given by $\vec{\upsilon}=(10\hat{i}+10\hat{j})$ ms^{-1}

(b) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L}=-(5/3)\hat{k}$ Nms

(c) The force $\vec{F}$ is given by $\vec{F}=(\hat{i}+2\hat{j})$ N

(d) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau}=-(20/3)\hat{k}$ Nm

7. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($> n_1$), as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l.



Which of the following statement(s) is(are) true ?

(a) ℓ is dependent on n_2

(b) $n_1 \sin\theta_i = n_2 \sin\theta_f$

(c) ℓ is independent of $n(z)$

(d) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$

8. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?

(a) The refractive index of the lens is 2.5

(b) The radius of curvature of the convex surface is 45 cm

(c) The faint image is erect and real

(d) The focal length of the lens is 20 cm

9. Highly excited states for hydrogen-like atoms (also called Rybderg states) with nuclear charge Ze are defined by their principal quantum number n, where $n >> 1$. Which of the following statement(s) is(are) true?

(a) Relative change in the radii of two consecutive orbitals does not depend on Z

(b) Relative change in the radii of two consecutive orbitals varies as $1/n$

(c) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

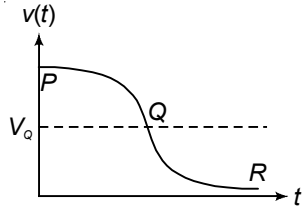
(d) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

10. A length-scale (ℓ) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for ℓ is (are) dimensionally correct?

(a) $\ell = \sqrt{\left(\dfrac{nq^2}{\varepsilon k_B T}\right)}$ (b) $\ell = \sqrt{\left(\dfrac{\varepsilon k_B T}{nq^2}\right)}$ (c) $\ell = \sqrt{\left(\dfrac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (d) $\ell = \sqrt{\left(\dfrac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

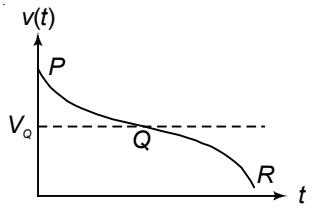
11. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $v(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let v_P, v_Q and v_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person?

(a) The plot below represents schematically the variation of beat frequency with time



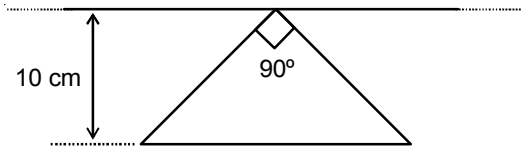
(b) $\nu_P + \nu_R = 2\nu_Q$

(c) The plot below represents schematically the variation of beat frequency with time



(d) The rate of change in beat frequency is maximum when the car passes through Q

12. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire.



The current in the triangular loop is in counterclockwise direction and increases at a constant rate of 10 As^{-1}. Which of the following statement(s) is(are) true?

(a) The induced current in the wire is in opposite direction to the current along the hypotenuse

(b) The magnitude of induced *emf* in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(c) There is a repulsive force between the wire and the loop

(d) If the loop is rotated at a constant angular speed about the wire, an additional *emf* of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

13. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?

(a) The temperature distribution over the filament is uniform

(b) The resistance over small sections of the filament decreases with time

(c) The filament emits more light at higher band of frequencies before it breaks up

(d) The filament consumes less electrical power towards the end of the life of the bulb

SECTION II
(Single Digit Integer Type)

This section contains **FIVE** questions.

This answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive

14. Two inductors L_1 (inductance 1 mH, internal resistance 3Ω) and L_2 (inductance 2 mH, internal resistance 4Ω), and a resistor R (resistance 12Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time $t = 0$. the ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is _____ .

15. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2 (P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

16. A hydrogen atom in its ground state is irradiated by light of wavelength 970Å. Taking $hc = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as –13.6 eV, The number of lines present in the emission spectrum is _____ .

17. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameter 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is _____ .

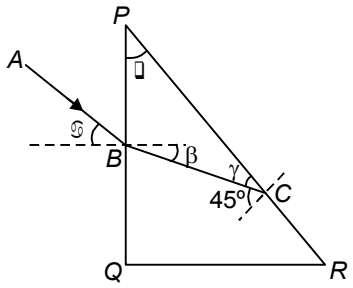
18. The isotope $^{12}_{5}\mathrm{B}$ having a mass 12.014 u undergoes β-decay to $^{12}_{6}\mathrm{C}$. $^{12}_{6}\mathrm{C}$ has an excited state of the nucleus $\left(^{12}_{6}\mathrm{C}^{*}\right)$ at 4.041 MeV above its ground state. If $^{12}_{5}\mathrm{B}$ decays to $^{12}_{6}\mathrm{C}^{*}$, the maximum kinetic energy of the β-particle in units of MeV is _____ . (1u = 931.5 MeV/c^2, where c is the speed of light in vacuum).

ANSWERS

1. (a)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (d)
6. (a), (b), (d)	**7.** (a), (b), (c)	**8.** (a), (d)	**9.** (a), (b), (d)	**10.** (b), (d)
11. (a), (b), (d)	**12.** (b), (c)	**13.** (c), (d)	**14.** 8	**15.** 9
16. 6	**17.** 3	**18.** 9		

Solutions

1. The adjoining figure shows the ray diagram. Applying Snell's law at B,



$$1.0 \sin 45^\circ = \sqrt{2} \sin \beta$$

$$\Rightarrow \quad \sin \beta = \frac{1}{2}$$

$$\Rightarrow \quad \beta = 30^\circ$$

For total internal reflection at C, the critical angle (i_C) is given by

$$\sin i_c = \frac{1}{\sqrt{2}} \Rightarrow i_C = 45^\circ$$

In triangle PBC,

$$\theta + 90^\circ + \beta + \gamma = 180^\circ$$

$$\Rightarrow \quad \theta + 90^\circ + \beta + (90^\circ - 45^\circ) = 180^\circ$$

$$\Rightarrow \quad \theta + \beta = 45^\circ$$

$$\Rightarrow \quad \theta + 30^\circ = 45^\circ$$

$$\Rightarrow \quad \theta = 15^\circ, \text{ which is option (a).}$$

2. $$h\nu = eV_0 + w_0$$

$$\Rightarrow \quad \frac{hc}{\lambda} = eV_0 + w_0$$

$$\Rightarrow \quad V_0 = \frac{hc}{e\lambda} - \frac{w_0}{e} \qquad ...(1)$$

So the graph $V_0 \, vs \, \frac{1}{\lambda}$ is a straight line of slope $m = \frac{hc}{e}$. Using the first two sets of observations given in the question, we have

$$(V_0)_1 - (V_0)_2 = m\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \quad (2.0 - 1.0) \text{ volt} = m\left(\frac{1}{0.3\times10^{-6}} - \frac{1}{0.4\times10^{-6}}\right)\text{m}^{-1}$$

$$\Rightarrow \quad m = 1.2 \times 10^{-6} \text{ Vm}$$

But $m = \frac{hc}{e}$. Hence

$$h = \frac{me}{c} = \frac{(1.2\times10^{-6})\times(1.6\times10^{-19})}{3\times10^8} = 6.4 \times 10^{-34} \text{ VCs}$$

$$\Rightarrow \quad h = 6.4 \times 10^{-34} \text{ Js}$$

So the correct option is (b).

3. Volume of water in the cooler is $V = 120$ litre $= 120 \times 10^{-3}$ m^3. Density of water is $\rho = 10^3$ kg m^{-3}. Therefore, the mass of water is

$$m = \rho V = (120 \times 10^{-3}) \times 10^3 = 120 \text{ kg}$$

The temperature of water decreases from 30ºC to 10ºC in time $t = 3$ hours $= (3 \times 60 \times 60)$s

Out of 3000 W, water loses heat energy $= 3000 - P$ in t seconds. Hence

$$(3000 - P)t = 120 \times 4200 \times (30 - 10)$$

$$\Rightarrow \quad (3000 - P) \times (3 \times 60 \times 60) = 120 \times 4200 \times 20$$

$$\Rightarrow \quad P = 2067 \text{ W}$$

So the correct choice is (b).

4. Let L be the length of the wire and Q_0 be the initial (i.e. at $t = 0$) charge distributed uniformly on it. The initial linear charge density is $\lambda_0 = \frac{Q_0}{L}$. Let Q be the wire at an instant t. The linear charge density of this instant is $\lambda = \frac{Q}{t}$. The magnitude of the electric field at a distance r from the wire placed in a medium of permittivity $\in$ at time t is

$$E = \frac{\lambda}{2\pi \in r} = \frac{Q}{2\pi \in rL}$$

From Ohm's law, the relation between current density j and electric field E is

$$j = -\sigma E = -\frac{\sigma Q}{2\pi \in rL} \qquad ...(1)$$

The negative sign indicates that the electrons drift in a direction opposite to that of E. Since the shell is hollow, the current flows only along the surface of the cylinder whose area is $A = 2\pi RL$. Since $j = \frac{I}{A}$,

$$I = jA = j \times 2\pi RL$$

where I is the current at time t. Since $I = \frac{dQ}{dt}$, we have

$$\frac{dQ}{dt} = j \times 2\pi RL = -\frac{\sigma Q}{2\pi \in rL} \times 2\pi RL$$

$$\Rightarrow \quad \frac{dQ}{dt} = -kQ, \text{ where } k = \frac{\sigma R}{\in r}$$

$$\Rightarrow \quad \frac{dQ}{Q} = -kdt$$

$$\Rightarrow \quad \int_{Q_0}^{Q} \frac{dQ}{Q} = -k \int_0^t dt$$

$$\Rightarrow \quad \ln\left(\frac{Q}{Q_0}\right) = kt$$

$$Q = \mathrm{Q}_0 e^{-kt}$$

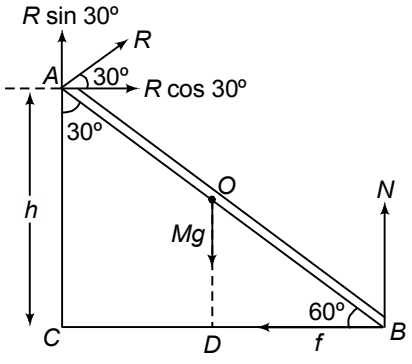
Hence the instataneous current density is

$$j = \frac{Q}{2\pi RL} = \frac{Q_0}{2\pi RL} e^{-kt}$$

Hence j falls exponentially with time.

It is clear that at $t = 0, j = \frac{Q_0}{2\pi RL}$ and as $t \to \infty$, $t \to 0$. So the correct graph is (a).

5. Refer to the adjoining figure which shows the forces acting on the stick AB with A touching the wall and end B touching the floor. To prevent slipping of the stick, the friction force must act at A in the upward direction and at B towards the left. Let R be the reaction force (which is the resultant of the normal reaction and the frictional force) at A and N be the normal reaction at B. The entire mass of the stick may be assumed to be concentrated at its centre O. Since the stick is in translational equilibrium, the total upward force = total downward force, i.e.



$$R \sin 30^\circ + N = \mathrm{Mg}$$

$$\Rightarrow \quad \frac{R}{2} + N = 1.6 \times 10 = 16$$

It is given that $N = R$. Hence

$$\frac{R}{2} + R = 16$$

$$\Rightarrow \quad R = \frac{32}{3} \text{ newton}$$

Also the net horizontal force on the stick must be zero, i.e.

$$f = R \cos 30^\circ = \frac{32}{3} \times \frac{\sqrt{3}}{2} = \frac{16\sqrt{3}}{3} \text{ newton}$$

Since the stick is also in rotational equilibrium, the total clockwise torque about B = total counterclockwise torque about B, i.e.

$$Mg \times BD = f \times 0 + R \times AB$$

$$Mg \times \frac{\ell}{2} \cos 60^\circ = R \times \frac{h}{\sin 60^\circ}$$

$$\Rightarrow \quad 1.6 \times 10 \times \frac{\ell}{2} \times \frac{1}{2} = \frac{32}{3} \times \frac{2h}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{h}{\ell} = \frac{3\sqrt{3}}{16}$$

So the correct option is (d).

6. Substituting the given value of α and β, we have

$$\vec{r} = \frac{10}{3} t^3 \hat{i} + 5t^2 \hat{j}$$

(a) $$\text{velocity } \vec{\upsilon} = \frac{d\vec{r}}{dt}$$

$$= \frac{d}{dt}\left(\frac{10}{3} t^3 \hat{i} + 5t^2 \hat{j}\right) = 10t^2 i + 10t\hat{j}$$

Putting $t = 1$s (given), $\vec{\upsilon} = 10(\hat{i} + \hat{j})\ \text{ms}^{-1}$. So option (a) is correct.

(b) Angular momentum with respect to the origin is

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times (m\vec{\upsilon}) = m(\vec{r} \times \vec{\upsilon})$$

$$= m\left[\left(\frac{10}{3} t^3 i + 5t^3 \hat{j}\right) \times \left(10\hat{i} + 10\hat{j}\right)\right]$$

$$= m\left(\frac{100}{3} t^3 - 50t^2\right)\hat{k}$$

Putting $t = 1$s and $m = 0.1$ kg, we get $\vec{L} = -\frac{5}{3}\hat{k}$ kgm^2 s^{-1}.

So option (b) is also correct.

(c) $$\vec{F} = m\vec{a} = m\frac{d\vec{\upsilon}}{dt}$$

$$= m\frac{d}{dt}\left(10t^2\hat{i} + 10t\hat{j}\right)$$

$$= m\left(20t\hat{i} + 10\hat{j}\right)$$

$$= 0.1\left[(20 \times 1)\hat{i} + 10\hat{j}\right]$$

$$= (2\hat{i} + \hat{j})\ \text{ms}^{-2}$$

So option (c) is not correct.

(d) $$\vec{\tau} = \left(\vec{r} \times \vec{F}\right) = \left(\frac{10}{3}t^3 i + 5t^2\hat{j}\right) \times m\left(20t\hat{i} + 10\hat{j}\right)$$

$$= m\left(\frac{100}{3}t^3\hat{k} - 100t^2 k\right)$$

$$= 0.1\left(\frac{100}{3} \times (1)^3\hat{k} - 100 \times (1)^2 k\right)$$

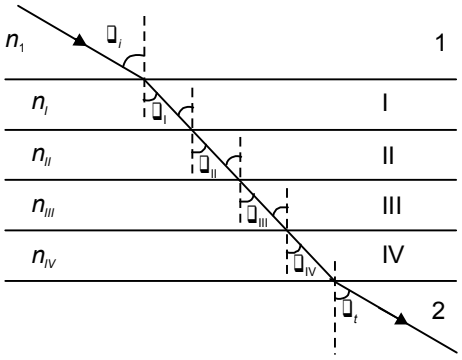
$$= -\frac{20}{3}\hat{k}\ \text{Nm, which is option (d).}$$

So the correct options are (a), (b) and (d).

7. The transparent slab can be assumed to consist of a large number N of slabs labelled I, II, III, ... etc. of refractive indices $n_{\text{I}} > n_{\text{II}} > n_{\text{III}}$ ... etc. as shown in the figure which shows refractions at the interfaces of I, and II, II and III and so on.

Applying Snell's law at each interface, we have

$$n_{\text{I}} \sin\ \theta_i = n_{\text{I}} \sin\ \theta_{\text{I}}$$
$$n_{\text{I}} \sin\ \theta_{\text{I}} = n_2 \sin\ \theta_{\text{II}}$$
$$n_2 \sin\ \theta_{\text{II}} = n_3 \sin\ \theta_{\text{III}}$$
$$\vdots$$
$$n_{N-1} \sin\ \theta_{N-1} = n_2 \sin\ \theta_f$$



From the above equations it follows that

$$n_1 \sin\ \theta_i = n_2 \sin\ \theta_f$$

So option (b) is correct.

Since N is large, the path of the refracted ray in the slab is a curve whose curvature depends on how n varies with z. Hence the lateral displacement ℓ depends on how n depends on z. So option (c) is also correct. Since the curvature of the curve does not depend on n_2, ℓ is independent of n_2. So option (a) is correct. Thus the correct options are (a), (b) and (c).

8. Given $u = -30$ cm and $\upsilon = +60$ cm. Using the lens formula

$$\frac{1}{\upsilon} - \frac{1}{u} = \frac{1}{f}\text{, we have}$$

$$\frac{1}{60} - \frac{1}{-30} = \Rightarrow f = 20\ \text{cm}$$

So option (d) is correct.

For reflection at the convex surface of radius of curvature R, $u = -30$ cm and $\upsilon = +10$ cm. Using the spherical mirror formula

$$\frac{1}{\upsilon} + \frac{1}{u} = \frac{2}{R}\text{, we have}$$

$$\frac{1}{10} + \frac{1}{-30} = \frac{2}{R} \Rightarrow R = 30\ \text{cm}$$

Using lens maker's formula

$$\frac{1}{20} = (\mu - 1)\left(\frac{1}{30} + \frac{1}{\infty}\right) \Rightarrow \mu = 2.5$$

So option (a) is also correct. Further, a convex reflecting surface never forms a real image. So option (c) is incorrect. Hence the correct options are (a) and (d).

9. The radius of nth orbit of the hydrogen like atom is

$$n = \frac{kn^2}{Z}, \text{ where } k = 0.53 \text{ Å}$$

$\therefore$ The relative change in the radii of two consecutive states is

$$\frac{r_{n+1} - r_n}{r_n} = \frac{\frac{k(n+1)^2}{Z} - \frac{kn^2}{Z}}{\frac{kn^2}{Z}} = \frac{(n+1)^2 - n^2}{n^2}$$

$$= \frac{2n+1}{n^2} = \frac{2}{n} + \frac{1}{n^2} \simeq \frac{2}{n} \qquad (\because n >> 1)$$

Thus $\frac{r_{n+1} - r_n}{r_n} \propto \frac{1}{n}$, which does not depend on Z. So options (a) and (b) are both correct. The energy of nth state is

$$E_n = -C\frac{Z^2}{n^2} \text{ where } C = 13.6 \text{ eV}$$

$\therefore$

$$\frac{E_{n+1} - E_n}{E_n} = \frac{-\frac{CZ^2}{(n+1)^2} + \frac{CZ^2}{n^2}}{-\frac{CZ^2}{n^2}}$$

$$= \frac{n^2}{(n+1)^2} - 1$$

$$= \frac{n^2 - (n+1)^2}{(n+1)^2}$$

$$= \frac{-2n-1}{n^2} \qquad (\because n >> 1)$$

$$= -\frac{2}{n^2} - \frac{1}{n^2}$$

$$= -\frac{2}{n} \qquad (\because n >> 1)$$

So option (c) is incorrect.

Angular momentum is $L = \frac{nh}{2\pi}$

$\therefore$

$$\frac{L_{n+1} - L_n}{L_n} = \frac{(n+1) - n}{n} = \frac{1}{n}$$

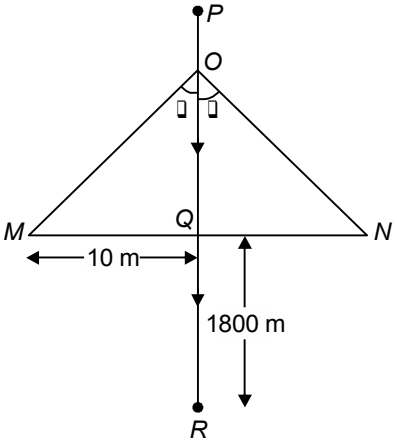
So option (d) is correct. Hence the correct options are (a), (b) and (d).

10. The dimensional formulae of the given quantities are:

$$[n] = [L^{-3}]$$
$$[q] = [AT]$$
$$[\in] = [M^{-1} L^{-3} T^4 A^2]$$
$$[T] = [K]$$
$$[\ell] = [L]$$
$$[k_B] = [M^1 L^2 T^{-2} K^{-1}]$$

Substituting these in the expression of ℓ given in options (a), (b), (c) and (d) we find that options (b) and (d) have dimension of length [L]. The dimension of the expression in option (a) is $[L^{-1}]$ and in option (c) is $[L^{3/2}]$. Hence the only correct options are (b) and (d).

11. Let ν_1 and ν_2 be the frequencies of the sound of loud speakers *N* and *M* respectively. Let the car be at point *O* at time *t*. The point *O* lies between *P* and *Q* as shown in the figure.



If *u* is the speed of the car (observer) and υ is the speed of sound. As the observer (in the car) travels from *O* to *Q*, he approaches the sources of sound *M* and *N* with speed u $\cos\theta$. At O, the observer hears sounds of apparent frequencies given by

$$\nu_1' = \nu_1\left(\frac{\upsilon + u\cos\theta}{\upsilon}\right)$$

and

$$\nu_2' = \nu_2\left(\frac{\upsilon + u\cos\theta}{\upsilon}\right)$$

The beat frequencies *v* is

$$v = \nu_1' - \nu_2' = (\nu_1 - \nu_2)\ 1\left(\frac{\upsilon + u\cos\theta}{\upsilon}\right)$$

$$v = (\nu_1 - \nu_2)\left(1 + \frac{u\cos\theta}{\upsilon}\right)$$

The rate of change of beat frequency with time is given by

$$\frac{d}{dt}(v) = -(\nu_1 - \nu_2)\frac{u}{\upsilon}\sin\theta\frac{d\theta}{dt} \qquad ...(1)$$

When the car crosses the point *Q,* the observer recedes from *M* and *N* with speed *u* cosθ and he hears a beat frequency *v*' given by

$$v' = (\nu_1 - \nu_2)\left(1 - \frac{u\cos\theta}{\upsilon}\right)$$

$$\therefore \quad \frac{d}{dt}(v') = (\nu_1 - \nu_2)\left(\frac{u}{\upsilon}\right)\sin\theta\frac{d\theta}{dt} \qquad ...(2)$$

When the car is at *P,* θ = 0º as *P* is far away from *Q*. Hence $\frac{dv}{dt} = 0$, i.e. the slope of *v* versus *t* graph is zero.

At *R,* θ = 0º. Hence the slope of the graph is again zero. But at *Q*, θ = 90º. Hence the slope of the graph at *Q* is maximum which is option (d). So the current graph is (a).

At *P*, θ = 0º.

$$v_p = (v_1 - v_2)\left(1 + \frac{u}{v}\right)$$

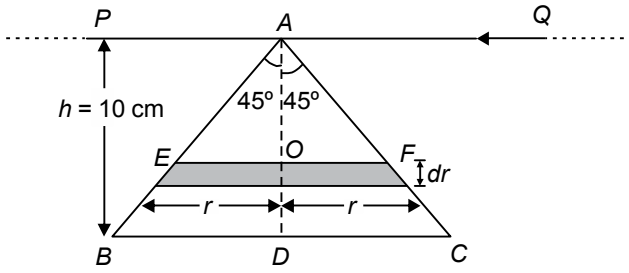
At R, $\theta = 0^\circ$.

$$v_R = (v_1 - v_2)\left(1 - \frac{u}{v}\right)$$

$\therefore$ $v_P + v_R = 2(v_1 - v_2)$, which is option (b).

So the correct options are (a), (b) and (d).

12. Refer to the adjoining figure.



Divide the conducting loop ABC into small elements each of small width dr. Consider one such element EF at a distance $r = AO$. Now $EO = AO \tan 45^\circ = AO = r$ and $OF = r$ so that $EF = 2r$. The area of the element is

$$dA = 2rdr$$

The magnetic field due to current I at a distance r from it is

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic flux through the element is

$$d\phi = BdA = \frac{\mu_0 I}{2\pi r} \times 2rdr = \frac{\mu_0 I dr}{\pi}$$

Magnetic flux through the triangular loop is

$$\phi = \int d\phi = \frac{\mu_0 I}{\pi} \int_{r=0}^{r=h} dr = \frac{\mu_0 I h}{\pi}$$

Induced emf is

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{\mu_0 h}{\pi}\frac{dI}{dt}$$

Given $\frac{dI}{dt} = 10\,\text{As}^{-1}$ and $h = 10$ cm $= 0.1$ m. Thus

$$|\varepsilon| = \frac{\mu_0}{\pi} \times 0.1 \times 10 = \frac{\mu_0}{\pi} \text{ volt}$$

So option (b) is correct.

It is given that the induced current in loop is counterclockwise. This is possible only if the current in the wire PQ is towards right because then the magnetic filed will be out of the plane of the loop.

The magnetic field due to the triagular loop at any point on the wire PQ is into the page. Hence the force on the wire PQ is directed away from the loop, i.e., the force is repulsive. So the correct options are (b) and (c).

13. It is given that the evaporation of tungsten from the filament is non-uniform, the temperature at different points on the filament must be different. Hence option (a) is incorrect.

Due to evaporation, the cross-sectional area of the wire decreases. Hence the resistance of different sections of the filament increases. So option (b) is also incorrect.

To check whether option (c) is correct or not, consider a small section of length ℓ and cross-sectional area A of the filament. If I is the current in the filament, the power radiated by the element is

$$P = I^2R = \frac{I^2\rho L}{A}$$

where ρ is the resistivity of tungsten. From Stefan's law,

$$P = \in \sigma T^4 A$$

where $\in$ is the emissivity. Equating the two expressions for P, we get

$$\in \sigma T^4 A = \frac{I^2\rho L}{A}$$

or $$T^4 = \left(\frac{I^2\rho L}{\in \sigma}\right)\frac{1}{A^2}$$

or $$T \propto \frac{1}{\sqrt{A}}$$

As the evaporation continues, A becomes smaller and smaller. Hence the temperature of the element becomes higher and higher. Hence the element radiates electro-magnetic waves of a high energy and hence of a high frequency (because $E = h\nu$). So option (c) is correct. The power is also given by

$$P = \frac{V^2}{R}$$

Since voltage V remains constant and R becomes very large (due to decrease in cross-sectional area caused by evaporation), the electrical power P consumed by the filament becomes small. So option (d) is also correct. Thus the correct options are (c) and (d).

14. Refer to the following figures.

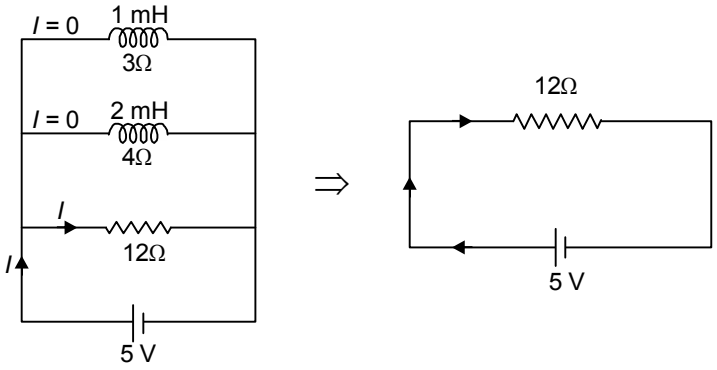


Fig. 1

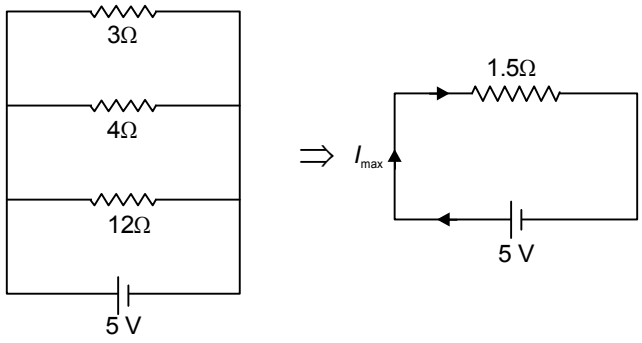


Fig. 2

Where the circuit is switched on at time $t = 0$, the current in both the inductors is zero because the induced back emf prevents the flow of current. The current flows only through the 12Ω resistor and is minimum (Fig. 1)

$$I_{\min} = \frac{5\text{V}}{12\Omega} = \frac{5}{12}A$$

After some time, the steady current flows in each inductor. Hence there is no back emf induced in the inductor (because $\frac{dI}{dt} = 0$) and, therefore, they behave as pure resistors. The equivalent resistance R is given by (Fig. 2)

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12} \Rightarrow R = 1.5\Omega$$

Since the resistance is minimum = 1.5 Ω, the current is maximum.

$$I_{\max} = \frac{5\text{V}}{1.5\Omega} = \frac{10}{3}\text{A}$$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{\frac{10}{3}}{\frac{5}{12}} = 8$$

The correct answer is 8.

15. Given $T_1 = 273 + 487 = 760$ K

$T_2 = 273 + 2767 = 3040$ K

First reading of the sensor is

$$S_1 = \log_2\left(\frac{\in \sigma A T_1^4}{P_0}\right)$$

Second reading of the sensor is

$$S_2 = \log_2\left(\frac{\in \sigma A T_2^4}{P_0}\right)$$

$$S_2 - S_1 = \log_2\left(\frac{\in \sigma A T_2^4}{P_0}\right) - \log_2\left(\frac{\in \sigma A T_1^4}{P_0}\right)$$

$$= \log_2\left[\frac{\in \sigma A T_2^4}{P_0} \times \frac{P_0}{\in \sigma A T_1^4}\right]$$

$$= \log_2\left(\frac{T_2}{T_1}\right)^4$$

$$= 4\log_2\left(\frac{T_2}{T_1}\right)$$

$$= 4\log_2\left(\frac{3040}{700}\right) = 4\log_2(4) = 4\log_2(2^2) = 4 \times 2 = 8$$

Given $S_1 = 1$. Hence

$S_2 = 8 + 1 = 9$

The correct answer is 9.

16. Energy of incident photon = $h\nu = \dfrac{hc}{\lambda}$

$$= \frac{1.237 \times 10^{-6}\,\text{eVm}}{970 \times 10^{-10}\,\text{m}}$$

$$= 12.75 \text{ eV}$$

Energy of hydrogen atom in the ground state = – 13.6 eV

When the hydrogen atom in the ground state absorbs the photon, the electron jumps to a state E_n given by

$$E_n = -13.6 + 12.75 = -0.85 \text{ eV}$$

Now, $$E_n = \frac{-13.6\,\text{eV}}{n^2}$$

$$\Rightarrow \quad -0.85 \text{ eV} = -\frac{-13.6\,\text{eV}}{n^2} \Rightarrow n = 4.$$

The following transitions can take place.

$n = 4 \rightarrow n = 3$, $n = 4 \rightarrow n = 2$, $n = 4 \rightarrow n = 1$,

$n = 3 \rightarrow n = 2$, $n = 3 \rightarrow n = 1$ and

$n = 2 \rightarrow n = 1$

So the number of spectral lines = 6.

17. Terminal velocity $V_t = \dfrac{2}{9}\dfrac{r^2(\sigma - \rho)g}{\eta}$

For sphere *P*: $r_1 = 0.5$ cm, $\sigma_1 = 8$ g cm^{-3}, $\rho_1 = 0.8$ g cm^{-3} and , $\eta_1 = 3\ P\ell$

For sphere *Q*: $r_2 = 0.25$ cm, $\sigma_2 = 8$ g cm^{-3}, $\rho_2 = 1.6$ g cm^{-3} and , $\eta_2 = 2\ P\ell$

$$\therefore \quad \frac{(V_t)_P}{(V_t)_Q} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{\sigma_1 - \rho_1}{\sigma_2 - \rho_2}\right) \times \left(\frac{\eta_2}{\eta_1}\right)$$

$$= \left(\frac{0.5}{0.25}\right)^2 \times \left(\frac{8 - 0.8}{8 - 1.6}\right) \times \left(\frac{2}{3}\right)$$

$$= 3$$

The correct answer is 3.

18. The nuclear reaction is represented by the equation

$${}^{12}_{5}\text{B} \longrightarrow {}^{12}_{6}\text{C} + {}^{0}_{1}\text{e} + \bar{\nu} + Q$$

By definition, mass of ${}^{12}_{6}\text{C}$ = 12.000 u

The energy released is

$$Q = (m_\text{B} - m_\text{A})c^2$$

$$= (12.014 - 12.000) \times 931.5$$

$$= 13.041 \text{ MeV}$$

Out of this 4.041 MeV of energy is taken by ${}^{12}_{6}\text{C}^*$. Therefore, the maximum kinetic energy of β particle $({}^{0}_{-1}\text{e})$ = 13.041 – 4.041 = 9 MeV. Then the kinetic energy of antinutrino $(\bar{\nu})$ is zero. So the correct answer is 9.

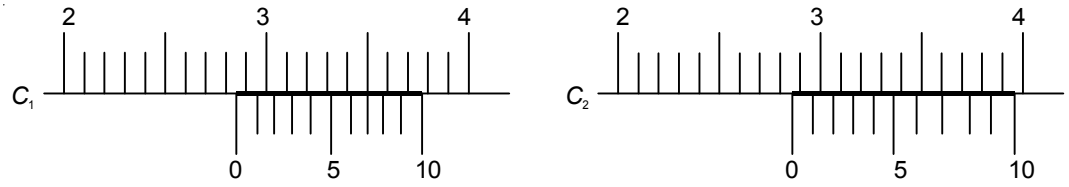
■■

JEE ADVANCED 2016: PAPER-II
(MODEL SOLUTIONS)

SECTION I
(Single Correct Answer Type)

This section contains **FIVE** questions.
Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.

1. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2, respectively, are :



(a) 2.87 and 2.86 (b) 2.85 and 2.82 (c) 2.87 and 2.87 (d) 2.87 and 2.83

2. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

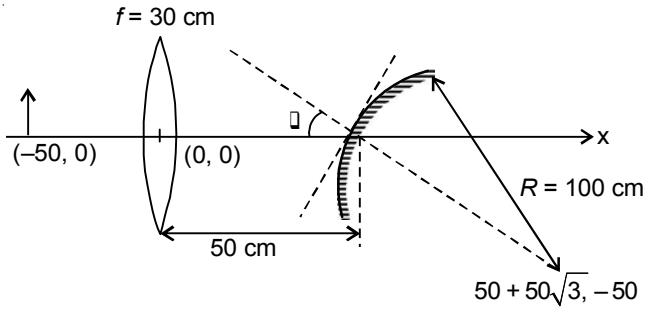
The measured masses of the neutron, ${}^{1}_{1}\mathrm{H}$, ${}^{15}_{7}\mathrm{N}$ and ${}^{15}_{8}\mathrm{O}$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the ${}^{15}_{7}\mathrm{N}$ and ${}^{15}_{8}\mathrm{O}$ nuclei are same, 1u = 931.5 MeV/ c^2 (c is the speed of light) and $e^2/(4\pi\varepsilon_0)$ = 1.44 MeV fm. Assuming that the difference between the binding energies of ${}^{15}_{7}\mathrm{N}$ and ${}^{15}_{8}\mathrm{O}$ is purely due to the electrostatic energy, the radius of either of the nuclei is : (1 fm = 10^{-15} m)

(a) 2.85 fm (b) 3.03 fm (c) 3.42 fm (d) 3.80 fm

3. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10ºC. Now the end P is maintained at 10ºC, while the end S is heated and maintained at 400ºC. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5}\mathrm{K}^{-1}$, the change in length of the wire PQ is :

(a) 0.78 mm (b) 0.90 mm (c) 1.56 mm (d) 2.34 mm

4. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30^\circ$ to the axis of the lens, as shown in the figure.



If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are :

(a) $(25, 25\sqrt{3})$ (b) $(0, 0)$ (c) $(125/3, 25/\sqrt{3})$ (d) $(50 - 25\sqrt{3}, 25)$

5. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^{5|}$ Pa and volume $V_i\ 10^{-3}$ m^3 changes to a final state at $P_f = (1/32) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately :

(a) 112 J (b) 294 J (c) 588 J (d) 813 J

6. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?

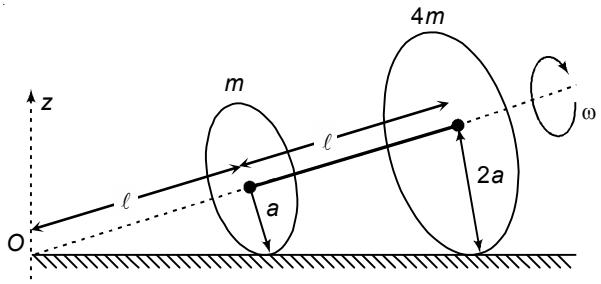
(a) 64 (b) 90 (c) 108 (d) 120

SECTION II
(One or More than One Options Correct Type)

This section contains **EIGHT** questions.

Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.

7. Two thin circular discs of mass m and $4m$, having radii of a and $2a$, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}a$ through their centres. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true?



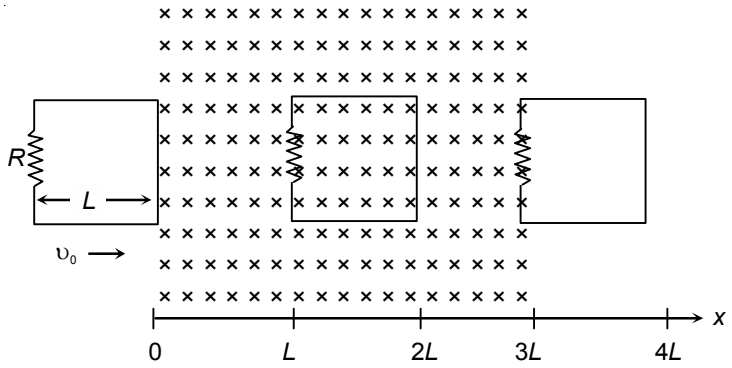
(a) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$

(b) The center of mass of the assembly rotates about the z-axis with an angular speed of $\omega/5$

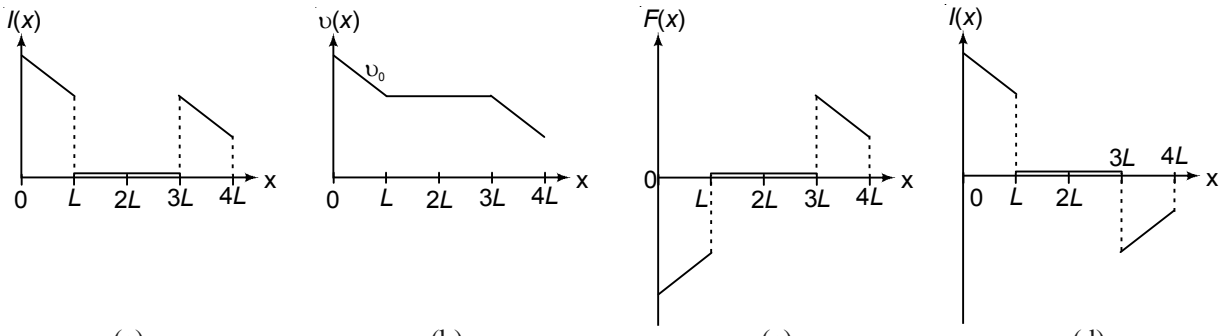
(c) The magnitude of the z-component of $\vec{L}$ is $55ma^2\omega$

(d) The magnitude of angular momentum of center of mass of the assembly about the point O is $81ma^2\omega$

8. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters a region of length $3L$ where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $\upsilon(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive.



Which of the following schematic plot(s) is(are) correct? (Ignore gravity)



9. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\dfrac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is(are) true?

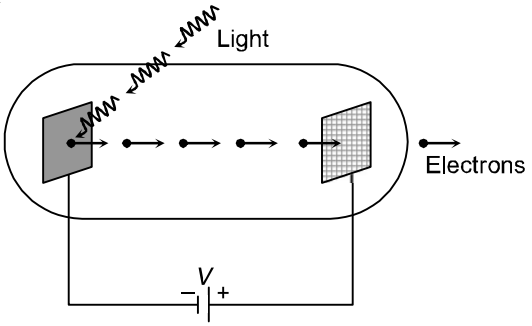
(a) The error in the measurement of r is 10%

(b) The error in the measurement of T is 3.57%

(c) The error in the measurement of T is 2%

(d) The error in the determined value of g is 11%

10. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statement(s) is(are) true?



(a) For large potential difference ($V >> \phi / e$), λ_e is approximately halved if V is made four times

(b) λ_e decreases with increase in ϕ and λ_{ph}

(c) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$

(d) λ_e is approximately halved, if d is doubled

11. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_C < R / 2$, which of the following statement(s) about any one of the galvanometers is(are) true?

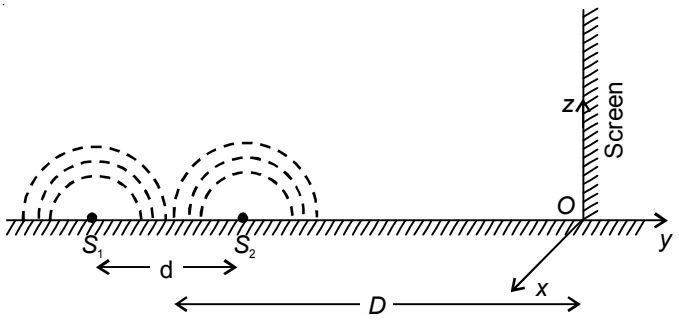
(a) The maximum voltage range is obtained when all the components are connected in series

(b) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer

(c) The maximum current range is obtained when all the components are connected in parallel

(d) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors

12. While conducting the Young's double slit experiment a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1,S_2) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for $z > 0$) at a distance $D = 3m$ from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the sources $d = 0.6003$ mm. The origin O is at the intersection of the screen and the line joining S_1 S_2. Which of the following is(are) true of the intensity pattern on the screen?



(a) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

(b) Straight bright and dark bands parallel to the x-axis

(c) Semi circular bright and dark bands centered at point O

(d) The region very close to the point O will be dark

13. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0; Consider two cases: (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass $m(<M)$ is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M?

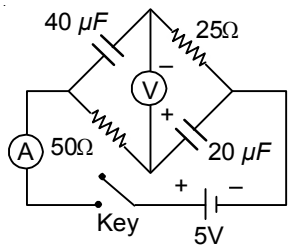
(a) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(b) The final time period of oscillation in both the cases is same

(c) The total energy decreases in both the cases

(d) The instantaneous speed at x_0 of the combined masses decreases in both the cases

14. In the circuit shown below, the key is pressed at time $t = 0$. Which of the following statement(s) is (are) true?



(a) The voltmeter displays –5V as soon as the key is pressed, and displays +5V after a long time

(b) The voltmeter will display 0 V at time $t = \ln 2$ seconds

(c) The current in the ammeter becomes $1/e$ of the initial value after 1 second

(d) The current in the ammeter becomes zero after a long time

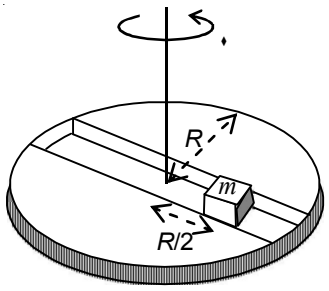
SECTION II
(Single Digit Integer Type)

This section contains **FIVE** questions.

This answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive

PARAGRAPH FOR QUESTIONS 15-16

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force *in* $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is



$$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{\upsilon}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}$$

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $(\vec{r} = (R/2)\hat{i}$ at $t = 0$ and is constrained to move only along the slot.

15. The distance r of the block at time t is :

(a) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (b) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$ (c) $\frac{R}{2}\cos 2\omega t$ (d) $\frac{R}{2}\cos \omega t$

16. The net reaction of the disc on the block is:

(a) $-m\omega^2 R\cos\omega t\,\hat{j} - mg\hat{k}$

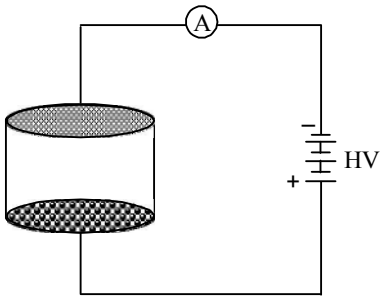
(b) $\frac{1}{2}m\omega^2 R\left(e^{2\omega t} - e^{-2\omega t}\right)\hat{j} + mg\hat{k}$

(c) $m\omega^2 R\sin\omega t\,\hat{j} - mg\hat{k}$

(d) $\frac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$

PARAGRAPH 2

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r << h$. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$.



Due to their conducting surface the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible.(Ignore gravity)

17. Which one of the following statements is correct ?

(a) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(b) The balls will stick to the top plate and remain there

(c) The balls will execute simple harmonic motion between the two plates

(d) The balls will bounce back to the bottom plate carrying the same charge they went up with

18. The average current in the steady state registered by the ammeter in the circuit will be :

(a) proportional to $V_0^{1/2}$

(b) zero

(c) proportional to V_0^2

(d) proportional to the potential V_0

ANSWERS

1. (d)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (c)
6. (c)	**7.** (a), (b)	**8.** (b), (d)	**9.** (a), (b), (d)	**10.** (a)
11. (a), (c)	**12.** (c), (d)	**13.** (a), (b), (d)	**14.** (a), (b), (c), (d)	**15.** (b)
16. (d)	**17.** (a)	**18.** (c)		

Solutions

1. If n vernier scale divisions coincide with $(n-1)$ main scale divisions (as is the case for vernier calipers C_1), the length of a small object is measured by the usual method which gives.

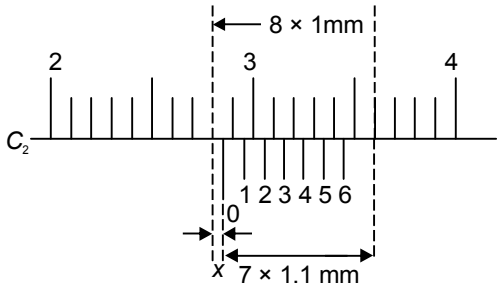
L_1 = main scale reading + $n \times$ V.C.

where n is the number of the vernier scale division which coincides with a main scale division and V.C. is the vernier constant. For C_1, $n = 7$ and V.C. = 0.1 mm = 0.01 cm. Hence [see figure]

$$L_1 = (2.8 + 7 \times 0.01)\text{ cm}$$

$$\Rightarrow \quad L_1 = 2.87\text{ cm}$$

For vernier calipers C_2, n vernier scale divisions concide with $(n + 1)$ main scale divisions. In this case, the length of a vernier scale division is greater than that of a main scale division. So, for C_2 the usual method is not applicable. The following method is to be used. Refer to the adjoining magnified diagram of C_2.



In ths case, the value of x is [see figure]

$$x = 8 \times 1\text{ mm} - 7 \times 1.1\text{ mm} = 0.3\text{ mm} = 0.03\text{ cm}$$

The length measured by C_2 is

$$L_2 = \text{main scale reading} + x$$
$$= 2.8\text{ cm} + 0.03\text{ cm} = 2.83\text{ cm}$$

Hence the correct option is (d).

2. For nitrogen ($Z = 7$). Therefore,

$$E_N = \frac{3}{5} \times \frac{7 \times 7 - 1)e^2}{4\pi \in_0 R} = \frac{126}{5}\frac{e^2}{4\pi \in_0 R}$$

For oxygen ($Z = 8$). Therefore,

$$E_O = \frac{3}{5} \times \frac{8 \times (8-1)e^2}{4\pi \in_0 R} = \frac{168}{5}\frac{e^2}{4\pi \in_0 R}$$

$$\therefore \Delta E = E_O - E_N = \frac{42}{5}\frac{e^2}{4\pi \in_0 R} \qquad ...(1)$$

Binding energy of $^{5}_{7}\text{N}$ is

$$(\text{B.E.})_N = 7 \times 1.007825 + 8 \times 1.008665 - 15.000109$$
$$= 0.123986\text{ u}$$

Binding energy of $^{15}_{8}\text{O}$ is

$$(\text{B.E.})_O = 8 \times 1.007825 + 7 \times 1.008665 - 15.003065$$
$$= 0.12019\text{ u}$$

Difference in binding energy $= 0.123986 - 0.12019$
$= 0.003796\text{ u}$

This must be equal to ΔE in Eq. (1). Hence

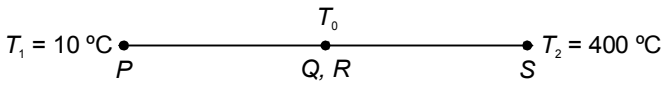
$$\frac{42}{5}\frac{e^2}{4\pi \in_0 R} = (0.003796) \times 931.5\text{ MeV}$$

$$\Rightarrow \quad \frac{42}{5} \times (1.44\text{ MeV fm}) \times \frac{1}{R} = 0.003796 \times 931.5\text{ MeV}$$

$$\Rightarrow \quad R = 3.42\text{ fm}$$

So the correct option is (c).

3. Let T_0 be the temperature of the junction of wires PQ and RS (see figure) below:



In the steady state, the rate of flow of heat in wire PQ = rate of flow of heat in wire RS. Hence

$$\frac{2k(T_0 - 10)A}{\ell} = \frac{2(400 - T_0)A}{\ell}$$

$$\Rightarrow \quad 2(T_0 - 10) = 400 - T_0$$

$$\Rightarrow \quad T_0 = 140^\circ\text{C}$$

Here we have assumed that the two wires have the same cross-sectional area A. The average temperature of wire PQ is $\text{T} = \frac{1}{2}(140 + 10) = 75^\circ\text{C}$. Hence the increase in the length of PQ is

$$\begin{aligned}\Delta\ell &= \alpha\ell(T - T_1)\\ &= (1.2 \times 10^{-5}) \times (1.0) \times (75 - 10)\\ &= 78 \times 10^{-5}\ \text{m} = 0.78\ \text{mm}\end{aligned}$$

So the correct choice is (a).

4. For the image formed by the lens, $u = -50$ cm, $f = +30$ cm. Using the lens formula $\frac{1}{\upsilon} - \frac{1}{u} = \frac{1}{f}$, we have $\frac{1}{\upsilon} - \frac{1}{-50} = \frac{1}{+30} \Rightarrow \upsilon = 75$ cm. This image is inverted and is formed on the principal axis of the lens. The (x, y) coordinates of the image are $x = 75$ cm and $y = 0$ (because the object is very small). This image, which is formed at as distance of $(75 - 50) = 25$ cm to the right of the convex mirror, acts as a virtual object for the mirror. If the mirror were not titled, then for the mirror $u = +25$ cm, $R = +100$ cm. Using the spherical mirror formula $\frac{1}{\upsilon} + \frac{1}{u} = \frac{2}{R}$, we have

$$\frac{1}{\upsilon} + \frac{1}{25} = \frac{2}{100} \Rightarrow \upsilon = -50\ \text{cm}$$

If the mirror is rotated by an angle of 30º as shown in the figure, the image will rotate by 60º. The new x-coordinate of the image will be

$$x' = 50 - 50\cos 60^\circ = 50 - 25 = 25\ \text{cm}$$

The new of y-cordinate will be

$$\begin{aligned}y' &= 0 - \upsilon \sin 60^\circ\\ &= 0 - (-50) \times \frac{\sqrt{3}}{2} = \frac{50\sqrt{3}}{2} = 25\sqrt{3}\ \text{cm}\end{aligned}$$

So the correction choice is (a)

5. For the first process (which is adiabatic), we have

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$\Rightarrow \quad \frac{P_i}{P_f} = \left(\frac{V_f}{V_i}\right)^\gamma$$

$$\Rightarrow \quad \frac{10^5}{\frac{1}{32} \times 10^5} = \left(\frac{8 \times 10^{-3}}{10^{-3}}\right)^\gamma$$

$$\Rightarrow \quad 32 = 8^\gamma$$

$$\Rightarrow \qquad 2^5 = 2^{3\gamma}$$

$$\Rightarrow \qquad 5 = 3\gamma \quad \Rightarrow \gamma = \frac{5}{3}$$

The second process is a two-step process. Work done in isobaric expansion at pressure P_i is

$$W = P_i(V_f - V_i) = 10^5\left(8\times10^{-3} - 10^{-3}\right) = 10^5 \times 7 \times 10^{-3}$$

$$\Rightarrow \qquad W = 7 \times 10^2 \text{ J}$$

The change in internal energy is the isochoric process at volume V_f is

$$\Delta U = \frac{1}{(\gamma - 1)}(P_f V_f - P_i V_i)$$

$$= \frac{1}{\left(\frac{5}{3} - 1\right)}\left[\left(\frac{1}{3}\times10^5\right)\times\left(8\times10^{-3}\right) - 10^5 \times 10^{-3}\right]$$

$$= \frac{3}{2}\left(\frac{1}{4}\times10^2 - 10^2\right)$$

$$\Rightarrow \qquad \Delta U = -\frac{9}{8}\times10^2 \text{ J}$$

$\therefore$ Heat supplied is

$$Q = W + \Delta U = 7 \times 10^2 - \frac{9}{8}\times10^2$$

$$\Rightarrow \qquad Q = \frac{47}{8}\times10^2 = 588 \text{ J}$$

So the correct choice is (c).

6. The radioactivity of a sample decreases to $\left(\frac{1}{2^n}\right)$ in n half lives. Since the initial activity is 64 times the permissible level, it must become $\frac{1}{64}$th of the initial value. Therefore,

$$\frac{1}{64} = \frac{1}{2^n} \Rightarrow n = 6$$

Thus in 6 half lives the activity will reduce to the permissible level. Therefore, the time taken is

$$t = 6\ T_{1/2} = 6 \times 18 \text{ days} = 108 \text{ days}$$

So the correct option is (c).

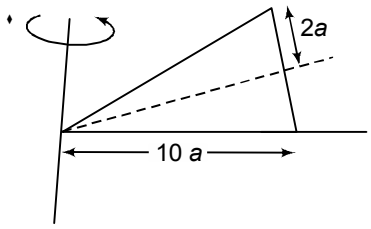
7. The magnitude of angular momentum of the system about its centre of mass is

$$L_{cm} = I_{cm}\omega$$

$$= \left[\frac{ma^2}{2} + \frac{4m(2a)^2}{2}\right]\omega$$

$$= \frac{17ma^2\omega}{2}$$

So option (a) is correct.



For option (b), refer to the adjoining figure.
The ratio of the angular speed of the discs is

$$\frac{\omega_2}{\omega_1} = \frac{2\pi(2a)}{2\pi(10a)} = \frac{1}{5}$$

Now $\omega_1 = \omega$, so $\omega_2 = \frac{\omega}{5}$. Hence option (b) is also correct.

Angular momentum about OZ is

$$L_z = \left[\frac{1}{2}m\ell^2 + \frac{1}{2}4m(2\ell)^2\right]\omega$$

$$= m\ell^2 \times \frac{17}{2}\omega$$

$$= m \times 24a^2 \times 17\omega \times \frac{1}{2}$$

$$= 204\ ma^2\omega$$

So choice (c) is incorrect. It can be shown that the magnitude of angular momentum about the centre of mass of the discs $= \frac{81}{5}m\ell^2\omega = \frac{81}{5} \times m \times 24a^2\omega = \frac{1944}{5}ma^2\omega$. So option (d) is also incorrect. Hence the correct choice are (a) and (b).

8. Induced emf = $BL\upsilon$, where υ is the velocity of the loop at a general position x. Induced current is

$$i = \frac{BL\upsilon}{R} \qquad ...(1)$$

Force on the loop is

$$F = -BiL = -B \times \frac{BL\upsilon}{R} \times L = -\frac{B^2L^2\upsilon}{R} \qquad ...(2)$$

Now
$$F = ma = m\frac{d\upsilon}{dt} = m\frac{d\upsilon}{dx}\frac{dx}{dt} = m\upsilon\frac{d\upsilon}{dx} \qquad ...(3)$$

Using (3) in (2), we have

$$m\upsilon\frac{d\upsilon}{dx} = -\frac{B^2L^2\upsilon}{mR}$$

$$\Rightarrow \quad d\upsilon = -\frac{B^2L^2}{mR}dx$$

$$\Rightarrow \quad \int_{\upsilon_0}^{\upsilon} d\upsilon = -\frac{B^2L^2}{mR}\int_0^x dx$$

$$\Rightarrow \quad \upsilon - \upsilon_0 = -\left(\frac{B^2L^2}{mR}\right)x \qquad ...(4)$$

From Equation (4) it follows that the velocity of the loop decreases linearly with x.
When the loop is completely in the region of magnetic field, the current i is zero ($\because$ there is no change in magnetic flux.) Hence $F = 0$, therefore, velocity υ is constant. The moment, the loop comes out of the field the current becomes clockwise, the force will be to the left (i.e. along negative x direction) and velocity υ decreases linearly with x. Hence the correct options are (b) and (d).

9. Mean time period is

$$\overline{T} = \frac{0.52 + 0.56 + 0.57 + 0.54 + 0.59}{5}$$

$= 0.556$ s $\simeq 0.56$ s (upto appropriate significant figures)

The absolute error in the five reading is.

$$\left|\overline{T} - T_1\right| = 0.56 - 0.52 = 0.04 \text{ s},$$

$$\left|\overline{T} - T_2\right| = 0.56 - 0.56 = 0.00,$$

$$\left|\overline{T} - T_3\right| = 0.56 - 0.57 = 0.01 \text{ s},$$

$$\left|\overline{T} - T_4\right| = 0.56 - 0.54 = 0.02 \text{ s},$$

$$\left|\overline{T} - T_5\right| = 0.56 - 0.59 = 0.03 \text{ s},$$

$$\text{Mean error } \Delta T = \frac{0.04 + 0.00 + 0.01 + 0.02 + 0.03}{5}$$

$= 0.02$ s

$$\text{Percentage error in } T = \frac{\Delta T}{T} \times 100 = \frac{0.02}{0.56} \times 100 = 3.57\%$$

$$\text{Percentage error in } r = \frac{0.001 \times 100}{0.010} = 10\%$$

$$\text{Percentage error in } R = \frac{0.001}{0.060} \times 100 = 1.67\%$$

Now $$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

$\Rightarrow$ $$g = \frac{4\pi^2 \times 7(R-r)}{5T^2}$$

$\therefore$ $$\frac{\Delta g}{g} = \frac{\Delta(R-r)}{(R-r)} + 2\frac{\Delta T}{T}$$

$$= \frac{0.002}{0.05} + 2 \times \frac{0.02}{0.56}$$

$$= 0.04 + 0.07 = 0.11$$

$\therefore$ Percentage error in $g = \frac{\Delta g}{g} \times 100 = 0.11 \times 100 = 11\%$

So the correct options are (a), (b) and (d).

10. Maximum initial kinetic energy of photoelectrons immediately after emission from the cathode plate is

$$k_i = h\nu - \phi = \frac{hc}{\lambda_{ph}} - \phi$$

The energy gained by the electrons on reaching the anode (wire mesh) = eV. Hence the final kinetic energy of the electrons on reaching the anode is

$$K_f = K_i + eV = \left(\frac{hc}{\lambda_{ph}} - \phi\right) + eV \quad ...(1)$$

For a very high V, equation (1) gives

$$K_f \simeq eV \quad ...(2)$$

The de-Broglie wavelength of the electron is (using Eq. 2)

$$\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2mK_f}} = \frac{h}{\sqrt{2me}} \times \frac{1}{\sqrt{V}} \quad ...(3)$$

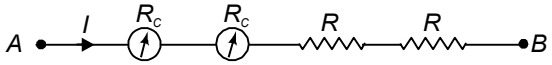
Hence if V is made four times, λ_e becomes half.

So option (a) is correct.

It follows the Eq. (1) that if λ_{ph} and ϕ both increase, K_f will decrease (as it becomes less then eV). Hence λ_e increases [See Eq. (3)]. So option (b) is incorrect.

It is easy to see that $\frac{d\lambda_e}{dt} \neq \frac{d\lambda_{Ph}}{dt}$. So option (c) is also incorrect. We know that λ_e is independent of d as it depend only on V and K_i. So option (d) is also incorrect. Thus the only correct option is (a).

11. Option (a)



Equivalent resistance between points A and B is

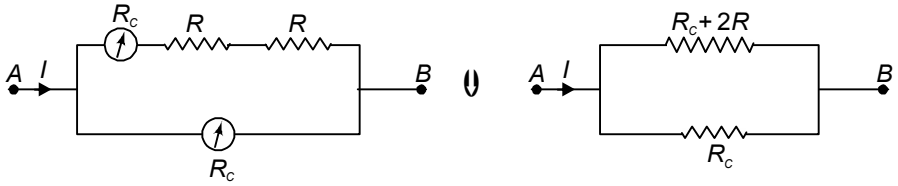
$$R_{eq} = R_C + R_C + R + R = 2\,(R_C + R)$$

Let I be the current. Since I is the same in each component of a series combination, the potential difference between A and B is

$$V_a = I\,R_{eq} = 2I\,(R_C + R)$$
$$\simeq 2IR \quad (\because R_C << R) \quad ...(1)$$

Since R_{eq} is maximum for a series combination, for a given current I, V_a is the maximum. So option (a) is correct.

Option (b)



The equivalent resistance between points A and B is

$$R_{eq} = \frac{R_C(R_C + 2R)}{R_C + (R_C + 2R)}$$

$$= \frac{R_C(R_C + 2R)}{2(R_C + R)}$$

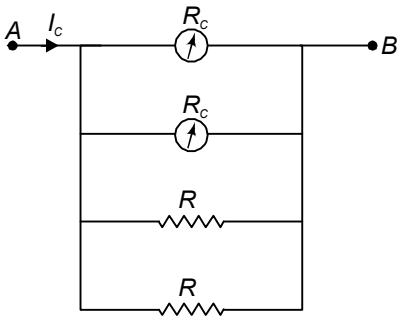
Since $R_C < \frac{R}{2}$, R_C is much less than $2R$ and R. So

$$R_{\text{eq}} \simeq \frac{R_C \times 2R}{2R} = R_C$$

and $$V_b = IR_{\text{eq}} = IR_{\text{C}} \qquad ...(2)$$

It follows from (1) and (2) that $\frac{V_b}{V_a} = \frac{R_C}{2R} << 1$. So option (b) is incorrect.

Option (c)



The equivalent resistance between points A and B is given by

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_C} + \frac{1}{R_C} + \frac{1}{R} + \frac{1}{R}$$

$$= \frac{2}{R_C} + \frac{2}{R}$$

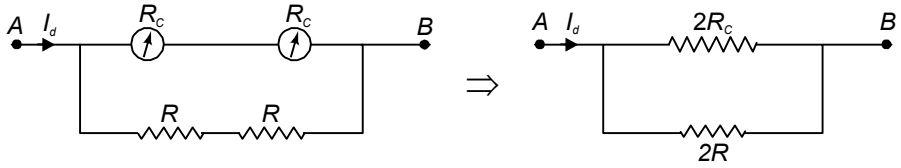
$$\Rightarrow \quad R_{\text{eq}} = \frac{R_C R}{2(R + R_C)} \simeq \frac{R_C}{2} \quad (\because R >> R_C)$$

For a given voltage V, the current I_C is

$$I_C = \frac{V}{R_{eq}} = \frac{2V}{R_C} \qquad ...(3)$$

Since the equivalent resistance of a parallel combination is the minimum, for a given voltage V, the current I_C is the maximum. So option (c) is correct.

Option (d)



The equivalent resistance between points A and B is

$$R_{\text{eq}} = \frac{2R \times 2R_C}{2R + 2R_C} \simeq \frac{4RR_C}{2R} \qquad (\because R >> R_C)$$

$$= 2R_C$$

$$\therefore \quad I_d = \frac{V}{R_{eq}} = \frac{V}{2R_C} \qquad ...(4)$$

12. It follows from (3) and (4) that $I_d > I_c$. So option (d) is incorrect. Thus the correct options are (a) and (c). Consider points on a circle of radius r on the screen. All points on this circle subtend the same angle on the centre of S_1 and S_2. So the interferance fringes are fringes of equal inclination. Hence the fringes are circular. At origin O, $r = 0$. Since there is no screen below O, fringes are semi-circular bright and dark bands centred at O. So option (c) is correct but options (a) and (b) are incorrect.

At O the path difference = d = 0.6003 mm = 0.6003 $\times 10^{-3}$ m which is equal to 1000.5 λ ($\because \lambda = 6 \times 10^{-7}$ m). So the path difference $\Delta = \left(1000 + \frac{1}{2}\right)\lambda$ which satisfies the conditon $\Delta = \left(n + \frac{1}{2}\right)\lambda$ with n = 1000. This is the condition for dark fringe. Hence there will be a dark fringe at O. So option (d) is also correct. Hence options (c) and (d) are correct.

13. Let ω be the angular frequency of the simple harmonic motion when the mass m is not placed on block of mass M and ω_1 when m is placed on M. Then

$$\omega = \sqrt{\frac{k}{M}} \text{ and } \omega_1 = \sqrt{\frac{k}{(M+m)}}$$

Case (i) When m is placed on M when M is at the mean position x_0.

Let υ be the velocity of block M when it passes through the mean position x_0 and υ_1 be the velocity of $(M + m)$ when the composite block passes through x_0. From conservation of momentum,

$$M\upsilon = (M+m)\upsilon_1$$

It A and A_1 are the amplitudes in the two situations, $\upsilon = \omega A$ and $\upsilon_1 = \omega_1 A_1$. Thus

$$M\omega A = (M+m)\omega_1 A_1$$

$$\Rightarrow \quad \frac{A_1}{A} = \frac{M}{(M+m)} \times \frac{\omega}{\omega_1}$$

$$= \frac{M}{(M+m)} \times \sqrt{\frac{M+m}{M}} = \sqrt{\frac{M}{(M+m)}}$$

$$\Rightarrow \quad A_1 = A\sqrt{\frac{M}{(M+m)}}$$

i.e. in this case the amplitude decreases by a factor of $\sqrt{\frac{M}{(M+m)}}$.

Case (ii) When m is placed on M when M is at the extreme position $(x_0 + A)$.

At the extreme position, the velocity of M and of $(M + m)$ is momentarily zero. Hence, in this case, the mean position x_0 and the extreme position $(x_0 + A)$ remain unchanged. Hence the amplitude of oscillation remains unchanged equal to A. So option (a) is correct.

Since the total mass in oscillation is $(M + m)$ in cases (i) and (ii), the time period in each case is

$$T_1 = \frac{2\pi}{\omega_1} = 2\pi\sqrt{\frac{(M+m)}{k}}$$

So option (b) is also correct.

The energy of the oscillator in case (i) is

$$E_1 = \frac{1}{2}(M+m)A_1^2\omega_1^2$$

Without m placed on M, the energy is

$$E = \frac{1}{2}MA^2\omega^2$$

$$\therefore \quad \frac{E_1}{E} = \frac{(M+m)}{M} \times \left(\frac{A_1}{A}\right)^2 \times \left(\frac{\omega_1}{\omega}\right)^2$$

$$= \frac{(M+m)}{M} \times \frac{M}{(M+m)} \times \frac{M}{(M+m)}$$

$$= \frac{M}{M+m}$$

Hence $E_1 < E$ is case (i).

In case (ii), $$E_2 = \frac{1}{2}(M+m)A_1^2\omega_1^2$$

and $$E = \frac{1}{2}MA^2\omega^2$$

$$\therefore \quad \frac{E_2}{E} = \frac{(M+m)}{M} \times \left(\frac{\omega_1}{\omega}\right)^2 \qquad (\because A_1 = A)$$

$$= \frac{(M+m)}{M} \times \frac{M}{(M+m)} = 1$$

Hence $E_2 = E$ in case (ii) so option (c) is incorrect.

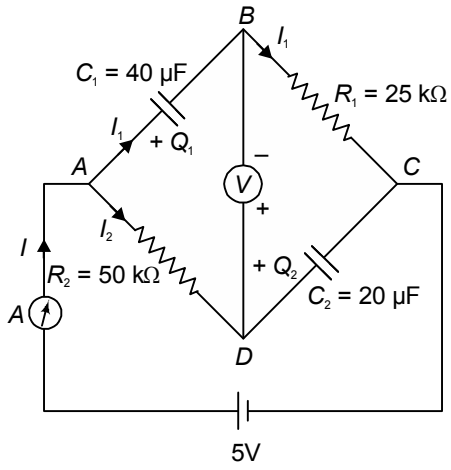
In case (i) the velocity at x_0 is $\upsilon_1 = \omega_1 A_1$ when m is placed on M at x_0 and $\upsilon = \omega A$ when m is not placed on M. Therefore,

$$\frac{\upsilon_1}{\upsilon} = \frac{\omega_1}{\omega} \times \frac{A_1}{A} = \sqrt{\frac{M}{(M+m)}} \times \sqrt{\frac{M}{(M+m)}} = \frac{M}{(M+m)}$$

Thus, $v_1 = v$.

In case (ii), the velocity = $\omega_1 A_1$, the same as in case (i). Hence velocity at x_0 decreases in both cases so the correct options are (a), (b) and (d).

14. Refer to the following figure.



At $t = 0$, when the key is pressed, there is no charge on capacitors C_1 and C_2. Therefore, current I_1 and I_2 are

$$I_1 = \frac{5\text{V}}{25000\Omega} = 0.2 \times 10^{-3}\ \text{A} = 0.2\ \text{mA}$$

and $$I_2 = \frac{5\text{V}}{50000\Omega} = 0.1 \times 10^{-3}\ \text{A} = 0.1\ \text{mA}$$

Therefore, $V_D + 50000\, I_2 = V_B$

$$\Rightarrow \quad V_D - V_B = -50000 \times I_2$$
$$= -50000 \times 0.1 \times 10^{-3}$$
$$= -5\text{V}$$

After a long time (i.e. when the capacitors are fully charged), $I_1 = I_2 = 0$. At this time charge on C_1 is

$$Q_1 = 5\text{V} \times 40\mu\text{F} = 200\ \mu\text{C}$$

The charge on C_2 is

$$Q_2 = 5\text{V} \times 20\mu\text{F} = 100\ \mu\text{C}$$

Therefore $$V_D - \frac{Q_1}{C_1} = V_B$$

$$\Rightarrow \quad V_D - V_B = \frac{Q_1}{C_1} = \frac{200\mu\text{C}}{40\mu\text{F}} = +5\text{V}$$

So option (a) is correct.

The charge on capacitor C_1 varies with t is

$$Q_1' = Q_1\left(1 - e^{-t/\tau_1}\right)$$

where τ_1 = time constant = $C_1R_1 = (40 \times 10^{-6}) \times 25 \times 10^3 = 1$s

$\therefore$ $Q_1' = Q_1(1 - e^{-t}) = 200\ (1 - e^{t})\mu C$

The current through C_1 varies with t as

$$I_1' = \frac{dQ'}{dt} = 200\ e^{-t}\ \mu A = 0.2\ e^{-t}\ mA$$

For capacitor C_2, $Q_2' = 100\left(1 - e^{-t/\tau_2}\right)$

where $\tau_2 = C_2R_2 = 20 \times 10^{-6} \times 50 \times 10^3 = 1$s. Hence

$Q_2' = 100(1 - e^{-t})\mu C$

and $I_2' = 0.1\ e^{-t}$ mA

Therefore $V_D' - V_B' = \frac{Q_2'}{C_2} - I_1' \times 25 \times 10^3$

$$= \frac{100(1 - e^{t}) \times 10^{-6}}{20 \times 10^{-6}} - 0.2 \times e^{-t} \times 10^{-3} \times 25 \times 10^3$$

$$= 5(1 - 2e^{-t})$$

At $t = \ell_n(2)$

$$V_D' - V_B' = 5(1 - 2e^{-\ell n(2)})$$

$$= 5\ (1 - 2 \times 0.5)$$

$$= 5\ (1–1) = 0$$

So option (b) is also correct.

At $t = 1s$, $I' = I_1' + I_2' = 0.2\ e^{-1}$ mA $+ 0.1\ e^{-1}$ mA

$$= \frac{0.3}{e} \text{mA}$$

Intial current at $t = 0$ is $I = (0.2 + 0.1)$ mA $= 0.3$ mA

So option (c) is also correct.

After a long time when steady state is reached, $I_1 = 0$ and $I_2 = 0$. So option (d) is also correct.

Hence all the four options are correct.

15. It is given that the rotational force experienced by the particle is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{\upsilon}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}$$

The term $m(\vec{\omega} \times \vec{r}) \times \vec{\omega}$ is directed radially outwards. Hence it increases $\vec{\upsilon}_{rot}$. The term $2m(\vec{\upsilon}_{rot.} \times \vec{\omega})$ is perpendicular to the edge of the slot and is hence cancelled by the normal reaction exerted by the edge of the slot on the particle.

The radial force is the centripetal force which is

$$F = m\omega^2 r$$

$\Rightarrow$ $$m\frac{d^2r}{dt^2} = m\omega^2 r$$

$$\Rightarrow \qquad \frac{d^2r}{dt^2} = \omega^2 r \qquad \ldots(1)$$

The solution of the differential equation (1) is

$$r = ae^{\omega t} + be^{-\omega t} \qquad \ldots(2)$$

where a and b are constants. This can be checked as follows. From Eq. (2)

$$\frac{dr}{dt} = a\omega e^{\omega t} - b\omega e^{-\omega t}$$

Differentiating again, we have

$$\frac{d^2r}{dt^2} = a\omega^2 e^{\omega t} + b\omega^2 e^{-\omega t}$$

Using (2), we get

$$\frac{d^2r}{dt^2} = \omega^2(ae^{\omega t} + be^{-\omega t}) = \omega^2 r$$

which is eq. (1).

It is given that at $t = 0$, $r = \frac{R}{2}$. Putting $t = 0$ in Eq. (1), we get

$$\frac{R}{2} = a + b \qquad \ldots(3)$$

Also $$\upsilon_{\text{rot}} = \frac{dr}{dt} = A\ a\omega e^{\omega t} - b\omega e^{-\omega t}$$

At $t = 0$, $$\upsilon_{\text{rot}} = (a - b)\omega$$

Given that at $t = 0$, $\upsilon_{\text{rot}} = 0$. This gives $(a - b)\omega = 0$

$\Rightarrow$ $$a = b.$$

So at $t = 0$ $$r = 2a \Rightarrow a = \frac{r}{2} = \frac{R}{4} \qquad \left(\because r = \frac{R}{2}\right)$$

Hence $$r = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

So option (b) is correct.

16. As stated in the above solution, the term $2m(\vec{\upsilon}_{\text{rot}} \times \vec{\omega})$ gives the normal reaction N from the edge of the slot. We have shown above that

$$r = \frac{R}{4}(e^{\omega t} + e^{-\omega t})$$

$\therefore$ $$\vec{\upsilon}_{\text{rot}} = \frac{dr}{dt} = \frac{R\omega}{4}\left(e^{\omega t} - e^{-\omega t}\right)$$

So the normal reaction is

$$N = 2m\frac{R\omega}{4}\left(e^{\omega t} - e^{-\omega t}\right) \times \omega \sin 90^{\circ}$$

$$= \frac{mR\omega^2}{2}\left(e^{\omega t} - e^{-\omega t}\right)$$

This force is along the y-axis. Hence

$$\vec{N} = \frac{mR\omega^2}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j}$$

The normal reaction exerted by the bottom of the slot = $mg\hat{k}$. Hence the net normal reaction from the slot is

$$\vec{N}_{net} = \vec{N} + mg\hat{k}$$

$$= \frac{mR\omega^2}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$$

So option (d) is correct.

17. Since the lower plate is connected to the positive terminal of HV, the balls wil acquire a positive charge and hence they will be attracted by the upper plate which is connected to the negative terminal of HV. On reaching the upper plate the balls will acquire a negative charge and hence they will be attracted by the lower plate and so on. Thus the balls will oscillate between the two plates. At any distance r from a plate, the force experienced by a ball is proportional to $1/r^2$. Thus the restoring force is not proportional to r. Hence the motion is not simple harmonic. So the correct option is (a).

18. If all the balls acquire the same charge q then

$$V_0 = \frac{q}{4\pi \in_0 r}$$

$$\Rightarrow \quad q = 4\pi \in_0 rV_0 \qquad ...(1)$$

If E is the electric field between the plates,

$$\frac{E}{h} = \text{p.d. between plates}$$

$$= V_0 - (-V_0) = 2V_0$$

$$\Rightarrow \quad E = 2hV_0$$

If m is the mass of each ball, the acceleration is

$$a = \frac{qE}{m} = \frac{2qhV_0}{m} \qquad ...(2)$$

Using (1) and (2) we get

$$a = \frac{8\pi \in_0 rhV_0^2}{m}$$

Time taken by a ball to reach the other plate is

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{m}{4\pi \in_0 rV_0^2}} = \frac{1}{V_0}\sqrt{\frac{m}{4\pi \in_0 r}}$$

If there are n balls, the average current is

$$I_{\text{av}} = \frac{nq}{t} = n \times 4\pi \in_0 rV_0^2 \times \sqrt{\frac{4\pi \in_0 r}{m}}$$

Thus $I_n \propto V_0^2$, which is option (c).

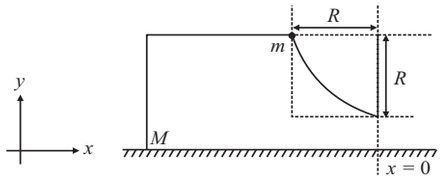
■■

JEE Advanced 2017: Paper - I
(Physics)

SECTION – I

Multiple Choice Questions will ONE or MORE THAN ONE correct options.

1. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at $x = 0$, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct?



(a) The velocity of the point mass m is:

$$v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$$

(b) The velocity of the block M is:

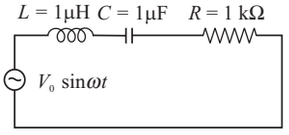
$$v = -\frac{m}{M}\sqrt{2gR}$$

(c) The position of the point mass is:

$$x = -\sqrt{2}\frac{mR}{M+m}$$

(d) The x component of displacement of the center of mass of the block M is: $-\dfrac{mR}{M+m}$

2. In the circuit shown, $L = 1\mu H$, $C = 1\ \mu F$ and $R = 1\ k\Omega$. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



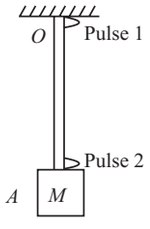
(a) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

(b) At $\omega \gg 10^6$ rad. s^{-1}, the circuit behaves like a capacitor

(c) The frequency at which the current will be in phase with the voltage is independent of R

(d) The current will be in phase with the voltage if $\omega = 10^4$ rad. s^{-1}

3. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse I) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



(a) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope

(b) The velocity of any pulse along the rope is independent of its frequency and wavelength

(c) The wavelength of Pulse 1 becomes longer when it reaches point A

(d) The time $T_{AO} = T_{OA}$

4. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true?

(a) The pressure difference between the leading and trailing faces of the plate is proportional to uv

(b) The plate will continue to move with constant non-zero acceleration, at all times

(c) At a later time external force F balances the resistive force

(d) The resistive force experienced by the plate is proportional to v

5. A human body has a surface area of approximately 1m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be $T_0 = 300\text{K}$. For $T_0 = 300$ K, the value of $\sigma T_0^4 = 460\ \text{Wm}^{-2}$ (where σ is the Stefan-Boltzmann constant). Which of the following options is/are correct?

(a) Reducing the exposed surface area of the body (*e.g.* by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

(b) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

(c) The amount of energy radiated by the body in 1 second is close to 60 Joules

(d) If the surrounding temperature reduces by a small amount $\Delta T_0 \ll T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

6. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

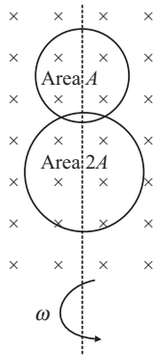
(a) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

(b) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(c) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

(d) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$

7. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. *At the point of crossing the wires remain electrically insulated from each other.* The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(a) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

(b) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(c) The net emf induced due to both the loops is proportional to $\cos \omega t$

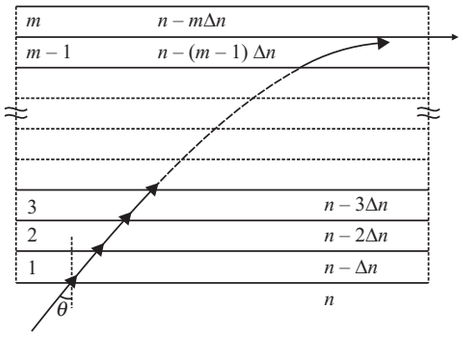
(d) The emf induced in the loop is proportional to the sum of the areas of the two loops

SECTION 2

Single Digit Integer Answer Questions

8. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$. then the *smallest possible* n_f is _____.

9. A monochromatic light is travelling in a medium of refractive index $n = 1.6$. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$. where n_m is the refractive index of the mth slab and $\Delta n = 0.1$ (see the figure).



The ray is refracted out parallel to the interface between the $(m - 1)^{\text{th}}$ and m^{th} slabs from the right side of the stack. What is the value of m?

10. ^{131}l is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately. (you may use $e^x \approx 1 + x$ for $| x | << 1$ and $\ln 2 \approx 0.7$).

11. A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is *reflected* by a large car *approaching* the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received).

12. A drop of liquid of radius $R = 10^{-2}$m having surface tension $S = \frac{0.1}{4\pi}$ Nm^{-1} divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of α is _____.

SECTION 3

Matching Type Questions

Answer Q.13, Q.14 and Q.15 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin ($x = 0$, $y = 0$, $z = 0$) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

Column 1		Column 2	Column 3
(I)	Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II)	Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = B_0\hat{x}$
(III)	Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV)	Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

13. In which case will the particle move in a straight line with *constant* velocity?

(a) (IV) (i) (S) (b) (III) (ii) (R)

(c) (III) (iii) (P) (d) (II) (iii) (S)

14. In which case would the particle move in a straight line along the negative direction of y-axis (*i.e.,* move along $-\hat{y}$)?

(a) (III) (ii) (P) (b) (III) (ii) (R)

(c) (IV) (ii) (S) (d) (II) (iii) (Q)

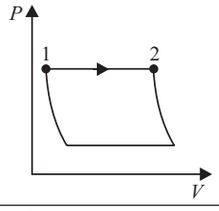
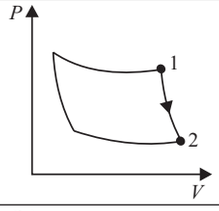
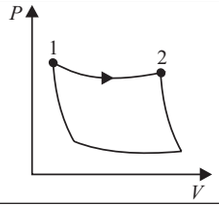
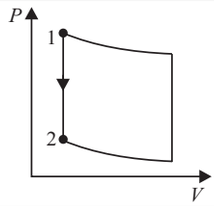
15. In which case will the particle describe a helical path with axis along the positive z direction?

(a) (III) (iii) (P) (b) (II) (ii) (R)

(c) (IV) (ii) (R) (d) (IV) (i) (S)

Answer Q.16, Q.17 and Q.18 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding P-V diagrams in column 3 of the table. Consider only the path from state 1 to state *2*. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas in n.

Column 1		Column 2	Column 3
(I)	$W_{1\to 2} = \frac{1}{\gamma - 1}(P_2V_2 - p_1V_1)$	(i) Isothermal	(P) 
(II)	$W_{1\to 2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III)	$W_{1\to 2} = 0$	(iii) Isobaric	(R) 
(IV)	$W_{1\to 2} = -nRT \ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

16. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

(a) (II) (iv) (R) (b) (III) (iii) (P)

(c) (II) (iii) (P) (d) (II) (iii) (S)

17. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

(a) (I) (iv) (Q) (b) (III) (iv) (R)

(c) (I) (ii) (Q) (d) (IV) (ii) (R)

18. Which one of the following options is the correct combination?

(a) (IV) (ii) (S) (b) (III) (ii) (S)

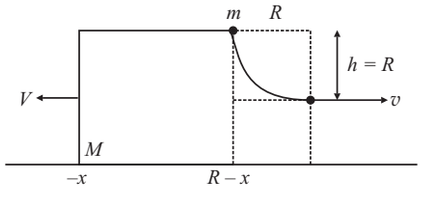
(c) (II) (iv) (R) (d) (II) (iv) (P)

ANSWERS KEY

1.	(a), (d)	**2.**	(a), (c)
3.	(a), (b), (d)	**4.**	(a), (c), (d)
5.	(a), (c), (d)	**6.**	(a), (c), (d)
7.	(a), (b)	**8.**	5
9.	8	**10.**	5
11.	6	**12.**	6
13.	(d)	**14.**	(b)
15.	(d)	**16.**	(c)
17.	(a)	**18.**	(b)

SOLUTIONS

1. Let V be the velocity of the block when mass m loses contact with the block (see figure).



Since no net external force acts on the system, the momentum is conserved, i.e.,

$$MV = mv \Rightarrow V = \frac{mv}{M}$$

From conservation of energy,

Loss in gravitational P.E. of m = gain in K.E. of M + gain in K.E. of m

$$\Rightarrow \quad mgh = \frac{1}{2}MV^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad mgR = \frac{1}{2}M\left(\frac{mv}{M}\right)^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad 2gR = \left(1+\frac{m}{M}\right)v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2gR}{\left(1+\frac{m}{M}\right)}}$$

Hence, option (a) is correct.

$$V = \frac{m}{M}v = \frac{m}{M}\sqrt{\frac{2gR}{\left(1+\frac{m}{M}\right)}}$$

along negative x-direction. So option (b) is incorrect.

Let Δx be the displacement of the centre of mass of m and ΔX be the displacement of the centre of mass of the block of mass M. Since no external force acts on the system, its centre of mass has no net displacement, i.e.,

$$m\Delta x + M\Delta X = 0$$

$$\Rightarrow \quad m\,(R - x) + M(-x) = 0$$

$$\Rightarrow \quad x = \frac{mR}{m+M}$$

Hence, option (d) is correct.

Now, $\Delta x = R - x$

$$= R - \frac{mR}{m+M} = \frac{MR}{M+m}$$

$\therefore$ Final position of $m = 0 - x$

$$= 0 - \frac{MR}{M+m} = -\frac{mR}{M+m}$$

Hence, option (c) is incorrect.

2. Capacitative reactance $X_C = \dfrac{1}{\omega C}$ and inductive reactance $X_L = \omega l$.

At $\omega = 0$, $X_C \to \infty$ and $X_L = 0$. Hence the current flowing through the circuit is nearly zero. So option (a) is correct. When $\omega >> 10^6$ rad s^{-1}, $X_C \to 0$ and $X_L >> 1$. Thus the circuit does not behave like a capacitor and option (b) is incorrect.

The current will be in phase with voltage at resonant frequency i.e. when $X_L = X_C$ or $\omega L = \frac{1}{\omega c} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$ which is independent of R. So option (c) is correct. The resonant frequency is

$$w = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1\times10^{-6})\times(1\times10^{-6})}} = 10^6 \text{ rad s}^{-1}$$

So option (d) is incorrect.

3. Mass of rope is $m = \mu L$ where μ is the mass per unit length of the rope and L is its length. The velocity of a pulse is given by

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension. Since the rope has a finite mass, the tension will not be the same at points on the rope. At A the tension is $T_A = Mg$ and at O, the tension is $T_O = (M + m)g$. At the mid-point, the tension is $\left(M + \frac{m}{2}\right)g$ which is the same for both pulses. Hence pulses 1 and 2 have the same velocity at the mid-point, so option (a) is correct.

Since μ is constant and T has nothing to do with wavelength or frequency, the velocity of any pulse along the rope is independent of wavelength and frequency.

So option (b) is correct.

Since $v = \nu\lambda$

$$\lambda = \frac{v}{\nu} = \frac{1}{\nu}\sqrt{\frac{T}{\mu}}$$

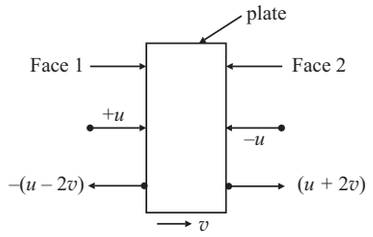
The frequency ν of the pulse cannot change as it depends only on the frequency of the source producing the pulse and since μ is also constant,

$$\frac{\lambda_1}{\lambda_2} \propto \sqrt{T}$$

Since the tension decreases as we go from O to A, the velocity of pulse 1 decreases as it travels from O to A. So the wavelength of pulse 1 becomes shorter when it reaches point A. Hence option (c) is incorrect.

Since the average speed of each pulse is the same, the time taken for pulse 1 to go from O to A = time taken for pulse 2 to go from A to O. So option (d) is correct. Thus the correct options are (a), (b) and (d).

4. Let m be the mass of each molecule. Let velocity directed to the right be taken as positive and velocity to the left be taken as negative.



The rate of change of momentum of a molecule due to collision with face 1 is

Δp_1 = Momentum after collision – momentum before collision

$= -m(u - 2v) - m(u + 2v)$

$= -mu + 2mv - mu - 2mv$

$= -2m\ (u + v)$ directed to the left

The rate of change of monentum of a molecule due to collision with face 2 is

$\Delta p_2 = m\ (u + 2v) - (-mu)$

$= 2m\ (u - v)$ directed to the right

Let τ_1 and τ_2 be the time of collision with face 1 and 2 respectively. Then the forces on face 1 and on face 2 will be proportional to $\frac{\Delta p_1}{\tau_1}$ and $\frac{\Delta p_2}{\tau_2}$

Let R_1 and R_2 be the number of collisions per unit time with faces 1 and 2 respectively, the $R_1 \propto \frac{1}{\tau_1}$ and $R_2 \propto \frac{1}{\tau_2}$. If F_1 and F_2 are the magnitudes of forces on faces 1 and 2, then

$$F_1 \propto R\ |\Delta p_1|$$

or $F_1 \propto R_1\ \Delta\ p_1$ and $F_2 \propto R_2\ \Delta p_2$

Now $R_1 \propto (u + v)$ and $R_2 \propto (u - v)$. Hence

$F_1 \propto 2m\ (u + v)^2$ and

$F_2 \propto 2m\ (u - v)^2$

$\therefore$ Net Force acting on the plate is

$F \propto F_1 - F_2$

$\propto 2m\ (u + v)^2 - 2m\ (u - v)^2$

or $F \propto 8muv$

Thus $F \propto uv$. So option (a) is correct.

Now $F \propto v$. Hence the plate will accelerate and will eventually acquire a terminal velocity because

the gas will tend to reduce the velocity until its acceleration becomes zero. Hence choice (c) is correct.

Froms Stokes' law, it follows that for small velocities the resistive force experienced by the plates is proportional to its velocity v. So option (d) is correct. Thus the correct options are (a), (c) and (d).

5. Assuming that human body behaves as a block body, the heat energy radiated per second from its surface is

$Q = \sigma A\ (T^4 - T_0^4)$,

σ = Stefan's constant

A = exposed surface area of the body

For small temperature difference

$$\Delta T = T - T_0 \Rightarrow T = T_0 + \Delta T$$

$$\therefore \quad Q = \sigma A\ \left[(T_0 + \Delta T)^4 - T_0^4\right]$$

$$= \sigma A\left[T_0^4\left(1+\frac{\Delta T}{T_0}\right)^4 - T_0^4\right]$$

$$= \sigma A\left[T_0^4\left(1+\frac{4\Delta T}{T_0}\right)^4 - T_0^4\right]$$

or $\quad Q = 4\sigma A\ T_0^3\ \Delta T$

It follows that Q decreases with decrease in A. So option (a) is correct.

From Wien's displacement law, it follows that the peak of the graph of Q versus λ shifts to shorter wavelength if the temperature T rises. So option (b) is incorrect.

$$Q = 4\sigma A\ T_0^3\ \Delta T = 4\sigma A\ T_0^3\ (T - T_0)$$

$$= (\sigma T_0^4) \times 4A \times \left(\frac{T}{T_0} - 1\right)$$

$$= 460 \times 4 \times 1 \times \left(\frac{310}{300} - 1\right)$$

$$= 61.3\ \text{Js}^{-1}\ \text{(or W)}$$

So option (c) is correct.

If T_0 decreases to $T_0 - \Delta T_0$, the heat energy radiated per second is given by

$$Q' = \sigma A\left[T^4 - (T_0 - \Delta T_0)^4\right]$$

or $$= \sigma A\left[T^4 - T_0^4\left(1 - \frac{\Delta T_0}{T_0}\right)^4\right]$$

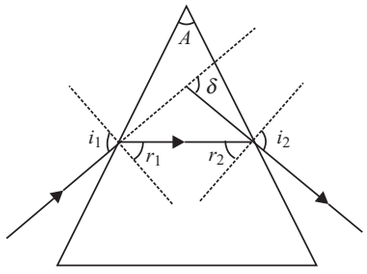
$$= \sigma A\left[T^4 - T_0^4\left(1 - \frac{4\Delta T_0}{T_0}\right)\right]$$

or $\quad Q' = \sigma A\ [T^4 - T_0^4 + 4T_0^3 \Delta T_0]$

$$= Q + 4\sigma A\ T_0^3 \Delta T_0$$

Thus to maintain the same temperature T, Q' must be greater than Q. So option (d) is correct. Hence the correct options are (a), (c) and (d).

6. Refer to the following figure.



$\delta = i_1 + i_2 - A$ and $r_1 + r_2 = A$

For minimum deviation, $i_1 = i_2$ and $r_1 = r_2$. Hence

$\delta_m = 2i_1 - A$ and $2r_1 = A$

Given $\delta_m = A$. Hence

$$A = 2i_1 - A \rightarrow i_1 = A = 2r_1 \Rightarrow r_1 = \frac{i_1}{2}$$

So, option (a) is correct.

From Snell's law,

$$\sin i_1 = \mu \sin r_1$$

$$\Rightarrow \quad \sin A = \mu \sin\left(\frac{A}{2}\right)$$

$$\Rightarrow 2\sin\left(\frac{A}{2}\right)\ \cos\left(\frac{A}{2}\right) = \mu \sin\left(\frac{A}{2}\right)$$

$$\Rightarrow 2\cos\left(\frac{A}{2}\right) = \mu$$

$$\Rightarrow \cos\left(\frac{A}{2}\right) = \frac{\mu}{2}$$

$$\Rightarrow A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

So option (b) is incorrect.

At minimum deviation, the ray inside the prism is parallel to the base. So option (c) is correct. Applying Snell's to the second refraction.

$$\mu \sin r_2 = \sin i_2$$

For tangential emergence, $i_2 = 90°$. Hence

$$\mu \sin r_2 = 1$$

$$\Rightarrow \quad r_2 = \sqrt{1-\frac{1}{\mu^2}}$$

Now, $r_1 = A - r_2$

$$\sin r_1 = \sin (A - r_2) = \sin A \cos r_2 - \cos A \sin r_2$$

$$= (\sin A)\ \frac{\sqrt{\mu^2 - 1}}{\mu} - (\cos A)\frac{1}{\mu}$$

From Snell's law applied at the first refraction,

$$\sin i_1 = \mu \sin r_1$$

$$\Rightarrow \quad \sin r_1 = \sin (A)\ (\mu^2 - 1)^{1/2} - \cos A$$

$$\text{or} \quad i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\left(\frac{A}{2}\right)} - 1 - \cos A\right]$$

So option (d) is correct. Thus the correct options are (a), (c) and (d).

7. At any time t, the area vector of each loop makes an angle $\theta = \omega t$ with $\vec{B}$. The instantaneous magnetic flux in loop 1 is $\phi_1 = BA \cos \omega t$ and in loop 2 is $\phi_2 = B(2A) \cos \omega t$. The magnitudes of induced emfs in loops 1 and 2 are

$$|\varepsilon_1| = \left|\frac{d\phi_1}{dt}\right| = BA\omega \sin \omega t$$

and

$$|\varepsilon_2| = \left|\frac{d\phi_2}{dt}\right| = 2\ BA\omega \sin \omega t$$

The net instantaneous flux is (because the two emfs oppose each other)

$$\varepsilon_{net} = 2BA\omega \sin \omega t - BA\omega \sin \omega t$$

$$= BA\omega \sin \omega t$$

The amplitude of the $\varepsilon_{net} = BA\omega$ which is equal to that due to loop 1 alone. So option (a) is correct.

When $\theta = \frac{\pi}{2}$, ε_{net} is maximum. So option (b) is correct. ε_{net} is proportional to $\sin\omega t$. So option (c) is incorrect. Option (d) is incorrect because the orientation of the loop is such that ε_1 and ε_2 oppose each other. Hence ε_{net} is proportional to the difference in areas. So the correct options are (a) and (b).

8. $V \propto \frac{1}{n^2}$. So

$$\frac{V_i}{V_f} = \left(\frac{n_f}{n_i}\right)^2$$

$$\text{or} \quad 6.25 = \left(\frac{n_f}{n_i}\right)^2 \Rightarrow \frac{n_f}{n_i} = 2.5 = \frac{5}{2}$$

So the smallest integral value of $n_f = 5$

9. The refractive index of layers decreases by a constant amount Δn as we go from lower to upper layers. For total reflection at the interface between the $(m - 1)$th and m^{th} layers, we have ($\because$ $n \sin\theta$ = constant)

$$n \sin \theta = (n - m\Delta n) \sin 90°$$

$$\Rightarrow \quad 1.6 \times \sin 30° = 1.6 - m\Delta n$$

$$\Rightarrow \quad 0.8 = 1.6 - m \times 0.1 \Rightarrow m = \frac{0.8}{0.1} = 8$$

10. Activity $R = R_o e^{-\lambda t}$ where the decay constant λ is given by

$$\lambda = \frac{\ln(2)}{T};\ T = \text{half life}$$

$$= \frac{\ln(2)}{8} \text{ per day} = \frac{0.7}{8} \text{ per day}$$

$$t = 12 \text{ hours} = \frac{1}{2} \text{day. So}$$

$$R = R_0 e^{-\frac{0.7}{8}\times\frac{1}{2}} = R_0 e^{-\frac{0.7}{16}}$$

Using $e^x = 1 + x$, we get

$$R = R_0\left(1 - \frac{0.7}{16}\right) = R_0 \times \frac{15.3}{16}$$

$$= \frac{2.4\times10^5 \times 15.3}{16}$$

$$= 2.3 \times 10^5 \text{ Bq}$$

Now, V litre is the volume of the blood, then

$$115 = \frac{R \times 2.5 \times 10^{-3}}{V}$$

$$= \frac{2.3\times10^5 \times 2.5\times10^{-3}}{V}$$

$$\Rightarrow \quad V = 5 \text{ litre}$$

11. Apparent frequency is

$$\mu' = \left(\frac{330+2}{330-2}\right) \times 492 = 498 \text{ Hz}$$

$\therefore$ Beat frequency = 498 – 492 = 6 Hz

12. Let r be the radius of each small drop. Since the total volume remains constant,

$$\frac{4\pi}{3}R^3 = K \times \frac{4\pi}{3}\ r^3 \Rightarrow R^3 = Kr^3$$

Initial P.E. is

$U_i = S \times 4\pi R^2$

Final P.E. is

$U_f = KS \times 4\pi r^2$

$\Delta U = U_f - U_i$

$\Rightarrow \quad 10^{-3} = KS \times 4\pi r^2 - 5 \times 4\pi R^2$

$= 4\pi S(Kr^2 - R^2)$

$= 4\pi SR^2\left(\frac{Kr^2}{R^2} - 1\right)$

$= 4\pi SR^2 (k^{1/3} - 1) \qquad (\because R^3 = Kr^3)$

or $\qquad 10^{-3} = 4\pi SR^2 \left(10^{\frac{\infty}{3}} - 1\right) \ (\because K = 10^{\alpha})$

Substituting the given values of S and R and solving we get

$$\alpha \simeq 6$$

13. The charged particle will move in a straight line with a constant velocity if no net force acts on it, i.e. if magnetic force = electrical force or

$$qvB = qE$$

$$\Rightarrow \qquad v = \frac{E}{B}$$

The electric and magnetic field must be perpendicular to each other.

If the particle is an electron, then = $\vec{v} = \frac{E_0}{B_0}\hat{y}$ if

$\vec{E} = -E_0\hat{x}$ and $\vec{B} = B_0\hat{z}$

In this case $\overrightarrow{F_B} = q(\vec{v} \times \vec{B})$

$= -e\left(\frac{E_0}{B_0}\hat{y} \times B_0\hat{z}\right) = -e\ E_0\hat{x}$

and $\overrightarrow{F_E} = q\vec{E} = -e \times \left(-E_0\hat{x}\right) = eE_0\hat{x}$

So the correct choices are (II), (ii) and (S) which is option (d).

14. If $v = 0$, the proton will move along the negative y direction if $\vec{E}$ is along negative y direction because if $\vec{v} = 0$, magnetic force is zero or parallel or antiparallel to $\vec{E}$. So the correct choices are (III), (ii) and (R) which is option (b).

15. The proton will describe a helical path with velocity $\vec{v} = 2\left(\frac{E_0}{B_0}\right)\hat{x}$ if $\vec{E} = E_0\hat{z}$ and $\vec{B} = B_0\hat{z}$ because then $\overrightarrow{F_E}$ will be along the $+z$ axis and $\overrightarrow{F_B}$ will be along $-y$ axis. so the correct choices are (IV), (i) and (S) which is option (d).

16. Since W represents work done on the gas, it is negative. For an isobaric process = (P = constant),

$$W = -\int PdV = -p\int_{V_1}^{V_2} dV = P\ (V_1 - V_2),$$

which is (II).

For an isobaric process, the P-V graph is parallel to the V-axis which is option (P). No other combination is possible. So the correct choices are (II), (iii) and (P), which is option (c).

17. Laplace gave the correct formula for the speed of sound in a gas. He corrected Newton's formula by assuming that the propagation of sound in a gas is an adiabatic process for which

$W = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)$ which is choice (I). Also choice (iv) is correct. In option (Q), process $1 \rightarrow 2$ is adiabatic. So the correct option is (a).

18. Option (a) is incorrect because process $1 \rightarrow 2$ is isochoric for which $W = 0$. Choice (c) is incorrect because $W = P\ (V_1 - V_2)$ is wrong for an adiabatic process. For the same reason choice (d) is also incorrect. Hence the only correct option is (b).

JEE Advanced 2017: Paper - II
(Physics)

SECTION – I

Multiple Choice Questions having ONLY ONE correction option.

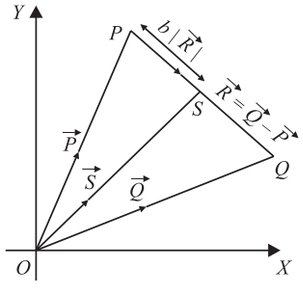
1. A photoelectric material having work function ϕ_0 is illuminated with light of wavelength $\lambda\left(\lambda < \frac{hc}{\phi_0}\right)$

The fastest photoelectron has a de Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then the ratio $\Delta\lambda_d/\Delta_\lambda$ is proportional to:

(a) λ_d^3/λ^2 (b) λ_d^2/λ^2

(c) λ_d/λ (d) λ_d^3/λ

2. Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b\,|\vec{R}|$. The general relation among vectors $\vec{P}$, $\vec{Q}$ and $\vec{S}$ is:



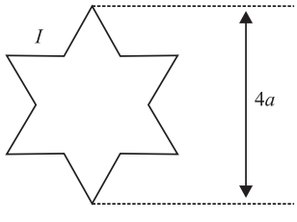
(a) $\vec{S} = (1 - b)\vec{P} + b^2\vec{Q}$

(b) $\vec{S} = (1 - b^2)\vec{P} + b\vec{Q}$

(c) $\vec{S} = (1 - b)\vec{P} + b\vec{Q}$

(d) $\vec{S} = (b - 1)\vec{P} + b^2\vec{Q}$

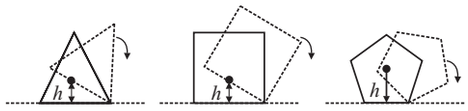
3. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is $4a$. The magnitude of the magnetic field at the centre of the loop is:



(a) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}-1]$ (b) $\frac{\mu_0 I}{4\pi a}3[\sqrt{3}-1]$

(c) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}+1]$ (d) $\frac{\mu_0 I}{4\pi a}3[2-\sqrt{3}]$

4. Consider regular polygons with number of sides $n = 3, 4, 5\ldots$ as shown in the figure. The centre of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the centre of mass for each polygon is Δ. Then Δ depends on n and h as:



(a) $\Delta = h \sin\left(\frac{2\pi}{n}\right)$ (b) $\Delta = h \sin^2\left(\frac{\pi}{n}\right)$

(c) $\Delta = h \tan^2\left(\frac{\pi}{2n}\right)$ (d) $\Delta = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$

5. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to:

(a) $R^{2/3}$ (b) R

(c) R^3 (d) $\frac{1}{R}$

6. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be $L = 20$ meters. Take the acceleration due to gravity $g = 10$ ms^{-2} and the velocity

of sound is 300 ms^{-1}. Then the fractional error in the measurement $\delta L/L$, is closest to:

(a) 0.2% (b) 5%
(c) 1% (d) 3%

7. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ kms^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to

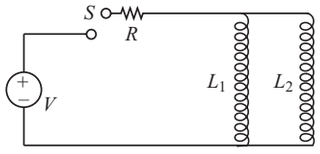
(Ignore the rotation and revolution of the Earth and the presence of any other planet)

(a) $v_s = 62$ kms^{-1} (b) $v_s = 42$ kms^{-1}
(c) $v_s = 72$ kms^{-1} (d) $v_s = 22$ kms^{-1}

SECTION 2

(Multiple choice Questions having ONE or MORE THAN ONE Correct Options)

8. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?



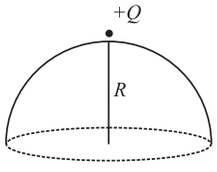
(a) At $t = 0$, the current through the resistance R is $\frac{V}{R}$

(b) After a long time, the current through L_2, will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(c) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

(d) The ratio of the currents through L_1, and L_2 is fixed at all times ($t > 0$)

9. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



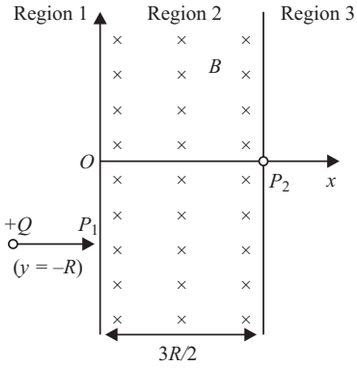
(a) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

(b) The component of the electric field normal to the flat surface is constant over the surface

(c) Total flux through the curved and the flat surfaces is $\frac{Q_0}{\varepsilon_0}$

(d) The circumference of the flat surface is an equipotential surface

10. A uniform magnetic field B exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+ Q$ and momentum p directed along x-axis enters region 2 from region 1 at point $P_1(y = -R)$. Which of the following option(s) is/are correct?



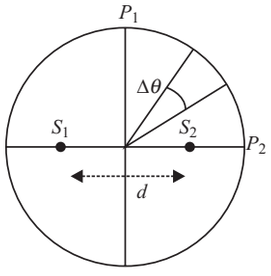
(a) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis

(b) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region I

(c) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle

(d) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from y-axis is $p/\sqrt{2}$

11. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance $d = 1.8$ mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?



(a) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

(b) A dark spot will be formed at the point P_2

(c) At P_2 the order of the fringe will be maximum

(d) The angular separation between two consecutive bright spots decreases as we move from P_1 and P_2 along the first quadrant

12. The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_x = V_0 \sin \omega t,$$

$$V_y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and}$$

$$V_z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

An ideal voltmeter is configured to read *rms* value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

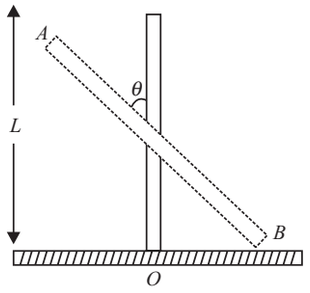
(a) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(b) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

(c) $V_{XY}^{rms} = V_0$

(d) independent of the choice of the two terminals

13. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct?



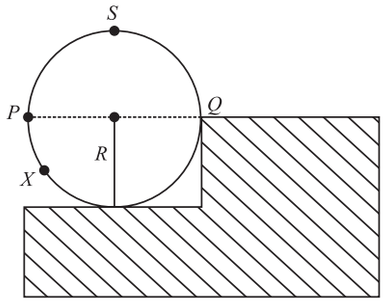
(a) The trajectory of the point A is a parabola

(b) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$.

(c) The midpoint of the bar will fall vertically downward

(d) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1 - \cos\theta)$

14. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?



(a) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step
(b) If the force is applied normal to the circumference at point P then τ is zero
(c) If the force is applied normal to the circumference at point X then τ is constant
(d) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

SECTION 3

(Paragraph Based Questions having ONLY ONE Correct Option)

Paragraph for Questions 15-16

Consider a simple RC circuit as shown in Figure 1.

Process 1 : In the circuit the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0, (i.e., charging continues for time $T \gg RC$). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T \gg RC$. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1. These two processes are depicted in Figure 2.

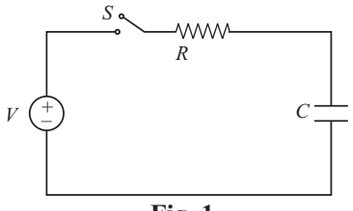


Fig. 1

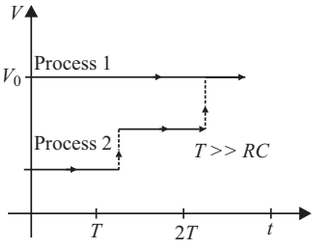


Fig. 2

15. In Process 2, total energy E_D dissipated across the resistance is:

(a) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (b) $E_D = \frac{1}{2}CV_0^2$

(c) $E_D = 3CV_0^2$ (d) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

16. In Process 1, the energy stored in the capacitor E_C and heat dissipated E_D across resistance are related by:

(a) $E_C = \frac{1}{2}E_D$ (b) $E_C = 2E_D$

(c) $E_C = E_D$ (d) $E_C = E_D \ln 2$

Paragraph for Questions 17-18

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring *rolls without slipping* on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

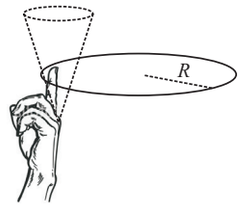


Fig. 1

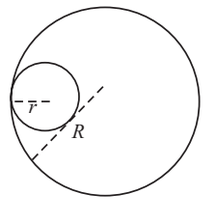


Fig. 2

17. The minimum value of ω_0 below which the ring will drop down is :

(a) $\sqrt{\frac{2g}{\mu(R-r)}}$ (b) $\sqrt{\frac{g}{\mu(R-r)}}$

(c) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (d) $\sqrt{\frac{g}{2\mu(R-r)}}$

18. The total kinetic energy of the ring is:

(a) $\frac{3}{2} M\omega_0^2 (R - r)^2$

(b) $\frac{1}{2} M\omega_0^2 (R - r)^2$

(c) $M\omega_0^2 (R - r)^2$

(d) $M\omega_0^2 R^2$

Answers

Section I

1. (a)	**2.** (c)
3. (a)	**4.** (d)
5. (b)	**6.** (c)

Section II

7. (b)	**8.** (b), (c), (d)
9. (a), (d)	**10.** (a), (b)
11. (a), (c)	**12.** (b), (d)
13. (b), (c), (d)	**14.** (b), (d)

Section III

15. (d)	**16.** (c)
17. (b)	**18.** None.

Hints and Solutions

1. If p is the momentum of the fastest photoelectron, its K.E. is

$$K_{max} = \frac{p^2}{2m}; \; m = \text{mass of electron}$$

$$= \frac{1}{2m}\left(\frac{h}{\lambda_d}\right)^2 \qquad (\because \quad \lambda_d = \frac{h}{p})$$

or $$K_{max} = \frac{h^2}{2m\lambda_d^2}$$

From Einstein's photo-electric equation,

$$\frac{hc}{\lambda} = K_{max} + \phi_0$$

$$\Rightarrow \qquad \frac{hc}{\lambda} = \frac{h^2}{2m\lambda_d^2} + \phi_0$$

$$\Rightarrow \qquad -\frac{hc}{\lambda^2}\Delta\lambda = -\frac{2h^2}{2m\lambda_d^3}\Delta\lambda_d$$

$$\Rightarrow \qquad \frac{\Delta\lambda_d}{\Delta\lambda} = \left(\frac{c}{2hm}\right)\left(\frac{\lambda^3{}_d}{\lambda^2}\right)$$

$\therefore \frac{\Delta\lambda_d}{\Delta\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$, which is option (a).

2. Applying triangle law to ΔOPS, $\vec{S}$ is the resultant of vectors $\vec{P}$ and $b\,|\vec{R}\,|$, i.e.

$$\vec{P} + b\,|\vec{R}\,| = \vec{S}$$

$$\Rightarrow \qquad b\,|\vec{R}\,| = \vec{S} - \vec{P}$$

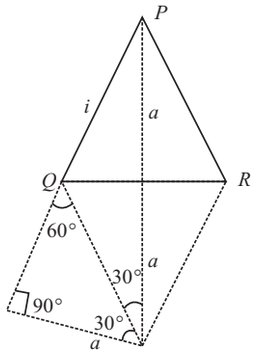
It is given in the figure that $\vec{R} = \vec{Q} - \vec{P}$

or $$\vec{P} = \vec{Q} - \vec{R}$$

or $$\vec{P} = \vec{Q} - \frac{1}{b}(\vec{S} - \vec{P})$$

$$\Rightarrow \qquad \vec{S} = (1 - b)\vec{P} + b\vec{Q}, \text{ which is option (c).}$$

3. Refer to the following figure.



The magnitude of the magnetic field due to one segment PQ of the star at the centre of the loop is

$$B = -\frac{\mu_0 i}{4\pi a}\int_{60°}^{30°} \sin\theta \, d\theta$$

$$= \frac{\mu_0 i}{4\pi a}\left|\cos\theta\right|_{60°}^{30°}$$

$$= \frac{\mu_0 i}{4\pi a}(\cos 30° - \cos 60°)$$

$$= \frac{\mu_0 i}{8\pi a}(\sqrt{3} - 1) \text{ directed into the page.}$$

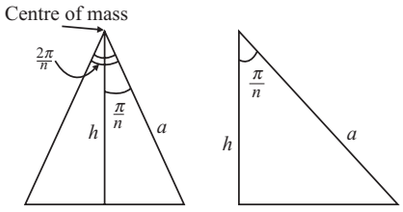
The magnitude of the magnetic field at the centre of the loop = 12B as all segments of stars produce magnetic field directed into the page. Hence

$$B_{loop} = 12B = \frac{12\mu_0 i}{8\pi a}(\sqrt{3} - 1)$$

$$= 6\left(\frac{\mu_0 i}{4\pi a}\right)(\sqrt{3} - 1)$$

So the correct option is (a).

4. Refer to the following figure. For n-sided regular polygon, the angle subtended at the centre of mass $= \frac{2\pi}{n}$



$$\frac{h}{a} = \cos\left(\frac{\pi}{n}\right) \Rightarrow a = \frac{h}{\cos\left(\frac{\pi}{n}\right)}$$

$$\Delta = a - h = \frac{h}{\cos\left(\frac{\pi}{n}\right)} - h = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$$

So the correct option is (d).

5. The total mass of the sphere is constant. Thus

$$\frac{4\pi R^3}{3}\rho = \text{constant}$$

or $$R^3\rho = \text{constant}$$

Differentiating with respect to time,

$$3R^2\rho\,\frac{d\rho}{dt} + R^3\frac{d\rho}{dt} = 0$$

or $$3R^2\rho v + R^3\,\frac{d\rho}{dt} = 0$$

$$\Rightarrow \qquad v = -\frac{R}{3}\frac{1}{\rho}\frac{d\rho}{dt}$$

It is given that $\frac{1}{\rho}\frac{d\rho}{dt}$ = constant. Hence $v \propto R$.

So the correct option is (b).

6. Time taken for the stone to reach the bottom of the well is given by

$$-L = -\frac{1}{2}gt^2$$

$$\Rightarrow \qquad t = \sqrt{\frac{2L}{g}}$$

Time taken for sound to travel from the bottom of the well to the observer is

$t' = \frac{L}{v}$; v = speed of sound

Total time taken is

$$T = t + t' = \sqrt{\frac{2L}{g}} + \frac{L}{v}$$

Differentiating partially with respect to L

$$\frac{\Delta T}{\Delta L} = \sqrt{\frac{2}{g}} \times \left(-\frac{1}{2\sqrt{L}}\right) + \frac{1}{v}$$

Since errors do not cancel each other,

$$\frac{\Delta T}{\Delta L} = \sqrt{\frac{2}{g}} \times \frac{1}{2\sqrt{L}} + \frac{1}{v}$$

$$= \sqrt{\frac{2}{10}} \times \frac{1}{2\times\sqrt{20}} + \frac{1}{300}$$

$$= \frac{1}{20} + \frac{1}{300} = \frac{16}{300}$$

$$\Rightarrow \qquad \frac{0.01}{\Delta L} = \frac{16}{300}$$

$$\Rightarrow \qquad \Delta L = \frac{0.01\times 300}{16} = \frac{3}{16}$$

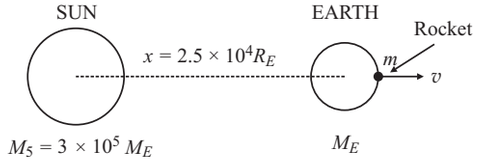
$$\therefore \qquad \frac{\Delta L}{L} = \frac{3}{16}\times\frac{1}{20}$$

Percentage error $= \frac{\Delta L}{L}\times 100 = \frac{3}{16}\times\frac{1}{20}\times 100$

$$= \frac{15}{16} \simeq 1\%$$

So the correct option is (c).

7. Refer to the following figure.



The escape velocity for earth is

$$v_e = \sqrt{\frac{2GM_E}{R_E}} = 11.2 \text{ kms}^{-1} \text{ (given)}$$

The gravitational potential energy of the SUN–EARTH system is (m = mass of the rocket)

$$U = -\frac{GM_Em}{R_E} - \frac{GM_Sm}{x}$$

$$= -\frac{GM_Em}{R_E} - \frac{G\times 3\times 10^5 M_Em}{2.5\times 10^4 R_E}$$

$$= -\frac{GM_Em}{R_E}\ (1 + 12)$$

or $$U = -\frac{13GM_Em}{R_E}$$

The minimum initial velocity v required for to escape from the sun-earth system is given by

$$\frac{1}{2}mv^2 = \frac{13GM_Em}{R_E}$$

$$\Rightarrow \quad v = \sqrt{13} \times \sqrt{\frac{2GM_E}{R_E}}$$

$$= \sqrt{13}\, v_e$$

$$= \sqrt{13} \times 11.2$$

$$= 40.4 \text{ kms}^{-1}$$

The closest option is (b).

8. At time $t = 0$ when the switch S is open, the source of voltage is not connected to the circuit. Hence no current flows in the circuit at time $t = 0$. So option (a) is incorrect.

Immediately after the switch is closed, a current begins to flow. Since the inductors offer reactance, it takes some time to grow to its steady state value $= i_0$. Since the inductors L_1 and L_2 are connected in parallel, the induced voltage V_L is the same across each inductor. If i_1 and i_2 are the currents through L_1 and L_2 at any time t during the growth of current, then

$$V_L = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$$

or $\quad L_1 di_1 = L_2\, di_2$

$$\Rightarrow \quad L_1 \int di_1 = L_2 \int di_2$$

$\Rightarrow L_1 i_1 = L_2 i_2 \Rightarrow \dfrac{i_1}{i_2} = \dfrac{L_2}{L_1}$, which is independent of time. So option (d) is correct.

Now, $i_0 = i_1 + i_2$ or $i_2 = i_0 - i_1$

or $\quad \dfrac{i_2}{i_1} = \dfrac{i_0}{i_1} - 1$

or $\quad \dfrac{L_1}{L_2} = \dfrac{i_0}{i_1} - 1 \qquad \left(\because \ \dfrac{i_1}{i_2} = \dfrac{L_2}{L_1}\right)$

$$\Rightarrow \quad i_1 = \frac{i_0 L_2}{L_1 + L_2}$$

Since, in the steady state, the inductors offer no reactance, $i_0 = \dfrac{V}{R}$. Hence

$$i_1 = \frac{V}{R} \frac{L_2}{(L_1 + L_2)}$$

Now, $\quad i_2 = i_0 - i_1$

$$= \frac{V}{R} - \frac{V}{R} \frac{L_2}{(L_1 + L_2)}$$

$$= \frac{V}{R} \frac{L_1}{(L_1 + L_2)}$$

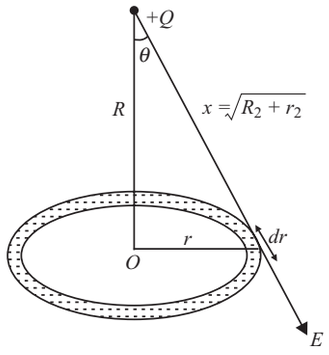
So option (b) and (c) are both correct. Thus the correct options are (b), (c) and (d).

9. The hemisphere consists of a curved surface of radius R and a flat (bottom) surface which is a disc of radius R. Since the charge + Q lies outside the hemisphere, the net electric flux (ϕ_e) through the curved surface and the flux (ϕ_f) through the flat surface is zero, i.e..

$$\phi_c + \phi_f = 0 \Rightarrow \phi_c = -\phi_f$$

So option (c) is incorrect.

To calculate flux through the flat surface (disc), we divide the disc into very small elements of length dr. Consider one such element at a distance r from the centre O of the disc as shown in the figure.



Electric Field due to + Q at the element is

$$E = \frac{Q}{4\pi\varepsilon_0 x^2} = \frac{Q}{4\pi\varepsilon_0 (R^2 + r^2)}$$

Area of element is $dA = 2\pi r\, dr$. The electric flux through the flat surface is

$$\phi_f = \int_0^R \vec{E} \cdot \vec{dA} = \int_0^R E\, dA \cos\theta = \int_0^R E\, dA \times \frac{R}{x}$$

$$= \int_0^R \frac{Q}{4\pi\varepsilon_0 (R^2 + r^2)} \times 2\pi r dr \times \frac{R}{(R^2 + r^2)^{1/2}}$$

$$= \frac{2\pi QR}{4\pi\varepsilon_0} \int_0^R \frac{r dr}{(R^2 + r^2)^{3/2}}$$

$$= \frac{QR}{2\varepsilon_0} \left[\frac{\frac{1}{2} (R^2 + r^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R$$

$$= \frac{QR}{2\varepsilon_0} \left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + R^2}} \right]$$

$$= \frac{Q}{2\varepsilon_0} \left(1 - \frac{1}{\sqrt{2}} \right)$$

Now $\phi_c = -\phi_f = -\dfrac{Q}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$

So option (a) is correct.

Electric potential at any point on the circumference of the flat surface is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+R^2}} = \frac{Q}{4\pi\varepsilon_0(\sqrt{2}R)}$$

which is constant. Hence the circumference of the flat surface is equipotential. So option (d) is correct.

The component of the electric field normal to the flat surface is

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{(R/\cos\theta)^2} = \frac{Q\cos^2\theta}{4\pi\varepsilon_0 R^2},$$ which

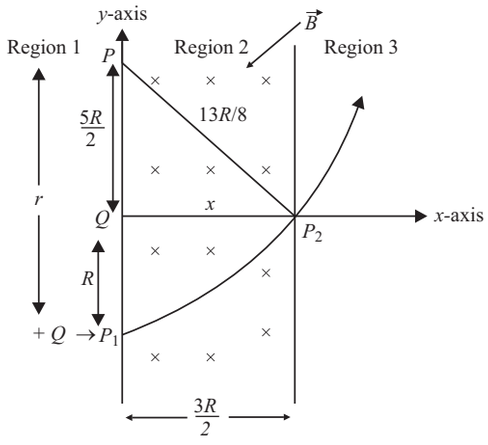
varies with θ and is constant. So option (b) is incorrect. Thus the correct options are (a) and (d).

10. The particle will describe a circle of radius r given by $r = \dfrac{mv}{QB} = \dfrac{p}{QB}$ in the anticlockwise sense.

Option (a): If $B = \dfrac{8}{13}\dfrac{p}{QR}$,

$$r = \frac{p}{Q}\times\frac{13QR}{8p} = \frac{13}{8}R$$

Refer to the following figure. It follows from the figure that



$$x = \sqrt{\left(\frac{13R}{8}\right)^2 - \left(\frac{5R}{2}\right)^2} = \frac{3R}{2}$$ which is equal to OP_2.

Hence the particle will enter region 3 at point P_2. So option (a) is correct.

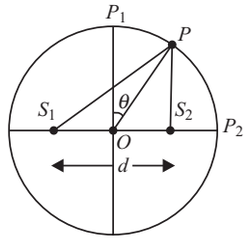
Option (b) The particle will re-enter region 1 if it describes a semicircle in region 2 of r a d i u s $r < x$ i.e. if $r < \dfrac{3R}{2}$, i.e. if $\dfrac{p}{QB} < \dfrac{3R}{2}$ or $B > \dfrac{2p}{3QR}$

. So option (b) is also correct.

Option (c): The distance between P_1 and P (the point of re-entry in region 1 $= 2r = \dfrac{2mv}{QB}$ which is directly proportional to m (for given v and B). So option (c) is incorrect.

Option (d): The particle leaves region 2 and re-enters region 1 moving horizontal along the negative x direction with momentum $-p$, it entered region 2 with momentum $+p$. So the change in momentum $= -p-(+p) = -2p$. So option (d) is incorrect. Thus the correct options are (a) and (b).

11. Refer to the following figure.



At point P_1, the path difference is $\Delta x = S_1P_1 - S_2P_1 = 0$. Hence there is a bright fringe at P_1. So option (b) is incorrect.

At point P_2, $\Delta x = S_1P_2 - S_2P_2 = S_1S_2 = d$. There will be nth bright spot at P_2 if

$$d = n\lambda$$

or $$n = \frac{d}{\lambda} = \frac{1.8\times10^{-3}}{600\times10^{-9}} = 3000$$

So there will be 3000th bright spot at P_2. So option (c) is correct. Hence there will be $n - 1 = 3000 - 1 = 2999$ bright spots between P_1 and P_2. So option (a) is correct.

Let P be the point on the circle for which angle between OP and OP_1 is, say θ. Assuming that the circle has a large radius, the path difference at point P is

$$\Delta x = d\sin\theta$$

The rate of change of path difference with angle θ is

$$\frac{d}{d\theta}(\Delta x) = d\cos\theta,$$

which decreases will increase in θ. So the angular separation between consecutive bright spots increases as we go from P_1 to P_2. Hence option

(d) is incorrect. Thus the correct option are (a) and (c).

12. When the voltmeter is connected between X and Y

$$V_{XY} = V_X - V_Y$$

$$= V_0 \sin \omega t - V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$= V_0 \left[\sin \omega t - \sin\left(\omega t + \frac{2\pi}{3}\right)\right]$$

$$= 2V_0 \left[\cos\left(\omega t + \frac{\pi}{3}\right)\sin\left(-\frac{\pi}{3}\right)\right]$$

$$= -\frac{\sqrt{3}\times 2}{2} V_0 \cos\left(\omega t + \frac{\pi}{3}\right)$$

$$= +\sqrt{3}\, V_0 \sin\left(\omega t - \frac{\pi}{3}\right)$$

Maximum value of $V_{XY} = \sqrt{3}V_0$

$\therefore$ rms value of $V_{XY} = \dfrac{\sqrt{3}V_0}{\sqrt{2}}$. So option (b) is correct and option (c) is incorrect.

When the voltmeter is connected between Y and Z.

$$V_{YZ} = V_Y - V_Z$$

$$= V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) - V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

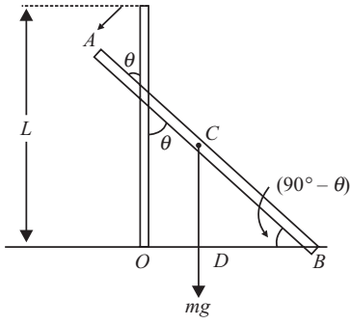
$$= V_0 \left[\sin\left(\omega t + \frac{2\pi}{3}\right) - \sin\left(\omega t + \frac{4\pi}{3}\right)\right]$$

$$= \sqrt{3}\, V_0 \cos \omega t$$

Maximum value of $V_{YZ} = \sqrt{3}\, V_0$

$\therefore$ *rms* value of $V_{YZ} = \dfrac{\sqrt{3}V_0}{\sqrt{2}}$, which is the same as the rms value of V_{xy}. So option (a) is incorrect and option (d) is correct. Hence the correct options are (b) and (d).

13. Refer to the following figure.



Option (a): The rod rotates about an axis passing through B and perpendicular to the plane of the page. Hence point A describes a circular trajectory. So option (a) is incorrect.

Option (b): At any instant of time, the magnitude of the torque about $B = mg \times BD = mg\ B < \sin\ \theta$

$$= \left(\frac{mgL}{2}\right) \sin\ \theta$$

This is so because the entire mass m of the rod can be assumed to be acting at its centre of mass C. Since the rod is uniform, $BC = \dfrac{AB}{2} = \dfrac{L}{2}$.

So option (b) is correct.

Option (c): Since there is no external horizontal force and the initial velocity of the rod is zero, the centre of mass will move vertically downwards. So option (c) is correct.

Option (d): Initial y-coordinate of the centre of mass = $\dfrac{L}{2}$. When the rod subtends an angle θ with the vertical, the y-coordinate = $\dfrac{L}{2}\cos\theta$. Shift in the mid-point = $\dfrac{L}{2} - \dfrac{L}{2}\cos\theta = \dfrac{L}{2}(1 - \cos\theta)$.

So option (d) is correct. Thus the correct options are (b), (c) and (d).

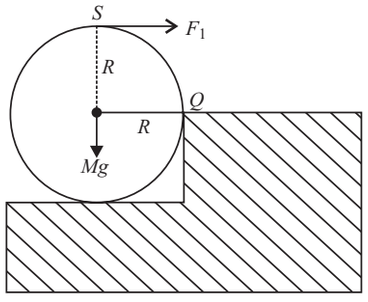


Fig. 1

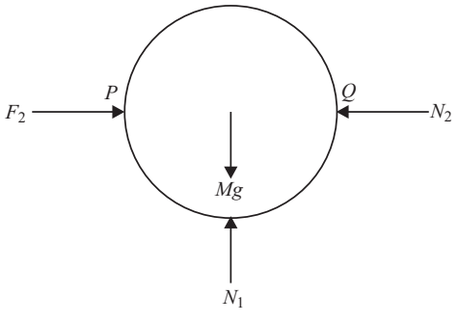


Fig. 2

Option (a): If the force F_1 is applied at S as shown in Fig. 1, torque of F_1 about $Q = F_1R$ (anticlockwise). Torque of Mg about $Q = MgR$ (clockwise). The net torque depends on the magnitudes of F_1 and Mg is given by

$$\tau = F_1R - MgR = (F_1 - Mg)\ R$$

If $F_1 > Mg$, $\tau \neq 0$ and the wheel will climb the step. So option (a) is incorrect.

Option (b): If the force F_2 is applied normal to point P as shown in Fig. 2, then F_2 balances with normal reaction N_2 of the wall at Q and force Mg balances with the normal reaction N_1 of the base of the step as shown.

Hence no net force acts on the wheel and $\tau = 0$. So option (b) is correct.

Option (c): If force F_3 is applied at X as shown in Fig. 3, the torque due to F_3 about $Q = F_3QT$ (clockwise). The torque due to Mg about $Q = MgR$ (anticlockwise). The net torque on the wheel is

$$\tau = F_3QT - MgR$$

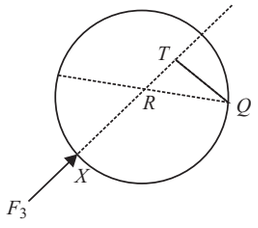


Fig. 3

If $F_3QT > MgR$, a net clockwise torque acts on the wheel, as a result the wheel begins to climb up the step. As it climbs QT increases. Hence τ does not remain constant. So option (c) is incorrect.

Option (d): If the force F_4 is applied tangentially at P, as shown in Fig. 4, the normal reaction at Q will be absent. Hence the frictional force is zero. As a result the wheel will start slipping as it rises up the step. It is given that there is no slipping, the wheel will rise up causing point Q to come down as shown in the figure.

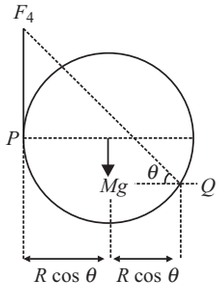


Fig. 4

Torque due to F_4 about $Q = F_4 \times 2\ R\ \cos\theta$ (clockwise)

Torque due to Mg about $Q = Mg \times R\ \cos\theta$ (anticlockwise)

Net clockwise torque

$= F_4 \times 2R\ \cos\theta - Mg \times R\ \cos\theta$ (clockwise)

$= (2\ F_4R - MgR)\ \cos\theta$.

Initially $\theta = 0$. As the wheel rises θ increase from zero to $\frac{\pi}{2}$. Hence $\cos\theta$ decreases from 1 to zero.

So option (d) is correct. Thus the correct options are (b) and (d).

15. At time $T > RC$, the capacitor is fully charged to the voltage of the battery (here RC is the time constant of the RC circuit).

When the voltage is $V_1 = \frac{V_0}{3}$, the charge supplied by the battery is $Q_1 = CV_1 = \frac{CV_0}{3}$

When the voltage is increased to $V_2 = \frac{2V_0}{3}$, the additional charge supplied by the battery is

$$Q_2 = C\left(\frac{2V_0}{3}\right) - C\frac{2V_0}{3} = \frac{CV_0}{3}$$

When the voltage is increased to $V_3 = V_0$, the additional charge supplied by the battery is $Q_3 = CV_0 - \frac{2CV_0}{3} = \frac{CV_0}{3}$

Total energy stored in the capacitor is

$$E = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2\ V_2 + \frac{1}{2}Q_3V_3$$

$$= \frac{1}{2}\left(\frac{CV_0}{3}\right) \times \frac{V_0}{3} + \frac{1}{2}\left(\frac{CV_0}{3}\right) \times \frac{2V_0}{3} + \frac{1}{2}\left(\frac{CV_0}{3}\right) \times V_0$$

$$= \frac{1}{18}CV_0^2 + \frac{1}{9}CV_0^2 + \frac{1}{6}CV_0^2$$

$$= \frac{1}{3}CV_0^2$$

Since the final voltage is V_0, final charge on the capacitor is $Q = CV_0$ and the final energy stored in the capacitor is

$$E' = \frac{1}{2}CV_0^2$$

$\therefore$ Energy dissipated through the resistor is

$$E_D = E' - E$$

$$= \frac{1}{2}CV_0^2 - \frac{1}{3}CV_0^2 = \frac{1}{6}CV_0^2$$

So the correct option is (d).

16. In this case, the charge supplied by the battery is $Q_0 = CV_0$

$\therefore$ Energy supplied by battery = $Q_0V_0 = CV_0^2$

Energy stored in the capacitor is

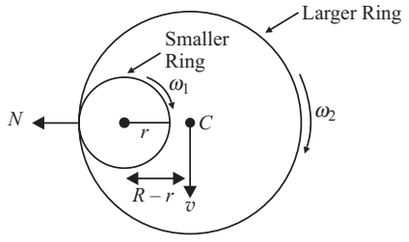
$$E_C = \frac{1}{2}Q_0V_0^2 = \frac{1}{2}CV_0^2$$

Energy dissipated through the resistor is

$$E_D = CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2$$

$\therefore$ $E_C = E_D$. So in this case the correct option is (c).

17. Refer to the following figure.



Let the angular speeds of the smaller and larger rings be ω_1 and ω_2 respectively. Let v be the linear speed of the centre C of the larger ring. The centre of the larger ring moves in a circle of radius $(R - r)$. So

$$v = (R - r)\omega_2 \qquad ...(1)$$

Since there is no slipping at the point of contact,

$$r\omega_1 = R\omega_2 - v \qquad ...(2)$$

Using (1) in (2), we have

$$r\omega_1 = R\omega_2 - (R - r)\ \omega_2 = r\omega_2$$

$$\Rightarrow \qquad \omega_1 = \omega_2 = \omega_0$$

Normal reaction at the point of contact is given by

$$N = Ma_c = M\omega_0^2\ (R - r)$$

For translational equilibrium in the vertical direction,

$$Mg = \mu N = \mu M\omega_0^2\ (R - r)$$

$\Rightarrow \omega_0 = \sqrt{\dfrac{g}{\mu(R-r)}}$, which is option (b).

18. The total kinetic energy = $\frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\,\omega_0^2$

$$= \frac{1}{2}M\omega_0^2(R - r)^2 + \frac{1}{2}(MR^2)\omega_0^2$$

$$= \frac{1}{2}M\omega_0^2\left[(R-r)^2 + R^2\right]$$

So no option is correct.

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